ANALYTIC AND NUMERICAL SOLUTIONS FOR FORCED CONVECTION HEAT TRANSFER FROM A SHEAROW PAST A ROTATING CYLINDER

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Analytic and numerical approximations to the thermaluid problem involving forced and convective convective heat transfer from a rotating isothermal cylinder placed in a non-uniform stream shear ow will be presented. The analytic approximations are obtained via series expansion of the scaled boundary layer equations in terms of appropriate boundary layer variable. The resulting approximations are valid not only for small time but also for moderate and large times provided that the Reynolds number of the ow is sufficiently large. The numerical method employed is based on a spectral-finite difference scheme in which the ow variables are approximated in terms of truncated Fourier series and integrated using the finite difference procedure. The behaviour of the ow and the heat transfer processes are investigated over a wide range of the ow parametric values including. The validity of the numerical scheme is tested via comparison with the analytic approximations as well as with existing empirical correlations that are based on other numerical and experimental observations.

It is shown that the numerical scheme yields results that are quantitatively as well as qualitatively consistent with both the analytical and experimental results.

A RESOLUTION OF 2-VARIATE GIBBS PHENOMENON

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Key Words and Phrases: Gibbs phenomenon, Fourier, Chebyshev, Gegenbauer
AMS Subject Classification: 42B05, 65T99, 42C05, 42C10

In this paper we introduce a method for the resolution of the Gibbs phenomenon for $f(x, y)$ based on Gegenbauer polynomials which are orthogonal with respect to weight function $(1 - x^2)^{\lambda - \frac{1}{2}}$, as an extension of the B.D. Shizgal and J.H. Jung (2003). It is shown that this method gives exact results for bivariate polynomial functions, and the result is independent of λ. Several numerical examples are considered and the results of applying this method to these examples illustrate the effectiveness of the method. The local property of the method is also discussed along with some illustrative examples.

LEAST ACTION PRINCIPLE

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Key Words and Phrases: Least Action Principle, Hamilton Principle, Minimum Time Principle
AMS Subject Classification: 97D40, 97D50

The application of basic differential calculus to the solution of a minimum time problem is presented. Under the experimental conditions here discussed it is shown that human beings choose movement trajectories that agree with the ones calculated using the minimum time principle. These conclusions are obtained from the analysis of a group of students who must run along a swimming pool and then jump and swim through the pool to reach an object placed on the opposite side. The mathematical analysis of the problem is presented, its solution and comparison with experimental results.

MULTICRITERIA OPTIMIZATION UNDER UNCERTAINTY

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Key Words and Phrases: multi-criteria, optimization, uncertainty, direct methanol fuel cell
AMS Subject Classification: 65K05

Design with partially unknown information remains an important problem in process systems engineering. While extensive surveys of the problem formulation and solution strategies are available, there is less discussion of different sources of unknown information and their impact on the design problem. In this presentation I will discuss the issue of parameteric uncertainty and two types of unknown information encountered for the design problem. We consider unknown input parameters in some domain for the process and we distinguish the following types of these parameters in the problem formulation:

- Uncertain parameters are never known exactly. Although expected values and confidence regions may be known for these parameters, the value of these parameters is not well known for the design problem. Examples of these include model parameters determined from experimental studies, as well as unmeasured and unobservable disturbances in the process.
- Variable parameters are not known at the design stage but can be determined accurately during the operation of the process. Examples of these include feed flow rates, process conditions and product demands. For these we can assume that control variables in the process can be adjusted to compensate for process variability.

Next we will consider the impact of uncertainty in multicriteria optimization (MCO). MCO is an important problem in many engineering disciplines. In process engineering, very often, process performance cannot be adequately captured using a single criterion. Therefore it has been necessary to consider several possibly conflicting performance criteria. For example optimizing process economics while minimizing environmental impact is a very desirable goal, but can only be partially achieved by trade-off. MCO determines the optimal trade-off in this case. Unfortunately MCO has largely been considered while ignoring uncertainty. This has motivated our development of methodologies to address MCO under uncertainty. In the presentation, case studies will include applications to the operation of a Direct Methanol Fuel Cell.

APPROXIMATION THEORY IN n-BANACH SPACES

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Key Words and Phrases: n-Banach Spaces, Approximation Theory , T-Convergence, Sequence of Linear Operator, Stability and Approach Conditions
AMS Subject Classification: 40A05, 46A70

In order to study the approximation theory in n-Banach Spaces, we define the concept of T-convergence by means of a sequence of linear operator in n-Banach Spaces, and we get some results by implosing the stability and approach conditions.

A DIFFERENTIAL CALCULUS FOR LINEAR OPERATORS WITH A VIEW TOWARDS APPLICATIONS

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Key Words and Phrases: Linear operator; signed kernel; derived process; Poisson process; Ramanujan's equation; median
AMS Subject Classification: 60G99, 41A35

We propose a differential calculus for linear operators represented by a family of finite signed measures as a unified methodology to obtain sharp estimates in classical limit theorems of probability theory, among other applications. Such a calculus is based on the notions of g-derived operators and processes and g-integrating measures, g being a right-continuous nondecreasing function. A striking feature is that, depending on the choice of g, this differential calculus also works for non-smooth functions and under weak integrability conditions. For linear operators represented by stochastic processes, we provide a characterization criterion of g-differentiability in terms of the characteristic functions of the random variables involved. We obtain explicit Taylor formulae in a variety of specific examples. As an illustration, we give sharp upper and lower bounds for a Ramanujan sequence connected with the median of the gamma distribution and solve two conjectures of Chen and Rubin referring to the median of the Poisson distribution.

A SELF ADJOINT EXPANSION OF A SYMMETRIC OPERATOR

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Key Words and Phrases: Hilbert space, self-adjoint operator, symmetric operator, closure
AMS Subject Classification: 47A70, 47A55

Let H be a separable Hilbert space with finite dimension. We will denote the inner product with $(.,.)$ and $(.,.)_{(0,\infty)}$ in the space H and in the space $L_2(0,\infty;H)$ respectively. Let $Q(x)$ be self adjoint operator from H to H for all x in $[0,\infty)$. Moreover, we consider $Q(x)$ as a continuous operator function in the interval $[0,\infty)$ with respect to the norm on the space $B(H)$. We suppose that there are positive constants c_1 , c_2 such that

$$c_1 \leq p(x) \leq c_2.$$

In this work, we prove that the closure of a symmetric operator L_0 which is formed by differential expression

$$(L_0 y)(x) = -(p(x)y'(x))' - Q(x)y(x)$$

and with the boundary condition

$$\cos\alpha.y(0) + \sin\alpha.y'(0) = 0$$

is self adjoint where $\alpha \in (-\infty,\infty)$ in the space $L_2(0,\infty;H)$. Moreover, we investigate some properties of this operator.

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NUMERICAL SOLUTION OF WANG-CHANG UHLENBECK EQUATION FOR FLOWS IN ROTATIONAL AND VIBRATION NONEQUILIBRIUM

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Key Words and Phrases: non-equilibrium hypersonic flows, generalized Boltzmann equation

The theoretical and numerical framework for computing flows of diatomic gases in rotational and vibrational nonequilibrium is presented. The algorithm is tested by computing the 1-D shock structure in Nitrogen at very high Mach numbers as well as the hypersonic flow past 2-D blunt bodies bathed in Nitrogen. The solutions are computed at Knudsen numbers covering all the three regimes: continuum, continuum-transition, and rarefied. In a previous paper AIAA 2007-0205, flow field simulations in a monoatomic gas were reported by employing several computational models namely the Navier-Stokes equations, Burnett equations, Direct Simulation Monte Carlo (DSMC), and the classical Boltzmann equation. The effect of Knudsen number Kn varying from 0.01 to 10 was investigated for Mach 3 flow past a blunt body. In this paper, the hypersonic flow field past a blunt body in a diatomic gas is computed using the Generalized Boltzmann (or the Wang-Chang Uhlenbeck) Equation (GBE) for Kn varying from 0.1 to 10. In the GBE, the internal and translational degrees of freedom are considered in the framework of quantum and classical mechanics respectively. The computational framework available for the standard Boltzmann equation (for a monoatomic gas with translational degrees of freedom) is extended by including the rotational and vibrational degrees of freedom in the GBE. The general computational methodology for the solution of the GBE for a diatomic gas is similar to that for the standard BE except that the evaluation of the collision integral becomes significantly more complex due to the quantization of rotational and vibrational energy levels. The solution of GBE requires modeling of transition probabilities, elastic and inelastic cross-sections etc. of a diatomic gas molecule, needed for the solution of the collision integral. An efficient computational methodology has been developed for the solution of GBE for computing the flow field in diatomic gases at high Mach numbers. There are two main difficulties encountered in computation of high Mach number flows of diatomic gases with rotational and vibrational degrees of freedom using the GBE: (1) a large velocity domain is needed for accurate numerical description of molecular velocity distribution function resulting in enormous computational effort in calculation of the collision integral, and (2) about 50 to 70 energy levels are needed for accurate representation of the rotational

spectrum of the gas. These two problems result in very large CPU and memory requirements for shock wave computations at high Mach numbers (¿ 6). Our computational methodology has addressed these problems, and as a result efficiency of calculations has increased by several orders of magnitude. These computations are performed on a 16 node SGI Origin 2000 cluster.

ON THE STABILITY OF THE FLOW BETWEEN CONCENTRIC CYLINDERS

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Key Words and Phrases: stability, flow, transition, Taylor-Couette-Dean

The study of the secondary motions induced by centrifugal forces in curved channel flows is an important area of theoretical, numerical or experimental investigations. The work we have to present is concerned by the dynamics of a viscous fluid between concentric cylinders when the gap between the cylinders is azimuthally opened, only partially filled or fully filled.

This last case constitutes the classical Taylor-Couette flow. The main sequence of its transition towards turbulence is now well known: Couette flow - cellular flow - wavy flow - modulated wavy flow - chaos. In contrast, the flows obtained in the azimuthally opened or partially filled annulus need much more investigations. Because they do not have rotational symmetry around the cylinder axis, the flows produced in the open system or in partially filled annulus undergo noticeable changes with spatial location along the stream. Two kinds of flows will be presented: the Dean flow and the Taylor-Dean flow.

* The Dean flow can be obtained by pumping a fluid around the annulus while the cylinders are at rest. Its study is compared to that of the Taylor-Couette flow. The main sequence of its transition towards turbulence is nearly the same as that of Taylor-Couette even thought the flow regimes are not so well defined.

*The Taylor-Dean flow is produced:

1) In a system of two coaxial circular cylinders azimuthally opened at θ=0 and $(2\pi - \theta_1)$ where θ_1 represents the region cut-off by a diaphragm. The basic flow is a combination of the inner cylinder rotation and a flow provided in the gap by external pumping. With increasing rotational velocity of the inner cylinder, the flow undergoes transitions from a laminar base flow to a chaotic state. This laminar-turbulent transition is studied for a wide range of τ, the ratio of pumping and rotation flow rates.

2) In a cylindrical cavity azimuthally bounded at $\theta_1 = \pi$. Both cylinders can rotate independently each of other.

A numerical method based on the finite volume method, the "Fluent" code is used to simulate the flow induced by rotation alone. The aim of this numerical simulation is to reproduce the instabilities obtained experimentally, at least those observed at the exit end.

RICCI CURVATURE AND TENSOR FIELDS ON THE QUOTIENT SPACES OF $Diff(S^1)$

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Key Words and Phrases: Ricci curvature, connection, infinite dimensional groups
AMS Subject Classification: 17B66, 30B50

As in [1], [2], the bracket on the Lie algebra $diff(S^1)$ is defined by $[u, v] = uv' - u'v$. Let V be the subspace of $diff(S^1)$ generated by $\{\cos(k\theta), \sin(k\theta)\}$ with $k \geq 2$. On V, we take the trigonometrical basis $(e_p)_{p>1}$ with $e_{2p-1} = \frac{1}{\sqrt{p^3-p}} \cos p\theta$, $e_{2p-2} = \frac{1}{\sqrt{p^3-p}} \sin p\theta$ and the scalar product which makes the basis $(e_p)_{p>1}$ an orthonormal basis of V. Let π be the projection operator from $diff(S^1)$ to V which consists in dropping the constant term and the terms in $\sin\theta$, $\cos\theta$ in the Fourier series of an element of $diff(S^1)$. For $v \in diff(S^1)$, the operator $\Gamma(v) : V \to V$ is defined by $2(\Gamma(v)u|w) = (\pi[w, v]|u) + (\pi[w, u]|\pi v)$, the operator $B(v) : V \to V$ is defined by $2\,B(v)u = \pi[v, u]$. We study the operators $\Gamma(v) + B(v)$, $\Gamma(v) - B(v)$ acting on V and composition of these operators.

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EVALUATION OF EFFICIENCY WITH AHP METHOD OF GSM OPERATORS IN TURKIYE:A CASE STUDY

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Key Words and Phrases: AHP, GSM System, Decision Making, Survey.

AHP is a proven decision support tool that allows the user to design a hierarchical structure for decision-making and weighs the trade-offs between decision criteria and alternatives. In this study, We will evaluate three GSM operators working in TURKEY market with this method.

AHP models are valued because they improve the quality of decision-making.

TURKIYE has tree GSM operators, TurkCell, Telsim (now VodaPhone), and AVEA.

Our research aim is to make future perspective by way of satisfying performance to customers

In our study , Matrix data obtained by a survey study. Next It was formed a hierarchy and operators ranks calculated with AHP priority vector.

GEOMETRIC INVESTIGATION FOR PHASE PORITRAITS OF PLABNAR SYSTEMS

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Key Words and Phrases: period function, isochonous center
AMS Subject Classification: 34C25, 34C15, 34C05

In this paper we study phase portrait of dynamical systems in plane and their corresponding vector fields from two point of views, the local and the global studies. Then we discuss the isochronicity of centers presenting some important techniques to investigate the monotonicity of period function then we specify conditions for isochronicity and monotonicity of the period function of some important differential systems like Lienard equation.

ON NUMERICAL DIFFICULTIES IN RIGID PLASTICITY

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Rigid plastic models are usually adopted for analysis and design of cold and hot metal forming processes as well as structures. In most of metal forming processes, there are frictional surfaces between deforming material and tool. An important friction law is the maximum friction law. Its remarkable property is that all possible frictional laws should lead to the friction stress which is less or equal to the friction stress following from the maximum friction law. For this reason, with the use of the maximum friction law it is possible to evaluate the load required to deform the material without knowing the actual friction stress, which is usually unknown. However, the velocity field may be singular in the vicinity of maximum friction surfaces (surfaces where the maximum friction law acts), which leads to certain difficulties with solving the corresponding boundary value problems by numerical methods. In the case of both metal forming processes and structures some models of rigid plasticity permit discontinuous velocity fields. The velocity discontinuity surfaces have the same properties as the maximum friction surfaces. In particular, the velocity field can be singular in the vicinity of velocity discontinuity surfaces. The present papers reviews the asymptotic behaviour of velocity fields in the vicinity of maximum friction surfaces and velocity discontinuity surfaces for several rigid plastic models such as classical rigid metal plasticity, several models of pressure-dependent plasticity and viscoplasticity. Several simple examples are given to illustrate the qualitative behaviour of solutions and corresponding numerical difficulties.

THE DATA STRUCTURES AND THE PARALLEL ALGORITHMS IN THE NUMERICAL SIMULATION OF THE TWO-DIMENSIONAL PROBLEMS OF GASDYNAMICS AND THERMAL CONDUCTION ON DUAL-CORE ARCHITECTURE

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Key Words and Phrases: concurrent and parallel computations, parallel overhead, shared memory systems, the compiler optimization for the latest versions of Intel C++, two-dimensional problems of mechanics, OpenMP Intel C++

Between a number of factors should be taken into consideration in the process of designing of multithreaded program for the shared memory systems we shall consider the data structures and the minimal data volume which is in the thread task in the parallel region. An application is characterized with high performance when these factors are taken into consideration simultaneously. Such a way the addition of the parallel and sequential overhead to the elapsed time is decreased. The factors influence on the speed up of the multithreaded applications for the two-dimensional problems of gas dynamics (Euler equations) or thermal conduction was considered. Study conserns the Intel C++ compilers and OpenMP technology on the Intel processors (Pentium(R) D, version 9.1 for the Dirichlet problem (the Gauss-Zeydel numerical algorithm); Duo T2400, version 9.0.019 for the gas dynamics problem (TVD-reconstruction); the operation system Windows XP). The elapsed time of the applications decreases significantly by means of the new features of the latest versions of the compiler and the processors. Therefore the ratio of the parallel overhead time to the time consuming for the thread task in the parallel region increases. We are thus led to the conclusion that the wave-schemes does not appropriate for the solving of the simple Dirichlet problem (without additional complicated calculation in each iteration of the numerical scheme; the array element amount is about N = 10000÷ 100000). Using of the compiler optimization such as /O2, /Qaxp is more effective than the wave parallel schemes. Multithreaded application is characterized with high performance if the thread task in the parallel region includes computation on the part of the array, which has the element amount is about a half of the array. Moreover, the large – grain parallelism model is more effective, because the compilers optimization and the parallel algorithm work together.

The supposed examples may be used in the courses of multithreaded programming for the shared memory systems. These examples may be used for demonstration of necessary of the parallel overhead estimation for the parallel OpenMP scheme choice.

ON GLOBAL PERIODICITY AND GLOBALLY ATTRACTING TWO CYCLES OF SECOND ORDER DIFFERENCE EQUATIONS

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This work consists of two main parts. In both of them we consider second order autonomous difference equations of the form:

$$y_{n+1} = f(y_n, y_{n-1}), \qquad n = 0, 1, 2, \ldots .$$

In the first part, we develop necessary and/or sufficient conditions on f so that every solution is periodic of the same period p. In the second part, we establish necessary and sufficient conditions so that all solutions are attracted to period-2 solutions.

A GENERALIZED SUMUDU TRANSFORM

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An integral transform, called the Sumudu transform was defined and studied by G. K. Watugala [Sumudu transform-a new integral transform to solve differential equations and control engineering problems. *Math. Engrg. Indust. 6 (iv) (1998),319-329]*. Units and scale preserving properties, makes the Sumudu transform an ideal tool for solving many engineering problems without restoring to a new frequency domain. In this paper, first we mention some properties of the Sumudu transform and then introduce a new integral transform involving confluent hypergeometric function as kernel. Some properties of this generalized transform are established and images of certain special functions, including Mittag-Leffler and H-function, under this transform are given. An inversion formula for this transform is obtained by using Laplace and its inverse operators.

THE SIMPLE ψ-θ MAP FOR DIVERTOR TOKAMAKS

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Key Words and Phrases: symplecticity, Hamiltonian system, tokamak
AMS Subject Classification: 37, 70

Trajectories of magnetic field lines in divertor tokamaks are a 1 1/2 degree of freedom Hamiltonian system. Magnetic field is divergence-free, so the motion of field lines in the phase space is symplectic. The simplest known Hamiltonian for the field lines is constructed in canonical coordinates (ψ,θ) that correctly describes the magnetic topology of divertor tokamaks [1]. A canonical transformation is used to construct a discrete map that is symplectic, preserves the universal rescaling renormalization invariance [2], and has the correct asymptotic form for reciprocal of rotational transform. This map is called the ψ-θ map. Method of maps [3, 4] is used to calculate stochastic broadening due to statistical topological noise, internal perturbations (called Edge Localized Modes), and external perturbations designed to induce chaos (from coils carrying currents, the I-coils). Topologically distinct subsets of magnetic footprint are calculated. A canonical transformation is applied to express results in the physical space (x,y) from the (ψ,θ) space. The practical implications of results for divertor tokamak magnetic confinement schemes such as ASDEX [5, 6] and the DIII-D [7] are discussed.

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EXACT SOLUTIONS OF THE STEADY STATE BOUNDARY LAYER EQUATIONS FOR FLUID FLOW OVER MOVING SURFACES

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Key Words and Phrases: nonclassical similarity transformation, moving fat plate, similarity solution
AMS Subject Classification: 76D05, 34B15, 34B40

Generally speaking, the process of obtaining analytical solutions of the boundary layer equations encounters considerable mathematical difficulties. The differential equations are nonlinear and solutions are obtained either by series expansion or by numerical techniques.

In this paper, we present the idea of non-classical similarity transformation to remodel the fluid flow over moving surfaces. Mainly, the fluid flow past a moving flat plate (Blasuis equation), the fluid flow past a moving wedge (Falkner-Skan equation) and the fluid flow past a rotating cylinder.

The non-classical similarity transformation employed takes into account the effect of the parameter λ, which is the ratio of the velocity of the moving surface to the fluid velocity,on the boundary layer thickness.

The transformation is applied to the above mentioned cases and a modified set of nonlinear equations will be derived.

In some special cases and exact analytical solutions are obtained. Numerical techniques are used to solve the other cases. Comparison of the new results with the existing results will be shown.

ON THE L^p BOUNDEDNESS OF CERTAIN ROUGH SINGULAR INTEGRAL OPERATORS

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Key Words and Phrases: singular integrals, maximal operators, L^p boundedness, rough kernel
AMS Subject Classification: 42B20, 42B15, 42B25

We establish the L^p boundedness for a class of singular integral operators $T_{\Omega,h}$ and a class of related maximal operatos$\mathcal{M}_{\Omega,h}$ when their singular kernels are given by functions Ω in $L(\log L)^{\alpha}(S^{n-1})$ and h satisfies a certain integrability condition.

Our results shows that the class of operators $T_{\Omega,h}$ behaves completely differtent from the classical class of Calderón-Zygmund operators T_{Ω}. Moreover, our results represent an improvement and extension over previoiusly known results.

ON THE OPERATOR TRANSFORM $\tilde{T} = |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$ AND ITS APPLICATIONS

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Key Words and Phrases: Aluthge Transform, p-hyponormal operators, w-hyponormal operators
AMS Subject Classification: 47B49, 47A63, 47B20, 47BA10

I will discuss the original idea behind the introduction of the operator transform $\tilde{T} = |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$. I will also discuss many properties of $\tilde{T}$ including properties involving the spectrum, numerical range, and the polar decomposition. Applications of this operator transform to several classes of operators including the classes of p-hyponormal and w-hyponormal operators will also be discussed.

REMARKABLE PROPERTIES OF DIFFERENTIAL SEQUENCES

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Key Words and Phrases: differential sequences, Lie symmetries, singularity analysis, hierarchies
AMS Subject Classification: 34M55, 47E05

We introduce the notion of Differential Sequences. In ordinary differential equations these are the analogues of the hierarchies found for partial differential equations, such as the KdV hierarchy. We initiate our study with the Riccati Sequence, of which the first two members are the Riccati equation and the Painlevé-Ince equation, in terms of symmetry analysis, singularity analysis and identification of the complete symmetry group for each member of the Sequence. The rich properties found in the Riccati Sequence stimulated the search and exploration of other Sequences. We mention the properties of some Sequences – Emden-Fowler, Kummer-Schwarz, Ermakov-Pinney to name a few – which we have recently investigated. The results are always intriguing. An explanation of the reason for the remarkable properties of Differential Sequences would be a most welcome development.

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PARABOLIC EQUATIONS WITH VARIABLE AND ANISOTROPIC NONLINEARITY

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Key Words and Phrases: nonlinear parabolic equation, nonstandard growth conditions
AMS Subject Classification: 35K65, 35K57

We present a review of the recent results in the theory of nonlinear parabolic equations of the type

$$u_t - \sum_{i=1}^{n} D_i \left(a_i |D_i u|^{p(x,t)-2} D_i u\right) + F(x,t,u,\nabla u) = 0,$$
$$u_t - \sum_{i=1}^{n} D_i \left(a_i |u|^{\alpha_i(x,t)} D_i u\right) + F(x,t,u,\nabla u) = 0,$$

where $a_i \equiv a_i(x,t,u)$ and $p_i(x,t)$ are given functions of their arguments. The following questions are discussed:

- existence of weak solutions in Orlicz–Sobolev spaces,
- L^{∞}–estimates and uniqueness of weak solutions,
- vanishing properties of solutions such as finite time extinction, formation of dead cores, directional localization caused by to the anisotropy of the diffusion operator,
- behavior of solutions in the limit cases when the equation eventually converts into a linear one; it happens that under certain conditions on the nonlinearity exponents the solutions may display the properties intrinsic to solutions of nonlinear equations.

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INTERFACES GENERATED BY DISCHARGE OF A HOT GAS IN A COLD ATMOSPHERE

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We consider a boundary layer approximation in the problem of the discharge of a laminar hot gas in a stagnant colder atmosphere of the same gas [2],[3]. This approximation leads to the nonlinear system of partial differential equations

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial r}(\rho v) = 0, \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = \frac{\partial}{\partial r}\left(\mu \frac{\partial u}{\partial r}\right) + G\left(1 - \frac{\varepsilon}{T}\right), \tag{2}$$

$$\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial r} = \frac{1}{Pr}\frac{\partial}{\partial r}\left(\mu \frac{\partial T}{\partial r}\right), \tag{3}$$

where Pr is the Prandtl number , G is the Froude number(are given positive numbers). The system is completed with the constitutive conditions $\rho = 1/T,\ \mu = T^{\sigma}$(for some $0 < \sigma < \infty$). Here the unknowns are the velocity, vector (v, u), and the temperature T. System (1)-(3) is considered in the domain $\Omega = \{(x, r) \in \mathbf{R}^2 : 0 < x < \infty, 0 < r < l \leq \infty\}$ and, so, the solutions must satisfy some auxiliary conditions which in our case are given by the boundary conditions

$$\frac{\partial u}{\partial r} = v = \frac{\partial T}{\partial r} = 0, \ \text{ for } r = 0, \ \text{ and for } x > 0, \tag{4}$$

$$u = \delta,\ T = \varepsilon,\ \text{ for } r = l, \ \text{ and for } x > 0, \tag{5}$$

$$u(0, r) = u_0(r) \geq \delta,\ T(0, r) = T_0(r) \geq \varepsilon \text{ for } x = 0 \text{ and for } r \in [0, l]. \tag{6}$$

Notice that, although arising in stationary regime, the system is of parabolic type and that condition (6) looks as an initial condition if we understood variable x as the "fictitious" time.

We prove existence and uniqueness of solutions of the nondegenerate problem (corresponding to the assumption $\delta > 0$ and $\varepsilon > 0$). We also study the limit case ($\delta = 0$ and $\varepsilon = 0$) leading to the degeneracy of the system for which we prove the generation of some interfaces given as the boundaries of the support of (u, T). Using the method of local energy estimate [1], we show that the solution possesses the localization properties such that- finite speed of propagation, waiting time property, extinction with respect to x. We discuss also some results of numerical calculations for this problem. To prove the existence and uniqueness of solutions we use the, so called, von Mises variables (x, ψ), (where ψ is the associated "stream function") which transform (1)-(3) into the purely diffusive system (eliminating the unknown v)

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial \psi}\left(T^{\sigma-1} u \frac{\partial u}{\partial \psi}\right) + \frac{TG}{u}\left(1 - \frac{\varepsilon}{T}\right), \frac{\partial T}{\partial x} = \frac{1}{Pr}\frac{\partial}{\partial \psi}\left(T^{\sigma-1} u \frac{\partial T}{\partial \psi}\right).$$

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WEYL SPECTRA AND GENERALIZATIONS

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For a bounded linear operator T on a Hilbert (separable) space, the Weyl Spectrum $w(T)$ is defined as

$$w(T) = \bigcap_{K \in \mathcal{K}} \sigma(T + K)\,,$$

the intersection being carried over the ideal $\mathcal{K}$ of compact operators.

In this lecture, we first discuss those classes of operators which satisfy

$$Re(w(T)) = w(Re(T)).$$

The study is carried over to introduce and discuss $w_\alpha(T)$, the Weyl spectrum of weight α over non-separable Hilbert spaces. The notion of index of Fredholm operators on separable Hilbert spaces is carried over to α-Fredholm operators, over non-separable Hilbert spaces and weighted weyl spectrum is discussed accordingly.

LATTICES WITH A MISSING BOND

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The calculation of the resistance of an infinite network of identical resistors is a classic well- studied problem in the electric circuit theory. The methods used in studying this problem vary from superposition of current distribution, random walk theory and Lattice Green's Function (LGF) method[1].

The analysis of the capacitance of an infinite network of identical capacitors has been arising recently. Tzeng and Wu[2]. introduced a formulation to determine the impedance between any two sites in an impedance network, where some numerical examples were given. Asad et. al[3] used the LGF method to calculate the capacitance between arbitrary lattice sites in a perfect infinite square lattice consisting of identical capacitors.

In this work, we shall use the Lattice Green's Function (LGF) approach to determine the capacitance for the so-called perturbed lattice obtained by removing one bond (capacitor) from the perfect infinite lattice. The content of this manuscript is helpful for electric circuit design and the method is instructive.

In a previous work the equivalent capacitance between the sites$i$and$j$in an infinite network consisting of identical capacitors each of capacitance C is given as:

$$C_o(i,j) = \frac{C}{2[G_o(i,i) - G_o(i,j)]}. \tag{1}$$

When removing one bond (i.e. capacitor) from the infinite network can be written as:

$$\frac{C_{o1}(i,j)}{C} = \frac{1}{\frac{1}{C_o(i,j)} + \frac{[\frac{1}{C_o(i,j_o)} + \frac{1}{C_o(j,i_o)} - \frac{1}{C_o(i,i_o)} - \frac{1}{C_o(j,j_o)}]^2}{4[1 - \frac{1}{C_o(i_o,j_o)}]}}. \tag{2}$$

Where the bond $(i_o j_o)$is removed.

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GENERALIZED KHINTCHINE INEQUALITY IN REARRANGEMENT INVARIANT SPACES

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Key Words and Phrases: independent random variable, Rademacher functions, Khintchine inequality, rearrangement invariant space, Boyd indices

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Let $r_1, r_2, \ldots$ be a sequence of independent random variables with symmetric two-point distribution $\mathbf{P}(r_i = 1) = \mathbf{P}(r_i = -1) = \frac{1}{2}$ (for example, r_i could be the classical Rademacher functions defined on $[0,1]$). By the famous Khintchine inequality, for every $p > 0$ there exists a constant $C_p > 0$ such that for arbitrary real a_k we have

$$\Big\|\sum_{k=1}^{\infty} a_k r_k\Big\|_{L_p[0,1]} \le C_p\Big(\sum_{k=1}^{\infty} a_k^2\Big)^{1/2}.$$

Necessary and sufficient conditions are found under which the following generalized Khintchine inequality

$$\Big\|\sum_{k=1}^{\infty} f_k\Big\|_X \le C\Big\|\Big(\sum_{k=1}^{\infty} f_k^2\Big)^{1/2}\Big\|_X$$

holds for arbitrary sequence $\{f_k\}_{k=1}^{\infty}$ of independent mean zero random variables from an rearrangement invariant space X on $[0,1]$. Moreover, a description of the subspace of X generated by Rademacher functions with independent vector coefficients is presented.

MATRIX WIENER–HOPF PROBLEMS AND FREDHOLM INTEGRAL EQUATIONS

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Key Words and Phrases: matrix, factorization, Riemann problem, Wiener–Hopf problem, Fredholm integral equation of the second type, existence of solution, uniqueness of solution, wave diffraction, impedance, thin elastic plate, numerical solution
AMS Subject Classification: 47A68

We consider the class of PDE problems that can be reduced to finding vector functions $\Phi_+(t) = \begin{pmatrix} \phi^1_+(t) \\ \cdots \\ \phi^n_+(t) \end{pmatrix}$ and $\Phi_-(t) = \begin{pmatrix} \phi^1_-(t) \\ \cdots \\ \phi^n_-(t) \end{pmatrix}$ which are analytical in the upper and lower half-planes of the complex variable t, if we know a linear relationship on the real axis

$$A(t)\Phi_+(t) + B(t)\Phi_-(t) + C(t) = 0, \qquad (1)$$

where A and B are given nondegenerate matrices, and C is a given vector. This problem is called Wiener–Hopf problem and the vector functions $\Phi_\pm(z)$ are solution of the problem.

The problem of finding $\Phi_\pm$ from the equation (1) is a special case of the so-called Riemann problem. To precisely formulate the Riemann problem, we, of course, should specify the relevant properties of the given matrices $A(t)$, $B(t)$ and $C(t)$, and the vectors $\Phi_\pm(t)$ we want to find.

The solution of the Riemann problem (1) is reduced to a so-called factorization of the matrix $M = A^{-1}B$, i.e., to a representation of M in the form of a product of two non-degenerate matrices $M(t) = K_+(t)K_-(t)$, K_+ being regular in the upper half-plane and K_- being regular in the lower half-plane. The factorization procedure, in turn, can be reduced to finding of linearly independent solutions of the homogeneous Riemann problem

$$\phi_+(t) = M(t)\phi_-(t). \tag{2}$$

It is known, that for $n > 1$, generally, this problem cannot be solved directly. However in this case it can be transformed to solution of an integral equation of the form

$$\psi(t) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{(M(t)M^{-1}(s) - E)\psi(s)}{t-s}ds + F(t), \quad t \in \mathbb{R}^1. \tag{3}$$

Here E is the unit matrix and $F(t)$ is a known vector function.

In many important classes of problems, equation (3) is of Fredholm type of second kind, and the corresponding homogeneous equations has only zero solution. Then equation (3) is uniquely solvable, and its solution yields the solution of the factorization problem and equation (1).

The given approach will be illustrated on an example of the scalar diffraction problem of a plane wave on an impedance half plane ($y = 0$, $x < 0$), with different impedances on both sides of the half plane. In this case the integral equations are solved numerically. In the future we plan to consider some more complicated problems, for example, the problems with thin elastic plate instead of impedance half-plane.

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WAVELET SOLUTION OF INTEGRO-DIFFERENTIAL EQUATIONS

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Key Words and Phrases: integro-differential equations, Chebyshev wavelets, op-erational matrix of integration
AMS Subject Classification: 34K28

A direct method for solving integro-differential equations by using Chebyshev wavelet basis is presented. We use operational matrix of integration (OMI) for Chebyshev wavelets to reduce this type of equa-tions to a system of algebraic equations. Some quadrature formula for calculating inner products have presented which can be operated by Fast Fourier Transform (FFT).The numerical examples and the num-ber of operations show the advantages of this method to some other usual methods.

CHARACTERISTICS OF OPTICAL AND PHOTOCONDUCTIVE PROPERTIES IN BULK AND THIN FILM OF TLS2 SINGLE CRYSTALS

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Key Words and Phrases: nonlinear properties
AMS Subject Classification: nonlinear physics

The photoconductivity measurements were carried out for bulk single crystals of TlS2 crystals by using the steady state (dc) method in order to elucidate the nature of the dc-photoconductivity (dc-PC) in these crystals. The photoconductivity measurements were carried out in the temperature range 77 to 300 K, excitation intensity range 2150 to 5050 Lux, applied voltage range 10 to 25V, and wavelength range 400 to 915 nm. As a result of the dc- PC, The temperature dependence of the energy gap width were described and the temperature coefficient of the band gap was determined. Reflectance and transmittance spectra of the TlS2 thin films were measured in the incident photon energy range 2.1 to 2.45 eV and in the temperature range 77 to 300 K. With aids of these spectra, the temperature dependence of optical transports and parameters were elucidated. In the low energy region of the studied incident photon energy range, the mentioned spectra were analyzed for describing the refractive index as a function of wavelength. As results of the refractive index-wavelength variations, both the oscillator and dispersion energies of the refractive index were thereafter estimated.

INFERENCE ABOUT THE MEAN OF A SKEWED POPULATION: EMPIRICAL LIKELIHOOD INFERENCE IN THE ONE SAMPLE PROBLEM

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Key Words and Phrases: empirical likelihood, Bartlett correction, Skewed population, t-test, population mean
AMS Subject Classification: 62G15

We consider the problem of hypotheses testing and interval estimation of the mean of a possibly skewed population. The usual procedures based on the large sample distribution of the studentized sample mean can be in error because of the violation of the nominal values of test sizes or confidence levels. Many attempts were made to overcome this problem. Most of them are based on correcting the studentized t variable with higher order terms and possibly using the bootstrap to set critical values. Another approach is based on the empirical likelihood and possibly using the bootstrap or the Bartlett correction to improve the calibration. In this paper, using simulation techniques, we investigate and compare these competing approaches in terms of the attainment of the nominal values of test sizes, confidence levels and the powers of the associated tests. It is found that intervals based on the Bartlett corrected empirical likelihood are very accurate even for small sample sizes from highly skewed populations. Its power performance is also comparable and in many times better than the other procedures considered besides its applicability for testing when all other procedures fail to attain the nominal sizes of the tests.

THE DUAL INTEGRAL EQUATIONS APPLICATION IN NONLINEAR THERMOELASTIC PROBLEMS

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Key Words and Phrases: thermoelastic stresses, dual integral equations, Crack closure, analytical solution

AMS Subject Classification: 74B20

Considering durability of structures with cracks it is necessary to take into account a possibility of cracks surfaces contact. Indeed, under arbitrary loading of structures there is no guarantee, that the cracks will be completely opened. The complete or partial closure can take place if the material is under compression. It leads to changing the stress and deformation states in the structure and influences the conditions of a crack growth and life-time.

Up to the present only few closed-form solutions about thermal cracks are available in the literature. Matczynski and Sokolovski, 1989, obtained an analytical solution for thermo-elastic problem about opening of a strait crack under two, symmetrical with respect to the crack plane, cooling sources, in plane two-dimensional statement. Gross and Heimer, 1993, considered the crack surface contact for curved cracks under thermal loading (also two-dimensional case and the problem was solved numerically). However, to the best knowledge of the authors, the analytical solution of axisymmetrical problem about a disk-shaped crack, and with contacting surfaces, is the first one, proposed in this paper.

Problem of partial opening of a penny-shaped crack due to concentrated heat sources was considered. Analytical results were obtained by means of Hankel transforms and corresponding dual integral equations. The closed form solutions of a heat flux across the crack's surfaces and opening of the crack were obtained. The solution was illustrated by several numerical results. Crack's openings as functions of the distance between heat sources and crack for different initial openings of crack were shown.

GAS DYNAMICS IN MICRO-SYSTEMS: AN ANALYSIS BASED ON KINETIC MODELS OF THE LINEARIZED BOLTZMANN EQUATION

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Key Words and Phrases: rarefied gas dynamics, kinetic models, discrete-ordinates method
AMS Subject Classification: 76P05, 76M22, 65N35

The increase interest in the technological development of micro and nano systems has brought attention to the study of gas flow in the transition regime [1]. In this case, alternative choices for modeling the flow, than the Navier-Stokes equations, have to be considered – as the Boltzmann equation and associated kinetic models [2].

In this talk some classical problems in rarefied gas dynamics [3] are introduced and discussed, on the basis of kinetic models of the linearized Boltzmann equation [4]. In addition, an spectral method [5] is used to develop expressions for physical quantities of interest. Analytical and computational aspects are discussed and numerical results [6, 7] are presented. Particular aspects relevant to the gas-surface interaction are also pointed out [8]. Special attention is given to a gas evaporation problem, where a quadratic eigenvalue problem arises [9].

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NON HERMITIAN OPERATORS IN QUANTUM MECHANICS WITH REAL SPECTRUM

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Key Words and Phrases: Krein spaces, non-hermitian operators, non-hermiticity and real spectrum
AMS Subject Classification: 46C20, 47A12, 15A60

We present simple examples of non-Hermitian models in quantum mechanics for which the Hamiltonian H has a real spectrum. An involutive J operator such that $H^* = JHJ$ is identified and explicitly constructed in each case. The operator J allows for the definition of an indefinite norm, which, obviously is not suitable for the standard quantum mechanical interpretation. We discuss the important role played by this operator in removing the arbitrariness in the definition of a positive definite norm operator N, suitable for quantum mechanical interpretation. In indefinite inner product spaces, J-Hermitian operators are known to have a spectrum which is symmetric relatively to the real axis or a real spectrum.

GENERALIZED TURING QUANTUM MACHINE AND ITS ALTERNATIVE BULK NMR IMPLEMENTATION

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Key Words and Phrases: quantum computing, NMR, Turing machine
AMS Subject Classification: 11K55, 68Q05, 81P68

The purpose of the present paper is twofold.

First, we introduce and study a generalized version of both the classical and quantum Turing computational model, namely, Turing machine able to act in a single step on any finite continuous segment of its infinite (to the left, to the right, or in both directions) band.

Second, we produce an alternative bulk NMR implementation of the corresponding quantum computational model. The qualifier "alternative" denominates our new and greatly simplified bulk NMR technique of experimental quantum computational modeling which offers the possibility to work with nuclear magnetic resonance on macroscopic materials, either liquid (non-viscous) or solid and as primitive as water, water solutions and solid Xenon.

The advantages of our theoretical and experimental approach are :

(i) greatly simplified experimental setting giving access to experimentation with quantum computations to a larger research community;

(ii) relatively long characteristic time of decoherence permitting the execution of up to several hundred quantum elementary operators in one coherence cycle;

(iii) high scalability potential, both for liquid and solid state NMR, with our initial experiments successfully conducted on cylinders of (pure) water carrying up to 30 qubits.

QUANTUM FILTERING AS AND OPTIMAL QUANTUM FEEDBACK CONTROL

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Many if not all tasks of quantum computation can be formulated as the problems of quantum control, i.e. control of the dynamics of qubits under certain constraints. These constraints are usually given in the form of admissible unitary gates, and the computational tasks are described as programming control of these gates without feedback.

In a more realistic non-unitary decoherence case programming control may not be optimal, and the best result can be achieved by using quantum feedback channels as controls by purifying in an optimal way the decohering gates. I will formulate the problem of feedback control for a simple quantum object, a single qubit say, having a feedback channel, measurement say, as a dynamical optimization problem on discrete steps. This can be thought as the modeling problem for the corresponding time-continuous quantum optimal feedback control. The latter was suggested by us in the 80's for open quantum systems like spontaneously radiating atom with the input-output quantum fields. Quantum filtering method was develpoped as a tool to form an optimal feedback for achieving a desirable final quantum state following closely a classical trajectory from each initial state.

A discrete time version of this quantum dynamical decision theory is applied to the optimal control problem in quantum automata. It is shown that quantum optimal filtering together with quantum dynamical programming for optimal quantum feedback give a universal error correction method, up to the best minimal error, in decohering systems.

The Gaussian case of quantum one-dimensional linear Markovian dynamical system with a quantum linear transmission line is studied and the optimal quantum multi-stage decision rule consisting of the classical linear optimal control strategy and quantum optimal filtering procedure is found. The latter contains the optimal quantum coherent measurement on the output of the line and the recursive processing by quantum Kalman-Bucy filter.

All the results are illustrated by an example of the optimal control problem for a quantum linear object and channels such as linear quantum transmission line, where the theory started from.

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OPTIMAL CONTROL OF TRUNCATED VOLTERRA INTEGRAL EQUATIONS

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We use the term "truncated Volterra integral equations" for integral equations in which the function to be integrated is truncated to become zero on parts of the interval of integration. The part of the interval of integration on which the kernel is truncated to zero is a decision variable or "control variable". The optimal control problem is to minimize a cost functional with integral and pointwise terms. We prove necessary conditions for optimality and provide methods for the solution of such control problems. Problems of this type arise in mathematical economics.

ON A CRYPTOGRAPHICAL CHARACTERIZATION OF CLASSICAL AND NONCLASSICAL EVENT SYSTEMS

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Key Words and Phrases: generalized event field, ring-like quantum logic, S-probability, cryptographic algebra, quantum behaviour, involution of event system

Systems of numerical events, defined by how the probability of occurrence of an event depends on the state of the physical system, and fields of events which are related to ring-like logics, are characterized under a common viewpoint by a property which is characteristic of symmetric ciphers. A method is established which allows to separate the classical and the quantum behavior in a way similar to the procedure related to Bell's inequalities, yet of a different nature.

UNIFIED FORMULAS FOR ARBITRARY ORDER SYMBOLIC DERIVATIVES AND INTEGRALS OF THE POWER-INVERSE TRIGONOMETRIC CLASS II

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This talk is a continuation of a series of papers on giving a complete solution to the problem of differentiation and integration of arbitrary (integer, fractional, real, or symbolic) order of elementary and special classes of functions. In general, the solutions are given through unified formulas in terms of the Fox H-function which in many cases can be simplified to less general functions. In this work, we consider two subclasses of the *power-inverse trigonometric class*. Namely, the *power-inverse sine class*

$$\left\{ f(x) : f(x) = \sum_{j=1}^{\ell} p_j(x^{\alpha_j}) \arcsin \beta_j x^{\gamma_j} \,, \alpha_j \in \mathbb{C}, \beta_j \in \mathbb{C}\backslash\{0\}, \gamma_j \in \mathbb{R}\backslash\{0\} \right\} \tag{1}$$

and the *power-inverse cosine class*

$$\left\{ f(x) : f(x) = \sum_{j=1}^{\ell} p_j(x^{\alpha_j}) \arccos \beta_j x^{\gamma_j} \,, \alpha_j \in \mathbb{C}, \beta_j \in \mathbb{C}\backslash\{0\}, \gamma_j \in \mathbb{R}\backslash\{0\} \right\}, \tag{2}$$

where p_j's are polynomials of certain degrees. One of the key points in this work is that the approach does not depend on integration techniques The arbitrary order of differentiation is found according to the Riemann-Liouville definition, whereas the generalized Cauchy n-fold integral is adopted for arbitrary order of integration.

The motivation of this work comes from the area of symbolic computation. The idea is that: Given a function f(x), can CAS find a formula for the nth derivative or integral of f(x)? This enhances the power of integration and differentiation of CAS. A software exhibit will be within the talk using the computer algebra system Maple.

WAVELET TRANSFORM FOR THE EVALUATION OF SEISMIC SURFACE WAVES

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The analysis of seismic surface waves, is a major issue in seismic signal processing, particulary in geology (civil engineering), the dispersion evaluation enable to characterize physics properties of the ground, but in geophysics, the surface waves are regarded as noise in the seismic profile and their extraction (denoising) increase the signal report on noise (SRN), consequently facilitate the interpretation of the seismic document. The evaluation of the dispersion parameters is a fundamental step of the seismic profile pretreatment, and usually a means to align and wave separation. Moreover, in practice, several technics of filtering and wave separation are based on the "good" estimation of dispersion parameters, in particular, for example the wave sep-aration by singular value decomposition (SVD). Historically, several methods allowing to estimate the delay and the phase shift. Among these methods, those based on the statis-tics of the higher order, the time-frequency, the time frequency-velocity representation and none of this approach estimate simul-taneously the three parameters of dispersion (in complex case). We propose in this paper, a new method based on the Contin-uous Wavelet Transform concept (CWT), and on mathematical modeling of the seismic signal. We demonstrate by choosing an adapted analysis wavelet (its form is "closer" to the seismic im-pulsion) and a scale factor, it will be possible to give a good estimation of dispersion parameters. Our method gives much improved numerical results, when com-pared to other classical methods, such as the gradient method. keywords: Continuous Wavelet Transform, surface seismic waves, mathematical modeling, delay, phase shift, dispersion coefficient.

STABILITY ANALYSIS AND INTERVAL APPROACH APPLIED TO BLADES OF WINDSCREEN WIPER

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Key Words and Phrases: mechanical system, friction induced vibration, stability analysis, interval approach

AMS Subject Classification: 70K50

Friction-induced vibration due to mode coupling is a major concern in a wide variety of mechanical systems. Effectively, the stability analysis and the associated non-linear amplitudes around a steady-state equilibrium point are two of the most important points in the study of non-linear dynamical systems depending on given control parameters.

Obviously, determining the stable and unstable regions is only one aspect of the problem of friction-induced flutter instability. The instability magnitude is a more significant design factor than the instability region. For example, the non-linear amplitudes resulting from physical parameters may be very small and hence the friction-induced vibration would be negligible for the mechanical system.

So in this paper, the first step is the static problem: the steady-state operating point for the full set of non-linear equations is obtained by their solution at the equilibrium point. Then, stability is investigated by calculating the eigenvalues of the linearized system at the equilibrium point. Secondly, we propose to apply interval analysis to obtain the maximum and minimum amplitudes of the complete non-linear mechanical system. This approach allows us to avoid numerical procedures that are both time consuming and costly to perform when parametric design studies are needed.

AN EXACT BOUNDED PML TECHNIQUE FOR TIME-HARMONIC WAVE EQUATIONS

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Key Words and Phrases: perfectly matched layer, time-harmonic scattering, Helmholtz equation
AMS Subject Classification: 65N30, 65N99, 76Q05

We consider the following Helmholtz problem which models the propagation of a wave of frequency $\omega > 0$ and velocity of propagation $c > 0$ in an unbounded homogeneous medium:

$$\begin{cases} \Delta u + k^2 u = 0 \qquad \text{in } \Omega_{\mathrm{E}}, \\ u = g \qquad \text{on } \Gamma, \\ \displaystyle\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial u}{\partial r} - iku\right) = 0, \end{cases}$$

where $k := \omega/c$ is the wave number, $\Omega_{\mathrm{E}} := \mathbb{R}^2 \setminus \overline{\Omega}_{\mathrm{I}}$, with $\Omega_{\mathrm{I}} \subset \mathbb{R}^2$ being a simply connected bounded domain with Lipschitz boundary Γ, and $g \in \mathrm{H}^{\frac{1}{2}}(\Gamma)$ is a given source function. The third equation is a typical *Sommerfeld* condition modeling the radiation of the wave at infinity. This is a classical scattering problem, whose existence and uniqueness of solution is well known (see for instance Colton [4]).

The typical first step for the numerical solution of such a problem is to truncate the unbounded computational domain, which entails an inherent difficulty: *how to choose boundary conditions to replace the Sommerfeld radiation condition at infinity* (see for instance [5]). There are several techniques to deal with this: boundary element methods, infinite element methods, Dirichlet-to-Neumann operators based on Fourier expansions, or the use of absorbing boundary conditions, for instance.

An alternative approach to deal with the truncation of unbounded domains is the so called *PML* (Perfectly Matched Layer) technique, introduced by Berenger Berenger

[1] for Maxwell's equations in electromagnetism. It is based on simulating an absorbing layer of damping material surrounding the domain of interest, like a thin sponge which absorbs the scattered field radiated to the exterior of the domain. The absorbing material is characterized by a damping function varying through the thickness of the layer. Typical examples are linear and quadratic damping functions ([1], [3]).

In a recent paper [2], we have introduced an 'exact' bounded PML, based on using a singular damping function. 'Exactness' must be understood in the sense that this technique allows exact recovering of the solution to time-harmonic scattering problems in unbounded domains. In spite of the singularity of the damping function, the procedure is shown to lead to a well posed conforming finite element discretization.

We analyze this approach in a simplified one-dimensional framework, which allows us to choose a convenient singular damping function. Subsequently, it is proved that a similar damping function acting on an annular layer also leads to an exact recovery of the solution on the physical domain. Finally, we report some numerical tests exhibiting the high accuracy of this technique, as well as its advantages as compared with other classical PML methods.

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DEVELOPMENT FOR AN IMPROVED PATTERN RECOGNITION SCHEME USING GENETIC ALGORITHM BASED FEATURE SELECTION: A STUDY ON SOFT COMPUTING APPROACHES

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The problem of feature selection consists of finding a feature subset of input training as well as test patterns that enable to describe all information required to classify a particular pattern. In present paper author focuses on particular problem for feature selection which plays a key role in machine learning. In fact before building a model for feature selection the goal is to identify and to reject the features that degrade the classification performance of a classifier. This is true when the available input feature space is very large and need exists to develop an efficient searching algorithm to combine these feature spaces to a few significant one which are capable to represent that particular class. Presently author has described two approaches for combining large feature spaces to efficient numbers using *Genetic Algorithm and Fuzzy Clustering* techniques. Finally the classification of patterns has been done using adaptive neuro fuzzy techniques. The aim of entire work is to implement the recognition scheme for classification of tumor masses appearing in tissue region as space occupying lesion identified by CT and MR images. The proposed model is adaptive in nature and indicates a promising direction for classification in a changing environment. The boundary detection of the region of interest ROI of a pattern is based on Fourier descriptors which introduces a large number of feature vector in a pattern recognition scheme. To classify different boundaries any standard classifier needs large number of inputs. To train the classifier large number of training cycles and huge memory are also required. A complicated structure of the classifier invites the problem of over learning and which may cause misclassification. This leads to develop significant feature selection for efficient pattern recognition scheme. Genetic algorithm is an efficient search algorithm based on mechanics of natural selection. GA based feature selection shows the improvement over fuzzy clustering due to natural selection mechanisms. Proposed feature selection methodology combined with adaptive neuro fuzzy classifier is an intelligent expert system that gives the user accurate detection even in presence of additive noise.

SEMIGRAPHS- DEGREE SEQUENCES

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AMS Subject Classification: 05C07, 05C65

Semigraphs were introduced by E.Sampathkumar in 1996, as a generalisation of graphs where an edge contains two or more vertices with some order and any two edges have at most one vertex in common. In hypergraphs(a well known generalisation of graphs) also, an edge contains vertices more than two. But the order among the vertices belonging to an edge is immaterial in hypergraphs. Thus while an edge is represented as a set in hypergraphs, it is represented as an ordered tuple in semigraphs. This difference leads us to get some applications of semigraphs. Though a wide theoretical base for semigraphs is developed by Sampathkumar, there are many problems in graph theory which are reposed for semigraphs. In this talk, we define semigraphs and characterise the degree sequences of semigraphs.

BIVARIATE OPERATORS WHICH INTERPOLATE SOME PARTIAL DERIVATIVES OF A FUNCTION ON THE VERTICES OF A SQUARE

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Key Words and Phrases: bivariate interpolation projectors, Hermite interpolation, Birkhoff interpolation, order of approximation
AMS Subject Classification: 41A05, 41A63

In this article we construct bivariate interpolations projectors using parametric extensions of univariate Hermite or Birkhoff interpolation projectors with two nodes which form the chains in a lattice of bivariate projectors. For this operators we give the range space, the interpolation properties and the remainder term. The projectors interpolate some partial derivatives of a function on the vertices of a square. We give some comparisons with tensor product operators about the approximation order and information used by operators (evaluations of the function and some partial derivative of functions in the vertices of the square).

ON THE POWER OF TESTS OF KOLMOGOROV–SMIRNOV TYPE TESTING THE TREND OF BROWNIAN BRIDGE/MOTION AND APPLICATIONS

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Key Words and Phrases: Boundary crossing probability, Brownian bridge with trend, Brownian motion with trend, tests of Kolmogorov–Smirnov type, signal–plus–noise model, Cameron–Martin–Girsanov formula, linear regression models, model checks for regression, change–point problem
AMS Subject Classification: 60G15, 60G70, 62G10, 62G32, 62J05

We consider a signal–plus–noise model $B_0 + h$ with Brownian bridge B_0 as noise and h as signal. We show a lower and an upper bound for the boundary crossing probability $P(\exists z \in [0,1] : h(z) + B_0(z) < \ell(z) \text{ or } u(z) < h(z) + B_0(z))$ where ℓ, u are boundary functions. The probability considered above corresponds with the power of tests of Kolmogorov–Smirnov type for testing

$$H_0 : h \equiv 0 \text{ against } K : h \neq 0.$$

Such tests can be used to check for regression by using a residual partial sums limit approach. Especially, in case an unknown constant is assumed as regression function in the linear model we consider the least squares residuals of this model. Then under the hypothesis H_0 that the linear model is true the residual partial sums limit process is a Brownian bridge. In case the hypothesis is not true then the residual partial sums limit process is a Brownian bridge with some trend $h \neq 0$. For more complicated linear regression models one gets more complicated Gaussian processes with mean zero if the assumed regression model is true and with trend $h \neq 0$ if the assumed regression model is not true.

Our bounds can be easily transformed to bounds for a boundary crossing probability of Brownian motion with trend. This result is also useful to check for a certain regression in an analogous way as described above.

STRUCTURE-PROPERTY BASED MODEL ESTIMATION OF ALKANES BOILING POINTS

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Key Words and Phrases: molecular descriptor family on structure-property relationships (MDF SPR), models assessment, boiling point, Alkanes

AMS Subject Classification: 03H05 (Nonstandard models in mathematics), 62P35 (Applications to physics), 93E24 (Least squares and related methods), 93E35 (Stochastic learning and adaptive control)

This study discusses a family of molecular descriptors on structure-property relationships (MDF SPR) to model the boiling points of alkanes based on their chemical structure.

The proposed approach uses the complex information obtained from the all alkanes from C3 to C9 structures in order to generate and calculate the molecular descriptors family. The structure-property relationship models were built based on the generated descriptors. The obtained models (model with one and two descriptors, respectively) were validated through the assessment of the cross-validation leave-one-out score. The comparison between the uni-varied model and the model with two descriptors was performed using Steiger's Z test. The best performing MDF SPR model was validated, and its correlation coefficient was compared with a previously reported model.

The analysis of the statistical characteristics of the obtained models demonstrated that the model with two descriptors has greater abilities in estimation and prediction compared with the model with one descriptor. This observation was also sustained by the results of training versus test analysis.

The results of this study revealed that the MDF SPR approach is a useful method to model the boiling points of alkanes providing stable models.

ARE CONFIDENCE INTERVALS FOR BINOMIAL DISTRIBUTED SAMPLES AN OPTIMIZATION METERS?

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Key Words and Phrases: optimization, confidence interval, binomial distribution, contingency table

AMS Subject Classification: 49M25 (Discrete approximations), 60A05 (Axioms; other general questions), 94B70 (Error probability), 62P10 (Applications to biology and medical sciences), 62Hxx (Multivariate analysis)

The aim of the research was to develop an optimization procedure of computing confidence intervals for binomial distributed samples based. An inductive algorithm is proposed method used to solve the problem of confidence intervals estimation for binomial proportions. The implemented optimization procedure uses two triangulations (varying simultaneously two pairs of three variables).

The optimization method was assessed in a simulation study for a significance level of 5%, and sample sizes that vary from six to one thousand and associated possible proportions. The obtained results are available online at the following address:

http://l.academicdirect.org/Statistics/binomial_distribution/

Overall, the optimization method performed better, the values of cumulative error function decreasing in average with 10%, depending on the sample sizes and the confidence intervals method with which it is compared.

The performances of the optimization method increase with increasing of the sample size, surprisingly because it is well known that the confidence interval methods that use the normal approximation hypothesis for a binomial distribution obtain good results with increasing of sample sizes.

ULTRA-NEWTONIAN GRAVITATION THEORY

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Key Words and Phrases: gravitation, Einsteinian law, redshift, universe, Hubble hypothesis, light velocity

In the talk a theory of the gravitation will be given that contradicts to the Einsteinian general relativity theory. The presented theory is based on the *postulate of flowing space* that was formulated by the author. As we show, from that postulate the well known Schwarzschild's metric (which explains the rotation of Mercouri's perihelion and the bend of the light trajectories near the Sun) is easily deduced. Furthermore, the gravitational redshift formula is deduced that is only postulated (without any proof) in the general relativity theory. Moreover, it is shown that a gravitational *blueshift* is possible. Some experiments are described two of which are able to discover the blueshift, whereas the third one can resolve the discussion between the Einsteinian gravitation law and the postulate of flowing space. Finally, the gravitational redshift of distant objects of the universe is explained without Hubble's hypothesis on the "extending universe". At the end of the talk it will be shown that the presented theory can be justified by consideration of a flux of some particles (as distinct from the Einsteinian theory that has, because of the influence of Hilbert, purely geometrical character).

RECONSTRUCTING POPULATIONS FROM FLAWED DEATHS BY AGE, BIRTHS, AND ONE OR TWO CENSUSES, WITH APPLICATION TO THE DON ARMY REGION, 1863-1915

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Key Words and Phrases: population reconstruction, simulated annealing, missing data
AMS Subject Classification: 91F10, 90C90, 37N40

Lotka-McKendrick population dynamics with migration, missing or flawed data, and possible change of territory is combined with simulated annealing to fit available censuses and vital statistics of births and deaths by age. The method is adapted to the position of the census, either at the beginning of the vital statistics series, at the end, or both. It is able to overcome the recurrent difficulty of missing data or miscounts, a case often encountered in real historical data. Simulations help calibrate the method and determine the weights associated to each data. The empirical case study of an administrative subdivision in southern Russia, orthodox Donskoi Khoperskii 1863-1915, which gathers all cases and difficulties, is treated.

A 3D MODEL FOR A STOCHASTIC SET OF SMALL SCALE PLUMES IN OPEN SEA

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Key Words and Phrases: convective plumes
AMS Subject Classification: Fluid mechanics

A model describing a process of formation of small scale plumes and their evolution in a very weakly stratified sea, under the effect of an external stochastic forcing, is here presented. It is assumed an unsteady process caused by space and time fluctuating wind bursts: these cool and evaporate a localized region of the surface water in such a way that a very slow buoyancy variability is allowed along the winds direction, but a space and time fluctuating surface buoyancy is transversally generated, such that a short horizontal correlation length is defined for it; on average a lot of minima, maxima and steep gradients in between is allowed in it; its vertical decaying depth $1/\lambda$ may have a large spatial range going from the radiation penetration depth to the thermal boundary layer depth. No fixed temperature or salinity boundary condition is given on the interaction surface, but a large variability is allowed on them. The process is mathematically described by the complete set of the non viscous Navier Stokes equations (in Boussinesq and hydrostatic approximation) coupled to the non diffusive mass conservation equation in a rotating frame. The wind stress has been disregarded because it does not operate in depth, where the convective motion is generated. A still sea initial condition is supposed, so that the space and time stochastic buoyancy horizontal variability, driven by the transverse winds, is the source of a convective process. Two space scales (a small plume scale and a collective perturbed region scale) have been recognized, acting together: so a multiple space scale method allows to decouple, in a streamfunction formulation, the vertical transverse plane (over which the plumes set is generating) from the winds direction line, along which the plume is deviated from the Coriolis force on longer times $t\ O(1/f)$(Coriolis parameter). Two time scales have been recognized, so that on the short time scale ($t < 1/f$) the plumes generation and first evolution process can be described in a Lagrangian representation on a $2D$ plane; on the long time scale shear horizontal instability allows a set of $3D$ small plumes to be defined: an enhanced region of perturbation can be recognized, due to stratification, driving to a different regime of scale laws. A kind of 'transformation' of the buoyancy allows the effect of the entrainment-detrainment to be analysed. After all we have:

a) a set of independent quasi periodical small scale plumes is generated, whose distance is given by the horizontal correlation length in the surface buoyancy. A critical distance, connected to a critical horizontal buoyancy gradient, is defined.

b)their evolution is described by an equation scalable with the penetration depth; it is ruled by time power 'one plume laws' depending on the statistics of the external events, their frequency, and by space-time buoyancy fluctuations power laws. The convective motion is driven by the mean horizontal unhomogeneity of the surface buoyancy

flux and its space-time variability. For short times a linear stability theory shows that the fastest growing in time internal perturbations take a very long time to grow, so that the stability of the described convective motion can be inferred for the interesting times. The analysis of the stability of the model, perturbed by vertical internal fluctuations on longer times, shows a weak intermittent behaviour: but plume evolution and scaling laws are ruled by random external forcing leading to a higher time power behaviour: this depends on the probability of the event $1/t$, which hides slower internal randomness; if the air-sea interaction statistics is such that it is impossible to define it, no self-similar behaviour is possible. Large internal fluctuations have a mixing and turbulence generation effect.

THE STABILITY AND STABILIZATION OF MOTION IN CRITICAL CASES

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AMS Subject Classification: 34D20, 35B35

In the first part of the presentation given a review of the works of the author devoted to stability of the motion [1-5]. Given new criterion of stability of nonlinear differential and difference equations in Banach spaces, of stability of solution of nonlinear systems of differential equations with lateness, nonlinear systems of differential equations with a small parameter attached to derivative, nonlinear systems of partial differential equations. Given criterion of stability of solution of nonlinear systems of differential equations with discontinuous right-hand sides. Stability of waves in nonlinear dissipative processes which are described by nonlinear parabolic equations are investigated too.

Criterion are applied to regular case and to all possible critical cases simultaneously.

In the second part of the presentation we give some new criterion of stabilization of solutions of differential equations.

In the third part of the presentation we give applications of these criterion to some tasks of ecology and economics. In particular, we offer new criteria of stability of Hotelling-Scellam models in ecology and economics. New criterion of stability of Kolmogorov model in ecology are given too.

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THE WIDTH AND ENTROPY OF SOME CLASSES OF FUNCTIONS AND ITS APPLICATIONS IN NUMERICAL ANALYSES

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The presentation consists of the three parts.

In the first part is given the review of the author works [1,2,3] devoted to evaluation of Babenko and Kolmogorov n-widths of functional classes $Q_{r,\gamma}(\Omega, M)$, $B_{r,\gamma}(\Omega, M)$ and construction of optimal with respect to order algorithms for approximation of functional classes $Q_{r,\gamma}(\Omega, M)$, $B_{r,\gamma}(\Omega, M)$.

In this part we evaluate the $\epsilon-$ entropy of the functional classes $Q_{r,\gamma}(\Omega, M)$, $B_{r,\gamma}(\Omega, M)$ and construct optimal with respect to order by memory algorithms for restoration functions from functional classes $Q_{r,\gamma}(\Omega, M)$, $B_{r,\gamma}(\Omega, M)$. also.

Let us remind the definitions of the functional classes $Q_{r,\gamma}(\Omega, M)$, $B_{r,\gamma}(\Omega, M)$.

Definition 1. Let $Q_{r,\gamma}(\Omega, M)$, $\Omega = [-1,1]^l$, $l = 1, 2, \ldots$ be the class of the functions $f(t_1, \ldots, t_l)$ defined on Ω and satisfying the following condititions:

$$\max_{t\in\Omega} |f^{(v)}(t)| \le M,\ 0 < |v| \le r,\ |f^{(v)}(t)| \le \frac{M}{(\rho(t,\Gamma))^{|v|-r-\zeta}},\ r < |v| \le s,$$

where $t = (t_1, \ldots, t_l)$; $v = (v_1, \ldots, v_l)$, $|v| = v_1 + \cdots + v_l$; $s = r + \gamma$, $\zeta = 0$, if γ is an integer; $s = r + [\gamma] + 1$, $\gamma = [\gamma] + \mu$, $\zeta = 1 - \mu$, if γ is non-integer; Γ is the bound of the domain Ω, $\rho(t, \Gamma) = \min_i \min(|1 + t_i|, |1 - t_i|)$ $f^{(v)}(t) = \frac{\partial^{|v|} f(t_1,\ldots,t_l)}{\partial t_1^{v_1} \cdots \partial t_l^{v_l}}$.

Definition 2. Let $\Omega = [-1,1]^l$, $l = 1, 2, \ldots$, $r = 1, 2, \ldots$, $0 \le \gamma < 1$. Function $f(t_1, \ldots, t_l)$ belongs to the class $B_{r,\gamma}(A, \Omega)$ if the following inequalities hold:

$$\left|\frac{\partial^{|v|} f(t_1, \ldots, t_l)}{\partial t_1^{v_1} \cdots \partial t_l^{v_l}}\right| \le A^{|v|} |v|^{|v|},\ 0 \le |v| \le r,$$

$$\left|\frac{\partial^{|v|} f(t_1, \ldots, t_l)}{\partial t_1^{v_1} \cdots \partial t_l^{v_l}}\right| \le \frac{A^{|v|} |v^{|v|}}{(\rho(t,\Gamma))^{|v|-r-1+\gamma}}$$

for $r < |v| < \infty$, where the constant A is independent of $|v|$.

In the second part of the presentation we show that conjugate functions as

$$\tilde{\varphi}(t) = \int_{-1}^{1} \frac{\varphi(\tau)}{(\tau - t)^p}, \quad -\infty < t < \infty,$$

$$\tilde{\varphi}(t_1, t_2) = \int_{-1}^{1}\int_{-1}^{1} \frac{\varphi(\tau_1, \tau_2) d\tau_1 d\tau_2}{(\tau_1 - t_1)^{p_1} (\tau_2 - t_2)^{p_2}},$$

$$\tilde{g}(t_1,t_2)=\int\limits_{-1}^{1}\int\limits_{-1}^{1}\frac{g(\tau_1,\tau_2)d\tau_1 d\tau_2}{((\tau_1-t_1)^2+(\tau_2-t_2)^2)^{p_3}},$$

where p, p_1, p_2, p_3 are integers, $p, p_1, p_2, p_3 \geq 1$, belong to functional classes $Q^*_{r,\gamma}(\Omega, M)$, $B^*_{r,\gamma}(\Omega, M)$ and construct optimal methods for approximation functions $\tilde{\varphi}(t)$, $\tilde{\varphi}(t_1, t_2)$, $\tilde{g}(t_1, t_2)$. (Functional classes $Q^*_{r,\gamma}(\Omega, M)$, $B^*_{r,\gamma}(\Omega, M)$ are axtension of classes $Q_{r,\gamma}(\Omega, M)$, $B_{r,\gamma}(\Omega, M)$.)

In the third part of the presentation we show that solutions of multidimensional Fredholm and Volterra weakly singular integral equations are devoted to functional classes $Q_{r,\gamma}(\Omega, M)$, $B_{r,\gamma}(\Omega, M)$ and construct optimal by order withe respect to accuracy algorithms for solution of these equations. Part of these results are given in the book [4].

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NONLINEAR INTERACTION AND RESONANCE OF COUNTERPROPAGATING WAVES

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Key Words and Phrases: nonlinear wave interaction, wave resoance, inhomogeneous material
AMS Subject Classification: 35L70, 74J25

In this paper a second order nonlinear hyperbolic partial differential equation with variable coefficients is considered. This equation governs nonlinear wave motion in inhomogeneous physically nonlinear elastic material.

Counterpropagation of two longitudinal waves is studied within a finite length of space under initial and boundary conditions at each ends where the initial wave profiles are determined in terms of particle velocity. Variation of material properties (density and elasticity) is assumed to be weak. The equation of motion is solved resorting to the perturbation technique. The solution is sought in series with small parameter. The result is that the equation of motion breaks down into a system of partial differential equations with r. h. s.

An analytical solution is derived making use of the software for symbolic computations *Maple*. This solution describes the initial stage of nonlinear propagation, interaction and reflection of longitudinal waves in weakly inhomogeneous elastic material. The solution in terms of stress is studied numerically. The aim is to clarify the peculiarities of boundary ocillations evoked by counterpropagating waves. It turns out that the maxima of boundary oscillation amplitudes are frequency dependent. Extensive numerical simulations indicate that the influence of variation of weakly inhomogeneous material properties on amplitude-frequency dependence is close to linear and the resonance phenomenon is involved. The research results may be used in algorithms for ultrasonic nondestructive characterization of considered materials.

NONLINEAR MAPS BETWEEN BESOV- AND SOBOLEV SPACES

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Key Words and Phrases: Lipschitz continuous maps, Besov spaces, Nonlinear wave equations
AMS Subject Classification: 35L70, 46T20, 46T25

We will present an extension of a result by Kumlin (thesis, 1985) from the case of analytic mappings to Lipschitz continuous maps. These results can be traced back to ideas of the late B E J Dahlberg (Proc. Symposia in Pure Math. XXXV, 1979).

Let $f \in C^s$ with $s > \frac{n}{2}$. Assume that

$$H_2^1(\mathbf{R}^n) \ni u \to f(u) \in L_2(\mathbf{R}^n)$$

is Lipschitz continuous.

Then either $n < 10$ or else $f(z) = cz$ for some constant c.

We will apply the result in the Main theorem to questions about existence, uniqueness and regularity of solutions to nonlinear wave- and Klein-Gordon equations.

Portions of this note represent joint work with P Kumlin (Department of Mathematical Sciences, Chalmers university of technology and Göteborg University).

PARSEVAL IDENTITIES INVOLVING THE HANKEL TRANSFORM AND THE $\mathcal{Y}$-TRANSFORM AND THEIR APPLICATIONS

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Key Words and Phrases: Hankel transforms, **H**-transforms, $\mathcal{Y}$-transforms, Parseval-Goldstein type theorems, Bessel functions, Struve functions
AMS Subject Classification: 44A10, 44A15, 33C10, 44A35

In the present paper the authors consider the integral transforms including the Hankel-transform, the **H**-transform and the $\mathcal{Y}$-transform. Using the well known properties of these transforms, a number of new Parseval-Goldstein type identities are obtained for these and many other well-known integral transforms. The identities proven in this paper give rise to useful corollaries for evaluating indefinite integrals of special functions. Some examples are also given as illustrations of the results presented here.

UNIQUENESS OF HARMONIC MAPPINGS WITH BLASCHKE DILATATIONS

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Let Ω be a bounded convex domain and let ω be a Blaschke product of order $n = 1, 2, \cdots$. It is known that the elliptic differential equation $\overline{f_{\overline{z}}}/f_z = \omega$ admits a one-to-one solution normalized by $f(0) = 0$, $f_z(0) > 0$ and maps the open unit disc $\mathbf{D}$ onto a convex $(n+2)-$gon whose vertices belong to $\partial\Omega$. In this talk it will be shown that this solution, which is of topological nature, is unique.

NUMERICAL ANALYSIS, LIMIT COMPUTATIONS, AND NEOCLASSICAL ANALYSIS

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Numerical analysis is the study of methods and algorithms for the problems of continuous mathematics. Many of such problems come from science and engineering. When scientists and engineers need numerical answers, they turn to computers. Nevertheless, as Trefethen (2006) writes, there is a widespread misconception about this process. Computers perform operations with finite strings of symbols and can do only a finite number of such operations. As a result, most problems of science and engineering in terms of continuous mathematical models cannot be solved by a finite sequence of computer operations even in principle. The reason is that convergence is a condition *sine qua non* for any numerical scheme, while even the most powerful models of the traditional computer science, such as Turing machines, stop after performing a finite number of operations with strings.

Situation changed with the development of the theory of super-recursive algorithms (Burgin, 2005). This theory includes many directions: inductive computations, computations with real numbers, infinite time computations, interactive computations, etc. One of these directions, limit computations, provides foundations for numerical analysis. Limit Turing machines form a class of powerful super-recursive algorithms and allow one to study methods and algorithms for finding numerical solutions to various mathematical, scientific and engineering problems by rigorous mathematical tools. Numerical methods, such as Newton's method, Lagrange interpolation polynomial, Gaussian elimination, and Euler's method, are modeled by limit Turing machines. The final solution emerges as the limit of intermediate results of a limit computation. Mathematical means for modeling such partially completed computations are provided by theory of approximations, interval analysis and neoclassical analysis (Burgin, 1995). The aim of the paper/lecture is to explicate properties of limit Turing machines in the context of numerical analysis.

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DUAL INTEGRAL CHARACTERIZATIONS FOR EXPONENTIAL STABILITY OF EVOLUTION FAMILIES ON BANACH SPACES

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Key Words and Phrases: operator semigroups, evolution families of bounded linear operators, uniform and nonuniform exponential stability

AMS Subject Classification: 47D06, 35B35

Let X be a real or complex Banach space and $\{A(t)\}_{t\geq 0}$ be a family of bounded linear operators acting on X. We shall denote by $S(\cdot)$ the solution of the following Cauchy Problem:

$$\dot{U}(t) = A(t)U(t), \quad t \geq 0, \quad U(0) = I. \tag{$A(t),0,I$}$$

The system

$$\dot{U}(t) = A(t)U(t) \tag{$A(t)$}$$

is called *uniformly exponentially stable* if there exist the positive constants N and ν such that

$$||S(t)S^{-1}(s)|| \leq Ne^{-\nu(t-s)} \quad \text{for all } t \geq s \geq 0. \tag{$\mathbf{UES}$}$$

It is clear that if the system $(A(t))$ is uniformly exponentially stable then the following four inequalities hold:

$$\sup_{s\geq 0} \int_s^\infty ||S(t)S^{-1}(s)||dt < \infty \tag{$\mathbf{UDC}$}$$

$$\sup_{s\geq 0} \int_s^\infty ||S(t)S^{-1}(s)x||dt = M(x) < \infty \quad \text{for every } x \in X \tag{$\mathbf{SDC}$}$$

$$\sup_{t\geq 0} \int_0^t ||S(t)S^{-1}(s)||ds < \infty \tag{$\mathbf{UBC}$}$$

$$\sup_{t\geq 0} \int_0^t ||S(t)S^{-1}(s)x||ds = N(x) < \infty \tag{$\mathbf{SBC}$}$$

The **Datko theorem:** states that **(UDC)** or **(SDC)**$\Rightarrow$ **(UES)** while that the **Barbashin theorem** asserts that **(UBC)**$\Rightarrow$ **(UES).**

At this moment we do not know if (SBC) implies (UES). Seems that this problem is unsolved even in the framework of the finite dimensional spaces.

It is easy to see that $(S^{-1})^*(\cdot)$ is the solution of the adjoint Cauchy Problem.

$$\dot{V}(t) = -A^*(t)V(t), \quad t \geq 0, V(0) = I \qquad (-A^*(t), 0, I)$$

Let $U(t,s) := S(t)S^{-1}(s)$ the evolution operator associated to the system $(A(t))$. The evolution operator associated to $(-A(t)^*)$ is $V(t,s) = [U(t,s)^*]^{-1}$. Thus is naturally to ask if **(UES)** is a consequence of the following **Strong Dual Barbashin's Condition**

$$\sup_{t\geq 0} \int_0^t ||U(t,s)^* x^*|| < \infty \text{ for every } x^* \in X^*. \qquad (\mathbf{DSBC})$$

Here we shall establish some links between **(DSBC)** and different kinds of stability of an evolution family of bounded linear operators acting on a Banach space X.

EXTENDED BREATHER SOLUTIONS IN A NONLINEAR LATTICE

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A nonlinear lattice model is investigated where anharmonic interactions between ionic cores and surrounding shells are considered. The model differs significantly from typical $\Phi_4$models since strong ionic mass dependences are present which may alter the effective potential from double-well to single well. In addition, exact solutions exist in the continuum limit which describe domain wall motion, kink formation and many other exotic states. An interesting new aspect of the model is revealed in the discrete lattice case where one type of exact solutions corresponds to so-called breathers. However, this kind of breather is distinctly different from those obtained withinΦ_4 models since its spatial extent spreads over several lattice constants. The coupling of these solutions to the harmonic lattice modes provides a temperature dependence to them and stabilizes the lattice against a polar instability.

