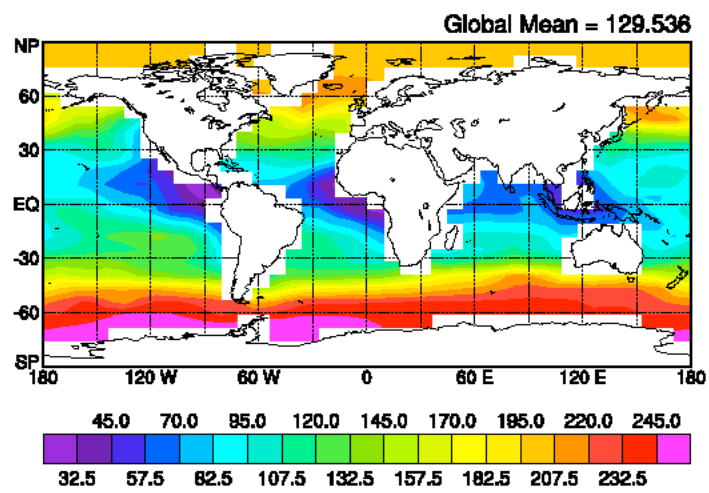
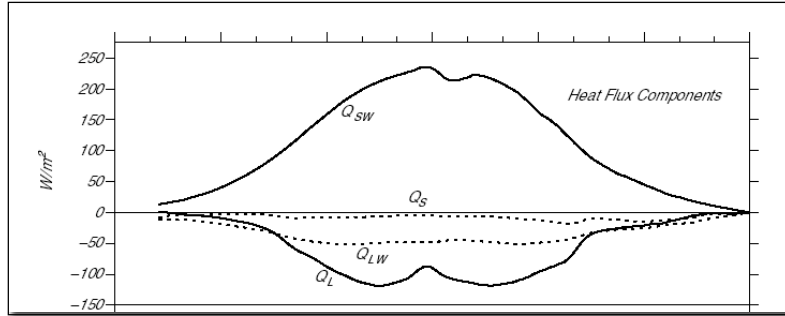


