A Proof-Theoretic Decision Procedure for the Finitary Lambda-Calculus

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Abstract. We describe a new decision procedure for $\beta\eta$-equality in the λ-calculi with finite product (pair) and coproduct (disjoint sum) types. Existing decision procedures for this problem use several techniques, such as normalization-by-evaluation and term rewriting. In this paper, we describe an alternative decision procedure, based on using the proof-theoretic notions of polarity and focusing to identify unique canonical representatives of $\beta\eta$-equivalence classes. Our decision procedure has two stages: First, we use Andreoli's notion of focusing to cut down on the number of equivalent ways to write a λ-term. However, focused proofs do not dictate the order of case-analyzes, and thus do not equate all terms that are coproduct-equal. To obtain unique canonical forms, we employ a multi-focusing language which imposes the further restriction that variables may be used only immediately after they come into scope.

Key words: Focusing, Polarity, Finitary lambda calculus, Proof theory, Coproduct equality