Possible applications to realistic systems are discussed.

SINGULARLY PERTURBED VECTOR FIELDS AND ITS APPLICATIONS

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Singularly Perturbed Vector Fields (SPVF) represents a coordinate free version of singularly perturbed systems of ordinary differential equations. Roughly speaking, a vector field is a singularly perturbed one if it can be decomposed to slow and fast sub fields due to a prescribed vector bundle structure. The main purpose of the SPVF concept is the proper identification of the fast sub field behavior that is crucial for a slow invariant manifolds evaluation and a correct description of the system dynamics. Locally a vector bundle is a direct product of a k-dimensional manifold onto an m-dimensional Euclidean space that represents a fast subspace for singularly perturbed vector fields. A choice of the vector bundle structure is the main technical obstacle for the SPVF applications. This problem can be divided onto two sub problems: an evaluation of the fast subspace dimension and an evaluation of local nonlinear coordinate system that permits to represent SPVF as a standard singularly perturbed system. These problems will be discussed in details. On the base of the Singular Perturbed Vector Field concept a novel numerical approach for simplifying complex kinetic models, the so-called Global Quasi Linearization method (GQL) is suggested. The GQL procedure allows us to check existence of global slow-fast decomposition for the original vector field (system). If such decomposition exists the GQL algorithm permits to evaluate a global coordinate system for which the original vector field (system) can be rewritten as a standard singularly perturbed one. Model test-examples will be discussed as well as applications to combustion models. The method is implemented within a code for the standard Intrinsic Low-dimensional Manifolds method (ILDM) and it is compared to the ILDM as well as to the detailed simulation.

DYNAMICS OF TRANSIENT CAVITIES

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Water entry problems are mainly composed of two different types of studies: the initial stages of contact [Howison1991] and the creation of the associated cavity [Birkhoff1957]. The reported work belongs to the second type.

We study the formation and collapse of a transient cavity of air in water created by the impact of a solid body (size R_0, velocity U). Experimentally, we first show that the digging of the cavity crutially depends on the wettability of the surface [Duez2007]. This unexpected effect is illustrated on figure 1.

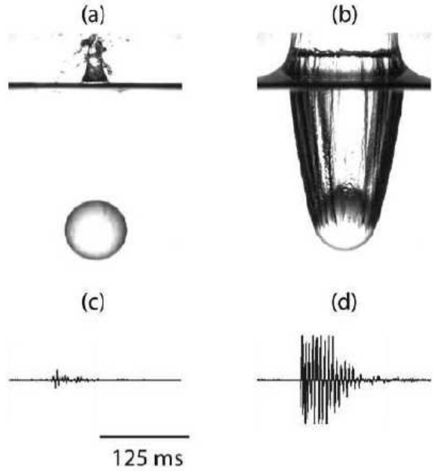

Figure 1.Top : Photography of the impact of two spheres ($R_0 = 1$ cm) differing only by their wettability via a nanometric coating on their surface; (a) impact of a perfectly wetting sphere, with contact angle $\theta \approx 15°$; (b) impact of a hydrophobic with contact angle $\theta \approx 100°$. The impact velocity is 5 m/s in both cases, corresponding to a 1.25 m height drop. The photography is taken 16 ms after impact beginning. Bottom : diagrams (c) and (d) are the time dependent audio recordings of the impacts, as measured by a microphone ~ 10 cm from impact point, for a hydrophilic (c) and a hydrophobic sphere (d). The signal is proportional to the acoustic pressure emitted during the impact. Units on the vertical scale are arbitrary (but identical). A big

"splash" is evidenced for the hydrophobic sphere, while a tiny "plop" is heard for the hydrophilic one. The sound is associated with the rapid closure of the cavity.

Once, the cavity is created, we characterise its dynamics from its creation ($t = 0$) until it collapses ($t = \tau$) in the limit where inertia dominates viscous and capillary effects. A typical impact sequence is presented on figure 2. This chronophotography presents the evolution of the cavity during the first 140 ms after the impact of a glass sphere covered with soot (hydrophobic surface) ($R_0 = 12$ mm) at $U = 2.1$ m/s ($Fr \equiv U^2/gR_0 \approx 39$). The digging of the cavity extends from image 1 to image 8 where it pinches. We note H the depth of the cavity at pinching and H_p the position of the neck. The diameter of the hole at the surface ($z = 0$) is referred to as $2R$. if $t = 0$ stands for the time at which the sphere touches the interface, the time of pinching is defined as $t = \tau$.

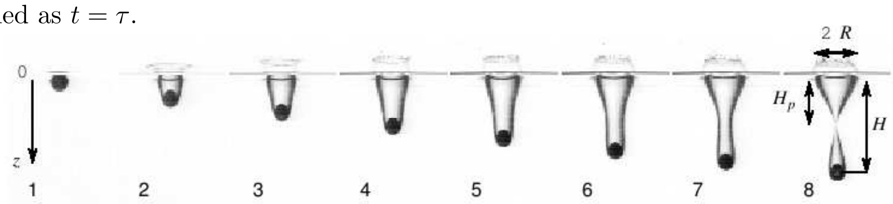

Figure 2. Chronophotography of the impact of a sphere ($R_0 = 12$ mm) at $U = 2.1$ m/s ($Fr = 39$). The time step between images is $\Delta t = 9.3$ ms. The sphere is coated with carbon soot.

Theoretically, we find an approximate analytical solution which describes the time evolution of the shape of the cavity from its creation to pinching. This approximate analytical solution predicts the existence of two very different cavities dynamics, one being characterised by a reduced depth evolution $H/R_0 \sim Fr^{1/2}$ and the other by the evolution $H/R_0 \sim Fr^{1/3}$. The reduced crater size is also different, one predicting $R/R_0 \sim Fr^{1/4}$, and the other $R/R_0 \sim 1$.

Both regimes have already been reported in the litterature in the case of the impact in water [Glasheen1996a] and in sand [Lohse2004]. Up to now, the two different scalings were associated to compressible effects. What we show is that both regimes can exist in the uncompressible limit, depending on the angle of the interface at the detaching point.

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VELOCITY FIELD CONSTRUCTION BASED ON LINEAR FINITE ELEMENT SOLUTION OF THE "DUAL FORMULATION" FLOW MODEL

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Key Words and Phrases: finite element discretization, "dual formulation" flow model, hydraulic head, streamfunction, advection-dispersion

AMS Subject Classification: 76M12,76S05

An accurate computation of flow in a heterogeneous isotropic formation is required in the description of contaminant advective transport. Solving the "dual formulation" flow model via linear finite element approximations leads to both hydraulic head and streamfunction distributions. Originated from the definitions of hydraulic potential and streamfunction, we present a method for velocity field construction using simultaneously linear hydraulic head and streamfunction solutions. The accuracy of our new approach is examined by three sets of numerical experiments on conservative advection-dispersion problem in highly heterogeneous formations, which demonstrate the advantage of our methodology over the traditional one that relies only on the linear hydraulic head solution.

VARIATIONAL INEQUALITIES INVOLVING PLN FUNCTIONS WITH APPLICATION TO EVOLUTION INCLUSIONS

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Let E be a separable reflexive Banach such that its strong dual is uniformly convex. We discuss the existence and the uniqueness of absolutely continuous solutions for the evolution inclusion

$$\begin{cases} 0 \in \dot{u}(t) + A(t)u(t) + F(t, u(t)), \\ u(0) = a \in D(A(0)); u(t) \in D(A(t)),\ \forall t \in [0, T] \end{cases}$$

where $A(t)$ is an m-accretive operator in E, $D(A(t))$ is the domain of $A(t)$, $F : [0, T] \times E \rightrightarrows E$ is a scalarly upper semicontinuous convex weakly compact valued mapping. As an application we prove the existence and the uniqueness of absolutely continuous solutions to

$$\begin{cases} 0 \in \dot{u}(t) + A(t)u(t) + \partial^p f_t(u(t)) \\ u(0) = x_0; u(t) \in \overline{B}_{\mathbb{R}^d}(x_0, r_0), \forall t \in [0, 1] \end{cases}$$

when $D(A(t)) = \overline{B}_{\mathbb{R}^d}(x_0, r_0)$, $\partial^p f_t$ is the proximal subdifferential of a nonconvex Lipchitzean pln function $f_t(.)$ defined on $\mathbb{R}^d$. We present a study of the preceding inclusion when $f_t(.)$ is l.s.c pln via new variational techniques in separable Hilbert spaces involving nonconvex l.s.c functions. Some existence results of periodic solutions in a class of evolution inclusions governed by the subdifferential of nonconvex l.s.c pln functions are also provided.

HIGHER ORDER BOUNDARY VALUE PROBLEMS ON TIME SCALES

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Key Words and Phrases: time scales, lidstone boundary value problem, upper and lower solutions, positive solutions

AMS Subject Classification: 39A10

In this study we are interested in the existence of positive solutions of the following Lidstone boundary value problem (LBVP),

$$(-1)^n y^{\triangle^{2n}}(t) = f(t, y^{\sigma}(t)), \qquad t \in [0,1], \tag{1}$$

$$y^{\triangle^{2i}}(0) = y^{\triangle^{2i}}(\sigma(1)) = 0, \qquad 0 \leq i \leq n-1 \tag{2}$$

where $n \geq 1$ and $f : [0, \sigma(1)] \times \mathbf{R} \to \mathbf{R}$ is continuous. We assume that $\sigma(1)$ is right dense so that $\sigma^j(1) = \sigma(1)$ for $j \geq 1$. Throughout this paper we let T be any time scale (nonempty closed subset of $\mathbf{R}$) and $[a,b]$ is a subset of T such that $[a,b] = \{t \in \mathrm{T} : a \leq t \leq b\}$. We shall state a fixed point theorem due to Schauder and we obtain existence of solutions for the LBVP $(1)-(2)$, also we prove the existence and uniqueness theorem for solutions which will lie between the lower and upper solutions when they are given in the well order, i.e. the lower solution is under the upper solution. Finally, we discuss the existence of a positive solution for the LBVP $(1)-(2)$ under $f_0 = 0, f_\infty = \infty$ or $f_0 = \infty, f_\infty = 0$.

A GENERALIZED LINEAR FRACTIONAL PROGRAM

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Key Words and Phrases: linear fractional, linear, regular set
AMS Subject Classification: 90C32

In this work we have developed an algorithm for solving a generalized linear fractional functionals programming problem. A generalized linear fractional functionals programming problem is a linear fractional functions programming problem in which there is some freedom in the choice of coefficients of an activity. Such problems arise when a system is being designed or when the input and output characteristics of a process depend on one or more parameters. Mathematically this problem is stated as

$$\text{Maximize } z = \frac{cx}{dx}$$

subject to

$$u_1x_1 + u_2x_2 \ldots + u_nx_n = b$$

$$x_j \geq 0, \ \ j = 1, 2, \ldots, n$$

Here c, $d \in R_n$ are fixed row vectors, b$\in R_m$ is a fixed column vector, and $x \in R_n$ is a column vector of variables; $u_j \in S_j$,and $S_j \subset R_m$ are polyhedrons.

In case the vectors u_j, are constants then this problem is same as a linear fractional functionals programming problem. Thus the problem considered here differs from a linear fractional functionals programming problem in the sense that column$u_j$are variables taking values from prescribed polyhedronsS_j. It is assumed that the problem has an optimal solution, which means that the feasible set of problem is regular i.e. nonempty and bounded.

It is shown that to arrive at the desired solution of problem one has to solve linear fractional functions programming problems and linear programming problems. A complete algorithm along with a numerical example is presented in this paper.

EULER EQUATIONS WITH NON-HOMOGENEOUS NAVIER SLIP BOUNDARY CONDITION

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Key Words and Phrases: Euler equations, flow through the boundary, vanishing viscosity, solvability

AMS Subject Classification: 35D05, 76B03, 76B47, 76D09

We consider the motion of an ideal fluid in a 2D-bounded domain, *admitting flows through the boundary* of this domain. The motion of the fluid in a domain $\Omega \subseteq \mathbb{R}^2$ is described by the Euler equations

$$\mathbf{v}_t + \operatorname{div}(\mathbf{v} \otimes \mathbf{v}) - \bigtriangledown p = 0, \qquad (\mathbf{x}, t) \in \Omega \times (0, T), \tag{1}$$

$$\operatorname{div} \mathbf{v} = 0, \qquad (\mathbf{x}, t) \in \Omega \times (0, T), \tag{2}$$

$$\mathbf{v}(\mathbf{x}, 0) = \mathbf{v}_0(\mathbf{x}), \qquad \mathbf{x} \in \Omega \tag{3}$$

with *non-homogeneous* Navier slip boundary conditions on the boundary Γ of the domain Ω:

$$\mathbf{v} \cdot \mathsf{n} = a, \qquad \mathbf{x} \in \Gamma \times (0, T), \tag{4}$$

$$2D(\mathbf{v})\mathsf{n} \cdot \mathsf{s} + \alpha \mathbf{v} \cdot \mathsf{s} = b, \qquad \mathbf{x} \in \Gamma^- \times (0, T). \tag{5}$$

Here $\mathbf{v}(\mathbf{x}, t)$ is the velocity of the fluid at $(\mathbf{x}, t) \in \Omega_T$; $p(\mathbf{x}, t)$ is the pressure; the tensor $D(\mathbf{v})$ is the rate-of-strain of the fluid's velocity $\mathbf{v}$; (n, s) is the pair formed by the outside normal and tangent vectors to the boundary Γ of Ω; Γ^- is the part of Γ, where $\mathbf{v}\,\mathsf{n} = a < 0$.

The results: *1) We establish the solvability of this problem (1)-(5) realizing the passage to the limit in the Navier-Stokes equations with vanishing viscosity;*

2) The solvability is proved in the class of weak solutions with L_p- bounded vorticity, $p \in (2, \infty]$;

3) It is shown that the weak solution satisfies the Navier slip boundary conditions (4)-(5).

EFFECT OF GRAVITY MODULATION ON STABILITY OF A HORIZONTAL DOUBLE-DIFFUSIVE LAYER

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Key Words and Phrases: double-diffusive instability, gravity modulation
AMS Subject Classification: 76R10

The instability characteristics of a horizonal stably stratified fluid layer being heated from below, including its subsequent nonlinear evolution under gravity modulation, have been investigated by experiments and two-dimensional numerical simulations. The fluid is contained in a horizontal test tank with an initial stable solute gradient and a constant-temperature gradient imposed by heating from below. Due to the non-diffusive boundaries, the vertical solute gradient slowly decreases and, eventually, the layer becomes unstable. From the time of the onset of instability, the critical solute Rayleigh number is determined. For the experiments with modulated gravity, the tank is fixed onto a platform that oscillates vertically at 1 Hz with an amplitude of 10 cm. The experiment is designed such that no internal wave mode of instability can be excited. The experimental results show that gravity modulation destabilizes the system slightly by increasing the solute Rayleigh number at onset by 8.4% and causes the oscillation frequency at onset to increase by 32.6%. Linear stability analysis and two-dimensional numerical simulations for the steady gravity case yield results that are in good agreement with the experiment. For the gravity modulation case, linear stability results do not show any effect of gravity modulation at the frequency of 1 Hz. Numerical simulations results show smaller increases in both the onset solute Rayleigh number and the oscillation frequency than those obtained in the experiment. The characteristics of the internal wave mode of instability and the interference effects between the internal wave mode and double-diffusive mode of instabilities are explored by numerical simulations. The financial support provided by NASA Grant NAG3-2354 for this research is gratefully acknowledged.

THE METHOD OF AVERAGING FOR SOLVING OF DIRECT AND INVERSE PROBLEM OF WAVE PROPAGATION

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In suggested approach the calculation of consecutive approximations of avera ging method starts with effective medium approximation. The determined fun ctions, describing the dependence of material coefficients of medium on space coordinates, are replaced by stochastic functions. By changing from determined to stochastic giving of material coefficients functions with the help of moments we calculate wave operator of effective medium. This operator defines integral properties of inhomogeneous medium. The finding of effective dynamic module is made with taking into account the multiple scattering in all wave length range.

In approximation of effective medium the structure of initial medium is des cribed integrally by correlation functions. First approximation of averaging method is made in form of superposition of plane waves.

The following approximations are found from the recurrent systems of effective medium equations with the right part, in which the deviation of given determi ned functions of material coefficients from effective values is taken into account.

The determination of physical and mechanical characteristic of medium by the scattered field measurement is of great interest for geophysics, hydrology, acoustics. There is a wide range of problems, connected with the reconstruction accuracy, taking account of multiply scattering and obtaining of stable solution. Various methods, which allow to receive evaluations for the reconstruction of field values, have been developed to solve inverse problems. The method of consecutive evaluation of physical and mechanical parameters of inhomogeneous medium, based on statistical variant of averaging method, is suggested. In the first approximation the reconstruction of macroparameters: expected value and correlation function, describing medium structure integrally, is realized. In the second and following approximations the characteristics of more fine structure of medium are determined.

ON ANISOTROPIC SINGULAR PERTURBATIONS PROBLEMS

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Let $\Omega = (-1,1)^2$. We would like to study the asymptotic behaviour of problems which model could be

$$\begin{cases} -\epsilon^2 \partial^2_{x_1} u_\epsilon - \partial^2_{x_2} u_\epsilon = f \quad \text{in } \Omega, \\ u_\epsilon = 0 \quad \text{on} \quad \partial\Omega, \end{cases}$$

when $\epsilon \to 0$ and show in particular that the solution converges toward the solution of the problem in lower dimension

$$\begin{cases} -\partial^2_{x_2} u_0 = f \quad \text{in } (-1,1), \\ u_0 = 0 \quad \text{on} \quad \partial\{-1,1\}, \end{cases}$$

with a local speed as big as we wish.

UNIFICATION OF A CLASS OF BILATERAL GENERATING FUNCTIONS FOR CERTAIN SPECIAL FUNCTIONS BY GROUP THEORETIC METHOD

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Key Words and Phrases: generating function, Laguerre polynomial, Bessel polynomial, Gegenbauer polynomial, Jacobi polynomial
AMS Subject Classification: 33C47

In this paper, a unified group-theoretic method of obtaining a class of bilateral generating functions involving some special functions has been suggested of course when suitable one-parameter continuous transformation groups can be constructed for those special functions.

In Section 2 of the paper, the method has been fully discussed and finally we have arrived at a conclusion in connection with the unification of a class of bilateral generating functions involving some special function,which is stated in the form of the following theorem.

Theorem : If

$$G(u,t) = \sum_{n=0}^{\infty} a_n \; p_m^{(n)}(u) \; t^n$$

then

$$\Omega(u,1) \; G\bigg(g(u,\ 1), wvh(u,\ 1)\bigg) = \sum_{n=0}^{\infty} w^n \; p_m^{(n)}(u) \; f_n(v)$$

where

$$f_n(v) \;=\; \sum_{q=0}^{n} a_q \; \frac{\prod_{i=0}^{n-q-1} \; C_{q+1}}{(n-q)!} \; v^q$$

and $p_m^{(n)}(u)$ is a special function of order m and of parameter n.

Finally in section 3 of the paper, we have obtained a good number of theorems and results on bilateral generating functions involving various special functions in course of application of the above theorem obtained in this investigation.

NUCLEAR CONTAMINATION IN A NATURALLY FRACTURED POROUS MEDIUM

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Key Words and Phrases: miscible compressible displacement; porous medium; fractured reservoir; double porosity; homogenization
AMS Subject Classification: 35B27, 35Q35, 76M50, 76N99

We consider a nonlinear coupled parabolic system arising in the flow of compressible nuclear waste disposal contamination in porous media. In particular the viscosity is concentration dependent. We adapt rigorously the model for a naturally fractured reservoir. The microscopic model consists of the usual equations describing Darcy flow in a reservoir except that the porosity and the permeability coefficients are highly discontinuous. Over the matrix domain, the coefficients are scaled by a parameter ϵ representing the size of the matrix blocks. This scaling preserves the physics of the flow in the matrix as ϵ tends to zero. Using homogenization theory, we derive rigorously the corresponding double porosity model. To this purpose, we mainly use the concept of two-scale convergence. The less permeable part of the rock then contributes as nonlinear memory terms. To specify them in spite of the strong nonlinearities and of the coupling, we then use a sequence of unfolding arguments.

TECHNOLOGICAL MODEL OF THE FLASH SMELTING FURNACE

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Key Words and Phrases: reaction kinetics, transport phenomena, mathematical modeling

On the base of general situations from the fluid mechanics and disperse systems, the technical thermodynamics, chemical reaction kinetics and transport phenomena are presented physicochemical and thermal processes in technologycal torch. The rate field is showed with method included mathematical processing of experimental data in a height of a physical model of the reaction shaft of a flash smelting furnace.

The information obtained from the applied model can be successfully used as part of a general model for control and research of the technologycal proses. It gives opportunity for tracing of basic parameters of gases and condensate phases.

DUAL-MESH CHARACTERISTICS IN PARTICLE-MESH METHODS

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Key Words and Phrases: Particle-mesh method, particle-in-cell (PIC) method, method of characteristics, dual-mesh characteristics

AMS Subject Classification: 65M06, 65M25

The particle-mesh method (PMM) is a powerful computational tool, particularly for the simulation of convection-dominated diffusion flows. The method introduces *computational particles* each of which is given a finite size and represents a large number of physical particles with similar properties. The convection part of the flow can be solved by moving the computational particles along the characteristics, while the diffusion part is carried out by utilizing a heat solver on a regular mesh. However, the method in practical applications shows the so-called *ringing instability*, an amplitude fluctuation in the computed solution. In this article, we suggest a novel numerical technique for particle movement, called the *dual-mesh characteristics* (DMC) of which the second mesh is formed by tracking back the cells along the characteristics. The particle movement is carried out by interpreting the particle positions (in the previous time level) in terms of the multi-linear coordinates of the second mesh. Strategies are also suggested in order to minimize numerical dissipation in the solution of convection-dominated diffusion flows. The resulting algorithm, *DMC-PMM*, turns out to be mass-conservative, non-oscillatory, of negligible dissipation, and more efficient than the conventional schemes. Numerical results are shown to demonstrate its accuracy and efficiency.

MATHEMATICAL MODELLING IN CONSTRUCTING KAMCHATKA REGIONAL TSUNAMI WARNING SYSTEMS

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Catastrophic tsunamis that crushed upon the ocean coast in the past had taken many human lives and destroyed the infrastructure of the coastal areas in several countries in Asia and in the Pacific. Recent tragic events motivated governments to develop new and improve the existing tsunami warning systems capable of mitigating the impact of catastrophic events. In future regional tsunami warning systems will be integrated into a network including both the systems currently being developed and the existing warning systems (in the past tsunami warning systems had been deployed to protect the coastal areas in Japan, USA, Australia, Chile and New Zealand).

We present the work conducted by the Institute of Computational Technologies in collaboration with other research institutes in Novosibirsk within the project to design a new generation of the tsunami warning system for the Pacific coast of Kamchatka. The aim of the project is to develop a technology that will allow us to build a database of possible occurrences of disastrous waves along the coast in a series of numerical experiments. In the first stage of the project a collection of basic model sources of tsunamigenic earthquakes will be defined. The model sources will be used to calculated initial elevation fields on the ocean surface. The next stage will employ numerical modelling to model the propagation and transformation of tsunami waves on the way from the source towards the coast. This information will be presented as a decision support system used by persons responsible for initiating disaster mitigation procedures such as evacuation of people and sending ships away from the dangerous areas of the coast.

The project will involve numerical solution of a large number of instances of wave hydrodynamics problems. At the same time interpretation of the results will require non trivial postprocessing and building specialized information systems. Numerical solution of wave hydrodynamics problems for multiple combinations of parameters of the earthquake source with high spatial resolution constitutes the major part of the computational requirements of the project. The amount of computations needed implies the use of high performance computers and may require adaptation of numerical algorithms to specific computing platforms.

In the future this approach may be extended to other areas of the basins of the Pacific and the Indian ocean. In the latter case we plan to consider the problem of finding the zones of the coast that carry a high level of risk of being affected by tsunamis. This would require to perform run-up calculations. To achieve the necessary precision we will need to take into account such features of the coastal line as small rivers, lakes and swamps. This will significantly increase the amount of computational power needed both during the development of numerical models and algorithms and in the production runs.

MATHEMATICAL MODELING AND NUMERICAL ANALYSIS OF MODELING AND APPLICATION OF NON-STANDARD LAWS IN THE SYSTEMS OF AUTOMATIC CONTROL

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AMS Subject Classification: 65k10, 90c90

In this paper we consider the non-standard laws in systems of automatic control, especially its mathematical descriptions and applications. The base of these laws are the classical continuous laws of control. Synthesis of controllers with non-standard laws is come down to modules, which contain logical functions and determinate mathematical operations. We set out its advantages in the case of technological processes with complete dynamic characteristics.

NONLINEAR DAMPING VIBRATIONS F TWO BINDING PENDULUMS

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The statement of fundamental problems and solution methods in a class of nonlinear dynamic systems are studied in the present paper. Theorical results are applied to the vibrations of two binding pendulums with viscoelastic spring. The effect of viscosity coefficients on the character of the solution is investigated. The conditions of stability, asymptotic stability and instability of the solutions are obtained and the orbits of the motion for the different values of parameters are driven by using Maple 10 program.

STABILITY ESTIMATES FOR AN INVERSE PROBLEM ASSOCIATED TO THE LINEAR BOLTZMANN EQUATION

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Key Words and Phrases: inverse problems, albedo operator, stability estimates
AMS Subject Classification: 35R30, 83D75

This work presents some stability estimates for the absorption and scattering coefficients in the inverse problem associated to the linear Boltzmann (Transport) equation

$$\frac{\partial u}{\partial t} + v \cdot \nabla_x u + q.u = \int_V f(x, v', v).u(t, x, v)dv',$$

related to the ALBEDO operator.

GRÖBNER BASES AND SOME OF THEIR APPLICATIONS

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Key Words and Phrases: Gröbner basis, polynomial equation, generalised Pell equation, permutation polynomial, Chebyshev polynomials
AMS Subject Classification: 13P10, 11D09, 12D45

Constructivism was present in mathematics since its inception. The advent of powerful computers in the last decades made this approach more appealing than ever. Effective methods feature nowadays a prominent role throughout mathematics. These techniques are appreciated because of the lucidity they bring to proofs and the countless applications in other sciences or industry. As large part of mathematics and a significant proportion of applications require solving polynomial equations, nowadays one currently makes use of several methods to explicitly find solutions of multivariate polynomials. One method allowing for exact representation of solutions is based on Gröbner bases computations. Gröbner bases encode structural information on the objects they describe. As soon as a Gröbner basis is known, a wealth of information on the object of interest is immediately available.

The first part of the talk will contain a brief description of the relevant definitions, results and algorithms. Then we present in some details several applications for Gröbner bases that have in common the presence of Chebyshev polynomials.

THE EXISTENCE AND APPROXIMATIONS OF COMMON FIXED POINTS OF MULTI-VALUED AND SINGLE-VALUED MAPPINGS

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Key Words and Phrases: convex metric space, non-self multi-valued mapping, common fixed point
AMS Subject Classification: 47H10, 54H25

Extending the Banach contraction principle, in 1968 Nadler and Markin first initiated the study existence of fixed points of multi-valued contraction self-mappings. In this exposition some results on coincidence and common fixed points of a pair of multi-valued and a pair of single-valued non-self mappings in complete convex metric spaces will be presented. By improving the earlier used methods of proofs, in [6] we obtain results for not necessarily compatible and not necessarily continuous mappings, which generalize some of the known results. In particular, a theorem by Rhoades [9] and a theorem by Ahmed and Rhoades [1] are generalized and improved.

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CONDITIONAL UNCERTAINTY MEASURES: A GENERAL APPROACH

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Key Words and Phrases: uncertainty measures, conditional events, conditioning
AMS Subject Classification: 60A05, 60A99, 28E99

We set up "reasonable" axioms for general conditional measures of uncertainty starting from a family $\mathcal{T}$ of *conditional events* $E|H$, each one being represented by a suitable *three–valued random variable* whose values are $1, 0, t(E|H)$. The latter turns out to be the relevant conditional measure $\varphi(\cdot|\cdot)$. In fact, given two commutative, associative, and increasing operations $\oplus$ and $\odot$ from $\mathbb{R}^+ \times \mathbb{R}^+$ to $\mathbb{R}^+$, we define suitable operations among the random variables of $\mathcal{T}$: the "result" is a random variable that does not, in general, belong to $\mathcal{T}$; then, if we consider only those elements of $\mathcal{T}^2$ such that the range of each operation is $\mathcal{T}$, we get conditions on $t(E|H)$ that can be taken as the "natural" axioms for a conditional measure. Then different (decomposable) conditional measures can be obtained by particular choices of the two operations $\oplus$ and $\odot$: for example, choosing ordinary sum and product, we get conditional probability. The main feature of this approach resides in the *direct* introduction of the conditional measure $\varphi(\cdot|\cdot)$ as a function whose domain is an arbitrary set of conditional events, so that it can be defined for any pair E, H, with $H \neq \emptyset$, and the knowledge (or assessment) of the "joint" and "marginals" unconditional measures $\varphi(E \wedge H)$ and $\varphi(H)$ is not required. Obviously, if the latter were already given, there must exist suitable rules that put them in relation with $\varphi(E|H)$, but the converse is not necessarily true! In particular, there is no need, as in the usual approaches (where the conditional measure is introduced by *definition* as a suitable function of the two aforementioned unconditional measures) of any specific assumption (e.g. the requirement, in the case of probability, of positivity for the measure of the conditioning event). Moreover, we search the minimal (necessary and sufficient) conditions on $\oplus$ and $\odot$ which render a conditional measure $\varphi(\cdot|\cdot)$ "similar" to a conditional probability, in the sense that it can be represented in terms of *classes* of *unconditional* uncertainty measures.

FAILURE OF MATERIALS CONTAINING NANOVOIDS

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Key Words and Phrases: fracture mechanics, atomistic simulations, nanostructured materials
AMS Subject Classification: 74R10 Brittle fracture

Cracks and voids - naturally formed during processing or introduced by design - affect the mechanical behaviour of brittle solids since they modify the overall materials strength. Such a feature is traditionally described by solid mechanics according to either *stress intensification* or *stress concentration* arguments. Both criteria unlikely work at the nanoscale since at least one among the continuum, or elasticity or linearity hypotheses may not hold.

In order to improve classical continuum models, modern theories of fracture are generally formulated so as to incorporate into their formalism a suitable material length scale, aimed at describing a process zone (nearby the crack tip) where at least one of the above constitutive hypotheses fails.

In this work we use atomistic simulations to investigate the failure criteria of a brittle material containing nanovoids. We compare different continuum models with the calculated failure strength in nano-defected crystalline β-SiC and we provide an estimate for the above process zone in the case of a crack, a cylindrical hole, and a spherical void.

Atomistic simulations here work as an "*ab initio* mechanical theory", which is able to discriminate among different fracture models since it is not based on a *a-priori*-guessed constitutive equation for materials behavior.

This work has been done in collaboration with M. Ippolito and A. Mattoni (Cagliari, Italy), and N. Pugno (Torino, Italy). Financial support by MiUR through the project "PON-CyberSar" and by INdAM "F. Severi" through the project "Mathematical Challenges in Nanomechanics" is warmly acknowledged.

IDENTIFICATION OF IMMERSED OBSTACLES VIA BOUNDARY MEASUREMENTS

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This Conference deals with the study of an inverse geometric problem in fluid mechanics. In particular, we are interested in the identification of a rigid body which is immersed in a cavity, filled with a fluid, by means of measurements of the Cauchy forces and the velocity of the fluid on one part of the exterior boundary. We will show an identifiability result, and using the so-called shape differentiation a stability result is obtained. Finally we present a numerical scheme which allow us to recover the rigid body in case of spherical and ellipsoidal geometries.

More precisely, we consider the following inverse problem: an inaccessible rigid body D is immersed in a viscous fluid, in such a way that D plays the rôle of an obstacle around which the fluid is flowing in a greater bounded domain Ω, and we wish to determine D (i.e. its form and location) via boundary measurement on the boundary $\partial\Omega$. Both for the stationary and the evolution problem, we show that under reasonable smoothness assumptions on Ω and D, one can identify D via the measurement of the velocity of the fluid and the Cauchy forces on some part of the boundary $\partial\Omega$. We show also that the dependence of the Cauchy forces on deformations of D is analytic, and give some stability result for the inverse problem.

RESOLVENT EQUATIONS FOR GENERALIZED MULTIVALUED NONLINEAR MIXED VARIATIONAL INCLUSIONS

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Key Words and Phrases: generalized multivalued nonlinear mixed variational inclusion; resolvent equation; iterative algorithm

In this paper, we study the existence and convergence properties of the generalized multivalued nonlinear mixed variational inclusion problem for finding the approximate solution. We established the equivalence between the varitional inclusion and the general resolvent equations, obtained three iterative algorithms, provided the convergence analysis of the algorithms. The results obtained in our paper improved and generalized a number of resent results.

Project support by the National Natural Science Foundation of China (No. 1987 10 48.), Natural Science Foundation of Jiangsu province (06KJD110072).

RESOLVENT EQUATIONS FOR GENERALIZED MULTIVALUED NONLINEAR MIXED VARIATIONAL INCLUSIONS

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A COMPERATIVE STABILITY ANALYSIS OF INTEGRATION SCHEMES APPLIED TO TEMPERATURE AND TIME-DEPENDENT GINZBURG-LANDAU MODEL OF SUPERCONDUCTIVITY

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We give conditions under which both Forward Euler and Modified Forward Euler method, which we proposed in a previous work, applied to Temperature and Time-dependent Ginzburg-Landau model of superconductivity(TTDGL), are stable.The practicality of the conditions are illustrated graphically where a small deviation from the proposed conditions results in undesirable results. TTDGL is also integrated with several schemes within the MATLAB ODE Suite and the results are compared and analyzed

THE DIRICHLET AND FRANKL PROBLEMS FOR MIXED TYPE EQUATIONS

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We consider the Dirichlet and Frankl problems for the Tricomi equation

$$(\Lambda-\lambda)u \equiv q(\eta)u_{\xi\xi}+u_{\eta\eta}+c_1(\xi,\eta)u_\xi+c_2(\xi,\eta)u_\eta+c(\xi,\eta)u-\lambda u=f,$$

$$\lambda\in R,\ q\in C^\infty(-\infty,\infty),\ q(0)=0,\ q'(\eta)>0 \text{ for } \eta\in(-\infty,\infty),$$

c_1, c_2, $c\in C^\infty(\overline{\Omega})$, in a plain region Ω, and their multidimensional analogs in a region $G\subset R^{m+1}$ for second order mixed type equations

$$(L-\lambda)u \equiv k(t,x)u_{tt}+2b_i(t,x)u_{tx_i}+b(t,x)u_t+$$
$$+(a_{ij}(t,x)u_{x_i})_{x_j}+a_i(t,x)u_{x_i}+(a(t,x)-\lambda)u=f,$$
$$\lambda\in R;\ k,\ b_i,\ b,\ a_{ij},\ a_i,\ a\in C^\infty(G);\ a_{ij}=a_{ji};$$
$$a_{ij}(t,x)\xi_i\xi_j\geq\theta\xi^2,\ \forall\xi\in R^m,\ (t,x)\in G,\ \theta=const>0,$$

summation from 1 to m is presumed with respect to repeated indices.

Sufficient conditions are found for the low order terms of the equations that imply existence and uniqueness of the strong solutions of the boundary value problems. Then the regularity of the solutions in the scale of Sobolev spaces is studied. We find conditions on the coefficients before first order derivatives that are close to the necessary ones, under which we have that the solution $u\in W^{l+1}(G)$ when $f\in W^l(G)$, $l\geq 0$ is integer. We clarify the influence of the low order terms for the wellposedness of the Dirichlet or Frankl boundary value problem and the regularity class of the solution.

A NON-LINEAR SCHRÖDINGER TYPE FORMULATION OF FLRW AND BIANCHI I&V SCALAR FIELD COSMOLOGIES

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Key Words and Phrases: Einstein field equations, Friedmann-Lemaître-Robertson-Walker, Bianchi I, Bianchi V, scalar field, perfect fluid, Ermakov-Milne-Pinney equation, non-linear Schrödinger equation

AMS Subject Classification: 83C05, 83C15

Recently there have been interesting reformulations of Einstein's field equations for scalar field cosmologies, both for isotropic and anisotropic models, in terms of generalized types of Ermakov-Milne-Pinney equations. Inspired by this work, we have discovered an alternative Schrödinger formulation of Einstein's equations in a Friedmann-Lemaître-Robertson-Walker universe. This provides for an alternate method of obtaining exact solutions of the field equations. After presenting this initial work, I will further demonstrate analogous Schrödinger models that have subsequently been found for both Bianchi I and Bianchi V cosmologies.

GREEN FUNCTION METHOD IN PHOTONIC CRYSTALS AND CLUSTERS

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Key Words and Phrases: integral-equations, Green's functions, self-consistent calculation

Diffractive phenomena are at basis of many interesting optical properties shown by the light propagation in complex systems; moreover, take a crucial role in the tailoring of optical devices, allowing the control of the local photon density of states. The light propagation in spatially periodic systems have been studied at very beginning of modern optics, when anomalous propagation in one dimensional optical grating was studied by R.W.Wood and J.W.S.Rayleigth (R.W.Wood, Philos.Mag.v.4 , pp369, 1902). Recently, photonic crystals and amorphous photonics, suggested by E.Yablonovich and S.John (E. Yablonovich Phys.Rev.Lett.58, pp.2059 , 1987) some years ago, are obtained by dielectric function modulation at micro or nano scale of composite dielectric materials, and interesting optical properties, strongly different from those of the corresponding bulk components, are observed. These systems have demonstrated all their optical potentialities for device realization due to their strong light localization (photonic gaps or quasi-gaps) and the scalability of their optical properties. From the theoretical point of view , in a semiclassical framework, photonic crystal and clusters computation requests to solve self-consistently the coupled Schroedinger-Maxwell equations in order to study the linear and non-linear optical properties of mesoscopic material system. Therefore, the propagation properties of polarization waves is usually computed by transforming integral-differential Maxwell equations in integral equations by Green function method. The main concepts on complex optics in mesoscopic materials are highlight by solving selected numerical examples, and a non exhaustive list of interesting optical effects, namely: nanomirror, super-radiance, Rabi energy enhancement, electric field localization, optical impurities and bands, are illustrated by computing the optical response, the local electromagnetic fields and the dispersion curves in many different spatially periodic systems and clusters. The effect of disorder on the optical properties of these systems is also briefly discussed.

STABILITY BOUNDS FOR N-LAYER HELE-SHAW FLOWS

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Key Words and Phrases: fluid dynamics, hele shaw flows, linear stability
AMS Subject Classification: 76

This talk is concerned with the stability analysis of N-layer Hele-Shaw flows in rectilinear channels. First, we will present an improved (tighter) upper bound on the growth rate of hydrodynamic disturbances in the case of 3-layer with constant-viscosity layers. We will present generalization of this result for the $N-$layer case. The $N-$layer problem is intricately more complicated to analyze for stability even when each layer has constant viscosity. Using a weak formulation, we will obtain in this talk upper bounds on the growth rate of hydrodynamic disturbances for such constant-viscosity $N-$layer flows. The practical use of these results will also be emphasized.

When each layer has variable viscosity, the $N-$layer case becomes very complicated even when $N = 4$. The upper bound problem on the growth rate for the case $N = 3$ is completely solvable here for arbitrary viscous profiles. For the four-layer case, result on the upper bound will be presented when at least one of the layer is individually stable. Results on more than 4-layer case here seems to be very difficult. The difficulties will be explained. This is a joint work with Gelu Pasa.

NONLINEAR STABILITY OF HELE-SHAW FLOWS WITH SMOOTH VISCOUS PROFILES

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Key Words and Phrases: fluid dynamics, spectral theory, Hele-Shaw Flows, non-linear instability
AMS Subject Classification: 76

In this talk, we will rigorously derive nonlinear instability of Hele-Shaw flows moving with a constant velocity in the presence of smooth viscosity profiles where the viscosity upstream is lower than the viscosity downstream. This is a single-layer problem without any material interface. The instability of the basic flow is driven by a viscosity gradient as opposed to conventional interfacial Saffman-Taylor instability where the instability is driven by a viscosity jump across the interface. The proof of nonlinear instability in this paper consists of the following steps.

(1) A variational characterization of the spectral problem for the unstable viscosity profile. The hyperbolic type of systems arising in mathematical physics may have continuum spectra (for example linearized Euler equation has a continuum spectrum) which makes it difficult to estimate the complicated spectra and the spectral radii [1]. In particular, only point or discrete spectrum estimate may not be sufficient for the estimate of the spectral radius due to the possible presence of the continuum spectrum. Therefore, a variational characterization of the spectral radius by eigenvalue estimates, not of eigenvalues only, has been used to locate a *dominant* eigenvalue from the spectrum. A similar approach has been used by Hwang and Guo [2] in the context of Rayleigh-Taylor instability.

(2) Construction of higher-order approximate solutions by solving an equation for approximate evolution of the dominant growing eigenmode in powers of initial amplitude. This method is similar in spirit to the one introduced by Grenier [3]. This step also has subtlety since one may encounter severe higher-order perturbations, unbounded in L^2 norms for instance. Although it is natural in the formal sense, it is not obvious that we could really construct such approximate solutions if we did not have a dominant eigenvalue. However, a dominant eigenvalue obtained from the analysis in the crucial step 1 allows control of higher-order perturbations in H^s norms for all $s \geq 3$.

(3) Showing that the actual solution remains close to the exponentially growing approximate solution of step 2 up to a time that scales logarithmically with initial amplitude but for times smaller than possible blow-up time of actual solution via a delicate bootstrap argument which was introduced by Guo and Strauss [4].

This is a joint work with Hyung Ju Hwang.

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SCATTERING DIAGRAM OF THE WAVE FROM A LOCAL DEFAULT OF PERIODICITY OF AN IMPEDANCE-STRIP REFLECTING GRATING

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The incident wave falls on an impedance-strip grating and the reflected wave propagates from the grating at infinity. The periodic structure of the grating is followed by the special structure of the reflected wave which can be considered consisting of finite number of propagating plane waves outgoing from a grating under different angles at infinity and of infinite number of exponentially vanishing waves. Such a behavior is standard for different types of reflecting gratings. However, provided a periodic structure of a diffraction grating is violated by a local default of periodicity (which we call "defect") the far field pattern changes. If sizes of a defect are relatively small, the interaction of the incident and reflected waves with the defect should cause also another type of wave. This wave propagates from the small defect and is similar to a wave propagating from a point source placed at a point of the defect. So the small defect could be considered as an imaginary source with the amplitude specified by the incident and reflected from the grating waves and depending on some integral characteristics of the small scatterer. The Dirichlet and impedance boundary conditions are considered on the defect.

We study the periodic grating with plane boundary, consisting of impedance strips with different surface impedances.[1] The value of the Green function of the periodic grating at hand can be computed at each point over the reflective grating provided it is known on the surface of the grating. The Green function on the surface solves the the Fredholm type integral equation of the second kind with the logarithmic singularity of the kernel. Having computed the Green function, one can calculate the leading term of the asymptotics of the scattering diagram of the wave from a small defect (default of periodicity). This characteristics is of prior importance for engineering practice.

In the case of the Dirichlet conditions the analysis of the resulting formula shows that the wave field scattered from the defect may have resonant behaviour, which means that the scattered field from the defect may have resonant increasing of the scattering pattern for some combinations of the parameters in the problem at hand.

[1]We assume that cylindrical symmetry holds so that there is no dependence on the coordinate along the impedance strips and the problem is actually two dimensional.

AN OPERATOR-CALCULUS APPROACH TO REDUCING LINEAR ODE/PDE PROBLEMS TO INTEGRAL EQUATIONS AND SOME OF ITS APPLICATIONS I: INITIAL-VALUE PROBLEMS FOR ODE

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Key Words and Phrases: differential equation, ordinary, partial, initial value, boundary value, integral equation, system of equations, operator calculus, lifting isomorphism

In a sequence of several papers, of which this is the first one, we shall consider a general approach to reduction of initial-value and boundary-value problems for (systems of) linear ordinary and partial differential equations with possibly non-smooth variable coefficients, right-hand side and initial/boundary value data, to (systems of) linear integral equations of Volterra (quasi-nilpotent) and Fredholm type. In the case when the ODE/PDE problems are non-degenerating and conserve their type (depending on the coefficients of the highest-order derivatives) the respective integral equations are of the second kind. In the case of degeneration/type-changing of the ODE/PDE, the respective integral equations are of the third kind. The operator calculi used are based on lifting isomorphisms between spaces of smooth functions (such as, e.g., Sobolev, Besov and Lizorkin–Triebel spaces) and spaces of integrable functions (such as Lebesgue and Lorentz spaces). This approach provides a somehow simpler and more explicit way to reduction of ODE/PDE problems to integral equations, compared to other reduction approaches known to us. We discuss a number of applications to linear and some nonlinear problems. In the present first work on this topic, we consider the simplest case: initial-value problems for ODE, when the respective integral equations are of Volterra type of the second and third kind.

K-FUNCTIONALS BETWEEN SEMI-HILBERT SPACES AND UNIFORMLY CONVEX BANACH SPACES WITH CONSTRAINTS

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Key Words and Phrases: Hilbert space, semi-Hilbert space, uniformly convex Banach space, K-functional, real interpolation functor, complex interpolation functor, differentiable norm, Fréchet derivative, variational calculus, Lagrange multiplier, canonical isometry, Hilbert adjoint operator, selfadjoint operator, unitary operator, strongly continuous operator semigroup, unbounded operator, densely defined operator, closed operator, operator range, Radon-Nikodým theorem, Borel-Stieltjes measure, Radon measure, Lebesgue-Stieltjes measure, projection-valued measure, spectral measure, spectral resolution of unity/identity, integral spectral representation, spectral asymptotics, Sturm-Liouville problem, Lebesgue space, Sobolev space, integral modulus of smoothness, sharp constant of embedding/equivalence

AMS Subject Classification: 46C05, 46M35, 47J30, 41A17, 41A44, 41A65, 46B04, 46C07, 46C05, 46C15, 46E35, 47A30, 47B15, 47B25, 47N99

This work is a continuation of the research done in [1] about explicit computation of a general class of Peetre K-functionals. While [1] was dedicated to the computation of 'classical' K-functionals via 'unconstrained' variational calculus, in the present study we consider the more complex case of K-functionals with a broad class of additional constraints, and discuss their explicit computation using 'constrained' variational calculus and optimal control with constraints on both the phase variables and the control parameters. In [2-4] J. Löfström studied real interpolation spaces generated by K-functionals between Hilbert spaces with some types of constraints up to isomorphism (norm equivalence). In the present study we shall consider K-functionals between spaces belonging to more general classes (including the category of all semi-Hilbert spaces), for more diverse types of constraints, and involving exact computation of the K-functionals – up to isometry (norm equality). In more detail, we discuss all topics on constrained K-functionals, as outlined in the program for future research proposed in Section 1 of [1], as follows:

- *K-functionals with equality constraints.* The results of [2,3] on constrained real interpolation of spaces with equality constraints can be sharpened and improved, at least in the case of Hilbert spaces and linear constraints, if the variational approach to the computation of the K-functional developed in [1] be extended to handle equality constraints by using *Lagrange multipliers.*
- *Constraints of type 'equality of functionals'.* This is the type of constraints which has been considered in [2,3]. Within the framework of the variational approach proposed here, this type of equality constraints correspond to *isoperimetric* constraints which are

incorporated in a standard way in the optimization via Lagrange multipliers which are *scalars.*

• *Constraints of type 'operator equality'.* This type of constraints has not been studied in [2,3], although this type of constraints have very important applications in the context considered in [2,3]. In the framework of the variational approach proposed here, this type of constraints can also be incorporated in a standard way in the optimization via Lagrange multipliers which in this case are *elements of the space (if the space is Hilbert or semi-Hilbert) or elements of the dual space (in the more general case when the space is a reflexive Banach or complete semi-normed space).* The results obtained for the Hilbert case can also be used as a heuristic guide towards extending the general, non-Hilbertian, case of the results in [3] to include also operator equality constraints, under some natural restrictions on the operators involved in these equalities, such as, e. g., closedness.

• *Constraints of type 'inequality'.* In principle, the variational approach allows also the incorporation of inequality constraints into the optimization. Inequality constraints of both functional and operator type can be included. In the case of functional inequalities, these are described with usual numerical inequalities; in the case of operator inequalities, these refer to the positive cone of the respective space. This can be used to further upgrade the theory developed in [3] and extend its applications to boundary problems for *variational inequalities.*

• *More than 2 spaces in the K-functional.* The above program can be extended also for the more general case of a K-functional between more than 2 spaces (see [4]) with constraints, with further extension of the results of [3].

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ATOMIC DECOMPOSITION OF WIENER AMALGAM SPACES

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Key Words and Phrases: atomic decomposition of a function space, unconditional base, Riesz base, Wiener amalgam, integral modulus of smoothness, averaged modulus of smoothness, global space, local space, A-space, Besov space, Wiener amalgam, K-functional, interpolation space, real interpolation functor, complex interpolation functor, closedness of a space scale with respect to an interpolation functor

AMS Subject Classification: 42C40, 46B07, 46B42, 46B70, 46E05, 46E30, 46E35, 46E40, 46M35, 47A57

In the 1980-s, as a generalization and extension of some ideas of Norbert Wiener (see, e.g., [1]), Hans-Georg Feichtinger introduced the concept of Wiener amalgam spaces and initiated the study of their properties. This study, truly to the original ideas of Wiener, has mainly been focusing on applications to harmonic analysis. For this type of applications it was not essential to specify the size of the window of the local space in the definition of the Wiener amalgam. Earlier and parallel to this, and without relevance to the above-said applications, the so-called averaged moduli of smoothness (or τ-moduli) were introduced and studied by the Bulgarian school in approximation theory and some other researchers, in connection to approximation problems. To the best of our knowledge, the close connection between Wiener amalgam spaces and the function spaces generated by the averaged moduli has so far been largely ignored, most probably, due to the following three reasons: 1) the ranges of applications of the two concepts have so far been almost disjoint; 2) the techniques related to τ-moduli were fairly different, because in this case it was essential to relate the size of the window of the local space with the resolution tolerance/step of the approximation method; 3) formulating the approximation problems in terms of one-sided approximation was, somehow, concealing the relevance to the Wiener amalgam space approach. The first signs of similarity were noted when attempts to diagonally interpolate Wiener amalgam spaces failed, similarly to the unsuccessful attempts to diagonally interpolate the A-spaces introduced by V. A. Popov (which are analogues of Besov spaces, with the integral moduli of smoothness being replaced by their averaged analogues). The first time when τ-moduli were represented as Peetre K-functionals between Wiener amalgam spaces was in [2,3], thus obtaining K-functionals involving spaces (Wiener amalgams) whose window of the local space was dependent on the step of the K-functional. As a consequence of this new viewpoint, in [2,3] was made a motivated conjecture that the A-spaces are not closed with respect to diagonal interpolation, and a method was proposed to interpolate directly between K-functionals rather than between the A-spaces induced by these K-functionals. Relatively recently, Krugljak and Matvejev proved in [4] that the aforementioned conjecture in [2,3] about the A-spaces is correct. In [5,6] we showed that it is possible to construct appropriate function norms of certain Wiener amalgam spaces which are based on coefficients of wavelets or other Riesz bases of atoms in the global space of the Wiener amalgam. In the present paper we extend and generalize the relevant results from [5,6], and discuss their relevance to the idea of proof in [4], as well as the possibility to extend and generalize the proof in [4] to more general Wiener amalgam spaces.

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GLOBAL NON-SMOOTH CONSTRAINED OPTIMIZATION VIA ASYMPTOTIC EXPANSION OF INTEGRALS DEPENDING ON A LARGE PARAMETER, I: COMPUTATION OF THE EXTREME VALUE

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Key Words and Phrases: non-smooth non-convex global optimization with non-smooth non-convex constraints, Integral representation of essential suprema and infima, Asymptotic expansion of integrals depending on a large parameter, Radon-Nikodym derivative, Frechet derivative, Morse's lemma, G-convergence, decreasing rearrangement, Edgeworth asymptotic expansion, Steepest descent, Saddlepoint approximation and asymptotic expansion, Laplace-type integrals, Integration on manifolds, Measures on manifolds, Stokes theorem for differential forms on smooth manifolds, Operator calculus over functions depending on large parameter, Best approximation, Operator norm, Minimax theory of non-differential games, Non-smooth non-convex optimal control problems
AMS Subject Classification: 28C15, 28C20, 41A50, 41A60, 41A80, 42B10, 42C40, 43A32, 44A05, 44A10, 44A35, 44A40, 46B70, 46G05, 47G10, 49N70, 49N99, 53C65, 53C65, 58E99, 58A10, 58C20, 58C25, 58C35, 65R10, 65T50, 65T60, 90C47, 90C56, 91A23

In [1] we made the first announcement, with detailed exposition and outline of some key proofs, of a very general method for finding the global extremum and extreme set (set of values of the argument for which the extremum is attained) of a (generally non-smooth non-convex) real-valued functional over a (generally non-smooth non-convex) constrained subset of its definition domain, provided that the extremum and extreme set exist. If the extremum and extreme set do not exist, the method yields a generalization of the concepts of extremum and extreme set, in a sense specified in the exposition and related to the essential supremum or infimum of the functional and its Lebesgue points with respect to a certain finite complete measure.

Unlike gradient-based methods where the extreme value is computed after finding the extreme set, in the new method proposed here the extreme value and the extreme set can be computed independently of each other via respective asymptotic expansions.

The present paper is a detailed exposition, with detailed proofs, of the first part of the results announced in [1], related to the computation of *the extreme value* of the functional by asymptotic expansion of integrals depending on a large parameter.

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GLOBAL NON-SMOOTH CONSTRAINED OPTIMIZATION VIA ASYMPTOTIC EXPANSION OF INTEGRALS DEPENDING ON A LARGE PARAMETER, II: COMPUTATION OF THE EXTREME SET

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In [1] we made the first announcement, with detailed exposition and outline of some key proofs, of a very general method for finding the global extremum and extreme set (set of values of the argument for which the extremum is attained) of a (generally non-smooth non-convex) real-valued functional over a (generally non-smooth non-convex) constrained subset of its definition domain, provided that the extremum and extreme set exist. If the extremum and extreme set do not exist, the method yields a generalization of the concepts of extremum and extreme set, in a sense specified in the exposition and related to the essential supremum or infimum of the functional and its Lebesgue points with respect to a certain finite complete measure.

Unlike gradient-based methods where the extreme value is computed after finding the extreme set, in the new method proposed here the extreme value and the extreme set can be computed independently of each other via respective asymptotic expansions.

The present paper is a detailed exposition, with detailed proofs, of the second part of the results announced in [1], related to the computation of *the extreme set* of the optimized functional by asymptotic expansion of integral operators depending on a large parameter. The computation of the extreme set via asymptotic methods is a much more complex and subtle problem than the computation of the extreme value (see part I), because now we have to compute - exactly or approximately - the characteristic function of a set, i.e., this problem is non-parametric in its essence, while computing the extreme value is essentially a parametric problem. For example, in the case of the extreme set, uniformness of the asymptotic expansion and other more advanced functional-analytic and operator-theoretical issues have to be addressed.

The spectrum of these integral operators must be (at least) dense on the domain of the optimized functional. A general way to generate such appropriate integral operators is by integral calculi related to operators with continuous spectra, such as the Fourier and Laplace transforms, and operators generated by continuous frames. From the point of view of numerical approximation, of particular interest are Rademacher-type bases and, in particular, Haar wavelets and their various generalizations.

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RELATIONS BETWEEN FUNCTIONAL NORMS OF A NON-NEGATIVE FUNCTION AND ITS SQUARE ROOT ON THE POSITIVE CONE OF BESOV AND TRIEBEL-LIZORKIN SPACES

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Key Words and Phrases: positive cone, isoperimetric constraint, shape-preserving, one-sided approximation, Besov space, Triebel-Lizorkin space, Hellinger metric, wavelet, embedding theorem, interpolation functor, pointwise multiplier

AMS Subject Classification: 41A29, 46B70, 46E35, 62G05, 62G07, 62G08

Wavelets have remarkable approximation properties, but the very essence of the construction of orthogonal or biorthogonal scaling functions and mother wavelets prevents the wavelet approximants from retaining certain shape-preserving properties such as positivity, monotonicity, k-monotonicity, one-sidedness of the approximation, etc. Wavelet approximation does not obey 'per se' also isoperimetric constraints such as preserving the value of their integral and higher-order moments. On the other hand, many engineering problems where wavelets find applications require preservation of some of the afore-mentioned types of constraints. One example is probability density estimation for speech recognition where the approximation is desirable to be a density itself, i.e., to be non-negative and to have integral equal to one. Another example is non-negative non-parametric regression-function estimation in positron-emission tomography (PET) imaging. In [1,2] shape-preserving statistical density estimators were proposed by considering $\sqrt{f} \in L_2$, where $f \in L_1$ is an unknown density. For this approach, optimal estimation rates of the risk were obtained under the assumptions that $\sqrt{f}$ belongs, more specifically, to certain function spaces with additional smoothness, continuously embedded in L_2. Since usually information is available about the smoothness of f itself, this required the study of the relations between the smoothness of f and $\sqrt{f}$. In this communication we study in detail these relations in the case when the smoothness of f is measured via Besov and Triebel-Lizorkin space scales. The results obtained can be considered also as embedding theorems for usual Besov and Triebel-Lizorkin spaces and their analogues in Hellinger metric.

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EVALUATION OF EXPO-RATIONAL B-SPLINES BY NUMERICAL QUADRATURES, I: ROMBERG INTEGRATION

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Key Words and Phrases: expo-rational B-spline, Newton-Cotes numerical quadrature, Romberg integration, error remainder in integral form, sharp error estimate
AMS Subject Classification: 41A15, 65D07, 65D32, 65G99

The definition of the new expo-rational B-splines (ERBS) (see [1]) involves integration over the interval between two subsequent knots. For a certain subset of the range of the intrinsic parameters of ERBS - the so-called 'scalable subset' (see [2]), this integration reduces to the pre-evaluation of a single integral; however, in the general case, the integral has to be computed approximately by quadratures, since it cannot be computed in closed form in terms of elementary functions, and its computation in terms of known special functions is a separate topic of ongoing research. In this communication we consider Newton-Cotes quadratures, with special emphasis on the controlled acceleration of their convergence via Richardson extrapolation. The main result is the derivation of sharp error estimates for the approximate computation of ERBS via Romberg integration.

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EVALUATION OF EXPO-RATIONAL B-SPLINES BY NUMERICAL QUADRATURES, II: GAUSSIAN QUADRATURES

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Key Words and Phrases: expo-rational B-spline, Newton-Cotes numerical quadrature, Romberg integration, Gaussian quadrature, Peano's kernel, error remainder in integral form, sharp error estimate
AMS Subject Classification: 41A15, 65D07, 65D32, 65G99

When applied to expo-rational B-splines (ERBS) (see part I of this study), Romberg integration has several advantages to Newton–Cotes quadratures: preservation of positivity (which is very important in the context of computation of B-splines), control over the order of approximation by varying the number of convergence-accelerating iterations of Richardson extrapolation (which is always meaningful because ERBS are infinitely smooth), uniformness of the sample-vector of the quadrature formula, and others. However, due to the nature of the expo-rational bell-shaped function (an exponent of a rational function which takes only negative values and has poles at the two adjacent knots) the precision of computation deteriorates for values near the knots. Because of this, it is of interest to consider quadrature formulae using non-uniformly distributed sample values, and, in particular, Gaussian quadratures. The main result in this part of the study is the the derivation of sharp error estimates for the approximate computation of ERBS via Gaussian quadratures, and comparison of the errors in the case of Romberg integration and Gaussian quadratures. Of special interest in this comparison is the distribution of the error near the knots. In a separate third part we shall carry through a similar comparative analysis for quadrature formulae using Peano's kernels.

APPROXIMATION ERROR OF EXPO-RATIONAL B-SPLINE INTERPOLATION, I: LAGRANGE, HERMITE AND BIRKHOFF INTERPOLATION

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Key Words and Phrases: expo-rational B-spline, Lagrange interpolation, Hermite interpolation, Taylor interpolation, multipoint Taylor interpolation, Abel–Goncharoff interpolation, Birkhoff interpolation, Pade interpolation, error remainder, approximation error
AMS Subject Classification: 41A05, 41A10, 41A15, 41A25

Due to the specific properties of expo-rational B-splines (ERBS) (see [1]), there is a very lucid correspondence between the type of interpolation at the ERBS knot-vector and the type of local parameter curves in the ERBS representation. For example, Hermite interpolation of certain multiplicity corresponds to a local curve which is the Taylor polynomial around the respective knot of order equal to the multiplicity of the Hermite interpolation in this knot. We study this correspondence for multipoint Taylor formulae, and, in particular, for Lagrange interpolation. We consider also the case of Abel-Goncharoff interpolation as a classical instance of Birkhoff interpolation. For all these cases we derive sharp error estimates of the local approximation, and compare this approximation 'far' and 'near' the knots. We provide also some animated graphical visualization.

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APPROXIMATION ERROR OF EXPO-RATIONAL B-SPLINE INTERPOLATION, II: EULER–MCLAURIN INTERPOLATION AND SAMPLING THEORY

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Key Words and Phrases: expo-rational B-spline, Euler-McLaurin formula, error remainder, approximation, sampling theorem, analytic, infinitely smooth

AMS Subject Classification: 30G30, 41A05, 41A10, 41A15, 41A25, 62D05, 65D07

In the second part of the study of approximation properties of expo-rational interpolants we study the error remainder of the Euler-McLaurin formula for the case of expo-rational B-splines (ERBS). We obtain sharp pointwise estimates for the error, and compare the error distribution 'far' and 'near' the knots. As a consequence, we obtain a sampling theorem for ERBS which shows that when the local curves in the ERBS presentation are analytic functions, the ERBS linear combination is exact on all analytic functions, and even on the broader class of all infinitely smooth functions which are analytic between the knots (and are infinitely smooth, but not necessarily analytic, in the knots).

VECTORIALLY NORMED SPACES

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Key Words and Phrases: vectorial norm, vectorial inner product, $\mathcal{B}$-regular Yosida space, bounded linear operators

AMS Subject Classification: 46A19, 46A22, 46A40, 46C50

The concept of pseudometric space was introduced by Kurepa. In a more or less explicit way, this concept has been used by several authors, namely when the pseudometric space is a linear space and the pseudometric is associated to a vectorial norm p. A vectorial norm is a mapping from a linear space into a Riesz space, with the properties of a usual norm, and is associated with the pseudometric ρ by the expression $\rho(u, v) = p(u - v)$. An approach to the subject, and its particular importance for Numerical Analysis, has been pointed out by Schroeder in his work "Das Iterations Verfahren bei allgemeinen Abstandsbegriff", by Collatz in his book " Functional Analysis and Numerical Mathematics", by F. Robert in his Ph.D. Thesis " Étude et Utilization des Normes Vectorielles en Analyse Numérique Linéaire , by E. Coimbra in her Ph.D.Thesis " Aproximação em Espaços V-Métricos" and also by Feingold and Varga among others. We have treated this subject in a context of general functional analysis; this was also the approach of Kantorovitch, Vulik and Pinsker in their work " Functional Analysis in Ordered Spaces ".

Some results that we have obtained are, in some sense, generalizations of some well-known theorems of the classical functional analysis. Generally we consider a vectorial norm p defined in a linear space E and with range into a Yosida space (i.e. a unitary Archimedean Riesz space) equipped with a certain concept of convergence. The notion of regular Yosida space, introduced by F. Robert, reveals itself as fundamental. However, we have shown that this concept implies necessarily the Yosida space to be finitedimensional. In this sense we have reformulated the concept and introduced the notion of $\mathcal{B}$-regular Yosida space (i.e. a unitary Archimedean Riesz space, Dedekind complete and such that the intersection of all its hypermaximal bands $\mathcal{B}$ is the zeroelement). We have also proved that $\mathcal{B}$-regular Yosida spaces are Riesz isomorphic to the space of all bounded real-valued mappings on a certain set. Of special interest among vectorially normed spaces are the vectorial inner product spaces i.e. when the vectorial norm p is defined in a natural way, in terms of a " vectorial inner product" F, by the formula $p(u) = (F(u, u))^{1/2}$. The theory of these spaces is richer and retains many features of Euclidean spaces, a central concept being orthogonality.

The principal results of our work concern the following areas:

1. Representation theorems of the elements of a vectorially normed space, of arbitrary dimension, when several kinds of topologies are considered in the space, namely when the space is a Banach space, when the space is equipped with a topology induced by a family of seminorms or when the space is equipped with the relatively uniform topology.

2. Extension theorems for bounded linear operators from a vectorially normed space into a $\mathcal{B}$-regular Yosida space (Hahn-Banach theorems), as well as characterization of bounded linear operators defined on a vectorial Hilbert space with range into a $\mathcal{B}$-regular Yosida algebra (Riesz representation theorems).

3. Discussion of orthonormal sets in vectorial inner product spaces, such as: the existence of maximal orthonormal sets; how an element of the space may be expressed in terms of the elements of an orthonormal set; necessary and sufficient conditions for an orthonormal set to be maximal. The main results in this area are: the Bessel inequality (countably infinite case), the Bessel inequality (general case) and the Parseval Identity.

On this lecture only one of the above areas will be focused with some detail.

A NONLINEAR MOVING-BOUNDARY PROBLEM FOR THE WAVE EQUATION IN A SEMI-INFINITE ONE-DIMENSIONAL DOMAIN

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Key Words and Phrases: moving boundary problems, perturbative expansions, touch-down point, wave equation

AMS Subject Classification: 35L05, 35L20, 35L70, 35L85

This work is aimed at studying the nonlinear oscillations of a mechanical systems resting on a unilateral elastic substrate reacting in compression only. We consider semi-infinite cables, subjected to a constant distributed load and to a harmonic displacement applied to the finite boundary. The time behaviour of the system is governed by the equations

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + 1 = 0, \qquad 0 \le x \le c(t) \tag{1}$$

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + u + 1 = 0, \qquad c(t) \le x < +\infty \tag{2}$$

$$u(c(t), t) = 0 \tag{3}$$

$$u(0, t) = U_0 \left(1 + \varepsilon \sin \omega t\right), \tag{4}$$

where $c(t)$, called Touch Down Point (TDP), is the location at which the system detaches from the substrate (assumed unique for simplicity). Since the position of the TDP is determined by the solution itself, we have a moving-boundary problem. After adopting a suitable change of variables which simplifies the analysis of the equations, we approach the problem by perturbative expansions in the smallness parameter ε. The zero-order solution corresponds to the static profile, while in the first- and second-order solutions we observe the presence of resonances for $\omega < 1$. In fact, two different regimes have been identified, one for $0 < \omega < 1$ and for $\omega > 1$. In the latter, energy is lost by radiation at infinity, which explains the absence of resonances in that range.

EFFICIENCY MEASUREMENT OF OECD FOR USING HEALTH RESOURCES

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Key Words and Phrases: technical efficiency, OECD, data envelopment analysis, health system performance

In this study, analyzes technical efficiency in the production of aggregate health outcomes of total fertility rate and increased life expectancy, using Organization for Economic Cooperation and Development (OECD) health data. Application of data envelopment analysis (DEA) reveals that some countries achieve relative efficiency advantages, including those with good health outcomes and those with modest health outcomes. Inefficient countries were determined. We defined the health resource usage for inefficient countries according to bench countries. In addition to, We conclude the TURKEY may learn from countries more economical in their allocation of healthcare resources that more is not necessarily better. Specifically, we find that the TURKEY can substantially reduce inputs while maintaining the current level of life expectancy.

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COMPLETE SEMIGRAPHS

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AMS Subject Classification: 05C07, 05C65

Semigraphs were introduced by E.Sampathkumar in 1996, as a generalisation of graphs where an edge contains two or more vertices with some order and any two edges have at most one vertex in common.

A semigraph G is a pair (V, X) where V is a finite nonempty set whose elements are called vertices of G, and X is a set of ordered n-tuples, called edges of G, of distinct vertices, for various $n \geq 2$, satisfying the following conditions:
SG1: Any two edges have at most one vertex in common.
SG2: Two edges $E_1 = (u_1, u_2, \cdots, u_m)$ and $E_2 = (v_1, v_2, \cdots, v_n)$ are considered to be equal if and only if
1. $m = n$ and
2. either $u_i = v_i$ for $1 \leq i \leq n$, or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

In this talk, we define complete semigraphs and give several interesting results about them alongwith a list of open problems.

MORE ABOUT WEIGHTED WEYL SPECTRUM

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Key Words and Phrases: Weighted spectrum, Weighted Weyl spectrum, α-Weyl Operator, Weighted Weyl operator

One of the most fundamental approaches to study Weyl's theorem is to consider the Weyl's spectrum $\omega(T) = \bigcap_{K \in K(H)} \sigma(T+K)$such that an operator T satisfies Weyl's theorem if $\sigma(T) - \omega(T) = \pi_{00}(T)$, where H is a separable Hilbert space, K(H) the two-sided ideal of compact operators in L(H), $\sigma(T)$ the spectrum of T and $\pi_{00}(T)$the set of all isolated eigenvalues of T of finite multiplicity.

Plenty of literature is available on various generalizations in case of separable Hilbert space of this concept, where the pioneering work in this area is due to M. Schechter, S. K. Berberian, L. A. Coburn, Youngah Yang etc. In contrast, there have been very few research articles on the generalization of these ideas in case of a non separable Hilbert space.

Using these ideas, Yadav and Arora developed the concept of α-Weyl spectrum '$\omega_\alpha(T)$' and α-Weyl's Theorem where $\omega_\alpha(T) = \bigcap_{k \in J_\alpha} \sigma(T+K)$ where J_α denotes the closure of the two-sided ideal of operator of rank less than α in case of a non separable Hilbert space of dimension h and α being a cardinal such that $N_0 \leq \alpha \leq h$.

In the present talk, our main concern will be to show that the weighted Weyl's theorem holds good for a hyponormal operator under some suitable conditions where an operator T is said to satisfy weighted Weyl's theorem if $\sigma(T) - \omega^0_\alpha(T) = \pi^i_{0\alpha}(T)$ with $\omega^0_\alpha(T)$denoting the weighted Weyl's spectrum introduced by Prof. S. C. Arora in his talk in this conference.

The second aspect of this talk is about $Re\omega^0_\alpha(T)$, the real part of $\omega^0_\alpha(T)$. It has been shown that $Re\omega^0_\alpha(T) \subset \omega^0_\alpha(Re(T))$.

FRAME OPERATOR IN THE BANACH CASE

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Key Words and Phrases: X_d-frame, frame operator

As it is well known, frames for Hilbert spaces play an increasing role in Pure and Applied Mathematics. During the last years, some generalizations of frames to Banach spaces were introduced and became topic for investigation (Banach frames, X_d-frames).

The present talk concerns generalization of the concept *Hilbert frame operator* and some of its properties to Banach spaces.

CHAOS GENERATING MATRIX FOR A LASER

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Key Words and Phrases: [A, B, C, D] matrix, waist of a Gaussian beam, recurrence relations, chaotic maps
AMS Subject Classification: 37N20

To be able to produce and even to predict chaos in a laser can be very helpful. In this work we find the matrix [a, b, c, d] for a chaos generating feature that enables us to obtain a matrix [A, B, C, D] for the whole laser, that will allow the waist of a Gaussian beam to follow a recurrence relation. The recurrence relations we are talking about are the logistic and the Hénon one among other chaotic maps.

STANDARDIZED CONDITIONAL EXPECTATION AND ITS APPLICATIONS IN FINANCE

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Key Words and Phrases: standardized conditional expectation, dependent random variables, bivariate distributions, CAPM
AMS Subject Classification: 62H20, 62P20, 91B82, 91B99

Given random variables X, Y, we call the following expression

$$SCE[X|Y = y] = \frac{E[X|Y = y] - E[X]}{Cov(X, Y)}$$

the standardized conditional expectation I believe this to be an importat notion deeper understanding of which is highly desirable and this paper is a move in that direction. Equally importantly there are good applications, one of them to finance, such as CAPM (the capital asset pricing model) and its generalizations.

EXPLICIT SOLUTIONS OF NONLOCAL BOUNDARY VALUE PROBLEMS FOR EVOLUTION EQUATIONS

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Key Words and Phrases: evolution equation, boundary value problem, convolution
AMS Subject Classification: 35K20, 44A40, 35C05

The class of linear PDE of the form

$$P\left(\frac{\partial u}{\partial t}\right) = Q\left(\frac{\partial^2 u}{\partial x^2}\right) + F(x,t)\,,$$

with real polynomials P and Q is considered in the strip domain $G = [0,\,1] \times [0,\,\infty)$ with nonlocal Cauchy conditions

$$\chi_t\left\{\frac{\partial^k u}{\partial t^k}\right\} = 0,\ \ k = 0,1,2,\,\ldots,\,\deg P - 1\,,$$

with a linear functional Φ on $C^1[0,1]$ is considered.

Rather unexpectedly, it is possible to find the solution $u(x,t)$ of the problem under consideration, provided we have on our disposal the solution of the same problem but for the special choice $F(x,t) \equiv x$. Denoting this solution by $U = U(x,t)$, the general solution can be expressed in a closed Duhamel-type form

$$u(x,t) = \frac{\partial^3}{\partial x^2 \partial t}\,(U * F)\ ,$$

where $* : C(G) \times C(G) \to C(G)$ is a non-classical convolution in $C(G)$. This convolution is written in explicit form and it could be used foth for theoretical study and for numerical calculation of the solution. As illustrative examples N. I. Ionkin and A. Beilin problems ($\Phi\{f\} = \int_0^1 f(\xi)d\xi$) and D. Ramakrishna and N. R. Amundson problems ($\Phi\{f\} = f'(0) + f'(1)$) are considered.

GRAPHICAL REPRESENTATION OF KIRKWOOD AND WIGNER DISTRIBUTION FUNCTIONS

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Key Words and Phrases: distribution functions, phase space, computer graphing
AMS Subject Classification: 130-04, 60-04

A great variety of quantum distribution functions (quasi probabilities) in phase space are widely used in many branches of quantum physics. One of them turned out to be a generating function for almost of them all and this is the Kirkwood distribution function (also known as Terletsky or Rihaczek distribution function). The goal of the work is to present some graphical examples (using computer graphing) of Kirkwood and Wigner distribution functions for some typical systems, such as harmonic oscillator, potential well and hydrogen atom, studied in physics

VARIOUS S-ADIC SYMMETRIC FUNCTIONS AND FORMAL GROUPS

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Key Words and Phrases: Hopf algebra, (quasi)symmetric function, noncommutative symmetric function, S-typification, (multi-)curve, sequence of divided powers, ghost component, triangular and diagonal primitives, pure primitive, Ditters conjecture, formal group, co(ntra)variant bialgebra
AMS Subject Classification: 16W30, 05E05, 05E10, 20C30, 14L05

Let S be an arbitrary set of integer primes. We focus our attention on the Hopf algebras of symmetric functions Λ, noncommutative symmetric functions $\mathcal{Z}$ and quasisymmetric functions QSym, considered over subrings $\mathbb{Z}_S$ of the field of rational numbers, consisting of fractions whose denominators in reduced form are not divisible by the primes of S, i.e. $\mathbb{Z}_S = \cap_{p\in S}\mathbb{Z}_{(p)}$. These Hopf algebras decompose into a categorical semidirect sum (or product) of mutually isomorphic Hopf subalgebras, called S-*typification*, a phenomenon in the commutative case due to P. Cartier and J.A. Dieudonn. We denote these Hopf subalgebras Λ_S, $\mathcal{Z}_S$ and QSym_S. The first two have the property that every finite sequence of divided powers can be extended to an infinite curve. For the special case S is the set of all primes, Hazewinkel introduced the so called Ditters conjecture 'QSym is a free polynomial ring'. He proved this conjecture using decomposition theorems for multi-curves, yielding a basis $\mathcal{H}$ for the Lie algebra $\mathcal{P}$ of primitives in $\mathcal{Z}$. Then he dualized these results using λ-ring theoretic methods. His basis $\mathcal{H}$ has the following *triangular property*: there is a total order for which the primitive elements have leading term $\gcd(l()z_l$ with l a Lyndon composition. Around the same time I observed that there exists a unique basis in $\mathcal{P}$, called *pure primitives*, for which this leading term is the *only* nonzero term having a Lyndon word as label, (diagonal property), and prepared tables for these pure primitives up to weight 9. This basis resulted from the idea to introduce for the monomial quasisymmetric function parametrized by $[nr_1,\dots,nr_k] =: [n\#r]$ with $r=[r_1,\dots,r_k]$, its *ghost components* $w_{d,r}$ by $[n\#r]=\sum_{d|n} w_{d,r}^{n/d}$. I have tables up to weight 10 (inclusive), expressing monomial quasisymmetric functions and pure primitives as polynomials in these ghost components, (2002). It is instructive to substitute these ghost components into the Hazewinkel determinants for $\lambda_n(r)$: it circumvents space-time consuming computations with overlapping shuffle products. The case of arbitrary S fits harmonically into this approach: letting $\mathbb{N}(S)$ be the subset of all positive natural numbers, multiplicatively generated by the elements of $S\cup\{1\}$, a slight generalization of a lemma, due to Dieudonn-Dwork affirms: if r is a Lyndon composition with all components in $\mathbb{N}(S)$, then the series $E_{S,r}(t) := \exp\left(\sum_{n\in\mathbb{N}(S)}[n\#r]\frac{t^n}{n}\right)$ has coefficients in $1+t\mathsf{QSym}_\mathsf{S}[[\mathsf{t}]]$. Free polynomial generation of QSym_S, (for arbitrary S), by ghost components is a consequence of this fact. **Application:** If G is an n-parameter formal group law defined over a base ring k, with S as the set of integer primes whose images in k are not invertible, then the covariant bialgebra $\mathcal{U}(G)$ is strongly isomorphic to a Hopf algebra quotient of the categorical semidirect sum of n copies of $\mathcal{Z}_S$, equivalently, the contravariant bialgebra $\mathcal{O}(G)$ is a Hopf subalgebra of the completion of the categorical direct product of n copies of QSym_S. Their structure may be described completely with curves.