Ocean Mixed Layer

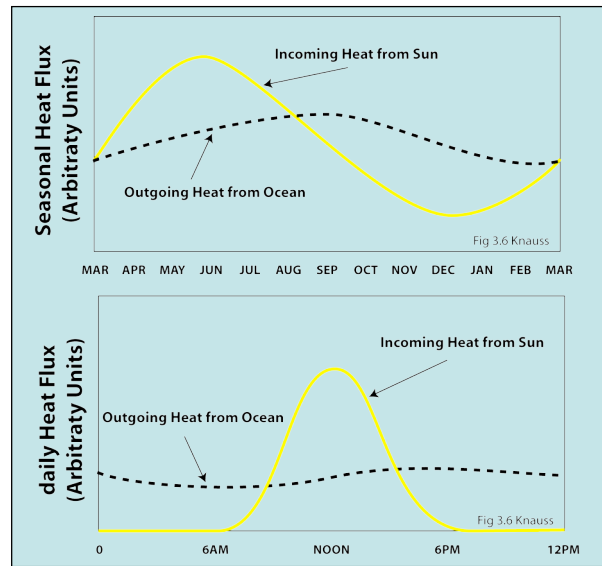
Maximum Mixed Layer Depth



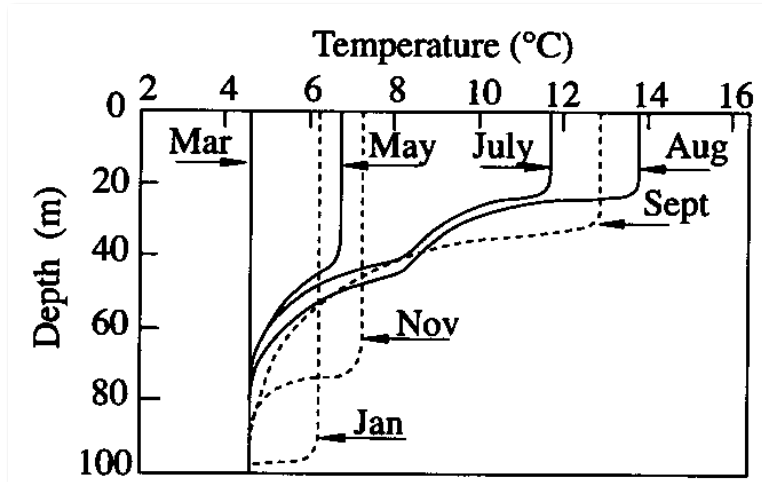
Heat Flux Components



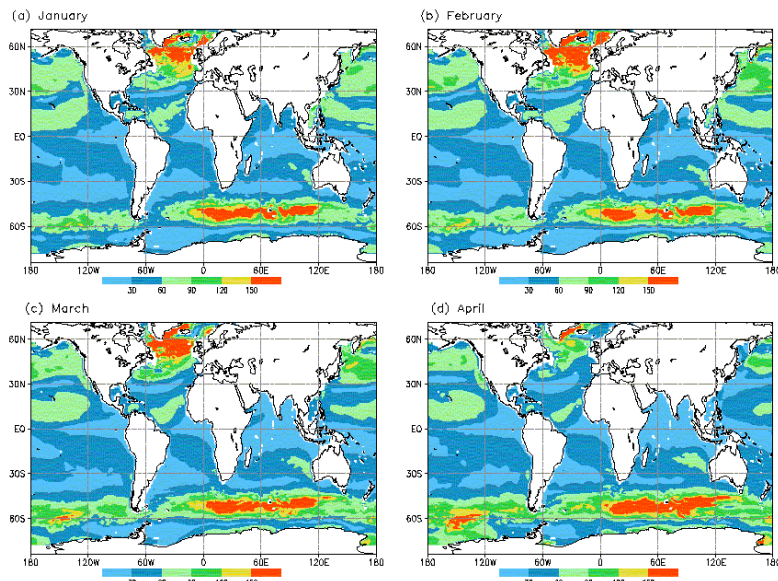
Temporal Dependence of Surface Heating



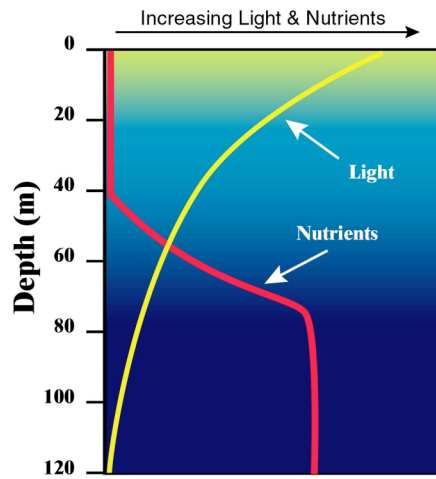
Variation in the Depth of the Seasonal Thermocline



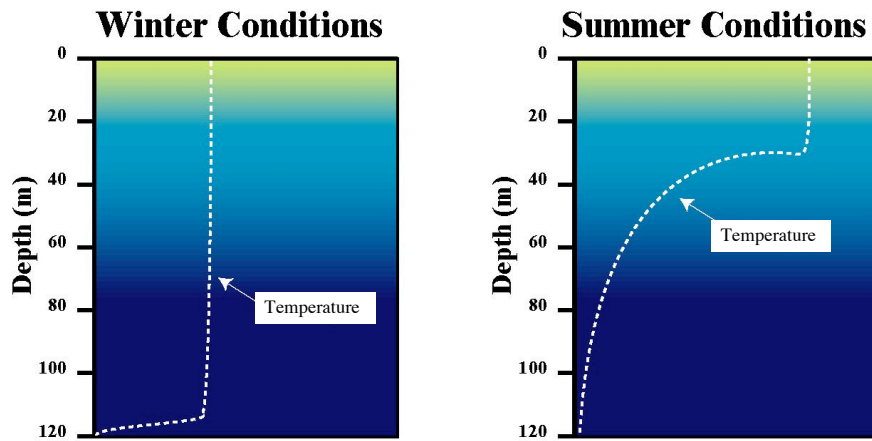
Ocean Mixed Layer Depth (m)



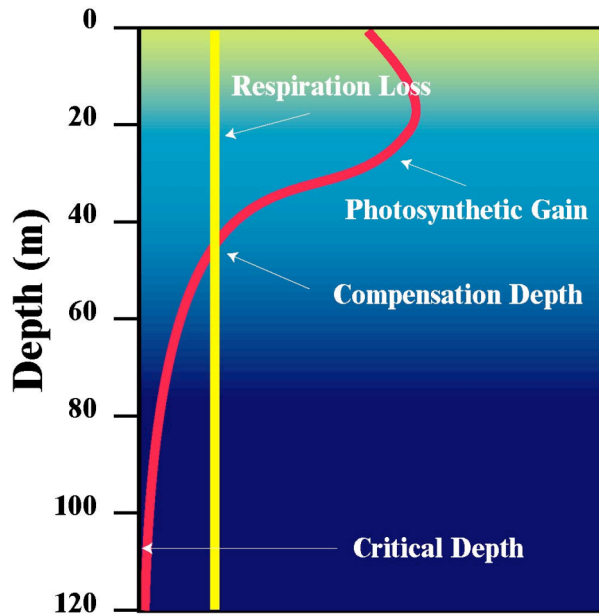
The Key Feature to Understanding Much About the Spatial and Temporal Distribution of Primary Production in the Ocean Can be Understood in Terms of Finding Places and Times When *Both* High Light and High Nutrients are Brought Together...