APPROXIMATION OF EIGENVALUES AND EIGENVECTORS OF LINEAR OPERATORS ON INFINITE DIMENSIONAL BANACH SPACE

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Key Words and Phrases: eigenvalues, eigenvectors, approximation
AMS Subject Classification: 34L05, 34L16

In this note we consider eigenvalue problem for operators on infinite dimensional Banach space. Exact eigenvalues, eigenvectors, and generalized eigenvectors of operators with infinite dimensional rang can rarely be found. It is imperative to approximate such operators with operators that belong to some well-known class of operators like as finite rang operators or normal operators, and solve the original eigenvalue problem approximately.

NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

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Key Words and Phrases: Finite element methods,The regularized long wave equation, Burgers' equation
AMS Subject Classification: 65M60

The main aim of this work is the study of finite element methods and their application to the numerical solution of transient non-linear partial differential equations.We use as examples numerical algorithms for the solution of the Regularized Long Wave equation and Burgers' equation.

Firstly the theoretical background to the finite element method is discussed.

Secondly we set up Galerkin's method,and the least squares method and A Petrov-Galerkin method containing a piecewise constant weight function.The appropriate element matrices are determined algebracially using the computer algebra package Maple.

The numerical algorithms for the RLW equation have been tested by studying the motion,interaction and development of solitary waves.We have shown that these algorithms can faithfully represent the amplitude of a single solitary wave over many time steps and predict the progress of the wave front with small errror.In the interaction of two solitary waves the numerical algorithms reproduce the change in amplitudes and the phase advance,and phase retardation caused by the interaction.The development of an undular bore is modeled and we demonstrate that its shape, height and velocity are consistent with earlier results.

Simulations arising from three different initial conditions for Burgers' Equation are studied using the Galerkin method including linear spacetime finite elements The results are compared with published data and found to be consistent.

WAVELETS GENERATED BY SPLINE SCALING FUNCTIONS

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Key Words and Phrases: wavelets, splines, multiresolution analysis
AMS Subject Classification: 65D07

The necessary and sufficient conditions for the (nonorthogonal) wavelet multiresolution analysis with arbitrary (for example B–spline) scaling function are established.

The following results are obtained:

1) the general theorem which declares necessary and sufficient conditions for the possibility of multiresolution analysis in the case of arbitrary scaling function;

2) the reformulation of this theorem for the case of B–spline scaling function from W_2^m;

3) the complete description of the family of wavelet bases generated by B–spline scaling function;

4) the concrete construction of the unconditional wavelet basis (with minimal support of wavelet) generated by B–spline scaling function which belongs to W_2^m.

These wavelet bases are simple and convenient for applications. In spite of their nonorthogonality, these bases possess the following advantages: 1) compactness of set suppψ and minimality of its measure; 2) simple explicit formulas for the change of level. These advantages compensate the nonorthogonality of described bases.

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ALTERNATIVES OF FUZZY SET THEORY IN THE INDUSTRIAL OPTIMIZATION

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Key Words and Phrases: fuzzy sets, optimization, decision making
AMS Subject Classification: 03E75

Quality standards in the manufacturing of a product are characterized for one or more response variables. These variables depend on a set of factors of both control and noise. The data, which is obtained by an experimental strategy, allow the building of mathematical models of second order of the responses based on the factors. With these models, it is possible to plan an optimization scheme that will result in a product with the best combination of these quality characteristics. An approximation to the solution of engineering problems of multiple response is to combine the individual responses in an unified objective function. In the presence of this situation, the fuzzy set theory presents alternatives in the procedure to optimize this function. However, there are statistical scenarios in which the procedures of optimization can give unappropriate results due to correlated multiple responses, the presence of outliers, data with no normal distribution or with high variance. One alternative to this situation it is to consider Fuzzy Multiple Objective Decision Making (MODM) methods, as they allow to explore with some flexibility the trade-off between improve the objective function and to meet the constraints. The difference for the different goals and constraints can be specified by the decision-maker. This model can be extended to satisfy the problem of constraints and goals analyzing a set of associate weights. In this paper, four methods are presented. These methods are built in the MODM framework and one is based on lineal programming. In addition, it is considered weighted objectives and constraints. A comparison is done in these methods and their efficiency is evaluated by the construction of a reference index. The experimental data used here were obtained from study cases performed in industrial processes.

COMPARISON AND PROPOSALS OF METHODS TO DETECT THE IRIS

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Key Words and Phrases: iris recognition, biometrics, iris segmentation
AMS Subject Classification: 68U10

Iris recognition has proved to have a superior performance than other biometric technologies. However, the acceptability of the iris identification is low. In order to increase the approval of the iris technology by the public in everyday life, it needs to be easy of use. For the recognition to succeed, it is vital to locate the iris (inner and outer boundaries). If the iris is not located correctly, the user has to start the process again, causing trouble. Accordingly, it is necessary robust algorithm to deal with no perfect conditions. Currently, the most used methodologies to locate the iris are based on an integrodifferential operator. These techniques work well but they are sensible to spot reflections. Hence, in the presence of reflections, the integrodifferential approaches fail. To solve this drawback, other methodology based on both integrodifferential operator and Hough transform has been proposed. This is a more robust approach but the Hough transform adds a large computational cost. Other methods for iris segmentation have poor performance as they are highly dependent of light conditions. Commonly, these approaches are based on the selection of a "reasonable" threshold. Thus, these techniques only work for a small set of conditions. Accordingly, in this paper, two methods for iris segmentation with lower computational cost and high effectiveness are proposed. One method is based on finding firstly the image centroid, secondly creating a subimage ($\frac{1}{4}$ of the original image size) with center in the last centroid and recalculating the subimage centroid. This step is repeated successively until consecutive centroids are near. The boundaries are detected moving away from the centroid in six different directions. Summing up, the idea to detect the part of the image that contain the limbus. Other method is based on spatial filtering with threshold. This approach showed superior results in comparison with all the methodologies.

ANTICIPATING INTEGRATION FOR GAUSSIAN PROSCESSES AND BOUNDARY VALUE PROBLEMS

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The talk is devoted to the stochastic anticipating equations with the extended stochastic integral with respect to the Gaussian processes of a special type. Such integrals are the generalizations of the well-known Skorokhod integral for a wide class of the Gaussian processes, which includes, for example, the fractional Brownian motion. In the particular cases the solutions of above mentioned equations are the well-known Wiener functionals after the second quantization. As an application, the stochastic Kolmogorov equation for the conditional distributions of the diffusion process is obtained. Also we will consider the conditional variant of the Feynman–Kac formula. The two last sections of the article are devoted to the smoothing problem in the case when noise is represented by the two jointly Gaussian Wiener processes, which can have not a semimartingale property with respect to the joint filtration.

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CONFIGURATIONS IN BINARY LINEAR CODES

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Key Words and Phrases: linear double error correcting codes, sphere packing bound
AMS Subject Classification: 94B05

It has long been a great mystery of coding theory, why there is a gap of a factor of $\sqrt{2}$ between the maximum length of double error correcting codes given by the sphere packing bound, and the length of the best known linear double error correcting codes. In an earlier paper the author observed that a better theory of these packings would seem to be a prerequisite to a theory of the analog of the Erdos-Turan conjecture for linear double error correcting codes. Some theorems were proved about configurations in such codes.

In this paper some further facts about configurations in linear double error correcting codes are proved. The main result relates the bound on the number of weight 5 vectors to the bound on the size of a neighborhood of a weight 5 vector. Unfortunately, presently the latter bound for linear codes seems no easier to obtain than the former.

It is conjectured that the bound on the size of the neighborhood of a weight 5 vector is smaller by a constant factor for linear codes than the bound for arbitrary codes. By the above mentioned result and the Johnson bound this would lead to an improved bound on the size of a linear double error correcting code.

Further results are presented on the configurations of the earlier paper also, in particular giving a bound on the size of the collection of weight 5 vectors, in cases not covered by the results of Brouwer and Toulhuizen. It is conjectured that the bound on the size is smaller by a constant factor for linear codes, for these configurations also.

Various empirical results are given relevant to these questions. An exhaustive computer search for cyclic double error correcting codes with $n^2 \geq 2^r$ was performed, for $n \leq 1025$; no codes other than a known family were found. Various properties of this known family are determined, including the configurations for some values of r. The configurations are also determined for some other codes; and some exhaustive searches for configurations are performed.

APPROXIMATE SOLUTION FOR 1-D COMPRESSIBLE VISCOUS MICROPOLAR FLUID MODEL IN DEPENDANCE OF INITIAL CONDITIONS

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Key Words and Phrases: micropolar fluid, strong solution, numerical solution
AMS Subject Classification: 35K55, 35Q35, 76N99, 42A16, 65M99

We consider a model for nonstationary 1-D flow of a compressible viscous heat-conducting micropolar fluid which is thermodynamically perfect and polytropic. A corresponding initial-boundary value problem has a unique strong solution on $]0,1[\times]0,T[$, for each $T > 0$ and for sufficiently small T this solution is a limit of approximate solutions which we get by the Faedo-Galerkin method.

Using the initial functions in the form of Fourier expansions we analyze the numerical approximate solutions in dependance of number of terms in Fourier series.

ON SPECTRAL DECOMPOSITION OF UNBOUNDED OPERATORS AND GELFAND'S THEOREM

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Key Words and Phrases: 47A60, 47A10, 47D03, 47D06, 47D62
AMS Subject Classification: 47A10

In this paper we are interested in spectral decomposition of an unbounded operator with discrete spectrum. We show that if A generates a polynomially bounded n-times integrated group whose spectrum set $\sigma(A) = \{i\lambda_k;\, k \in \mathbb{Z}^*\}$ is discrete and satisfies $\sum \frac{1}{|\lambda_k|^\ell \delta_k^n} < \infty$ (n and ℓ nonnegative integers), then there exists projectors $(P_k)_{k\in\mathbb{Z}^*}$ such that $\sum P_k x = x$ ($x \in D(A^{n+\ell})$), where $\delta_k = \min\left(\dfrac{|\lambda_{k+1}-\lambda_k|}{2}, \dfrac{|\lambda_{k-1}-\lambda_k|}{2}\right)$.

THE ACCESSIBILITY NUMBERS OF DENSE NETWORKS AND AN ALGORITHM

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Key Words and Phrases: connectivity, graph theory applications, graph operations, network vulnerability and stability, deterministic network models

AMS Subject Classification: 05C40, 05C90, 68M10, 90B10, 90B18, 90B35

The vulnerability value of a communication network shows the resistance of the network after the disruption of some centers or connection lines until the communication breakdown. In the design of a network, the vulnerability of the network must be taken into consideration. It requires less vulnerability or greater degrees of stability. A network should be designed as possible as stable. How can this be done? Since a graph is considered as the model of a communication network, we can use the notions of graph theory. In graph theory, deterministic measures of the stability which are called as parameters of graphs are connectivity, integrity and tenacity. For a long time in graph theory any vertex is considered with its neighborhood. By means of this idea, the accessible set and the accessibility number of a connected graph are defined. First, it is studied in the accessibility number of a graph and its relation between other graph parameters. Now,in this paper we define the dense product of two graphs which increases connectivity of the graph. We prove that the accessibility number of dense product of two graphs is equal to product of their accessibility numbers. Also, in this paper we offer an algorithm for finding the accessibility number of given graph whose complexity is O(n^3).

CAVITATION, INDENTATION AND PENETRATION

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Key Words and Phrases: plastic materials, large strains, cavitation, indentation, penetration

Elastoplastic deformation patterns induced by expanding cavities, under internal pressure, provide useful analytical models in applied mechanics and engineering. Applications range from material hardness indentation tests to dynamic penetration of projectiles. The lecture will start with a brief review of experimental and numerical simulation data exposing cavitation fields in elastoplastic media.

Recent advances in deriving analytical solutions for steady state expanding cavities enable the construction of highly accurate formulae for cavitation pressure, accounting for strain hardening (softening) and elastic compressibility. For dynamic fields, solutions are described by Mach number (relative to medium wave velocity) power series with coefficients that depend on elastic and plastic moduli. Cylindrical cavitation solutions have been obtained for both Mises and Tresca plastic solids.

Analysis of conical indentation is facilitated with the spherical cavity static expansion eigen-cell leading to elegant results for the hardness of strain hardening solids. Comparison with finite element calculations reveal good agreement over a wide range of material properties. We examine the relation between hardness and cavitation pressure and show that the cylindrical cavitation yield stress, in a Tresca material, serves as an appropriate scaling stress for hardness.

Dynamic cavitation solutions are used to estimate penetration depth of rigid projectiles at normal impact of elastoplastic media. Both spherical and cylindrical fields give reasonable predictions of penetrating depth and, in fact, produce practical bounds on experimental data. Influence of friction, impact velocity and penetrator profile is illustrated by a few examples.

Future research directions will be discussed, including pressure sensitivity of porous solids, models of deep plastic friction, strain gradient effects and rate dependence.

THE FOURTH-ORDER EULER-BERNOULLI BEAM EQUATION – GLOBAL GALERKIN FINITE ELEMENT AND ANALYTICAL SOLUTIONS

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Key Words and Phrases: Euler-Bernoulli Beam, Galerkin Finite Element Method

This paper explores the global Galerkin finite element solution of the fourth-order differential equation for the displacement w of an Euler-Bernoulli supported cantilever beam

$$\frac{d^2}{dx^2}\left[EI\left(x\right)\frac{d^2w}{dx^2}\right] = f\left(x\right) + \hat{p}\left(x\right) + \hat{m}\left(x\right) \quad \text{for} \quad 0 < x < L \tag{1}$$

subject to the boundary conditions $w\left(0\right) = dw/dx|_{x=0} = 0$, $w\left(L\right) = 0$ and $\bar{M}_R = M\left(L\right) = 0$. A distributed load $f\left(x\right)$, a point load $\hat{p}\left(x\right) = \hat{P}\delta\left(x - x_p\right)$ at the point $x = x_p$ and a concentrated moment $\hat{m}\left(x\right) = -\hat{M}\,D\left(x - x_m\right)$ at the point $x = x_m$ act on the beam. $EI\left(x\right)$ is the bending stiffness of the beam, $\delta\left(x - x_p\right)$ a Dirac delta function and $D\left(x - x_m\right)$ a unit doublet function.

In the traditional Galerkin finite element approach taught to many of the first time students of the finite element method, the method starts, after the definition of the problem, with the discretization of the domain into a collection of contiguous non-overlapping elements. The finite element formulation over an element is then developed. This leads amongst others to the appearance of boundary fluxes or forces associated with each element. After the assembly of the global equations a decision must then be made about the treatment of the fluxes (or forces) at the interface between adjacent elements. Normally it is assumed that the fluxes (forces) cancel, unless a point source (point load or concentrated moment) occurs at the interface. In such a case the point source (point load or concentrated moment) is assumed to represent the jump in flux (forces).

In the global Galerkin finite element method the point sources (point loads or concentrated moments) are included in the the source term(s) of the differential equation(s) during the definition of the problem, e.g. Eq. (1). The Galerkin formulation over the entire domain is then developed. This entails the weak formulation, the selection of the approximate solution over the domain and the formation of the resulting system of global equations. In order to evaluate the integrals to obtain the coefficients of the global set of equations, we make use of the property that an integral can be written as the sum of a number of integrals. We therefore discretise the domain into a collection of sub-domains (elements) which leads to the GGFEM formulation. In this approach the inter-element fluxes do not appear in the formulation and the point sources (point loads or concentrated moments) are treated in a natural way as part of the source term(s) or system of external forces. Although the differences between the traditional GFEM and the GGFEM approaches at a first glance may appear to be subtle, from a philosophical and physical point of view they are significant. The GGFEM can be seen as a systems or holistic approach to the finite element solution of a problem.

In this overview of the global Galerkin finite element method we will first focus on the development of the finite element formulation and the subsequent finite element solution. The solution of two simple examples will then be considered and a comparison will be made between the finite element and analytical solutions. This will lead to a summary of the strengths and weaknesses of the two approaches.

ON SQUAREFREE COMPOSITE NUMBERS

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Key Words and Phrases: prime number, Carmichael Number, squarefree Composite
AMS Subject Classification: 11Y16, 11Y11, 11A51

Let $\alpha \in \mathbb{Z} \setminus \{0\}$. A composite squarefree number N is said to be an α-*Korselt number* (K_α-*number*, for short), if $N \neq \alpha$ and $p - \alpha$ divides $N - \alpha$ for each prime divisor p of N. This paper is devoted to both a numerical and theoretical study of Korselt numbers. Let N be a squarefree composite number; we say that N is an α-*Tunisian number* (T_α-*number*, for short) if N is both a K_α-number and a $K_{(-\alpha)}$-number. This paper deals, also, with the study of Tunisian numbers and some related typical prime numbers. Many computer programs are given, in order to understand Korselt and Tunisian Numbers.

THE STABLE PARETIAN HYPOTHESIS OF MANDELBROT AND THE GAUSSIAN HYPOTHESIS OF BACHELIER: TWO CONCEPTS APPLIED IN MATHEMATICAL FINANCE

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AMS Subject Classification: 91B28, 91B30, 91B84

Bachelier (1900) had the path-breaking idea to model the returns of security and commodity markets as a random walk, meaning that successive differences of returns are Gaussian distributed. Despite of this fundamental concept, named the *Gaussian hypothesis*, Mandelbrot (1963) and Fama (1965) showed that the distributions of the returns do not behave like this, but behave like the more general stable distributions, containing skewness and kurtosis. Mandelbrot called his concept the *stable Paretian hypothesis*. Rachev (2000) and many other researchers confirmed Mandelbrot's theses. Since about forty years, a controversy between these both distributional hypotheses continue to exist in mathematical finance.

The economic and mathematical background, the methods and the tools relied upon both hypotheses are presented. The usefulness of characteristic functions, see Jondeau (2007) and Cizek (2005), and of Fast Fourier Transforms (FFT) to calculate the densities of α-stable distributions is underlined and an accent is put on the estimation of their parameters.

Serbinenko and Emmenegger (2007) have investigated the distributions of the returns of some financial market indices of the new emerging markets in Eastern Europe after 1991, confirming also in this new area the *stable Paretian hypothesis*. The Sharpe ratio (*Gaussian hypothesis*), the STARR (Stable Tail Adjusted Return Ratio) and the Rachev-ratio (*stable Paretian hypothesis*) are calculated for the evaluation of the risk.

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UTILITY MAXIMIZATION AND STOCHASTIC PDE'S

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Key Words and Phrases: generalized utility function, random fields, stochastic backward partial differential equations, feedback formulae, stochastic Hamilton-Jacobi-Bellman equation
AMS Subject Classification: 93E20, 60H15, 91B28, 91B16, 35R60

This work studies the preference problem of maximizing total expected utility from consumption in the general context of random coefficients. We identify the associated value function as a generalized utility function, and exploit the interplay between dynamic programming and Feynman-Kac results via the theory of random fields and stochastic partial differential equations (SPDE's). In particular, the resulting value random field of the optimization problem satisfies a non-linear, backward SPDE of parabolic type, widely referred to as the stochastic Hamilton-Jacobi-Bellman equation. In addition, the dual value random field is characterized in terms of a linear, backward parabolic SPDE. Employing the generalized Ito-Kunita-Wentzell formula, we obtain adapted versions of stochastic feedback formulae for the optimal portfolio and consumption choices. In the case of deterministic model coefficients, these formulae become deterministic functions and establish the current level of wealth as a "sufficient statistic" for the maximization problem. Finally, several open problems concerning future research are discussed.

APPROXIMATION PROPERTIES OF BIVARIATE GENERALIZATION OF BLEIMANN, BUTZER AND HAHN OPERATORS BASED ON THE Q-INTEGERS

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Key Words and Phrases: positive linear operators, bivariate Korovkin theorem, bivariate modulus of continuity, bivariate Lipschitz type maximal function, q-integers
AMS Subject Classification: 41A36

In this presentation, bivariate case of BBH operators based on the q- integers is constructed as follows:

$$\begin{aligned} L_{n_1,n_2}(f;q_{n_1},q_{n_2},x,y) &= \frac{1}{l_{n_1,q_{n_1}}(x)}\frac{1}{l_{n_2,q_{n_2}}(y)} \\ &\times \sum_{k_1=0}^{n_1}\sum_{k_2=0}^{n_2} f\left(\frac{[k_1]_{q_{n_1}}}{[n_1-k_1+1]_{q_{n_1}} q_{n_1}^{k_1}}, \frac{[k_2]_{q_{n_2}}}{[n_2-k_2+1]_{q_{n_2}} q_{n_2}^{k_2}}\right) \\ &\qquad \times q_{n_1}^{\frac{k_1(k_1-1)}{2}} q_{n_2}^{\frac{k_2(k_2-1)}{2}} \left[\begin{array}{c} n_1 \\ k_1 \end{array}\right]_{q_{n_1}} \left[\begin{array}{c} n_2 \\ k_2 \end{array}\right]_{q_{n_2}} x^{k_1} y^{k_2}. \end{aligned}$$

Here, $l_{n_1,q_{n_1}}(x) = \prod_{s=0}^{n_1-1}(1+q_{n_1}^s x)$, $l_{n_2,q_{n_2}}(y) = \prod_{s=0}^{n_2-1}(1+q_{n_2}^s y)$, and

$$[k]_q = \begin{cases} \frac{1-q^k}{1-q}, & q \neq 1 \\ k, & q = 1 \end{cases}$$

Then Korovkin type approximation properties of this generalization are obtained with the help of Volkov's theorem. Lastly,we obtain rates of convergence of these operators by means of bivariate modulus of continuity and Lipschitz type maximal functions. Also the Voronovskaja type asymptotic estimate is given for these operators.

THE MATHEMATICAL PROTEIN FOLDING PROBLEM

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Key Words and Phrases: constrained optimization, closed surface, mean curvature
AMS Subject Classification: 49Q10, 53A10

Proteins are nature's robots. They participate in every life phenomenon: regulate physiological processes; catalyze chemical reactions; form supporting structures; transfer signals so that we can feel, hear, and see; form immune system to deal with intruders; etc. Proteins are polymers consist of amino acids in sequences joined by peptide bonds. Protein folding is the process of nascent proteins taking their special native structures. The three-dimensional shapes of proteins' native structures hold the key to understanding their ubiquitous functions in life, hence is called the "second genetic code". Anfinsen showed that the native structure of a protein only depends on its amino acid sequence and hypothesized the thermodynamic principle for the protein folding phenomenon. Since then, the protein folding problem, predict the natives structure of a protein with only the knowledge of its amino acid sequence, occupies a more and more important position in biological study. Rapid DNA sequencing technic advances have supplied many protein amino acid sequences but the structural knowledge of them legs far behind. For example, there are 259,034 sequence entries in the website UniProtKB/Swiss-Prot, but there are only 41,952 protein structures (including many structures obtained from different experiments of the same sequences) in the Protein Data Bank. This should be no surprise, since the genetic code for amino acid sequences of proteins is carried by DNA sequences, a discrete (zero-dimensional) code; but the "second genetic code", the native structure of a protein, is continuous and three-dimensional. This jump from discrete, zero-dimension to continuous three-dimension of course brings far more complexity.

Protein folding is the focus of biophysics and biochemistry researches and the holy grail of computational biology. After over four decades of intensive research the holy grail is still being pursued. To grab the holy grail, ideas from other disciplines, especially mathematics, may shed new insight and make a breakthrough. A mathematical model of protein folding has been established, and initial implementation showed the power and potential of this model. As had happened in physics, mathematics may also be "unreasonably effectiveness" in grabbing the holy grail that is the aim of this project.

We will present the new mathematical model for protein folding and protein structure prediction. Mathematically, this model is a constrained geometric optimization problem, we call it the mathematical protein folding problem. This problem is a generalization of classical (constrained) isoperimetric problem. We will discuss how to create the model from the well-known structure features of the native structures of globular proteins and the related mathematical problems such as the existence of minimizers, characterization and regularity of minimizing surfaces, and the first variational formula.

MULTIPLE SCATTERING OF WAVES AND DYNAMIC STRESS FROM A CIRCULAR CAVITY IN A SEMI-INFINITE FUNCTIONALLY GRADED PIEZOELECTRIC/PIEZOMAGNETIC MATERIAL

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Key Words and Phrases: Semi-infinite piezoelectric/piezomagnetic materials; Multiple scattering of elastic waves; Dynamic stress concentration factor; Circular cavity; Image method
AMS Subject Classification: Applications in mechanics

Nowadays, composites consisting of piezoelectric and piezomagnetic components have found increasingly applications in engineering structures, particularly in so-called smart materials/intelligent structure systems. This is because that these composites possess some new product properties such as the magneto-electric and the secondary pyroelectric effects which are not demonstrated with their individual components. On the other hand, the development of functionally graded materials (FGM) has demonstrated that they have the potential to reduce the stress concentration and increase fracture toughness. Consequently, the concept of functionally graded materials can be extended to the piezoelectric/piezomagnetic materials to improve the reliability of piezoelectric/piezomagnetic materials and structures.

During tailoring, connection and serving of these structures, it is inevitable to make cavities, and some failures such as holes and cracks may also occur inside the structures. Under dynamic loads, the stress around and near the discontinuities may increase sharply, which causes the decrease of the strength of structures and the fatigue and fracture of structures. With the advent of FGMs, significant efforts have been made in the study of the dynamic stress around the cavities and cracks in the materials under dynamic loading. These include analytical, numerical and experimental investigations.

In this paper, an analytical method is applied to investigate the multiple scattering of shear waves and dynamic stress around a circular cavity in a semi-infinite functionally graded piezoelectric/piezomagnetic composite material. The analytical solutions of wave fields are expressed by employing wave function expansion method and the expanded mode coefficients are determined by satisfying the boundary conditions of the cavity. Image method is used to satisfy the free boundary condition of the semi-infinite structure. According to the analytical expression of this problem, the numerical solutions of the dynamic stress concentration factor around the cavity are presented. The effects of the material properties, the buried depth of the cavity, the incident wave number and the nonhomogeneous parameter of materials on the dynamic stress around the cavity are analyzed. Analyses show that the piezoelectric and piezomagnetic properties have great effect on the dynamic stress in the region of intermediate frequency and the effect increases with increasing wave number. When the nonhomogeneous parameter of materials is less than zero, it has less influence on the maximum dynamic stress around the cavity; however, it has greater influence on the distribution of the dynamic stress around the cavity. When the nonhomogeneous parameter of materials is greater than zero, it has greater influence on both the maximum dynamic stress and the distribution of dynamic stress around the cavity, especially in the case that the buried depth is comparatively small.

THE MEAN MOTION AND LEVIN'S SECULAR CONSTANT THEOREM

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Key Words and Phrases: almost periodic function, mean motion, secular constant, Picard's type theorems
AMS Subject Classification: 42A75, 30B50

Let $P(z)$ be an exponential polynomial $\sum_{j=1}^{n} c_j e^{i\lambda_j z}$, $c_j \in \mathcal{C}$, $\lambda_j \in \mathcal{R}$. The famous mean motion problem which goes back to Lagrange is the existence of the limit

$$c(y) = \lim_{\beta-\alpha\to\infty} \frac{\Delta_{\alpha<x<\beta} \arg P(x+iy)}{\beta-\alpha}$$

This problem was proved by B. Jessen and H. Tornehave (1945). They also proved the existence of the limit for every holomorphic almost periodic function $f(x+iy)$ on a strip $a < y < b$ for all $y \in (a, b)$ outside of some countable set. Moreover, $c_f(y) = -J'_f(y)$, where $J_f(y) = \lim_{S\to\infty}(2S)^{-1}\int_{-S}^{S} \log|f(x+iy)|dx$ is *Jessen function* of f.

Furthermore, if $\Lambda^- = \inf \mathrm{sp} f > -\infty$ for an almost periodic function f, then f extends to the upper half-plane as a holomorphic almost periodic function and

$$-\Lambda^- = \lim_{y\to+\infty} \frac{\log|f(iy)|}{y} = \lim_{y\to+\infty} \frac{J_f(y)}{y} = \lim_{y\to+\infty} J'_f(y) = -\lim_{y\to+\infty} c_f(y).$$

If $\Lambda^- \notin \mathrm{sp} f$, the proof of the second equality is rather complicated; it is the contents of Levin's Secular Constant Theorem (1941).

In our talk we suggest multidimensional analogues of the mean motion and Levin's Theorem; for application we prove some new Picard's type theorems for holomorphic almost periodic functions with spectrum in a cone.

AN EXPANSION RESULT FOR A STURM-LIOUVILLE EIGENVALUE PROBLEM WITH IMPULSE

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Key Words and Phrases: Impulsive boundary value problem, Eigenvalue, Eigenfunction
AMS Subject Classification: 34A37

The paper is concerned with an eigenvalue problem for second order ordinary differential equations with impulse. Such a problem arises when the method of separation of variables applies to the heat conduction equation for two-layered composite. The existence of a countably infinite set of eigenvalues and eigenfunctions is proved and a uniformly convergent expansion formula in the eigenfunctions is established.

THE LQR TRACKING AND VIBRATION CONTROL OF A NONLINEAR ONE-LINK FLEXIBLE ROBOTIC MANIPULATOR

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Key Words and Phrases: LQR, flexible manipulators, nonlinear systems, linear control
AMS Subject Classification: control theory

The dynamical system studied in this work comprises a one-link flexible robotic manipulator capable of translational and rotational motion. Linear curvature is considered for the flexible link but coupling with rigid body degrees of freedom produces nonlinear terms in the governing equations of motion. These equations are obtained via the Lagrangian formalism. The present developments can be used for the mathematical modeling of a space manipulator designed grasp and conduce objects (or an astronaut) up to some part of a space station in order to execute a determined task. In the investigation presented in this work, different angular velocities are considered in order to gradually increase the importance of the nonlinear terms in the governing equation of motion and verify the influence of these terms on the performance of the control law. The technique proposed here for the tracking and vibration control of the nonlinear robotic manipulator is the LQR. The optimal gains are obtained by neglecting all the nonlinear terms (by considering small velocities, smal beam deflections,). The gains so obtained are then used in the control of the complete system (including the nonlinear terms). The optimal gains are obtained considering the finite time approach. Here one considers that all the states are at disposal for feedback in the control law. The basic idea here is to verify the robustness of the LQR plus tracking control by considering its application to nonlinear systems and assuming that the nonlinear terms are not considered in the design fo the control law.

AN ITERATIVE PRE-CONDITIONED METHOD TO ADJUST WIND FIELDS

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Key Words and Phrases: Pre-conditioner, Conjugate gradient, Saddle-point problem
AMS Subject Classification: 65N30, 76B75

A variational method was developed by Sasaki [1] to be used as a diagnostic model in meteorology. This method deals to an elliptic problem for the multiplier and the motivating idea is to minimize a functional subject to the constraint of mass conservation. In this work we present an innovative method to generate a vector field that can be applied to obtain an adjusted wind field satisfying such constraint. This method arises from the re-formulation of the minimizing problem into a saddle-point one. This saddle-point problem is solved by an (iterative) Uzawa's method. This proposal is inspired from Glowinski's approach to solve Stokes-like problems in computational fluid dynamics [2]. It turns out that the elliptic problem for the multiplier is an optimal pre-conditioner for this iterative method.

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INTERPOLATION OF MULTIINDEX SEQUENCES

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Key Words and Phrases: interpolation, group action, universal surjectivity of the Cantor set
AMS Subject Classification: 54E99, 57S30, 54H05

In 1998, Y. Benyamini published the following theorem. There exists a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that for any sequence $(x_n)_{n\in\mathbb{Z}} \in [0,1]^{\mathbb{Z}}$, there is $t \in \mathbb{R}$ such that $x_n = f(t+n)$ for all $n \in \mathbb{Z}$. That is, the function f interpolates, so to speak, all sequences in $[0,1]^{\mathbb{Z}}$ "simultaneously". Benyamini proved also that it is not possible to interpolate in this way all bounded $\mathbb{Z}$-sequences but it is possible to interpolate all bounded $\mathbb{N}$-sequences.

In 2005, R. Naulin M. and C. Uzcátegui united and generalized these results to the case of M-sequences, where M is an arbitrary subset of $\mathbb{Z}$.

In the talk, we will consider the case of topological spaces X and Y, with an abelian group acting totally disconnected on X. We will state an analogous problem of "simultaneous interpolation" of all "generalized sequences" using continuous mappings $X \to Y$. We will give further generalizations of the Benyamini and Naulin–Uncátegui theorems, in particular, their multidimensional analogues.

THE VAN ROOSBROECK SYSTEM, BOUNDED DISCRETE STEADY STATE SOLUTIONS ON BOUNDARY CONFORMING DELAUNAY GRIDS

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Key Words and Phrases: stability, bounded discrete solutions, semiconductor models, Delaunay grids
AMS Subject Classification: 65N12

The paper summarizes properties of Delaunay grids, Voronoi diagrams and presents a weak formulation of the the Scharfetter-Gummel-scheme on d dimensional simplex grids. Together with a discrete weak maximum principle it is possible to obtain the identical bounds for the discrete solutions on boundary conforming Delaunay grids as for the analytic system. This qualitative stability result holds for any spatial step size h. Sufficiently close to the unique equilibrium follows the existence of a unique steady state solution by proving that the Schur complement for the two continuity equations is a M-matrix.

Dissipativity is obtained in case of a backward Euler time discretization, too.

Based on the proofs, averaging procedures for parameters can be derived, which preserve the properties of the discrete system. To study algorithmic issues, the scheme is implemented in an experimental - with respect to algorithms - device simulator.

NEW CLOSURE RELATIONS IN LARGE EDDY SIMULATION

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Key Words and Phrases: LES, closure relations, SGS viscous dissipation
AMS Subject Classification: 76F65

The present - day Large Eddy Simulation models based on the Smagorinsky assumption and the drawbacks of the dynamic calculation of the closure coefficient for the generalised subgrid scale turbulent stress tensor are presented. Generally, the numerical integrations of motion equations, on generalized curvilinear grids, are not able to conserve kinetic energy. The relations between numerical scheme conservation property of mass, momentum and kinetic energy and the drawbacks of the dynamic Smagorinsky - type turbulence models are shown. In this paper the relation between the Noll formulation of the principle of Material Frame Indifference and the principle of Turbulent Frame Indifference in Large Eddy Simulation is revised; the definition of a new Rule of Turbulent Closure Relations is proposed. In this paper we demonstrate that the transport equation of the generalised sub-grid scale (SGS) turbulent stress tensor is form-invariant but not frame-indifferent under Euclidean transformations of the frame. A new closure relation for the generalized SGS turbulent stress tensor is proposed: this closure relation complies with the rule of Turbulent Closure Relations. In the proposed model the generalized SGS turbulent stress tensor is related exclusively to the generalized SGS turbulent kinetic energy (which is calculated by means of its balance equation) and the modified Leonard tensor. The viscous dissipation of the generalized SGS turbulent kinetic energy is calculated by solving its balance equation. It is demonstrated that the balance equation of the viscous dissipation is form-invariant but frame-dependent under Euclidean transformations of the frame; the closure relations proposed in this paper allow the modelled balance equation of viscous dissipation respect the properties of Euclidean form-invariance and frame-dependence of the exact equation.

SECURE COMPUTER COMMUNICATION BASED ON CHAOTIC RÖSSLER OSCILLATOR

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Key Words and Phrases: synchronization, chaos, secure computer
AMS Subject Classification: 93C10, 37N35

We present the application of chaotic Rössler oscillator in the transmission of secure computer messages. The secure communication is obtained with chaotic unidirectional-coupled Rössler oscillator improved with a computer algorithm. The algorithm is based in a scheme that synchronizes via one of the variables, while a signal is transmitted through another variable. We show that this scheme allows more stable communications. Audio signal, text and image can be transmitted.

DERIVATIVE FREE METHOD IN OPTIMIZATION. IMPLEMENTATION ISSUES IN A PRACTICAL PROBLEM

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Key Words and Phrases: non monotone, derivative free, space decomposition
AMS Subject Classification: 49M05, 49M27, 90C56

We are dealing with the problem of finding a minimizer $x_* \in C$, and the minimum value $f(x_*)$ of a real valued function of n variables $f : \mathcal{R}n \to \mathcal{R}$. Ideally we wish to find a global minimum; i.e., $f(x_*) \leq f(x), x \in C$, but in general most algorithms only ensure that limit points of a subsequence of estimates, i.e., $\{x_i\}_{i\in I}$ satisfy some local necessary conditions for minimality. Another technical hindrance that arises frequently is the lack of derivative information, which prevents the use of powerful quasi Newton methods. We instead must rely on a sample of function values on certain directions of search that ensure convergence.

Our exposition will be confined to unconstrained optimization and will focus on derivative free non monotone (DF NM) algorithms that try to circumvent the aforementioned difficulties: DF only relies on function values and NM tries to escape from a local minimum. After a brief exposition that points out the advantages of these algorithms, we study a particular implementation to the real problem of *allocating the Access Points (APs) of Wireless Local Area Networks (WLANs) that maximize coverage*.

This paper has a twofold objective: on the one hand, we note that the DF NM algorithm is highly competitive with known techniques, which in some cases do not even ensure convergence to a local minimum; on the other hand, we recognize that DF NM algorithms may be improved *significantly* with a judicious choice of search directions, that depends on the problem's structure.

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THE MONOTONICITY METHOD FOR SOLVIG SDE'S IN THE DUAL OF A MULTI-HILBERTIAN SPACE

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Key Words and Phrases: Finite and infinite dimensional stochastic differential equations, multi-Hilbertian spaces, nuclear spaces, exsistence, uniqueness, coercivity, monotonicity
AMS Subject Classification: 60H10

We will begin with a short review of conditions and techinques for constructing solutions of stochastic differential equations, including Lipshitz coefficients, linear growth and monotonicity conditions, and compactness. We will demostrate their application to the problem of constructing a solution in a multi–Hilbertian space.

Consider an SDE $dX_t = L(X_t)\,dt + A(X_t)\,dB_t$, driven by a d–dimensional Brownian motion, with coefficients $L : \Phi^{'} \to \Phi^{'}$, $A = (A_1, ..., A_d)$, $A_i := \Phi^{'} \to \Phi^{'}$, whose solution X_t, is a $\Phi^{'}$–valued process. The space Φ is multi–Hilbertian, not necessarily nuclear, and it's dual $\Phi^{'} = \bigcup_p H_p^{'}$, where H_p are Hilbert spaces. This general set–up can be useful in applications to quantum mechanics, where non–nuclear multi–Hilbertian structure may arise in a construction of a rigged Hilbert space representation of a Lie group.

Nuclearity assumpion may be added to reduce the assumptions on the coefficients of the SDE, and extend the analysis to a $\Phi^{'}$-valued Brownian motion.

We allow the coefficients $L, A_i : H_p \to H_q$, with $H_p \hookrightarrow H_q$, which creates a complex problem when the approximate solutions "move" from one Hilbert space to another. We require monotonicity and the following weakened linear growth condition on the (linear) coefficients:

$$\|L(t,\omega)u\|_q + \|A(t,\omega)u\|_{HS(q)} \leq \theta \|u\|_p,$$

so that the growth of L and A in H_q is controlled by the larger norm in H_p. This assumption allows to study cases when L and A are differential operators, which in general do not satisfy the usual growth condition assumed in the literature. However, the weakened growth condition brings some challenges in establishing the exsistence result. When the initial condition is an element of Φ, assuming "uniform" monotonicy and growth, we can realize to solution in Φ.

A stronger growth condition together with the nuclearity assumption, coercivity, and a Lipshitz-type monotonicity condition, allow for obtaining solutions in the non–liear case.

As one application, we consider $\Phi = \mathcal{S}^{'}$, $A = \nabla$, and $L = \frac{1}{2}\triangle$, and establish the monotonicity and growth properties to show that the solution X_t is the translation $\rho_{B_t}\phi$ of the initial condition ϕ by the Borwnian motion.

An interestig case arises when X_t is a diffusion in $\mathbb{R}^d$, given by the SDE

$$dX_t = b(X_t)dt + \sigma(X_t)dB_t,$$

with $\sigma = (\sigma_{i,j})_{i,j=1}^d$, and $b = (b_1, ..., b_d)$ are $C^\infty\left(\mathbb{R}^d\right)$ functions. Since the coefficients are not necessarily of compact support, the corresponding infinite dimensional SDE needs to be considered in $\mathcal{E}^{'}$, the dual of $C^\infty\left(\mathbb{R}^d\right)$, and the infinite dimensional coefficients will be the operators $A : \mathcal{E}' \to L\left(\mathbb{R}^d, \mathcal{E}'\right)$ and $L : \mathcal{E}' \to \mathcal{E}'$ defined by

$$A\psi(h) := -\sum_{j=1}^{d}\sum_{i=1}^{d}\bigl\langle \sigma_{i,j}, \psi\bigr\rangle(\partial_i\psi)h_j$$

$$L\psi := \frac{1}{2}\sum_{i,j=1}^{d}\left(\sigma\sigma^*\right)_{i,j}(\psi)\partial_{i,j}^2\psi - \sum_{i=1}^{d}\langle b_i, \psi\rangle\,\partial_i\psi,$$

$\psi \in \mathcal{E}'$, $h = (h_1, ..., h_d) \in \mathbb{R}^d$, and $\left(\sigma\sigma^*\right)_{i,j}(\psi) = \sum_{k=1}^d \langle \sigma_{i,k}, \psi\rangle\,\langle \sigma_{j,k}, \psi\rangle$. If we consider an infinite dimensional process of the form $Y_t = \delta_{X_t}$, with the diffusion X_t possibly exploding in a finite time, we arrive at the concept of solutions "locally of compact support" and define an explosive $\mathcal{E}^{'}$–valued solution as such, which exits $\mathcal{E}^{'}$ in a finite time with a positive probability.

Those concepts facilitate discussion on one–to–one correspondence of possibly explosive solutions to finite dimensional SDE's with $C^\infty\left(\mathbb{R}^d\right)$–coefficients and their infinite dimensional counterparts.

RANK OF A MAXIMAL SUBGROUP IN $H^1(M, Z)$ WITH TRIVIAL CUP-PRODUCT

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Let M be a smooth closed oriented manifold, $h(M)h^{max}(M)$ be the maximal rank of a maximal subgroup in $H^1(M, Z)$ with trivial cup-product, and $h^{min}(M)$ the minimal rank of such a subgroup. It has been shown that the value of $h(M)$ characterizes the topology of Morse form foliations on M: e.g., if $rk\omega > h(M)$, where ω is a Morse form on M, then its foliation has a minimal component. We give upper and lower bounds on $h^{max}(M)$ and $h^{min}(M)$ in terms of the first and second Betti numbers. In addition, we calculate these values for a connected sum and direct product of manifolds.

NEW TIME-FREQUENCY TRANSFORMS FOR TRANSIENT COMPLEX SIGNALS

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The classical time-frequency transforms (e.g. Wigner distribution, wavelet transform, etc.) have been widely used for processing of complex chirps, i.e. transient signals with *constant* frequency speeds (chirp rates). These signals have *linear* variation of the instantaneous frequency in time. The classical techniques are suitable for the chirps. the most time-frequency transforms ideally follows the linear variation of the instantaneous frequency.

However, in practical applications, for radar, sonar and mechanical systems, e.g., change of the instantaneous frequency of radar and sonar signals, start-up or shut-down of rotating machinery or change of the shaft instantaneous frequency during machinery operation, one has to process transient complex signals with any *nonlinear* polynomial and non-polynomial variation of the instantaneous frequency in time. These signals could characterize radar, sonar signals and the shaft and blade passing vibrations of rotating machinery during nonlinear change of shaft frequency.

New time-frequency transforms have been proposed and investigated for *amplitude estimation* of transient signals (on the background of interference) with the nonlinear polynomial and non-polynomial time variation of the instantaneous frequency and its derivatives. The main advantage of the proposed transforms is that transforms could ideally follows the nonlinear polynomial frequency variation without amplitude errors.

THERMODYNAMIC BEHAVIOR FROM SCHROEDINGER'S DYNAMICS IN INCOMPLETELY OBSERVED QUANTUM SYSTEMS

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Key Words and Phrases: quantum mechanics, statistical mechanics, non-equilibrium

Thermodynamic behavior is characterized by the approach to equilibrium as stated by the second law. The Schroedinger equation, however, neither features any fix point nor does it allow for a change of entropy with respect to the full system. This raises the question how quantum systems can, in general, exhibit thermodynamic behavior. We basically consider two scenarios: i) Quantum systems coupled to other (larger) quantum systems which need not to be reservoirs in the traditional sense. The observation is incomplete since only observables corresponding to the primary system are considered. ii) Interacting quantum systems placed on a (finite) lattice. Only the particle number or the local energy at each site is observed. This allows for an investigation of the possible emergence of diffusion from Schroedingers dynamics. In both scenarios thermodynamic behavior with respect to the considered observables is found. Key concepts are entanglement, projection techniques and the Hilbert Space Average Method (HAM).

HYPERBOLIC OUTER BILLIARDS

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Key Words and Phrases: dynamical systems, hyperbolic dynamics, billiard-type dynamical systems

AMS Subject Classification: 37D40, 37D50, 34D08

Mathematical billiards are extensively studied and in many respects well un- derstood dynamical systems. For instance, the question of chaotic behavior in billiard dynamical systems has been given much attention since Sinai described the first class of hyperbolic billiards in 1970. Outer billiards, on the other hand, are relative new comers in dynamical systems and have received comparatively little attention. They appear to have many interesting connections to ordinary billiards and some other physical systems, like impact oscillators. After a brief survey of chaotic ordinary billiards I will describe the first example of a chaotic outer billiard.

ON EMBEDDING ORDERED PARTIAL GROUPOIDS INTO PARTIALLY ORDERED SEMIGROUPS

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Key Words and Phrases: ordered partial groupoids
AMS Subject Classification: partial algebras

Partial algebras have proven useful in applications such as computer science, mathematical linguistics, and the theory of measurement (Burmeister, Chomsky, Cohn, Krantz et al., Ljapin and Evseev). In applied areas, operations are not always defined, nor are such structures understood through the usual algebraic methods. More specifically, observations do not necessarily fit readily into the groupoid model with its complete binary operation. Observations are often incomplete, leading to a more appropriate model as a partial groupoid, with its partial binary operation. Ljapin and Evseev provide the first treatise with a systematic treatment of partial groupoids.

Research began some time ago on the general problem of embedding partial groupoids in semigroups (Tamari, Ljapin, Ljapin and Evseev). Some work has been done on determining axioms which guarantee that such a partial algebra can be embedded in a semigroup (Conrad, Schmidt, Gensemer and Weinert).

This study takes that research one step further and asks what additional axioms can be articulated to guarantee an ordered partial groupoid's embedding in a partially ordered semigroup. Gornostaev provides a general characterization of such an embedding, but does not provide axioms guaranteeing such an embedding. We develop axioms systems incorporating various definitions of associativity in a partial groupoid context which, along with other axioms, guarantee an embedding into a partially ordered semigroup. Extensions are explored regarding axioms which allow an ordered partial groupoid's embedding in a partially ordered commutative semigroup and other partially ordered semigroups with special properties.

This study could be viewed as a step toward embedding such structures in the real numbers with addition and the usual ordering. This may prove to be useful in economics and other areas where real number representations are of interest.

SYMMETRIC ONE-FACTORIZATIONS OF THE DIRECT PRODUCT OF GRAPHS

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Key Words and Phrases: direct product, one-factor, one-factorization
AMS Subject Classification: 05C70, 05C25

A one-factor of a graph is a set of edges forming a spanning one-regular subgraph (a perfect matching); a one-factorization is a partition of the edges of the graph into one-factors.

The direct product of two graphs, denoted $G \times H$, is that graph whose vertices are $\{(g, h) : g \in V(G), h \in V(H)\}$ whose edges are given by $(g, h) \sim (g', h') \Leftrightarrow (g \sim g')$ and $(h \sim h')$. This product has been studied for many years under a variety of names, including *Kronecker product*, *tensor product*, and *conjunction*. We examine the question of conditions on the two graphs G and H that assure that the product $G \times H$ has a one-factorization. In particular, we look at the product of a cycle C_n with a regular graph, and demonstrate sufficient conditions for the existence of a one-factorization that possesses an elegant group of symmetries.

AN ISING-LIKE MODEL FOR A NONEQUILIBRIUM SYSTEM

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Key Words and Phrases: nonequilibrium statistical mechanics, Ising model, matrix mechanics
AMS Subject Classification: 82D30, 81P15, 82C20, 82C26

We consider a quantum model for a finite, Ising-like system, in the context of 0-dimensional matrix mechanics. Our focus is on developing a model in the context of nonequilibrium statistical mechanics. The evolution of disorder and frustration in this system is associated with a characteristic phase transition. An unusual aspect of the model is a two-dimensional time, which from an experimental standpoint would be observed to exhibit the ordinary sequential behavior that we associate with time as a one-dimensional quantity. We will discuss the relationship of this model to the random field Ising model and to the Ising spin glass. The relationship of 0-dimensional matrix mechanics to the field theoretic formulation of quantum mechanics will also be discussed.

ON FINITE MOMENT CONDITIONS FOR THE LADDER EPOCHS OF RANDOM WALKS

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Key Words and Phrases: Random Walk with Negative Drift, Ladder Epoch, Generalized $GI/G/1$ Queue, Busy Period
AMS Subject Classification: 60G50

Let $\{X_n : n \geq 1\}$ be a sequence of independent identically distributed random variables with $-\infty \leq E(X_n) < 0$, and $Z_n = \sum_{i=1}^{n} X_i$, $n = 1, 2, \ldots$. Then $\{Z_n\}$ is a random walk with negative drift. Let $\{(S_n, T_n)\}$ be an independent identically distributed sequence of vectors of non-negative random variables, where for each n, S_n and T_n may be *dependent*, such that

$$S_n - T_n \sim X_n, \quad n = 1, 2, \ldots.$$

By studying the finite moment conditions for busy periods of a generalized $GI/G/1$ queue in which S_n is the service time of the nth arrival, and T_n is the time between arrivals n and $n + 1$, we derive conditions for the finite moments of the ladder epochs of the random walk $\{Z_n\}$.

NONLINEAR THERMODYNAMIC ASPECTS UPON FAZE TRANSITION MODELS

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The considerations upon faze transformation are made for a steel with 0,8% C, molded in ingot mould at 1500 Celsius degrees, in water cooling conditions. It is supposed a non-isotherm process. On huge scales of temperatures the steel changes various types of structure: cubic with centered volume, cubic with centered faces, cubic with centered volume again, related to the various fazes: liquid – δ, austenite, pearlite, martensite, ledeburite, etc. (the microstructure reveals a dendrite disposal), in conformity with phases diagram.

The macrostructure is determined by the dendrite structure, as a result of the competition of two intimate phenomena of the cooling process: nucleation and crystals' growing. Priority evolution's circumstances of one or the other phenomenon were analyzed by metallurgists and physicians. Mathematical literature tried to accord the dendrite structure of the metal with a lattice structure of the process and, implicit, of the elementary domains of the structure, which is placed in the faze mixture of the intermediary zone.

For the local study a representative elementary volume is cut (*ver*) which is attributed thermodynamic and geometrical parameters, is suffering some transformation processes, which defines particular constitutive laws of the faze transformation of the ver. The mathematical models of the faze transformation circumscribe to some nonlinear problems of heat diffusion (the cooling of metallic mould), to the problems of heat and mass transfer (solidification), elasto-plastical deformation with faze transition in the solid materials.

The faze transitions in solid material (solid – solid) are linked to thermodynamics process with hysteresis, which lead to dissipation models; usually the liquid – solid transformations are conducted by reversible process (without internal energy dissipation). The analysis of a hysteresis lens reveals characteristics regarding elasto - plasticity or material hardening.

BOUNDARY VALUE PROBLEMS: IMPACT OF PARALLEL PROCESSING IN BOUNDARY ELEMENT METHODS

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Key Words and Phrases: Parallel Processing, BEM
AMS Subject Classification: 65N38

For Practical Engineering problems of applied mathematics with irregular shaped boundaries, it is difficult to obtain closed formed solutions; these types of problems can be solved by various numerical methods of which Boundary Element Method (BEM) is most accurate and efficient with respect to other methods. In this paper, we consider the two dimensional problem of water flow through a T-shaped pipe. The problem was solved using Boundary Element Method by considering single domain as well as multi-domain problems. Parallel processing concepts have been used and the impact shown graphically. Menu based GUI software has been design for taking various inputs and to show output numerically as well as graphically for different problems.

CONSTRUCTIVE REPRESENTATION THEORY FOR THE FEYNMAN OPERATOR CALCULUS

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Key Words and Phrases: 46T12, 47DO6, 28C20
AMS Subject Classification: Feynman operator calculus, time-ordered, semigroups, Hille-Yosida theorem

In this talk, I survey recent progress on the constructive theory of the Feynman operator calculus. The theory is constructive, in that operators acting at different times actually do commute. First, I discuss a new Hilbert space, which makes it easy to construct the elementary path integral in the manner originally envisioned by Feynman. After developing the time-ordered operator theory I show how to extend a few of the important theorems of semigroup theory (including the Hille-Yosida theorem). This means that the formulation of physical theory using this approach is a natural extension of basic operator theory to the time ordered setting, so that the problematic disentanglement method used by Feynman to justify his theory, by relating it to standard methods is not required in this setting. I then show how the theory can be reformulated as a physically motivated sum over paths, and use this version to extend the Feynman path integral to include more general interactions and prove a substantial generalization of the Feynman-Kac Theorem. If time permits, I will discuss the solution to the two open conjectures of Dyson concerning QED, and the impact of this approach to the foundations for relativistic quantum theory.

POSITIVE SOLUTIONS OF EQUATIONS WITH NONLINEAR CAUSAL MAPPINGS

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Key Words and Phrases: causal mappings, positive solutions, differential equations, functional differential equations, difference equations with continuous time
AMS Subject Classification: 34K20, 34K99, 93D05, 93D25

We consider positive solutions of scalar equations with nonlinear causal mappings (operators). These equations include differential, differential-delay, integro-differential, difference equations with continuous time and other traditional equations. Many books and papers are devoted to positive solutions of various concrete classes of equations, such as functional differential equations and difference equations. But to the best of our knowledge the positivity conditions for equations with causal mappings were not explored in the available literature.

For a positive $T \leq \infty$, let E be a Banach space of functions defined on $[0, T]$ with a norm $\|.\|_E$ and the unit operator I. For all $\tau \in [0, T)$ and $w \in E$, let the projections P_τ be defined by

$$(P_\tau w)(t) = \begin{cases} w(t) & \text{if } 0 \leq t \leq \tau, \\ 0 & \text{if } \tau < t \leq T \end{cases} .$$

In addition, $P_T = I$. A mapping $F : E \to E$ satisfying the condition $P_\tau F P_\tau = P_\tau F$ ($\tau \in [0, T]$) is called a causal mapping (operator). Let $B(0, T)$ be the space of real bounded measurable functions defined on $[0, T]$ with the sup-norm $|.|_{B(0,T)}$. By K_+ the cone of nonnegative functions from $B(0, T)$ is denoted.

For a positive number $R \leq \infty$, put $K_R := \{w \in K_+ : 0 \leq w(t) \leq R,\ \ 0 \leq t \leq T\}$. Let F be a continuous causal operator mapping K_R into K_+ . We prove the existence of positive solutions to the equation

$$x(t) = f(t) + \int_0^t Q(t, t_1)(Fx)(t_1)dt_1 \ \ (0 < t \leq T < \infty),$$

where $Q : [0 \leq s \leq t \leq T] \to [0, \infty)$ is a nonnegative measurable kernel and $f \in K_+$ is given. Applications of the main result to differential, difference and differential-delay equations are also discussed. The suggested approach enables us to consider various classes of equations from the unified point of view.

THE PARABOLIC COMPACTIFICATION AND APPLICATIONS

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The properties of a compactification that maps the n dimensional Euclidean space onto a "parabolic bowl" are studied. Unlike the stereographic projection this compactification distinguishes among the different directions "at infinity". This compactification represents the n dimensional Euclidean space in terms of rational functions.

A few applications of the parabolic compactification will be discussed. These applications include: the rational approximation of unbounded functions. The approximation of unbounded periodic functions by quotients of trigonometric polynomials. The identification of critical points at infinity of polynomial dynamical systems, The representation of solutions of dynamical systems and their rate of blow up. Benefits to the Lorenz equations will also be pointed out.

TWO AGENT BASED MODELS AND MARKET STYLIZED FACTS

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Key Words and Phrases: agent based models; time series; financial market
AMS Subject Classification: 62P05

Traditionally two types of models of financial markets are explored. In the models of the first type financial time series are treated as whole empirical entities. In the models of the second type the interaction between agents is taken into account explicitly and the market is considered as a complicated dynamical system. A validation of an agent based model normally consists in matching model generated time series with real data of financial market. The key question is the criterion of matching. In this paper we consider two agent based models of the financial market: a self-fulfilling Ising model developed by D. Sornette and W.-X. Zhou [1] and a modified model introduced by A.-H. Sato and H. Takayasu [2]. In [1] five metrics are considered which make possible validating of the model. We add one more metric using the index of variation (close to fractal dimension) introduced by M. M. Dubovikov [3]. We evaluate the index of variation of RTS (Russian Trading System) and determine the values of model parameters under which time series generated in the model have the index of variation close to that of RTS.

Further, we modify the model of [2] introducing the influence of the market history on the agents' strategy. The generalized model follows the market stylized facts. For a significant domain of the model parameters the system of the agents drives itself into the critical state. In the critical state the artificial market usually rises. The occasional crises change the periods of the rise.

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HIGH-ORDER ABSORBING BOUNDARY SCHEMES

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Key Words and Phrases: Waves, High-order, Artificial boundary, Absorbing boundary condition, Higdon, Auxiliary variables, Finite elements, Finite Differences, Wave guide, Dispersive waves, Stratified medium, Evanescent waves

AMS Subject Classification: 35L05

The subject of high-order absorbing boundary schemes for the solution of wave problems in unbounded domains will be reviewed. These types of schemes have begun in 1993 with Collino's work, and have seen important progress in the last few years. In particular, the Hagstrom-Warburton local high-order Absorbing Boundary Condition (ABC) for the time-dependent wave equation is presented. This ABC, which is based on a modification of the Higdon ABC, has been further developed and extended in a number of ways: (1) New analysis of the ABC is discussed, which leads to an adaptive scheme for dynamically choosing the ABC free parameters; (2) The ABC is extended to the case of a dispersive medium, for which the Klein-Gordon wave equation governs; (3) The case of stratified medium is considered and the way to apply the ABC to this case is explained; (4) The ABC is extended to take into account evanescent modes in the exact solution. The analysis is applied to two-dimensional wave guides and to exterior scattering problems. The numerical schemes incorporating the ABC include a high-order finite difference scheme and a standard finite element scheme. Numerical examples are presented to demonstrate the performance of the extended ABC.

EFFICIENCY OF MULTIQUADRIC COLLOCATION METHOD FOR SOLVING PDE

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Key Words and Phrases: radial basis function, finite element method, multivariate interpolation

In this paper, we demonstrate the efficiency of the multiquadric radial basis functions (MQ-RBFs) collocation method for solving partial differential equations (PDEs), as theoretically compared to the finite element method (FEM). The MQ-RBF has the property of exponential convergence with respect to the shape parameter. Although the optimal choice of shape parameter is still an unsettled issue, there exists a wide range c of values in which the RBF solution has high accuracy. Error estimation of the approximate solution is also given and the numerical results indicate that the method provides accurate approximations.

COMPUTER-AIDED KINETIC THEORY AND GRANULAR GASES

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Key Words and Phrases: Boltzmann equation, Chapman-Enskog expansion, Granular gases, homogeneous cooling state, transport coefficients, symbolic processing
AMS Subject Classification: 76T25, 82C40, 76Axx, 76P05, 82C70

Sonine polynomials are commonly used to expand the velocity distribution function in the Boltzmann equation. As is well known they are derivable from generating functions. The novel finding we wish to report is that all non-linear and linear coupling constants in the Sonine expanded Boltzmann equation (both for elastically and inelastically colliding gases, the latter representing granular gases) can be expressed as derivatives of a “super-generating” function, an analytic expression for which is presented. Using a symbolic processor we are able to transform the Boltzmann equation, as well as the equations arising in the Chapman-Enskog expansion thereof into a set of algebraic equations. Using this method we have computed (for the first time) the distribution function of the homogeneous cooling state, confirming the existence of its conjectured exponential tail. We have also computed to high accuracy all linear transport coefficients, obtaining excellent agreement with results of numerical simulations. Results corresponding to the Enskog-Boltzmann equation (for moderately dense gases) as well as for binary mixtures are now available as well, and time allowing, will be discussed in the talk alongside possible other applications.

HOW (NOT) TO APPLY THE HIROTA BILINEAR METHOD

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Key Words and Phrases: solitons, Hirota's method, bilinear equations
AMS Subject Classification: 35Q51, 35Q55, 37K10

Integrable nonlinear partial differential equations have attracted much attention of mathematicians as well as physicists. While the former are mainly interested in new methods of solving the initial and/or boundary value problems, the latter look for special solutions representing physical phenomena. Solitons belong to the most important solutions from the point of view of applications to physics and technology. The Hirota bilinear method and its multilinear refinements provide simple tools for construction of such solutions.

Hirota's method consists of several stages, each of which requires some invention and attention. The proper substitution in order to express the equation in the bilinear variables (1), reduction of the excess degrees of freedom which transforms the equations into their bilinear form (2), the perturbation scheme (3) and finally solution of the system of equations at its successive orders of magnitude (4), all these operations have to be done with caution.

The first step may be facilitated by singularity analysis. In the second step careless reductions may easily lead to an overdetermined system even though the result at first sight looks like normal consistent system of equations. On the other hand solving overdetermined reductions may provide useful information. The perturbation scheme requires proper assumptions on the orders of magnitude of the unknown functions. Finally, choice of proper solutions is necessary at the last stage if we want to obtain a physically relevant model.

Each of these steps will be discussed and illustrated with examples from equations of mathematical physics.

STABILITY RESULTS IN LINEAR SEMI-INFINITE PROGRAMMING WHEN COST AND RIGHT-HAND-SIDE COEFFICIENTS ARE PERTURBED

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Key Words and Phrases: sensitivity, stability, parametric optimization
AMS Subject Classification: 90Cxx

This work analyzes the effect on the optimal value of a given linear semi-invite programming problem of the perturbation type which more frequently arise in practical applications: those which affect the objective function and the right-hand-side coefficients of the constraints. In particular, we give some results with respect the exact value of a perturbed problem as a linear function of the perturbation.

CAN THE CG METHOD YIELD CONSERVATIVE FLUXES?

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Key Words and Phrases: continuous Galerkin methods, conforming finite element method, hybridization, elliptic problems, conservation
AMS Subject Classification: 65M60, 65N30, 35L65

Conservative fluxes are often needed in many applications involving second order elliptic partial differential equations. The failure of the standard continuous Galerkin (CG) finite element method to provide such fluxes is well known. This talk is devoted to the development of a new postprocessing technique to overcome this failure. The first step is the computation of a numerical flux trace defined on element interfaces and is motivated by the structure of the numerical traces of discontinuous Galerkin methods. This computation is non-local in that it requires the solution of a symmetric positive definite system, but the system is well conditioned independently of mesh size, so it can be solved at asymptotically optimal cost. The second step is a local element by element postprocessing of the CG solution incorporating the result of the first step. This leads to a conservative flux approximation with continuous normal components. We provide a theoretical error analysis of the postprocessed flux approximation and also display numerical evidence suggesting that the approximation is competitive with the approximation provided by the Raviart-Thomas mixed method of corresponding degree. (This is joint work with B. Cockburn and H. Wang.)

COMBINING PERIODIC SEQUENCES

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Key Words and Phrases: Binary periodic sequences, Stream Ciphers, Correlation attacks, Nonlinear shift registers, Boolean functions, Linear complexity
AMS Subject Classification: 94A60

A classical keystream generator for stream ciphers consists of several linear feedback shift registers whose output sequences are combined by a Boolean combining function to produce the final keystream. These generators are vulnerable to various cryptographic attacks, like algebraic attacks and fast correlation attacks. We study similar keystream generators in which the linear feedback shift registers are replaced by nonlinear shift registers. Due to the avoidance of linearity such keystream generators are more resistant towards mentioned attacks.

If the deployed shift registers have a simple cycle structure, then the minimal polynomials of the produced shift register sequences have a simple algebraic structure. This allows us to prove various properties of the keystream: least period, linear complexity, bias, number of translation distinct keystreams.

In the analysis, it is necessary to know lower bounds for the linear complexities of the individual nonlinear shift register sequences. Such nontrivial lower bounds are not known but must be established computationally. With the Berlekamp-Massey or the Euclidean algorithm this is feasible up to the shift register length 26. We present a probabilistic algorithm which can to the task up to the shift register length 45.

We investigate a new correlation attack which aims to recover the initial states of the driving shift registers. In this attack, the keystream is subjected to certain linear transformations which have the effect that the new sequence is no longer balanced. The attack can be applied to all generators which produce the final sequence out of given binary periodic sequences of relatively short periods, if the periods satisfy certain congruence relations.

EQUATION SOLVING GENERALIZED INVERSES

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Key Words and Phrases: Rings, Moore-Penrose inverse, Drazin inverse, linear and nonlinear systems of equations
AMS Subject Classification: 15A09

In the literature the concept of pseudoinverse (or generalized inverse) was first introduced by I.Fredholm in 1903, where a generalized inverse of an integral operator was given. The generalized inverse of matrices was first introduced by E.H.Moore in 1920. Moore constructed the general reciprocal by means of projectors, established its uniqueness and justified its application as an equating solving. In the next 30 years little was done until mid-1950s when discoveries of the least-squares properties of certain generalized inverses and its relationship to linear equation solving appeared. In 1955, R.Penrose gave a definition of a pseudoinverse of a singular matrix which corresponds to an algebraic equivalent to Moore's definition. In honor to both contribution, this uniquely defined generalized inverse is called *the Moore-Penrose inverse.* A major characteristic of the MP-related inverses is the fact that they provide solutions or least squares solutions for a system of linear algebraic equations. However, depending on specific applications, some other generalized inverses were defined such as *Drazin inverse* and, as a special case, the Group inverse, which possess some inverse-like spectral properties. Moreover, they provide solutions for linear differential and linear difference equations .

During the last 50 years applications of generalized inverses have been developing rapidly, and in spite most of the recent studies focus on non-equation solving generalized inverses, this work will be centered on properties and applications of such item, mostly considering finite matrices over an associative ring with unity.

DECAY ON INFINITY AND UNIFORM ANALYTIC REGULARITY FOR TRAVELLING WAVE TYPE SOLUTIONS

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Key Words and Phrases: decay on infinity, uniform analytic regularity, semi–linear equations
AMS Subject Classification: 35B65, 76B25, 35Q51

The talk will be focused on some recent results on the decay on infinity and the uniform analytic regularity of travelling wave type solutions for evolution PDEs appearing as models in Mathematical Physics and Applied Sciences. We outline an abstract approach (based on perturbative methods in functional spaces, nonlinear composition estimates, and techniques from the theory of pseudo-differential operators) for deriving precise estimates for the decay on infinity and the uniform analytic–Gevrey regularity of solutions to semi-linear PDEs in $\mathbb{R}^n$. As particular cases we recapture large classes of solitary wave and homoclinic type solutions of semi-linear evolution equations like the Schrödinger equation, the KdV equation, the KdV-Burgers equation. We exhibit explicit solutions which show that our estimates are sharp. A crucial ingredient of our investigations is the choice of the functional frame: the scale formed by the Gelfand–Shilov spaces $S^{\mu}_{\nu}(\mathbb{R}^n)$, $\mu, \nu > 0$, $\mu + \nu \geq 1$.

The results are obtained in collaboration with Marco Cappiello (Università di Ferrara) and Luigi Rodino (Università di Torino).

WINDOWED OSCILLATORY INTEGRAL: AN APPROACH TO COMPUTE HIGH-FREQUENCY WAVE FIELDS

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Key Words and Phrases: geometrical optics, uniform asymptotics, Gaussian beams, integral representations
AMS Subject Classification: 35C15, 74G15, 34E10

High-frequency asymptotic methods in wave-propagation problems (quasi-classical in quantum mechanics) are known as physically-based approaches to obtain, in particular, numerical results for the range of parameters which are not accessible by finite-deference or finite-element techniques. However, despite of the physical clearness of asymptotic methods their implementation is not always straightforward: for example, in the simplest form known as the *geometrical optics* the approach is said to be *non-uniform* and fails on manifolds carrying the singularities of the geometrical objects connected with classical trajectories (rays).

Starting the middle of 20th century a lot of work was done to understand the uniform generalizations of the geometrical optics. The *Maslov's theory* (also known as the *canonical operator method*) provides the most common and uniform integral representation for a high-frequency wave field. However, this representation is not effective for numerics. Another uniform integral representation known as *Gaussian beams method* is highly promising because of its simplicity and computational efficiency but has certain internal limitations.

In the talk we give a brief review of known asymptotic techniques describing high-frequency wave processes and suggest the new uniform representation entitled *windowed oscillatory integral* (WOI). The approach preserves all attractive features of Gaussian beams method but can be applied beyond the limitations of the latter. WOI also is asymptotically equivalent to both Maslov's and Gaussian beams methods and thus serves as an "asymptotic bridge" between these two approaches.

A HYBRID ASYMPTOTIC METHODS. TECHNIQUE OF APPLICATION

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Key Words and Phrases: asymptotic methods, hybrid WKB-Galerkin method, deformable solid mechanics

Along with development of computer engineering, application of asymptotic methods to the set of problems increases. A number of complex mechanical problems can be significantly simplified due to application of theses approaches. Asymptotic methods can be also used as a zero-order approximation for the further numerical calculations. A wide variety of deformable solid mechanics problems, reduced to differential equations with variable coefficients and boundary problems, can be solved analytically only in exceptional cases. If differential equation of the problem contains a small dimensionless parameter, it is reasonable to find an approximate solution with predetermined value of parameter. Though, obtained solution exhibits a high accuracy in a small interval of parameter variation, including the solution of a singular differential equation. Since the field of asymptotic solutions application is limited by the value of parameter, hybrid asymptotic methods are more accurate. There are a number of mentioned approaches based on the idea of approximate solution improvement [1-7]. It is possible that one of the first publications devoted to the hybrid Wentzel-Kramer-Brillouin (WKB)-Galerkin method is [6]. WKB-Galerkin method has already shown its advantages in different branches of mechanics and it enables to find an approximate solution as an asymptotic one. This technique permits to obtain especially good results in solution of differential equations with a parameter (small or large) near the higher order derivative [6, 7]. Hybrid methods have proved to be useful in a variety of applications such as slender body, thermal and structures problems, deformable solid mechanics, deformation control and shape control problems, nonlinear dynamics under the deterministic and stochastic external loading and so forth [1, 5-7]. The hybrid WKB-Galerkin method was successfully applied in solution of mechanical boundary problems described by a singular differential equation with variable coefficients. Presented WKB-Galerkin method has the potential of overcoming some drawbacks of the WKB and Bubnov-Galerkin methods applied separately, while combining some of the good features of each. References

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COMBINATORIAL IDENTITIES AND GEOMETRIC SUMS

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Key Words and Phrases: combinatorial identities
AMS Subject Classification: 05A19

There are many examples of combinatorial identities that involve geometric sums. Two such examples are

$$\frac{1}{2^{n+2}}\sum_{k=0}^{n}\frac{1}{2^k}\binom{n+k}{2k} = \frac{1}{4^{2n+1}}\sum_{k=0}^{n}9^k\binom{2n+1}{2k+1} = \frac{1}{6}\left(1+\frac{2}{4^{n+1}}\right),$$

$$\frac{1}{2^{n+2}}\sum_{k=0}^{n-1}\frac{1}{2^k}\binom{n+k}{2k+1} = \frac{1}{4^{2n}}\sum_{k=0}^{n-1}9^k\binom{2n}{2k+1} = \frac{1}{6}\left(1-\frac{1}{4^n}\right).$$

In this talk we show how to extend these kinds of identities to obtain new identities. One such identity is given by

$$\frac{1}{4^{2(n+1)}}\sum_{k=0}^{n}9^k\left(\binom{2n+1}{2k}-\binom{2n+1}{2k+1}\right) = \frac{1}{12}\left(1-\frac{1}{4^n}\right)$$

$$= \sum_{i=1}^{n}\left[\sum_{k=0}^{i-1}\sum_{j=2k+2}^{i+1+k}\binom{i+1+k}{j}\frac{(-1)^{j+1}}{2^{i+2+k}} - \sum_{k=0}^{i}\sum_{j=2k+1}^{i+1+k}\binom{i+1+k}{j}\frac{(-1)^{j}}{2^{i+2+k}}\right].$$

We discuss algebraic as well as a computer-generated proofs of such identities.

DESCRIPTION OF ROTATIONAL PHENOMENA USING MATRIX SOLUTIONS OF THE NONLINEAR WAVE EQUATIONS

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Key Words and Phrases: matrix solution, nonlinear wave equation, particles collision, simultaneous rotations, vortex ring
AMS Subject Classification: 35Q58

About ten years ago the nonlinear Klein-Gordon equation was considered over the field of hyper-complex numbers and the corresponding exact solutions were constructed. Using the matrix representation of the hyper-complex numbers these solutions were converted to matrix solutions of the Klein-Gordon equation and others wave equations considered as the matrix equations.

In the subsequent years the intense attention was concentrated on the studying of basic properties of the constructed matrix solutions and their applications to mathematical modelling. The rotational property of matrix solutions is a topping property which is used for the modelling of the vortex rings in fluid flows and can be used for the describing the toroidal form of the basic elementary particles.

More interesting application of matrix solutions at this time is their using for simulation of the particles collision. Key point in this simulation is based on the theorem of the composition and decomposition of simultaneous rotations in a plastic matter such as gas-like ether.

ON REGULARIZED TRACE OF A DIFFERENTIAL OPERATOR OF SECOND ORDER WITH UNBOUNDED OPERATOR COEFFICIENTS

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Key Words and Phrases: Hilbert space, self-adjoint operator, kernel operator, spectrum and resolvent

AMS Subject Classification: 47A10, 47A70, 47A55, 47B34

Let H be a separable Hilbert space and let $H_1 = L_2(H; [0, \pi])$ denotes the set of all measurable functions f with values in H and such that

$$\int_0^\pi \|f(x)\|_H^2 \, dx < \infty.$$

We consider the operators L_0 and L in H_1 which are formed by the differential expressions

$$l_0(y) = -y''(x) + Ay(x) \qquad \text{and} \qquad l(y) = -y''(x) + Ay(x) + Q(x)y(x)$$

and the same boundary conditions $y(0) = y'(\pi) = 0$ respectively. Suppose that A and $Q(x)$ in the above expressions satisfy the following conditions:

(1) $A : D(A) \to H$ is a self adjoint operator. Moreover, $A \geq I$ and $A^{-1} \in \sigma_\infty(H)$ where I is an identity operator in H and $\sigma_\infty(H)$ is the set of all compact operators from H to H.

(2) For every $x \in [0, \pi]$, $Q(x) : H \to H$ is a self-adjoint compact operator. It is also a kernel operator $(Q(x) \in \sigma_1(H))$.

(3) The functions $\|Q^{(i)}(x)\|_{\sigma_1(H)}$ $(i = 0, 1, 2)$ are bounded and measurable in the interval $[0, \pi]$.

(4) For every $f \in H$, $\int_0^\pi (Q(x)f, f)_H \, dx = 0$.

In this work we will firstly show how the concept of regularized trace for operator L is constructed and later will obtain a formula for this regularized trace.

EFFICIENCY MEASUREMENT OF PRIVATE TEACHING

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Key Words and Phrases: Data Envelopment Analysis, efficiency measurment, Optimization

Comparing data groups is an integral part of our daily life. Efficiency and productivity, because resources are limited and human wants unlimited, are important and going on in most area. It is the biggest problem for every economy how the scarce resources distribute suitably in the society. Performance is the quantitative and descriptive explanation of what an individiual, a group or a company can provide regarding the business done. The main objective of the economical growth and development is to improve the performance. Regarding the performance, there are performance measurement models prepared for various purposes and applications. However, there is not one single performance measurment model to be applied for all sectors. Data Envelopment Analysis(DEA), which has been widely used lately is one these models.

Data Envelopment Analysis is a well-known technique for efficiency analysis of business and entities or organizations. In this study, relative efficiencies of private teaching institutions for year of 2006, were measured using DEA.

ANALYSIS OF WIND-INDUCED VIBRATION OF OVERHEAD TRANSMISSION LINES

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Key Words and Phrases: overhead transmission lines, long span with in-span fittings, wind-induced aeolian vibrations, subspan oscillations
AMS Subject Classification: 70

Overhead transmission lines with high-voltage levels of 20 - 500 kV may be subjected to wind induced conductor motion in form of

- high frequency vibration, also known as Karman or aeolian vibration, with the critical frequency range between 5 - 50 Hz,
- low frequency vibration or conductor galloping, where icing is one of the triggering factor and the frequency range between 0,1 - 3 Hz,
- subspan or bundle conductor oscillation, which is a wake induced oscillation of flutter type.

All kind of motions may cause extensive damage to components of the conductor support system or the conductor itself. In the present paper attention will be focused on conditions in very long spans in range of a few thousand meters (e.g. fjord or river crossing), where there often is the need for large number of in-span fittings, such as dampers and aircraft warning spheres. This adds additional complexity to the wind-induced vibration problems for the conductor.

For conductors with a cable length of some hundered meters for estimating the vortex excited vibrations of the lines, in the case of aoelian vibrations a simple method, the Energy Balance Principle is well established for spans damped near the end. For long spans with in-span fittings this simple approach is no longer feasible, since the location of the fittings in the span as well as their dynamic characteristics are of importance. The vibration levels may now differ considerably along the span and this is reflected in the vibration modes. A modified approach to this problem is described.

In the case of wake-induced vibrations, the critical wind velocity (bifurcation point) of incipient flutter will be changed. New conditions for which flutter in the line is triggered are specified.

DYNAMIC ANALYSIS OF QUEUEING MODELS

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Key Words and Phrases: Queueing models, C_0-semigroup, spectrum, adjoint operator
AMS Subject Classification: 60K25, 47D03, 47A10

In this lecture, we will introduce our work about queueing models. This lecture is devided into 5 parts. In Section 1, we will state problems that we will study. In Section 2, we will introduce results about well-posedness of queueing models. In Section 3, we will introduce results about asymptotic properties of time-dependent solutions of queueing models. In Section 4, we will introduce asymptotic properties of queueing length and other indices. In Section 5, we will state further research problems.

ON THE HISTORY OF INVESTIGATION OF CONVEX POLYHEDRA WITH REGULAR FACES

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Below the term "regular-faced polyhedron" refers to any convex polyhedron in which all faces are regular polygons, possibly different.

Esaulova's initiative. In 1946 L. N. Esaulova (Tashkent, Uzbekistan) sent a letter to A. D. Aleksandrov, Chairman of the Seminar on geometry at Leningrad University (USSR). In this letter she presented a list of possible types of vertices of regular-faced polyhedra, and proved the theorem: In addition to 5 regular polyhedra, 13 semiregular polyhedra and two infinite series (prisms with regular bases and square lateral faces, and antiprisms with a belt of lateral faces consisting of regular triangles), there can exist only a finite number of other regular-faces polyhedra. Diagrams of several polyhedra were attached to the letter.

A. D. Aleksandrov sent this letter to V. A.Zalgaller only in 1961. We can guess why Aleksandrov recalled Esaulova's letter after 15 years. In 1960 N. W. Johnson [1] published a brief communication about his list of regular-faces polyhedra different from prism and antiprisms. He and Grunbaum [2] proved that the only regular-faced polyhedra that contain polygons not occurring in Archimedean polyhedra are prisms and antiprisms. Bernal [3] noticed a relation between regular-faced polyhedra and the structure of some liquids . Aleksandrov may have seen the article [3] in the journal "Nature" that was available in the USSR. He may then recalled Esaulova's letter and sent it to Zalgaller, without however mentioning Johnson's list.

Zalgaller's method. V. A. Zalgaller [4] calls a regular-faced polyhedron simple if it cannot be dissected into two regular regular-faced polyhedra by a plane intersecting the surface of the polyhedron only in edges of the polyhedron.

The problem posed in [4] was to find all simple regular-faces polyhedra $\{M_i\}$ different from prisms and antiprisms. After the problem has been solved, finding all regular-faces polyhedra different from prisms and antiprisms is no longer a challenge. Zalgaller offered a strategy based on Cauchy's theorem [6] for solving the problem posed in [4]. In 1962 while arranging the 239^{th} Mathematical School in Leningrad, USSR, he organized a group of students [5] to pursue the starting steps of this strategy. The investigation was completed in 1966 [7] and published in a monograph [8].

Johnson's list. N. Johnson, a US geometer, approached, in parallel and independently of Esaulova, the same question and advanced much further. It is likely that he took a heuristic way and in 1960 he announced [1] a set of 92 regular-faces polyhedra in addition to the regular and semiregular polyhedra, prisms and antiprisms.

Johnson's complete publication [9] includes the list of these 92 polyhedra. He named each of them. In most cases the name also points to the construction method for a polyhedron. Diagrams of all 92 polyhedra are available in the Internet [10].

Johnson conjectrures in [9] that his list is complete. After reading [4], Johnson sent his article [9] to Zalgaller. The main result of Zalgaller's monograph [8] is the proof that only 28 simple regular-faces polyhedra can exist in addition to prisms and antiprisms. There is another important result in the monograph [8]: This is the theorem that Johnson's list is complete. The theorem was formulated in [8] on page 18, but because of the limitation on length of the publication, the procedure of proving is presented only briefly.

Disciples. After publishing [8] and [11], Zalgaller entrusted his pupils to continue the investigations of convex polyhedra with regular faces. B. Ivanov and Yu. Pryakhin considered the development of polyhedra for the cases when some dihedral angles are straight [12], [13]. The case when some vertices of the polyhedra are inside the face planes or on the edges is considered in [4].

Application and generalization. In 1988 P. Makarov [15] employed Zalgaller's polyhedra to deduce semiregular polyhedra in four-dimensional space. In 1989 L. Aslanov [16] used Zalgaller's polyhedra to simulate the structure of a substance. After Aslanov's investigation [16] Zalgaller understood that the researchers' interest was not confined to only convex polyhedra with regular faces. Even more attention was given to arbitrary polyhedra with regular or close-to-regular faces. The finiteness of the number of such polyhedra is shown in [17]. While looking for the list of the polyhedra mentioned in 17], it was found [18] that in addition to the polyhedra determined in [12]and [13], there was an abundance of other polyhedra with regular faces having some pairs of faces within one plane. V.Zalgaller took an interest in this study. He approved of restoring his proof of completeness of the list of polyhedra by N. Johnson. And, after checking [12], [13] and [14], started a search for a complete list of polyhedra.

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Substitute into the position of the dots without breaks the name of one of 92 polyhedra the picture of which you want to see, e.g.GyroelongatedCupola.html and you will have the picture of polyhedron N 92.

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RELIABILITY-BASED SCHEDULING MODELS FOR SYSTEMS SUBJECT TO FAILURE

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Key Words and Phrases: Reliability, Fixed Job Scheduling, Maintenance Scheduling, k-out-of-n Systems, Optimization
AMS Subject Classification: 90B25

Reliability models have been widely used in scheduling, which is a well-defined and well-studied area in Operations Research. In this study, we consider two different types of reliability-based scheduling problems in stochastically failing systems of components. In the first problem, we consider the scheduling of tasks in a system of servers subject to stochastic failures. We assume that we have a predetermined and fixed number of servers to process a given set of jobs during a given time span. Once a server fails, it is assumed that it remains in a nonfunctional condition until the end of the time span. We further assume that the system of servers can only operate as long as at least k of the m servers are in functioning state. We analyze the cases for a general k, and a special case where k=1 that corresponds to the parallel server structure. The jobs have fixed ready times and deadlines, and the objective is to maximize the total weight of the processed jobs. Hence, the problem can be modelled as an operational fixed job scheduling problem with stochastically failing servers. We also present our solution procedures.

The second part of our study is on maintenance scheduling, which is another popular area of scheduling. Specifically, we investigate the preventive maintenance (PM) schedules for a system subject to failure. Our concern is to find the optimal intervals between the PM activities. We formulate the problem of finding optimal maintenance intervals, T_i, until replacement. We provide the optimal solution based on analysis of the problem using the reliability threshold value.

GRUNSKY IDENTITIES AND INEQUALITIES

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Gronwall discovered the area theorem for conformal maps in 1914. Later, in the 1930s, Grunsky found a general form of the area theorem, now known as the Grunsky inequalities. Here, we explain how to understand the Grunsky inequality in terms of Beurling-type operators. The approach also supplies a "Grunsky identity", which has the Grunsky inequality as an immediate consequence. We further discuss how more general Grunsky identities may be obtained.

BORDERLINE BEHAVIOR FOR 2 × 2 ITERATIVE SYSTEMS

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Key Words and Phrases: iterative systems, linear classification, generalized eigenvectors
AMS Subject Classification: 39B12

The study of iterative systems of the form $\overrightarrow{x}_{n+1} = A \times \overrightarrow{x}_n$ for an $n \times n$ matrix A has been greatly enhanced by the use of spreadsheet techniques and specialized computer software. Some of the phase portraits for iterative systems were outlined in a systematic fashion by Hubbard and West in the companion book to their pioneering software MacMath. As part of a complete categorization of such systems, the author, in collaboration with Professor Jean-Marie McDill of California Polytechnic University and Hubert Hohn of Art Media Lab (Boston, MA), worked on extending the linear classification tool for ordinary differential equations, which was part of *Interactive Differential Equations* (IDE), to a tool for iterative systems. The attempt to do so led the author to discover that the iterative case exhibits more complicated behaviors, some of which are quite remarkable. The talk will begin by introducing the colorful tool from IDE, followed by an introduction of the new iterative tool designed by Hohn. Many of the borderline cases in the Trace-Determinant plane of the iterative systems will be exhibited, and then a complete classification of the corresponding phase portraits will be supplied in detail. We will look at the general solutions for the most interesting cases in terms of eigenvectors and find generalized eigenvectors when required. As an application, we will see that some behaviors of Markov chains are predicted by the parameter plane results.

ON PATH PARTITIONS AND COLOURINGS IN DIGRAPHS

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Key Words and Phrases: digraphs, path partitions, colourings, k-colourings, orthogonality
AMS Subject Classification: 05C15, 05C20, 05C38, 06A07

The well known Dilworth's Theorem [Dil] states that in a Partially Ordered Set (Poset) the minimum number of chains in a chain partition equals the maximum size of an antichain. The Greene-Kleitman Theorem [GK] extends Dilworth's Theorem to the case where a set of k antichains is considered instead of one antichain. Berge's [Be82] strong path partition conjecture from 1982 generalizes and extends Dilworth's Theorem and the Greene-Kleitman Theorem to all digraphs.

The conjecture is known to be true for all digraphs only for $k = 1$ (by the Gallai-Milgram Theorem [GM]) and for $k \geq \lambda$, by the Gallai-Roy Theorem [Gallai], [Roy], where λ is the cardinality of the longest path in the graph. It was also proved for all acyclic digraphs (see [Linial81], [Cam94], [Saks], [Cam86] and [AHH]), and recently, it was proved by Berger and Hartman [BH] for $k = 2$, and by Hartman and Kartin for $k = \lambda - 1$.

The 'dual' problems are defined by exchanging between the roles of 'independent sets' and 'paths'. So the 'dual' of Dilworth's Theorem states that in a Poset the minimum number of antichains covering the Poset (i.e., an optimal colouring) equals the maximum size of a chain. This result is easy to prove from Dilworth's Theorem by considering the complement of the Poset. However, for general digraphs, there is no apparent relation between the 'dual' problems and the original problems.

In this talk I will give an overview of the related problems and various proof techniques.

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ON TERNARY ADDITION CHAINS

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Key Words and Phrases: addition chain
AMS Subject Classification: 11Y16, 11Y55

A p-ary addition chain of n is, by definition, a sequence of integers $a_1, a_2, \ldots, a_r = n$ under the condition that for any $1 \leq k \leq r$ there exists $i_1, i_2, \ldots, i_p < k$ such that $a_k = a_{i_1} + a_{i_2} + \cdots + a_{i_p}$. When $p = 2$, they are binary addition chains and were introduced in sight of optimization of power multiplications. The interest centers to find $\ell(n)$ the minimum length r of binary chains. In literature several method are proposed when $p = 2$ to find $\ell(n)$ by explicit and approximate way. One of the way to find them approximately is known as a fixed window method. 2^t-ary addition chain may be found in this context.

In this talk, we would like to discuss p-ary addition chains in general positive integers p theoretically. Most of the notions introduced in $p = 2$ has its p-version and some properties are easily generalized, some, become involved. For example, binary addition chains containing just one small step are known and are classified in 6 types. It is true such chains can be classified in finite fashion in general p, but to count up them is not so easy even when $p = 3$. On the other hand, the pruning bounds used in a back tracking algorithm to find minimal chains can be generalized into very clarified form. We also show numerical examples for $p = 3$, which one might have an interest in.

IMPLEMENTATION OF S-STAGE IMPLICIT RUNGE-KUTTA METHOD OF ORDER 2S FOR SECOND ORDER INITIAL VALUE PROBLEMS

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Key Words and Phrases: implicit Runge Kutta method, iterative technique, initial value problem

This study is concerned with the approximate solution of the special second order initial value problem. For this purpose we use the s-stage (s=2,3,4) Implicit Runge-Kutta methods of order 2s. Fourth order method is used by two iterative technique, perfect square iterative scheme and Cooper and Butcher iterative scheme while sixth and eighth order methods are solved by using Cooper and Butcher iterative scheme. We have converted the scheme in such a way that only two function evaluations are required per iteration for fourth order method, three for sixth order method and four for eighth order method. Finally we present the numerical results.

ON VASICEK STOCHASTIC INTEREST RATE PROCESS WITH STOCHASTIC VOLATILITY

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This paper investigates Vasicek Stochastic interest rate processes with stochastic volatility described by a Vasicek model with constant parameters.The paper treats the discretized version of these processes as a system of recurrence relations and a closed form solution for the interest rate process is obtained. The density of the stochastic term (in this discretized version of the interest rate process) is obtained as an improper integral, as an infinite series in terms of modified Bessel functions and as an infinite series with a remainder term, where an upper bound for the remainder term is given. Finally, the characteristic function of the distribution having the above mentioned densities is found. The remarkable result is that this characteristic function appears as the generating function for Laguerre polynomials Ln-1/2 .Also the connection between Ln-1/2 and Hermite polynomials stated. Using this characteristic function moments of all orders are obtained.

HIERARCHICAL PARALLEL PROCESSING WITH LINEAR RECOGNITION UNITS

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Key Words and Phrases: parallel computation, factorization formulae, parallel algorithms, representations, pattern recognition, induction

AMS Subject Classification: 66R05, 11B34, 03H05

We present an *"example"* topology that allows study of the capabilities of a hierarchical parallel processor each of whose units is capable of simple linear recognition. We show that such a hierarchical parallel processor has the capacity to quickly identify complex factorizations. For example, it can discover formulae associated with well known number-theoretic results such as the closure of integers that are the sums of two squares or the sums of four squares. This parallel processor has similarities with A. Uttley's Informon.

We briefly illustrate the topology and parallel processor using the simple example of factorization of squares: (x^2-y^2)=$(x-y)(x+y)$. Any equality that can be obtained from (x^2-y^2)=$(x-y)(x+y)$ by making numerical substitutions for variables will be called an *example*. We allow the vacuous example with no variable substitutions. We further classify *examples* based on the *number of variables*. Thus (x^2-y^2)=$(x-y)(x+y)$, (x^2-9^2)=$(x-3)(x+3)$, and $(10^2$ - $8^2)$ = (2)(18) are *2-variable, 1-variable* and *0-variable examples* respectively. We define the smallest topology on the set of *examples* where *(k+1)-variable examples* are *limit points* of the set of *(k)-variable examples* that may be obtained from them by arbitrary numerical substitution. The resulting topology is homeomorphic to an initial segment of the ordinals.

This topology models the natural process of pattern recognition. For example the statement "(x^2-3^2)=$(x-3)(x+3)$ generalizes the infinite set of numerical examples, $(10^2$ - $3^2)$ = (10 - 3)(10 + 3), $(11^2$ - $3^2)$ = (11 - 3) (11 + 3), $(12^2$ - $3^2)$ = (12 - 3)(12 + 3)," mirrors the statement that "(x^2-3^2) = $(x$ - 3)$(x+3)$ is the limit point of the infinite set of numerical examples, $(10^2$ - $3^2)$ = (10 - 3)(10 + 3), $(11^2$ - $3^2)$ = (11 - 3) (11 + 3) etc. "

The topology naturally motivates the following parallel processor algorithm: At stage n, n =0,1,2, linear recognition units recognize the "pattern" in n-variable examples which results in identifying the $(n+1)$-variable formula which generalizes them. For example at stage 0, the *examples* $(10^2$ - $3^2)$ = (10 - 3)(10 + 3), $(11^2$ - $3^2)$ = (11 − 3)(11 + 3), $(12^2$ - $3^2)$ = (12-3)(12+3), are recognized as being generalized by the formula $(x^2$ - $3^2)$ = $(x$ - 3)$(x+3)$." The algorithm allows quick discovery of complicated factorizations without a need of knowing underlying algebraic structures.

LINEAR FINITE DYNAMICAL SYSTEMS OVER $\mathbb{Z}_{p^r}$

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Key Words and Phrases: Linear Finite Dynamical Systems, Cycles' formulas, Galois Rings
AMS Subject Classification: 37B99, 05C38, 99B20, 94C99

A Finite Dynamical System (FDS) is a pair (X, f) where X is a finite set and f is a function of X in itself. The dynamics of a FDS is the study of the orbits of its elements, that is, the study of the set $\{x, f(x), f^2(x), \ldots\}$. We associate to each FDS a graph, whose nodes are the points of X and there is an arrow from x to y, whenever $f(x) = y$. Such graphs are composed only of cycles and trees.

Finite Dynamical systems have appeared in several contexts: recursive finite differences equations over finite fields, some special types of electrical circuits (sequential networks, modular linear systems and control systems), models of neuronal networks, learning neuronal nets, and more recently in genetic networks.

Linear Finite Dynamical Systems (LDFS) are FDS (E, f) where E is a vector space over a finite field and the map is linear. It has been known, for a longtime, a formula (Elspas' formula) relating the sizes and amount of cycles of the graph with the bijective linear map defining the graph. The author extended such results to arbitrary linear maps over fields.

Two further extension will be presented here. Finite fields will be replaced by rings of integer module some power of a prime and vector spaces by free modules. There are several new formulas relating the graph structure (cycles and trees) to the linear map. This extension allows for easy simulations with many valued state variables, since there is no need of additional computational resources (languages usually have built–in facilities for taking module).

A second extension will be the considerations of affine maps. This would allow for direct extension of the complement in 2–element boolean algebra, where $x \mapsto x' = 1 + x$ is an affine map over $Zsub2$.

PRIM DIFFUSIONS OVER SAMPLED DATA

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Key Words and Phrases: minimal spanning trees, heat diffusion equation, level set estimation, data clustering
AMS Subject Classification: Combinatorial optimization, probability, pattern recognition

Prim diffusions are time varying graphs that are grown from a single point, the "root," on a set of measured data points or feature vectors. These discrete time diffusions are constructed by applying Prim's algorithm [1] for determining a minimal spanning tree (MST) over the entire set of data points. Prim diffusions generalize the entropic graph clustering and pattern matching methods developed in our previous work [2], [3], [4]. In this talk we show how Prim diffusions can be used to solve non-parametric inference problems over high dimensional feature spaces. Such inference problems include clustering, classification, and detection. Our approach is motivated by strong convergence of the time varying span of a Prim diusion to an evolving set whose contours are described by partial dierential equations of anisotropic unit. In the presentation we will discuss these convergence results and illustrate the approach for applications including medical imaging [5], sensor networks [6], and other applications.

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THE LAMBDOMA KEYBOARD: APPLYING AND EXPERIENCING MATHEMATICS

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Key Words and Phrases: Pythagoras, Georg Cantor, Helmholtz, Lambdoma, harmonic, sub-harmonic, symmetric, asymmetric, musical intervals, interdisciplinary, electronics, scales, matrix, ratios, algorithms, reference octave, topological knot theory, keynote generating frequency
AMS Subject Classification: Communication Systems

Similarities in matrices of Pythagoras' ChiX, Georg Cantor's array, and the Lambdoma matrix indicate a connective link of 2,500 years. These matrices of ordered sets of ratios may be translated into a musical keyboard that applies to interdisciplinary relationships in visual art, mathematics, music and electronics. Algorithms that generate Lambdoma matrices in Csound and C++ are presented. The Lambdoma matrix of whole number ratios is an application of mathematics to some of Helmholtz musical intervals of unison (1:1), octave ((2n);(2-n)), ratios (p/q) and frequencies (Hertz). Each keynote frequency, such as (341 Hz), keynote (F), interval (Major Fourth) and ratio (4/3) generates its own harmonic and a sub-harmonic field of frequencies based upon its rows and columns of ratios that form an anagram. Within the 16 by 16 Lambdoma matrix' fourth quadrant is revealed a measurement tool for an eight-scale reference octave of ratios, frequencies and musical notations. Pattern recognition using Pythagoras' color-coding of musical notation indicates the non-linear convex aspect of whole numbers (n/1) and their concave inverses (1/n) in an expansion of the Lambdoma matrix. Lissajous figures of the Lambdoma matrix may point to a numerical ratio solution to topological knot theory through the algorithm (arc tan (i/j)).

FEM VERSUS BEM

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We consider the following Problem:

$$-\Delta u(x) = f(x) \qquad x \in \Omega$$
$$u(x) = 0 \qquad x \in \partial\Omega$$

We will use this Benchmark Problem to explain the main differences between FEM and BEM and to show where the advantages and disadvantages are for both methods.

We introduce both methods in details and show the main differences in the background of the methods and in the applications.

The main equation of the FEM is

$$\int_\Omega \nabla u(x) \cdot \nabla v(x)\, dx = \int_\Omega f(x) \cdot v(x)\, dx$$

The main equation of the BEM is

$$\int_{\partial\Omega} E(x,\xi) \cdot \underbrace{\frac{\partial u(\xi)}{\partial n}}_{??}\, d\sigma_\xi = \underbrace{\frac{u(x)}{2}}_{g(x)/2} + \int_{\partial\Omega} \frac{\partial E(x,\xi)}{\partial n_\xi} \cdot \underbrace{u(\xi)}_{g(\xi)}\, d\sigma_\xi$$

$$\forall\ x \in \partial\Omega$$

We will discuss Plus and Minus of FEM and BEM regarding these main equations.

The differences in the convergence analysis are not so important as we will show.

ANALYSIS OF FRICTION-INDUCED SELF-GENERATED VIBRATIONS ORIGINATED FROM MODE COUPLING - APPLICATION TO CLUTCH SQUEAL NOISE

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Key Words and Phrases: Mode coupling, instability, damping, gyroscopic effects, limit cycle
AMS Subject Classification: 70K42

The present study deals with an audible disturbance known as automotive clutch squeal noise. A two-degrees-of-freedom phenomenological model is exposed highlighting a non-conservative coupling due to the friction forces, as well as gyroscopic effects.

Then, a parametric study of the stability domain is performed by application of the Hartman-Grobman theorem, analytically and numerically. Important information is obtained on the role of the structural damping regarding the fluttering destabilization, relatively to the coupling actions and especially the gyroscopic coupling.

Furthermore, a non-linear method is proposed in order to determine the limit cycle arising beyond the Hopf's bifurcation point and allowing characterizing the steady vibrations. This method is based on the estimation of the non-linear response by using approximate Fourier series.

Finally, some practical design recommendations are provided in order to reduce the propensity of clutches to squeal.

GENERALIZED EULERIAN FRACTIONS AND THEIR APPLICATIONS TO COMBINATORICS AND SPLINES

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Key Words and Phrases: symbolic operator, power series, generalized Eulerian fractions, generalized Eulerian polynomials, generalized exponential splines, Sheffer-type polynomials, Stirling number of the second kind

AMS Subject Classification: 05A15, 65B10, 65D07, 33C45, 39A70, 41A80

Here presented is a type of generalized Eulerian fractions that can be used to construct the generalized exponential splines and the transformation formulas and summation formulas for various types of power series. Some related topics such as the generalized Eulerian polynomials and Sheffer-type polynomials are also discussed.

PRESSURIZED FLUID PIPELINE LEAK DETECTION METHODS

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Key Words and Phrases: fluid leaks detection, signal filtration, correlation function, neural network
AMS Subject Classification: 00A69

Prompt detection of fluid leaks is an important problem for many technical systems, for example, water leakage from water- and heat-supply networks, oil product leakage from transit pipelines, and so on. The possibility of ecological pollution and other reasons make it important to detect leak events promptly and reliably. In order to mend the pipeline as soon as possible, it is also important to locate the leakage area with great accuracy. Although there are numerous methods that detect leak events, their successful implementation is often hindered due to various reasons. The growing list of requirements for leak diagnostics systems requires continuous improvement of the technical equipment and the data processing algorithms. The efficiency of diagnostics can be increased by using new physical parameter measurement systems, data transfer and processing systems, and efficient algorithms for event recognition. From a mathematical standpoint, the task of improving diagnostics quality belongs to the class of signal recognition problems. The presence of intensive noise requires special approaches to leak event recognition. The authors offer several methods for this case that are based on the theory of filtration and correlation function, as well as methods based on the application of a neural network. These solutions have been applied to real technical systems with good results in leak event detection.

HIERARCHICALLY STRUCTURED BRANCHING-DIFFUSING SYSTEMS

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Key Words and Phrases: branching process, measure-valued process, superprocess, multilevel superprocess
AMS Subject Classification: 60K35, 60G57

The two-level superprocess is the diffusion limit of a two-level branching Brownian motion, where particles are grouped into superparticles which themselves duplicate or vanish according to a branching dynamic, in addition to the motion and branching of the individual particles themselves. We define three classes of initial states for two-level superprocesses and describe the corresponding patterns of longtime behavior, including two very different types of equilibria. Specifically, we show that two of these classes of initial states lead to longtime behavioral patterns in high dimensions that do not exist for ordinary, single-level branching systems or superprocesses. (Joint work with A.Greven.)

MAGNETOHYDRODYNAMIC CYLINDRICAL TAYLOR-COUETTE FLOW

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Key Words and Phrases: hydrodynamic stability, spectral methods
AMS Subject Classification: 76E07, 76E25, 76M22

We consider the flow of an electrically conducting fluid between differentially rotating cylinders, in the presence of an externally imposed magnetic field, and present a three-dimensional spectral solution of the governing momentum and induction equations. Depending on whether the magnetic Reynolds number is small or large, the induction equation may be treated as either diagnostic or predictive. These codes are used to study magnetorotational instabilities in the presence of combined axial and azimuthal magnetic fields. Detailed comparisons with liquid metal experiments will also be presented.

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SMART TECHNOLOGIES FOR STRUCTURAL SAFETY

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This presentation demonstrates progress in applications of so-called Smart Technologies to structural safety and particularly to Structural Health Monitoring and Adaptive Impact Absorption. The monograph (Ref.[1]), under preparations, will present soon more detailed discussion of the considered problems and research results obtained recently in the Smart-Tech team. Smart Technologies are understood as incorporation of imbedded systems (distributed hardware with driving electronics and software tools) into engineering structures in order to improve their safety or performance.

One of typical Structural Health Monitoring (SHM) problems leads to damage detection systems based on array of piezoelectric transducers sending and receiving strain waves. The signal-processing problem is the crucial point in this concept and a neural network based method is one of the most often suggested approaches to develop a numerically efficient solver for this problem. An alternative approach to the inverse dynamic analysis can be based on generalised so called VDM (Virtual Distortion Method) concept making use of pre-computed, time dependent influence matrix. It allows decomposition of the dynamic structural response on components caused by external excitation in undamaged structure (the linear part) and on components describing perturbations caused by the internal defects (the non-linear part). In the consequence, analytical formulas for calculation of these perturbations and the corresponding gradients can be derived.

Motivation for the undertaken research on Adaptive Impact Absorption (AIA) is to respond to requirements for high impact energy absorption e.g. in the structures exposed for risk of extreme blast, vehicles with high crashworthiness, protective barriers, etc. In contrast to the standard passive systems the proposed approach focuses on active adaptation of energy absorbing structures (equipped with sensor system detecting and identifying impact in advance) with high ability of adaptation to extreme overloading. A semi-active or fully-active solutions can be realised via controllable dissipative devices (structural fuses) with no need for important power supply. Feasible, dissipative devices under considerations in the discussed applications can be based on the following technologies: MR fluids or hydraulic and pneumatic piezo-valves.

The following AIA applications are currently under development: Adaptive Landing Gears, Adaptive wind turbines, Adaptive Inflatable Structure.

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COMBINED MESH SUPERPOSITION METHOD AND HOMOGENIZATION APPROACH FOR A CRACK PROBLEM IN PERIODIC COMPOSITES

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Key Words and Phrases: homogenization, elasticity, mesh superposition, crack
AMS Subject Classification: 65M60, 74B05, 74Q05, 74Q15, 74R10

We are concerned with the numerical computation of a crack problem posed on microstructural heterogeneous materials that contain multiple phases in the microstructure. The mechanical failure of such materials is a natural multi–scale effect since cracks typically nucleate in regions of defects on the microscopic scale. The modeling strategy for solving the crack problem considers simultaneously the macroscopic and microscopic models. Our approach is based on an efficient combination of the homogenization technique and the mesh superposition method (s-version of the finite element method). The homogenized model relies on a double–scale asymptotic expansion of the displacements field. The mesh superposition method uses two independent (global and local) finite element meshes and the concept of superposing the local mesh arbitrarily onto the global continuous mesh. The crack is treated by the local mesh and the homogenized material model is considered on the global mesh. Numerical experiments for problems on biomorphic microcellular ceramic templates with porous microstructures of multiple materials constituents are presented.

INVERSE PROBLEMS FOR LINEAR DIFFERENTIAL OPERATORS

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Key Words and Phrases: inverse spectral theory, distribution of eigenvalues, inverse scattering problems
AMS Subject Classification: 34B05, 34L15, 81U40

Consider the one-dimensional Schrödinger operator $Ly = -y" + q(x)y$ on an interval of the real line. By applying boundary conditions we get a system of eigenvalues. Our first topic is the inverse eigenvalue problem: we describe the sets of eigenvalues containing enough information for the unique recovery of the operator. We give an overview of classical and new results, briefly discuss the spectral theoretic background and the problem of stability. Some extremal properties of the eigenvalues will be presented as well. It turns out that this topic is closely related to some inverse scattering problems in quantum mechanics. We show the connections and discuss how to get the operator from the (measured) scattering data. The instability of the inverse problem is the main difficulty here. We present some stability results and some reconstruction procedures.

CSEGGRAPH: COLUMN SEGMENT GRAPH GENERATOR

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Key Words and Phrases: column segment graph, sparse matrix, mathematical derivative, graph coloring

AMS Subject Classification: 68N99, 68R10, 90C27, 90C30

A graph generator associated with the determination of mathematical derivatives is described. The graph coloring instances are obtained as intersection graphs $G_\Pi(A)$ of $m \times n$ sparse pattern matrix A with row partition Π. The size of the graph is dependent on the row partition; the number of vertices can be varied between the number of columns (using single block row partition Π_1) and the number of nonzero entries of A (using m block row partition Π_m). The chromatic number of the generated graph instances satisfy $\chi(G(A)) \equiv \chi(G_{\Pi_1}(A)) \leq \chi(G_{\Pi_m}(A))$.

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RATIO INEQUALITIES FOR HEAT KERNELS AND THEIR APPLICATION TO SPECTRAL GAP

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Key Words and Phrases: Brownian Motion, Heat Kernels, Ratio Inequalities, Spectral Gap
AMS Subject Classification: 60J65, 35K05, 35P15

We will discuss a collection of inequalities related to the first exit time of Brownian motion and the heat kernel, and their relationship to the spectral gap. A recent addition to this collection by Burgess Davis and me provides explicit lower bounds for the difference between the gap of certain convex domains and that of the smallest oriented rectangle containing them. We will sketch the proof of this result and consider ways in which it might be extended.

COMPUTATIONAL AND DYNAMIC SYSTEMS APPROACHES TO PERITONEAL DIALYSIS ASSOCIATED PERITONITIS

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Key Words and Phrases: peritoneal dialysis, peritonitis
AMS Subject Classification: Mathematical Biology

Peritoneal dialysis uses transport through the peritoneal membrane to clear poisons from the blood in patients with kidney failure. Salt- and sugar containing fluid (dialysate) is instilled into the peritoneal cavity, and waste products diffuse into the dialysate. The dialysate is subsequently drained, clearing or removing waste products from the body. Peritoneal dialysis is the predominant mode of dialysis in children with acute or chronic kidney failure, and is commonly used to support adults with chronic kidney failure. The technique itself is relatively simple and does not require expensive facilities or equipment, so it is well suited for use in areas with very limited medical resources to treat individuals with kidney failure. A major problem in peritoneal dialysis is that of peritonitis: the fluid becomes infected. Peritonitis can cause serious morbidity, and contributes to patient mortality. In addition, peritonitis can render the peritoneal membrane unsuitable for further use in dialysis. The antibiotics used to treat peritonitis can compromise whatever kidney function the patient still has. Moreover, long or repeated courses of antibiotics can lead to the development of antibiotic resistance, a very serious and current problem. Unfortunately, pharmacokinetic / pharmacodynamic guidance for the most effective, least costly, and safest ways to treat this infection is not available. In this talk I will describe my recent work on developing mathematical models to help in better managing patients who are supported with this valuable approach.

DYNAMICAL MODELS FOR PLANT PATTERN FORMATION

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Key Words and Phrases: dynamical system, morphogenesis, pattern formation, phyllotaxis
AMS Subject Classification: 37G35, 37N25, 92C15, 92C80

Phyllotactic patterns occur in a wide variety of plants and are the result of a regularly occurring process in shoot apical meristems. Primordia form in succession with the position of the newest primordia determined by the existing primordia. Processes like this are well suited to being modeled with discrete dynamical systems. This talk will present new dynamical systems based on developmental rules proposed by Hofmeister and the Snows. These dynamical systems are compatible with recent discoveries on auxin efflux transporters in meristematic cells. We show that these models have sufficient complexity to provide insight into the developmental process while at the same time they are simple enough that their behavior can be rigorously analyzed. In particular the fixed points of these dynamical systems correspond to commonly observed spiral lattice patterns observed in plants. We will also discuss the role initial conditions have in selecting which fixed points of the dynamical systems are exhibited by plants thereby accounting for the prevalence of Fibonacci numbers as well as less commonly observed phyllotactic patterns.

ON $(p, q, \mu, \nu, \phi_1, \phi_2)-$GENERALIZED OSCILLATOR ALGEBRA AND RELATED BIBASIC HYPERGEOMETRIC FUNCTIONS

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This paper provides with the generalization of the work by Floreanini *et al* . (1993 *J. Phys.* A: Math. Gen. **26** 611 –4) who generated bibasic hypergeometric functions from (p, q)-oscillators. We consider a six-parameter deformed oscillator algebra realized from the $(p, q)-$deformed boson oscillators. We build the corresponding Fock space representation in an infinite dimensional subspace of the Hilbert space of a harmonic oscillator. We then define its realization in terms of a generalized derivative and investigate the relation between this representation and generalized bibasic Laguerre functions and polynomials.

P-MOMENT EXPONENTIAL STABILITY OF IMPULSIVE DIFFERENTIAL EQUATIONS WITH RANDOM IMPULSES AT RANDOM MOMENTS

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Key Words and Phrases: differential equations, random impulses, stability
AMS Subject Classification: 34K10, 34B15, 34K25

Differential equations with impulses are a basic tool to study evolution processes that are subjected to abrupt changes in their state. There are three kinds of typical impulsive differential equations: systems with impulses at fixed time, systems with impulses at variable time, and autonomous systems with impulses (see the monographs [1, 2] and the references cited therein). However, actual jumps don't always happen at determined points but often at random points. In these cases the impulsive moments are random variables. Owing to the effect of random impulsive moments any solution of these systems is a stochastic process, which behavior is very different from the piecewise continuous solutions of the determined impulsive differential equations. At the same time, the qualitative investigation of the impulsive differential equations with random impulses at random moments is absolutely different that the stochastic differential equations. All these cause the necessity of deep studying of different properties of the solutions of the impulsive differential equations at random moments and their applications. In the recent years only partial theoretical results are obtained for such kind of equations ([3]).

In the present paper nonlinear differential equations subject to random impulses are studied. Randomness is introduced both through the time between impulses, which is distributed exponentially, and through the amount of the impulses. Such models arise naturally in the study of a number of physical phenomena, particularly impacting systems. The stability of the solution is defined and studied.

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DYNAMIC PRISONER'S DILEMMA ON SCALE-FREE NETWORK

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Key Words and Phrases: game theory; cooperation frequency; scale-free network
AMS Subject Classification: 91A22, 91A25, 93A10

Evolutionary game theory has been used to characterizing and understanding the cooperative behavior in systems consisting of selfish individuals. Since the groundwork on repeated games by Axelrod, the evolutionary Prisoner's Dilemma Game (PDG) has drawn much attention from scientific communities.

Complex networks can describe a wide range of systems ranging from nature to society and biological systems. Recently, much empirical evidence of real social networks has revealed that they are associated with a scale-free, power-law degree distribution.

In this paper, we study the PDG in a scale-free social network where the agents play the game with a probability proportional to the power of their degree, i.e., $P_i \sim k_i^\alpha$. In this way, the agents' participation in the game change with time, and our study reveals some properties of PDG in a dynamic social structure. Simulation shows the dynamic attending of agents has an important effect on the evolutionary game and reveals that in order to enhance cooperation behavior, we need to constrain participant of low-degree agents and encourage participant of high-degree agents in the game. When α is set to be slightly higher than zero, a maximum cooperation frequency is achieved. Our study may also shed some light on the policy construction of government.

DESIGN ISSUES IN INFRASTRUCTURE WI-FI MESH NETWORKS

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Key Words and Phrases: wireless mesh networks, Wi-Fi networks, infrastructure design, system architecture

AMS Subject Classification: 94-02

Infrastructure Wi-Fi mesh networks are foreseen to be a major technology in future wireless systems. A major characteristic of this type of networks is that it provides a wireless backbone, that is, the wireless access points in the backbone communicate with each other. Wireless backbone provides many benefits which include low infrastructure wiring cost, high capacity and coverage, and better system scalability. With these benefits, infrastructure Wi-Fi mesh networks can be used in many environments such as public hot zones and urban area. However, designing this type of networks requires the considerations of many issues. In this paper, we study these issues in details and give insights into the future research direction of the wireless system development.

SENSITIVITY ANALYSIS FOR A NEW SYSTEM OF GENERALIZED VARIATIONAL INCLUSIONS INVOLVING RELAXED LIPSCHITZ MAPPING

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Key Words and Phrases: system of generalized variational inclusions, relaxed Lipschitz mapping, sensitivity analysis, iterative algorithms, Hilbert spaces
AMS Subject Classification: 49J40, 47H10, 90C30

In this paper,we study the behavior and sensitivity analysis of a solution set of a new system of generalized variational inclusions involving relaxed Lipschitz mapping.The approach is new and generalizes many known results in this field.

APPROXIMATING INTERVAL FUNCTIONS

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Key Words and Phrases: approximation, interval function, least squares
AMS Subject Classification: 65D10, 65G40, 65K10

An closed interval $\mathbf{a} = \{a|\underline{a} \leq a \leq \overline{a}$, where, $\underline{a}, \overline{a} \in \Re\}$ is a set that consists of all real numbers between its lower and upper bounds, inclusively. We use boldfaced letters to represent intervals. The lower and upper bounds of an interval $\mathbf{x}$ are specified by an under- and an over-line $\underline{x}$ and $\overline{x}$, respectively. Ever since interval analysis first introduced by Moore [6], it has become an active research branch in both applied mathematics and computer science [4].

An interval valued function $\mathbf{y} = f(\mathbf{x})$ maps an interval $\mathbf{x}$ to an interval $\mathbf{y}$. If $\mathbf{x} = \{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n\}$ is an interval vector in $\Re^n$, then $f(\mathbf{x})$ is a multivariate function from $\Re^n$ to $\Re$ for $n > 1$. In real world applications, observations of both independent and dependent variables are often interval valued. Therefore, it is important to approximate interval valued functions when a collection of interval valued pairs $(\mathbf{x}, \mathbf{y})$ is provided.

In a previous study, we proposed an approach to approximate an univariate interval function with Lagrange polynomial interpolation [3]. Provided a collection of interval pairs $(\mathbf{x}_i, \mathbf{y}_i)$ both $\mathbf{x}_i, \mathbf{y}_i \subset \Re$ and $i \in \{0, 1, \cdots, N\}$. Then, we can approximate $f(\mathbf{x})$ with degree $d \leq N$ polynomials as the follow:

$$f(\mathbf{x}) \approx \bigcap_S \left[\sum_{i \in S} \mathbf{y}_i \left(\prod_{\substack{j \in S \\ j \neq i}} \frac{\mathbf{x} - \mathbf{x}_j}{\mathbf{x}_i - \mathbf{x}_j} \right) \right] \tag{1}$$

where S is a subset of $\{0, 1, \cdots, N\}$ with cardinality $d+1$. The intention of the intersection is to reduce possible overestimation. It is also assumed that $\mathbf{x}_i \cap \mathbf{x}_j = \emptyset$ whenever $i \neq j$.

In this talk, after review our previous work, we propose algorithms that approximate multivariate interval functions $\mathbf{y} = f(\mathbf{x})$ with a collection of interval pairs $(\mathbf{x}_i, \mathbf{y}_i)$ where $\mathbf{x} \subset \Re^n$, $i \in \{0, 1, \cdots, N\}$, and $\mathbf{y} \in \Re$. We assume that $\mathbf{y}(\mathbf{x}) \approx \alpha_0 + \sum\limits_{1 \leq j \leq m} \alpha_j \phi_j(\mathbf{x})$ where $\phi_j(\mathbf{x})$ is a preselected set of m interval basis functions, and $\alpha = (\alpha_0, \alpha_1, \cdots, \alpha_m)^T$. Approximating an interval function then becomes to determine the vector α.

Let Y be the N-interval vector $(\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_N)^T$. Through evaluating the basis functions at each pair $(\mathbf{x}_i, \mathbf{y}_i)$, we obtain the model $Y \approx \Phi\alpha$ where Φ is an $N \times (m+1)$ interval matrix such that $\Phi_{ij} = \phi_j \mathbf{x}_i$. Hence, a reasonable approach to determine the vector α is to minimize the norm of $||\Phi\alpha - Y||$. Using the principle of the least squares, we can find a vector α by solving the interval normal equation

$$\mathbf{A}\alpha = \mathbf{b} \tag{1}$$

where $\mathbf{A} = \begin{pmatrix} N & \Sigma\phi_1 & \Sigma\phi_2 & \cdots & \Sigma\phi_m \\ \Sigma\phi_1 & \Sigma\phi_1^2 & \Sigma\phi_1\phi_2 & \cdots & \Sigma\phi_1\phi_m \\ \Sigma\phi_2 & \Sigma\phi_2\phi_1 & \Sigma\phi_2^2 & \cdots & \Sigma\phi_2\phi_m \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \Sigma\phi_m & \Sigma\phi_m\phi_1 & \Sigma\phi_m\phi_2 & \cdots & \Sigma\phi_m^2 \end{pmatrix}$, and
$\mathbf{b} = (\Sigma\mathbf{y}_i \ \Sigma\mathbf{y}_i\phi_1(x_i) \ \Sigma\mathbf{y}_i\phi_2(x_i) \ \cdots \Sigma\mathbf{y}_i\phi_m(x_i))^T$.

There are ways other than the normal equations to solve the least squares. For example, a method based on the QR decomposition is available [5] for overcoming possible ill-condition matrices in normal equations. However, we take the normal equation approach for its convenience. The challenge now is how to appropriately bound the solution sets of the interval normal equations since they are mostly irregular shaped even non-convex [7]. Naive applications of interval arithmetic may cause serious overestimation and negatively affect the quality of approximation. We will discuss practical techniques such as inner approximation and volume expansion with examples from real applications [1], [2].

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UPPER AND LOWER BOUNDS OF TEMPORAL AND SPATIAL DECAYS FOR THE NAVIER–STOKES EQUATIONS

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Key Words and Phrases: heat solution, Navier-Stokes, temporal-spatial decay, upper bound, lower bound, weights

AMS Subject Classification: 35Q30, 76D07

We obtain the lower bounds of the temporal-spatial decays for weak solutions of the Navier-Stokes equations

$$C_0(1+t)^{-\frac{5-2i}{4}} \leq \|(1+|\cdot|^2)^{i/2}\mathbf{u}(\cdot,t)\|_{L^2} \leq C_1(1+t)^{-\frac{5-2i}{4}}$$

for $i = 1, 2$. The upper bound parts are estimated in several papers, for example Bae and Jin 2005, and He and Xin 2001. By the interpolation arguments, we also have

$$C_0(1+t)^{-\frac{5}{4}+\frac{\alpha}{2}} \leq \|(1+|x|^2)^{\alpha/2}\mathbf{u}(\cdot,t)\|_{L^2} \leq C_1(1+t)^{-\frac{5}{4}+\frac{\alpha}{2}}$$

for $0 \leq \alpha \leq 2$.

ON THE PROPAGATION OF DISCONTINUITIES OF A SYSTEM OF THE WAVE EQUATIONS

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Key Words and Phrases: the propagation of discontinuities, high-frequency asymptotic, wave equations, smooth characteristics of the variable multiplicity
AMS Subject Classification: 35L15, 34E05

We consider the following problem on the propagation of discontinuities of a system of two wave equations interacting via a potential:

$$\begin{pmatrix} \frac{\partial^2}{\partial t^2} - a_1^2 \frac{\partial^2}{\partial x^2} & 0 \\ 0 & \frac{\partial^2}{\partial t^2} - a_2^2 \frac{\partial^2}{\partial x^2} \end{pmatrix} \overline{u}(x,t) = \begin{pmatrix} 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & 0 \end{pmatrix} \overline{u}(x,t)$$

$$\overline{u}(x,0) = \overline{0}, \quad \frac{\partial \overline{u}(x,0)}{\partial t} = \overline{0}, \quad \overline{u}(0,t) = \begin{pmatrix} \delta(t) \\ 0 \end{pmatrix}.$$

We consider the case where the velocities $a_1(x)$ and $a_2(x)$ coincide at a point (called a singular point). The presence of the partial space derivatives in the potential matrix makes the problem rather complicated. By the ray method an asymptotic solution of a stationary problem is constructed outside some neighborhood of the singular point. Next the stationary problem is reduced to a system of integral equations convenient for the iteration process. By the analysis of a decrease of the iterative kernels a high frequency asymptotic of the solution is found which is also valid near the singular point. Applying the Fourier transform in the frequency to the high frequency asymptotic of the stationary problem, we obtain the parametrix of non–stationary problem, which describes the propagation of its discontinuities.

SINGULAR SPECTRUM ON THE CONTINUOUS ONE OF THE FRIEDRICHS MODEL OPERATORS OF ABSOLUTE TYPE

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Key Words and Phrases: analytic functions, eigenvalues, Friedrichs model, linear system, modulus of continuity, selfadjoint operators, singular point, zeros
AMS Subject Classification: 47B06, 47B25

In $L_2(\mathbb{R})$ we consider a family of selfadjoint operators A_m , $m > 0$, given by $A_m = |t|^m \cdot + V$. Here $|t|^m\cdot$ is the operator of multiplication by the function $|t|^m$ of the independent variable $t \in \mathbb{R}$, and V (perturbation) is an integral operator with a continuous Hermitian kernel $v(t,x)$. We assume that V is non-negative and belongs to the trace class σ_1 : $V \geq 0$, $V \in \sigma_1$. The kernel $v(t,x)$ is assumed to satisfy the following smoothness condition

$$v(t+h,t+h) + v(t,t) - v(t+h,t) - v(t,t+h) \leq \omega^2(|h|) \;,\; |h| \leq 1 \,,$$

with the function $\omega(t)$ (the modulus of continuity of V) monotone and satisfying a Dini condition: $\omega(t) \downarrow 0$ as $t \downarrow 0$, and $\int_0^1 (\omega(t)/t)\, dt < \infty$.

The continuous spectrum of A_m coincides with $[0\,,+\infty)$. We study a problem of finding sharp, in a sense, conditions on the kernel that guarantee that the singular spectrum is absent near the origin. It is shown that such sufficient conditions are given in terms of asymptotic behavior of the modulus of continuity $\omega(t)$ as t tends to zero. It appears that for $m \in (1\,,3]$ these conditions also depend on a rank of the perturbation operator V. Namely, if rank$V < \infty$, then provided that $\omega(t) = O(t^{(m-1)/2})$, $t \to 0$, the spectrum near zero is purely absolutely continuous. But if rank$V = +\infty$, then the structure of $\sigma_{sing}(A_m)$ depends on the value of a constant C in the condition $\omega(t) = Ct^{(m-1)/2}$. The sharpness of these conditions is confirmed by counterexamples. For $m \leq 1$ the spectrum is always purely absolutely continuous in some neighborhood of the zero point on the interval $[0\,,+\infty)$. At the same time for $m > 3$ the singular spectrum may appear near zero for any modulus of continuity $\omega(t)$.

MINIMUM ENERGY CONTROL AND ITS APPLICATIONS

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Key Words and Phrases: null controllability, mimimum energy, Riccati equation, sampled-data system

AMS Subject Classification: 93B05, 93C05, 93C57

A linear system is said to be null controllable with vanishing energy (NCVE) [3] if any state can be steered to the origin with arbitrarily small amount of energy in the sense of L^2-norm. Two sets of necessary and sufficient conditions for null controllability with vanishing energy in Hilbert space are given in [3]. In finite dimensions a linear system is NCVE if and only if it is controllable and zero is the unique nonnengative solution of the singular Riccati equation, which corresponds to the regulator problem with zero penalty on state. A linear system is NCVE if and only if it is controllable and all eigenvalues of the system matrix have nonpositive real parts. The linearized relative motion of a satellite with respect to another in a circular orbit is NCVE if the former has three independent thrusters and the relative orbit transfer problem is considered in [4]. Necessary and sufficient conditions for NCVE are also obtained for discrete-time systems in [2] and applied for periodic systems in [1].

In this paper the minimum energy problem to steer an arbitrary initial state to the origin is considered. It is shown that the infimum is obtained by the maximal solution of the singular Riccati equation. Then a design method of a stabilizing feedback controller which steers a given initial state to the origin with energy arbitrarily close to the infimum is proposed. As an application a circular restricted three-body problem is considered and the stabilization of an equilibrium point is discussed.

The generalization of the minimum energy problem to sampled-data systems and impulse control systems is also considered and their infima are compared. They coincide when the linear system is NCVE.

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GEODESIC MODELING OF A QUASIGEODESIC FLOWS GEODESIC MODELING OF A QUASIGEODESIC FLOWS

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Key Words and Phrases: Quasigeodesic flow, geodesic modeling, dynamical system, ordinary differential equations, affine connection, principal fibre bundle, Riemannian gauge structure, trajectory and point morphism

Quasigeodesic flow (QF) on manifold **M** is the flow **f=(M,f)** (on tangent bundle **TM**) of the second order dynamical systems (or ordinary differential equations) on **M**.

The problem of geodesic modeling of QF has formulated by E. Cartan in other terminology (Sur les varietes a connecxion projective. – Bull. Soc. Math. France, 1924, T.52, P.205-241). This problem is closely connected with T.Levi-Civita's problem of mapping, conserving trajectories of QF for Lagrangian dynamical systems (Sulle Transformazioni delle eqiazioni dinamiche. – Ann. Di Mat. Ser.2. – 1896. – T.24. – P.255-300). The term "geodesic modeling" is belongs to A.Z.Petrov, Ya.L.Shapiro and V.A.Igoshin.

For every QF **f** on **M** has constructed the some generalized affine connection Γ on space **M** $\times$**R,** which geodesic line will be coinciding with integral curve of QF (**f** on **M** $\times$**R).** The space of paths-geodesics of the connection Γ is geodesic (or pulverization) model of the QF **(M,f)** (Igoshin V.A. Pulverization modeling of quasigeodesic flows. – Dokl. Akad. Nauk. SSSR, 1991. – V.320. – N.3. – P.531-535).

By means of geodesic modeling theory, which belongs to the first author, the series of results is obtained. Some of them are connected with point symmetry by S.Lie (Lie S., Engel F. Theorie der Transformations gruppen. – Bd. I – III. – Leipzig: B.G. Teubner, 1888 – 1893). In particular, the problem of a local point triviality of QF is completely solved (Igoshin V.A. Pulverization modeling. I (II and III). – Izv. Vyssh. Uchebn. Zaved., Mat. – 1992. – N 6. – P.63 – 70 (accordingly, 1994. – N 10. – P.266 – 32 and 1995. – N 5. – P.39 – 50). Besides, it is necessary to note results, which are based on Egorov I.P.'s researches and concern to search of dimensions of the maximal Lie algebras of infinitesimal transformations – symmetries QF (Igoshin V.A., Kitaeva E.K. Maximum mobile quadratic quasigeodesic flows of nonzero curvature. Differentsial'naya geometriya mnogoobrazii figures. Kaliningrad university. – 2000. – V.33. – P.41 – 44).

The part of other results concerns to trajectory morphismes of QF. For example, it is obtained geometrical criterions of inclusion of an electromagnetic field in Riemannian Gauge Structure (Igoshin V.A., Ya.L. Shapiro. Quasigeodesic mapping and Riemannian Gauge Structure. – Dokl. Acad. Nauk SSSR. – 1989. – V.305. – 5/ - P.1035 – 1038).

There are also the researches concerning to geometry foliations and fibre bundle with connections (Igoshin V.A. Matem. Zametki. – 1980, V.27 – N 5 and V.28. – N 6) and to geometry of pseudo-riemannian manifolds in general theory of relativity (Igoshin V.A. J. of Math. Sci., 2003. – V.113 – N 3. – P. 471 – 488).

At last, finishing theses (resume), we shall note, that now (at present) we investigate Riemannian geodesic models of QF and, besides, geodesic models of QF of the supreme orders.

A NONSTANDARD DIFFERENCE APPROACH FOR A 2-DIMENSIONAL SINGULAR PERTURBATION PROBLEM

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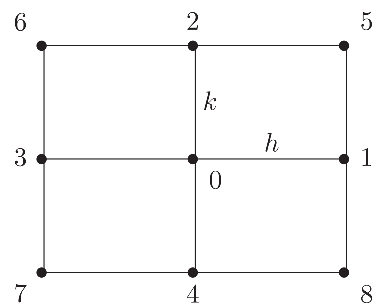
Key Words and Phrases: singular perturbation, finite difference
AMS Subject Classification: 65N06, 74H15, 74S20

We consider

$$\begin{aligned} -\epsilon u_{xx}(x,y) - \epsilon u_{yy}(x,y) + u_x(x,y) &= 2\epsilon\pi^2 \sin(\pi x)\sin(\pi y) + \pi cos(\pi x)\sin(\pi y) \\ u(0,y)=0, \quad u(1,y)=0, \quad & \quad u(x,0)=0, \quad u(x,1)=0, \end{aligned}$$

where the exact solution is $u(x,y) = \sin(\pi x)\sin(\pi y)$.

We combine the two ideas of writing high order derivatives coming from Taylor series expansions in terms of the lower order derivatives, and rewriting the error of the expansions to reinforce diagonal dominance of the resulting system. We develop two techiques using nine equally spaced points as in the figure:



Finally, we present numerical results and investigate stability.

MATCHING METHOD FOR SOME BISINGULAR BOUNDARY VALUE PROBLEMS

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Key Words and Phrases: differential equations, asymptotic expansions, small parameter
AMS Subject Classification: 34E05, 34E10, 35J25

Boundary value problems for differential equations with small parameter ε are considered. Solutions of many those problems depend on small parameter non regularly. We will consider only those problems which solutions $u(x,\varepsilon)$ admit asymptotic expansions $u(x,\varepsilon) \overset{as}{=} \sum\limits_{k=0}^{\infty} \varepsilon^k u_k(x),\ \varepsilon \to 0$, everywhere in the domain Ω except a small neighborhood of a set Γ of lesser dimension.

Sometimes one can obtain uniform asymptotic expansions of the solution as the sum of the above-mention series and inner expansion $v(\xi,\varepsilon) \overset{as}{=} \sum\limits_{k=0}^{\infty} \varepsilon^k v_k(\xi)$ (ξ are new stretched coordinates.) Boundary layer functions $v_k(\xi)$ are exponentially vanish far from the set Γ. But this well known way is impossible for many interesting problems when coefficients $u_k(x)$ of outer asymptotic expansion have their own singularities at Γ. We call these boundary value problems as bisingular ones.

Lecture will be devoted to some bisingular boundary value problems which were investigated by author and his students some years ago. In addition two problems which were investigated recently will be discussed.

The first problem concerns inner asymptotic expansion of the solution of the equation $\varepsilon^2(U_{x_1x_1} + U_{x_2x_2}) + \varepsilon b U_{x_1} + f(x_1, x_2, U) = 0$.

The equation $f(x_1, x_2, U) = 0$ has the cusp point in the origin so its solution $U(x_1, x_2)$ has one or two stable roots in the neighborhood of the origin in dependence of the point (x_1, x_2). The general terms of the inner asymptotic expansion were constructed and investigated in the cases $b = 0$ or $b \neq 0$. One of them is the solution of Abelian equation.

The second problem concerns the initial problem $\varepsilon \dfrac{dx}{dt} = f(x,t,\varepsilon)$, $x(t_0) = A$, $t_0 \leqslant t \leqslant t_1$. We suppose that there exist functions $\varphi_1(t) \in C^\infty[t_0,t_1]$ and $\varphi_2(t) \in C^\infty[t_0,t_1]$ such that $f(\varphi_j(t),t,0) \equiv 0,\ j = \overline{1,2}$,

Let solution $\varphi_1(t)$ is stable and $\varphi_2(t)$ is unstable for $t_0 \leqslant t^* < t_1$. It means that inequalities $\dfrac{\partial f}{\partial x}(\varphi_1(t),t,0) < 0$, nd $\dfrac{\partial f}{\partial x}(\varphi_2(t),t,0) > 0$ for $t < t^*$ are fulfilled.

For $t > t^*$ we consider that solution $\varphi_2(t)$ is stable, nd $\varphi_1(t)$ is unstable i.e. $\dfrac{\partial f}{\partial x}(\varphi_1(t),t,0) > 0$, and $\dfrac{\partial f}{\partial x}(\varphi_2(t),t,0) < 0$.

So we have loss of the stability solution near the point $t = t^*$. Uniform asymptotic expansion of the solution is constructed and proved.

TEMPERATURE FIELD CREATED BY LASTING VARIABLE EXTERNAL INFLUENCE

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Key Words and Phrases: time, initial conditions, method of separation of variables
AMS Subject Classification: 34B05, 34B60, 35K55

Dependance of solution of heat conduction equation on initial conditions decreases exponentially with time increasing. At sufficiently long time, influence of initial conditions is not significant, and temperature field is determinated only by variable external influence. To find such temperature field method of separation of variables can be developed that differ from the generally known one. Such metod is presented and obtained results are analysed.

DYNAMIC STABILITY OF VISCOELASTIC SHELLS

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In this research, the parametric sensitivity analysis in the problem of flutter of viscoelastic cylindrical shells, with an arbitrary difference function of relaxation, is examined by the Laplace integral transform method. The critical value of free stream velocities and vibrations frequencies are determined from the condition that the real parts of the poles in Bromwich integral must be zero, which corresponds to the harmonic motion. Exact value of critical speed and corresponding frequency for a general isotropic viscoelastic constitutive relation with constant Poisson ratio are obtained. The solutions are analyzed for critical, subcritical and supercritical cases. It is shown that the viscoelastic flutter speed is smaller than the corresponding elastic ones if elastic module of material is equal to the initial value of relaxation function. The limit cases for short and long time are analyzed. Influence of aerodynamical damper is studied assuming the parameter of viscous property of material is smaller enough in comparison with the parameter of aerodynamical damper and vice versa. A new analytic method for solution of considered problems of linear aero-viscoelastistisity is developed. The results for the time dependent Poisson ratio are obtained using the elastic-viscoelastic correspondence principle.

FAULT DETECTION FOR UNCERTAIN SYSTEMS USING ADAPTIVE SLIDING-MODE DISTURBANCE OBSERVER

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In industry, to detect faults in plants in early stage is an important issue. This paper considers failure or malfunction as an non-zero input of an unknown fault signal and proposes a method to detect the fault signal for plants with uncertainties. The plant considered in this paper is supposed to have two kinds of uncertainty; unstructured one and structured one. The unstructured uncertainty is unknown disturbance with known upper bound. The structured uncertainty is unknown disturbance of a sum of unknown parameters multiplied by known functions. The method in this paper estimates the unknown fault signal robust to the two kinds of the uncertain signals. The method is to use a disturbance observer, estimating state variables of a system augmented by a dynamic model of the fault signals. Hence the fault signals are a part of state variables of the augmented system, when the state variables of the augmented system is estimated, then the signal also is estimated. To make the observer robust to the unstructured uncertainty, this paper includes sliding mode scheme in the observer, and to be robust to the structured uncertainty, the observer in this paper has an adaptive law to estimate the unknown parameters. The feature of the observer in this paper is to treat the two kind of unknown signals, that is, the fault signals and the disturbances, in the different way and it estimates the fault signals exactly and reject the effect of the disturbances completely.

FACTORIZATION OF ELEMENTS IN CLASSICAL GROUPS INTO A PRODUCT OF INVOLUTIONS

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Key Words and Phrases: involution, structure theorem of modules over PID, factorization of linear maps and matrices
AMS Subject Classification: 15A04, 15A23, 15A33

An element τ in an algebraic system is called an involution if $\tau^2 = 1$. Our purpose is to factorize elements in various algebraic systems into a product of as small number of involutions as possible.

In this direction we present a proof for the excellent Djocović's two involution theorem for linear automorphisms by using system of invariants of a finitely generated module over a principal ideal domain. As a result the proof will become rather simpler.

Also we shall state some results on factorizations of elements in some classical groups into a product of involutions.

DIMENSIONALITY REDUCTION FOR HIGHER-ORDER TENSORS: ALGORITHMS AND APPLICATIONS

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Key Words and Phrases: multilinear algebra, higher-order tensor, rank reduction, singular value decomposition
AMS Subject Classification: 15A69, 15A18

Higher-order tensors are generalizations of vectors (order 1) and matrices (order 2) to order 3 or higher. They have various application areas, such as biomedical engineering, image processing, scientific computing, system identification, control, and signal processing. The mode-n ($n = 1, 2, \ldots$) vectors of a tensor are its columns, rows, etc. The dimension of the vector space spanned by the mode-n vectors is called mode-n rank. This is a generalization of the column and row rank of a matrix. Contrary to the case of matrices, different mode-n ranks are not necessarily equal to each other.

We look for the best rank-(R_1, R_2, R_3) approximation of third-order tensors. In the matrix case, the best low-rank approximation can be obtained from the truncated singular value decomposition (SVD). However, in the tensor case, the truncated higher-order SVD (HOSVD) gives a suboptimal low-rank approximation of the tensor, which can only be used as a starting value for iterative algorithms. We consider three such algorithms based on the orthogonal iterations, trust-region on manifolds and Rayleigh quotient iteration methods. We touch on some of the applications.

SOLVING INTERIOR RIEMANN-HILBERT PROBLEM VIA BOUNDARY INTEGRAL EQUATION WITH CORNERS

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Key Words and Phrases: boundary integral equation, Riemann-Hilbert problem, Fredholm's alternative theory, Dirichlet problem
AMS Subject Classification: 30E25, 45B05, 30C30

We are presenting an effective technique involving boundary integral equations for seeking numerical solution of the interior Riemann-Hilbert problem, briefly RH problem, a class of boundary value problems for analytic functions on a region with a finite number of corners. We begin the talk by presenting an integral equation related to the RH problem and proofs of the solvability and uniqueness of the integral equation and its equivalence to the RH problem will be included. By this proof, the integral equation is suitable for uniquely solvable RH problem. The case of non-uniquely solvable RH problem will also be addressed by modification of the integral equation resulting in a new integral equation associated with the non-uniquely solvable RH problem that is uniquely solvable. The establishment of its solvability will also be included. The developed numerical technique for the problem involves an iterative formula that abolishes singularities during numerical integrations, providing solutions at off-corner points and an interpolation formula for solutions at the corners. Here, we illustrate through numerical examples that the method is successfully employed using Gaussian quadrature as the underlying integrating rule for both uniquely and non-uniquely solvable RH problem with excellent accuracy. Finally, we show that Dirichlet problem falls under the non-uniquely solvable RH problem and obtain a special case of the integral equation for the Dirichlet problem and its advantage over Swarztrauber's integral equation for the Dirichlet problem is given.

THE FISHER INFORMATION METRIC AND POISSON KERNEL MAPS

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Key Words and Phrases: Fisher information metric, Poisson kernel, negatively curved manifold
AMS Subject Classification: 53C21

1. Let (M, dv) be a compact, connected oriented manifold of unit volume form dv. The space $\mathcal{P}(M)$ of smooth probability measures of positive density defined over M is endowed with the Fisher information metric g:

$$g(\tau_1, \tau_2) = \int_M \frac{d\tau_1}{d\rho}\frac{d\tau_2}{d\rho}\rho, \quad \rho \in \mathcal{P}(M), \quad \tau_1, \tau_2 \in T_\rho\mathcal{P}(M).$$

As a Riemannian manifold, this space $(\mathcal{P}(M), g)$ is of constant sectional curvature $c = 1/4$ (T.Friedrich). The Fisher information metric is a generalization of the Fisher information matrix (g_{ij}).

2. Let (X, h) be a simply connected, complete n-dim. Riemannian manifold satisfying $-b^2 \leq K_X \leq -a^2$ for certain positive constants a, b. The boundary ∂X of X at infinity, diffeomorphic to S^{n-1}, carries the standard volume form $d\theta$. The Dirichlet problem at infinity;

$$\Delta u(x) = 0, \quad u|_{\partial X} = f$$

can then be solved in terms of Poisson kernels $\Phi(x, \theta)$, $x \in X$, $\theta \in \partial X$:

$$u(x) = @\int_{\theta\in\partial X} \Phi(x, \theta) f(\theta) d\theta.$$

Since, for any fixed point $x \in X$, $\Phi(x, \theta)d\theta$ yield probability measures on ∂X, we define, associated with the Poisson kernels, the map

$$\varphi : X \longrightarrow \mathcal{P}(\partial X); \quad \varphi(x) = \Phi(x, \theta)d\theta$$

as the family of probability measures on the ∂X parametrized with the Riemannian manifold X.

We would like to report several remarkable results of geometry concerned with the Fisher information metric and the Poisson kernel map φ.

SHORTEST DIRECTING WORDS OF NONDETERMINISTIC DIRECTABLE AUTOMATA

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A *nondeterministic automaton* $\mathcal{A} = (S, X, \delta)$ consists of the following data: (1) S is a set of states. (2) X is an alphabet. (3) δ is a relation such that $\delta(s, a) \subseteq S$ for any $s \in S$ and any $a \in X \cup \{\epsilon\}$.

We will deal with nondeterministic directable automata and their related languages. For nondeterministic automata, the directability can be defined in several ways.

Let $\mathcal{A} = (S, X, \delta)$ be a nondeterministic automaton. In the following definition, $Sw^{\mathcal{A}}$ denotes $\bigcup_{s\in S} sw^{\mathcal{A}}$ for $w \in X^*$.

Definition (1) A word $w \in X^*$ is D_1-*directing* if $sw^{\mathcal{A}} \neq \emptyset$ for any $s \in S$ and $|Sw^{\mathcal{A}}| = 1$. (2) A word $w \in X^*$ is D_2-*directing* if $sw^{\mathcal{A}} = Sw^{\mathcal{A}}$ for any $s \in S$. (3) A word $w \in X^*$ is D_3-*directing* if $\bigcap_{s\in S} sw^{\mathcal{A}} \neq \emptyset$.

In the talk, the new results on the shortest directing words will be presented.

EQUATIONS FOR THE RESPONSE PROBABILITY DENSITY OF A DYNAMIC SYSTEM UNDER N-COMPONENT NON-POISSON IMPULSE PROCESS EXCITATION

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State vector of a dynamic system under a Poisson train of impulses is a non-diffusive Markov process and its joint probability density function satisfies an integro-differential generalized Fokker-Planck-Kolmogorov equation which is also called Kolmogorov-Feller equation. If the train of impulses is driven by non-Poisson, for example renewal, counting processes, the state vector is not a Markov process. Non-Markov pulse problems can be converted into Markov ones by augmenting the state vector of the dynamic system by auxiliary variables driven by Poisson processes. Exact techniques of this kind have been developed for trains of impulses driven by Erlang renewal processes (Iwankiewicz and Nielsen, 1999) or by a generalized Erlang renewal process (Iwankiewicz, 2002). For these classes of random impulse processes the techniques of equations for response moments have been developed.

Recently the explicit equations governing the response probability density have been derived for oscillators under random trains of impulses driven by a single non-Poisson, renewal processes (Iwankiewicz, 2005, 2006).

The excitation considered in the present paper is more general, it is a number of **n** random trains of impulses, each of whom is driven by a non-Poisson, renewal process.

Each of these processes is a renewal process with inter-arrival times being the sum of two independent negative-exponential distributed random variables. These driving processes are assumed to be statistically independent. Each of the impulse processes is recast into a Poisson driven impulse process, with the aid of an auxiliary, purely jump stochastic variable. Hence there are **n** additional state variables. Each auxiliary variable is governed by the stochastic differential equation driven by two independent Poisson processes, with different parameters and it is tantamount to two Markov states.

The Markov chain for the whole problem is constructed by considering the coincidences of the states of the individual jump processes. Thus the total number of Markov states equals $\mathbf{2}^n$. The jump probability intensity functions pertinent to this problem are formulated. The equations governing the joint probability density-distribution function of the response and of the Markov states of the auxiliary variables are derived from the general integro-differential forward Chapman-Kolmogorov equation.

The resulting equations form a set of integro-partial differential equations.

HIERARCHICAL VORTICAL STRUCTURES IN A HOMOGENEOUS ISOTROPIC TURBULENT FLOW

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Key Words and Phrases: hierarchical vortices, filtering, tracking, skeleton

Our recent efforts to understand the hierarchical vortex motions in a turbulent flow are presented. First, multi-scaled vortices are extracted from a forced homogeneous isotropic turbulent field using the Fourier and Wavelet filters. Fourier filter is applied to the velocity field, while the Wavelet filter is applied to the field of the second invariant of the characteristic equation of velocity gradient tensor Q. It is shown that both filtering methods can extract vortices of various scales, and that the structures captured by the two methods are almost the same. The division of a vortex takes place more slowly for the smaller-scaled vortex structures, and the fine-scaled vortices are more actively stretched compared to the larger-scaled vortices. Next, the growth of each extracted vortex is tracked automatically using a simple algorithm, which calculates the overlapping ratio of vortical structures between two sequential times. It is shown that this tracking scheme is capable of handling vortices even when they split into pieces. Finally, the skeleton of each vortex is identified and the characteristics of vortices of different scales are briefly mentioned. In the study, the centerline of a vortex is defined as lines connecting the geometrical centers of the sections. It is found that the vorticity vectors are aligned with the directions of the central axes and that they also have a tendency to be aligned with the vector $\omega_j S_{ij}$. The vector $\omega_j S_{ij}$ appears in the vorticity equation, whose role is to stretch, i.e. intensify, and distort the vortices.

ANALYTIC SOLUTION FOR THE CAUCHY-RIEMANN EQUATION WITH NON-LOCAL BOUNDARY CONDITIONS IN THE FIRST QUARTER

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Key Words and Phrases: Cauchy-Riemann equation, analytic solution, non-local boundary condition

Laplace equation and Cauchy-Riemann equations are the best kind of ellipitic equations. Boundary value problems for the laplace and possion equations are often reduced to the second kind of Fredholm integral equations [3] and [4]. Then the solving process can be continued by numerical methods. But this reduction process to the integral equation about Cauchy-Riemann equation in the first quarter is not possible. Because for this equation in this region, we can not determine and obtain the Green's function [5]. Since the fundamental solution of Cauchy-Riemann equation is not depend on distance in plane, and therefore , we can not determine the Green's function. In this paper, we give an analytic solution for this equation in the first quarter.

For this, we will use the analytic solution of Cauchy-Riemann equation in the upper half plane which have been presented in [5].

STABILITY OF MARRIAGE AND DOWRY

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Key Words and Phrases: non-cooperative game theory, mechanism design, matching markets, double-sided auctions

AMS Subject Classification: 91A10, 91B26, 91B68, 93D99

We consider a variation of the Gale-Shapley [1] matching problem and the Shapley-Shubik [2] assignment game. The problem is best thought of in the following way: There are m men to be matched with n women. Each man has a preference over the various women quantifiable as some M-value. Similarly, each woman has a preference over the various men, quantifiable as some W-value. Each man and each woman is to be matched with at most one woman and one man respectively. All unmatched players remain unmarried. All participants are asked to express their preferences. Moreover, it is allowed that a woman (or her family on her behalf) can offer to pay some dowry to the man. We assume that both men and women have quasi-linear utility functions, and are strategic. The problem is then to determine a matching of men and women that is socially optimal (in the sense of maximizing sum of payoff's of all players) and stable (in the sense of no player reneging on the matching or the *engagement*).

The key requirements are that the designed mechanism be *ex ante individual rational*, i.e., it is rational for every player to participate in such a matching system, it is *strongly budget-balanced*, i.e., the sum of payments paid by women equals the sum of payments paid to the men, and that it is *socially optimal* at equilibrium. Moreover, it would be desirable to have *incentive compatibility*, i.e., it is a dominant strategy for each player to reveal his or her preferences truthfully. A key motivation for this problem is an advertiser-host matching mechanism.

Despite it being a seemingly classical problem, it remains unresolved (see [3], [4] for setup with strategic players but no dowry). Moreover, it is well-known that it is impossible to achieve all the four properties mentioned above in a mechanism (Hurwicz impossibility theorem). We propose a non-Vickrey-Clarke-Groves type, ex ante individual rational mechanism that is able to achieve strong budget-balance. Further, we show that there exists a Nash equilibrium that is efficient.

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STATISTICAL PHYSICS AND FINANCIAL MARKETS

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Key Words and Phrases: Ising spins, computer simulations, financial markets, persistence
AMS Subject Classification: 82C44, 82D80, 91B82, 35B33

After a general overview of the persistence (or "first passage") problem, we discuss the phenomenon at zero-temperature in the disordered Ising model on a square lattice. Results from extensive numerical simulations will be presented. We find strong evidence for 'blocking' regardless of the amount or type of disorder present in the system. In particular, the blocking probability (the fraction of spins which *never* flips) is discussed in both the bond diluted and the random bond cases. The latter case displays interesting non-monotonic, double-humped behaviour as the concentration of ferromagnetic bonds p is varied from zero to one. The peak is identified with the onset of the zero-temperature spin glass transition in the model. Our results are completely consistent with the observation that for infinite systems this model has 'mixed' behaviour, namely positive fractions of spins that flip finitely and infinitely often, respectively. [Gandolfi, Newman and Stein, Commun. Math. Phys. **214** 373, (2000).]

In the second part of the talk, the persistence phenomenon is studied in a financial context by using a novel mapping of the time evolution of the values of shares in a portfolio onto Ising spins. The method is applied to historical data from the London Financial Times Stock Exchange 100 index over an arbitrarily chosen period. By following the time dependence of the spins, we find evidence for a power law decay of the proportion of shares that remain either above or below their 'starting' values. As a result, we estimate a persistence exponent for the underlying financial market to be ≈ 0.5. We also study the persistence of sequences of consecutive positive/negative daily log returns (known as runs).

PARABOLIC BOUNDARY VALUE PROBLEMS IN NONSMOOTH CYLINDERS WITH DATA IN BESOV SPACES

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We adapt the method of boundary layer potentials to the Poisson problem for the heat operator $\partial_t - \Delta$ in a bounded Lipschitz cylinder, with Dirichlet and Neumann boundary conditions. When the lateral datum has a fractional amount of smoothness measured at the Besov scale with parabolic anisotropy, the well-posedness of these problems is obtained in a constructive fashion. More specifically, the solution can be represented as a double layer potential in the Dirichlet case, and as a single layer potential in the Neumann case.

The main theorems we prove extend, generalize and bring together many earlier results, such as the work of E. Fabes and N. Rivière (1978) for C^1 domains; R. Brown (1990) for integer amount of smoothness; D. Jerison and C. Kenig (1995), and E. Fabes, O. Mendez and M. Mitrea (1998) who dealt with the case of the Laplacian.

THE GEOMETRY OF CANTOR SETS DERIVED FROM CONTINUED FRACIONS

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This paper will treat Cantor like sets derived from continued fractions following the geometric approach of Harold Stark as well as the Cantor-like functions derived from such sets. I will explore and show connections to Hausdorf dimension, absolute continuity, category (as in Baire), approximate continuity, the approximate derivative and the related category results.

COUPLING COMPUTATIONAL FLUID DYNAMICS WITH OPTIMIZATION THROUGH GENETIC ALGORITHMS

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Key Words and Phrases: heat and mass transfer, combustion, optimization
AMS Subject Classification: 80A20, 80A25, 80M50

Designing optimal shapes and optimal configurations for practical engineering applications is of high practical importance. Optimization involving computational fluid dynamics (CFD) computations is an intensive field of research and CFD have a high potential to explore a large number of different configurations.

In the present study first a two-dimensional model of a cross-flow tube bank heat exchanger is considered to find the optimal placement in order to enhance the heat exchange and reduce the pressure loss at the same time. Next the optimal configuration for a problem involving coupled fluid flow and heat transfer as well as chemical reactions is investigated and laminar gas flames in low-power domestic burner are considered. The optimisation problem consists in finding the minimal mass-flow rate of the pollutant species CO, while the fuel and oxidizer mass flows of the inlets are varied.

The numerical optimization is carried out using genetic algorithms (GA). The basic idea associated with the GA approach is to search for optimal solutions using an analogy to the evolution theory. During the iteration (or "evolution" using GA terminology) procedure, the decision variables or genes are manipulated using various operators (selection, combination, crossover or mutation) to create new design populations, i.e., new sets of decision variables. The computation time is successfully reduced through the parallel implementation of GA.

The main goal of this work is to achieve cost-efficient design optimization of problems involving complex flows with heat transfer or chemical reactions using computational fluid dynamics.

A FORMULA FOR VERTEX CUTS IN B-TREES

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Key Words and Phrases: graph theory, B-tree, polynomial formula
AMS Subject Classification: 05C05 (Trees), 05C10 (Topological graph theory, embedding), 05C85 (Graph algorithms), 05C90 (Applications), 11T06 (Polynomials)

The paper presents a polynomial formula giving the number and size of substructures that result after removing of one vertex from a b-tree.

The solution proposed for this problem is presented by using of a polynomial formula. Two particular cases are presented.

The obtained polynomial formulas for vertex cuts in b-trees can be generalized, allowing calculations of any structures of interest. The obtained formula works also as limit formulas for trivial trees, which are paths.