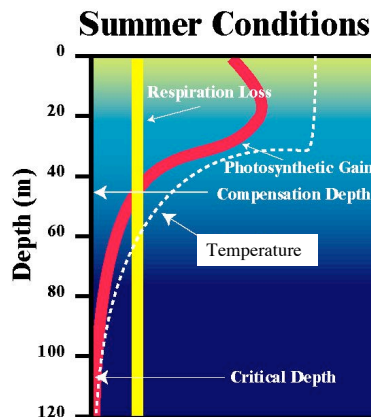
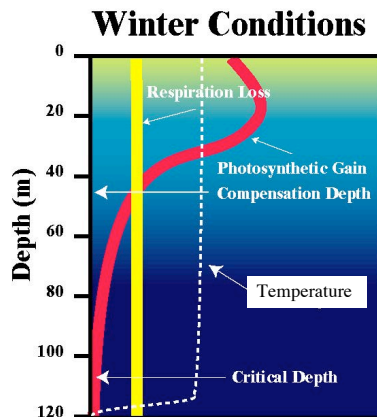
Changing Mixed-Layer Depth in Temperate Oceans



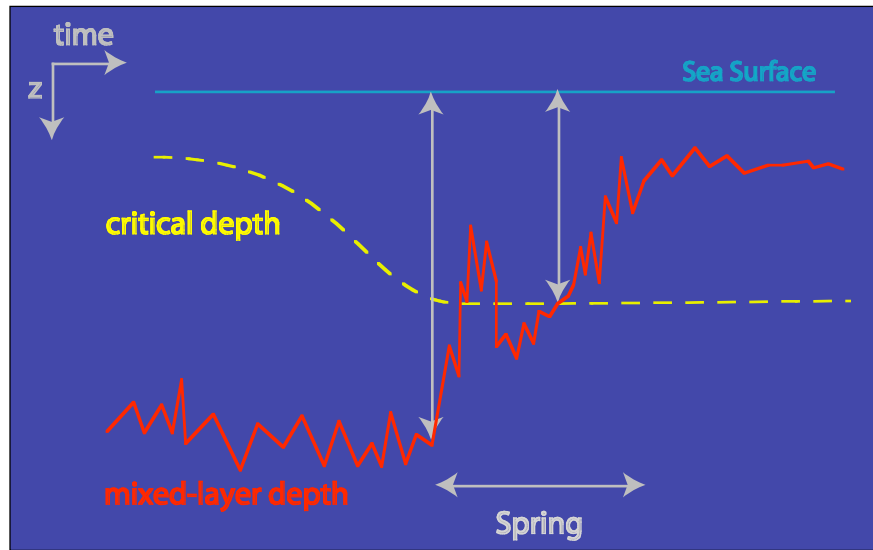
The *critical depth* concept forms the foundation of our understanding of seasonal phytoplankton dynamics in the ocean



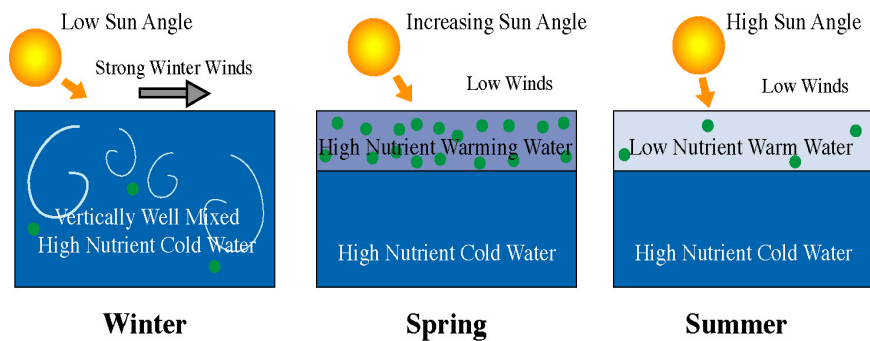
- During winter: the mixed layer is deeper than the critical depth and respiration exceeds photosynthesis
- During summer: the mixed layer is above the critical depth and photosynthesis exceeds respiration