CHARACTERISTIC AND COUNTING POLYNOMIALS ON MODELING NONANE ISOMERS PROPERTIES

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Key Words and Phrases: characteristic polynomial, counting polynomials, nonane isomers, Henry's law constant

AMS Subject Classification: 05C10, 05C85, 05C90, 11T06

The major goal of this study was to investigate the broad application of graph polynomials to the analysis of Henry's law constants of nonane isomers.

In this context, Henry's law constants of nonane isomers were modeled using characteristic and counting polynomials and the characteristic and counting polynomials on the distance matrix, on the maximal fragments matrix, on the complement of maximal fragments matrix, and on the Szeged matrix were calculated for each compound.

One of included nonane isomers, 4-methyloctane, was identified as an outlier and was withdrawn from further analysis. This report describes the performance and characteristics of top five significant models. The results show that Henry's law constants of nonane isomers can be modeled by applying characteristic and counting polynomials.

ON THE BASIS NUMBER AND THE MINIMUM CYCLE BASES OF THE WREATH PRODUCT OF SOME GRAPHS

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Key Words and Phrases: cycle space; basis number; (minimal) cycle basis; wreath product
AMS Subject Classification: 05C38, 05C75

A cycle basis $\mathcal{B}$ of a graph G is called a d-fold if each edge of G occurs in at most d of the cycles in $\mathcal{B}$. The basis number, $b(G)$, of G is the least non-negative integer d such that $\mathcal{C}(G)$ has a d-fold basis. The length $l(\mathcal{B})$ of a cycle basis $\mathcal{B}$ is the sum of the lengths of its elements: $l(\mathcal{B}) = \sum_{C\in\mathcal{B}} |C|$. A minimum cycle basis is a cycle basis with minimum length. A construction of a minimum cycle bases for the wreath product of some classes of graphs is presented. Moreover, the basis numbers for the wreath product of the same classes are determined.

A CLASS OF INITIAL VALUE METHODS FOR THE DIRECT SOLUTION OF SECOND ORDER INITIAL VALUE PROBLEMS

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Key Words and Phrases: collocation, interpolation, initial value methods, initial value problems, matrix inversion, multiple finite difference methods, second order
AMS Subject Classification: 65L99

In this paper, we propose a class of Linear Multistep Methods (LMMs) which is applied as Initial Value Methods (IVMs) to obtain the direct solution for second order initial value problems. The class of methods is derived by the interpolation and collocation of the assumed approximate solution and its second derivative at $x = x_{n+j}$,$j = 1, 2, ..., r-1$ and $x = x_{n+j}$, $j = 1, 2, ..., s-1$ respectively, where r and s are the number of interpolation and collocation points. The interpolation and collocation procedures lead to a system of (r+s) equations involving (r+s) unknown coefficients, which are determined by the matrix inversion approach. The resulting continuous coefficients are used to construct the approximate solution from which Multiple Finite Difference Methods (MFDMs) are obtained and simultaneously applied to provide a direct solution to Initial Value Problems (IVPs). In particular, the methods are implemented without the need for either predictors or starting values from other methods. Numerical examples are given to illustrate the efficiency of the methods.

A SOLUTION OF PDE WITH VARIABLE COEFFICIENTS AND ITS APPLICATION IN CIVIL ENGINEERING PROBLEM

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Key Words and Phrases: PDE, fluid mechanics

In this study, an analytical solution for a Partial Differential Equation (PDE) with variable coefficients will be outlined. This solution was based on the one first proposed by Varley & Seymour (1988). In the approach, PDEs with rapidly change coefficients can be solved with an arbitrary function. With the new solution, we further develop appropriate approximations for numerous civil engineering problems.

- Wave-induced pore pressure variation in a seabed with variable permeability: In this approach, soil permeability is considered to be a function of vertical direction. An analytical solution is obtained for the prediction of wave-induced pore pressure and liquefaction in marine sediments.
- Rough oscillatory turbulent boundary layer flow: In this approach, eddy viscosity is considered as a function of vertical distance from the seabed surface. Most previous analytical approach, assumed the eddy viscosity as a constant within either 2-layered or 3-layered approaches. Here, we consider it to be an arbitrary function.
- Tide-induced groundwater fluctuation in coastal aquifers: All existing analytical approaches are limited to homogeneous coastal aquifer. Here, we consider it to be a function of horizontal and vertical distance from the reference point, and obtain a set of analytical solution.

SINGULAR SPECTRUM ANALYSIS AND THE TAKENS MODEL FOR ANALYZING MULTISIGNAL PROCESSES

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Many applications in engineering require the analysis of multiple signals from sensors at different locations on a system. Singular spectrum analysis is an effective if little used method for describing the harmonic properties of a system while pulling out slow-moving trends as the system ages. We describe here a new formulation for singular spectrum analysis which gives more accurate estimates of the harmonic structure while not being tied to computational parameters of previous formulations. In addition, we show how this analysis can be combined with the Takens approach to nonlinear modeling of dynamic processes for important tasks of fault classification and regression.

ON THE THEORY OF FUNCTIONS OF OMEGA–BOUNDED TYPE IN THE HALF–PLANE

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Key Words and Phrases: weighted spaces of regular functions
AMS Subject Classification: 32A35, 31A05

The lecture gives the basic representations of the general theory of functions of ω-bounded type in the upper half-plane. The starting point are the canonical representations of some Banach spaces $A^p_{\omega,\gamma}$ of holomorphic functions. Also, there is a theorem on the projection from L^p_ω to A^p_ω. For $p = 2$ (i.e. in the case of Hilbert spaces) there is a theorem on the orthogonal projection from the corresponding L^2_ω to A^2_ω, a Paley-Wiener type theorem and a theorem on a natural isometry between A^2_ω and the Hardy space H^2, which is an integral operator along with its inversion. Then the canonical representations of Nevanlinna-Djrbashian type classes of δ–subharmonic functions are given. The functions from the considered spaces and classes can have arbitrary growth near the finite points of the real axis.

CORRELATION FUNCTIONS AND KINETIC COEFFICIENTS FROM SIMULATION OF DIMER REACTION

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Key Words and Phrases: dimer simulation, kinetic coefficients, correlation functions, stochastic analysis

AMS Subject Classification: 00A72, 70F05, 82B05, 82B31, 82C05

The last three quarters century of work in the statistical mechanics of rate coefficients in equilibrium kinetics has concentrated on the fluctuation dissipation theorem, diffusion phenomena, Brownian motion, the Fokker-Planck and Langevin equations and various other combinations of these that serve as building blocks. Kramer's theory of chemical rate equations, and modern elaborations incorporating the transition state theory of chemical reaction rates are examples [1,2].

Although published theories abound, actual simulations of reactions to study rate expressions are more rare, especially those relating to stochastic analysis. One pioneering example [3] showed that the correlation analysis of the rate constant expressed as time covariance functions as claimed or associated with D. Chandler and others suffered a discrepancy of the order of 300% from the actual simulation results of the rates from an isomerism model of a molecule in solution depicted A↔B, which is strictly speaking an internal transformation of the same molecular species. A less restricted case of a reaction is the dimerization reaction 2A $\leftrightarrow A_2$ where the particles A can exchange with similar A atoms to form the dimer, where the interparticle potential for this model are all 2-body only. Correlation or covariance functions from actual simulation runs of a dimer reaction with exchange characteristics are presented, together with models using elementary Ito calculus to attempt to relate fluctuations in the species numbers of dimer and particle to the kinetic rate coefficients, and such a treatment is not the standard approach to this class of problems in the physical sciences.

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PARTIALLY ASYMMETRIC EXCLUSION PROCESS WITH CREATION AND ANNIHILATION

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Key Words and Phrases: partially asymmetric exclusion process; Meanffeld theory; Monte-Carlo simulation

AMS Subject Classification: 90B20, 60K30, 65C35, 65C05

In recent years, asymmetric simple exclusion processes (ASEPs) have become an important tool for investigation of many processes in chemistry, physics and biology. ASEPs are discrete non-equilibrium models that describe the stochastic dynamics of multi-particle transport along one-dimensional lattices. Each lattice site can be either empty or occupied by a single particle. Particles interact only through hard core exclusion potential. If particles can only move in one direction, we have the totally asymmetric simple exclusion process (TASEP). In contrast, in partially asymmetric exclusion process (PASEP), particles are allowed to hop into both directions, but with dierent rates. Recently, the coupling of ASEPs with non-equilibrium processes has led to many unusual and unexpected phenomena. For example, Parmeggiani et al. investigated the interplay of TASEP with the creation and annihilation of particles. The phenomenon of localized density shocks was produced and was explained by applying a phenomenological do- main wall theory. The aim of the present paper is to investigate the coupling of PASEP with particle creation and annihilation. The shock formation could also be observed. The phase diagram is presented from both meaneld approximation and Monte-Carlo simulations. A comparison with the counterpart of TASEP is made.

SPECTRALLY ACCURATE KRYLOV DEFERRED CORRECTION TIME INTEGRATORS

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Key Words and Phrases: Krylov subspace methods, Krylov deferred correction, spectral deferred correction, method of line, spectral integration, preconditioner, parabolic PDE
AMS Subject Classification: 65B05, 65F10, 65L80

In this talk, we introduce a new class of spectrally accurate time stepping methods for efficient and accurate solutions of ordinary differential equations (ODEs), differential algebraic equations (DAEs), and partial differential equations (PDEs). The methods are based on applying spectral deferred correction techniques as preconditioners to Picard integral collocation formulations, least squares based orthogonal polynomial approximations are computed using Gaussian type quadratures, and spectral integration is used instead of numerically unstable differentiation. For ODE problems, the resulting Krylov deferred correction (KDC) methods solve the preconditioned nonlinear system using Newton-Krylov schemes such as Newton-GMRES method. For PDE systems, method of lines transpose (MoL^T) couples the KDC techniques with fast elliptic equation solvers based on spectral methods or integral equation formulations and fast algorithms. Preliminary numerical results show that the new methods are of arbitrary order of accuracy, extremely stable, and very competitive with existing techniques, particularly when high precision is desired.

PERIODIC ORBITS WITH COLLISIONS IN A RESTRICTED 4-BODY PROBLEM

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Key Words and Phrases: Hamiltonian systems, Sitnikov problem, restricted 4-body problem
AMS Subject Classification: 70F10, 70F35

In this work, we study a generalization of the classical Sitnikov problem with four bodies in a 2+2 configuration, moving under the action of the Newtonian potential as follows: two bodies with equal positive masses (primaries) $m_1 = m_2 = \frac{1}{2}$ follow planar keplerian orbits, and two bodies with negligible masses (secondaries) move on the perpendicular line passing by the primaries' center of mass. When the total energy is negative, every solution is a collision orbit.

We regularize the singularities due to collisions using a Levi-Civita type regularization. In the limit case $m_3 = m_4 = 0$, we obtain two uncoupled Sitnikov problems. In this case we find several families of periodic orbits.

GLOBAL STABILITY FOR THE N-SPECIES LOTKA-VOLTERRA TREE SYSTEMS OF REACTION-DIFFUSION EQUATIONS

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Key Words and Phrases: Cauchy problem for parabolic systems, Global stabolity
AMS Subject Classification: 35K57, 35B35, 92D25

We study Cauchy problem for the n-species Lotka-Volterra tree systems of reaction-diffusion equations as follows

$$(u_i)_t - D\Delta u_i = u_i\left(b_i - \sum_{j=1}^{n} a_{ij}u_j\right), \quad (t,x)\in[0,\infty)\times R_+^m,$$
$$u_i(0,x) = u_i^0(x), \qquad x\in R_+^m, \qquad i\in\mathcal{N},$$

where $\mathcal{N}=\{1,\cdots,n\}$ and n is the number of species. Applying invariant region method and with the aid of quadratic form theory we obtain a set of sufficient conditions for the globally asymptotic stability of the solutions. The criteria in this paper are in explicit forms of the parameters, and thus, are easily verifiable. Moreover, this criteria is applicable to competition model, cooperation model, as well as to predator-prey model.

MULTITYPE LINEAR FRACTIONAL BRANCHING PROCESSES AND APPLICATIONS

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Key Words and Phrases: linear fractional birth laws, multitype branching, mini-demography, pedigree studies of diseases
AMS Subject Classification: 60J80, 15A48

Recently with G. Letac we have computed explicitly some classical quantities related to a multitype branching process when the generating functions of its birth law are fractional linear functions with the same denominator. To achieve this, the main tool is an adapted parameterization of these functions by the mean matrix M and an element w of the first quadrant.

In this talk we will present some of those results with applications to the Kin number problem: at time n an individual (ego) is chosen at random in a multitype branching process: one is interested in the distribution of the tree stemmed from some ego's ancestor. This gives the relationship between sibship sizes and offspring numbers. The fact that the distributions are different is well known to demographers.

Those results could be applied for instance in mini-demography and in pedigree studies of diseases.

In spite of its limitations the multitype linear fractional process is the only one for which we can do extensive explicit computations.

A DISCRETE TIME EPIDEMIOLOGICAL SEIRS MODEL

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Key Words and Phrases: epidemic model, delay-difference equations, non-linear incidence, stability analysis, oscillatory and quasi-periodic dynamics
AMS Subject Classification: 92D30, 39A11, 92B05

A SEIRS epidemiological model is developed in terms of a system of delay-difference equations. It is assumed a non-linear incidence and constant periods of latency, infectiousness and immunity. Local stability conditions are determined related to the threshold parameter R_0. When the transmission parameter β increases, the qualitative behavior change from monotonic to damped oscillations, and then to a quasi-periodic dynamics. For some parameters values the relation between the inter-epidemic periods and immunity periods turns out to be linear. The qualitative behavior of SIRS and SEIS models are different from the ones displayed by our SEIRS model. Comparisons with other epidemiological models are given and it is shown that our SEIRS model is able to exhibit a wider range of dynamics than previous epidemiological models. In particular, the conditions for obtaining different probability distributions of the infected individuals are explored.

OPTIMAL DESIGN OF LMSE FIR DIGITAL FILTERS BY NUMERICAL OPTIMIZATION OF GENERALISED RAYLEIGH QUOTIENT

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Key Words and Phrases: FIR digital filters, LMSE criterion, Wiener filter, overlapped signal and noise spectra, generalised Rayleigh quotient, gradient optimisation
AMS Subject Classification: 93A25, 65F30

Electrical filters act as input-output systems, and purify input signals by attenuating unwanted spectral components. If signal and noise spectra overlap, the least mean square error (LMSE) at a filter's output is provided by the Wiener filter. However an ideal transfer function of the Wiener filter could be impossible to implement using physically available components in many cases. Conventional design methods for finite impulse response (FIR) digital filters aim to approximate an ideal transfer function (for example, using least square or minimax criteria), but during this approximation loose the connection to the targeted LMSE criterion. An alternative approach is to account for constraints, imposed by realisable devices (e.g., FIR filters), from the beginning and optimise LMSE numerically [1]. This approach leads to a need of optimisation of a generalised Rayleigh quotient (GRQ). The latter could be done numerically using a gradient optimisation representing a numerical linear algebra algorithm. The presentation discusses an ideal Wiener filter, the constraints imposed by FIR filters, conversion of the applied problem into a problem of numerical optimisation of GRQ, efficient computational algorithm to conduct such an optimisation, examples of various FIR filter designs, and their advantages comparing to filters designed by conventional techniques.

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MATHEMATICAL ANALYSIS OF THE AUTORESONANCE PHENOMENON

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Key Words and Phrases: nonlinear oscillator, resonance, perturbation, asymptotics, averaging
AMS Subject Classification: 34C15, 34E99

This report is about resonance phenomenon occuring in the nonlinear oscillating system which is forcing by an oscillating driver with a small amplitude. A simple mathematical model is given by the nonlinear Hamiltonian system under small oscillating force. We are interesting for: is it possible that energy of the system, starting near stable equilibrium, grows up to order of unity while the external force remains weak. The well known "nonlinear resonance" phenomenon is not related directly to the autoresonance. The autoresonance is more complicated phenomenon occuring in nonlinear system. In this case a remarkable feature of nonlinear oscillation is discovered. Namely, the free frequency of the system depends on the energy. So the free frequency is varying when the oscillating amplitude is growing. Hence we must to vary the driver frequency in order to use resonance effect and this is a basic idea of the autoresonance. But how to do it? It is very nice that autoresonance arises under different driver frequency. The principal role plays the direction of change of frequency and not the way of change with time. The autoresonance look like that under appropriate driver the system selfadjusts to the varying external conditions so that it remains in resonance with the driver for a long times. This long time resonance leads to a strong increase in the response amplitude under weak driving perturbation. There is else one feature of nonlinearity which is discovered. The autoresonance is occurred if the driver amplitude exceeds some crucial value. Such type thresholds were suggested by V.Veksler in 1944 and recently they were discovered by simulation of different mathematical model. The report is devoted to asymptotic analysis of mathematical models of the autoresonance phenomenon which is occurred in the finite dimensional dynamical system.

NUMERICAL COMPARISON BETWEEN MAGNUS METHOD AND CLASSICAL METHOD FOR SOLVING OSCILLATORY ODES

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Key Words and Phrases: geometric integrators, magnus expansion, nonlinear system
AMS Subject Classification: 65L05

In this paper, the differential equation $Y' = A(t, Y)Y; \quad t \geq 0, \quad Y(0) = Y_0$, known as Lie-type equation where $Y_0 \in \mathcal{G}$, $\mathcal{G} \rightarrow$ Lie-Group, $A : \mathbb{R} \rightarrow \mathbf{g}$, and $\mathbf{g} \rightarrow$ corresponding Lie-algebra of Lie-Group $\mathcal{G}$ is considered. The solution of this equation can be represented as an infinite series whose terms consist of integrals and commutators, based on the Magnus Series. This expansion is used as a numerical geometrical integrator called Magnus Series Method, to solve this type of equations. This method which is also one of the Lie-Group methods, has slower error accumulation and more efficient computation results during the long time interval than classical numerical methods such as Runge-Kutta, since it preserves the qualitative features of the exact solutions. Moreover, we introduce a fifth order method based on the magnus expansion and provide some examples as a numerical tests in order to illustrate the efficiency of the method.

AN APPLICATION OF THE DOMAIN DECOMPOSITION METHOD TO THREE-DIMENSIONAL LARGE-SCALE MAGNETIC FIELD ANALYSES

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Key Words and Phrases: domain decomposition method, finite element method, magnetic field analysis, the conjugate gradient method

We have applied the domain decomposition method to magnetic field analyses. Also we have developed ADVENTURE_Magnetic using results of our research. ADVENTURE_Magnetic is a module developed in ADVENTURE project for analysis of three-dimensional (3-D) magnetic fields. The hierarchical domain decomposition method is implemented for computing in parallel environments using PC clusters or network-connected computers. ADVENTURE_Magnetic supports two analyses. The first is non-linear magnetostatic analysis. This capability is formulated by A method, where the magnetic vector potential A is used as an unknown function. The Newton method is mainly used to solve the simultaneous nonlinear equations. The Conjugate Gradient (CG) method is used as the symmetric solver. In each subdomain, CG method is again used as the solver for the symmetric system arising in approximations. Then, a shifted incomplete Cholesky factorization is used as the preconditioner. The second is eddy current analysis. This capability is formulated by two methods. One of them is A-phi method, where the magnetic vector potential A and the electric scalar potential phi are used as unknown complex functions. Another one is A method. Conjugate Orthogonal Conjugate Gradient (COCG) method is used as the complex symmetric solver. In each subdomain, COCG method is again used as the solver for the complex symmetric system arising in approximations. Then a similar shifted incomplete Cholesky factorization is again used as the preconditioner. In this talk, magnetic field analyses with domain decomposition method are introduced and some numerical examples are shown.

This work was supported by the Revolutionary Simulation Software project (http://www.rss21.iis.u-tokyo.ac.jp/en/index.html).

A DEFINITION AND BASIC PROPERTIES OF MULTIMODAL FUNCTIONS AND THEIR OPTIMIZATION

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Key Words and Phrases: multimodal function, unimodal function, nonconvex optimization, nonlinear optimization
AMS Subject Classification: 65K05, 90C26

A minimization problem can be formulated as follows:

$$\text{(NMP)} \quad \text{minimize} \quad f_o(\boldsymbol{x}), \qquad \text{subject to } \boldsymbol{x} \in S.$$

In such a problem, if the objective function f_o to be minimized is unimodal, efficient descent-based methods (e.g., quasi-Newton methods and trust region methods) based on properties of the convex functions have been proposed. Various methods (e.g., branch and bound methods, clustering method, GA and SA) for the case in which the function is multimodal and has multiple isolated local minima have been also proposed, but these methods are less effective and have weaker convergence properties than descent-based methods. This seems to be mainly related to the fact that the definition and properties of multimodal functions have not been investigated for a long time.

In this paper, we consider the following extended minimization problem (ENMP):

$$\text{(ENMP)} \quad \text{minimize } f(\boldsymbol{x}) \equiv f_o(\boldsymbol{x}) + \delta_S(\boldsymbol{x}),$$

where $\delta_S(\boldsymbol{x})$ is an *indicator function*, which is defined by

$$\delta_S(\boldsymbol{x}) = 0, \text{ if } \boldsymbol{x} \in S, \qquad \delta_S(\boldsymbol{x}) = +\infty, \text{ if } \boldsymbol{x} \notin S.$$

In this minimization problem, we introduce a new concept of *local minimal value set* (*l.m.v.s.*) and the following basic properties for a function with flat regions: (1) A set of local minima includes the set of *l.m.v.s.* and (2) the set *l.m.v.s.* includes a set of strictly local minima.

Based on the number of connected elements ($|\,l.m.v.s.|_c$) of *l.m.v.s.*, a weak lower unimodal function ($|l.m.v.s.|_c = 1$) and a (lower) multimodal function ($|l.m.v.s.|_c \geq 2$) are defined. We show that the weak lower unimodal function includes a quasi-convex function.

We propose an algorithm for finding *l.m.v.s.* of a function with flat regions of one variable. For a multivariate function with flat regions, local minimization methods usually stop in a flat region without *l.m.v.s.*. We show that this difficulty can be overcome by modifying the stop condition of a local minimizer in a certain case.

ON CONTINUATION OF SOLUTIONS OF A CLASS OF DIRICHLET BOUNDARY PROBLEMS

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Key Words and Phrases: elliptic PDE, boundary problems, continuation of solution
AMS Subject Classification: 35J67, 35J25

We consider the real-analytic (C^ω) continuation of solutions of radiation Dirichlet boundary problems on subsets of $\mathbb{R}^n$, $n \geq 2$, involving elliptic real-analytic second-order differential operators, C^ω boundaries and C^ω boundary data. In addition to being of purely mathematical interest, this topic is highly relevant for the construction and implementation of modern numerical solution schemes (such as the Method of Auxiliary Sources [KA02]) for, e.g., acoustic and electromagnetic scattering problems. The real-analyticity of the setup can be used [SS94, Ch. 6] to complexify the original boundary problem and obtain a corresponding holomorphic Cauchy problem on a complex subset of $\mathbb{C}^n$. This Cauchy problem is incomplete, in that only one of the two elements of the Cauchy data vector is specified. If an estimate is available of the domain of holomorphic continuation of the missing Neumann boundary datum, then - at least in certain relevant special cases - an existence result such as the classical theorem of Cauchy-Kovalevskaya can be used to establish an estimate of the domain of C^ω continuation of solution to the original problem. We show the following: let Γ be a level set in $\mathbb{R}^n$ of $x_n - \psi(x')$ for some $\psi \in C^\omega(\mathbb{R}^{n-1}, \mathbb{R})$, and let $\Omega = \{x \in \mathbb{R}^n, x_n \geq \psi(x')\}$. Let P be a second-order elliptic differential operator on Ω° with coefficients in $C^\omega(\Omega)$. Then the Dirichlet-to-Neumann operator associated with P, μ_+ and a suitable radiation condition in μ_+, and mapping Dirichlet boundary data to corresponding Neumann boundary data, preserves the domain of holomorphic continuation of its argument - within the domain of holomorphic continuation of ψ and P.

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THE DIFFERENTIAL TRANSFORM APPROXIMATION FOR THE SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

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In this paper, numerical solution of system of the ordinary differential equations (ODEs) presents considered by the differential transform method.

In engineering practices, the governing equations of many physical phenomena can be modelled by the ODEs that describe the physical system.

In this study, we consider an initial value problem for the system of the first order differential equations,

$$\begin{aligned} y_1' &= f_1(x, y_1, ..., y_n) \\ y_2' &= f_2(x, y_1, ..., y_n) \\ &\vdots \\ y_n' &= f_n(x, y_1, ..., y_n) \end{aligned} \tag{1.1}$$

with the initial conditions,

$$y_i(x_0) = \alpha_i \quad i = 1, ..., n \tag{1.2}$$

where each equation in (1.1) represents the first derivative of the dependent variable y_i, $i = 1, ..., n$ as a function of the independent variable and

A variety of methods (e.g. exact and approximate methods) can be applied to find the solutions of (1.1) systems. In some cases, these equations may be too complicated to solve analitically. Therefore, numerical techniques can be efficiently used in these cases, for example, differential transform method.

The concept of differential transform method was first proposed by [Zhou, 1986]. The method is based on Taylor expansion.

In this paper, the definitions and the processes of the differential transform method is given for ODEs. Different ODEs are solved these method and compared with analytic solution and numerical solution.

FITTING GLD DISTRIBUTIONS

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Key Words and Phrases: fitting distributions, generalized lambda distribution, Johnson system, method of moments, method of percentiles, L-moments
AMS Subject Classification: 62

Fitting statistical distributions to a data set is a frequently occuring problem in a wide variety of applications. To find a distribution that provides a good fit one must first decide what family of distributions to consider and then determine how best to estimate parameters in order to identify a specific distribution within that family. This paper will discuss some of the current research regarding both the choice of distribution systems, with particular emphasis on the Generalized Lambda Distribution (GLD) family. Additionally, three parameter estimation methods (a method based on moments, one based on percentiles and a third based on L-moments) is discussed and an example that illustrates GLD fits obtained through these three methods is included.

EXPERIMENTAL AND ANALYTICAL ASPECTS OF END EXCITATION

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Key Words and Phrases: elastic waveguide, end excitation, energy partition
AMS Subject Classification: 74-05, 74J05, 74H15

In the present communication a crossing point between the mechanics of end excitation and the mathematical solution of its model is placed into focus. At least half a dozen of methods are cited in the literature in conjunction with finding the solution of end problem with a particular form of end excitation. Among these the following titles can be found: variational principle, collocation, bi-orthogonality, residual boundary-value minimization, reciprocal work, and projection. These methods appear to be valid both for dynamic and for quasi-static problems with no explicit summarized clarification of the advantages or the limitations of each method.

In the first part of the talk experimental results for dynamic end excitation by collision of short and long rods, within the elastic limit of the material properties, will be presented. The emphasis here is on the effect of various forms of excitations on the near and the far fields in the rod. The experiments show that the far field dynamic response of the input bar is practically insensitive to the variation of the contact area and form of the striker (within the limits of axially symmetric forms).

The steady state response of an elastic, semi-infinite strip to harmonic, non-uniform excitations is given next. In particular, bi-orthogonality property of the Lamb wave modes was used to derive the energy partition among the propagating modes for various end excitations for a range of excitation frequency. These propagating modes convey energy from the excited end and therefore comprise the far field response. It is shown that for moderately non-uniform excitations, the energy partition pattern is similar to uniform excitation. This result can be related to low sensitivity of the far field to spatial form of the contact area observed in the collision experiments.

MOMENT EQUATIONS FOR THE BOLTZMANN GAS OF INELASTIC SPHERES

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Key Words and Phrases: Boltzmann equation, Hamburger moment problem, granular gas, moment equations, boundary conditions

AMS Subject Classification: 82C40

We describe a hierarchy of formal expansions that represent the Fourier transform $\mathcal{F}[f](t,\mathbf{x},\mathbf{k})$ of a solution $f(t,\mathbf{x},\mathbf{v})$ of the Boltzmann equation for a granular gas with a constant coefficient of restitution α. The approximations are based on a formal solution of the Hamburger moment problem where one attempts to recover f knowing the finite sequences of its first moments. In particular, we describe a hierarchy of the weighted power series representations for $\mathcal{F}[f]$ with coefficients that depend on the moments of f alone. The constructed expansions can be Fourier inverted term by term, to recover the series representation of f. In three dimensions the solution of the Hamburger moment problem culminates in a exact description of the Chapman- Enskog hypothesis for the density f.

The first two representations correspond to the Maxwellian and Gaussian expansions. They have been exploited by Grad, Jenkins and Levermore in their study of the elastic and inelastic version of the Boltzmann equation. The next representation has a weight that depends on the first 13 moments of the Boltzmann density f and it yields modified Grad's 13 moment equations for the granular gas. The principal tools in deriving the moment equations are the exact form of the Fourier transform of the nonlinear, inelastic, Boltzmann equation -analogous, in spirit, to Bobylev work on the Fourier transform of the Boltzmann equation for the Maxwell molecules- and the precise truncation criterion that is based on the reminder term for the finite Hamburger expansion. We also show that the solution of the Hamburger problem yields the boundary conditions for the moments that are computed from the microscopic boundary conditions for the Boltzmann equation itself.

ON A PROPERTY OF BASES IN BANACH AND HILBERT SPACES

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Key Words and Phrases: base, stability, expansion by root vectors
AMS Subject Classification: 34L05, 34L10

In this work we consider the problem of conservation of the bases' in Banach spaces after small perturbations for the purpose of applying obtained results to investigation of spectral expansions associated with differential operators.

STABILITY, BIFURCATION AND CHAOS IN A DISCRETE-TIME HOPFIELD NEURAL NETWORK

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Key Words and Phrases: stability, bifurcation, chaos, Hopfield neural network
AMS Subject Classification: 39A11, 58F13, 58F14, 92B20

A stability and bifurcation analysis is undertaken for the following discrete-time Hopfield neural network of two neurons:

$$\begin{cases} x_{n+1} = a_1 x_n + T_{11} g_1(x_n) + T_{12} g_2(y_n) \\ y_{n+1} = a_2 y_n + T_{21} g_1(x_n) + T_{22} g_2(y_n) \end{cases} \qquad \forall n \geq 0$$

where $a_1, a_2 \in (0,1)$ are the internal decays of the neurons, $T = (T_{ij})_{2\times 2}$ is the interconnection matrix, $g_i : \mathbb{R} \to \mathbb{R}$ $(i = 1, 2)$ are continuous transfer functions and $g_i(0) = 0$. Conditions ensuring the asymptotic stability of the null solution are found, with respect to two characteristic parameters of the system. It is shown that for certain values of these parameters on the boundary of the stability domain, Fold, Flip or Neimark-Sacker bifurcations occur, but codimension 2 bifurcations are also present. The direction and the stability of the Neimark-Sacker bifurcations are investigated by applying the center manifold theorem and the normal form theory. The theoretical results are followed by a numerical example, which also reveals chaotic behavior near the origin in this type of networks.

GEOMETRIC THEORY OF DEFECTS

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A description of dislocations and disclinations defects in terms of Riemann–Cartan geometry is given, with the curvature and torsion tensors being interpreted as the surface densities of the Frank and Burgers vectors, respectively. A new free energy expression describing the static distribution of defects is presented, and equations of nonlinear elasticity theory are used to specify the coordinate system. Application of the Lorentz gauge leads to equations for the principal chiral $\mathbb{SO}(3)$-field. In the defect-free case, the geometric model reduces to elasticity theory for the displacement vector field and to a principal chiral $\mathbb{SO}(3)$-field model for the spin structure. As illustrated by the example of a wedge dislocation, elasticity theory reproduces only the linear approximation of the geometric theory of defects. It is shown that the equations of asymmetric elasticity theory for the Cosserat media can also be naturally incorporated into the geometric theory as the gauge conditions. As an application of the theory, phonon scattering on a wedge dislocation is considered. The energy spectrum of impurity in the field of a wedge dislocation is also discussed.

SOME EQUALITIES WHICH HOLD IN (n, m)-GROUP (Q, A) FOR $n \geq 2m$

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Key Words and Phrases: (n, m)-group, $\{1, n-m+1\}$-neutral operation, inverse operation
AMS Subject Classification: 20N15

In this paper, we have proved two equalities which hold in (n, m)-group (Q, A) for $n \geq 2m$. The first of them is the generalization of equality $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$, which holds in binary group $(Q, \cdot)$.

The second of them is equality

$$A\left(x_1^m, b_1^{n-2m}, y_1^m\right) = A\left(A\left(x_1^m, a_1^{n-2m}, \left(a_1^{n-2m}, e\left(b_1^{n-2m}\right)\right)^{-1}\right), a_1^{n-2m}, y_1^m\right).$$

The above discussed equality for $m = 1$, was proved in the monograph J. Ušan: "n-groups in the light of the neutral operations".

ON THE REAL HILBERT SPACE REPRESENTATIONS OF BARRELLED REAL LOCALLY C^*-ALGEBRAS AND LOCALLY JB-ALGEBRAS

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Key Words and Phrases: Real locally C^*-algebras, locally JB-algebras, Hausdorff Projective limit, Hilbert space representations

AMS Subject Classification: 46K10, 16W10, 46A08, 46H35, 46H70

Real Hilbert space representations of the barrelled Hausdorff Projective limits of real locally C^*-algebras and locally JB-algebras are studied. It is shown that in the case of the real barrelled locally C^*-algebras a realization by the *-algebra of closable operators defined on a dense subspace of a real Hilbert space always exists. In the case of the barrelled locally JB-algebras it is shown that a factor-algebra modulo Jordan ideal can be as well realized as an operator algebra.

RECENT DEVELOPMENTS IN THE THEORY OF FRAMES IN BANACH SPACES

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Key Words and Phrases: frames, Banach frames, atomic decompositions
AMS Subject Classification: 42C15, 42A38, 46B26

A brief introduction to the theory of frames, highlighting its uses in various other fields will be given. This will be followed by basic definitions and results on frames in Hilbert spaces. Various generalizations of frames for Banach spaces, namely, Atomic Decompositions, Banach Frames and Retro Banach frames will be given. Also the implications and non- implications among these concepts will be supported by examples. Some recent results obtained in the theory of Banach frames will also be given. Finally, some new directions and some open problems in the theory of frames will be given.

APPROXIMATION BY K-TUPLES FROM CHEBYSHEV SPACES

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Key Words and Phrases: Chebyshev, k-tuple, best approximation
AMS Subject Classification: 41A28, 41A50

The problem of approximating a fixed function $f \in C[a,b]$by a k-tuple $(p_1, p_2, \ldots, p_k)$ $\in$ $(C[a,b])^k$is explored. A natural error function $g_f$is defined and approximating k-tuples are chosen from X^k, where Xrepresents a Chebyshev space of dimension n. The questions of existence of best approximations, characterization of such approximations, degree of approximation, and uniqueness of best approximations are examined. With mild assumptions, best approximations are shown to exist. A partial characterization of local best approximations is developed along with some local uniqueness results. For certain types of functions, this form of approximating function can provide approximations with small error using a modest number of parameters. There is some similarity to the segmented approximation introduced by Lawson in 1964, but neither theory subsumes the other.

GARDEN OF EDEN AND COLONIES

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Key Words and Phrases: grammar system, colony, garden-of-eden
AMS Subject Classification: 68Q45

In the paper grammars and grammar systems will be treated as rewriting systems for the states of the environment, which are represented by strings over the fixed alphabet.

Typical states known from the theory of cellular automata, the *Garden-of-Eden, Life, Doomsday* and *Non-life*, reflect basic developmental stages of the environment, which are determined by (im)possibility of each state to produce next state as well as by (im)possibility of each state to be produced by another state. A state is in the *Garden-of-Eden*, if it cannot be derived from another state and it can produce next state. Analogically are defined *Doomsday, Life* and *Non-life*.

In the talk we present the study of the above mentioned sets of states (languages) for *colonies*, the grammar systems with the simplest components introduced in [1]. Components of a colony, each of which is able to produce a finite language, rewrite the common string by given protocol of the cooperation.

In the talk we illustrate results obtained for Garden-of-Eden for sequential b and t mode colonies and for PM colonies. We will characterize the *Garden-of-Eden* language, discuss its emptiness, (in)finiteness and introduce the density of the *Garden-of-Eden* as a function of the length of the states. The details can be found in [2] and [3].

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DETERMINATION OF THE EARTH'S GRAVITATIONAL FIELD FROM SATELLITE ORBITS

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Key Words and Phrases: harmonic function, SO(3) representation, satellite—to–satellite tracking
AMS Subject Classification: 70F15, 83B05, 20C35

The determination of the Earth's gravitational potential is one of the main issues of Geodesy. Usually, in Geodesy the gravitational potential is represented as a spherical harmonics series. Therefore gravitational potential determination means the computation of the coefficients of this series from the observation of quantities, which are influenced by the gravitational field.

Typically, those observations are related to the orbits of satellites with a low orbital altitude (Low Earth Orbiters). Starting in 2000 a series of dedicated gravitational field satellite missions was launched leading to an important improvement in the knowledge about the static and the time-variable gravitational field of the Earth.

While processing the data from these missions it became apparent that the usual spherical harmonics representation is not able to exploit the full information content of the data –a certain amount of smoothing is unavoidable when using spherical harmonics.

Therefore, the geodetic research concentrates on regional refinement of the global spherical harmonics solution. This refinement means

- The use of short satellite arcs only, which are crossing the area of interest
- The use of rapidly decaying base functions for the representation of the incremental gravitational potential, so called localizing base functions.

The paper aims at the establishment of relationships between the satellite dynamic along the short arcs and the gravitational potential representation by localizing base functions. This will be done in four steps:

(1) Choice of harmonic splines on the sphere as localizing base functions.
(2) Selection of a rotating coordinate system, which x-axis follows the movement of the satellite.
(3) Transformation of the localizing base function representation into the rotating coordinate system, using the representation theory of $SO(3)$.
(4) Establishment and solution of the equations of satellite motion in the rotating system.

Having performed these steps the parameters of the localizing gravitational potential representation can by determined as the solution of a nonlinear lest-squares problem. The nonlinear problem is solved by genetic algorithms and simulated annealing algorithms.

PSEUDO-TRANSIENT CONTINUATION

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Key Words and Phrases: nonlinear equations, pseudo-transient continuation, optimization
AMS Subject Classification: 65H10, 65H20, 65K10, 65L05

Pseudo-transient continuation is a method for finding dynamically stable solutions of nonlinear equations. The approach mimics temporal integration, but uses large time steps toward the end of avoiding the cost of a fully time-accurate simulation. In this talk we will compare pseudo-transient continuation to conventional damped Newton method approaches, discuss convergence results and time step control, explain how one can use it for optimization and connect it to classic trust region methods. We will present some examples including inverse problems and nonlinear equations on manifolds.

FIXED POINT THEOREMS

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Key Words and Phrases: fixed point, increasing, and axiom of choice
AMS Subject Classification: 03E25

Some new fixed point results are given in this paper.

Also, the Fundamental Theorem of Calculus will be discussed.

The FTC involving the Riemann Integral is one of the most pervasive and basic results in Real Analysis that almost every student of Mathematics has been exposed to and has extensively used in great variety of Mathematics. FTC appears in every book on Calculus and Real Analysis, however, with an ever-present gap. It seems that nowhere in the literature one can find a nontrivial and easily accessible to the reader an "if and only if" theorem of the FTC involving the Riemann Integral. My result is intended to eliminate this centuries old gap.

GENERALIZED THUE-MORSE SEQUENCES AND THE VON KOCH CURVE

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Key Words and Phrases: Thue-Morse sequence; von Koch curve; recurrent curves
AMS Subject Classification: 28A80, 37B10, 68R15

In a recent paper, Ma and Holdener used turtle geometry and polygon maps to realize the Thue-Morse sequence as the limit of polygonal curves in the plane. After proving that a sequence of such curves converges to the von Koch curve in the Hausdorff metric, they asked whether or not there exist certain generalized Thue-Morse sequences that also encode the curve. Here we answer their question in the affirmative, providing an infinite family of words that generate generalized Thue-Morse sequences encoding the von Koch curve.

ANALYSIS OF WIND-INDUCED VIBRATION OF OVERHEAD TRANSMISSION LINES

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Key Words and Phrases: overhead transmission lines, long span with in-span fittings, wind-induced aeolian vibrations, subspan oscillations
AMS Subject Classification: 70

Overhead transmission lines with high-voltage levels of 20 - 500 kV may be subjected to wind induced conductor motion in form of

- high frequency vibration, also known as Karman or aeolian vibration, with the critical frequency range between 5 - 50 Hz,
- low frequency vibration or conductor galloping, where icing is one of the triggering factor and the frequency range between 0,1 - 3 Hz,
- subspan or bundle conductor oscillation, which is a wake induced oscillation of flutter type.

All kind of motions may cause extensive damage to components of the conductor support system or the conductor itself. In the present paper attention will be focused on conditions in very long spans in range of a few thousand meters (e.g. fjord or river crossing), where there often is the need for large number of in-span fittings, such as dampers and aircraft warning spheres. This adds additional complexity to the wind-induced vibration problems for the conductor.

For conductors with a cable length of some hundered meters for estimating the vortex excited vibrations of the lines, in the case of aoelian vibrations a simple method, the Energy Balance Principle is well established for spans damped near the end. For long spans with in-span fittings this simple approach is no longer feasible, since the location of the fittings in the span as well as their dynamic characteristics are of importance. The vibration levels may now differ considerably along the span and this is reflected in the vibration modes. A modified approach to this problem is described.

In the case of wake-induced vibrations, the critical wind velocity (bifurcation point) of incipient flutter will be changed. New conditions for which flutter in the line is triggered are specified.

N-DIMENSIONAL DIFFERENTIAL TRANSFORMATION METHOD FOR SOLVING PDES

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The differential transform method which is applied to solve eigenvalue problems and to partial differential equations (PDE) is proposed in this study, using the n-dimensional transform method.

In this work, n-dimensional transform method has been introduced and fundamental feature have been defined for the first. Besides, as an aplication of two and three – dimensional differential transform, exact solution of linear and nonlinear partial diferential equations have been investigated.

In many engineering practices,we encounter linear or nonlinear partial diferential equations (PDEs) that describe the physical system.

The method can be easily applied to linear or nonlinear problems and is capable of reducing the size of computational work.

ZERO (SUB)SETS OF HOLOMORPHIC FUNCTIONS AND COMPLETENESS OF EXPONENTIAL SYSTEMS

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Key Words and Phrases: completeness, exponential system, entire function, Gamma function, zero set, uniqueness set, balayage, (sub)harmonic minorant
AMS Subject Classification: 30E10, 30D15, 33B15, 31A05

Let $\Lambda = \{\lambda_k\}$ be a sequence in the complex plane $\mathbb{C}$. Let Γ denote the Gamma function. Denote by $H\bigl(D(r)\bigr)$ the space of holomorphic functions on $D(r) := \{z \in \mathbb{C} \colon |z| < r\}$ with the topology of uniform convergence on compact subsets of $D(r)$. *The radius of completeness* $R(\Lambda)$ in $\mathbb{C}$ is equal to

$$\inf\bigl\{r > 0 \colon \bigl\{\exp(\lambda_k z)\bigr\} \text{ is complete system in } H\bigl(D(r)\bigr)\bigr\}.$$

All main results on $R(\Lambda)$ before 2006 can be found in [Kh06]. We use general results from [Kh07I] to estimate of $R(\Lambda)$. A typical result is
Theorem ([Kh07II; Theorem C]). *Let* $\Lambda \subset (0, +\infty)$, $\pm\Lambda := \Lambda \cup \{-\lambda_k\}$*;*

$$\Lambda(t) := \sum_{0<\lambda_k\le t} 1; \quad \overline{D}_P(\Lambda) := \frac{2}{\pi} \limsup_{y\to+\infty} \int_0^{+\infty} \frac{y}{y^2+t^2} \frac{\Lambda(t)}{t}\, dt$$

is the upper Poisson density of the sequence Λ (see [M–R61]). *Then* $(0{,}8472\dots)\cdot \pi\overline{D}_P(\Lambda) = \sqrt{\pi}\bigl(\Gamma(3/4)\bigr)^2\overline{D}_P(\Lambda) \le R(\pm\Lambda) \le \pi\overline{D}_P(\Lambda)$.

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ON THE PLANAR DIRICHLET PROBLEM FOR THE HELMHOLTZ EQUATION IN THE LATTICED DOMAINS AND ITS APPLICATION TO THE STATIONARY SCHRÖDINGER'S EQUATION

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Key Words and Phrases: Dirichlet problem, Laplace equation, Doubly-periodic functions, Cauchy type integrals
AMS Subject Classification: 30E20, 30E25,45E

The Dirichlet problem in the plane with the doubly-periodically distributed halls for the Helmholtz equation is considered. The halls are bounded by the Liapunov's contours.

By the Carleman and Magnaradze representations [1] the problem is reduced to the Dirichlet problem for the Laplace equation in the latticed domain. By means of the conformal mapping method the problem is reduced to the singular integral equation. By the theory of singular integral equations [2] the existence of the solutions of the class of exponentially doubly-quasi periodic functions is proved.

In several cases the effective solutions are obtained.

The results are applied to the planar stationary Schródinger's equation [3], which is considered in a ring ($z_1 < r < r_2$, r_1 and r_2 are radiuses of the concentric circles with the center at the point $(0, 0)$).

The following problem is considered: in the ring find the solution of the equation

$$\Delta\psi + f(r)\,\psi = 0,$$

satisfying the boundary condition $\psi|_{r_1} = \psi|_{r_2} = 0$, where $r = \sqrt{x^2 + y^2}$, f is the given Holder continuous function.

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THE UNIQUENESS OF THE SOLUTION OF DUAL EQUATIONS FOR STURM-LIOUVILLE EQUATION WITH TURNING POINT OF ORDER $4m + 1$

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Key Words and Phrases: inverse problem, dual equation, turning point, Sturm-Liouville problem, indefinite
AMS Subject Classification: 34E

In this paper, we prove the uniqueness of the solution of dual equations that arise in studying inverse Sturm-Liouville equation with turning point of order 4m+1. This result is necessary for expressing inverse problem of indefinite Sturm-Liouville equation.

THE NUMERIC-ASYMPTOTIC METHOD OF SOLUTION OF GAMES IN A METRIC SPACE

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Key Words and Phrases: Differential game theory, quasidifferential equations, delay, metric space
AMS Subject Classification: 93C99

Many economical, mechanical systems and operations researches are considered by the following necessary factors: presence of several controlled systems interconnected among themselves which purposes are various and the change of systems is in time. The mathematical device of research of such control systems is the theory of differential games of N persons. For the description of processes developing time in nonlinear metric spaces are used the quasidifferential equations. In the present paper the algorithm of numerically - asymptotic construction of equilibrium strategy of games of N persons which movement in nonlinear metric space is considered.

Let change of a position $\{t, x\}$ in game of N persons is described by the quasidifferential equation in metric space with delay

$$\delta(x(t+h), f(\varepsilon, h, t, x(t), x(\alpha(t)), u_1, ..., u_N)) = o(h), \quad x(0) = x_0, \tag{1}$$

and the payoff function of the i-th player looks like

$$I_i(u_1, ..., u_N)) = \Phi_i(T, x(T)), \quad i = \overline{1, N}, \tag{2}$$

with $u_i \in U^i \in Z^i$ here Z^i are metric spaces with the metric $\delta_{1i}(\cdot,\cdot)$, $f : (0, \varepsilon_0] \times (t_0, t_0 + T] \times X \times X \times U_1 \times \cdots \times U_N \to X$ is a map assigning at all $\varepsilon \in (0, \varepsilon_0]$ and $u_i \in U_N$ is the local quasimovement.

To the problem (1)-(2) the following averaged problem

$$\delta(y(t+h), f^0(h, y(t), y(\alpha(t)), v_1, ..., v_N)) = o(h), \quad y(0) = x_0, \tag{3}$$

$$I_i(v_1, ..., v_N)) = \Phi_i(T, y(T)), \quad i = \overline{1, N} \tag{1}$$

is presented. Here $v_i \in V^i \in Z_1^i$, Z_1^i – metric spaces with the metric $\delta_{2i}(\cdot,\cdot)$, $f^0(h, y(t), y(\alpha(t)), v_1, ..., v_N) = \lim\limits_{\varepsilon \to 0} f(\varepsilon, h, t, x(t), x(\alpha(t)), u_1, ..., u_N)$,

Since the equation (3) is autonomous, the investigation of differential game (3), (4) is much easy.

A NOTE ON THE NON-CONSTANT COEFFICIENT LINEAR SECOND ORDER PARTIAL DIFFERENTIAL EQUATION

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In this study, we consider the general case of linear second order partial differential equations

$$[u_{xx} + u_{xy} + u_{yy} + u_x + u_y + u] = f(x, y)$$

then we convolute the same differential equation $u_{xx} + u_{xy} + u_{yy} + u_x + u_y + u$ by polynomial $p(t, x) = \sum_{j=1}^{m} \sum_{i=1}^{n} x^i t^j$ in order to make the differential equations with non constant coefficient then we solve the new equation by using the double Laplace Transform. It was proved that if $F(x, y)$ is solution of non- homogenous differential equation $u_{xx} + u_{xy} + u_{yy} + u_x + u_y + u = f(x, y)$ then the solution of new equation is in the following form

$$\frac{1}{i!j!} p(t, x) *^x *^t F(t, x) + \frac{1}{j!} t^j F(x, t) \quad \text{if} \quad j > i$$

and

$$\frac{1}{i!j!} p(t, x) *^x *^t F(t, x) + \frac{1}{i!} x^i F(x, t) \quad \text{if} \quad i > j$$

where $p(t, x) = \sum_{j=1}^{m} \sum_{i=1}^{n} x^i t^j$ is a polynomial. In particular case, we consider heat and wave equations and solve by using the same technique.

MATHEMATICAL AND COMPUTER MODELING OF FUNCTIONAL DIFFERENTIAL EQUATIONS

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Key Words and Phrases: functional differential equations, mathematical modelling, numerical methods simulation, software
AMS Subject Classification: 34K05, 34K07

The report presents basic principles and methods of an approach to mathematical and computer modelling of systems with delays, described by functional differential equations. The approach is based on constructions and methods of i-smooth calculus (i-smooth analysis) and some methodological principles.

The approach is the foundation of the general approach to constructing numerical methods for functional differential equations.

Some aspects of elaborating software and results of computational experiments are discussed.

The report presents Software developed on the basis of the elaborated numerical and control algorithms.

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MULTI-COMPONENTS MODELS OF COMBINED DRY FRICTION BASED ON PADE APPROXIMATIONS

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Key Words and Phrases: combined dry friction of the rolling and sliding, Pade approximations, rheological models, dynamics of the heavy rolling ball
AMS Subject Classification: 70E18, 70F40, 74H10

Essentially new multi-components models of combined dry friction of the rolling and sliding are presented. Under the models of friction are understudied the interrelations between friction force components, torque and velocities. The models involves the replacement of exact integral expressions for the net vector and torque of the dry friction forces, formed with the assumption that Coulomb's friction law is valid at each point of the contact area, by appropriate Pade approximations. This approach substantially simplifies the combined dry friction modelling, making the calculation of double integrals over the contact area unnecessary. Unlike available models, the model based on the Pade approximations enables one to account adequately for the relationship between force and kinematical characteristics over the entire range of angular and linear velocities. The approximate models preserve all properties of the models based on the exact integral expressions and correctly describe the behavior of the net vector and torque of the friction forces and their first derivatives at zero and infinity. Moreover, one does not have even to calculate the integrals to determine the coefficients of the Pade approximation. The corresponded coefficients can be identified from experiments. Consequently, the models based on Pade approximations may be considered as rheological models of combined dry friction. A number of kinematics parameters defined forced interaction determined the dimension of models. If the area of contact of rubbed solids has the circle form and the motion is combination of pure sliding and whirling then the dimension of models equals two, but in the case of non symmetric areas of contact or if besides combination sliding there is a rolling then dimension of models equals three. With aid of developed models are presented the new qualitative explanation of the dynamics of the heavy ball rolling on the rubbed surface.

NUMERICAL SOLUTION OF GENERALIZED PANTOGRAPH EQUATIONS WITH DIFFERENTIAL TRANSFORM METHOD

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Delay differential equations are important in applied science like natural or control systems. Many events can be successufuly modelled by delayed differential equations. There are different kinds of DDEs studied by various methods. Especially special type of delayed differential equations which named as pantograph equations arises in quite different fields of pure and applied mathematics such as number theory, dynamical systems, probability, quantum mechanics and electrodynamics. In particular, it is used by Ockendon and Taylor to study how the electric current is collected by the pantograph of an electric locomotive, from where it gets its name.

In this paper, we deal with numerical solution of generalized pantograph equation which a special type of delayed differential equations presents considered by the differential transform method. Differential transform method differs from the high order Taylor series method, was first proposed by [Zhou, 1986]. This technique uses n th order polynomials as the approximation to the exact solution and requires the computation of higher-order derivatives.

The following equation

$$y^{(m)}(x) = \sum_{l=1}^{m-1} p_l(x) y^{(l)}(\alpha_l x + \beta) + f(x) \tag{1}$$

is presents generalized of pantograph equations with

$$y^{(l)}(0) = \lambda_l, \quad l = 0, 1, ..., m-1 \tag{2}$$

initial values. Where $p_k(k)$ and $f(x)$ are analiytic functions and α_l, β_l, λ_l 's are real or complex constants.

In this paper, the definitions and the processes of the differential transform method is given for pantograph equations and any problems are solved with these method and compared with analytic solution and numerical solution.

LIST COLORING OF N-MONOPHILIC GRAPHS

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In 1990, Kostochka and Sidorenko proposed studying the smallest number of list-colorings of a graph G among all assignments of lists of a given size n to its vertices. We say a graph G is n-monophilic if this number is minimized when identical n-color lists are assigned to all vertices of G. Kostochka and Sidorenko observed that all chordal graphs are n-monophilic for all n. Donner (1992) showed that every graph is n-monophilic for all sufficiently large n. We show that cycles are n-monophilic for all n; G is not 2-monophilic iff all its cycles are even and it contains at least two cycles whose union is not $K_{2,3}$; for every $n >= 2$ there is a graph that is n-choosable but not n-monophilic.

SIMULATION OF GROWTH AND AGE STRUCTURE OF ECOLOGICAL POPULATIONS ON THE BASE OF ODE AND PDE

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Key Words and Phrases: forest ecology, dynamical system, cellular automata, forecasting, optimal control

AMS Subject Classification: 92D25

We represent a family of models describing growth of ecological populations, in particular a multiform forest plant as an example of practical use. We employ a sequential step-by-step constructing system of dynamical equations: (1) a basic and very simple ODE model based on the classic Volterra model allowing analytical solution or numerical testing of phase portraits. (2) a more complicated model which takes into account age structure (the Leslie matrixes approach) based on differential and appropriate difference equations. (3) several expanded models described by ODEs and PDEs for obtaining additional information, e.g., provided by the advection equation (where one coordinate is the time and another is age). (4) a final spatially-distributed cellular automata model and additional models based on PDE (the advection type equations). The dynamical equations are formulated for biomass density in each spatial point. As the basic model we employ the simplest Volterra model addressing interaction resourseconsumer with competition between different species of forest. Various methods of nonlinear dynamics are used to investigate the equations describing evolution of forest population as a whole, i.e. without an account of spatial distribution and migrations. For a more precise calculation of plant age structure we use a vector of forest biomass density instead of scalar biomass density. This vector contains elements describing density of biomass of different age groups. For instance, "0" element corresponds to newborn biomass, "1" and "2" to trees between 1...10 year and 11...20 years, respectively, and so on. Meanwhile, such an advanced approach implies modification of the initial system according to age distribution of plant population. This technique is based upon the well-known Leslie matrices. The behavior of age-structured solution is rather similar to the basic model solution. However, a typical solution in this case contains small oscillations because of periodic temporal change of population age structure. A practically significant simulation deals with the cellular automata model, i.e. numerical solving the dynamical system of differential equations for the forest biomass density and soil factor. Spatial effects of forest growth are taken into consideration by dynamical equations for each spatial point. The final system represents evolution of biomass of certain form and age group in a fixed spatial point. By this manner 2-form forest is divided into 10 age groups system, giving 2x10x10000 equations and providing 100x100 spatial grid to be used. Additional equations describe resources, water balance, and other factors for addressing the relief and spatial irregularities. Groups of equations for different spatial points are slightly bound, basically by the process of new biomass birth. Typical questions of planning ecological and economic clearances are to be solved by the methods of optimal control over the dynamical systems under discussion. The numerical solution obtained by virtue of optimization algorithms allows one to organize management of forest plants from both the ecological and economical viewpoint. All the models are being built-up on the base of natural data and real actions during the past 15 years in one of the forest plants of Moscow region. The original software for ecological forecasting is developed as well.

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PARTIAL AVERAGING FOR OPTIMAL CONTROL PROBLEMS WITH IMPULSIVE EFFECTS

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Key Words and Phrases: method of averaging, differential inclusion, impulsive differential inclusion, small parameter, controlled systems
AMS Subject Classification: 49N25, 49J24, 49J25

In this paper are presented some results connected to the applications of the averaging method for solving of optimal control problems, where the models are systems of differential equations with impulsive effects. We suppose additional control in the impulses.

PROGRESS IN CROSSING NUMBERS RESEARCH

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Key Words and Phrases: graph, drawing, crossing number, Cartesian product
AMS Subject Classification: 05C10, 05C38

It was Paul Turán who introduced the concept of crossing numbers. In his Brick Factory Problem he asked on the minimal number of crossings among edges of the complete bipartite graph $K_{m,n}$. The *crossing number* $cr(G)$ of a graph G is the minimum possible number of edge crossings in a drawing of G in the plane. Computing the crossing number of a given graph is in general an elusive problem. Garey and Johnson have proved that this problem is NP–complete. The exact value of the crossing number is known only for some families of graphs. The structure of Cartesian products of graphs makes Cartesian products of special graphs one of few graph classes, for which exact crossing number results are known.

In 1973, Harary, Kainen, and Schwenk established the crossing number of $C_3 \times C_3$ and conjectured that $cr(C_m \times C_n) = m(n-2)$ for $3 \leq m \leq n$. Recently has been proved by Glebsky and Salazar that for any fixed $m \geq 3$, the conjecture holds for all $n \geq m(m+1)$. Besides the Cartesian products of two cycles, there are several other exact results. In 2006, Bokal proved the conjecture given by Jendroľ and Ščerbová that $cr(K_{1,n} \times P_m) = (m-1)\lfloor \frac{n}{2} \rfloor \lfloor \frac{n-1}{2} \rfloor$ for the path P_m of length m. Applying the capped Cartesian product operation in combination with a newly introduced $\pi-$ subdivision he established the crossing numbers for the Cartesian products of $K_{1,n}$ with a tree of maximum degree 3 and for the product $W_n \times P_m$, where W_n is the wheel on $n+1$ vertices.

In the talk, we summarise the known crossing numbers for the Cartesian products of small graphs of order for, five, and six with cycles, paths and stars. Besides, we present several new results concerning crossing numbers of two graphs.

VIRTUAL ENVIRONMENT APPLICATIONS IN SCIENCE, EDUCATION, AND INDUSTRY

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Key Words and Phrases: visualization, virtual environment, physically based modeling
AMS Subject Classification: 97U80, 68U05, 65D18, 68U20

Since Virtual Environment (VE) has become a major R&D area worldwide within the recent decade, it can be noticed that this human-machine interface technology is of high advantage for a large number of application areas. With the availability of less expensive infrastructure for performing VE research it must be expected that research in VE at the universities will significantly increase and that this technology will be established in the educational area as a basic tool for communication and teaching.

The talk presents some results of cooperative project performed by ICPT, Russia and Fraunhofer IMK, Germany. We offer to create contemporary VE installation for physical modeling, industrial design, and educational purposes using widely available standard components. Visualization can be of great significance in mathematics and physics issues where new insight may be inferred from images showing hidden properties. Different panels on MathVis at Visualization'xx conferences have shown a permanent interest in this area. Our experience shows that VE technology provides a great benefit in fundamental research, and in practical problem solving, using immersive visualization technique and direct data manipulation in virtual space.

In this talk we present a several topics in visualization and animation of topologically non-trivial objects as an example for scientific and educational application. As an example of industrial application we present a new approach for the simulation of elastic objects under real-time conditions.

We conclude that Virtual Environment has successful applications in various fields of science, education and technology.

DISTRIBUTED ORDER CALCULUS AND EQUATIONS OF ULTRASLOW DIFFUSION

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We consider equations of the form

$$\left(\mathbb{D}^{(\mu)}u\right)(t,x) - \Delta u(t,x) = f(t,x), \quad t > 0, x \in \mathbb{R}^n,$$

where $\mathbb{D}^{(\mu)}$ is a distributed order derivative, that is

$$\mathbb{D}^{(\mu)}\varphi(t) = \int\limits_0^1 (\mathbb{D}^{(\alpha)}\varphi)(t)\mu(\alpha)\,d\alpha,$$

$\mathbb{D}^{(\alpha)}$ is the Caputo-Dzhrbashyan fractional derivative of order α, μ is a positive weight function.

The above equation is used in physical literature for modeling diffusion with a logarithmic growth of the mean square displacement. In this work we develop a mathematical theory of such equations, study the derivatives and integrals of distributed order.

RAMSEY THEORY IN TOPOLOGY AND ANALYSIS

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Key Words and Phrases: partition relations, game theory, moving-off family, regular variation
AMS Subject Classification: 05C10, 91A05, 54D20, 26A03

We present a variety of results on some applications of (generalized) Ramsey theory to topology (covering properties and function spaces) and analysis (divergent processes). Relations of this theory with (2-person) game theory will be also considered.

A BRIEF LOOK TO ROBUST PARAMETER DESIGN AND OFF-LINE QUALITY CONTROL

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Key Words and Phrases: robust design, quality improvement, response surface methodology, dual response optimization
AMS Subject Classification: 62N10, 62K99, 65K10

In today's increasingly competitive marketplace more attention is being paid to off-line quality control and the idea of robust product design. Quality improvement starts at the very beginning, i.e., during the design stages of a product or a process, and continues through the production phase. The poor quality cannot be improved by the process of inspection. Recent advances in quality technology have resulted from considering the variation of a quality characteristic as well as its mean value. Variability in quality characteristics often results in rejection of a product or poor process performance. Robust parameter design (RPD) is an approach to reduce variability and reach intended targets of a product or process.

A response surface approach to robust design, sometimes called dual response system (DRS) optimization, has drawn much attention in the current literature. In our context, the dual response refers to the mean and the standard deviation of the process. The DRS is usually addressed by a single objective optimization problem, in which the principal response (i.e., the mean) is considered as an objective function and the secondary response (i.e., the standard deviation) is converted into a constraint. However, restrictions on the secondary response may rule out better conditions, since an acceptable value for the secondary response is usually unknown. In fact process conditions that result in a smaller standard deviation are often preferable.

In this study, we will briefly review the ideas of robust parameter design and so called "off-line" quality control. Then, we will discuss about more flexible formulations of the problem.

BLOW-UP FOR A PARABOLIC MODEL UNDER BOUNDARY NONLINEARITY

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Key Words and Phrases: blow-up solution, reaction-diffusion equation, quasilinear parabolic equation
AMS Subject Classification: 35K20, 35K50

We study a quasilinear parabolic PDE with initial and nonlinear boundary value data,

$$\begin{aligned} u_t - Lu + f(t,x,u) = 0, &\quad (t,x) \in Q \equiv (0,T] \times \Omega, \\ \frac{\partial u}{\partial \nu} = \psi(u), &\quad (t,x) \in \Gamma \equiv (0,T) \times \partial\Omega, \\ u(0,x) = \varphi(x), &\quad x \in \Omega, \end{aligned}$$

where u is the unknown function, t is the time, and x - the space, $\Omega \subset \mathbb{R}^n$ is a smooth bounded domain.This problem arises in chemical engenering, technologies and nuclear reaction dynamics.

We study the behavior of the solutions of the problem under consideration. A low solution which blows up at some point of $(0,T] \times \Omega$ is constructed. A criterion for existence of blow-up is established.

BLOW-UP PHENOMENON FOR PARABOLIC SYSTEMS WITH IMPULSIVE PERTURBATIONS

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Key Words and Phrases: blow-up, impulsive parabolic system, impulsive initial and boundary value problem

AMS Subject Classification: 35R12, 35K50

We consider a class of systems of parabolic PDEs (reaction-diffusion equations) with initial and boundary value data.

$$\mathbf{w}_t - L\mathbf{w} = f(\mathbf{w})$$

where $\mathbf{w} = (u, v)^T$,u, v are the components of the unknown function $\mathbf{w}$, L is some elliptic operator - for instance it could be the Laplacian $\triangle = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \cdots + \partial^2/\partial x_n^2$, t is the time, and $x = (x_1, x_2, \ldots, x_n)$ - the space ($x \in \Omega \subset \mathbb{R}^n$).

We study the behavior of the positive solutions of the system under consideration.The main result is the existence of at least a singular point $M_0(T^*, x^*) \in (0, +\infty) \times \Omega$, such that the solution $u(t, x)$ of the considered system blows up, i. e. tends to infinity when $(t, x) \to M_0$.

This system is a mathematical model that describes discrete dynamical processes in biology, ecology, chemistry, nuclear reactor dynamics, neutron transport etc.

STOCHASTIC APPROXIMATION AND OSCILLATIONS

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Key Words and Phrases: averaging principle, diffusion approximation, dynamical systems, oscillation
AMS Subject Classification: 60F17

We will discuss the influence of high oscillation to the Stochastic Approximation Theorems: the Averaging Principle for the systems of diffusion processes, and the Diffusion Approximation for the random dynamical systems.

In the lecture, we consider the systems of random differential equations. The coefficients of the equations depend on a small parameter. The first equation, "slow" component, Ordinary Differential Equation or Diffusion Process, has unbounded highly oscillating in space variable coefficients and random disturbances, which are described by the second equation, "fast" component, with periodic coefficients.

The sufficient conditions for weak convergence as small parameter goes to zero of the solutions of the "slow" components to the certain random process are stated.

The examples are given.

PIECEWISE CONTINUOUS SYSTEMS USED FOR CONTROL AND IDENTIFICATION

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Key Words and Phrases: recursive parameter estimation, on-line identification, adaptive systems, state-space modls, piecewise functionning systems
AMS Subject Classification: Control theory

This paper proposes an original method for recursive on-line plant identification. The method identifies directly a linear continuous time state space model of the plant from full state measurements through a black box approach. The basic principle consists in using a reference model, called clone, which runs in parallel so as to reproduce the behavior of the plant. To do so, the state error between the plant and its clone is minimized by means of an adaptive algorithm that tunes the clones varying parameters iteratively. These parameters represent the estimators of those of the plant. The originality lies in the particular structure of the clone. The latter is in fact a Piecewise Continuous System (PCS) characterized by exogenous switching of its state. The method is appropriate for real time applications, where real plants are controlled by digital calculators. Results from computer simulation and a real time application are given.

MULTI-FACTOR RATIONAL LOGNORMAL MODELS FOR VALUATION OF CREDIT SWAPTIONS AND BASKET DEFAULT SWAPS

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We apply the multi-factor rational lognormal approach to model the interest rate process and default intensity processes of risky obligors. Under the rational lognormal framework, positivity of the default intensities and interest rate are guaranteed. In our model, pairwise correlation of the default processes is introduced through correlation among the stochastic factors. For pricing of single-name credit default swaps and swaptions, we manage to obtain analytic representation of the fair swap premium and option price. For pricing of basket default swaps, we derive the joint distribution of default times and the present value of the cash flows up to the time of the kth-to-default under the assumption of conditional independence. Numerical simulation experiments were performed to demonstrate the dependence of the pricing of basket default swaps on default correlation among the risky obligors in the basket.

REMOTE SENSING IMAGE ANALYSIS USING NEURAL NETWORKS

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Key Words and Phrases: remote sensing, neural networks, classification
AMS Subject Classification: Pattern recognition

The new technique using neural networks has been recently proposed. This Is new in the points that the classification could be achieved in a nonlinear discrimination and the classification functions could be determined by learning. In this paper, we adopt a neural network of layered type to classify the classify remote sensing data into several categories. The data are SPOT and ENVISAT-1/ASAR@(C-band), and ALOS/PALSAR(L-band). The aim of the classification is to detect the rice field and estimate the rice product in autumn.

The input of the neural networks are three bands remote sensing data observed by SPOT and the output is five categories like urban area, water region, two types of forest, and rice filed. Then we apply the similar technique to the detection of the rice filed by using SAR data and the classification results are compared with a conventional technique of classification like maximum likelihood method.

SINGULAR NONLINEAR PROBLEMS IN THE INFLATIONARY COSMOLOGY MODELS AND THEIR MULTIPLE SOLUTIONS

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Key Words and Phrases: nonlinear wave equations, singular problem without initial data, self–similar soliton–type solutions, singular BVPs to the second–order nonlinear ODEs, a multiplicity of solutions, accompanying singular spectral problems
AMS Subject Classification: 83F05, 34B15, 35L70

We set up and study a singular problem without initial data for a system of N nonlinear wave equations connected with the models in the inflationary cosmology where the space–time is postulated as the de Sitter space. The $(D+1)$–dimensional de Sitter space, in the conventional system of units, is described by the metric $ds^2 = dt^2 - \exp(2Ht)\sum_{i=1}^{D} dx_i^2$ where $0 < H$ is the Hubble constant. In this space–time, we consider a system of N interacting scalar fields $\{\varphi_j\}_{j=1}^N$ with the Higgs self–action potential $U = \lambda^2(\sum_{j=1}^N \varphi_j^2 - \nu^2)^2$ where $\lambda > 0$ and $\nu > 0$ are the parameters. For the dimensionless variables $\varphi_j = \varphi_{j,old}/\nu$, $\vec{r} = (H/\nu)\vec{r}_{old}$ and *conformal time* $\tau = -\exp(-Ht)/\nu$, we formulate the following singular problem for the Higgs fields in the de Sitter space:

$$\frac{\partial^2\varphi_j}{\partial\tau^2} - \frac{(D-1)}{\tau}\frac{\partial\varphi_j}{\partial\tau} - \Delta_D\varphi_j + \frac{4C^2\varphi_j}{\tau^2}\left(\sum_{s=1}^N \varphi_s^2 - 1\right) = 0, \tag{1}$$

$$\vec{r} \in \mathbb{R}^D, \qquad \tau \in I_\tau, \qquad j = 1, \ldots, N,$$

$$\lim_{|\vec{r}|\to\infty} \sum_{j=1}^N \varphi_j^2(\vec{r},\tau) = 1 \qquad \forall \vec{r},\tau: \quad \vec{r} \in \mathbb{R}^P, \quad \tau \in I_\tau. \tag{2}$$

Here Δ_D is the D–dimensional Laplace operator, $C = \lambda\nu/H > 0$ is the dimensionless parameter, $\mathbb{R}^P \subseteq \mathbb{R}^D$, $I_\tau = \mathbb{R}_-$ or $I_\tau = \mathbb{R}$, for a geodesic complete space.

The problem (1), (2) has a global $SO(N)$ symmetry in the space of the fields $\{\varphi_j\}_{j=1}^N$. For $P \geq 2$, we introduce in $\mathbb{R}^P$ the polar coordinates r, θ_1, …, θ_{P-1} ($r \geq 0$, $0 \leq \theta_1 < 2\pi$, $0 \leq \theta_k \leq \pi$, $k = 2, \ldots, P-1$) and look for the solutions as the domain walls with the different space symmetries: we replace x_j, as a function of polar coordinates, by φ_j and a radial variable r by unknown function $\varphi(r,\tau)$. For these constructions, generating in particular such objects as *the (hyper)bubbles*, *(hyper)strings* and *(hyper)monopoles*), the condition (2) implies $\lim_{r\to\infty}\varphi^2(r,\tau) = 1$ $\forall \tau \in I_\tau$.

In particular we look for the self–similar solutions of the indicated above types, setting $\psi(\xi) = \varphi(r/\tau)$. Then we get the singular problem

$$[(1-\xi^2)\psi']' - [(D-1)\xi - (P-1)/\xi]\psi' = Q\psi/\xi^2 + 4C^2\psi(\psi^2-1), \tag{3}$$

$$-\infty < \xi < -1, \qquad -1 < \xi < 0,$$

$$\lim_{\xi\to-\infty}\psi^2(\xi)=1, \tag{4}$$

where a value of parameter Q is connected with N, D and P.

For singular points $\xi=-1$ and $\xi=0$, we set the limiting conditions

$$|\lim_{\xi\to-1+0}\psi(\xi)|<\infty \quad \lim_{\xi\to-1+0}[(1+\xi)\psi'(\xi)]=0; \tag{5}$$

$$|\lim_{\xi\to-0}\psi(\xi)|<\infty, \quad \lim_{\xi\to-0}[\xi\psi'(\xi)]=0. \tag{6}$$

We discuss analytic–numerical approach to singular BVP (3), (5), (6). In particular we obtained that this BVP is solvable, its nontrivial solutions are continuable with no limit as $\xi\to-\infty$ and satisfy condition (4), moreover there occurs a multiplicity of solutions. For the input problem (1), (2), they correspond to the self–similar soliton–type objects collapsing in $\tau=0$. A number of the solutions depends on the magnitude of C: the critical bifurcation values of C are the eigenvalues of associated spectral problem for linear ODE obtained by the linearization of (3) on a trivial solution.

For the models and their investigations, see, e.g., [1]–[3] and references therein.

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GENERALIZED QUASI-LINEAR ORDINARY DIFFERENTIAL EQUATION OF EMDEN-FOWLER TYPE WITH PROPERTY A

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Key Words and Phrases: generalized ordinary differential equations, property **A**
AMS Subject Classification: 34K15

Consider the differential equation

$$u^{(n)}(t) + \sum_{i=1}^{m} p_i(t)\,|u(t)|^{1+\frac{d_i}{\ln t}}\ \mathrm{sign}\,u(t) = 0, \quad t \geq a > 1, \tag{1}$$

where $n \geq 2$, $p \in L_{loc}(R_+; R_+)$, $d_i \in R$ $(i = 1, \dots, m)$.

Definition [1]. We say that Eq. (1) has property **A** if any proper solution u is oscillatory if n is even, and is either oscillatory or satisfies $|u^{(i)}(t)| \downarrow 0$ as $t \uparrow +\infty$ $(i = 0, \dots, n-1)$, when n is odd.

Theorem 1. *Let $p_i(t) = c_i\,p(t) + o(t^{-n})$, where $p \in L_{loc}(R_+; R_+)$, $c_i \in (0; +\infty)$ $(i = 1, \dots, m)$. For Eq. (1) to have Property* **A**, *it is sufficient that*

$$\liminf_{t\to+\infty}\ t \int_t^{+\infty} s^{n-2}\,p(s)\,ds >$$
$$> \max\left\{ -\lambda(\lambda-1)\cdots(\lambda-n+1)\Big(\sum_{i=1}^{m} c_i\,e^{\lambda d_i}\Big)^{-1} : \lambda \in [0, n-1] \right\}.$$

Theorem 2. *Let $c_i \in (0, +\infty)$, $p_i(t) = c_i\,t^{-n}$ $(i = 1, \dots, m)$. Then Eq. (1) has Property* **A** *if only if*

$$\max\left\{ -\lambda(\lambda-1)\cdots(\lambda-n+1)\Big(\sum_{i=1}^{m} c_i\,e^{\lambda d_i}\Big)^{-1} : \lambda \in [0, n-1] \right\} < 1.$$

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Key Words and Phrases: density of sets, ideal of sets, porosity, I-convergence, I^*-convergence, congruence field

AMS Subject Classification: 40A05

Let $P(\mathbb{N})$ stand for the family of all subsets of positive integers $\mathbb{N}$. A family of subsets of $\mathbb{N}$ is said to be the ideal if it is additive and hereditary. The ideal $I \subset P(\mathbb{N})$ is said to be non-trivial if $I \neq \emptyset$ and $\mathbb{N} \neq I$.

Definition. Suppose that $I \subset P(\mathbb{N})$ is a non-trivial ideal of subsets of $\mathbb{N}$. A sequence $x = (x_n)_1^\infty$ of real numbers $\mathbb{R}$ is said to I-converge to $\xi \in \mathbb{R}$ (shortly I-$\lim x_n = \xi$) provided that for each $\varepsilon > 0$ the set $A(\varepsilon) = \{n \in \mathbb{N} : |x_n - \xi| \geq \varepsilon\}$ belongs to I.

It can be verified that I-convergence has some "good" properties, e. g., the uniqueness of the limit, the convergence of the sum and the product of two I-convergent sequences.

Examples. (a) Let I_f denote the family of all finite subsets of $\mathbb{N}$. Then I_f is a non-trivial ideal and I_f-convergence coincidents with the usual convergence.

(b) Let $\mathrm{d}(A)$ be the asymptotic density of $A \subset \mathbb{N}$, i. e., $\mathrm{d}(A) = \lim_{n\to\infty} \frac{A(n)}{n}$, where $A(n) = |A \cap \{1, 2, \ldots, n\}|$. Denote by D the class of all $A \subset \mathbb{N}$ for which $\mathrm{d}(A) = 0$. Then D is a non-trivial ideal and D-convergence is the statistical convergence, i. e., D-$\lim x_n = \lim \text{ stat } x_n$.

The set l^∞ of all real bounded sequences endoved with the supremum norm is a Banach space. There is a natural question how "big" is the convergence field $F(I) = \{x = (x_n) \in l^\infty : \text{there is } I\text{-}\lim x_n \in \mathbb{R}\}$. The answer usues notions of porosity and maximal ideal.

INEQUALITIES FOR TRIGONOMETRIC SUMS AND APPLICATIONS

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Key Words and Phrases: Positive trigonometric sums, inequalities for special functions, starlike functions, subordination
AMS Subject Classification: 42A05, 42A32, 26D05, 26D15, 33B15, 33C45, 33C45

We give several sharp inequalities for trigonometric sums. Our main result provides a generalization of a famous theorem of Vietoris. These results have remarkable applications in the theory of starlike functions and other problems in geometric function theory. Some other application will be presented. In particular we give new positive sums of Jacobi polynomials and their significance in problems dealing with quadrature. Several other related inequalities for special functions will be discussed.