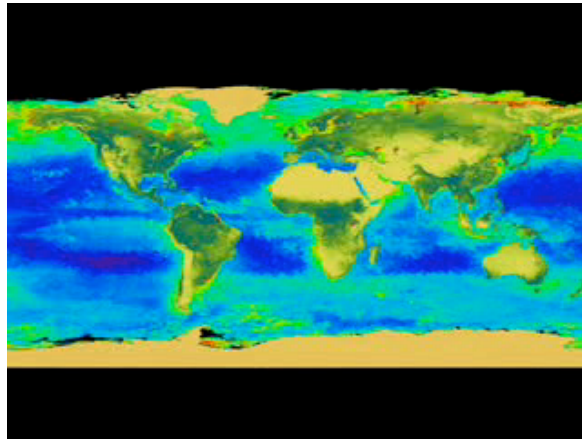
Spring Bloom Time Series



Seasonal Pattern of Water Column Stratification, Near-Surface Nutrient Replenishment and Spring Phytoplankton Blooms



Temporal Variability in Global Chlorophyll



There are two approaches to characterizing mixed layer dynamics:

1. Kraus-Turner-Niiler (Bulk Mixed Layer Model)
2. Mellor-Yamada- (Vertically Resolved Turbulent Transport Model)

Change in Mixed Layer Temperature Due to Surface Heat Flux
Without a Change in Depth

$$T(t + \Delta t) - T(t) = \frac{Q}{\rho h C_p} \Delta t$$

$$\frac{\partial T}{\partial t} = \frac{Q}{\rho h C_p}$$

T = Temperature

t = time

Q = surface heat flux

C_p = specific heat of water
4217 J kg⁻¹K⁻¹

ρ = density of seawater

h = mixed layer depth

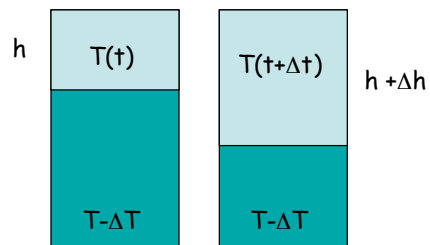
Change in Mixed Layer Temperature Due to Entrainment

New temperature after mixed layer deepening is just the weighted average of two masses of water...

$$T(t + \Delta t) = \frac{Th + (T - \Delta T)\Delta h}{h + \Delta h}$$

$$T(t + \Delta t) - T = \frac{-\Delta T \Delta h}{h + \Delta h}$$

$$\frac{\partial T}{\partial t} = -\Delta T \frac{1}{h} \frac{dh}{dt}$$



Change in Internal Energy (Heat Content) Due to Heat Flux from Top and Entrainment from Bottom

$$\frac{\partial T}{\partial t} = \frac{Q}{\rho h C_p}$$

Change in ML temperature (heat energy) due to surface heat flux

$$\frac{\partial T}{\partial t} = -\Delta T \frac{1}{h} \frac{\partial h}{\partial t}$$

Change in ML temperature (heat content) due to entrainment of cold deep water

$$\frac{dT}{dt} = \frac{Q}{\rho h C_p} - \frac{\Delta T}{h} \frac{dh}{dt}$$

Change in ML temperature (heat energy) due to surface heat flux and entrainment

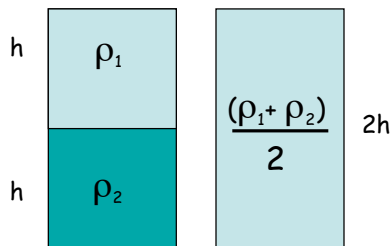
$$\frac{dT}{dt} = \frac{Q}{\rho h C_p} - \frac{\Delta T}{h} w_{entrain}$$

$$\frac{\partial h}{\partial t} = w_{entrain} = \text{Entrainment Velocity}$$

Change in Potential Energy of a Water Column Due to Entrainment

$$\frac{PE}{Area} = \int_0^h \rho g z dz$$

Simple Example...



$$PE = \rho_2 g \int_0^h z dz + \rho_1 g \int_h^{2h} z dz = \rho_2 g \frac{h^2}{2} + \rho_1 g \left(\frac{4h^2}{2} + \frac{h^2}{2} \right) = gh^2 \left(\frac{3\rho_1 + \rho_2}{2} \right) \quad \text{before}$$

$$PE = \frac{(\rho_1 + \rho_2)}{2} g \int_0^{2h} z dz = \frac{(\rho_1 + \rho_2)}{2} g \frac{(2h)^2}{2} = gh^2 (\rho_1 + \rho_2) \quad \text{after}$$

**Change in Potential Energy (Water Column Center of Mass)
By Entrainment**

Simple Example (continued)...