DECREASE OF ERRORS INFLUENCE IN SPEECH ACTIVITY DETECTION IN SPEECH RECOGNITION ON THE BACKGROUND OF NOISE

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Key Words and Phrases: speech recognition, speech activity detection, statistical modeling, autoregressive mathematical model, intensive acoustical noise
AMS Subject Classification: 60K40

One of the methods to recognize speech commands, SC, on the background of intensive noise is to compare SC with their models, stored in the database. Models do not contain noise that is why their temporary end points are determined precisely. SC under recognition are hardly noised, so their end points are determined with errors. For example, a SC may not have a beginning. As a result of these errors SC and their models turn out to be uncoordinated and it negatively influences the quality of recognition.

It is rather difficult to increase the accuracy end points of speech activity detection, i.e. SC end points. That is why another approach is suggested in this paper. It is the coordination of SC end points and its models that is needed in the process of recognition. For temporary coordination of SC with models it is suggested to add noise to the models and to determine the model end points again using the same method, which is used for the determination of SC end points. Added noise is to be similar to actual noise. Autoregressive mathematical model of noise was used. The conducted statistical modeling has shown that the suggested method provides to increase the quality of SC recognition.

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One of the best ways for students to learn and apply mathematics is through data about sports and statistics. IN various sports tournaments, such as the World Cup in soccer (football), or the country-regionUS National Collegiate Association of placecountry-regionAmerica (NCAA) college basketball Division 1 championship tournament, there are some nice applications to tree diagrams, probabilities and predictions as to who will win. I have done some work on this, and presented at the International Conference on Teaching Statistics (ICOTS) in placecountry-regionSouth Africa in 2002. There are also good areas of applied mathematics and statistics in the area of baseball, where I have done many presentations.

In this lecture, I will expand upon these ideas, and offer some thoughts for classroom and research use.

BASIC PROCESS ALGEBRA

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Key Words and Phrases: process algebras, BPA, action relations, bisimulation, Noetherian quotients

AMS Subject Classification: 68Q85, 06F05, 16Y99

Basic process algebra (BPA) is an algebraic structure $(P, +, \cdot)$ such that $(P, +)$ is a semilattice with least element 0, $(P, \cdot)$ is a semigroup with identity 1, and satisfies the following equations: $(a + b)c = ac + bc$, $0a = 0$, and $1a = a1 = a$. BPAs − as generated algebras − are very fundamental structures which are introduced in algebra of communicating processes [1]. This version of BPA is often referred as $\mathrm{BPA}_\delta^\varepsilon$, with deadlock δ and empty process ε [2]. One may notice that BPA as a special case of a near-semiring with unity [3]. In order to study the algebraic properties of action relations of BPAs, we first formulate the notion of P-semilattice (P-Sℓ) as a special case of S-semigroup of near-semiring S. The concrete nature of P-Sℓ captures the properties of intermediate states between processes; whereas P-Sℓ abstracts the notion of BPA as well. In fact, the concept of P-Sℓ is an algebraic representation of BPAs. For $\alpha, \beta \in \Gamma$, a P-Sℓ; and $a \in P$, we define the action relation $\alpha \xrightarrow{a} \beta$ if and only if α has a summand $a\beta$, i.e., $\alpha = a\beta + \alpha$. This work targets to observe an algebraic abstraction of action relations using Noetherian quotients; and establishes some important properties of action relations. Also, in this abstract setup, this work addresses bisimulation congruence of BPA.

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PATH PLANNING FOR MOBILE ROBOTS USING ANNEALING NEURAL NETWORK

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Key Words and Phrases: path planning, autonomous vehicles, neural networks, simulated annealing

AMS Subject Classification: 62M45, 68T40, 83C50

This paper presents a novel potential field based path planning technique for differential drive mobile robots. The path planning is realized in two- and three-dimensional environments filled with static obstacles whose shapes and positions are known. The path planners are based on description of the obstacles by simulated annealing neural networks. The generated paths are piecewise linear with changing directions at the corners of the obstacles. Algorithms for 2D and 3D environments are derived. The local minima problem—an important cause for inefficiency of potential field methods—is treated and solved also. The algorithm allows parallel computation which decreases the time for path generation. Comparison with existing algorithms is discussed and results of benchmark tests for the calculation speed in single and multiprocessor environments are summarized. It is shown that the calculation speed of the proposed algorithm depends lineraly on the total number of vertices of the obstacles—a feature which places the algorithm among the fastest ones.

The algorithm is practically realized as independent software environment allowing simulations and control of differential drive robots like “Pioneer P3-DX”, “Khepera”, and “ER-1”. It can be successfully applied to snake type robots, flying robots, and control of Gantry cranes. The planner is tested on about 3000 random problems, which cover hundreds of obstacles geometries, and solves every problem in seconds. The proposed algorithm is a significant improvement of the existing potential field algorithms because it generates optimized in length paths without being trapped in local minima and has high calculation speed.

DESIGN PARAMETERIZATION AND SHAPE SENSITIVITY ANALYSIS OF CAD-BASED FLUME SECTIONS

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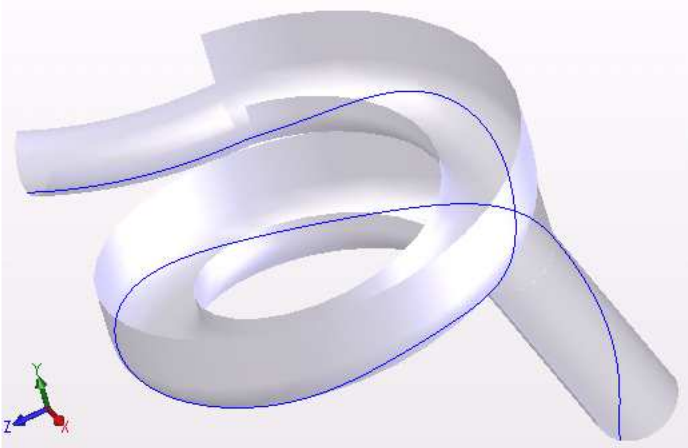
Key Words and Phrases: computer-aided design, differential equations, Lagrange equations, waterslides
AMS Subject Classification: 70-02

This paper presents design parameterization and shape sensitivity analysis (DSA) methods for design of recreational waterslides, in which the flume sections are represented in Computer-Aided Design (CAD) environment. These flume sections serve as the building blocks for general waterslide configurations. Key geometric dimensions of the flume sections and the overall waterslide configurations are identified as design variables at both global and local levels. A set of differential equations based on Lagrange's equation of motion that describe the motion of the riding object are derived. These coupled ordinary differential equations are solved numerically using *Mathematica*. In addition, an analytical shape DSA method has been developed and employed to support design of the waterslide configurations. The DSA expressions are stated in different set of coupled differential equations. These equations are solved again using *Mathematica*. The method proposed is general and is not restricted by the continuity requirements at the junction of the flume sections. This is because that the derivatives of position and velocity of the riding object at the entrance of a flume section are obtained from those at the exit of the previous neighboring flume section. A different set of DSA equations are solved for respective flume sections. Note that the proposed method overcome the design for manufacturing (DFM) issues of the waterslides, where the waterslide configurations are presented in B-spline surfaces with their control points chosen as design variables. A real-world water slide configuration is presented to demonstrate the feasibility of the proposed methods, as shown in figures below.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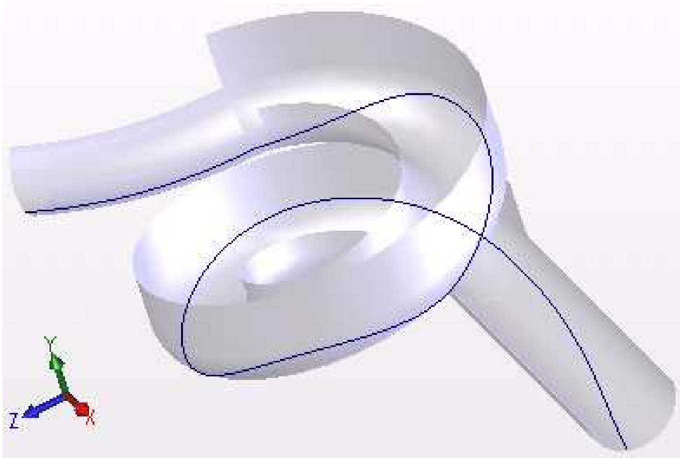
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COMPUTER-AIDED MODELING AND SIMULATION FOR RECREATIONAL WATERSLIDES

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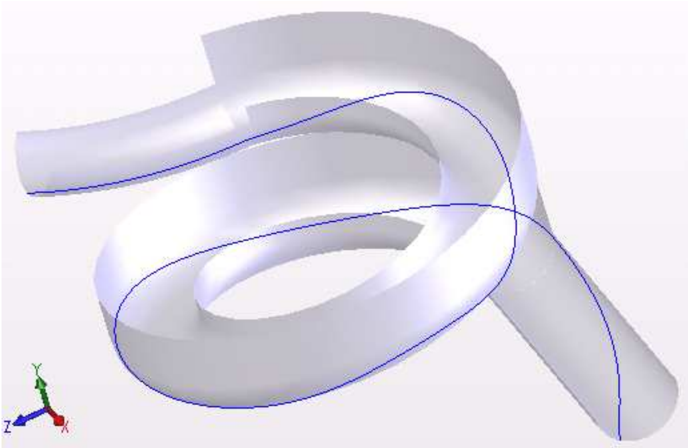
Key Words and Phrases: computer-aided design, differential equations, Lagrange equations, waterslides

AMS Subject Classification: 70-02

This paper presents a computer-aided modeling and simulation method for analyzing position, velocity, and acceleration of riding objects on recreational waterslides. Safety and excitement levels of the object riding on the waterslide are the two common criteria for waterslide design. Between the two, safety is far more critical than excitement. Safety aspect of the water slide design can be reasonably ensured by restricting the riding object stays within the physical boundary of the slide. In this paper, mathematic representations of various common flume sections are first created in parametric surfaces. These basic building blocks have also been implemented in Computer-Aided Design (CAD) tool, such as *SolidWorks*. A set of coupled differential equations based on Lagrange's equation of motion that describe the motion of the riding object are derived, in which friction forces are included. These second order ordinary differential equations are then solved using *Mathematica*. Initial position and velocity are specified for the entire waterslide. The position and velocity of the riding object at the entrance of the following flume section are obtained from those at the exit of the previous neighboring flume section. A different set of differential equations are solved for respective flume sections. The solutions are then brought back to CAD for visualization, similar to the ones shown in the figure below. A real-world waterslide configuration is presented to demonstrate the feasibility of the modeling and simulation method. The major contribution of the paper is extending waterslide simulation to truly CAD-based flume sections, and bringing frictions into the formulations that make the simulations more realistic.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ACTIVE OPTIMAL CONTROL OF THE KdV EQUATION

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Key Words and Phrases: optimal control, open-loop control, feedback control, variational iteration method, KdV equation
AMS Subject Classification: 49J20

In this paper, the optimal point-wise control of the KdV equation is investigated with an objective of minimizing a given performance measure. The performance measure is specified as a quadratic functional of the final state and velocity functions along with the energy due to open- and closed-loop controls. The minimization of the performance measure over the controls is subjected to the KdV equation with periodic boundary conditions and appropriate initial condition. In contrast to standard optimal control or variational methods, a direct control parameterization is used in this study which presents a distinct approach toward the solution of optimal control problems. The method is based on finite terms of Fourier series approximation of each time control variable with unknown Fourier coefficients and frequencies. He's well-known variational iteration method for the nonlinear partial differential equations is applied to the problem and thus converting the optimal control of lumped parameter systems into a mathematical programming.

A numerical simulation is provided to exemplify the proposed method.

VORTEX PARTICLE METHODS IN FLUID DYNAMICS

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Key Words and Phrases: vorticity, vortex particle, vortex method
AMS Subject Classification: 76M23

The importance of the vortex method in fluid dynamics stem from the fundamental role that plays of the vorticity in fluid dynamics. The vorticity is a main key for understanding on nearly all real fluid dynamics phenomena. That fact is enclosed in popular phrases like *"the sinews and muscles of the fluid motion"*, *"the sinews of turbulence"* or *"the voice of fluid motion"*. In vortex methods vortex particles are used. The calculations are carry out in Lagrangian variables. One can study the evolution of the vorticity by tracing the position of that vortex particles. In the paper will be presented the vortex methods applied to very different situation: from 1-D case to 3D. One dimensional case is related to the modelling of evolution of the interface between two mediums with different densities. Assuming that such interface can be treated as vortex sheet its evolution can be express by a integro-differential equations. The methods is very particular but very accurate and encompasses such phenomena as the Rayleigh-Taylor instability, the evolution of the thermals bubble. At present, the vortex methods constitute important method for the solutions of Navier-Stokes equations. It will demonstrated their usefulness for the modelling of the flow over the profiles and in the channel flows. We used this method for the study of the vortex layer eruption from the walls induced by the vortex patch. The 3D vortex method will be demonstrated for the study of the vortex ring interactions like the vortex leap-frogging and vortex reconnection phenomena. All vortex methods involve several subsidiary algorithms like velocity evaluations, displacements of the particles, diffusion and realisation of the boundary conditions. In our calculation we used the vortex-in-cell method. This method is alternative to direct vortex method that based on the Biot-Savart law. For the same number of particles ($N \approx 10^5$) vortex-in-cell method is about 1000 times quicker than direct method.

PARALLEL UNSTRUCTURED GRID GENERATION FOR PROBLEMS OF COMPUTATIONAL MECHANICS

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Key Words and Phrases: unstructured grids, parallel algorithms, automatic grid generation, domain decomposition

AMS Subject Classification: 65N50, 65Y05

The desire to simulate even more geometrical and physical features of technical structures and the availability of parallel computers and parallel numerical solvers which can exploit the power of these machines have led to a steady increase in the number of grid elements used. Memory requirements and computational time are too large for usual serial computers.

An algorithm for automatic parallel generation of three-dimensional unstructured computational meshes based on geometrical domain decomposition is proposed in this paper. Software package build upon proposed algorithm is described. Several practical examples of mesh generation on multiprocessor computational systems are given. It is shown that developed parallel algorithm enables us to reduce mesh generation time significantly (dozens of times). Moreover, it easily produces meshes with number of elements of order $5 \cdot 10^7$, construction of those on a single CPU is problematic. Questions of time consumption, efficiency of computations and quality of generated meshes are also considered.

NUMERICAL SIMULATION OF AERODYNAMIC FLOWS WITH HIGH-ORDER SHOCK-CAPTURING SCHEMES

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Key Words and Phrases: computational fluid dynamics, supersonic flows, high-order schemes for hyperbolic conservation laws

AMS Subject Classification: 76J20, 76L05, 76N15, 35L65, 35L67

The present paper is aimed at development and practical application of high-resolution algorithms and codes for numerical simulation of supersonic flows of the compressible fluid. They should robustly capture shocks and other flow discontinuities and, simultaneously, accurately simulate the smooth part of flows, which can contain complicated shock/shock, shock/vortex, shock/instability wave interactions. It can be expected that high-order shock-capturing schemes will be used as basic numerical tools in next-generation scientific and commercial CFD solvers.

The modern essentially non-oscillatory (ENO) and weighted ENO (WENO) schemes are natural candidates for this role. We describe Euler and Navier-Stokes solvers based on WENO schemes and give examples of their application to numerical simulation of complicated 2D and 3D shock-dominated flows. Computations performed for a number of different problems confirm that high-order WENO schemes are powerful tools for simulation of compressible fluid flows. They can be also considered as very promising candidates for Direct Numerical Simualtion (DNS) and Large Eddy Simulation (LES) of turbulent supersonic flows.

TRAVERSABLE WORMHOLES SUPPORTED BY CHAPLYGIN GAS

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Key Words and Phrases: wormholes, Chaplygin gas
AMS Subject Classification: 83C05

The existence, or possible existence, of wormholes may be viewd as a natural outgrowth of the general theory of relativity. Wormholes may be defined as handles or tunnels in the spacetime topology connecting different universes or widely separated regions of our own universe. (In other words, the universe may be multiply connected.) The study of traversable wormholes was initiated in 1988. Renewed interest in the subject is due to the discovery that our universe is undergoing an accelerated expansion, caused by a negative pressure, referred to as *dark energy*. A special case, called *phantom energy*, violates the so-called null energy condition, a key requirement for the existence of traversable wormholes.

This talk begins with a brief discussion of a class of exactly solvable models of phantom energy wormholes and continues with wormholes supported by another exotic form of matter, *Chaplygin gas*, usually viewed as a candidate for combining dark matter and dark energy. Not being an exactly solvable model, there is greater reliance on numerical techniques. The sheer abundance of solutions lacking certain undesirable features such as large tidal forces or event horizons implies that such wormholes might actually occur naturally. A rough measure of traversability then suggests that the subset of traversable wormholes may be surprisingly large.

THE EFFICIENCY OF ESTIMATORS IN THE NONLINEAR ERRORS-IN-VARIABLES MODELS WITH UNKNOWN NUISANCE PARAMETERS, AND MODEL SELECTION

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We consider consistent estimators in nonlinear measurement error models, where the error in covariate is normally distributed with known variance. While structural methods, in particular the quasi-score (QS) method, take advantage of the knowledge of the regressor distribution (if available), functional methods, in particular the corrected score (CS) method, discard such knowledge and work even if such knowledge is not available. We deal with efficiency of the estimators comparing in Lowener order the asymptotic covariance matrices and computing the rank of the difference of the matrices. It has been shown that QS is more efficient than CS as long as regression distribution is completely known [1], [2].

We focus on the situation where nuisance parameters in the regression distribution have to be estimated. We show a proper way how to construct the QS estimator in this case and prove that QS is still more efficient than CS. Suppose additionally that the latent variable is normally distributed, with unknown mean and variance. In Poisson, Gamma, and polynomial models the QS estimator of the mean and variance are constructed via the empirical mean and variance of the observed surrogate data, which is equivalent to pre-estimation. And in logit model the QS estimator of the mean is just the same, while the variance should be estimated together with regression parameters.

For a linear structural errors-in-variables model, a new kind of goodness-of-fit test is constructed based on the QS estimator. The power of the test is discussed. The proposed procedure is rather general and can be modified for any statistical model, where the parameters are estimated using score equations.

The results are joint with Prof. H. Schneeweiss (Munich) and my Ph.D. students S. Shklyar and A. Malenko.

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NEW SQUARE-ROOT ALGORITHMS FOR LOG-LIKELIHOOD GRADIENT EVALUATION

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Key Words and Phrases: system identification, maximum likelihood estimate; log-likelihood gradient; Kalman filter, square-root filtering algorithms

AMS Subject Classification: 93E12, 93E11, 62M20

Method of maximum likelihood is a general method for parameter estimation often used in system identification. To implement it, it is necessary to maximize the likelihood function that is usually done using the gradient approach. It involves the computation of the likelihood gradient with respect to unknown system parameters. For linear dynamical system models these lead to the implementation of Kalman filter, which is known to be numerically unstable. The aim of this work is to present new efficient algorithms of likelihood gradient evaluation. They are more reliable in practice and improve robustness of computations against roundoff errors. The paper is also supplied with numerical examples which clearly confirm the presented theory. The comparison with the conventional Kalman filter approach will be also given.

VARIABLE-STEPSIZE NORDSIECK FORMULAS: STABILITY AND ORDER REDUCTION PHENOMENON

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Key Words and Phrases: nordsieck methods, order reduction phenomenon, stability, modified formulas
AMS Subject Classification: 65L05, 65L06, 65L20

In the paper we study an order reduction phenomenon arising in Nordsieck methods when they are applied to ordinary differential equations on nonuniform grids. It causes some difficulties of using stepsize selection strategies in practical computations. We prove that the problem mentioned above is just a consequence of the fact that the concepts of consistency and quasi-consistency are not equivalent for such sort of methods. Therefore we show how to improve the Nordsieck methods with this property in order to avoid the order reduction phenomenon. We consider both explicit and implicit ways of doing that and study zero-stability of modified Nordsieck Adams-Moulton methods. The paper is also supplied with numerical examples which clearly confirm the presented theory.

WAVE PROPAGATION IN MICROSTRETCH VISCOELASTIC SOLID

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Key Words and Phrases: microstretch viscoelastic solid, imperfect boundary, amplitude ratios, symmetric & skew-symmetric modes
AMS Subject Classification: 74A, 74B, 74D, 74E, 74J

In the first part, propagation of plane waves at an imperfect interface between two microstretch viscoelastic solid half-spaces of different material properties has been investigated. Correspondence principle has been used to obtain the equations of microstretch viscoelastic solid from microstretch elastic solid. The amplitude ratios of different reflected and transmitted waves are obtained for an imperfect boundary and deduced for normal stiffness, transverse force stiffness and transverse couple stiffness, slip and welded boundary. The variations of amplitude ratios with the angle of incidence have been shown graphically. It is noticed that the amplitude ratios of reflected and transmitted waves are affected by the stiffness and material properties of the media. In the second part, the propagation of waves in microstretch viscoelastic plate, subjected to stress free conditions is investigated. The secular equations in closed form for symmetric and skew-symmetric modes propagation are derived. The amplitude of the displacement components, microrotation and microstretch are also obtained analytically. At short wavelength limits, the secular equations for symmetric and skew-symmetric waves in a stress free plate reduce to Rayliegh type surface frequency equations and the finite thickness plate appears as a semi-infinite medium. The results for micropolar elastic and elastic material are obtained as particular cases. The phase velocities and attenuation coefficients with wave number and amplitude of displacement components, microrotation and microstretch in case of symmetric and skew-symmetric modes are presented graphically. The analytical and numerical results are found to be in closed agreement.

REFLECTION AND DEFORMATION IN MAGNETO-THERMO-MICROSTRETCH ELASTIC SOLID

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Key Words and Phrases: magneto-thermo-microstretch elastic solid, amplitude ratios, time harmonic sources, Fourier transform

AMS Subject Classification: 74A, 74B, 74G, 74J

The present investigation is concerned with the reflection and deformation in magneto-thermo-microstretch elastic solid in the presence of a transverse magnetic field, at the boundary surface. The generalized theories of thermoelasticity developed by Lord and Shulman and Green and Lindsay has been used to investigate the problem. Magnetic effect on the amplitude ratios of various reflected waves with the angle of incidence have been depicted graphically. Fourier transform technique is used to study the deformation due to time harmonic distributed thermomechanical sources. Uniformly and linearly distributed sources have been taken to illustrate the utility of the apporach. The integral transform has been inverted by using a numerical technique to obtain the components of normal strain, force stress, tangential couple stress, microstress, temperature distribution and induced electric and magnetic fields. Magnetic effect on the resulting quantities has been depicted graphically for different sources. Some particular cases of interest are also deduced from the present investigation.

PRELIMINARY REPORT ON EDGE DETECTION METHODS AND ITS APPLICATION IN EARTHQUAKE RESEARCH

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Key Words and Phrases: edge detection, signal processing, surface deformation, semiology
AMS Subject Classification: 86A17, 86A15, 94A08, 41A10

Most damage from earthquakes is due to surface deformation. Thus, if the displacement can be predicted, the amount of damage might be lowered through issuing a warning. In order to issue an early warning, the algorithm must be able to process the data efficiently and predict the displacement relatively closely.

In earthquake research, strong motion records can be used to predict displacement. Since the strong motion data is typically non-smooth and has many location-dependent issues, many related studies are often rely on experts' opinions, i.e., one or more parameters in finding displacement are decided by the experts case by case. This makes it difficult to develop an automatic algorithm for predicting displacement.

In this study, we develop an algorithm to predict displacement without the use of experts' opinion. The algorithm combines methods from signal process and edge detection. We adopt methods to filter noise and detect the jumps in the strong motion data, then determine the baseline and integrate the data to find the displacement.

A set of strong motion records of earthquakes that occurred in Taiwan is used to test the algorithm. The results will be compared to the close-by GPS records. Because of the loss of experts' opinions, the result may seem to have larger errors. However, the purpose of this algorithm is to create a general process that can efficiently predict any displacement regardless the location of the strong motion, therefore, we will be accepting a larger margin of error when comparing the computing results and the GPS records.

AN ITERATIVE NUMERICAL METHOD FOR THE LASER WELDING PROBLEM

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Key Words and Phrases: level set, boundary elements, Stefan problem
AMS Subject Classification: 65N38, 35Q99, 35J55

In the laser welding process, a sheet of metal moves at a constant rate under a laser. The sheet melts forming a small pool of molten metal point where the laser strikes the sheet. The boundary between solid and liquid metal is determined by a steady state, two-phase Stefan problem. We present an iterative numerical method for finding this boundary. A boundary element method is used to solve the Stefan problem, and level set techniques are used to move the boundary between iterations. The method is described and numerical results presented with an emphasis on the observed accuracy and computational complexity.

METHODS OF STABILITY THEORY AND SINGULARLY PERTURBED SYSTEMS

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The work is connected with development of the effective approximate methods in the nonlinear analysis on the basis of A.M.Lyapunov methods. The methods of the stability theory in a combination with asymptotic approach allow to provide the development of comparison method with reference to problems of mathematical modelling in dynamics of complex systems, to get the analogue of reduction principle, that is well-known in the stability theory, in the general qualitative analysis, including critical cases (in A.M.Lyapunov sense) for the shortened systems (quasi-Tikhonov systems in N.N.Moiseev sense).

Initial, the well detailed model adequate to real process, as a rule, is very complex, and it is giving the nonlinear highly-dimensional, multiconnected problems. The presence in system of motions with strongly distinguished from each other characteristic times gives the additional difficulties in investigation by numerical methods; there are special problems connected the stability of computing process (a case of poorly caused matrixes). It generates the necessity of decomposition of initial system, with reduction to the shortened system, giving equivalent shortened model, with the reduction of an initial problem to the shortened dynamic problem. Thus the strict substantiation of validity of such reduction is necessary. There is a fundamental problem: the establishing of reduction principle with development of regular algorithm of constructing and substantiating of the acceptability of the shortened models in dynamics for singularly perturbed systems.

The worked out approach, based on stability postulate, on methodology of parametrical stability and singularity postulate, allows to develop the effective manners of modelling, to get the approximate methods, acceptable for the exact analysis, with generalization of a principle of reduction. Here the variables and the parameters of system are separated on essential and non-essential; the problem of decomposition - reduction for initial system is solved; the initial dynamic problem is solved as singularly perturbed one; the problem of modelling is reduced to a problem of stability with irregular perturbations. With reference to engineering practice the developed approach gives a strict theoretical substantiation of using of the approximate methods and approximate models (approximate theories). For systems with multiple-time scales the methods of investigation in stability problems are discussed (with extending of statements and problems in a context of a principle of reduction and postulate of stability in sense A.M.Lyapunov, N.G.Chetayev, K.P.Persidsky). The regular algorithm for constructing of the acceptable reduced models (simplified systems) by a strict mathematical way is developed, the problem of their substantiation is formulated; the conditions of a reduction in investigation of stability for nominal (in N.N.Moiseev sense) model are determined. The solution of singularly perturbed stability problem (when spectra of the appropriate matrixes are critical both for slow, and for fast variables) is obtained; nominal systems are quasi-Tikhonov systems (in N.N.Moiseev sense). The regular manners for estimations of permissible parameters are constructed. Using the approach, based on methods of A.M.Lyapunov in a combination with ideas of N.G.Chetayev, it is examined the singularly perturbed systems, that are near the boundary of stability domain. With division of variables and motions in system on components of various groups and classes (critical and basic ones; slow and fast ones) the conditions of the reduction in the solving of qualitative problems (stability; ε, η-estimation; speedness, optimality, $\ldots$) to investigation of the constructed subsystems are determined. The conditions of a correctness (in the accepted here sense) of constructed shortened models are formulated. With extending of traditional statements of the classical theory of stability and theory perturbations, with introduction of hierarchy variables the sequence of the shortened (reduced) systems is constructed. Developed methods give the regular algorithm for obtaining of new shortened models as asymptotic approximations (asymptotic models), with decomposition of dynamic properties (including stability property, with generalization of A.M.Lyapunov results for singularly perturbed systems).

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MULTIVALUED FUNCTIONS WITH THE (H) PROPERTY

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Key Words and Phrases: set-valued maps; derivative set-valued maps; generalized differential equations

AMS Subject Classification: 26B40, 54C60, 28A20, 34A99

Let $I = [a, b] \subset \mathbb{R}$, Y let be a reflexive Banach space. Let $F : I \times Y \to Y$ be a multivalued function and let $(x_0, y_0) \in I \times Y$. We shall be concerned with the initial value problem for the differential inclusion

1. $\frac{dy}{dx} \in F(x, y(x)), \quad y(x_0) = y_0.$

By a solution of (1) we mean any absolutely continuous function $f : [x_0, b] \to Y$, such that $f(x_0) = y_0$ and $f'(x) \in F(x, f(x))$ for almost all $x \in [x_0, b]$.

Filippov in [3] proves the existence of solution when values of F are compact convex subsets of $\mathbb{R}^n$, F is continuous at any point $(x, y) \in I \times Y$ and satisfies a Lipschitz condition of the form $d_H(F(x, y), F(x, y')) \leq k(x)||y - y'||$, where d_H denotes the Hausdorff metric in the space of closed bounded subsets of Y and $k(\cdot)$ is integrable. Hermes in [4] showes the existence of a solution of (1) when the Lipschitz condition is removed and replaced by a weaker condition involving the variation of the multivalued function. Castaing in [2] does not require continuity of F. He showes the existence of solution of (1) when F is convex compact valued, $F(x, \cdot)$ is upper semicontinuous for each $x \in [a, b]$, $F(\cdot, y)$ is Lebesgue measurable for each $y \in Y$ and F is integrably bounded, i.e. there is an integrable function g on $[a, b]$ such that $||u|| \leq g(x)$ for any $u \in F(x, y)$, $x \in [a, b]$ and $y \in Y$.

In the present paper we obtain the existence of a solution of (1) when $F : I \times Y$ has the (H) property, i.e. $F(\cdot, y)$ is a derivative for each $y \in Y$, the family $\{F(x, \cdot)\}_{x \in I}$ is equicontinuous and F fulfil less restrictive condition than integrably boundness, i.e. we suppose that family $\{G_f\}_{f \in \mathcal{C}(I,Y)}$ is uniformly integrably bounded where $G_f(x) = F(x, f(x))$ for $x \in I$ and $\mathcal{C}(I, Y)$ denotes the family of all continuous vector functions on the interval I.

In order to give effect we need suitable definition of derivative multivalued function. For that reason we give some concept of differentiability of multivalued function.

The notion of differentiability is developed by taking advantage of some ideas used by Hukuhara in [5], to give our definition of differentiability for a reasonably generous class of multivalued function. But the purpose of this paper is not the study of differentiability of multivalued functions. We give only some properties needed later on. We consider multivalued function $\Phi : I \to Y$ whose values are closed convex and bouded subsets of Y, where Y is a real reflexive normed linear space. In this case derivative of multivalued function in a poit $x \in I$ is also some closed convex and bounded subset of Y, what is essential for further considerations. π-differentiability of multivalued functions discused in [1] is given by taking advantage of Rädstrom's embedding theorem. In this case derivative of multivalued function at a point $x \in I$ is a continuous linear mapping (a comparision of both notions of differentiability is given). Furthermore some notion of a derivative multivalued function is introduced. In order to give effect we adopt the notion of integrability of multivalued functions given by Banks and Jacobs in [1]. Some definition of difference of sets (which we shall need later on) is introduced.

Finally we define the (H) property of multivalued function and show that the Carathéodory superposition of multivalud function with the (H) property with any continuous function $f : I \to Y$ is a derivative. We finish our consideration giving some application of this property for existence of a solution of differential inclusion (1).

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THREE A POSTERIORI ERROR ESTIMATES FOR APPROXIMATE SOLUTIONS TO NONLINEAR CONSERVATION LAWS

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Key Words and Phrases: error, a posteriori, hyperbolic, conservation law, adjoint, stability
AMS Subject Classification: 65M15, 35L65, 65M12

Conservation laws model such diverse applications as multiphase flows, elastics, automobile traffic, and gravitational waves in general relativity, but the modeling of compressible fluid flow by Euler's equations has been the impetus for their mathematical development. Despite the importance of conservation laws, our lack of understanding of the stability of solutions to such nonlinear systems has made it difficult to construct (i) rigorous and accurate error estimators for numerical schemes and (ii) rigorous adaptive numerical schemes. Three new approaches are presented for rigorous a posteriori error estimation of numerical solutions to nonlinear hyperbolic conservation laws. These approaches are interesting because they quantify the essential processes of error *generation*, *propagation* and *cancellation*.

The first error estimate is the most general and applies to all hyperbolic systems of nonlinear conservation laws [1]. For numerical approximations generated by the front-tracking method, we extend the stability result of Bressan, Liu, and Yang and identify the leading order term of the error as entropy production. Unfortunately, this estimate does not account for error *cancellation*. The second estimate is an extension of Liu's wave tracing method for Glimm's scheme that allows for explicit error *cancelllation* [2]. This second estimate is reliable, even for large times. The third error estimate is a rigorous application of the adjoint-based method of error estimation for front-tracking approximations. Error *propagation* and *cancellation* is in this way quantified by solving an intermediate adjoint problem.

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A SURVEY ON POTENTIALLY $K_{r+1} - G$-GRAPHICAL SEQUENCES

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Key Words and Phrases: graph, degree sequence, potentially $K_{r+1} - G$-graphic sequences
AMS Subject Classification: 05C35

The set of all non-increasing nonnegative integers sequence $\pi = (d(v_1), d(v_2), ..., d(v_n))$ is denoted by NS_n. A sequence $\pi \epsilon NS_n$ is said to be graphic if it is the degree sequence of a simple graph G on n vertices, and such a graph G is called a realization of π. The set of all graphic sequences in NS_n is denoted by GS_n. A graphical sequence π is potentially H-graphical if there is a realization of π containing H as a subgraph, while π is forcibly H-graphical if every realization of π contains H as a subgraph. Let K_k denote a complete graph on k vertices. Let $K_m - H$ be the graph obtained from K_m by removing the edges set $E(H)$ of the graph H (H is a subgraph of K_m). This paper summarizes briefly some recent results in potentially $K_{r+1} - G$-graphic sequences, and presents some open questions and conjectures.

RECENT ADVANCES IN KRYLOV SUBSPACE SPECTRAL METHODS

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Key Words and Phrases: spectral methods, Gaussian quadrature, variable-coefficient, Lanczos method, stability, heat equation, wave equation
AMS Subject Classification: 65M70, 65M12, 65D32

This talk summarizes recent and ongoing work on an alternative approach to the solution of diffusion problems and wave propagation problems in the variable-coefficient case that leads to a new numerical method, called a Krylov subspace spectral method.

The basic idea behind the method, applied to a PDE of the form $du/dt + L(x, D)u = 0$, is to use Gaussian quadrature in the spectral domain to compute Fourier components from elements of $\exp[-L\Delta t]$ for a matrix L discretizing $L(x, D)$ and time step Δt, using algorithms developed by Golub and Meurant, as opposed to applying Gaussian quadrature in the spatial domain as in traditional spectral methods.

This strategy allows accurate resolution of all desired components, for both high and low frequencies, without having to resort to smoothing techniques to ensure stability. In fact, by describing the Fourier components in terms of directional derivatives of moments, we can demonstrate unconditional stability given sufficient smoothness of the coefficients of $L(x, D)$.

We also discuss various generalizations of these methods, such as application to systems, which yields a simple high-order scheme for the second-order wave equation. Numerical results will be presented for both parabolic and hyperbolic problems in one, two and three space dimensions.

This talk includes joint work with Patrick Guidotti, Knut Solna, and Margot Gerritsen.

ON SOME FRACTALS IN NUMBER THEORY

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Key Words and Phrases: fractal, number theory, self-similarity, Morse-Thue sequence, Cantor set, coloring of the unit disk, trinomial arc, trinomial equation

AMS Subject Classification: 11A99, 11B57, 12D10, 14H45, 14P05, 26C10, 28A80, 30C15, 60G18

In the past, sets or functions that are not sufficiently smooth or regular have tended to be ignored. But in the recent years, it has been realized that the mathematics of non-smooth and irregular sets, called *fractals*, are worth studying. One of the most frequented fractals is the type of fractals of *self-similarity.* A set of this category is a set which contains copies of itself at arbitrarily small scales. The question now is what has number theory got to do with all this? Are there any bonds between fractals and number theory? Surprisingly, the answers to these questions are affirmative. Take the simplest concept from number theory, the sequence of Kronecker's integers. After some operations, we obtain the resulting sequence called the *Morse-Thue sequence.* It is manifestly aperiodic; it never repeats, but it has a much more interesting property: the infinite sequence is self-similar. In fact, fractals can often be defined in number theoretic terms; for instance, the middle third Cantor set; one of the best known fractals. It is constructed from the unit interval $[0, 1]$ by a sequence of deletion operations. The Cantor set provides a good example of fractals in number theory. In the end, we present in this paper one of the most surprising examples of fractals in number theory; the *coloring of the unit disk.* The color of a complex number is defined as the number of vertices of the convex hull of powers of that number. It is proved that the set Γ of points where the color changes is the union of some families of trinomial arcs solutions of the trinomial equation $z^n = \alpha z^k + (1-\alpha)$, where $z = \rho\, e^{i\theta}$ is a complex number, n is an integer larger than one, $k = 1, 2, \ldots, n-1$ and α is a real number between 0 and 1. This effective study asserts that Γ is a fractal set and that one of the types of trinomial arcs of Γ called *Farey arcs* is defined through a Farey sequence; notion of the number theory.

A DISCRETE TIME EPIDEMIOLOGICAL SEIRS MODEL

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Key Words and Phrases: epidemic model, delay-difference equations, non-linear incidence, stability analysis, oscillatory and quasi-periodic dynamics

A SEIRS epidemiological model is developed in terms of a system of delay-difference equations. It is assumed a non-linear incidence and constant periods of latency, infectiousness and immunity. Local stability conditions are determined related to the threshold parameter R_0. When the transmission parameter β increases, the qualitative behavior change from monotonic to damped oscillations, and then to a quasi-periodic dynamics. For some parameters values the relation between the inter-epidemic periods and immunity periods turns out to be linear. The qualitative behavior of SIRS and SEIS models are different from the ones displayed by our SEIRS model. Comparisons with other epidemiological models are given and it is shown that our SEIRS model is able to exhibit a wider range of dynamics than previous epidemiological models. In particular, the conditions for obtaining different probability distributions of the infected individuals are explored.

STATISTICAL INFERENCE FOR PARTIALLY OBSERVED MULTITYPE BRANCHING PROCESSES. APPLICATION TO PLANT DYNAMICS MODELS

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Key Words and Phrases: oilseed rape, populations dynamics, stage-structured populations, multitype branching processes, immigration, statistical inference, partial observations

Oilseed rape (Brassica Napus) is a well developped crop in Europe. This species is able to grow outside cultivated fields and possesses numerous populations observed on field margins or on roadside verges. The release of genetically modified oilseed rape may involve some undesirable effects for the environment: the risk of transgene spread is amplified by the presence of abundant *feral populations* of oilseed rape growing on the road verges. In order to study the dynamics of these populations, a ground survey has been conducted in an agricultural region of oilseed rape production, Sélommes, located in the center of France: almost each month from 2001 to 2003, feral oilseed rape populations have been observed on 3 roads and 3 paths. Each population is accurately described with count data containing the populations sizes in each stage of their development.

We use multitype branching processes with immigration in one type to model the dynamics of these stage-structured populations. We first study the parametric inference for multitype branching processes in this context. However, in practice, a new problem occurs when modelling the population dynamics of feral oilseed rape. Some stages (i.e. types), necessary to derive consistent estimates for the parameters of these multitype branching processes cannot be observed. This is not a peculiar situation. It is indeed a quite generic problem for many population dynamics models. This is to be linked to statistical inference for state-space models, although this problem does not belong to this class of models. This leads to new estimation problems. We develop a general framework to study the parametric inference for these partially observed multitype branching processes. We then apply it to a first specific case: the Poisson case. We obtain results that shed light on the various questions addressed here. We use simulated data departing from this model and study the performances of this theoretical approach. Results are really good. However, results are not quite satisfactory when applied to the feral oilseed rape data. This has required further work.

A STUDY OF LINEAR OPERATORS IN THE DEVELOPMENT OF GEOMETRIC FUNCTION THEORY

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Key Words and Phrases: univalent functions, differential operators and linear operators
AMS Subject Classification: 30C45

The study of linear operators and their operations in the class of univalent functions finds an important place in the field of Geometric function theory not only in the past, but even recently. Let $\mathcal{N}$ denote the class of analytic functions defined in the unit disc $\mathcal{U} = \{z : |z| < 1\}$ normalized by the conditions $f(0) = f'(0) - 1 = 0$. Numerous linear operators like $\Gamma(f)(z) = f'(z)$, $\Phi_\gamma(f)(z) = \dfrac{\gamma+1}{z^\gamma}\displaystyle\int_0^z \zeta^{\gamma-1} f(\zeta)\, d\zeta$ for $\Re\{\gamma\} > 0$ and $\Omega_t(f)(z) = (1-t)f(z) + tzf'(z)$ for $0 \leq t \leq 1$ are defined and studied for their preservation or transmission properties. In this note we give a brief survey of different operators such as Generalized Bernardi- Libera Livingston operator, Srivastava -Owa fractional derivative operator, Ruscheweyh derivative operator, Carlson -Shaffer operator, Hohlov linear operator, Dziok-Srivastava operator and study the role of these operators in the recent developments of Geometric function theory.

ABSOLUTELY IRREDUCIBLE GROUP ACTIONS AND BIFURCATION

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Key Words and Phrases: group action, bifurcation, isoptropy subgroups
AMS Subject Classification: 37J15, 37J20, 58E07, 58E40

Symmetry is a phenomenon which can be seen in may applications of mathematics in nature and technology. Many of these applications can be modeled as bifurcation problems, where the change of a parameter changes the structure of solutions.

Bifurcation problems with symmetry lead after standard reduction techniques (Lyapunov-Schmidt, Center manifold) to a low dimensional problem with a group action on the state space. Generically this action is absolutely irreducible if we study steady state bifurcation, or simple if we study Hopf bifurcation. In this talk we concentrate on the steady state case and hence we look at absolutely irreducible group actions.

Despite a long history of detailed studies of specific problems some of the fundamental questions are still open. In general steady state bifurcation problems, a change of stability generically leads to one–dimensional kernels. it is well known, that such problems produce bifurcating equilibria. The straight forward generalization to equivariant problems leads to the following question: does a change of stability through an absolutely irreducible group action always lead to bifurcating equilibria. In the talk we will discuss some aspects of this problem.

Some of these group actions give rise to equivariant Hamiltonian systems, where the symplectic structure and the Hamiltonian function are not invariant under the group action.

APPLICATIONS OF CLUSTERS AND FRACTALS IN MATHEMATICAL PHYSICS

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Clusters and fractals are discussed from the point of view of:

(i) the role of special C?-algebras in the infinite-dimensional complex analysis,

(ii) the role of fractal gradation and inoculation, in particular in the physics of condensed matter,

(iii) fractal gemmae and the atomization of structure fractals,

(iv) relationship with hyperkählerian structures and twistor-like structures.

MATCHING EXTREMAL THEORY AND PRACTICE IN RISK MODELING AND INFERENCE

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Key Words and Phrases: risk, extreme values, structural safety, capsize, environmental regulation
AMS Subject Classification: 60G70, 60F05

The methods of Extreme Value Theory provide important – even pivotal - tools in the modeling and statistical inference for situations involving rare events with high level risks, as for example for the design of dikes to prevent flooding, assessment of risks of core meltdown in a nuclear reactor, and the setting of environmental standards. However in routine applications of the theory basic underlying statistical principles may be overlooked, leading to conclusions of dubious value. One common pitfall is the misidentification of level (such as a critical stress level in a vessel or spacecraft) as being "extreme" in the sense required by Extreme Value modeling, or that observation periods may be too short or mismatched with the levels of concern.

In this paper we give a brief introduction to the standard methods of Extreme Value Theory for the reader (listener) unfamiliar with the topic. Our main focus will be with common pitfalls such as those mentioned above, with particular reference to three areas (a) Structural safety of vessels and ocean structures in high seas, (b) Stability of ships in stormy conditions, and (c) Environmental inference exemplified by tropospheric ozone regulation. Alternative methods will be suggested where appropriate for dealing with high values which are of concern (or even physically critical), but not necessarily as high as required for realistic use of Extreme Value Theory.

USING SOFTWARE EMULATION IN FPGAS TO IMPROVE CO-DESIGN DEVELOPMENT TIME

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Key Words and Phrases: hardware/software co-design, custom instructions, FPGA

The focus of this research is to determine when utilizing custom instructions, written for a Field Programmable Gate Array (FPGA), will improve execution time while not greatly increasing development time. The act of developing a single integral part of custom hardware is often more complex than that of developing a comparable single custom instruction. Hardware can require large design changes to reorder the execution of its parts. Software algorithms can reorder the use of custom instructions, in many combinations, in far less time. Custom instructions allow for many of the benefits afforded to custom hardware while reducing the amount of development time and hardware skills required.

To remove the need to rebuild the FPGA for each algorithm we choose to build a generic FPGA client that sends a list of supported custom instructions to a managing server. The server then decides which algorithm, with data yet to process, to compile. The FPGA test-bed used is an Altera Cyclone EP1C12, on it is implemented a standard NIOSII CPU and a simple 64bit ALU. The custom instructions are added to this ALU. Space is a problem when there is more than one algorithm to be processed, or the needed collection of the various specialized sub-sets of custom instructions do not fit into the FPGA.

Software-emulation is used to allow algorithms to be compiled for FPGAs that do not contain all of the custom instructions required. A 32 bit Universally Unique Identifier (UUID) is used to identify the instructions. At the linkage stage of compilation, the custom instruction's UUIDs are resolved to be either the supported real instructions or to be software-emulated. The binary file that is created is an overlay that is sent to the client and run as a function.

A profiler can be used to determine the portions of the algorithm that should be replaced by custom instructions. Validation of the replacement process can be done using the software-emulated calls. Benchmarking relative speed increases can be done using single-tick stubs, to emulate the speed of execution on the FPGA. This can all be done on a development machine requiring no access to, or skill in, FPGAs.

The approach being tested is designed to facilitate faster and less costly exploration of computationally intensive algorithms, of a discrete nature. This includes: image processing, pattern recognition, key generations of encryption/decryption, and many others. The goal of our research is to find the dataset size range in which this co-design approach will be more efficient, in terms of development and processing time, than the two design extremes of exclusively software or custom hardware design.

DYNAMICS OF TWO MODELS OF PEPTIC ULCERS AND H. PYLORI

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Key Words and Phrases: age-dependent mortality, structured model, prevalence
AMS Subject Classification: 92D25, 92D30

We consider two different models, one unstructured and one structured, for the dynamics of H. pylori infection and related peptic ulcers. The structured model accounts for age-dependent mortality. One of our main goals is to establish the existence criteria of an endemic disease equilibrium analytically, and the other is to compare two models on the effect of changes in treatment rate (ρ) and infection transmission rate (β) on ulcer prevalence and annual incidence. Here, the treatment means antibiotic therapy. Although it has been proved that most ulcers are caused by H. pylori infection rather than stress or spicy foods, changing medial belief and practice take time, and some doctors are still treating ulcers without antibiotics. We show that the effect of changes in ρ on ulcer prevalence and annual incidence is the same for both models and that the effect of changes in β is qualitatively similar, but quantitatively different for both models.

EXACT CONTROLLABILITY OF SEMILINEAR EVOLUTION EQUATION IN HILBERT SPACES AND APPLICATIONS

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Key Words and Phrases: semilinear evolution equation, controllability,damped wave equation
AMS Subject Classification: 93B05, 93C25

In this paper we characterize the exact controllability for the following semilinear evolution equation

$$z' = Az + Bu(t) + F(t, z, u(t)), t > 0, z \in \boldsymbol{Z}, u \in U$$

where Z, U are Hilbert spaces, $A : D(A) \subset Z \longrightarrow Z$ is the infinitesimal generator of strongly continuous semigroup $\{T(t)\}_{t\geq 0}$ in $Z, B \in L(U, Z)$, the control function u belong to $L^2(0, \tau; U)$ and $F : [0, \tau] \times Z \times U \longrightarrow Z$ is a suitable function. First, we give a necessary an sufficient condition for the exact controllability of the linear system $z' = Az + Bu(t)$. Second, under some conditions on F, we prove that the exact controllability of the linear system is preserve by the semilinear system. Finally, we apply these results to the controlled damped wave equation.

BENFORD'S LAW FOR RANDOM VARIABLES

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AMS Subject Classification: 60A10, 60E05

The First Digit Law is the empirical observation that in many tables of numerical data the leading significant digits are not uniformly distributed as one might at first suspect. The following law was first postulated by Simon Newcomb [Ne] in 1881:

$$\text{Prob(first digit} = d) = \log_{10}(1 + 1/d),$$

where $d = 1, \ldots, 9$. Since the rediscovery of this distribution by physicist Frank Benford [Be] in 1938, an abundance of additional empirical evidence has appeared, see [Ra] and [H1] for a review. What has become known as "Benford's law" has found applications e. g. in the distribution of one-day return on stock market idexes, the distribution of the population of U.S. counties, or the detection of accounting fraud.

So far, little attention was paid to the behavior of random variables in connection to Benford's Law. As Hill stated in [H1]: *An interesting open problem is to determine which common distributions (or mixtures thereof) satisfy Benford's law...* In [Le] the conformance of several survival distributions to Benford's law was tested using computer simulations. The special case of exponentially distributed random variables was studied in [EL]: exponential variables satisfy the first digit law only approximatively, but precise estimates can be given.

In my talk – partially based on joint work with Lutz Duembgen, University of Berne, Switzerland, and Rudolf Riedi, Rice University, USA – I focus my attention to sequences of random variables that obey Benford's law in the limit. I'll explain how such asymptotically Benford sequences can be constructed in various ways. Most survival distributions are shown to come close to Benford's law for a suitable choice of the parameters.

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ON SOLVING AEROACOUSIC PROBLEMS USING MODELED BOLTZMANN EQUATION

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Key Words and Phrases: aeroacoustics, Boltzmann equation
AMS Subject Classification: 76Q05, 76N15, 76M99

Numerical simulations of aeroacoustic problems has attracted the attention of the computational physics community in recent years. Commonly used numerical methods for aeroacoustics solve the full set of unsteady compressible Navier-Stokes equations; thus allowing the far-field sound and the near-field aerodynamics to be determined without modeling the source terms in the wave equation. Since the acoustic field has very low energy contents, a low dispersive and low dissipative scheme is required if wave propagation were to be resolved accurately in a direct aeroacoustic computation. A direct numerical simulation (DNS) scheme, made up of a 6th-order compact finite-difference scheme and a 4th-order Runge-Kutta time marching, and has been used to study aeroacoustic problems where the acoustic field with velocity fluctuations five orders of magnitude smaller than any mean field fluctuations was successfully resolved.

Recently, alternative numerical schemes based on modeled Boltzmann equation (MBE) has been proposed for dynamic simulations of dense fluids. In these schemes the Bhatnagar-Gross-Krook (BGK) model is usually invoked to approximate the complex Boltzmann collision integral. These schemes, however, cannot be used to tackle aeroacoustic problems because the simple BGK model failed to reproduce the fluid transport coefficients correctly, let alone the associated energy scale disparity. Thus, an improved BGK model, which correctly replicates the specific heat ratio γ (or Mach number, M) for air, the fluid viscosity μ (or Reynolds number, Re), and the thermal conductivity κ (or Prandtl number, Pr), has been proposed. With these improvements it can be shown that the recovery of the full set of unsteady compressible Navier-Stokes equations is possible, at least for diatomic gases, thus lending MBE a feasible model for simulating aeroacoustic physics. In this paper the rationale for the recovery of these important transport coefficients and the selection of appropriate numerical schemes for solving MBE are discussed.

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CONTINUOUS LOGIC IN THE AUTOMATA DYNAMICS

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Key Words and Phrases: continuous logic, logical modelling, automata theory, automata dynamics, deterministic and nondeterministic automata
AMS Subject Classification: 03B70, 68R99

In 1971 the author has established, that operations of continuous logic (CL), defined on an interval $C = [A, B], a \vee b = max(a, b)$, $a \wedge b = min(a, b)$, allow to describe in the analytical form dynamic processes in the circuits of finite (digital) automata. In 1972 the author has shown, that quasiboolean algebra of CL $\{C; \vee, \wedge\}$ is the adequate mathematics for the description of automata dynamics. With the help of CL-operations $\vee, \wedge$, it is possible to express the moments of changes of a signal in any unit of the circuit of any automata, if the moments of changes of its input signals are known. This result generalize and expand results of Shannon, Nakashima and Shestakov, established in 1938-1941 the conformity between Boolean algebra of logic and statics of work of the circuits of automata without memory (relay-contact circuits). Analytical dynamics of finite automata developed on CL allows to find in the analytical form, to analyze and synthesize dynamic processes in automata of rather large complexity with enough complex input processes. CL and dynamic automata today describe various systems in engineering, economy, sociology, politology etc.

Most simply objects in this theory are deterministic dynamic automata $y_k(t) = G_k[x_1(t), \ldots, x_n(t)]$, $k = 1, \ldots, m, x_i \in X, y_k \in Y$. Here $x_i(t)$ and $y_k(t)$ are input and output processes, X and Y are input and output alphabets, $(G_1, \ldots, G_m)$ is operator without memory. Such automata are realized by logic and inertial elements which has no feedback. $X = Y = 0, 1$ (all the processes are binary). Logic element i realizes on output Boolean function of the inputs $\{0, 1\}^R \to \{0, 1\}$. Inertial elements are delay and filter. The delay moves together the moment of change of input process on constant τ, the filter does not pass on output change of input process, with time lag less than on τ.

ON SCALAR DIFFERENTIAL EQUATIONS WITH A FINITE NUMBER OF PERIODIC SOLUTIONS

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Key Words and Phrases: ordinary scalar differential equations, Abel equation
AMS Subject Classification: 34C35

We present a proof of the following statement. Let $f(t,x)$ be a periodic function of t with period $T > 0$, such that $F = \partial f/\partial x$ continuous. If, for every t, $F(t,x)$ is a convex function in x, then the equation $dx/dt = f(t,x)$ has at most 3 periodic solutions of the period T. Particular case is the classical Abel differential equation. We use in the proof tools from one-imensional dynamics.

AN EFFICIENT OPTIMIZATION ALGORITHM FOR THE CONFIGURATION OF PHYSICS SYSTEMS UNDER UNSYMMETRIC FORCE FIELDS

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Key Words and Phrases: unconstrained optimization, Hessian-free Newton-Raphson method, BiCGstab, central-difference

AMS Subject Classification: 65K10, 11D04

To configure the ground state of a large scale physics system within unsymmetric force field (i.e., the numerically formulated Hessian might be inaccurate and unsymmetric), this paper proposed a Hessian-free Newton-Raphson method that employs a central-difference based bi-conjugate gradient stabilized (BiCGstab) strategy to solve Newton equations (denoted as HFNR-BiCGstab-diff). As the major contributions, this work first proves that a BiCGstab strategy using central-difference technique to approximate matrix-vector product (denoted as BiCGstab-diff) converges and the accuracy of the resulted solution (with respect to the exact solution of BiCGstab based on exact matrix-vector product) is controllable; Second, this work proves that Newton-Raphson method based on BiCGstab-diff converges at superlinear rate even with inaccurate and unsymmetric Hessian matrix. The whole work is implemented in Pthread/C language on IBM SP2. With three-dimensional clustered particle simulation problems (the degree of freedom may be up to $122,807$) as benchmarks, HFNR-BiCGstab-diff was critically assessed by comparing with alternative unconstrained optimization methods (e.g., preconditioned nonlinear conjugate gradient).

SOME RESULTS ON ABSOLUTE STABILITY OF DYNAMICAL SYSTEMS APPLICATIONS IN MECHANICS, TECHNOLOGY, INDUSTRY

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Key Words and Phrases: absolute stability, dynamical system, inner, flying vehicle, electroplasticity
AMS Subject Classification: 93-02, 93D05, 93D10, 93C95

One of the developments of the variational approach in the theory of absolute stability is based on the theory of inners of square matrix. The inner method for analysis of the absolute stability of dynamical systems is described in details in the present paper. Algebraic conditions of absolute stability obtained using inner method are shown and discussed. Different ways of continuation of development of the inner method in the absolute stability theory are suggested. Several questions and open problems in this area are formulated. Applications of inner conditions of absolute stability are presented.

The motion of an axisymmetric rotating flight apparatus in the air is investigated under conditions of unknown and incomplit information. The problem is to insure regularity of the flight which means sufficiently close matching during the flight of the vector of velocity of the center of mass of the vehicle with its axis of symmetry. Magnus effect, nonstationary air flow, possibility of unexpected wind, and other possible perturbations are considered. To solve this problem a mathematical model in the form of a system of differential equations with nonstationary coefficients and incomplete information on parameters is constructed and investigated for its absolute stability.

The electroplasticity phenomenon means ability to improve many important properties (technological plasticity, constructional solidity, ability for diffusion welding) of conductive materials (metals, alloys, flaky and composite materials), by special treatment of an electromagnetic field or an impulse current with untraditional parameters. The electroplasticity phenomenon and its effects described above are being utilized in industry: in production of metallic sheets, strips, bars, wires, as well as bi-metals and tri-metals from different metals and alloys which is in use in aviation and space industry, military and other areas. Construction of special technological processes for this production and investigation of stability of these processes lead to the absolute stability problem because of incomplete information on technical parameters. This problem is solved on the basis of the inner approach.

Reference

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FAST SOLUTION OF CHROMATOGRAPHIC PARTICLE AND COLUMN MODELS ON PARALLEL COMPUTERS

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Chromatography is commonly applied for protein separation in biopharmaceutical industry. Purification of pharmaceutical products typically requires several chromatographic steps, whose design and optimization for new products are time consuming and expensive. Quantitative simulation of chromatographic biomolecule separation can help speed up process design and cut down costs. However, chromatography models comprise various parameters whose values need to be estimated from experimental data.

Modern experimental techniques, such as Confocal Laser Scanning Microscopy (CLSM), provide comprehensive data of increasing quantity and quality. However, model based analysis of these data is computationally very demanding, particularly for multi-component systems, since parameter estimation involves repetitive solution of nonlinear and stiff partial differential-algebraic equation systems. Hence, fast solution of chromatography models is crucial for both rational analysis and model based design of separation processes for biomolecules.

Chromatography models exhibit equation structures that allow efficient solution on parallel computers. Parallelization is particularly important as the present trend towards multiprocessor and multicore technology even for standard personal computers can not be economically utilized without suitable algorithms. Hence, two submodels of the general rate model are chosen for assessment of parallel solution techniques. These submodels describe isolated porous particles, and chromatographic columns with nonporous particles, respectively.

The model equations are partial differential-algebraic and stiff when spatially discretized. In order to avoid numerical instabilities, the first derivatives in the column model are approximated with the WENO method (weighted essentially non oscillatory). The resulting large systems of ordinary differential-algebraic equations are solved with the implicit differential-algebraic solver IDA. All calculations were performed in Jülich on the supercomputer JUMP.

The results prove that numerical solution of chromatography models can be sped up tremendously with a beneficially decreased computational overhead, and encourage implementation of a parallel solver also for the general rate model, which describes column chromatography with porous particles.

NEW TRENDS IN HAUSDORFF OPERATORS

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Key Words and Phrases: Hausdorff operator, Fourier transform, real Hardy space, BMO, power series

AMS Subject Classification: 47B38, 42B10, 46E30, 30B10

Hausdorff operators (Hausdorff summability methods) appeared long ago aiming to solve certain classical problems in analysis. Modern theory of Hausdorff operators was inspired in complex analysis setting by work of A. Siskakis and in the Fourier transform setting by the work of F. Móricz and the author. Their interest in this subject was first concentrated on one-dimensional problems. While Hausdorff operators for power series are still studied in dimension one only, most interesting problems for the Hausdorff operators of Fourier integrals are multi-variate. The most general definition of the Hausdorff operator reads as

$$(\mathcal{H}f)(x) = (\mathcal{H}_\Phi f)(x) = (\mathcal{H}_{\Phi,A} f)(x) = \int_{\mathbb{R}^n} \Phi(u) f\big(xA(u)\big)\, du,$$

where $A = A(u) = (a_{ij})_{i,j=1}^n = \big(a_{ij}(u)\big)_{i,j=1}^n$ is the $n \times n$ matrix with the entries $a_{ij}(u)$ being measurable functions of u. This matrix may be degenerate at most on a set of measure zero; $xA(u)$ is the row n-vector obtained by multiplying the row n-vector x by the matrix A.

We concentrate ourselves on open problems in the subject. In a recent paper by Lerner and the author conditions on the matrix A are found to provide the boundedness of the operator in the real Hardy space. The proof is based on the H^1-BMO duality. To extend this result to product Hardy spaces a new proof is needed. The whole stuff aims to clarify whether a new complete characterization of various Hardy spaces by means of Hausdorff operators is possible.

"COMBINED MANIFOLDS" AS SETS OF SINGLETONS WITH DUAL COORDINATES

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Key Words and Phrases: product manifolds, lorentz manifolds, Minkowski space
AMS Subject Classification: 53C15, 53C25, 83C05, 53C80

This paper introduces the term, "combined manifold," $\mathcal{M}^{[3]}$, as the graph of a diffeormorphism from one $n-manifold$, $\mathcal{M}^{[1]}$, to another, $\mathcal{M}^{[2]}$, so that their tangent spaces $T_p\mathcal{M}\ \forall p \in \mathcal{M}$ are related by

$$T_{p^{[3]}}\mathcal{M}^{[3]} \subset T_{p^{[3]}:=\left(p^{[1]},p^{[2]}\right)}\left(\mathcal{M}^{[1]} \times \mathcal{M}^{[2]}\right) \simeq T_{p^{[1]}}\mathcal{M}^{[1]} \oplus T_{p^{[2]}}\mathcal{M}^{[2]}; \tag{1}$$

consequently, $\forall$ bilinear forms $g^{[1]}$ and $g^{[2]}$ defined respectively on $T_{p^{[1]}}\mathcal{M}^{[1]}$ and $T_{p^{[2]}}\mathcal{M}^{[2]}$, $\forall w^{[1]}, w^{[2]} \geq 0$,

$$g^{[3]} := w^{[1]}g^{[1]} + w^{[2]}g^{[2]} \tag{2}$$

is a bilinear form on $T_{p^{[3]}}\mathcal{M}^{[3]}$. An elemental example of a combined manifold is that of a diagonal map (see, e.g., Isham): $f(x) = (x, x)$, which renders a clear illustration of the idea of a singleton with dual coordinates. In many systems, the same object engages in two activities, e.g., serving both as a receiver and sender, input and output, buyer and seller, etc. As such, combined manifolds lend themselves to wide applications. Among many examples, consider especially the following two parametrizations, where $\eta :=$ the Minkowski metric, $i = 1, 2$:

$$f^{[i]} : U \subset \left(\mathbb{R}^4, \eta\right) \longrightarrow \mathcal{M}^{[i]}, \tag{1}$$

then $\mathcal{M}^{[3]} := \left\{p^{[3]} := \left(p^{[1]}, p^{[2]}\right) = \left(f^{[1]}(u),\ f^{[2]}(u)\right) \mid u \in U\right\}$ is a combined 4-manifold. This pair of parametrizations (1) yields the familiar corresponding two sets of equations

$$R^{[i]}_{\mu\nu} - \frac{1}{2}R^{[i]} \cdot g^{[i]}_{\mu\nu} = \frac{-8\pi G^{[i]}}{c^2}T^{[i]}_{\mu\nu}. \tag{3}$$

Define $w^{[1]} := \omega\left(G^{[1]}, G^{[2]}\right)$ by a certain specific $\omega : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow (0, 1)$ and set $w^{[2]} = 1 - w^{[1]}$; then

$$g^{[3]} := w^{[1]}g^{[1]} + w^{[2]}g^{[2]} \tag{4}$$

is the unique metric for $\mathcal{M}^{[3]}$ that yields

$$R^{[3]}_{\mu\nu} - \frac{1}{2}R^{[3]} \cdot g^{[3]}_{\mu\nu} = \frac{-8\pi G^{[3]}}{c^2}T^{[3]}_{\mu\nu}, \tag{5}$$

for some $G^{[3]}$ and $T^{[3]}_{\mu\nu}$, with the contained dynamics determined by the geodesics.

SYMPLECTIC ELASTICITY APPROACH FOR FREE VIBRATION OF RECTANGULAR THIN PLATES

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Key Words and Phrases: eigenvalue, eigenvector, elasticity, Hamitonian principle, Legendre transformation, symplecticity

AMS Subject Classification: applications in mechanics, physics, chemistry, biology, technology, economics and industrial problems

Using a new symplectic method commonly applied by theoretical physicists, a new symplectic elasticity approach is developed for deriving exact analytical solutions to some long standing basic problems in free vibration of rectangular thin plates with any boundary conditions where exact solutions are hitherto unavailable. The models are built in a new symplectic space and the analytical solutions are derived using symplectic geometry. Employing the Hamiltonian principle with Legendre's transformation, analytical free vibration solutions could be obtained by eigenvalue analysis and expansion of eigenfunctions in both lengthwise and widthwise directions. Unlike the classical semi-inverse approaches using trigonometric, hyperbolic and/or Bessel functions where a trial amplitude function is pre-determined, this new symplectic approach is completely rational without any guess functions and yet it renders exact solutions beyond the scope of applicability of the semi-inverse approaches. In short, the symplectic approach developed in this paper presents a new approach in an area previously unaccountable in classical mechanics and the semi-analytical approach forms a limited sub domain of this new approach. Examples for plates with selected boundary conditions are solved and the exact solution is discussed. Comparison with the classical solutions shows excellent agreement. As the derivation of this new approach is fundamental, further research can be conducted not only for other types of boundary conditions, but also for thick plates as well as bending, buckling, wave propagation, etc.

CENTRAL LIMIT THEOREMS FOR MARKOV CHAINS

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Let $\{X_n\}$ be a stationary ergodic Markov chain on the state space (S, Σ, m) with transition probability P, Markov operator $Pg(x) = \int g(y)P(x, dy)$, and P-invariant probability m (the invariant initial distribution). We assume that the chain is defined on the space of trajectories $(\Omega, \mathcal{A})$, where $\Omega := \Pi_{n=0}^{\infty} S$ and $\mathcal{A} := \otimes_{n=0}^{\infty} \Sigma$. We denote by $\mathbb{P}_x$ the probability on $\mathcal{A}$ corresponding to the initial probability δ_x (the chain starts from x), and put $\mathbb{P}_m = \int \mathbb{P}_x dm$ for the probability of the chain with initial distribution m. The P-invariance of m is equivalent to the shift-invariance (in Ω) of $\mathbb{P}_m$.

Given $f \in L_2(S, m)$ with $\int f\,dm = 0$ we obtain an additive functional on the chain defined by $S_n(f) := \sum_{k=1}^n f(X_k)$. The original problem is to find conditions on f such that $\frac{1}{\sqrt{n}} \sum_{k=1}^n f(X_k)$ converges in distribution, in $(\Omega, \mathbb{P}_m)$, to a centered normal distribution (possibly degenerate). This is the *annealed* Central Limit Theorem (CLT) for f.

The second problem is to find conditions on f such that for a.e. $x \in S$, the CLT holds for $S_n(f)$ in $(\Omega, \mathbb{P}_x)$; i.e., for a.e. $x \in S$, $\frac{1}{\sqrt{n}} \sum_{k=1}^n f(X_k)$ converges in distribution, in $(\Omega, \mathbb{P}_x)$, to a centered normal distribution. This is the *quenched* CLT. If the quenched CLT holds and the variance of the limiting normal distribution is a.s. a constant σ_f, then the annealed CLT holds, and the variance of the limiting normal distribution is also σ_f.

In this talk I will survey the developments on both problems, and present some of the recent results. This talk is based on several joint papers with Yves Derriennic.

A PRACTICAL IMPLEMENTATION FOR BANDWIDTH SELECTION IN KERNEL DISTRIBUTION FUNCTION ESTIMATORS

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Key Words and Phrases: Bandwidth selection; Bayesian estimator; Kernel estimation; Nonparametric distribution estimator; Prior distribution; Smooth density estimator; Smooth parameter; Survival Function

AMS Subject Classification: 62, Statistics

Two kernel smooth distribution estimators for a distribution function, $F(x)$ which has density function $f(x)$, are considered. Both estimators depend on the sample size n, the smooth bandwidth h, and the kernel function, $k(x)$, which could have unbounded support. It is shown that the bandwidth $h_0 = C(k, f, k, x)n^{-1/3}$ is asymptotically optimal in the sense that $\left[\hat{\phi}(x, n, h_0) - F(x)\right]^2 / \inf_h \left[\hat{\phi}(x, n, h) - F(x)\right]^2 \to 1$ in probability as $n \to \infty$, where $\hat{\phi}(x, n, h)$ indicates both kernel smooth estimators under regular conditions on the kernel function. It is also shown that h_0 is asymptotically optimal bandwidth for the kernel smooth quantile estimators defined as the inverse functions of kernel smooth distribution estimators. The regular conditions for the kernel function here are different from the conditions given by Lio and Padgett (1991). However, the drawback of h_0 is that it contains unknown quantities $F(x)$ and $f(x)$. To implement the optimal bandwidth selection, a bayesian bandwidth is proposed. The empirical study of computer simulation shows that the proposed smooth distribution estimators with bayesian bandwidth could have mean squared errors that are almost the same as the mean squared errors for the smooth estimators with the optimal bandwidth h_0. Finally, the proposed estimators are used to fit the distribution of tensile strength of 6061-T6 aluminum sheeting from Birnbaum and Saunders (1958, 1969).

ARTIN'S CONJECTURE FOR COMPOSITE MODULI

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Key Words and Phrases: primitive root, Artin's conjecture, distribution of primitive roots, composite/prime moduli
AMS Subject Classification: 11N37, 11A07, 11A25, 11L40

It is well-known that $(\boldsymbol{Z}/p\boldsymbol{Z})^*$ is a cyclic group for any prime p. A generator of the group is called a primitive root modulo p. There are many attractive theorems and conjectures concerning distribution of the primitive roots. This issue can be investigated in two perspectives. First, we can fix a modulus p and look at the number of primitive roots for p, which turns out to be $\phi(p-1)$. Study on this function shows that $\phi(p-1)/(p-1)$ possesses a continuous distribution function. Secondly, we can fix an integer a and look at the number $P_a(x)$ of prime moduli up to x that have a as a primitive root. Artin's conjecture claims that $P_a(x) \sim A(a)\pi(x)$ for some constant $A(a)$ depending on a, where $\pi(x)$ is the number of primes up to x. Our focus in the talk is on whether or not it makes sense to ask the same questions for composite moduli. Classically primitive roots are not defined for composite moduli n as $(\boldsymbol{Z}/p\boldsymbol{Z})^*$ is mostly not cyclic. However, if we look at elements with the maximum order, then primitive roots can be naturally extended for arbitrary moduli.

Although there can be many questions for primitive roots for composite moduli, we would like to investigate issues on distribution of the primitive roots similarly as above. First, let us fix a modulus n and consider the number $R(n)$ of primitive roots for n. A closed form for $R(n)$ is not too hard. But what can be concluded about the the relative asymptotic density of the set $\mathcal{R}_u = \{n : R(n)/\phi(n) \leq u\}$? The talk would show nonexistence of the density of $\mathcal{R}_u$ for certain real numbers u, due to an oscillation in the distribution of values of $R(n)$. Secondly, let us fix an integer a and consider the number $N_a(x)$ of the moludi up to real number x that have a as a primitive root. The talk would show the connection between average value of $R(n)$ and average value of $N_a(x)$. The oscillation in the distribution of $R(n)$ results in an oscillation of the average value of $N_a(x)$, which is quite different from Artin's conjecture on average, namely the average value of $P_a(x)$ over integers a in a certain interval depending on x. The exotic behavior in the average values of $N_a(x)$ promotes one to conjecture similar behavior for individual function $N_a(x)$ for most integers a. What is known for this problem will be discussed. Should time allow, a brief view on source of the oscillations would be attempted.

TRIGONOMETRIC SPLINES AND SPHERICAL SPLINES

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In this paper, univariate and bivariate trigonometric splines of arbitrary parameters with compact supports are formulated. The approximation properties of trigonometric splines are explored. Furthermore, interpolating trigonometric splines on small compact supports are formed and approximation properties are established. The spherical splines are constructed using the trigonometric splines. Key words: spline functions, trigonometric spline, spherical spline

EXISTENCE AND UNIQUENESS RESULTS FOR A CLASS OF NONLINEAR DIFFERENTIAL SYSTEMS

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Key Words and Phrases: nonlinear differential system, Cauchy problem, monotone operator, strong solution, weak solution, existence of solutions, asymptotic behaviour
AMS Subject Classification: 34G20, 35L55, 47H05, 39A12

We investigate the existence, uniqueness and asymptotic properties of the strong and weak solutions to the nonlinear differential system

$$\begin{cases} u_j'(t) + \mathrm{d}\,\frac{v_{j+1}(t)-2v_j(t)+v_{j-1}(t)}{h_j^2} + c_j A(u_j(t)) \ni f_j(t) \\ v_j'(t) - \mathrm{d}\,\frac{u_{j+1}(t)-2u_j(t)+u_{j-1}(t)}{\overline{h}_j^2} + d_j B(v_j(t)) \ni g_j(t), \end{cases} \tag{S}$$

$$0 < t < T,\ \ j = \overline{1,N},\ \text{ in } H,$$

with the extreme conditions

$$\begin{gathered} \begin{pmatrix} u_1(t) - u_0(t) \\ -v_1(t) + v_0(t) \end{pmatrix} = \alpha \begin{pmatrix} v_1(t) \\ u_1(t) \end{pmatrix}, \\ \begin{pmatrix} -u_{N+1}(t) + u_N(t) \\ v_{N+1}(t) - v_N(t) \end{pmatrix} = \beta \begin{pmatrix} v_N(t) \\ u_N(t) \end{pmatrix},\ \ 0 < t < T, \end{gathered} \tag{EC}$$

and the initial data

$$u_j(0) = u_{j0},\ \ v_j(0) = v_{j0},\ \ j = \overline{1,N}. \tag{ID}$$

Here H is a real Hilbert space, $n \in I\!N$, $T > 0$, $c_j,\ d_j,\ h_j,\ \overline{h}_j > 0$, for all $j = \overline{1,N}$, α, β and A, B are multivalued operators in H^2, respectively H, which satisfy some assumptions.

This problem is a discrete version with respect to x (with $H = I\!R$) of some nonlinear second-order partial differential systems, subject to boundary conditions and initial data (see [1], [2], [3]). In our proofs we use several results related to maximal monotone operators and nonlinear evolution equations in Hilbert spaces (see the monographs [4], [5]).

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SINGULAR EQUILIBRATED STRESS FIELDS FOR NO–TENSION BODIES

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Key Words and Phrases: Masonry panels, equilibrium, divergence measures
AMS Subject Classification: 74G70, 49Q15

We study the equilibrium problem for bodies made of a no-tension material, subjected to distributed or concentrated loads on their boundary and possibly subject to gravity. Admissible and equilibrated stress fields are interpreted as tensor valued measures with distributional divergence represented by a vector valued measure. Such stress fields are generalizations of ordinary functions, which allows us to consider stress concentrations on surfaces and lines. The general framework for this approach is presented first and then illustrated on examples of 2-dimensional panels under different loads. In the general framework we present some classes of stress fields represented by measures and determine their weak divergences and the surface tractions. Combinations of these stress fields will be shown to give the solutions for the panels. The analysis of panels subjected to concrete loads leads to explicitly determined admissible stress fields with one or more curves of concentrated stress. The explicit nature of the stress field often enables us to determine the collapse value of the loading parameter.

BIOINFORMATICS: PROBLEMS AND MODELS — REGULATION OF GENE EXPRESSION

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Key Words and Phrases: mathematic modeling, bioinformatics, transcription regulation model, attenuation in bacterial cell
AMS Subject Classification: 93A30, 92D10, 92D20

In applied mathematics and computer science, bioinformatics (or, otherwise, computational molecular biology) becomes a field of ever increasing importance. Its goal is to develop realistic models of basic biological processes by analyzing tremendous amount of genetic data accumulated on organisms as distant as viruses and human, and to infer evolutionary history of organisms, thus providing us with novel biological evidence. No doubt is left that such evidence can be reliably obtained in silico. Situation here becomes akin to that in experimental and theoretical physics. The talk will introduce the first model of a basic process in the bacterial cell — classic attenuation regulation of gene expression. Computer program realizing the model produces output that is in good agreement with the experiment. Constructing the model required original developments in complex geometric structures description, algorithmic theory and programming. The model was used to make important biological predictions.

Modeling mechanisms of gene expression regulation to predict quantitative characteristics of expression (such as estimation of the expression level and substrate concentration) is an important challenge. Studies [Lyubetsky et al, J. of Bioinformatics and Computational Biology, 5(1), 2007; Lyubetsky et al, Molecular Biology, 40(3), 2006] introduce a model of one particular kind of regulation, classical attenuation regulation. There, dynamics of secondary RNA structure in the leader region of a gene, progression of the ribosome and the polymerase along RNA/DNA strands, and premature termination of the polymerase, are modeled by a very special, elaborated in detail Markov chain. In the chain, transition rate constant of the ribosome progression depends on a "control variable" — concentration of charged tRNA molecules in the cell. All other transition rate constants do not depend on the control variable and are defined through parameters of state energy and interaction of variable mRNA secondary structure with the polymerase. Termination and antitermination of gene expression correspond to particular random events in the Markov chain. Due to large size and complex structure of the chain its simulation is a heavy computational task. It was solved, and the program simulates one trajectory in fractions of a second, [http://lab6.iitp.ru/rnamodel/].