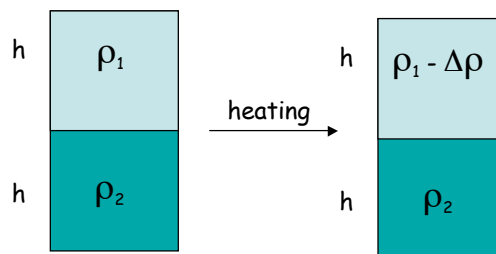
$$\Delta PE = gh^2(\rho_1 + \rho_2) - gh^2 \frac{(3\rho_1 + \rho_2)}{2}$$

or...

$$\Delta PE = gh^2 \left(\frac{\rho_2 - \rho_1}{2} \right) > 0$$

Entrainment produces a positive change in the potential energy corresponding to a rise in the center of mass of the water column

A similar argument can be made regarding the change in potential energy due to heat flux addition. Heating the Mixed Layer lowers the center of mass and reduced the Potential Energy



The Main Point: Mixing Increases the Potential Energy of the Water Column and Heating Lowers the Potential Energy of the Water Column

Mechanical Energy Input

Work = Force x distance so the Rate of Work = Force x (dx/dt) or...
Rate of Work = Force x U

$$\frac{dW}{dt} = \tau \cdot U_{ML} = \rho u_*^2 \cdot U_{ML} = m_o \rho u_*^3$$

This expression gives the rate of work done by the wind to drive mixing and thereby change the potential energy of the water column.

$$PE = \int_0^h \rho g z dz = \frac{1}{2} \rho g h^2$$

Potential Energy (PE)

$$\frac{dPE}{dt} = \frac{1}{2} g h^2 \frac{d\rho}{dt}$$

$$\frac{dPE}{dt} = \frac{1}{2} g h^2 \frac{d\rho}{dt} = m_o \rho u_*^3$$

Rate of change in potential energy is equal to rate of work supplied by wind mixing and heating by surface flux

$$\frac{dPE}{dt} = \frac{1}{2} g h^2 \left(\frac{d\rho}{dT} \right) \left(\frac{dT}{dt} \right) = m_o \rho u_*^3$$

$$\frac{1}{2} g h^2 (-\alpha \rho) \left(\frac{1}{h} \frac{Q}{\rho C_p} - \frac{\Delta T}{h} w_{entrain} \right) = m_o \rho u_*^3$$

$$\frac{dT}{dt} = \frac{1}{h} \frac{Q}{\rho C_p} - \frac{\Delta T}{h} w_{entrain}$$

$$\frac{1}{\rho} \frac{d\rho}{dT} = -\alpha$$

$$\frac{1}{2}gh^2(-\alpha\rho)\left(\frac{1}{h}\frac{Q}{\rho C_p} - \frac{\Delta T}{h}w_{entrain}\right) = m_o\rho u_*^3$$

$$-\frac{1}{2}\alpha gh\left(\frac{Q}{\rho C_p} - \Delta T w_{entrain}\right) = m_o u_*^3$$

$$\frac{1}{2}\alpha gh\Delta T w_{entrain} = m_o u_*^3 - \alpha gh\frac{Q}{2\rho C_p}$$

Note: this sign is correct, but I had to change it from positive to negative - somewhere above there is a sign error. I'll give extra credit (bragging rights) to anyone who finds the sign error.

The Main Point: Change in potential energy resulting from entrainment results from an increase caused by wind mixing and a decrease caused by heat flux

During **Mixed Layer Deepening** the entrainment velocity is given by...

$$w_{entrain} = \frac{2}{\alpha gh\Delta T}\left(m_o u_*^3 - \alpha gh\frac{Q}{2\rho C_p}\right)$$

During **Mixed layer Shoaling** it is assumed that heating and wind mixing are in balance so that...

$$m_o u_*^3 - \frac{\alpha gh}{2\rho C_p}Q = 0$$

$$h = \frac{2C_p\rho m_o u_*^3}{g\alpha Q}$$

h = **Monin-Obukhov** = mixed layer depth during the warming period given as a function of heat input and wind mixing