THE PRONY METHOD OF IDENTIFYING DYNAMICALLY BALANCED SYSTEMS RELAXATION CHARACTERISTICS

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Key Words and Phrases: ordinary differential equation, quasipolynomial, Prony algorithm, approximation, discrete function, uniform net
AMS Subject Classification: 34, 41, 92

A way of investigating medico-biological, biophysical, physico-chemical or other dynamically balanced systems is to study the relaxation of certain system parameters ("variables") after an external impact. The variables either regain their original levels or pass to new (adaptation) levels. Their graphs are called *relaxation characteristics*. The mathematical analysis of homeostasis relaxation characteristics is reduced to the problem of constructing models, that describe the behavior of these characteristics, and to evaluating the parameters of these models. Among the most common classes of models used to study relaxation characteristics we name the class of models describing the dynamics of these characteristics by means of an ordinary linear differential equation with constant real coefficients:

$$y^{(k)} + a_1 y^{(k-1)} + \ldots + a_k y = f, \tag{1}$$

where f is a finite exponentially harmonic sum. The solutions of equation (1) (quasipolynomials) they yield a good approximation of many dynamical processes occurring in investigation practice. If the coefficients of equation (1) and some numerical parameters of its right-hand side are unknown, the input information to find the quasipolynomials is its moments, i.e., its values at a finite number of equidistant time instants. Among the approaches to solving the above problem of identifying equation (1), we name the Prony algorithm and its modifications [1, Chapter 11], [2, Chapter 2]. In the talk we present in detail a modifications of this algorithm and, on the basis of it, develop a method of approximating a discrete function defined on a finite number of nodes of a uniform net by quasipolynomials of fixed order.

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HIGH ORDER COMPACT AND UPWIND METHODS

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Key Words and Phrases: compact methods, upwind methods, Navier-Stokes equations
AMS Subject Classification: 74S20, Finite difference methods

We consider two type of high order numerical methods for solving the Navier-Stokes equations: compact and upwind methods.

Firstly, we analyse procedures for obtaining compact fourth order method to the steady 2D Navier-Stokes equations in the streamfunction and streamfunction-vorticity formulations. To test the procedure we solve many problems, including cavity and channel problems, and the results are compared with results obtained by second order central differences to moderate Reynolds numbers.

Secondly, we apply high order upwind methods to free surface flow problems. A crucial point is the discretization of the advective terms in the transport equations, which is of primordial importance for the prediction of the flow. In the context of the normalized variable formulation and total variation diminishing, we present the development and implementation of the bounded high order upwind adaptative QUICKEST scheme in the 2D and 3D robust codes for incompressible Navier-Stokes equations. Numerical results for the 1D shock tube, 2D impinging jet and 2D/3D broken dam flow are compared with existing analytical and experimental data.

THE APPLICATION AND ANALYSIS OF FRACTAL STRUCTURES TO ESTIMATE RESPONSE IN RANDOM STRUCTURES: PAST AND PRESENT INNOVATIONS

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Key Words and Phrases: Random Structure, Fractal Structure, Porous Media, Diffusion-Limited Aggregation, Structural Analysis, Thermodynamic Analysis, Atomic Vibration
AMS Subject Classification: 28A80

The increasingly diversified application of Fractal Theory has had important ramifications in the field of Condensed Matter Physics. Models such as diffusion-limited aggregation (DLA), among others, use simple stochastic algorithms that approximate physical systems in great detail. The increasing accuracy of the Physical Sciences requires model results that are proportionately accurate; however, the current trend of continuous quantization of Euclidean models has an accuracy upper bound based on computing time requirements. Various methods in Fractal Theory propose solutions to this problem by providing highly accurate approximations of physical systems which are computationally less intensive, yet stochastically precise. It is suggested that fractal models have enormous potential to approximate actual responses in random structures, providing an important advancement in complex model-response computation.

GENERALIZED METHOD OF SUCCESSIVE APPROXIMATIONS

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Key Words and Phrases: order-theoretical fixed point theorems, metric fixed point theory, topological fixed point property, methods of successive approximations
AMS Subject Classification: 03F60, 06A06, 35R05, 45L05, 47H10, 54H25

Let (A,¡) be an asymmetrical relational system, i.e. x ¡ y implies y *non*¡ x for all x, y $\in$ A, and let z $\in$ C $\subseteq$ A. Then z is the smallest (greatest) element in C whenever each other element x of C satisfies z ¡ x (x ¡ z , respectively). Let D $\subset$ C. Then z is a successor of D in C whenever z is the smallest ¡ - bound of D in C, i.e. z is the smallest element of C such that
x ¡ z for all x $\in$ D; if D = $\{$x$\}$ then z is simply the successor of x in C, briefly z = succ_C(x). The supremum $\sup_C$(D) of D in C is either the greatest element in D if it exists in D or the successor of D in C in the opposite case.

For an arbitrary function f: A $\rightarrow$ A and a set-point function σ: **D** $\rightarrow$ A, with **D** composed of some sets S $\subseteq$ A, and for an arbitrary a $\in$A, we define the orbit O(a) to be the $\subset$-greatest among the well-orderings (C,¡) which emanate in (A,¡) from the point a , in which f is the successor function and σ is the supremum set-point function, i.e.: (1) a is the smallest element in C (2) f(x) = succ_C(x) for each x $\in$ C which is ¡ - bounded in C (3) S$\in$ **D** and σ(S)= $\sup_C$(S) for each $\emptyset \neq$S$\subset$C which is ¡ - bounded in C. Namely, O(a) is the union of all such C's. Our first theorem states under assumption O(a) $\in$ **D** : (i) if σ(O(a)) is a $\leq$ - bound of O(a) in (A,¡) then σ(O(a)) is a greatest element in O(a) (ii) if O(a) has a greatest element b then f(b) is not a ¡ - bound of O(a), i.e. if f(b) is a $\leq$ - bound of O(a) then f(b) = b (the symbol $\leq$ stands of course for the alternative " either ¡ or = ").

We define I(a) to be the $\subset$-smallest among all B $\subseteq$ A which satisfy the following three conditions (4) a $\in$ B (5) f(B) $\subseteq$ B (6) $\emptyset \neq$ S$\subseteq$B and S$\in$**D** imply σ(S)$\in$B. Namely, I(a) is the intersection of all such B's. Our second result is: O(a) $\subseteq$ I(a), and the equality I(a) = O(a) holds true if and only if the following two conditions are satisfied (iii) if O(a) $\in$ **D** then σ(O(a)) $\in$ O(a) (iv) if there is a greatest element b in O(a) then f(b) $\in$ O(a).

As a corollary we obtain immediately the following result: If O(a) $\in$ **D**, and σ(O(a)) and f(σ(O(a))) both are $\leq$ - bounds of O(a) in (A,¡), then (v) σ(O(a)) = f(σ(O(a))) (vi) σ(O(a)) is a greatest element in O(a) (vii) O(a) = I(a).

Finally, using the orbits O(a) we prove that the approximation procedures by Abian and Brown 1961, Fuchssteiner 1977 and 1986, Heikkilä 1996, Heikkilä and Laksmikantham 1994, Büber and Kirk 1996 and 1995 (cf. also Appendix A.8 in the book by Khamsi and Kirk 2001) - all constitute the same generalized iteration procedure (and the iteration function $\mathrm{It}_f(\mathrm{a})$ of f at a by Fuchssteiner 1977 is just the dual to the function $\mathrm{It(a)} = \sigma(\mathrm{O(a)})$ with $\sigma = \sup_A$). Simultaneously, these are the same iteration procedures as in the books by Moschovakis 1974 and Schröder 2003 (cf. his interpretation of the iteration by Abian and Brown 1961), where they are defined by using ordinal numbers and transfinite induction. The transfinite methods which are surveyed in the article by Kirk 2003 are under consideration.

ASYMPTOTIC FORM OF THE SOLUTION OF STURM-LIOUVILLE PROBLEM WITH m TURNING POINTS

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Key Words and Phrases: Sturm-Liouville problem, Turning point, Asymptotic solution
AMS Subject Classification: 34B24

We consider the Sturm-Liouville equation

$$U'' + (\rho^2\phi^2(x) - q(x))U = 0 \ (*)$$

with any initial conditions, on a finite interval I, say $I = [0,1]$, where I contains m turning points $x_1, x_2, ..., x_m$, which are zeros of ϕ. Using fundamental solutions constructed in [W. Eberhard, G. Freiling, A. Schneider, Connection formulae for second-order differential equations with a complex parameter and having an arbitrary number of turning points, Math. Nachr. 165 (1994), 205-229.], we show that if one obtains the asymptotic solution of initial value problem (*) in the interval $[0, x_1)$, the asymptotic solution in the remaining intervals can be obtained by special relations according to the type of turning points.

TOPICS IN PHYSICAL MATHEMATICS: GEOMETRIC TOPOLOGY AND FIELD THEORY

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In recent years the interaction between geometric topology and classical and quantum field theories has attracted a great deal of attention from both the mathematicians and the physicists. We discuss some topics where this has led to new viewpoints as well as new results. They include categorification of knot polynomials and a special case of the gauge theory to string theory correspondence in the Euclidean version of the theories where exact results are available.

RECENT TRENDS ON THE ASYMPTOTIC PROPERTIES OF ORTHOGONAL POLYNOMIALS IN SOBOLEV SPACES

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It is very well known that orthogonal polynomials associated with a nontrivial probability measure supported on the real line satisfy a three-term recurrence relation (second order linear difference equation) translating the fact that the multiplication operator by x is a symmetric operator with respect to the standard inner product defined by such a measure.

Let $\{\mu_k\}_{k=0}^m$ be a vector of positive Borel measures supported on the real line. In the linear space **P** of polynomials with real coefficients we introduce the following Sobolev inner product

$$\langle p, q\rangle_S := \sum_{k=0}^{m} \int_{\mathbb{R}} p^{(k)}(x)\, q^{(k)}(x)\, d\mu_k(x), \tag{1}$$

In general, the sequence of polynomials orthogonal with respect to the above inner product does not satisfy a recurrence relation involving a fixed number of terms, independently of the degree of the polynomials.

In [1] the authors proved that there exists a polynomial multiplication operator H, symmetric with respect to (1), if and only if $\{\mu_k\}_{k=1}^m$ are discrete measures supported in subsets of the real line, associated with the zeros of the polynomial H. Taking into account the behaviour of the coefficients in the corresponding higher order difference equation, then relative asymptotic properties for such polynomials are studied in [3] when the measure μ_0 belongs to the class M(0,1), an important example of bounded measures including the Jacobi weight probability measure, among others,. An extension of these results, concerning strong and ratio asymptotics when the support of the measure μ_0 is unbounded will be analyzed in this talk. In particular we will emphasize the study of these questions for the measure $d\mu_0(x) = exp(-x^4)$, supported on the wide real line, which constitutes a seminal case of Freud weight.

On the other hand, when the components of the vector of measures are nontrivial probability measures supported on unbounded subsets of the real line, very few contributions (see [2], [4], and [5]) have been done concerning the asymptotic properties of the corresponding sequences of Sobolev orthogonal polynomials, mainly when m = 1. Here We will summarize some of the basic results and we will suggest new problems in this direction for m greater than 1.

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ON THE EXISTENCE AND THE ASYMPTOTIC BEHAVIOUR OF REGULARLY (SLOWLY) VARYING SOLUTIONS OF SOME CLASSES OF SECOND ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENT

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We present some results recently obtained jointly with Professor Kusano Takasi, Fukuoka Univ. Japan in the study of equations

$$(A) \qquad x'' = q(t)x(g(t)), \quad q(t) > 0$$

$$(B) \qquad x'' \pm p(t)x(g(t)) \pm q(t)x(h(t)) = 0, \quad p(t), q(t) > 0$$

in the framework of Karamata regular (slow) variation. (N.H. Bingham, C.M. Goldie, J.L. Teugels, Regular Variation, Encyclopedia of Math. and its Appl., Vol. 27 Cambridge U.P., 1987).

A necessary and sufficient condition is obtained for equation (A) to possess a slowly varying solution both in the advanced and retarded case, and also a sufficient one to possess a regularly varying solution in the retarded case. As an application some precise asymptotic formulas as $t \to \infty$ for such solutions are derived.

Some similar results are obtained for equation B where $g(t)$ is a retarded and $h(t)$ is an advanced argument.

Schauder-Tychonoff fixed point theorem, and some result on the equation $x'' = q(t)x(t)$ (without deviating argument) (V. Marić, Regular variation and Differential Equations Lecture Notes in Math. 1726. Springer, Berlin 2000), serve as the main tool.

AUTOMATED PROGRAM WRITING SYSTEM BASED ON PER MARTIN-LÖF'S TYPE THEORY

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Key Words and Phrases: Martin-Löf's Type Theory, automation of inference
AMS Subject Classification: 03 Mathematical Logic, 68 Computer science

In [1] and [2] Per Martin-Löf has introduced his Type Theory (hereafter MLTT). Any problem defined in the "language" of MLTT is a type in MLTT. To solve the problem, that is, to infer the type that represents the problem in MLTT is the same as to construct an object of that type. Based on the construction of that object, a program can be created that produces a solution of the problem. The automation of the above mentioned procedures will result in a system for automated program writing. To achieve this goal first the theoretical basis should be established. It requires a large number of scholastic proofs and results.

The consistency of MLTT is the first requirement. One of my results is a proof of consistency of Martin-Löf's Type Theory. It has been submitted for publication in Springer Archive for Mathematical Logic. Another problem is the efficiency of the generated programs. This problem can be reformulated into a problem of comparison of complexity of inferences in MLTT and in other known systems.

Next is the building of the algorithm that automates the construction of an object of the type that represents the problem under question. Construction of this algorithm raises fixed-point type problems.

The last step for the ultimate goal is the creation of the automated program writing system.

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A NEW MEASURE FOR DISTANCE TO UNCONTROLLABILITY

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Key Words and Phrases: distance to uncontrollability, approximate common divisor, structured low-rank approximation
AMS Subject Classification: 93C15, 93B05, 93B40

Let $\operatorname{dist}(\mathcal{B}, \widehat{\mathcal{B}})$ be a measure for the distance between the linear time-invariant systems $\mathcal{B}$ and $\widehat{\mathcal{B}}$. The distance $d(\mathcal{B})$ of $\mathcal{B}$ to the set of uncontrollable systems w.r.t. the distance measure "dist" is defined as

$$d(\mathcal{B}) := \min_{\widehat{\mathcal{B}}} \operatorname{dist}(\mathcal{B}, \widehat{\mathcal{B}}) \quad \text{subject to} \quad \widehat{\mathcal{B}} \text{ is uncontrollable.} \tag{1}$$

The definition of Paige [1], Section X, matches our definition with

$$\operatorname{dist}(\mathcal{B}, \widehat{\mathcal{B}}) = \left\| \begin{bmatrix} A & B \end{bmatrix} - \begin{bmatrix} \widehat{A} & \widehat{B} \end{bmatrix} \right\|_2^2, \tag{$*$}$$

where A, B and $\widehat{A}, \widehat{B}$ are parameters in state space representations $\sigma x = Ax + Bu$, $y = Cx + Du$ of $\mathcal{B}$ and $\widehat{\mathcal{B}}$, respectively. (σ is the shift operator in discrete-time or the derivative operator in continuous-time.) Computing $d(\mathcal{B})$ w.r.t. (*) received a lot of attention in the literature, but has the drawback that it *depends on the choice of the state space basis* and is therefore not a property of the pair $(\mathcal{B}, \widehat{\mathcal{B}})$.

We consider problem (1) with an alternative distance measure that is representation invariant. Let $p(\sigma)y = q(\sigma)u$, be a representation of $\mathcal{B}$ as a difference or differential equation. In the SISO case, normalizing p to be monic, the parameters p, q are unique and the distance measure

$$\operatorname{dist}(\mathcal{B}, \widehat{\mathcal{B}}) := \|p - \widehat{p}\|_2^2 + \|q - \widehat{q}\|_2^2 \tag{$**$}$$

is a property of $(\mathcal{B}, \widehat{\mathcal{B}})$. The resulting problem of computing $d(\mathcal{B})$ w.r.t. (**) is a Sylvester structured low-rank approximation problem.

References

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EXPONENTIAL DECAY BOUNDS FOR NONLINEAR HEAT PROBLEMS WITH ROBIN BOUNDARY CONDITIONS

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Key Words and Phrases: nonlinear parabolic equation, decay bounds, maximum principles
AMS Subject Classification: 35K60, 35B40, 35B50

We investigate the behavior of the solution of the following nonlinear heat problem:

$$\begin{cases} \Delta u + f(u) = u_{,t}, & (\mathbf{x}, t) \in \Omega \times (t > 0), \\ \dfrac{\partial u}{\partial n} + \alpha u = 0, & (\mathbf{x}, t) \in \partial\Omega \times (t > 0), \\ u(\mathbf{x}, 0) = g(\mathbf{x}), & \mathbf{x} \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in R^2 with $C^{2+\epsilon}$ boundary $(0 < \epsilon < 1)$, f and g are two nonnegative and differentiable functions, and the constant $\alpha > 0$. In (1.1) $\dfrac{\partial u}{\partial n}$ is the exterior normal derivative of u and the condition

$$\frac{\partial g}{\partial n} + \alpha g = 0 \tag{1.2}$$

is satisfied on $\partial\Omega$.
We determine conditions on the geometry and data sufficient to preclude the blow up of the solution and to obtain an exponential decay bound for the solution and its gradient.

MIRROR SYMMETRY AND DERIVED CATEGORIES

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To any physicist Calabi Yau, that is, a compact complex manifold with trivial canonical bundle, equipped with a Kahler class and a B-field, we can attach a $N = 2$ SCFT. Two CYs, X, X' are mirror if there is a mirror morphism between the SCFT's. We will explain Mirror Symmetry from different points of view.

METRICS FOR THE CANTOR SET

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Key Words and Phrases: Cantor set, zero dimensional compact metric space, fractal dimension; Hausdorff dimension

AMS Subject Classification: 54E45, 28A78, 28A80

Any zero dimensional separable metric space (X, d) may be topologically embedded in the Cantor Set C, and if the metric d is totally bounded, then, for some suitable compatible metric for C, (X, d) can be isometrically embedded as well. In particular, if (X, d) is any zero dimensional compact metric space, then there is a compatible metric $d*$ for C such that $d*$ restricted to X is the metric d. Understanding the structure of the collection $M(C)$ of all compatible metrics for C is therefore of interest. We show that $M(C)$ is actually a lattice $(M(C), U, \cap)$. Here, rUs is defined by $rUs(x, y) = max(r(x, y), s(x, y))$, and is obviously a metric. And $r \cap s$ is the sup of all metrics d for C such that $d(x, y) \leq min(r(x, y), s(x, y))$ for all x and y in C. The question of the existence of a compatible metric d such that $d \leq min(r, s)$, (taken pointwise), is answered affirmatively for the space C, so the metric $r \cap s$ exists. From the fact that $M(C)$ is a lattice, it follows easily that if X is any zero dimensional compact metric space, $M(X)$ is a lattice. Zero dimensionality plays an important role in the proof of these results. If I denotes the closed unit interval, then it can be shown that $M(I)$ is not a lattice. (The proof that $M(I)$ is not a lattice is combinatorial, and uses the Catalan numbers.) For a zero dimensional compact metric space X, connections between the lattice structure of $M(X)$ and the fractal dimension of X are explored.

THE GENERALIZED SCHUR ALGORITHM AND APPLICATIONS

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Key Words and Phrases: Generalized Schur Algorithm, QR factorization, eigenvalues, structured matrices

AMS Subject Classification: 65F05, 65F15

The generalized Schur algorithm is widely used to compute the QR and LDU decomposition of structured matrices. It relies on the knowledge of the generators of the matrix with respect to a shift operator rather than the elements of the involved matrix. In this talk the main features of the generalized Schur algorithm are described as well as applications in signal and image processing.

PAGE MIGRATION ON RING NETWORKS

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Key Words and Phrases: page migration, data management, online algorithm, ring network
AMS Subject Classification: 68M10

The *page migration problem* is to compute dynamic allocation of a page on a network for a given sequence of nodes issuing requests for the page. The goal is to minimize the total communication costs of services for requests and of migrations of the page. Formally, the problem is, given a graph $G = (V, E)$ with edge weight $w : E \to \mathbb{R}^+$, $D \in \mathbb{Z}^+$, and a sequence $s_0, c_1, \ldots, c_k \in V$, to compute a sequence $s_1, \ldots, s_k \in V$ so that the cost function $\sum_{i=1}^{k}(\mathrm{dist}(s_{i-1}, c_i) + D\,\mathrm{dist}(s_{i-1}, s_i))$ is minimized, where $\mathrm{dist}(u, v)$ $(u, v \in V)$ is the minimum sum of weights of edges in a path connecting u and v on G.

An *online* page migration algorithm determines s_i without knowing $c_{i+1}, \ldots, c_k$ for $1 \leq i < k$, while an *offline* algorithm determines $s_1, \ldots, s_k$ based on the knowledge of $c_1, \ldots, c_k$. Let $\mathrm{cost}_A(\sigma)$ be the cost of a page migration algorithm A for an instance $\sigma = (G, D, s_0, c_1, \ldots, c_k)$. An online page migration algorithm $\textsc{Alg}$ is *ρ-competitive* if there exists a value α independent of k such that $\mathrm{cost}_{\textsc{Alg}}(\sigma) \leq \rho\,\mathrm{cost}_{\textsc{Opt}}(\sigma) + \alpha$ for an optimal offline algorithm $\textsc{Opt}$ and for any σ.

We did not know so far any deterministic online algorithm with competitive ratio less than 4 for networks other than trees, uniform networks, and Cartesian products of those networks. In this note we give a $2 + \sqrt{2}(\simeq 3.4142)$-competitive deterministic algorithm on ring networks for the setting that $D = 1$. We can also derive algorithms for trees of rings and tori with the same competitive ratio and with the same setting. Moreover, we show a lower bound of 3.1639 for general networks, improving on the previously best bound of 3.1481, and a lower bound of 3.1213 for rings. Our lower bound for rings is the first result which gives an explicit lower bound greater than 3 for ring networks.

IDENTIFICATION AND SIMULATION OF BOOLEAN FUNCTIONS WITH EXOTIC SYMMETRIES

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Key Words and Phrases: symmetric functions, conjugate symmetry
AMS Subject Classification: 68W35

Symmetric Boolean functions play an important role in the design of electronic circuits. Symmetric functions simplify the routing, and can also simplify the simulation of circuits. Over the years many different algorithms have been proposed for detecting symmetric functions. We recently proposed a simulation technique for complex circuits that performs significantly faster for symmetric circuits than for non-symmetric circuits. Unfortunately, symmetric circuits are quite rare. There 65,536 4-input Boolean functions, only 32 of which are symmetric. By using conjugacy along with our new detection algorithm, we can expand the number of symmetric functions more than 100 fold. In addition to conjugate symmetries, there are additional types of symmetry that can be useful in simplifying the simulation of Boolean circuits. This talk will focus on the detection and exploitation of exotic symmetries in the simulation of Boolean circuits.

Exotic symmetries are defined as follows. The standard representation of the symmetric group S_n in the set of $n \times n$ matrices over some field F consists of all those matrices that have a single 1 in each row and each column and zeros elsewhere. Conjugate symmetries are generated by those matrix groups that are conjugate the standard representation. We are primarily concerned with matrices over the field $GF(2)$, the integers modulo 2. In most cases, there are matrix groups that are isomorphic to S_n that are *not* conjugate to the standard representation. It is these matrix groups that give rise to the exotic symmetries in Boolean functions. This talk will discuss detection algorithms for these exotic symmetries, and adaptations of these algorithms for symmetries conjugate to exotic symmetries.

COOPERATIVE INCENTIVE EQUILIBRIUM FOR A BIORESOURCE MANAGEMENT PROBLEM

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Key Words and Phrases: fishery, resource managament problem, cooperative games, incentive equilibrium

AMS Subject Classification: 91A12, 91A25, 91A80

A dynamic game model of bioresource management problem is considered. Let us divide the water area into two parts: s and $1-s$, where the two players exploit the fish stock during T time periods. The center, which divide the water area (determines s) and two fishing firms are the participants of this game.

The dynamics of the fishery is described by the equation:

$$x'(t) = F(x(t)) - q_1E_1(t)(1-s)x(t) - q_2E_2(t)sx(t),\ x(0) = x_0\,, \tag{1}$$

where $x(t) \geq 0$ – size of the population at time t; F – natural growth function of the population; $E_i(t) \geq 0$ – firm's fishing efforts at time t and $q_i > 0$ – catchability coefficients $(i = 1, 2)$.

The players net revenues over a fixed time period $[0, T]$ are:

$$\begin{aligned} J_i &= g_i(x(T)) + \int\limits_0^T [\tfrac{1}{2}a_iE_i^2(t)s_i^2x^2(t) + b_iE_i(t)s_ix(t) + \\ &+ c_iE_i(t)E_j(t)s_is_jx^2(t) + d_iE_j(t)s_jx(t) + \tfrac{1}{2}l_iE_j^2(t)s_j^2x^2(t)]dt\,, \end{aligned} \tag{2}$$

where $i = 1, 2$, $i \neq j$, $s_1 = 1-s$, $s_2 = s$, the coefficients a_i, b_i, c_i, d_i, l_i correspond for the concrete problem and include discount rate as $e^{-\rho_i t}$.

We defined the cooperative and incentive equilibriums in the problem (1)-(2). Some results of computer simulations are presented.

ON RECENT ADVANCES FOR THE NUMERICAL SOLUTION OF THE PRIMITIVE DIFFERENCE EQUATIONS IN INCOMPRESSIBLE FLUID FLOW PROBLEMS

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Segregated solution procedures played an important role for decades for the solution of the coupled finite difference equations resulting from the discretization of the equations for the incompressible fluid flow equations. The very important problem to be addressed in this context is the treatment of the strong coupling of the momentum and continuity equations, so-colled 'the velocity pressure coupling problem', despite the fact that there doesn't exist an explicit equation for the 'pressure difference' variable which plays a very important role in the coupling.

For long time, there has been a belief that utmost has been done in this respect, and certain classical 'segregated type' methods address the issue efficiently. However, recent advances demonstrated that incomparably robust, highly efficient and much less sophisticated methods can be devised using implicit block discretization techniques. These types of methods reduce the overall cost of simulation to some twenty percent of those with 'classical' approaches.

In this text, a brief summary of the 'state of the art' of the new trends is presented.

HOLOMORPHIC CHERN-SIMONS AND BF THEORIES ON SUPER TWISTOR SPACES

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Key Words and Phrases: twistor, Chern-Simons,BF theory, supermanifolds, super Yang-Mills equations, conformal gravity, supersymmetry
AMS Subject Classification: 32L25, 58A50

In 2003, Witten introduced the twistor string which is a string theory in super Twistor space, $\mathbf{CP}^{3|4}$. One of the key ideas behind the twistor string is holomorphic Chern-Simons theory. It is of interest to extend the idea of twistor strings and thus holomorphic Chern-Simons theory beyond $\mathbf{CP}^{3|4}$ to other spaces in twistor theory such as super ambitwistor spaces.

In this talk, we shall begin with an introduction to twistor and ambitwistor spaces. We quickly review various results from twistor theory such as Penrose-Ward transforms. We also present various ideas from supergeometry which we will be needing.

After a short introduction to Chern-Simons theory and its holomorphic analog, we discuss holomorpic Chern-Simons theory on $\mathbf{CP}^{3|4}$. BF theory, another topological gauge theory and its extension by A. Popov to holomorphic BF theory are reviewed. We also give its extension to complex supermanifolds. Finally, we investigate holomorphic BF theory on super ambitwistor spaces and the role of holomorphic and almost complex bundles in holomorphic Chern-Simons and BF theories.

THE SHALLOW WATER EQUATIONS IN LAGRANGIAN COORDINATES

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The shallow water equations are frequently used as a model for both atmospheric and oceanographic circulation. They are a simple form of the equations of motion that describe the evolution of an incompressible fluid in response to gravitational and rotational accelerations. In this work the shallow water equations are written in Lagrangian coordinates, and the positions of fluid particles are identified for all time. These coordinates do no vary with time because the independent variables are the particles' fixed initial position. Accurate numerical solutions of the Lagrangian shallow equations will be shown using traditional numerical methods.

A FUNCTIONAL BAYESIAN METHOD FOR THE SOLUTION OF INVERSE PROBLEMS: AN APPLICATION TO THE CALIBRATION OF A SOIL MODEL

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Key Words and Phrases: Bayesian, Inverse Problem, Polynomial Chaos Decomposition, Random Fields, Markov-Chain Monte Carlo, Soil Model, 3D Finite Element Method, 3D Digital Image Correlation

AMS Subject Classification: 62F15, 60G20, 65J22, 65C05, 74S05, 68U10

This work introduces a Functional Bayesian (FB) formulation as a probabilistic methodology for the calibration of constitutive models that incorporates material random responses and local effects into the assessment of constitutive parameters. This particular calibration process is known as the probabilistic solution to the inverse problem. Estimates of the statistics required for the Bayesian solution are obtained from a series of standard triaxial tests which are coupled with 3-Dimensional (3D) stereo digital images allowing for the capturing of material local effects. In addition, the probabilistic method includes the spatial representation of elemental 'material properties by introducing spatially varying parameters within a 3D Finite Element Model (3D-FEM) to reproduce to the extent possible the actual heterogeneous response of the material. The sampling of spatial 'material realizations is performed by the Polynomial Chaos (PC) method, which permits the simulation of multi-dimensional non-Gaussian and non-stationary random fields. Integration of the random parameters is performed via Markov Chain Monte-Carlo and Metropolis-Hastings algorithms. The calibration of a soil sample is presented as a case study to illustrate the applicability of the method when the soil response lies within the linear elastic domain. Calibration results show a probabilistic description of the spatially distributed parameters and of the coefficients of the chaos representation that defines it. Inferences retrieved from the MCMC sampling include the analysis of the 'material properties and of the coefficients of the PC representation which enhances understanding of the randomness associated with the material composition and response.

THE MICROPOLAR FLUID MODEL FOR BLOOD FLOW THROUGH A TAPERED ARTERIES WITH A STENOSIS

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A micropolar model for axisymmetric blood flow through an axially nonsymmetreic but radially symmetric mild stenosis tapered artery is presented. To estimate the effect of the stenosis shape, a suitable geometry has been considered such that the axial shape of the stenosis can be changed easily just by varying a parameter (referred to as the shape parameter).

The model is also employed to study the effect of the taper angle. Flow parameters such as velocity, the resistance to flow (the resistance impedance), the wall shear stress distribution in the stenotic region and its magnitude at the maximum height of the stenosis (stenosis throat) have been computed for different values of the shape parameter, the taper angle, the coupling number and the micropolar parameter. It is shown that the resistance to flow decreases with increasing values of the parameter determining the stenosis shape, also the resistance to flow increases (in the stenotic region) with the coupling parameter and decreases with the micropolar parameter. The magnitudes of the resistance to flow are higher in the case of a micropolar fluid model than in the case of a Newtonian fluid model. The wall shear stress distribution in the stenotic region and its magnitude at the maximum height of the stenosis have the same character with respect to coupling parameter and micropolar parameter. Finally, the effect of the coupling number, the micropolar parameter and the taper angle on the velocity profile are discussed.

SOLUTION OPERATORS FOR DIFFERENTIAL EQUATIONS OF INFINITE ORDER ON CONVEX SETS IN THE PLANE

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Key Words and Phrases: holomorphic functions, differential equations of infinite order, continuous linear right inverse
AMS Subject Classification: 34A35, 46A13, 46E10

Let Q be a bounded convex subset of $\mathbb{C}$ with nonempty interior int$Q$. We assume that Q has a countable fundamental system of its neighborhoods consisting of convex domains Q_n, $n \in \mathbb{N}$. A geometric characterization of this property is that the intersection of Q with its boundary ∂Q is closed and the intersection of $(\partial Q)\backslash(Q \cap \partial Q)$ with each supporting straight line to the closure cl$Q$ of Q is compact. This setup covers in particular bounded convex domains Q and convex compact sets Q with nonempty interior in $\mathbb{C}$. The space $H(Q)$ consists of all functions holomorphic on some open neighborhood of Q. We endow $H(Q)$ with the inductive topology of $H(Q) = \text{ind}_{n\to}H(Q_n)$ where $H(Q_n)$ is the Fréchet space of all holomorphic functions on Q_n. If $a(z) := \sum\limits_{n=0}^{\infty} a_n z^n$ is a nonzero entire function on $\mathbb{C}$ of at most order one and zero type, we consider the differential equation of infinite order

$$a(D)f := \sum_{n=0}^{\infty} a_n f^{(n)} = g,$$

$g \in H(Q)$. It is well know that this equation has a solution f for each function $g \in H(Q)$.

In the present lecture we investigate whether there is a solution $f = R(g)$ of this equation which depends continuously and linearly on g. In other words, we solve a problem on the existence of a continuous linear right inverse

$$R : H(Q) \to H(Q)$$

for the surjective differential operator

$$a(D) : H(Q) \to H(Q).$$

The necessary and sufficient conditions of the existence of a continuous linear right inverse R for a *fixed* operator $a(D)$ will be given in terms of the boundary behavior of conformal mappings of the open unit disc $\mathbb{D}$ onto int$Q$ and onto $\mathbb{C}\backslash$cl$Q$ in directions of the accumulation of the zero set of the function a. From the mentioned criterion it follows that each nonzero differential operator

$$a(D) : H(Q) \to H(Q)$$

has a continuous linear right inverse if ∂Q is of Hölder class $C^{1,\lambda}$ for some $\lambda > 0$.

Our results extend the similar results which were obtained previously by S.Momm (for convex domains Q) and by S.Momm and the author (for compact convex sets and locally closed convex sets Q).

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BOUNDARY SINGULAR CHARACTERISTICS OF HAMILTON-JACOBI EQUATION

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Key Words and Phrases: method of singular characteristics, solution on the boundary
AMS Subject Classification: 49L25, 35L60

Two types of characteristics, classical and singular ones, are associated with a first-order partial differential equation, such as the Hamilton-Jacobi or Bellman-Isaacs equation. Differential-geometric description for both types is similar but they have different analytical expressions in a given coordinate frame, and they may intersect. In many cases singular characteristics allow to construct nonsmooth solutions to a first order PDE. In some situations singular characteristics run along the boundary of the domain under consideration in which case they may supply a special extension of the boundary conditions, which originally are not specified on the whole boundary. After these natural boundary conditions are obtained, the solution can be constructed by the standard method of integrating the equations of the classical characteristics. In this presentation we will sketch the theory of singular (boundary) characteristics and give some applications in state constraint differential game (optimal control) problems and in image reconstruction algorithms. The theory of singular characteristics is developed in a number of papers and a book by the author.

THE STEINER PROBLEM, TEICHMÜLLER SPACE, AND THE COMPUTATION OF FUCHSIAN GROUPS

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AMS Subject Classification: 30F35, 30F60, 68Q45, 30F10, 53A10

The classical Steiner problem is that of finding length minimizing graphs (or networks) spanning a fixed set of nodes in the Euclidean plane. Analogs of this problem in two-dimensional flat tori and compact hyperbolic surfaces will be considered.

An *admissible graph* G in a complete, compact Riemannian surface M is defined to be an embedded graph in M with edges consisting of geodesic segments. It is further required that $M \setminus G$ be diffeomorphic to an open cell. It will be shown that the length functional on the space of admissible graphs $\mathcal{G}(M)$ always has at least one local minimum. These minima must be trivalent at each node, and any two edges adjacent to a node must meet at an angle of $2\pi/3$. Consequences for flat tori will be discussed first to motivate the results for higher genus surfaces.

When M is a compact hyperbolic surface of genus $g \geq 2$, the length minimizing graphs in $\mathcal{G}(M)$ lift to geodesic polygons in the Poincaré disk D with $12g-6$ edges and interior angles of $2\pi/3$. These polygons, which we call *Steiner domains*, give rise to a presentation of a Fuchsian group Γ such that M is isometric to D/Γ. Production rules for generating unique representatives of the elements of Γ will be presented, and it will be seen that these rules are considerably simpler than the production rules that arise from the standard one-vertex presentation of the fundamental group of M. We will also explore the connection between the space $\mathcal{S}_g$ of all Steiner domains and the Teichmüller space $\mathcal{T}_g$ of compact hyperbolic surfaces of genus g.

Finally, applications of these results to the computation of meromorphic functions on Riemann surfaces will be discussed. Ultimately, this computational method will be applied to the graphical rendering of minimal surfaces.

RECENT ADVANCES IN THE CONSTRUCTION OF GREEN'S FUNCTIONS FOR APPLIED PDE

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Key Words and Phrases: partial differential equations, Green's functions
AMS Subject Classification: 35C15, 35J25, 35Q72, 65N38

The method of eigenfunction expansion [1] represents one of the most effective approaches to the construction of Green's functions for elliptic partial differential equations that are frequently used in simulating real life phenomena and processes in engineering and science. The method brings compact series representations of Green's functions for a vast variety of boundary value problems for Laplace, Klein-Gordon and biharmonic equation. The method is not limited, however, to these equations, it is also effectively applicable, for example, to a number of problem statements in the theory of elasticity.

But the method of eigenfunction expansions has a quite unfortunate limitation which is especially notable in its numerical implementations. The series representing Green's functions cannot uniformly converge due to the presence of a singularity in Green's functions. This makes inappropriate the truncation of series, which is used in computing, when values of Green's functions are to be obtained for a close location of observation and source points, which is unavoidable in computer routines based on such methods as the boundary element.

This lecture aims at the reviewing of a recent progress made in obtaining of computer-friendly series representations of Green's functions constructed by the method of eigenfunction expansion. The progress is based on the convergence improvement of the series. This became possible when in [2] a special technique was proposed that allows for the series representing Green's functions to be split onto a regular and singular component. The latter was analytically summed up making the ultimate form of Green's function accurately computable for any mutual location of the observation and the source point.

The lecture also reviews the author's recent work on the extension of the Green's function notion to partial differential equations with piecewise continuous coefficients. It is evident that the Green's function notion cannot directly be applied to such equations. This gives a rise to the so-called [3] matrix of Green's type notion. In addition, recent advances will also be reviewed in the construction of Green's functions for regions of irregular configuration.

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SOME PROPERTIES OF THE OPTIMAL DISCOVERY PROCEDURE

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Key Words and Phrases: multiple hypothesis testing, optimal discovery procedure, unbiased tests
AMS Subject Classification: 62H15, 62F03

The optimal discovery procedure (ODP) is a new approach to multiple tests of significance that was recently introduced by Storey (2007). Similarly to the Neyman-Pearson lemma in the single hypothesis setting, the ODP was shown to possess optimality when testing multiple hypotheses. Thus, it provided more power than the procedure based on separate UMP unbiased tests in a situation where several tests on normal population means were performed. We consider how the behavior of the ODP varies for certain classes of distributions. In particular, we investigate the unbiasedness of this testing procedure.

RESTRICTED PATH INTEGRALS IN THEORY OF DECOHERENCE AND DISSIPATION OF QUANTUM SYSTEMS

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Key Words and Phrases: quantum measurements, path integrals, decoherence, dissipation
AMS Subject Classification: 81P15, 81S40

Decoherence and dissipation of a quantum system is caused by entanglement of the system with its environment. The dynamics of such a system is usually obtained by considering the system together with the environment and subsequent tracing the environment out. Instead, quantum decoherence (and dissipation) may be presented phenomenologically in the framework of theory of continuous quantum measurements. The approach to continuous quantum measurements based on Restricted Path Integrals (RPI) provides a selective description of decoherence of a quantum system in terms of this system only, without any reference to explicit models of the environment [1].

The environment is characterized in this approach by the information about the system's state recorded in the environment's state. This information is interpreted as the measurement readout. Technically this is done by restricting the system's path integral onto the set of paths compatible with the measurement readout. In the Markovian approximation, the restriction of the path integral is equivalent to an readout-dependent imaginary term in the Hamiltonian of the system. The measurements are naturally classified as minimally disturbing onces (leading to only decoherence) and non-minimally disturbing onces (leading to both decoherence and dissipation [2].

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FRACTIONAL BANACH ALGEBRAS AND LAPLACE TRANSFORM

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It is well-known the relationship between the convolution algebra $L^1(R^+, *)$ and the Laplace transform $\mathcal{L}$ defined by

$$\mathcal{L}(f)(z) = \int_0^\infty f(t)e^{-zt}dt, \qquad f \in L^1(R^+, *).$$

In the last twenty years, some subalgebras of $L^1(R^+, *)$ are defined by usual derivatives and fractional derivation. In this talk we present a survey about main results in this setting and new ones recently proved. We consider scalar and vector-valued results which include integrated semigroups in Banach spaces, pseudo-resolventes in Banach algebras and algebra homomorphisms.

GENERALIZATION OF THE MÜNTZ SEQUENCES

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In this talk we discus a generalization of the Müntz sequences, called uniqueness sequences, and the role they play in the characterization of the Laplace Transform. We prove an extension of Theorem 1.7.3 in “Vector-Valued Laplace Transforms and Cauchy Problems” by W. Arendt, C.J.K. Batty, M. Hieber and F. Neubrander, which was first mentioned by Pastor in 1919 and is a special case of a result of Yu-Cheng Shen, see “The identical vanishing of the Laplace integral”. We also give an extensive characterization of the uniqueness sequences.

THE RADON-GAUSS TRANSFORM

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Key Words and Phrases: radon transform, infinite dimensional gaussian, hyperplanes
AMS Subject Classification: 28C99

The Radon-Gauss Transform is a generalization of the Radon transform to infinite dimensions.

Since there is no useful version of Lebesgue measure in infinite dimensions, and the Gauss measure is the most useful standard measure in this setting, we use Gauss measure as the background measure for the transform. We construct Gaussian measure for any given hyperplane in an infinite-dimensional Hilbert space, and this is used to define the Radon-Gauss transform.

An inversion formula is obtained.

In the finite dimensional case, it is known that there is an inversion formula using powers of the Laplacian and another formula using Fourier transform. We have not been able to give an appropriate meaning to an infinity-power of the Laplacian in our context (though the possibility of a meaningful definition remains), so we proceeded using Segal-Bargmann transform, and we establish a relation which allows inversion of the Radon-Gauss transform by inverting the Segal-Bargmann transform.

APPLICATIONS OF COMPOUND MATRICES

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Key Words and Phrases: compound matrices, exterior algebra
AMS Subject Classification: 15A75, 65F20

Compound matrices are formed by exterior products of the rows or columns of a matrix. We offer some applications: the explicit generation of all possible linear relations in linear systems, including the underdetermined and overdetermined cases; the pseudoinverse; the eigenvalue/eigenvector problem; and some other considerations of linear and multilinear algebra.

HAMILTONICITY OF THE CARTESIAN PRODUCT OF TWO DIRECTED CYCLES MINUS A SUBGROUP

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Key Words and Phrases: Directed cycles, Cartesian product, Hamiltonian circuit
AMS Subject Classification: 05C

Let $\mathbb{Z}_a \times \mathbb{Z}_b - \mathbb{Z}_c \times \mathbb{Z}_d$ be the product of two directed cycles minus a subroup. Also, let $A = \frac{a}{c}$ and $B = \frac{b}{d}$. We show that this digraph is Hamiltonian if and only if there exists positive integers M and n such that $Am + Bn = AB - 1$ and $\gcd(dm, cn) = 1$. Our result is a natural extension of work by Curran, who studied Hamiltonian paths in the Cartesian product $\mathbb{Z}_a \times \mathbb{Z}_b$, and by Penn and Witte, who studied Hamiltonian paths in the Cartesian product of $\mathbb{Z}_a \times \mathbb{Z}_b - \{(0,0)\}$. This is joint work with Victoria Barone and Matthew Mauntel.

MAGNETOHYDRODYNAMICS IN NON-UNIFORM FLOW FIELDS

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Key Words and Phrases: magnetohydrodynamic, Lorentz, flow-control, non-uniform flows, deformable shapes, bubbles and drops

A general method for evaluating the MHD Lorentz induced pressure loads, acting on an arbitrary moving deformable shape, embedded in a non-uniform ambient flow of a conducting fluid in the presence of a uniform external magnetic field, is presented. The analysis is valid during a short time after the body is impulsively introduced into the unperturbed linear flow field (constant strain and vorticity). It is shown that a moving spherical shape affected by an inclined magnetic field will always experience both drag and lift. These reactions are linear in the velocity (such as in low Re creeping flows) even though the analysis in the sequel is based on the assumption that the ambient fluid is incompressible and inviscid.

SELF-VALIDATED METHODS FOR THE SIMULTANEOUS INCLUSION OF POLYNOMIAL ZEROS

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Key Words and Phrases: polynomial zeros, simultaneous methods, approximate zeros, interval methods, error bounds

AMS Subject Classification: 65H05, 65G20, 30C15

The computed roots of algebraic equations are only approximations to the exact roots since there are errors originating from discretization, truncation and from rounding. For this reason, it is important to apply a root-finding procedure which simultaneously improves the approximations to the roots and also gives error bounds of the improved approximations. In this paper we study self-validated methods that automatically provide upper error bounds of the computed approximations. The following techniques for the simultaneous inclusion of the zeros are considered: (I) interval methods which deal with disks as arguments, (II) hybrid methods that combine simultaneous methods in ordinary complex arithmetic and only one final iteration step of an interval method in circular complex interval arithmetic, and (III) *a posteriori* bound error methods. Numerical examples illustrate each of the presented approaches.

MULTIPLICITY AND LOCATION RESULTS FOR BEAM-COLUMNS SIMPLY SUPPORTED WITH ONE-SIDED NAGUMO CONDITIONS

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In this work it is presented some existence, non-existence and locion results for the problem composed by the fourth order nonlinear fully equation

$$u^{(4)}(x) + f(x, u(x), u'(x), u''(x), u'''(x)) = s\ p(x) \tag{E}$$

for $x \in [a,b]$, $f : [a,b] \times \mathbb{R}^4 \to \mathbb{R}$, $p : [a,b] \to \mathbb{R}^+$ continuous functions and s a real parameter, with the boundary conditions

$$\begin{array}{r} u(a) = A \\ u'(a) = B \\ u'''(a) = C \\ u'''(b) = D \end{array}, \quad A, B, C, D \in \mathbb{R} \tag{BC}$$

This problem models several phenomena, such as, the bending of an elastic beam simply supported at the endpoints. It will be done an Ambrosetti-Prodi type discussion on s. That is, there are $s_0, s_1 \in \mathbb{R}$ such that:

· for $s < s_0$ or $(s > s_0)$ there is no solution of (E)-(BC).
· for $s = s_0$ problem (E)-(BC) has one solution.
· for $s \in]s_0, s_1]$ or $(s \in [s_1, s_0[)$ there are two solutions of (E)-(BC).

The arguments used apply lower and upper solutions technique, *a priori* estimations and topological degree theory. In this paper we replace the usual Nagumo condition $|f(x, y_0, y_1, y_2, y_3)| \leq \varphi(|y_3|)$ by the one-sided condition $f(x, y_0, y_1, y_2, y_3) \leq \varphi(|y_3|)$.

An application to a continuous model of the human spine, used in aircraft ejections, vehicle crash situations and some forms of scoliosis, will be presented.

A GROUP THEORY APPROACH FOR ASSEMBLY/DISASSEMBLY MECHANICAL SIMULATION

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Key Words and Phrases: assembly/disassembly; contact; bi-quaternions; family of trajectories

Nowadays, products assembly/disassembly scheduling is a key part of the product design. This paper dials with a mathematical model able to represent all the possible displacements for each couple of surfaces defining a mechanical contact. The method is based on a model of elementary contact mobility representing rotations, translations and helical movements in a mechanism. In order to determine the families of trajectories for each combination of elementary contacts, a bi-quaternion is associated to each type of kinematic joints in a mechanism. After a brief introduction of Clifford algebra, a general scheme to evaluate the resulting families of trajectories when combining two bi-quaternions reflecting the mobilities resulting from two functional contacts is proposed. Then, various possible combinations between elementary contacts with a general one are investigated. Thus, the families of possible trajectories are obtained allowing breaking the contact between two components of a mechanism during the simulations of disassembly operations. Considering a finite set of functional contacts, it is demonstrated that the combination of any two of them produces a solution belonging to the initial set.

SPECTRAL ANALYSIS ON NATURAL CONVECTION HEAT TRANSFER FROM A RECTANGULAR CYLINDER PARTIALLY HEATED IN AN INFINITE EXTENSION

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Key Words and Phrases: natural convection, doubly-connected, spectral finite difference
AMS Subject Classification: 33E05, 65M70, 80A20

Two-dimensional steady-state natural convection around a rectangular cylinder placed horizontally is analyzed, using a spectral finite difference scheme [1] by a time marching algorithm. Fluid is assumed to be Newtonian, substantially incompressible except temperature dependence of density. That is, the system of equations consists of

$$\frac{J}{r^2}\frac{\partial}{\partial t}\zeta + \frac{1}{r}\frac{\partial(\zeta,\psi)}{\partial(r,\beta)} = \frac{1}{\sqrt{Gr}}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \beta^2}\right)\zeta + \frac{1}{r}\ \frac{\partial(T,y)}{\partial(r,\beta)}$$

$$\frac{J}{r^2}\zeta + \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \beta^2}\right)\psi = 0,$$

$$\frac{J}{r^2}\frac{\partial}{\partial t}T + \frac{1}{r}\ \frac{\partial(T,\psi)}{\partial(r,\beta)} = \frac{1}{Pr\sqrt{Gr}}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \beta^2}\right)T$$

where ζ : vorticity, ψ : stream function, T : temperature, Pr : Prandtl number, Gr : Grashof number, $J \equiv \partial(x,y)/\partial(\alpha,\beta)$, and (x,y) is a Cartesian coordinate such that y is vertically upward and

$$x + iy = Z(u,k) + \frac{\mathrm{cn}(u,k)}{\mathrm{sn}(u,k)}\ \mathrm{dn}(u,k) + \left\{\frac{E(k)}{K(k)} - 1 + k^2\right\}u\ ,$$

$$|\Re u| \le K(k)\ ,\ |\Im u| \le K(k')$$

$$i\ \mathrm{sn}\left\{\omega(u + iK(k')), k^*\right\} = b\,(\xi - 1)\,/\,(\xi + 1)\ ,\ \xi \equiv \mathrm{e}^{\alpha + i\beta}\ ,\ r \equiv \mathrm{e}^{\alpha}\ ,\ -\infty < \alpha \le 0$$

$$\omega = \frac{1 + k'}{2}\ ,\ b = \frac{1 + k'}{k}\ ,\ k^* = \frac{1 - k'}{1 + k'}\ ,\ k' \equiv \sqrt{1 - k^2}$$

$Z(,)$ is a Jacobian zeta-function. $K()$ and $E()$ are the complete elliptic integral of the first kind and of the second kind respectively. The surface is given by $\alpha = 0$ and the aspect ratio of the cylinder, λ, is given by $1/\lambda = \left\{E(k') - k^2K(k')\right\} / \left\{E(k) - k'^2K(k)\right\}$. Reference length L is so defined that (half of the horizontal width) / $L = E(k) - k'^2K(k)$. Introduced is an auxiliary condition (doubly-connectedness): $\oint(\partial p/\partial\beta)d\beta = 0$, i.e., $\oint(\partial\zeta/\partial\alpha)d\beta/\sqrt{Gr} + \oint T(\partial y/\partial\beta)d\beta = 0$ (at the surface).

Figure 1 shows the flow boundary layer thickness δ (90% flow rate points) along the cylinder surface s (measured in the clockwise direction, $s = 0$ at the top center) at $Gr = 2 \times 10^6, \lambda = 1, Pr = 0.7, T(s) = 1(0.4972 < s < 1.1972)$, otherwise 0.

$\boldsymbol{F}$= $0.03 - 0.369i$: total force on the cylinder except static buoyancy $/(\rho U^2 L)$, $U \equiv (\nu/L)\sqrt{Gr}$, ν: kinematic viscosity. $Nu_m = 2.78$: mean Nusselt number on the heated part. $C_M = -0.077$: moment of force (except stationary moment) to the origin (center) $/(\rho U^2 L^2)$.

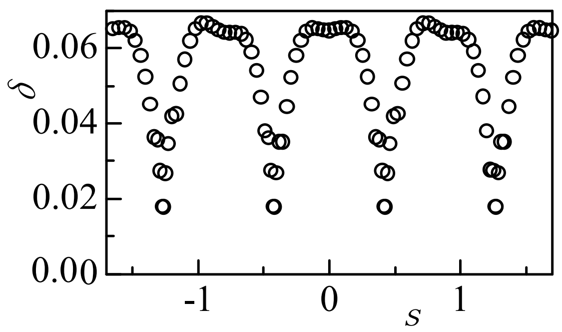

Figure 1. Boundary layer thickness

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A GLANCE AT CONSTRUCTING THE MATRICES WITH RECURSIVE ENTRIES ALONG WITH EVALUATING THEIR DETERMINANTS

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Key Words and Phrases: determinant, LU-factorization, recurrence relation, generalized pascal triangle, Fibonacci and Lucas k-number
AMS Subject Classification: 11B39, 15A36, 15A15, 11C20

We prove several evaluations of determinants of matrices, the entries of which are given by some recurrence relations, with various choices for the first rows and columns. We also generalize some results about evaluating determinants which can be noticed in [1]-[5], and obtain the new proofs for some Theorems in [1]-[3] and [5]. Moreover, we find some infinite matrices the principal minors of which form the Fibonacci or Lucas sequence.

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FIXED CHARGE MULTI COMMODITY FLOW PROBLEM AND ITS CO-OPERATIVE GAME

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Key Words and Phrases: discrete mathematics, optimization, graph theory, computing complexity theory, co-operative game theory
AMS Subject Classification: 90C27, 91A12

We introduce " Fixed Charge Multi Commodity Flow Problemh and its variations. The variations include the problem combined with "Facility Location Problemh. These problems have so many applications especially for computer science, telecommunications and transportations.

Firstly, we will show how to solve these problems efficiently using Lagrangian relaxations and good cuts (i.e. Branch and Cut method). Unfortunately, these problems are an $\mathcal{N}P$-hard Problem on a complete graph in terms of Computing Complexity Theory. But we can solve them on some underlying graphs in polynomial time. We will discuss about the structure of the graphs. (e.g. "series-parallel graphh)

Secondly, we define the co-operative games (N, v) arising from these problems. Generally, it is hard to calculate Shapley-value, nucleolus, kernel, core etc. Because we must calculate $v(S)\ \forall S \in 2^N$. I propose some ideas for other solutions of the co-operatives.

A COLLOCATION METHOD FOR THE ONE-DIMENSIONAL SCHRODINGER EQUATION

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Key Words and Phrases: time dependent schrodinger equation, collocation method, radial basis function

This paper addresses the problem of the construction of collocation method for the one dimensional time dependent schrodinger equation in an unbounded domain. Artificial boundary conditions are introduced to reduce the original problem into an initial boundary value problem in a bounded domain with transparent boundary conditions. These boundary conditions are generated from two integro-differential boundary operators with Singular Kernel. Then we use collocation method with radial basis functions(RBFs) to solve this problem.

SOME REMARKS ON CONTROLLABILITY AND OBSERVABILITY OF COMPLEX SYSTEMS

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In applying the theory of linear systems in the analysis of economic systems a special decomposition has an important role. In the first part of the lecture the controllability, observability, realisation and optimisation of such decompositions will be analysed. F. Szigeti and the Author have proved a Kalman's type of rank condition for the controllability and the observability of time-dependent linear systems with structure matrix. It was prooven that it was a unique and non-commutative Lie-algebra $\mathcal{L}$ of dimension 2 which could be generated from the structure matrices In the second part of the lecture we generalise this result for the finite dimensional systems. Finally some applications of these results will be mentioned.

LEFT INVARIANT OPTIMAL CONTROL SYSTEMS AND SUB-RIEMANNIAN GEOMETRY

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Key Words and Phrases: optimal control, invariant vector field, sub-riemannian geometry
AMS Subject Classification: 49K15, 49Q99, 93B29

We present in this lecture results about the following optimal control problem: minimize $\int u_1^2 + \cdots + u_n^2$, among the solutions $t \mapsto (g, u_1, \ldots, u_n)$ of the control system $\dot{g} = u_1 X_1(g) + \cdots + u_n X_n(g)$, $g \in G$ and $u_i \in L^2(\mathbb{R})$. Here G is a Lie group and $\Delta = \{X_1, \ldots, X_n\}$ with $n < \dim G$, is a family of left invariant vector fields on G. This is a generalization to the Lie-group theoretical framework, of the classical affine-in-controls-quadratic optimal control problem.

The group G is endowed with a sub-Riemannian geometric structure, by declaring the set $\Delta(g)$ orthonormal for each $g \in G$, for then, a smooth varying inner product $g \mapsto \langle \cdot, \cdot \rangle_g$ is defined in the planes span$[\Delta(g)]$. Horizontal curves are absolutely continuous curves $t \mapsto g(t)$, satisfying $\dot{g}(t) \in \Delta(g)$ a.e. For horizontal curves parametrized by arc-length, the above optimal control problem is equivalent to the sub-Riemannian geodesic problem.

The three-dimensional case corresponds to the Heisenberg group $\mathbb{R}^3$ with a contact distribution Δ, it is well known and it stands as the prototype of the theory, see [1]. We have studied higher dimensional Heisenberg groups as well as some other nilpotent groups, see [2] and [3].

We approach the problem by means the Pontryagin Maximum Principle and the associated Hamiltonian formalism. In some nilpotent cases, we present a detailed characterization of the geodesics, small radius spheres and conjugate locus. As an application we present some low dimensional cases that model certain non-holonomic dynamical systems.

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THE STATISTICS OF PRECIPITATION

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Key Words and Phrases: Precipitation, Statistics, Spatial Distributions, Time Series, Teleconnections, EOFs, Spectral Analysis
AMS Subject Classification: 62-07, 62F-99, 86A10

For decades, meteorologists and climatologists have researched the physical properties of precipitation. While the physical mechanisms that create convective and large-scale precipitation events have been known for quite some time, the statistical properties of precipitation remain illusive. Consequently, most researchers have used physical parameters to estimate and/or forecast precipitation (on various spatial and time scales) using known formulas. Fewer researchers have tackled this problem using purely statistical methods.

Nevertheless, several statistical techniques have been used to a) show relationships between the precipitation signal at one location and at other locations around the globe b) show relationships between the precipitation signal and other physical parameters and c) explain spatial and temporal variability in the fields. For instance, teleconnection maps have been used to show spatial relationships in precipitation patterns and EOFs have been used to isolate modes of precipitation. While these and other nonparapetric methods attempt to explain precipitation characteristics, estimating future precipitation events become troublesome because of the nonlinearities involved in the field.

In this study, distributions for precipitation will be proposed based on the empirical characteristics of the field. Afterwards, various inferential statistics will be done using the hypothesized distributions and compared with the historical record.

INDUSTRY CYCLONE'S DESIGN (CFD SIMULATION)

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Key Words and Phrases: cyclones, Navier Stokes, fluent, multiphase, scaling

The objective of this work is to use the adjusted time variation equations $_{(1)}$ to optimize the geometry of cyclones used in the industry, obtain a mathematical model to predict the profile if the gas and suspension particles speed, calculate the pressure fall and the efficiency of collection.

The mathematical model will be used to simulate the behavior of different geometries to the work conditions of a cyclone, the adjusted time variation equations in turbulent multiphase regime (Gas – particles) are resolved with Fluid Dynamic - Fluent Computer program; finally results are compared against data obtained through field work.

Objective:

The objective of this work is to use the adjusted time variation equations in order to optimize the geometry of cyclones used in the industry $_{(2)}$, obtain a mathematical model to predict the profile if the gas and suspension particles speed, calculate the pressure fall and the efficiency of collection.

Methodology:

The work is divided in four phases:

To construct a proposed geometry with a mesh of 70,000 to 250,000 cells (Make the adjusted time variation equations discrete)

To select frontier conditions and physical properties, for gases and particles (Turbulence Model for instant).

To propose a solution strategy for the equations.

To analyze the results and provide feedback to the model.

Introduction:

Cyclones are very used in the cement industry due to their construction ease, low cost and reduced maintenance (Without movable parts). Nonetheless, the design is complicated and it involves a high degree of uncertainty because of the fluid flow analysis in multiphase (solid-gas, solid-liquid) where the concentration of particles is high. The "traditional" design method uses semi-empirical equations obtained from low temperature, clean gas experiments (low concentration of particles). This gives problems when scaling equipment in the cement industry, because the gases handles have a high concentration of suspended dust, and it involves heat exchange.

Currently, with the development of faster and more capable computers, the Computer Aided Design has become popular. It allows users to resolve mathematical models for complex geometries that represent real physical systems. For the case of turbulent regime flow fluid problems with suspended solids and heat transfers, a simulation technique known as CFD (Computer Fluid Dynamic) and with the computer program Fluid can be used to resolve the resultant models in a fast and friendly way. This program will be used to evaluate the system's response to the different operational conditions and various proposed geometries, selecting the most viable geometries to physically model.

Results:

The product of the physical model simulation and of the CFD Hill be the speed's profiles, the pressure fall and the efficiency of particle collecting for different proposed geometries. The CFD's results are compared against experimental dates published, which will be used as targets to compare the precision of the used mathematical models.

Conclusions:

At the moment, cyclones have been designed and installed at an industry level. Nonetheless, the studies have been carried out using low dust concentrations. Through this investigation is intended to optimize and standardize the procedure for the cyclone's design for gases with high content of suspended particles.

Notes:

Navier's Equation - Stokes for turbulent flow, equations of mass and energy preservation.

Fluent. - Software to simulate fluid flow, heat transfer in complex geometries. Simulations will be conducted with the Fluent package in a SUN 2500 workstation.

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DISCRETE DIFFERENTIAL GEOMETRY OF POLYHEDRON TILES AND LOCAL PROTEIN STRUCTURE

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Key Words and Phrases: discrete mathematics, differential geometry, tetrahedron sequence, local protein structure

AMS Subject Classification: 52B99, 52C99, 92B99, 92E10

This talk proposes a new mathematical framework, *the discrete differential geometry of polyhedron tiles*, that can be applied to analysis of the local structure of proteins.

The basic idea is the following observation: pile unit cubes up and view the resulting surface from above. If diagonal lines are printed on the upper faces of each cube, one obtains a drawing made up of the diagonal lines, which defines a flow of triangle tiles (Fig. 1a).

If we consider unit cubes in the four-dimensional space, we obtain a flow of tetrahedrons. And one can encode local protein structures using the "second derivative" of the tetrahedron flow (Fig. fig1b).

With this method at hand, it becomes possible to describe the structure of a protein without any subjective hierarchical classification nor clustering by machine learning.

The programs are available from `http://www.genocript.com`.

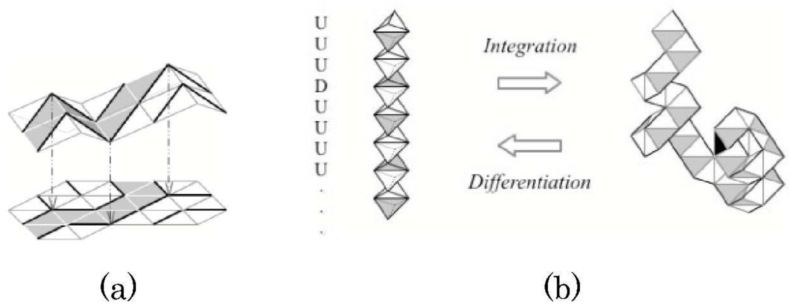

Figure 1. Basic ideas.

RECENT ADVANCEMENTS IN EVOLUTIONARY ALGORITHMS FOR DYNAMIC PROBLEMS

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Key Words and Phrases: optimization, heuristic search, evolutionary algorithm
AMS Subject Classification: 65K10, 90C59, 68T20

Evolutionary algorithms (EAs) are iterative, heuristic search methods that have found broad application in a variety of industries. They are commonly applied to complex, multi-modal, and multi-dimensional optimization problems where the exact functional form is unknown. EAs accumulate information at each iteration to adaptively improve their ability to solve some problem of interest. Since this information is accumulated during successive iterations, there is an assumption of consistency in the evaluation function in traditional EA application.

Many real-world optimization problems involve dynamic environments where this consistency assumption does not hold. Common examples of problems where environmental changes would cause EA fitness evaluations to change over time include: scheduling problems, where available resources vary over time; financial trading models, where market conditions change abruptly; and data mining, where the contents of databases are continuously updated. Application of EAs to these dynamic problems involves unique challenges.

Recent advancements in the field of evolutionary computation have enhanced the ability of EAs to perform in dynamic environments. Improvements in environment change detection, new techniques for diversity maintenance, and the addition of memory mechanisms are all examples of recent innovations. This lecture will review these recent advancements, discuss the capabilities and limitations of each, provide the basic understanding necessary to extend evolutionary algorithms to dynamic problems, and suggest research areas for future investigation.

ON THE STRUCTURE OF BURGERS' TURBULENCE

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Key Words and Phrases: dynamical systems, fluid turbulence

The concept of an approximate inertial manifold introduced some time ago by Temam and his co–workers, does not describe all the behaviour of fluid turbulence. This is illustrated for the special case of the one dimensional Burgers equation on space $\mathbb{H}$:

$$\partial v/\partial t + v\,\partial v/\partial x = \nu \partial^2 v/\partial x^2 + f; \quad x \in [0,1]$$

if $f(x)$ is an imposed body force as a function of position. $v(x,t)$ is the velocity field and ν the viscosity coefficient. Only two wave number bands are associated with such manifolds. Thus, let $\mathbf{P}$ project onto the subspace $\mathbb{H}_p$ of $\mathbb{H}$ and $\mathbf{Q}$ projects onto the orthogonal complement, $\mathbb{H}_q$, of $\mathbb{H}_p$ in $\mathbb{H}$. Hence $\mathbf{P}v = v_p$ defines the low mode motion and $\mathbf{Q}v = v_q$ the motion associated with the high wave number modes. The total velocity is given as $v = v_p + v_q$.

Let the forcing function be defined by eigenmodes of the diffusion operator. The present study considers the interaction of two such eigenmodes for different wave numbers. The objective being to obtain some understanding of the different structure of the equation in different wave number bands. As shown in my paper *'Theorems and Computation in Fluid Turbulence', Plovdiv, 2003*, the decay of $\|v\|^2(t) \equiv \int_0^1 v^2\,dx$ for periodic boundary conditions is dependent upon the forcing function wave number.

The present numerical studies clearly show a triple region structure (which is more in line with classical turbulence theory). That is, the low mode equation has an internal structure but no simple decomposition of the form indicated above in terms of projection operators. The numerical study is augmented with theoretical estimates.

KANTOROVICH VERSION OF A CLASS OF MIRAKJAN-SZÁSZ OPERATORS

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Key Words and Phrases: linear and positive operators, Kantorovich type operators, rate of convergence
AMS Subject Classification: 41A36

We construct and investigate a Kantorovich version of a class of Mirakjan-Szász operators type.

The operators are defined as follows:

$$(\tilde{K}_{n,\alpha}f)(x) : C([0,\infty)) \to C([0,\infty))$$
$$(\tilde{K}_{n,\alpha}f)(x) = ne^{-nx}\sum_{k=0}^{\infty}\left(\frac{nx}{\alpha_n}\right)^k \frac{1}{k!}\int_{k/n}^{(k+1)/n} f(t)dt$$

where α_n is a sequence of positive numbers satisfying the conditions

$$\alpha_n = 1 + O(n^{-h}),\ h > 0,\ \alpha \geq 1\ .$$

The paper contains also a convergence theorem of this operators and a quantitative estimation of the approximation process in terms of first order modulus of smoothness of function f.

INFORMATION SYSTEMS VIA DOMAINS

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Key Words and Phrases: Scott-continuous functions, effectively represented domain, powerdomain, epistemic structure

We offer a shell for the construction of information systems which are tolerant to contradictory information. The idea we use as a starting point was born in the many-valued logic community in the mid-1970s. In [3] Belnap outlined of how contradictory information might be handled if an artificial agent (the computer) is placed in information flow. Despite republication of [2] and [3] in [1], Belnap's idea, as it seems, has not caused much enthusiasm among logicians, though it has been picked up under the name of a *bilattice* by some in the logic programming community (cf., e.g., [4] for an early account). The reason for such an attitude, we think, is that Belnap's approach did not define any new logic and did not improve on any old logic. However, it used a structure which in turn gave rise to other well-organized structures, which could be treated as a data storage space and a data operational space. The latter is a must, when only a special kind of data transformers is allowed to operate over the former. In our case we allow only *Scott-continuous functions* [5] to be used. Some justified data transformers and their Scott-continuity will be discussed.

We describe how this can be implemented by using the powerdomain construction [5]. This is applied to an *effectively represented domain* based on an *epistemic structure* (to be defined), that is, the structure aimed to maintain the information system if contradictory information occurs. This idea will be illustrated by a few examples.

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SOME INTEGRAL EQUATIONS RELATED TO CONFORMAL MAPPING OF DOUBLY CONNECTED REGIONS INVOLVING THE KERZMAN-STEIN AND THE NEUMANN KERNELS

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Key Words and Phrases: conformal mapping, doubly connected regions, integral equations, Kerzman-Stein kernel, Neumann kernel, Gauss-Newton algorithm, Lavenberg-Marquardt algorithm, Fletcher's algorithm, Cauchy's integral formula

AMS Subject Classification: 30C30, 30C40, 45G15, 65E05, 65H10

Based on a boundary relationship satisfied by a function which is analytic in a doubly connected region bounded by two closed Jordan curves, an integral equation is constructed. Some applications considered are the conformal mappings from a doubly connected region bounded by two closed smooth Jordan curves onto: (a) an annulus, and (b) a unit disk with a circular slit. Among the kernels involved are the Kerzman-Stein and the Neumann kernels. Numerical implementations on some test regions are also presented.

FIXED POINT RESULTS IN UNCOMPLETED G-METRIC SPACES

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Key Words and Phrases: D-metric spaces, generalized metric spaces, G-metric spaces, incomplete G-metric spaces, fixed point

AMS Subject Classification: 54H25, 47H10

In 2003, in collaboration with Brailey Sims we introduced more robust concept of a generalized metric spaces which we call a G-Metric Space, and we developed a topological structure in such spaces. Also we explore the fixed point theory of contractive mappings, and mappings satisfied various related conditions in Complete G-Metric Spaces.

In this paper we prove some fixed point Results for mappings satisfied different sufficient conditions defined in Uncomplete G-metric spaces.

ANISOTROPIC RADIAL BASIS FUNCTIONS

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AMS Subject Classification: 41A05, 41A63, 42A82

The essential property of a "Radial" Basis Function is the positive definiteness, or conditional positive definiteness, which ensures that the system of equations determining the coefficients in the representation has a unique solution.

There are multiple reasons why anisotropic basis functions may be needed or be more appropriate. The most obvious is that if the basis function is to be defined on $\mathrm{R}^n \times \mathrm{T}$ then there is no natural norm on this space that would reflect the unique properties of time. A second reason is that function being interpolated or approximated may incorporate a directional dependence, i.e. a rotation and a stretching or shrinking. Thirdly, differentiability of the basis function is often critical, i.e., partial differentiability. Separating the differentiability from one dimension to another may be necessary, e.g., differentiability with respect to time as contrasted with differentiability with respect to a space coordinate. This is important in the case of meshless methods for solutions of differential and partial differential equations.

Positive definiteness (or conditional positive definiteness) is often dependent on the dimension of the space. That is, some radial basis functions are only positive definite for small n. Alternatively the form of the radial basis function may be dimension dependent, e.g., the thin-plate spline. In general it is important that the function be strictly positive definite, i.e., not just non-negative definite. Bochner's Theorem only pertains to the latter.

Thus construction of non-radially symmetric basis functions which can easily be shown to be strictly positive definite is important, a number of examples and general methods will be given. Some examples are only applicable to the spatial-temporal problem and others are more general.

L_p-APPROXIMABILITY AND ITS APPLICATIONS IN TIME SERIES ANALYSIS AND SPATIAL ECONOMETRICS

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Key Words and Phrases: L_p-approximability; OLS estimator; asymptotic distribution; dependent errors; deterministic regressors; polynomial trends
AMS Subject Classification: 37M10, 62F12, 62H11, 62J05

Several functional-theoretical tools have been developed lately to solve some problems in the asymptotic theory of econometric models.

(1) Mynbaev (2001) has proposed to approximate sequences of vectors by functions from $L_p(0,1)$. The most important results are: a series of properties allowing one to approximate integrals with sums, a study of convolution operators on the sets of L_p-approximable sequences, related central limit theorems for linear and quadratic forms of random variables and a criterion of L_p-approximability. These results allowed the author to solve the long-standing problem of OLS asymptotics for an autoregression with polynomial trends (Mynbaev, 2006, 2007a). They have also been instrumental in the study of spatial models (Mynbaev & Ullah, 2006 and Mynbaev, 2007b) where a new effect has been discovered: the asymptotic distribution may not be normal and may include quadratic forms of independent standard normal variables, in addition to linear forms.

(2) Johansen (2000) and Nielsen (2005) have used matrices defined recurrently. They have been successful in the study of consistency of OLS estimators.

(3) Phillips (2001) has employed the properties of slowly varying functions. In a regression with asymptotically collinear regressors he has revealed fine differences between the regressors making the transformed OLS estimator convergent.

Our purpose is to review and compare these approaches, with the emphasis on the first one.

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MODELLING AND SIMULATION OF LARGE SCALE COMPLEX INDUSTRIAL SYSTEMS

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Computer modelling, 3D technologies and simulation provide a foundation upon which industrial processes and systems can be transformed and innovation dramatically accelerated. Simulation in industry has yet to meet its full potential. The development of models are very time consuming , particularly for geometries of complex engineering systems such as manufacturing plants, automobiles, aircraft and ships. This talk will expose issues and challenges associated with modelling and simulation of large scale systems through a number of case studies to highlight the benefits gained through the use of simulation. This is e-TeX, Version 3.141592-2.1 (MiKTeX 2.4) (preloaded format=latex 2000.11.28) 18 JUN 2007 11:43 entering extended mode **E:/Svety/Conference2007/abstracts/data/najafi.tex (E:/Svety/Conference2007/abstracts/data/najafi.tex LaTeX2e ¡2001/06/01¿ Babel ¡v3.7m¿ and hyphenation patterns for english, french, german, ngerman, du mylang, nohyphenation, loaded. ! Undefined control sequence. l.1

STABILITY ANALYSIS OF FLUID-STRUCTURE INTERACTION PROBLEMS USING BOUNDARY CONTROL METHOD

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Key Words and Phrases: structure, elastic structure, boundary control, ideal fluid

In this paper, the boundary control approach is used to provide a mathematical proof of asymptotical stabilization of fluid– structure interaction problems. The boundary control method is a new approach for stabilizing the systems with continuous media. In this paper is shown "**how to stabilize the structure which maybe in contact with the fluid**". Other boundary control researchers paid no attention to the interaction of fluid and structure or they used the numerical method for their aims, and they paid no attention to the analytical method. They only assumed the free vibration of structures. They used the Euler-Bernoulli beam or Timoshenko beam or Kirchhoff plates for their stability analysis. We used the analytical method for stabilizing the structure and for the first instance, we used the boundary control in fluid–structure interaction for the stabilizing the vibrating structure.

Our aim is to achieve the stabilization of elastic structures in contact with fluid. It is used lyapunov–based method for this problem. It is supposed that the fluid is ideal. The two dimensional fluid is assumed for this problem. The governing equation for solid can be linear or nonlinear. In both cases we can stabilize the structure.

It is assumed that the governing equation for the fluid is the two dimensional wave equation.

BLOWING-UP BEHAVIOR OF THE PERIODIC SOLUTION TO THE SEMILINEAR PARABOLIC EQUATION WITH AN IMPULSIVE CONTROL

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Key Words and Phrases: impulsive control, blow-up time, semilinear parabolic equation
AMS Subject Classification: 35R12, 35K57, 35B05

We consider the following initial and boundary value problem.

$$\begin{aligned} u_t - u_{xx} &= u^2 \qquad t>0,\ \ x\in R^1 \\ u(x+2\pi,t) &= u(x,t) \qquad t>0,\ \ x\in R^1 \\ u(x,0) &\equiv \lim_{t\to+0} u(x,t) = \frac{1}{2}\{\lim_{x'\to x-0} u_0(x') + \lim_{x'\to x+0} u_0(x')\} \end{aligned}$$

where $u_0(x+2\pi) = u_0(x)$ $x\in R^1$ and u_0 is a nonnegative function of bounded variation.

We add the usual impulsive control;

$$\begin{aligned} \max_{x\in R^1} u(t,x) &< S \quad \text{for } t\in[t_{k-1},t_k) \ \ (k=1,\ 2,\ \ldots), \\ \max_{x\in R^1} u(t_k-,x) &= S \qquad (k=1,\ 2,\ \ldots), \\ u(t_k,x) &= \alpha u(t_k-,x) \quad \text{for } x\in\ R^1 \ \ (k=1,\ 2,\ \ldots), \\ 0 &= t_0<t_1<\cdots<t_m<\cdots, \end{aligned}$$

where $\max_{x\in R^1} u(t_k-,x) = \lim_{t\to t_k-0}[\max_{x\in R^1} u(t,x)]$. Let S and α $(0<\alpha<1)$ be constants. Let u_0 satisfies $\max_{x\in R^1} u_0(x) < S$. Then the solution $u(t,x)$ will be continued globally in time.

We add another impulsive control instead of the above.

$$\begin{aligned} \min_{x\in R^1} u(t,x) &< S \quad \text{for } t\in[t_{k-1},t_k) \ \ (k=1,\ 2,\ \ldots), \\ \min_{x\in R^1} u(t_k-,x) &= S \qquad (k=1,\ 2,\ \ldots), \\ u(t_k,x) &= \varphi(x)u(t_k-,x) \quad \text{for } x\in\ R^1 \ \ (k=1,\ 2,\ \ldots), \\ 0 &= t_0<t_1<\cdots<t_m<\cdots, \end{aligned}$$

where $\min_{x\in R^1} u(t_k-,x) = \lim_{t\to t_k-0}[\min_{x\in R^1} u(t,x)]$ and $\varphi(x)=1$ for $-a+2n\pi \le x \le a+2n\pi$ $(0<a<\pi,\ n=1,2,\ldots)$ and $\varphi(x)=0$ otherwise. Let S be a constant. Let u_0 satisfies $\min_{x\in R^1} u_0(x) < S$. In this talk, we consider whether the solution $u(t,x)$ can be continued globally under this condition.

PARALLEL PROGRAMMING MODELS FOR FINITE-ELEMENT APPLICATIONS ON SMP CLUSTER ARCHITECTURES

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Key Words and Phrases: parallel programmnig models, MPI, OpenMP, FEM
AMS Subject Classification: 68W10

In order to achieve minimal parallelization overhead for symmetric multiprocessor (SMP) cluster architectures and PC clusters with multi-core processors, a multi-level *hybrid* programming model is often employed. The aim of this method is to combine coarse-grain (message passing) and fine-grain (thread, OpenMP) parallelism. Another often-used programming model is the single-level *flat MPI* model, in which separate single-threaded MPI processes are executed on each processing element (PE). Although a significant amount of research on this issue has been conducted in recent years, it remains unclear whether the performance gains of the hybrid approach compensate for the increased programming complexity. Some examples show that flat MPI is rather better, although the efficiency depends on hardware performance, features of applications, and problem size.

The author has developed an efficient parallel iterative method for Finite-Element Methods (FEM) for SMP cluster architectures with vector processors such as the *Earth Simulator (ES)*. The method is based on a three-level hybrid parallel programming model, including message passing for inter-SMP node communication, loop directives by OpenMP for intra-SMP node parallelization and vectorization for each PE.

In this presentation, results of recent development and comarison between *flat MPI* and *hybrid* parallel programming models on various types of parallel computers will be described.

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INFINITESIMAL BEHAVIOR OF 2-DIMENSIONAL DYNAMICAL SYSTEMS

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Key Words and Phrases: dynamical system, minimal set
AMS Subject Classification: 37B05, 37E30

In this talk, we will consider the problem when a 2-dimensional dynamical system keeps a codimension one foliation invariant. If there exists a codimension one invariant foliation, then the dynamical system is strictly restricted. On the other hand, many important dynamical systems, for example, Anosov systems and irrational transformations, admit invariant foliations.

For convenience, we will restrict our attention to diffeomorphisms of the torus T^2. In order to consider the invariant foliations, the projectivized bundle is useful. Let TT^2 denote the tangent bundle of the torus. We define the projectivized bundle by

$$PT^2 = \{(z,v) \in TT^2\,;\, v \neq 0\}/v \sim kv \quad (k \neq 0).$$

The derivative $Df : TT^2 \to TT^2$ induces a diffeomorphism Pf of PT^2. Then f is tangent to a C^r foliation if and only if there is a C^r section $\Gamma : T^2 \to PT^2$ such that $Pf(\Gamma(z)) = \Gamma(f(z))$. Among several candidates to construct such sections, we choose minimal sets of Pf, i.e. closed Pf-invariant sets of PT^2 which are minimal with restrict to the inclusion:

Theorem. (S. Matsumoto–H. Nakayama) If $f : T^2 \to T^2$ is tangentially distal and minimal, then there is an invariant C^0 1-dimensional foliation.

Theorem (H. Nakayama–S. Noda) If Pf has more than two minimal sets, then Pf is topologically equivalent to an irrational translation. In the case when Pf has exactly two minimal sets, then there are two C^∞ sections which separate these minimal sets if Pf is not topologically equivalent to an irrational transformation.

Here the cross ratio of the straight lines in the plane passing through the origin plays an important role.

INTEGRAL OPERATORS ASSOCIATED WITH FOURIER SUMS AND THEIR MEANS

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Key Words and Phrases: weighted norm inequalities, Fourier sums and their means
AMS Subject Classification: 26D15, 42B08

Let $L_v^p(Q_N), Q_N = [-\pi, \pi]^N$, be weighted Lebesque space. The problem of boundedness of some integral operators (from L_v^p in L_u^p) and related problems are considered. One of the typical results is the next. Let $f_h(x) = |\Omega(h)|^{-1} \int_{\Omega(h)} |f(x+t)|dt$ be Steclov integral means over parallelepipeds $\Omega(h) = \otimes_{j=1}^{N}[-h_j, h_j]$ and (u, v) - a pair of weighted functions, $u_h = u_h(0), v_{h,p} = (v_h^{-1/(p-1)}(0))^{p-1}$, $A_p(u, v; h) = u_h \cdot v_{h,p}$. The inequality

$$\sup_h \int_{Q^N} (f_h(x))^p u(x)dx \leq C \sup_h A_p(u, v; h) \int_{Q^N} |f(x)|^p v(x)dx$$

holds for every $p \geq 1$; the constant C depends on u, v, p, N only; result remains valid for averages over oblique parallelepipeds, which are obtained by triangular-matrix- transform of $\Omega(h)$.

Let $\{K_m(x)\}$ be a sequence of kernels, possessing by even, nonincreasing (on each variable x_j) majorants $K_m^* = K_m^*(x)$. Then the relation

$$|(f * K_m)(x)| \leq C \int_{Q_N} K_m^*(h) f_h(x)dh$$

holds for every m. The such type estimates are the base for researches of grows of Fourier sums over rectangular and oblique parallelepipeds and their linear means. Thus, if $\{\sigma_m^\alpha(f)\}$ be a sequence of (C, α) -means (of positive order), then the following statements are equivalent for every $p \geq 1$:

a)$sup_h A_p(u, v; h) < \infty$;

b)$||\sigma_m^\alpha(f)||_{L_u^p} \leq C||f||_{L_v^p}$.

SYMMETRIC α-STABLE SUBORDINATORS AND CAUCHY PROBLEMS

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Key Words and Phrases: iterated Brownian motion, PDE connection, $\alpha-$ stable process, α-time process, Fractional diffusion, Lévy process, Cauchy problem, Brownian subordinator, Caputo derivative, fractional derivative in time

AMS Subject Classification: 60J65, 60K99

We introduce a class of stochastic processes based on symmetric α-stable processes, for $\alpha \in (0, 2]$. These are obtained by taking Markov processes and replacing the time parameter with the modulus of a symmetric α-stable process. We call them α-time processes. They generalize Brownian time processes studied by Allouba and Zheng and they introduce new interesting examples. We establish the connection of $\alpha-$time processes to some higher order PDE's for α rational.

An apparently unrelated class of processes, emerging as the scaling limits of continuous time random walks, involve subordination to the inverse or hitting time process of a classical stable subordinator. The resulting densities solve fractional Cauchy problems, an extension that involves fractional derivatives in time. For processes subordinated to Brownian motion, the case $\alpha = 2$, we will show a close and unexpected connection between these two classes of processes, and consequently, an equivalence between these two families of partial differential equations. (This part of the talk is work done with Baeumer and Meerschaert). In particular, we show that iterated Brownian motion has a connection with a fractional in time Cauchy problem.

FINDING THE MINIMUM SEMI-DEFINITE RANK OF A GRAPH

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Key Words and Phrases: graph of a matrix, positive semi-definite matrices, rank
AMS Subject Classification: 15A18, 15A57, 05C50

A matrix $A \in M_n(\mathbb{C})$ is called *Hermitian* if $A = A^*$. A Hermitian matrix with non-negative eigenvalues is called a *positive semi-definite (PSD)* matrix. Given a Hermitian matrix A we associate a simple, undirected graph G with vertices $V(G) = \{1, \cdots, n\}$ and edges $E(G) = \{(i, j) \mid a_{ij} \neq 0, i \neq j\}$. This associated graph is independent of the diagonal entries of A. Given a graph G, the *minimum semi-definite rank* of G, denoted $msr(G)$, is the minimum of rank of A as A varies over all PSD matrices with graph G.

In this talk we present results on the upper and lower bounds for $msr(G)$, and the effect of topological changes such as vertex or edge modifications on $msr(G)$. In addition, we will discuss the $msr(G)$ for some classes of graphs, including bipartite graphs and chordal graphs.

RANK CHANGE IN POISSON DYNAMICAL SYSTEMS

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Key Words and Phrases: rank change, Poisson geometry, Casimirs, subcasimirs, symmetry breaking, invariant theory

AMS Subject Classification: 37K05, 37K20, 37K25, 37K35, 37K45, 37J15, 37N10, 53D17, 53D20

It is shown in this paper how a connection may be made between the symmetry generators of the Hamiltonian (or potential) invariant under a symmetry group G, and the subcasimirs that come about when the rank of the Poisson structure of a dynamical system drops by an even integer. This *kinematics-dynamics* connection is made by using the algebraic geometry of the orbit space in the vicinity of rank change, and the extra null eigenvectors of the mass matrix (Hessian with respect to symmetry generators) of the Hamiltonian (or potential). Some physical interpretations of this point of view include a control-theoretic prescription to study stability on various symplectic leaves of the Poisson structure. Methods of Invariant Theory are utilized to provide parametrization for the leaves of a Poisson dynamical system for the case where a compact Lie group acts properly on the phase space, which is assumed to be modeled by Poisson geometry.

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ROBIN CAPACITY AND ITS APPLICATIONS TO FLUID MECHANICS

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Key Words and Phrases: Robin capacity, aerodynamic lift of an airfoil, extremal length of curve-family, variation

AMS Subject Classification: 30C85, 31A15, 76B10

Robin capacity is a characteristic of a part A of the boundary ∂D of a plane domain D containing ∞. It plays an important role in geometric function theory and its applications. If $A = \partial D$, then Robin capacity of A coincides with the Robin constant of D. We apply Robin capacity to investigation of a problem in fluid mechanics.

M. A. Lavrent'ev in 1934 proved that a circle arc provides the maximal aerodynamical lift P in the class of all arcs γ of fixed length l with curvature $K(s) \leq C/l$, $0 \leq s \leq l$, where C is sufficiently small. He found that it is true for $C = 1/21$.

Problem. *To find the maximal value of the constant C for which the statement of Lavrent'ev remains valid.*

We show that the Lavrent'ev constant (1/21) can be increased by a factor of 6 at least.

The proof uses variation of the Robin capacities for the edges of the slit along γ under increase of $K(s)$ on an infinitesimal segment. (Here D is the complement to γ.) The sum of these Robin capacities equals the transfinite diameter of γ, and the difference coincides with the lift P for γ.

We prove that variation of the Robin capacity, under sufficiently smooth change of the boundary ∂D, is proportional to area of the changeable part of D in some extremal metric and obtain explicit formula for the variation.

MATHEMATICAL PROBLEMS FOR PIEZOELECTRIC-METALLIC COMPOSITES

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The recent years have seen an growing interest in the investigation of mathematical models of an elastic medium which take into account the influence of various physical fields such as thermal, electric, magnetic and other ones. An impetus for such studies was the creation of many new materials possessing properties which are not characteristic of usual elastic bodies. Among these are piezoelectric materials.

Despite a great number of works on thermo-electro-elasticity for bodies with piezoelectric effects, which have appeared in the last years, not many strict mathematical results are available for interaction problems of elastic complex structures possessing piezoelectric properties. Here in different regions of the composed body different mathematical models are considered describing the corresponding different physical and mechanical properties.

We study the following problem related to the above mentioned industrial applications: Given is a three-dimensional composite consisting of a piezoelectric matrix with metallic inclusions (electrodes). We derive a linear model with regard thermal stress effects for the interaction of the thermoelastic and electrical fields and perform a rigorous mathematical analysis by the potential method.

The main difficulty in modelling is to find appropriate boundary and transmission conditions for the composed body and to formulate them in an efficient way.

At the end we got strongly elliptic non-self-adjoint linear systems of partial differential equations in the metallic and ceramic parts coupled by transmission conditions and endowed with mixed boundary conditions. The mathematical analysis includes the study of existence, uniqueness and regularity of the resulting elliptic boundary-transmission problem assuming the metallic and ceramic materials occupy smooth or polyhedral domains.

The solutions are constructed with the help of an indirect method, writing the solutions as layer potentials in the ceramic and metallic parts with unknown densities. The densities are to determine in such a way, that the interface and boundary conditions are satisfied. This reduces the initial transmission problem to the equivalent strongly elliptic system of pseudodifferential equations involving pseudodifferential operators on manifold with boundary.

The solvability and regularity of solutions to the resulting boundary integral equations and the original problem are analyzed in Sobolev-Slobodetski H^s_p and Besov $B^s_{p,t}$ spaces.

We investigate also the stress singularities which appear near zones, where the boundary conditions change and where the interfaces meet the exterior boundary. We show that the order of the singularity is related to the eigenvalues of the symbol matrices of the corresponding pseudodifferential operators and study their dependence on the material constants of the composite.

GEOMETRICAL FOUNDATIONS FOR THE INVARIANT (ABSOLUTE) OPERATOR THEORIES OF THE FUNDAMENTAL PROCESSES

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In this paper we discuss the elements of **quantum geometrodynamics (QGD).**
In particular we establish the following **fundamental** theorem: The **dimensionality** (that is the number of linearly independent dynamical variables that determine the states of a physical system) of all free particle processes is **eight**, and the **geometry** is either **Euclidean** or **pseudoeuclidean**. Ipso facto a change of dimensionality signals physical **interaction**, and a change of geometry heralds **instability** and **decay** processes.

ON VIZING'S CONJECTURE

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Key Words and Phrases: graph products, Vizing's conjecture, domination
AMS Subject Classification: 05C69

A dominating set D of a graph G is a subset of vertices of G such that every vertex of G is in D or is adjacent to a vertex in D. The minimum cardinality of such a set D is called the domination number, denoted $\gamma(G)$.

In 1965 V. G. Vizing conjectured that $\gamma(G \,\Box\, H) \geq \gamma(G)\gamma(H)$, where $\Box$ denotes the Cartesian product of G and H. The vertex set of $G \,\Box\, H$ consists of pairs (x, y) where x is a vertex of G and y is a vertex of H. Two vertices (x_i, y_i) and (x_j, y_j) are adjacent in $G \,\Box\, H$ if and only if (x_i, x_j) is an edge in G and $y_i = y_j$ or (y_i, y_j) is an edge in H and $x_i = x_j$.

While Vizing's conjecture has been proved for a variety of restricted classes of graphs (e.g., trees), it remains open in the general case. We extend previously published work in several ways, obtaining new results on the minimum degree of G and bounds on $\gamma(G)$ for graphs G satisfying Vizing's conjecture, and on forbidden subgraphs of G in the case Vizing's conjecture is not true. These results and techniques may serve as building blocks and may provide promising directions for proving Vizing's conjecture.

THE HERMITE-HADAMARD INEQUALITY

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Key Words and Phrases: convex function, signed measure, Hermite-Hadamard inequality
AMS Subject Classification: 26A51, 26D15

The classical inequality of Hermite-Hadamard [6] gives us an important estimate of the mean value of a convex function over a compact interval. A far reaching generalization in the framework of locally convex spaces is offered by Choquet's theory [6], [7], based on the concept of majorization. A.M. Fink [3], C. P. Niculescu [4], [6], J. de la Cal and J. Cárcamo [1], P. Czinder and Z. Páles [2] have obtained a series of results even more general.

The aim of our talk is to discuss the present state of art in this field as well as to outline some connections with PDE and probability theory.

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SOME DYNAMICAL SYSTEMS WITH CHAOTIC STATES

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Key Words and Phrases: complex systems, nonintegrability, chaotic states, prediction
AMS Subject Classification: 37D25

Shchur & Nikolaevski [1] proved nonintegrability of Yang-Mills dynamics, therefore physical vacuum has chaotic states. They proved that in low-dimensional subspace, analytic integrals other than the Hamiltonian don't exist, resulting in the absence of a complete set of integrals for the Yang-Mills system.

Uzing Okuda's [2] versions of neuron/neural network definitions and Smale's [3] horseshoe, Nikolaevski [4] proved that a neuron/neural network is nonintegrable. Chaotic states are manifestly built in the proof. The role of chaotic states in the human brain was also investigated, concluding that chaotic states refresh the human brain. The brain of a person suffering from epilepsy, is often cycling around the same items and can't refresh itself. The parts of the brain, involved into the cycling, become very tired ("overheated"), which can provoke seizures. Chaotic states are not localized in the phase space; they attend almost every spot in it, which can help against "overheat" of local parts of the brain. It furthered progress in understanding of the mechanism of epilepsy.

A common belief exists that chaotic behavior is impossible to predict, because close trajectories diverge exponentially in time. Gurvitz & Nikolaevski [5] developed and implemented in Government Projects two new methods that allow to forecast chaotic behavior with error growing only as square root from time. One method is based on semi-Markov chains, the other - on time series forecasting. In the lecture, explanation of both methods, how and why they work, will be given.

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MODELLING OF PARTICLE POPULATION DYNAMICS: FORMATION OF NANOSCALED TITANIA

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Particulate systems of nanosized titania have a high innovation potential for industry. Today there is a variety of applications of these materials such as catalyst supports, photo catalyst, ceramics, adsorbents, pigments, cosmetics ingredients as well as carriers for medical diagnostics and pharmaceutical products.

High dispersed titania nanoparticles have been produced by sol-gel precipitation process of titanium tetra isopropyl orthotitanate as an organic precursor material in a nitric acid solution, followed by a mechano-chemical redispersion reaction of the spontaneously precipitated agglomerate particles.

The polydisperse particle size distributions of titania were measured by dynamic light scattering and laser diffraction to get kinetic information about the process progress. In this study we focused the effect of shear rates on the redispersion process. Experimental data indicates that a higher characteristic shear rate or dissipation rate, respectively, results in smaller agglomerate sizes at the initial state of particle formation.

The reversible process of redispersion (deagglomeration, disintegration) and agglomeration (coagulation) of the sol-gel process of titania nanoparticles was modelled and simulated by discrete population balance equations focussing on classical process kinetics. These equations used for a polydisperse particle system refer to the theory of Smoluchowski for convection-controlled (orthokinetic) coagulation, the deagglomeration is considered as the reverse process. The redispersion rate constant of one agglomerate disintegrating is assumed to be proportionally to the volume of the aggregates.

The applied characteristic shear rates (specific power consumption/energy dissipation rates) in the process are connected with the agglomeration rate constants (population balance kernels). To analyse the disintegration process of titania, these agglomeration rate constants were compared with agglomerate constants theoretically calculated.

As the agglomerates in the acid suspension are stabilized electrostatically by forming a positively charged double layer (zeta-potential), the stability ratio according Fuchs for a slow coagulation process was introduced in the model to take into account the agglomerate interaction.

Simulations to describe the particle size distributions during the disintegration process have been done. The discrete population balance model has been solved as an inverse problem, the process rate constants for agglomeration and redispersion, the equilibrium constant of the process, and the stability ratio were calculated from experimentally obtained particle size distributions.

COCYCLES OVER CERTAIN HYPERBOLIC HIGHER RANK ABELIAN ACTIONS

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We consider cocycles over certain hyperbolic higher rank abelian actions and show rigidity properties for cocycles with values in a Lie group or a diffeomorphism group, which are close to identity on a set of generators, and are sufficiently smooth. The actions we consider are Cartan actions on compact quotients of SL(n,R) or SL(n,C), for n greater than 3. The results rely on a technique developed recently by D. Damjanovic and A. Katok.

This work is joint with A. Katok.

HOW TO BEAT THE MARKET AND SLEEP WELL AT NIGHT OR SHARPE-OPTIMAL SPDR PORTFOLIOS

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The Sharpe Ratio of an investment portfolio is, loosely speaking, the ratio of its reward to its risk. We seek portfolios of maximum Sharpe Ratio from a fixed universe of Exchange Traded Funds (ETFs).

It is convenient to look at this problem in a geometric setting. Then a portfolio is identified with its risk vector in a high-dimensional Euclidean space, and the Sharpe Ratio of the portfolio is proportional to the cosine of the angle between the risk vector and an expected-reward axis. Now we seek to maximize this cosine (and thus the Sharpe Ratio) over a convex polytope of portfolios. The maximization is accomplished by a simplex-type algorithm using updated QR-factorizations.

STRUCTURE THEOREM FOR FUNCTIONALS SPACE $\mathcal{S}'_{\omega_1,\omega_2}$

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We introduce the space the $\mathcal{S}_{\omega_1,\omega_2}$ of all C^∞-functions ϕ such that

$$\sup_{|\alpha|\leq m}||e^{k\omega_1}\partial^\alpha\phi||_\infty$$

and

$$\sup_{|\alpha\leq m}||e^{k\omega_1}\partial^\alpha\hat{\phi}||_\infty$$

are finite for all $k \in \mathbf{N}_0$,$\alpha \in \mathbf{N}_0^n$ where ω_1 and ω_2 are two weights satisfying the classical Beurling conditions. Moreover, we give a topological characterization of the space $\mathcal{S}_{\omega_1,\omega_2}$ without conditions on the derivatives. For functionals in the dual space $\mathcal{S}'_{\omega_1,\omega_2}$, we prove an structure Theorem by using the classical F. Riesz representation thoerem

ON PHYSARUM SOLVER ON PLANAR GRAPH

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Key Words and Phrases: adaptive network, Physarum polycephalum , the shortest path problem, global asymptotical stability, local Lyapunov functions
AMS Subject Classification: 34D05, 34D23, 37N25

Physarum polycephalum is an amoeba-like organism that exhibits path-finding behavior in a maze. Its body contains a tube network by means of which nutrients and signals circulate through the body in effective manner. Physarum solver is a model equation which was proposed by Tero *et al.*. It describes the adaptive dynamics of a transport network of the true slime mold Physarum polycephalum. However, according to the results of numerical simulations, it can also used for path-finding in a complicated network. In this paper, we prove mathematically rigorously for two specified vertices s, t on the same face of any planar graph Physarum solver can find the shortest s-t path. In other words, we give a rigorous proof of the fact that the equilibrium point corresponding to the shortest path is globally asymptotically stable for Physarum solver. As telling from the mathematically technical point of view, it is a very interesting point that we prove the gllobally asymptotical stability without using usual Lyapunov function, which is defined globally in the whole domain. Instead of this, we use a kind of "local" Lyapunov function to operate a graph under consideration on which Physarum solver is defined. From the viewpoint of the dynamical system, such an operation of graph is a kind of reduction of the system to an inertial manifold. We make the reduction of the system by using of "exponential tracking" property of Physarum solver. This property is proved by our "local" Lyapunov function. As telling about an application of Physarum solver, they have attempted to apply Physarum solver to navigation system of road map. In fact, Physarum solver has found the shortest path connecting between Seattle and Houston admirably. It is, therefore, meaningful very much from the viewpoint of application to verify that Physarum solver solves the shortest path decision problem mathematically rigorously.

BOUNDEDNESS AND CONVERGENCE OF CODED CHARACTER SETS

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Key Words and Phrases: coded character set, Bolzano-Wierstrass theorem, digital computer architecture

AMS Subject Classification: 26A06, 40A05, 68P30, 68R99, 94B05

A coded character set is a set containing sequences of binary digits or bits (i.e sequences of 0s and 1s) which enable character data to be represented, manipulated and stored in a digital computer architecture. As a mathematical function, a CCS is a mapping of a set of bit patterns into a set of characters eg in the popular 7-bit American Standard Code for Information Interchange (ASCII), the bit pattern 1000001 is mapped into the character 'A'. Although many authors have studied the algebraic properties of (error detection and correction) codes, relatively little attention has been paid to the general study of codes(especially CCSs) from the point of view of real analysis. In this paper, the author studies the basic real analytic properties of coded character sets when viewed as sets of sequences. One of the results established in the paper is that the bit pattern of a coded character set is not generally convergent but satisfies the Bolzano-Weierstrass theorem i.e it is bounded and has a convergent subsequence.

INTELLIGENT METHOD FOR BANK NOTE CLASSIFICATION

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Key Words and Phrases: Bill classification, neural networks, reliability measure
AMS Subject Classification: Pattern recognition

For the pattern classification problems the neuro-pattern recognition which is the pattern recognition based on the neural network approach has been paid an attention since it can classify various patterns like human beings.

In this paper, we will explain the competitive neural networks that are used to classify the bill money. The structure of a LVQ competitive network is two layers. One is input layes and the other is competetive layer. The input for the LVQ is bill money data where an original image consists of 128x64 pixels and the input data to the network is compressed as 64x15 pixels to decrease the computational load. The output of the network consists of the Italian Liras of 8 kinds, 1,000, 2,000, 5,000, 10,000, 50,000 (new), 50,000 (old), 100,000 (new), 100,000 (old) Liras with four directions A,B,C, and D.

In the input layer the original bill money data are applied and all the units at the input layer are connected to all the neurons at the output layer. The competitive layer will output only one neuron which is called winner neuron. The winner neuron is selected as the neuron with the minimum distance between an input vector and its connection weight vector. The connection weights are set by the random number at the beginning. Here, we set the mean vector of the cluster plus small random number. Then the following learning algorithm of the connection weight vector is used.

The experimental results show the effectiveness of the proposed algorithm compared with the conventional pattern matching method.

SYMMETRY GROUP APPROACH TO THE GENERALIZED BURGERS EQUATION $u_t + uu_{tt} = \lambda u_{xx}$

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Key Words and Phrases: symmetries, Lie group, Burgers equation, invariant, global solutions

Symmetry of a system of differential equations is a transformation that maps any solution to another solution of the system. In Lie's framework such transformations are groups that depend on continuous parameters and consist of point transformations (*point symmetries*), acting on the system's space of independent and dependent variables, or, more generally, contact transformations (*contact symmetries*), acting on independent and dependent variables as well as on all first derivatives of the dependent variables. Lie groups, and hence their infinitesimal generators, can be naturally *prolonged* to act on the space of independent variables, dependent variables, and derivatives of the dependent variables. We present a Lie symmetry approach in solving Burgers Equation: $u_t + uu_{tt} = \lambda u_{xx}$ which is a nonlinear partial differential equation, which arises in model studies of turbulence and shock wave theory. In physical application of shock waves in fluids, coefficient λ, has the meaning of viscosity. So far in both analytic and numerical approaches the solutions have only been established for $0 \leq \lambda \leq 1$. In this paper, we give a global solution to the Burgers equation with no restriction on λ, i.e. for $\lambda \in (-\infty, \infty)$.

DIFFUSION OF A PULSED POLARIZED LASER BEAM: MODELLING AND SIMULATION

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Key Words and Phrases: radiative transport, partially deterministic Markovian jump process, polarized light, diffusion of a laser beam
AMS Subject Classification: 60, 92

We describe the transport of light through the atmosphere as a stochastic process of corpuscular multiple scattering. Multiple scattering is considered as a sequence single scatterings of polarized photons at particles of the atmosphere. Each single scattering of a photon is decomposed into a random collision, a random selection of the type of scattering particle, a random directional scattering, and a deterministic transformation of the Stokes vector and its reference vector. The associated Markov kernels yield a Markov kernel for a time discrete Markov process with a high dimensional state space. To this time discrete Markov process a time continuous process is associated which turns out to be a cadlag partially deterministic Markovian jump process (PDMP). The infinitesmal generator of this process may be obtained from the Markov kernels of the time discrete process. The associated backward and forward Kolmogorov differential equations may be considered as radiative transfer equations. In the case of unpolarized light with directional scattering distributions which are rotational invariant with respect to the incident beam (which is not true for laser light), the backward Kolmogorov differential equation turns out to be the well-known Chandrasekhar radiative transfer equation.

The description of the transport of polarized light by a PDMP and the application of the method of iterated fictitious collisions make it possible to design Monte Carlo algorithms which allow for the simulation of the diffusion of pulsed polarized laser beams in with respect to density, orientation and mixture of particles structured clouds, broken clouds and exhaustion plumes, such as lidar and CCD lidar returns with transversal and longitudinal diffusion. We shall show some simulations of this type.

APPROXIMATION METHODS FOR FIXED POINTS OF HEMICONTRACTIVE OPERATORS IN HILBERT SPACES

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AMS Subject Classification: 47H04, 47H06, 47H10

In this work, the improved generalized iteration method introduced by Owojori and Imoru[7] is considered to investigate the fixed points of hemicontractive operators in Hilbert space. Our results here represent some improvements and extensions of the earlier results of Ishikawa[4], Deng and Ding[3], Chidume[1], Chidume and Osilike[2], Owojori and Imoru[6], Qihou[8], Liu[5] and Xu[10] from Lipschitz or continuous pseudocontractive operators to slightly more general continuous hemicontractive operators and from the Mann and Ishikawa iteration methods, with and without errors, to more general improved iteration method.

EXPONENTIAL APPROXIMATION ON MULTIPLICATIVE CALCULUS

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Key Words and Phrases: multiplicative, calculus, interpolation
AMS Subject Classification: 65D05

Multiplicative calculus is considerably different from the ordinary calculus created by Newton and Leibnitz. This calculus is based on a multiplicative rate of change whereas the usual calculus is based on an additive rate of change. It can provide different perceptions for applications in science and engineering as well as in mathematics. In this paper, we use the multiplicative calculus to develop alternative methods to approximating and interpolating a function by using exponentials.

FINE REGULARITY FOR PARABOLIC SYSTEMS WITH DISCONTINUOUS COEFFICIENTS

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Key Words and Phrases: parabolic systems, a priori estimates, Morrey spaces, Hölder regularity
AMS Subject Classification: 35R05, 35B45, 35B65

We are dealing with $2b$-order linear systems of the form

$$\mathfrak{L}\mathbf{u} := D_t\mathbf{u}(x,t) - \sum_{|\alpha|=2b} \mathbf{A}_\alpha(x,t) D^\alpha \mathbf{u}(x,t) = \mathbf{f}(x,t) \tag{1}$$

for the vector-valued function $\mathbf{u}\colon Q \to \mathbb{R}^m$ where $(x,t) \in Q = \Omega \times (0,T)$ with an n-dimensional domain Ω, $n \geq 2$, and $T > 0$. Here $\mathbf{A}_\alpha(x,t)$ stands for the $m \times m$ matrix $\left\{a_\alpha^{kj}(x,t)\right\}_{k,j=1}^m$ of the measurable coefficients $a_\alpha^{kj}\colon Q \to \mathbb{R}$ which are supposed to belong to $VMO(Q) \cap L^\infty(Q)$ and $\mathbf{f}\colon Q \to \mathbb{R}^m$ is given.

Assume the system (1) is uniformly parabolic in the sense of Petrovskii, that is, the ζ-roots of the m-degree polynomial

$$\det\left\{\zeta\,\mathbf{I}_m - \sum_{|\alpha|=2b} \mathbf{A}_\alpha(x,t)(i\xi)^\alpha\right\} = 0 \qquad (i = \sqrt{-1})$$

satisfy, for some $\delta > 0$ and all $s = 1, \dots, m$, the inequality

$$\operatorname{Re}\zeta_s(x,t,\xi) \leq -\delta|\xi|^{2b} \qquad \text{for a.a. } (x,t) \in Q,\ \forall \xi \in \mathbb{R}^n, \tag{2}$$

where $\mathbf{I}_m$ is the identity $m \times m$ matrix and $\xi^\alpha := \xi_1^{\alpha_1}\xi_2^{\alpha_2}\cdots\xi_n^{\alpha_n}$.

We will derive *a priori* estimates in Sobolev and Sobolev–Morrey spaces for the strong solutions to (2) by means of potential analysis and boundedness of certain singular integral operators with kernels of mixed homogeneity. As a byproduct, precise characterization of the Morrey, BMO and Hölder regularity will be given for the solutions and their derivatives up to order $2b-1$.

The results are obtained in collaboration with Lubomira Softova.

GENERALISED STABILITY ANALYSIS AND TRANSIENT GROWTH IN LASERS

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It is known that there are hydrodynamic systems where perturbations are able to extract very efficiently energy from a background state even when such background state is asymptotically stable. This results in strong transient amplification of the perturbations that are not (and cannot be) predicted by standard linear stability analysis. Transient noise amplification and other discrepancies between linearization theories and experiments are explained by the fact that the eigenvectors of the linear stability operator are not orthogonal to one another. Similar phenomena are known in optics as transient gain and are usually associated to laser systems with cavity geometries such that the cavity modes are not orthogonal. In this work we show that there can be transient growth of perturbations of a lasing state even for laser systems with orthogonal cavity modes. We show that whenever the background state is a lasing state, the linear stability operator has, in general, non–orthogonal modes. This means that the short term dynamics and the spectrum of the perturbations are always different from those predicted by a standard stability analysis. It does not, however, guarantees the presence of transient gain in the region where the background state is stable. We show the universality of this phenomenon by proving that the linear stability operator of the complex Swift-Hohenberg equations (CSH) for semiconductor lasers always presents non–orthogonal eigenvectors when the background state is a lasing state.

In this presentation we analyse and discuss non–orthogonal eigenvectors of linear stability operators and their physical significance for lasers with asymmetric gain line. The lasing solution may show transient growth even in regions where it is linearly stable. For standard semiconductor lasers, characterised by negative line–enhancement factors, this result changes the dynamics over short times but not the long term stability because the two instability curves are tangent at the threshold point $k = 0$. However, for configurations with positive line–enhancement factors transient growth of perturbations should be observed in spite of the lasing solution being linearly stable. Such situation is analogous to Reynolds turbulence and may induce chaotic behaviour in linearly stable regimes.

ON THE VALUE FUNCTION FOR AN OPTIMAL CONTROL PROBLEM

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AMS Subject Classification: 49-01, 49L20, 49L25

An infinite horizon singular optimal control problem with unbounded control set leads to a dynamic programming equation called the Hamilton Jacobi Bellman equation that the value function must satisfy. We find explicitly the corresponding value function.

AN EXACT MACROSCOPIC EXTENDED MODEL WITH MANY MOMENTS

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Key Words and Phrases: extended thermodynamics
AMS Subject Classification: 26A33

Extended Thermodynamics (ET) is a very important theory: for example, it predicts hyperbolicity, finite speeds of propagation waves as well as continuous dependence on initial data. Therefore, it constitutes a significative improvement of ordinary thermodynamics. Here its methods are applied to the case of an arbitrary, but fixed, number of moments.

Originally, ET was studied with a macroscopic approach but the exact solution of the conditions which are present in the theory with many moments is still lacking. This gap is here filled and the constitutive functions appearing in the balance equations are determined up to whatever order with respect to thermodynamical equilibrium.

In the meanwhile, it has been studied with the kinetic approach which is more restrictive of the above one. But it leads to an ill-posed system of equations, and we hope that our less restrictive approach may face this difficulty. To this end, another approach to the problem has been developed and is denoted by COET (Consistent Order Extended Thermodynamics), but some problems remains open: In COET of order 2 we find Extended Thermodynamics with 13 moments, which has no sense in Ordinary Extended Thermodynamics with the Kinetic Approach; moreover, the non relativistic limit of Relativistic Extended Thermodynamics gives the 14 moments theory instead of the 13 moments one. This aspect has to be clarified.

In the meanwhile, we retain useful the present alternative approach.

FAMILY OF ITERATIVE METHODS FOR COMPUTING THE ZEROS OF ANALYTIC FUNCTIONS

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Key Words and Phrases: family of iterative methods, simultaneous methods, zeros of analytic functions, convergence

AMS Subject Classification: 65H05

We consider a one parameter family of iterative methods for the simultaneous determination of complex zeros of a class of analytic functions which have only simple zeros inside a simple smooth closed contour in the complex plane. The proposed family is based on a cubically convergent family of iterative methods for solving nonlinear equations. The presented convergence analysis shows that the order of convergence of the considered family is four. Numerical examples demonstrate a good convergence properties, fitting very well theoretical results.

MATHEMATICAL AND COMPUTER SOFTWARE FOR PATH-AMALGAMATION OF THE PLANAR GRAPHS

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Key Words and Phrases: φ-transformations of planar graphs, genus, reachability number
AMS Subject Classification: 05C10, 05C85

Denote G_i as a simple connected nonorientable planar graph without loops and multiply edges, and remark L_i as a path length n_i of this graph, where $n_1 \geq n_2$, $n_i > 0, i = 1, 2$. Define a path-amalgamation of planar graphs G_i as an amalgamation of all vertexes of L_1 with corresponding vertexes of L_2 or interior points of here edges and deletion of double edges in the new planar graph G as a result of path-amalgamation of planar graphs $G_i, i = 1, 2$. Introduce a number reachability of all vertexes in the new planar graph G as a minimal number of 2-cells of the plane on whose boundaries lied set of vertexes graph G. Task: the investigation of structural properties of the m-minimal graph G with minimal set of there edges and with given number reachability m, $m > 1$, of all there vertexes on the base of building mathematical and computer software for path-amalgamation of the planar graphs. Result: was given a package of programs on Visual Age PL/I 2.1 from IBM and was obtained a none full list of 4-minimal planar graphs from path-amalgamation of the some 3-minimal planar graph, which was obtained early, and one of K_4 or $K_{2,3}$. Method: φ-transformations of graphs by Dr. Khomenko N.P.. Mathematical content: for given m-minimal planar graph G will be existed φ-transformation defined on planar graphs G_1 and G_2,as in one of follow cases: a1 where G_1 is a $m-1$-minimal planar graph with path L_1 and G_2 is a K_4,or $K_{2,3}$ with path L_2 by means following: $\varphi(G_1 + G_2, \sum\limits_{i=1}^{n} a_i + g_i) \rightarrow (\Im, \{a_i^*\}_1^n)$, where $M_1 = \{a_i\}_1^{n_1}, t_{G_1}(M_1) = 1, \cdot M_2 = \{g_i\}_1^{n_2}, t_{G_2}(M_2) = 1\cdot$ are points sets of G_1 and G_2 with reachability numbers $1, 1$, accordingly; a2) where G_1 is a none $m-1$-minimal planar graph with closely path L_1, but he was obtained as φ-image by mean case a1. A full list of nonizomorphic 3-minimal planar graphs was obtained with help of this software.

BIJECTIVE CODES FOR TREES AND *K*-TREES

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Key Words and Phrases: labeled trees, labeled k-trees, bijective coding, optimal algorithms
AMS Subject Classification: 68W, 68R

We consider the problem of coding labeled trees and labeled k-trees by means of strings of labels. This codification is an interesting alternative to the usual representation of graph data structures in computer memories and it has many practical applications (e.g. evolutionary algorithms over trees, random generation of trees and k-trees, data compression).

This representation was first used in 1918 by Prüfer in the proof of Cayleys theorem to show a one-to-one correspondence between free labeled trees on n nodes and strings of length $n-2$. In 1970 Rényi and Rényi generalized Prüfers bijective proof obtaining a redundant code for a subset of labeled k-trees.

After 1918, the problem of coding labeled trees has been widely studied both from a mathematic and an algorithmic point of view. Several different bijective codes that realize associations between labeled trees and strings of labels have been introduced. Optimal encoding and decoding algorithms are known for most of these codes. Moreover it is shown that only some of these codes satisfy properties like locality (small changes in the tree correspond to small changes in the string and viceversa) and heritability (edges of a tree corresponding to two mixed strings belong to one of the two existing trees), requested for special applications.

Subsequentely to the Rényis result, attempts have been made to obtain an algorithm with linear running time for the redundant Prüfer code. Concerning with non redundant codes, during the nineties appeared in literature results realizing bijection between well defined set of strings and Rényi k-trees and/or general k-trees; however optimal encoding and decoding algorithms were produced only for Rényi k-trees. The first encoding and decoding algorithms running in linear time with respect to the size of the k-tree have been published this year (joint work with S. Caminiti and E.G. Fusco).

OSCILLATIONS OF CERTAIN DIFFERENTIAL EQUATIONS OF SECOND ORDER

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Key Words and Phrases: oscillation, monotonicity
AMS Subject Classification: 34A30, 34A40, 34C10

We are interested in the following equations: $z''(t)+$

$$+\sum_{i=1}^{n}\theta_i(t)z'(t-\tau_i)+\sum_{k=1}^{\tilde{n}}\beta_k(t)z(t-\sigma_k)+Mz(t)=F(t).$$

We assume that $M>0,\ \tau_i\geq 0,\ \forall\ i=\overline{1,n},\ n\in\mathbf{N}$ and $\sigma_k\geq 0,\ \forall\ k=\overline{1,\tilde{n}},\ \tilde{n}\in\mathbf{N}$ are constants as well as that $\theta_i(t)\in C([T,\infty),[0,\infty)),\ \forall\ i=\overline{1,n},\ n\in\mathbf{N}$ and $\beta_k(t)\in C([T,\infty),[0,\infty)),\forall\ k=\overline{1,\tilde{n}},\ \tilde{n}\in\mathbf{N}$, where $T\geq 0$ is a large enough constant. In general, we prove sufficient conditions for the absence both of eventually positive and monotincally non-decreasing solutions and eventually negative and monotincally non-increasing solutions. Also, we find sufficient conditions for the existing of oscillating solutions only in the particular case, where $\theta_i(t)\equiv 0,\ \forall\ i=\overline{1,n}$. Further, we comment the distributions of their zeros.

PATENTS IN NEW TECHNOLOGIES

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Key Words and Phrases: game theory, optimization, management decision making, strategic *R&D*, endogenous spillovers, patents
AMS Subject Classification: 37N40, 90B50, 91A10

Our analysis add some new insights to the importance of the use of patents in new technologies. We present a new R&D investment in a Cournot model and we thoroughly analyze the short and long term economical effects of the Nash R&D investments in the profits of the firms involved in the Cournot competition. For old technologies, the long term economical effects are not very sensitive to changes in the efficiency of the R&D programs and also to changes in the market structure. However, for new technologies, the long term economical effects are very sensitive to small changes in the efficiency of the R&D programs and also to small changes in the market structure. We find a favorable economical region to firm F_1, a favorable economical region to firm F_2 and a recovery region for both firms characterized by the production costs of both firms. These three main economical regions are dynamically characterized by the stable manifolds of two saddle points.

CONTROLLING MULTISTABILITY BY ATTRACTORS ANNUHILATION

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Key Words and Phrases: coexisting attractors, periodic modulation, control of multistability
AMS Subject Classification: 34C28

The coexistence of several stable states (attractors) for a given set of parameters has been observed in many natural and experimental systems as well as in complex theoretical models. Such multistability is not convenient for many applications, for example, if one desires to create a stable device with a fixed output. In this lecture I will demonstrate how it is possible to control multistability in order to make the system monostable. A small perturbation of any system parameters may not in general create any significant qualitative change in dynamics of a multistable system. However, a slow periodic modulation with properly adjusted amplitude and frequency can do so. In particular, it can control the number of coexisting attractors. The basic idea of the controlling mechanism is to introduce a collision between an attractor with its basin boundary. As a consequence, the attractor is destroyed and the transients settle down to an adjacent attractor. With the appropriate choice of modulation parameter values, the control can induce boundary crisis to undesirable attractors, and thus resulting in controlled multistability. These features are general for a wide class of dynamical systems with coexisting attractors. It was observed first theoretically in the Hénon map and laser equations and then confirmed experimentally in modulated CO_2 and fiber lasers. Such a control is robust against small noise as well.

REGULARIZED STRUCTURED TOTAL LEAST SQUARES ALGORITHM IMPLEMENTATION FOR RECONSTRUCTION OF A TRUE IMAGE AFTER BLURRING

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Key Words and Phrases: high-resolution image reconstruction, image deblurring, Gaussian blur, regularization, ill-conditioned matrices, software implementation
AMS Subject Classification: 65F22, 65F10, 68U10

Many investigators have recently turned to the problem of reconstructing a high resolution image from the blurred one. Image deblurring problem has been shown to become a focus in image processing because some other problems of the field are closely related or even can be reduced to it, for example, the reconstructing a high resolution image from multiple undersampled frames. Mathematically, these problems are described and successfully solved by means of the total least squares (TLS) method taking into account that error exists not only in the blurred (measured) image but also in the blurring matrix. More to it, the TTL needs to be the structured and regularized one because the blurring matrix is Block Toeplitz and ill-conditioned.

That much interest in the regularized structured total least squares as applied to image processing has inspired confidence into us that an efficient software product would be of demand and enable wider computational experimenting. This paper presents such a software implementation supported by a number of numerical experiment results to verify the design and validate the method.

MULTISCALE MATHEMATICAL MODELLING IN CHEMICAL ENGINEERING

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Mathematical modeling has been used in chemical engineering for many decades. Recently it developed even further, covering different length and time scales.

In the paper different branches of applied mathematics and computing, used for the mathematical modeling of chemical processes, shall be briefly reviewed, and typical problems shall be discussed.

The scope of the subject covers problems in molecular modeling, linear algebraic problems in the description of chemical stoichiometry and chemical kinetics, the use of vector and polyadic analysis, description of the momentum, heat and mass transfer problems, a short survey of numerical methods applied to the above problems, coupled diffusion - chemical reaction and heat transfer problems, and mathematical methods used in the description of chemical and biochemical reactors.

GROUP-THEORETIC NON-UNIFORM PSEUDO AND QUASI-RANDOM SEQUENCES FOR SIMULATION

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Key Words and Phrases: pseudo and quasi-random sequences, non-uniform sequences, quasi-Monte Carlo integration, importance sampling, Weyl sequences
AMS Subject Classification: 11K60, 14H40, 65C10, 65C05, 65D32

Many complex mathematical problems can be solved by Monte Carlo (MC) methods. Thus, there are many algorithms for generating pseudo-random sequences, almost all of which are for the uniform distribution. However, many applications require non-uniform random sequences. These are usually created by using a techniques such as rejection sampling or transformation on a uniform sequence. In this talk we introduce a new method to directly generate, without transformation or rejection, some non-uniform pseudo-random sequences. This method is a group-theoretic analogue of linear congruential pseudo-random number generation. We provide examples of such sequences, involving computations in Jacobian groups of plane algebraic curves, that have both good theoretical and statistical properties.

We will also discuss the related problem of generating non-uniform quasi-random sequences for use with Quasi-Monte Carlo (QMC) methods. In practice, QMC integration is often applied to integrands on unbounded domains with non-uniform probability measures, integrals for which there is little theoretical validation. We will introduce group-theoretic methods to generate some non-uniform deterministic Weyl-like (quasi-random) sequences. We also introduce a new importance sampling technique, which can be used with these group-theoretic sequences or lattice rules to create QMC integration rules with a high asymptotic order of convergence.

ASYMPTOTIC EXPANSIONS OF SINGULAR SOLUTIONS FOR $(3+1)-D$ PROTTER PROBLEMS

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In this talk we investigate some boundary value problems for the wave equation, that are multi-dimensional analogues of Darboux-problems (or Cauchy-Goursat problems) on the plane. Originaly they were proposed by Murray Protter in the fifties. He studied the 3-D wave equation in a domain Ω, bounded by two characteristic cones S_1 and S_2 and the plain region S_0. The homogeneous data are prescribed on S_1 and the noncharacteristic circle S_0. Many authors studied these problems using different methods, like: Wiener-Hopf method, special Legendre functions, a priori estimates, nonlocal regularization and others. Initially the expectation was that such BVPs are classical solvable for very smooth right-hand side functions. Contrary to this traditional belief, soon it became clear that unlike the plane Darboux problem, the Protter's problem is not well posed. The reason is that the adjoint homogeneous problem has infinitely many smooth classical solutions. Popivanov and Schneider (1995) showed that for each $n \in \mathbb{N}$ there exists a right-hand side function $f \in C^n(\overline{\Omega})$ of the wave equation, for which the uniquely determined generalized solution of the Protter problem has a strong power-type singularity like $|x|^{-n}$. An important aspect is that the singularity is isolated only at a single point – the vertex O of the characteristic light cone S_2 and does not propagate along the bicharacteristics. Still, there are many open questions, that naturally arise from the previous works on the Protter problem. In the present talk we examine the analytical behavior of the unique generalized solution of the Protter problem and discus some of the following open problems:

1. To study the general case when the right-hand side function $f \in C^1(\overline{\Omega})$:

∘ Find some appropriate conditions for the function f under which there exists a generalized solution.

∘ Our a priori estimates show that the generalized solutions can have at most an exponential growth at the isolated singular point. The natural question is: are there any singular solutions with exponential growth or do they all have only polynomial growth?

∘ To find some appropriate conditions for the function f under which the Protter problem has only regular, bounded or even classical $C^2(\overline{\Omega})$ solutions.

2. What happens with the ill-posedness of the Protter problems in a more general domain when the maximal symmetry is lost?

3. Why there appears a singularity for such smooth right hand side even for the wave equation? It is interesting to find the reason for this phenomenon, which is not traditional for the wave equation. Can we numerically model it?

From analytical point of view we give the exact asymptotic expansion of the generalized solution at the singularity point O. On the other hand, as a result of numerical computations we present some graphics of solutions with different power of singularity.

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SINGULAR SOLUTIONS TO PROTTER'S PROBLEM FOR A CLASS OF 3-D WEAKLY HYPERBOLIC EQUATIONS

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Key Words and Phrases: weakly hyperbolic equations, boundary value problems, generalized solutions, singular solutions, propagation of singularities
AMS Subject Classification: 35L20, 35L50

In 1952 M. Protter formulated new boundary value problems (BVP) for a class of hyperbolic equations, as well as for some hyperbolic-elliptic equations. Actually, in the case of the wave equation, or for some weakly hyperbolic equations (like Tricomi equation in the hyperbolic part of the domain) the new problems are three-dimensional analogues of the Darboux problems (or Cauchy-Goursat problems) on the plane. In the contrast of the well-posedness of the Darboux problem in 2-D case, the new problems are stromgly ill-posed. According to the statement of Protter problems for mixed hyperbolic-elliptic equations, they are some 3-D analogues of the Guderley-Morawetz plane problem, appeared from the transonic fluid dynamics models. Are that 3-D Protters problems well posed, or ill posed, for hyperbolic-elliptic equations, is still open question.

In the present lecture we study such Protter problems for weakly hyperbolic equations in 3-D domain Ω_m, bounded by two characteristic surfaces Σ_1^m and Σ_2^m, and by a plane region Σ_0. Now, it is well known that, for the infinite number of smooth functions in the right-hand side, some of Protter's problems do not have classical solutions. Popivanov and Schneider (1993) found the reason of this fact in the case of Dirichlet's condition on Σ_0: the strong power-type singularity appears in the generalized solution on the characteristic surface Σ_2^m. Now, for weakly hyperbolic equation involving lower order terms we study the BVP, with the Dirichlet's condition on Σ_1^m and third BV data on $\Sigma_0 : u_t + \alpha\,(x)\,u = 0$. Under some conditions on the lower order terms we prove that for each $n \in \mathbb{N}$ there exists a right-hand side function $f_n \in C^{n-3}(\bar{\Omega}_m) \cap C^{\infty}(\Omega_m)$, for which the corresponding unique generalized solution of the Protter problem has strong power type singularity $(|x|^2 + |t|^{m+2})^{-n/2}$ at the point O. It is interesting that this singularity is isolated at the vertex O of the characteristic surface Σ_2^m and does not propagate along it.

GUDERLEY-MORAWETZ-PROTTER BOUNDARY PROBLEM FOR WEAKLY HYPERBOLIC EQUATIONS, NONEXISTENCE FOR SUPERCRITICAL EXPONENT

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It is well known, starting from the seminal paper of Pohozaev (1965), that the homogeneous Dirichlet problem for semi-linear elliptic equations such as $\Delta u + u|u|^{p-2} = 0$ in Ω a bounded subset of $\mathbf{R}^n$, with $n \geq 3$, will permit only the trivial solution $u \equiv 0$ if the domain is star-shaped, the solution is sufficiently regular, and $p > 2^*(n) = (n+2)/(n-2)$ the critical exponent in the Sobolev embedding of $H_0^1(\Omega)$ into $L^p(\Omega)$ for $p \leq 2^*(n)$, which fails to be compact at the critical exponent. For semi-linear partial differential equations of mixed elliptic-hyperbolic type with various boundary conditions, the nonexistence of nontrivial solutions is shown for domains which are suitably star-shaped and for nonlinearities with supercritical growth in a suitable sense. The results follow from integral identities of Pohozaev type which are suitably calibrated to an invariance with respect to anisotropic dilations in the linear part of the equation. For the Dirichlet problem, in which the boundary condition is placed on the entire boundary, the technique is completely analogous to the classical elliptic case as first developed by Pohozaev in the supercritical case. At critical growth, the nonexistence principle is established by combining the dilation identity with another energy identity. For "open" boundary value problems in which the boundary condition is placed on a proper subset of the boundary, sharp Hardy-Sobolev inequalities are used to control terms in the integral identity corresponding to the lack of a boundary condition. Especially, for the three dimensional Guderley-Morawetz-Protter problem for weakly hyperbolic equations, the nonexistence of nontrival solutions in the supecritical case is proved. Let mention, that in this case the critical value $p = 2^*$ is different from the critical value $2^*(n)$ in the case of strongly elliptic type of equations.

ON THE HYPOELLIPTICITY OF SOME CLASSES OF OVERDETERMINED SYSTEMS OF PSEUDODIFFERENTIAL OPERATORS

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Key Words and Phrases: hypoellipticity, subellipticity, overdetrmined system
AMS Subject Classification: 35H10, 35A27, 35H20, 35B65

This short communication deals with the/micro/local hypoellipticity and subellipticity of the solutions of overdetermined systems of pseudodifferential operators having simple and double characteristics. As an application we study the hypoellipticity of systems of complex valued vector fields. The most general results are proved when the loss of regularity in the /microlocalized/Sobolev spaces is less than or equal to 1. The case when the minimal loss of regularity is > 1 is discussed and illustrated by several examples too.

MAGNETIC GEOMETRY IN THE DIII-D TOKAMAK

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Key Words and Phrases: symplecticity, Hamiltonian system, tokamak
AMS Subject Classification: 37, 70

Trajectories of magnetic field lines are a 11/2 degree of freedom Hamiltonian system. The authors have published a number of papers that calculate the stochastic layer and magnetic footprint in divertor tokamaks using method of maps [1, 2] with a generating function that correctly describes the magnetic topology [see ref. 3 and references therein]. Here a generating function is developed that quite accurately describes the magnetic geometry in the DIII-D tokamak. The DIII-D tokamak is one of the leading divertor tokamaks for advanced magnetic fusion research in the world [4]. The equilibrium fit (EFIT) data for the DIII-D shot 115467 at the time instant 3000 ms is used to develop the generating function. The EFIT data is converted from the (x,y) space to (ψ,θ) space using the canonical transformation $x = \sqrt{\frac{2\psi}{B_0}}\cos(\theta) \text{and} y = \sqrt{\frac{2\psi}{B_0}}\sin(\theta)$. The level curves of Hamiltonian are expressed as a bivariate polynomial in x and y using nonlinear regression. It is found that a bivariate polynomial with 5th power in $\sqrt{\psi}\cos(\theta)$ and 6th power in $\sqrt{\psi}\sin(\theta)$gives a quite accurate representation of magnetic geometry of the DIII-D shot. The hyperbolic fixed point differs by less than 3 cms in distance from the EFIT data for a loss of less than 0.15% in the generating function. A canonical transformation is applied to construct a symplectic map in canonical variables (ψ,θ). This map can quite accurately trace the trajectories of magnetic field lines in physical space in the DIII-D using the transformations (ψ,θ) ? (x,y) ? (R,Z) . This new approach is quite accurate in describing the magnetic geometry of a specific divertor tokamak. The initial results of this promising construction and its implications for realistic divertor tokamaks will be presented.

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BRANES IN CONDENSED MATTER PHYSICS

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Key Words and Phrases: fermions on surfaces
AMS Subject Classification: 81T20, 81T30, 81T40, 74A50

String theory is a well known catalyst for seminal interactions between mathematics and physics. Theoretical physicists became aware of powerful mathematical tools like Lie algebra cohomology or the theory of complex manifolds when they had to use these methods to understand the quantum dynamics of relativistic strings.

It is not so widely known that some of the mathematical techniques used in string and brane theories can also be adapted for calculations in low-dimensional electron systems. The dynamics of electrons at surfaces, interfaces, and quantum wires is important for materials science and for the experimental modeling of magnetism and statistical physics in low-dimensional systems.

The talk will specifically discuss the mapping between half-order differentials and spinors for the description of fermions in low-dimensional systems, and the use of brane techniques to describe the transition between surface and bulk properties of electrons in the presence of a surface or interface.

VISUALIZATION OF INFINITESIMAL DEFORMATION OF SOME CLASS OF TOROID

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Key Words and Phrases: infinitesimal bending, infinitesimal deformation, rigidity, toroid, C++, OpenGL

AMS Subject Classification: 53A05, 53C45, 68U05

The basic aim of the deformation theory is to point out to a class of rigid or non-rigid surfaces. In this paper are considered torus like surfaces obtained by revolution of quadrangular meridian. Infinitesimal bending of generated surfaces is explored using Cohn-Vossen's method. We present visualization of such surfaces and their infinitesimal deformation obtained by our programm. It is written in C++ and for modelling three dimensional surfaces and curves is used OpenGL graphic library.

COUPLING OF FLUID AND STRUCTURE VIA THE COMPONENT TEMPLATE LIBRARY

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Key Words and Phrases: partioned methods, strong coupling, fluid-structure interaction
AMS Subject Classification: 81V05, 74F10

Problems of fluid-structure interaction contain of a fluid and structure part. Both problems are coupled by the boundary conditions on a common interface. In the simulation of fluid-structure interaction often so-called partioned methods are used, i.e. both problems are solved by different software packages. This approach has the advantage that for every subproblem the best solvers can be used.

In this talk both codes are controlled by the help of the component template library (CTL). To use the CTL component-based software systems are needed. Component-based software systems have the advantage that the implementations have a longer lifetime and that linkage of may be incompatible libraries is avoided. It is possible to exchange the software components without any difficulty.

In this talk algorithmic aspects of the interaction of the fluid-, the structure-code and the CTL are explained. The CTL has the main task to distribute the subproblems and to control the loops for solving the linear systems of equations and the loop for the time discretization [2].

Practical examples from the ship building industry show the interaction of the software and that our approach gives good and fast results.

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RESIDUAL FINITENESS OF INFINITE AMALGAMATED PRODUCTS OF CYCLIC GROUPS

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A group G is called *residually finite* if the trivial subgroup of G is closed in the profinite topology of G. The residual finiteness of groups has been extensively studied, mainly for finitely generated groups. If a group is residually finite, then there exists an algorithm to solve the word problem.
We classify the residual finiteness of infinite amalgamated free products of infinite cyclic groups. As a side result we also classify residual finiteness and subgroup separability of the subgroups of $(\mathcal{Q}, +)$.

Joint work with V.Metaftsis.

ON LEAST-SQUARES FINITE ELEMENT FORMULATIONS FOR INCOMPRESSIBLE FLUID FLOWS AND SHEAR DEFORMABLE SHELLS

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We present finite element formulations based on least-squares variational principles for the numerical solution of (a) the stationary and non-stationary Navier-Stokes equations governing viscous incompressible fluid flows and (b) shear-deformable plates and shells [1-5]. The equations governing viscous incompressible flows or the shear deformable plates and shells are expressed as an equivalent set of first-order equations by introducing the vorticity or the velocity gradients as independent variables in fluid flow problems and bending moments and shear forces in the case of shear deformable plates and shells. The use of least-squares principles leads to a variational unconstrained minimization problem where the approximation spaces for the dependent variables (e.g., velocity and pressure) can be chosen independently, i.e. stability requirements such as the inf-sup condition never arise. For the non-stationary case, we present a formulation where the effects of space-time are coupled. This results in a true space-time least-squares minimization procedure, as opposed to a space-time decoupled formulation where a least-squares minimization procedure is performed in space at each time step.

Although not commonly emphasized, low order nodal expansions tend to lock and reduced integration techniques must be used to obtain acceptable numerical results. As shown and concluded in our previous works [3-5], use of high order nodal/modal expansions and full integration are the appropriate way to truly minimize the least-squares functional. The quality of the numerical solution may be judged by the value of the least-squares functional, which decays exponentially fast as the expansion order of the nodal/modal basis is increased.

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ILL-POSED INVERSE PROBLEMS: APPLICATIONS IN SIGNAL RESTORATION AND METHODS OF TOTAL LEAST SQUARES

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I present a brief overview of some signal restoration problems arising in medical imaging and seismic data analysis, demonstrating the need for utilization of regularization in the solution of ill-posed inverse problems. New algorithms which use regularized total least squares for completely unstructured parameter estimation problems are then developed. Future extensions of the research will be described.

THE STABILITY PROBLEM FOR EVOLUTIONARY GAMES VIA VECTOR LYAPUNOV METHODS

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Key Words and Phrases: evolutionary games, stability, Lyapunov methods, continuous time replicator dynamics
AMS Subject Classification: 34D20, 93D05

In this paper the stability problem for single and multipopulation evolutionary games is addressed. The formulation is given in terms of the continuous time replicator dynamics which is a system of ordinary differential equations that describe how the mixed strategies vector changes. After recalling some preliminaries about stability, we proceed by introducing the evolutionary stable strategy concept and the continuous time replicator dynamics, some needed characterizations are also provided. The overall presentation is given for the single and multipopulation settings. This results are in a more detailed form discussed and when it is possible new approaches are considered. Finally, the stability problem for the general multipopulation case is solved. The fact that being a strict Nash equilibrium implies asymptotic stability is shown to hold by means of a vector Lyapunov approach.

NONLINEAR DYNAMICAL CONTROL SYSTEMS: DISCRETIZATION, ESTIMATION, AND CONTROL

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Key Words and Phrases: continuous–time, discrete–time, nonlinear dynamical control systems, discretization, sampling, sampling frequency
AMS Subject Classification: 93C15, 93C55, 93C57, 93C62, 93C83

Nonlinear dynamical control systems (NLDCS) are mostly continuous – time (CT) systems; yet, the omnipresence of digital computers in control engineering imposes transforming nonlinear CT control and observation laws into discrete–time (DT) algorithms for implementation purposes. Discretizing NLDCS persists as an interesting and challenging problem because, among other reasons, there is no supporting Shannon's sampling theorem prescribing how to choose the sampling frequency implicitly involved in any discretization.

The Euler or standard discretization of NLDCS might be described as "periodic sampling + linear interpolation". On looking for discretizations with better transient–state fitting properties at low sampling frequencies, *Euler-Picard discretization*, i.e., "periodic sampling + Picard interpolation", and *Euler-Taylor-Picard discretization*, i.e., "periodic sampling + Taylor approximation + Picard interpolation", were proposed in a previous work. Afterwards improvements of these two discretization methods, that reduce computation times, have also been more recently developed and reported.

Our search for DT models of CT NLDCS was originally oriented towards the digital implementation of nonlinear controllers and observers. However, such models might also be thought of as state estimators of CT systems, what might be used for different purposes. In this work we start considering our previous work from this new perspective, and describe some of its applications on estimation and control of nonlinear dynamical control systems. The reconstruction problem of CT signals from DT ones will also be approached.

MODELLING OF EXCITED STATE KINETICS IN FLUORESCENCE NANOMETROLOGY

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Key Words and Phrases: fluorescence, lifetime, data analysis, inverse problem, special functions

Fluorescence spectroscopy of nano- to femtosecond time resolution and how they relate to spatial resolution is an important non-invasive method for determining molecular structures and monitoring processes occurring on the nanometre scale. Numerous applications in materials and life sciences include: studies of polymer/gel/membrane formation, protein conformations, ligand binding, ion transport, etc.

The relationships between the detected fluorescence responses and the sought for system parameters (e.g. molecular separation, concentration, conformation), are established on the basis of the relevant models of excited-state kinetics of the fluorophores involved. Extracting these parameters from raw lifetime data usually requires solving an inverse problem and various theoretical approaches are attempted in order to reveal the maximum of useful information.

During the lecture, some solutions of inverse problems appearing in fluorescence nanometrology will be presented and illustrated using real data and the new challenges, related to the applications of fluorescence to biological and medical problems, will be discussed.

VERY SLOW DECAYING AUTO-CORRELATIONS: APPLICATION TO VOLATILITY IN FINANCIAL MARKETS

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Key Words and Phrases: long-time auto-correlations, fractional Brownian motion, financial markets, volatility
AMS Subject Classification: 60G50, 62M10, 62P20, 65Q05

We present a simple model for simulating a long-time memory in time series characterized in addition by a stochastic variance. The model is based on a combination of fractional Brownian motion concepts, for dealing with the long-time memory, with an autoregressive scheme with conditional heteroskedasticity, responsible for the stochastic variance of the series. These ideas are applied to the description of long-time autocorrelations of volatility ubiquitously observed in stock markets. The theoretical results reproduce stylized features of the financial time series.

A FAMILY OF HIGHER ORDER ITERATIVE PROCESSES AND THE CATALAN TRIANGLE

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Key Words and Phrases: Newton-type methods; iterative processes; quadratic equations; Catalan triangle

AMS Subject Classification: 65B99, 65H05, 65J15, 65P99

We construct and study numerically and dynamically a new family of convergent iterative root-finding algorithms

$$t_{k+1} = t_k - H_q(L_f(t_k))f(t_k)/f'(t_k), \quad \text{where} \quad L_f(t) = f(t)f''(t)/f'(t)^2$$

and $H_q(t) = \displaystyle\sum_{j=0}^{q-2} \frac{1}{2^j} C_j t^j,\ q \geq 4,\quad C_j = \frac{1}{j+1}\binom{2j}{j}$ (Catalan numbers, see [Hilton]) when f is a quadratic polynomial in the complex and real cases. This family includes Newton's method (see [GER]), Chebyshev's method (see [HERN7]) and as special case $q \to \infty$ the well known Euler's or Cauchy's method, (see [Melm]). We prove that these methods converge with prefixed order of convergence q to the roots of the polynomial f. We provide global results depending on the prefixed order of convergence is even or not. We show the relation between the Catalan triangle [Sh74] and the general convergence of the family [Smale]. Moreover, we present some computer graphics showing the intricate dynamical structure of some of these methods which converge with order q to the roots of the quadratic polynomial f. Finally, the extension to Banach spaces of this family is analyzed, so that, under certain natural modifications, we obtain iterative processes with any R-order of convergence, when they are applied to approximate solutions of quadratic equations in Banach spaces.

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STATISTICAL ANALYSIS OF KEY COMPARISON DATA

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Key Words and Phrases: key comparisons, metrology and data statistical analysis
AMS Subject Classification: 62-07, 62P35

In Primary Metrology, the nations world-wide are continuously participating in key comparisons (KC) in several quantities, trough their National Metrology Institutes (NMIs). Those KC allow the participants to verify the accuracies of their reference standards or measurement systems, directly against their counterparts of the same metrological quality in other countries.

The results allow the participants to correct for any deviation from the obtained key comparison reference value (KCRV), which is accepted as an international reference and therefore serves as an objective and sometimes legal basis in trade disagreements. This is the reason why the KCs demand a high responsibility from the pilot NMI when analyzing the data and proposing the KCRV.

The KCs are organized either by the corresponding Consultative Committee (CC) of the *Bureau International des Poids et Mesures* (BIPM), of the quantity to be compared, when a reference value is sought; or by any Regional Metrology Organization (RMO), like the Sistema Interamericano de Metrolgía (SIM) for the Americas, when a link to a previous KCRV has to be established.

In this talk, I present the proposed statistical analysis procedure for the data of KCs to be coordinated by the Consultative Committee of Photometry and Radiometry (CCPR) of the BIPM; emphasizing those aspects on which the definition of the KCRV depends. Also, a particular example is presented for a regional KC on luminous flux in progress at the SIM, where the CENAM, the NIM of Mexico, is acting as the pilot NMI. This luminous flux SIM KC is intended to provide the link to the luminous flux CCPR KC previously performed.

ON $m_X - \Pi$GS- CLOSED SETS IN m_X-SPACES

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In the literature, the notions of g-closed set, gs-closed set, sg-closed set, π-closed set, πg-closed set, πgs-closed set, gp-closed set, πgp-closed set, and their relationships, are studied in a topological space as well as the different notions of continuous functions and irresolute functions, where they use the concepts mencioned previously. The fundamental idea of this talk is to define the above notions on a $m-$space [2], that is a set X, together with a collection of subsets of X called a $m-$structure that contain $\emptyset$ and X. Also, we look for conditions on the $m-$structure in order to generalize the well known results in this matter. Moreover, we find the existent relation between the different notions of continuity and irresoluteness. Finally, we show that the obtained results are a generalization of many of the results obtained by G. Aslim et al. in [1].

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SIMULATING SPACE-TIME WITH COMPUTERS

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Recent developments in quantum gravity and string theory have raised lot of debates about the very concept of space-time and causality at Planck scale. The length and time The very concept of space, time and causality loose their meaning below this scale. The space-time behaves discretely at Planck scale.

The metaphor that Nature behaves discretely at the Planck scale is not at all clear to 21st century scientists. One of the present authors (SR) along with Requardt described how macroscopic space-time or its underlying mesoscopic substratum emerges from a more fundamental concept, a fluctuating cellular network around the Planck scale. Henceforth, we shall call it the RR model of space-time after Requardt and Roy. It is generally believed that no physical laws that are valid in continuum space-time will be valid below or near the Planck scale. RR proposes that geometry emerges from a purely relational picture a la Leibniz. The discrete structure at the Planck scale consists of elementary nodes which interact or exchange information with each other via bonds, playing the role of irreducible elementary interactions. Here, the bonds states are dynamical degrees of freedom which, a fortiori, can be switched off or on. The wiring i.e. the pure geometry of the network, is also an emergent , dynamical property and is not given in advance. Consequently, the nodes and bonds are not arranged in any regular way e.g., a lattice, and there is no fixed near / far order. This implies geometry will become to some extent a relational (Machian) concept and is an a priori element of our formalism.

First we will model the dynamical cellular network, QX, with its cellular automaton-like dynamics, as described in RR.

We introduce an extension of the theory by interpolating one step.

The purpose of this extension is to achieve a manageable computational task.

Spatial geometry is going to evolve from the dynamics of the QX network.

For the emergence of spatial organization we use a neural network approach, based on the differences of finite sets, rather than the random metric of RR based on fuzzy sets.

Here, we have described a process called condensation which determines the instantaneous states for the macroscopic system which appears to be continuum.

Even so, the network QX , is changing rapidly by a time-discrete process. We are going to regard stepwise increasing network time as an internal process variable, microscopic time, that is distinct from the continuous physical time aspect of general relativity. Thus, we can envisage two dimensions of time.

LIFETIME IN STOCHASTIC MODELLING AND STATISTICAL PHYSICS. STATISTICAL DISTRIBUTIONS WITH LIFETIME

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Lifetime as achievement of the given level of random process has been introduced in work [1] where the equations for density of probability of a lifetime distribution have been received. In work [2] the review of results for Kramers problem, a special case of a lifetime of system in case of its transition through a potential barrier [3] is presented. Research of a lifetime is conducted in many stochastic models of the theory of random processes, for example, in stochastic storage processes [4]. Their application to physical problems is considered in [5]. In work [6] it is shown, that the nonequilibrium statistical operator introduced in works of Zubarev [7], it is possible to present as averaging of the quasi-equilibrium statistical operator on distribution of a lifetime of the system. In work [8] the statistical distributions containing a lifetime as thermodynamic parameter of system are introduced and it is shown how from these distributions it is possible to receive of superstatistics distributions [9]. We give a ground to the entered distribution which contains a lifetime of the system as thermodynamic parameter.

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COMMON FIXED POINT THEOREM IN PROBABILISTIC QUASI-METRIC SPACES

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Key Words and Phrases: probabilistic metric spaces; quasi-metric spaces, fixed point theorem, R-weakly commuting maps, triangle function
AMS Subject Classification: 54E70, 54H25

In this paper, we consider complete probabilistic quasi-metric space and prove a common fixed point theorem for R-weakly commuting maps of types (A_f) and (A_g) in this space.

HIGHER-ORDER ASYMPTOTIC DISTRIBUTIONS OF NON-DEFINITE STURM-LIOUVILLE PROBLEM

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Key Words and Phrases: turning point, Sturm-Liouville, non-definite, asyemptotic distribution, eigenvalue, higher-order

AMS Subject Classification: 34L20

In this paper, we consider the differential equation

$$y'' + (\rho^2\varphi^2(x) - q(x))y = 0$$

on the interval$[0, 1]$,where$[0, 1]$ contains m zeros of $\varphi^2(x)$, the so called turning point,ρ^2 is a real parameter and the function$q(x)$is bounded and integrable in$[0, 1]$. Using a technique used previously in [1], we derive the higher-order asymptotic distribution of the positive eigenvalues associated with this equatoin for the Dirichlet problem(i.e,$y(0) = y(1) = 0$). Note that in similar case, the leading term of the asymptotic distribution of positive and negative eigenvalues was derived previously by Atkinson,and Mingarelli [2].

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VOXELABLE GRAPH RECOGNITION IS NP-COMPLETE

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Key Words and Phrases: voxelable graphs, four regular graphs, NP-complete problems
AMS Subject Classification: 68Q15 Complexity Classes, 05C10 Topological Graph Theory

A *vox-solid* is a connected subset V of R^3 made from the union of a finite set of unitary cubes with gravity centers in Z^3 and such that the boundary of V is a surface. Vox-solids are related to the digital representation of 3-dimensional objects. The *face adjacency* graph of a vox-solid has the faces on the boundary of the vox-solid as vertices, and the edges are the pairs of faces sharing more than one point. A *voxelable graph* is a graph isomorphic to the face adjacency graph of some vox-solid. In this talk we proof that the problem of recognition of voxelable graph spherical embeddings is NP-complete by reducing 3-SAT to it.

SCATTERING AND SPECTRAL THEORY OF SOME DIFFERENTIAL OPERATORS

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Key Words and Phrases: wave operators, scattering theory, spectral theory, schrodinger operator, random potentials

AMS Subject Classification: 34L05, 47A40, 47B80

We introduce a model in perturbation theory, We consider the Schrodinger operator H with certain potentials and study the scattering theory and spectral properties of H, in particular, the existence and completeness of the wave operators $W_{\pm}(H, H_0)$ and for the coincidence of the essen-tial spectra of the operators H_0 and H and also we find conditions for the negative spectrum of the operator H to be finite.

THE DUAL PAIRING PRINCIPLE OF QUANTUM FIELD THEORY: COOPER PAIRS AND BOSE–EINSTEIN CONDENSATES

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Key Words and Phrases: quantum field theory of complex line bundles; $\mathbf{U}(1,\mathbf{C})$ gauge symmetry; Cooper pairs, Bose–Einstein condensates; atom laser; metaplectic symmetries of the symplectic spinor bundle configuration; quantum information transfer
AMS Subject Classification: 81T40, 43A80, 81T75

The non–invasive diagnostic protocols of magnetic resonance imaging have been made effective for the clinical routine by means of the time–reversing concept of fast spin echo. Because Cooper pairs in a spin singlet are formed from time–reversed quantum states, the coadjoint orbit picture of the unitary dual $\hat{N}$ of the real Heisenberg step 2 nilpotent Lie group N gives rise to the symmetry group $\mathbf{SU}(2,\mathbf{C}) \cong \mathbf{Spin}(3)$. Due to the Hopf fibration of the unit sphere $\mathbf{S}_3 \hookrightarrow \mathbf{C} \oplus \mathbf{C}$ over $\mathbf{S}_2 \hookrightarrow \mathbf{R} \oplus \mathbf{C}$, the compact Lie group $\mathbf{SU}(2,\mathbf{C})$ acts via $\mathbf{U}(1,\mathbf{C})$ gauge transformations on the complex line bundle associated with N. The metaplectic symmetries of the symplectic spinor bundle configuration, conjugated by indistinguishable pairs of contragredient flat leaves occurring in the foliation of $\hat{N}$, permit a natural approach to the quantum information transfer within pairs of Bose–Einstein condensates. The phase factor $\mathbf{U}(1,\mathbf{C}) \cong \mathbf{S}_1$ of the Hopf bundle controls the quantum information retrieval performed by an application of the symbolic calculus of pseudodifferential operators on the symplectic plane $W \cong N/\text{center}$. Equivalently, the Weyl calculus of phase–coherent states applies, because it admits a natural interpretation in terms of the coadjoint orbit picture of the equivalence classes of irreducible unitary linear representations of nilpotent Lie groups of which the central extension $N = \text{Heis}(W) \cong W \oplus \mathbf{R}$ is one copy.

Numerical Treatment OF FRACTIONAL DIFFERENTIAL EQUATIONS BASED ON THE GRÜNWALD-LETNIKOV APPROACH

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Key Words and Phrases: fractional derivatives, fractional differential equations, Grünwald-Letnikov approach, difference methods, numerical schemes, stability properties, error estimates
AMS Subject Classification: 65M06, 65M12, 26A33, 35K05

The subject of fractional calculus and its applications has gained considerable popularity and importance during the past three decades, mainly due to its attractive applications in numerous seemingly diverse and widespread fields of science and engineering. Fractional differential equations have been used for the mathematical modelling in potential fields, diffusion problems, waves in liquids and gases, in heat equations and in Maxwell's equations. The main advantage of the fractional calculus is that fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes.

This paper is devoted to the numerical treatment of some fractional extensions of ordinary and partial differential equations. Based on Grünwald-Letnikov's approach to a fractional derivative, finite difference schemes for the approximation of the solution are discussed and difference schemes are derived. The main properties of the explicit and implicit numerical methods developed, related to stability, convergence and error behaviour are also studied. Stability conditions as extensions of the CLF condition are derived and numerical experiments are provided.

TRANSFORMS, FILTERS AND EDGE DETECTORS IN IMAGE PROCESSING

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This is an expansion of my previous presentation in our *Third International Conference of Applied Mathematics and Computing* in 2006 in Plovdiv. Again, filter design is an integral component to enhance desired aspects within an image such as clarity of edges. This paper provides an introduction to filter design and image edge detection using matrices, partial derivatives, convolutions and frequencies that enter into identifying components in the problem. We are especially addressing the notion of edge detection, which has far reaching applications in all areas of research, including medical research. For example a patient can be diagnosed as having an aneurysm by studying an angiogram. An angiogram is the visual view of the blood vessels whereby the edges are highlighted. This process is completed through convolution, filters and special frequency techniques using the software, MATLAB 7.2 (2006). Some illustrations included will be vertical, horizontal and Sobel Edge Detectors together with some wavelet transforms to locate the edges in an image. We also include the Fast Fourier Transform to obtain the frequency space together with filtering such as the use of the Butterworth Filter.

SOME BOUNDARY VALUE PROBLEMS FOR DEGENERATING HYPERBOLIC EQUATIONS IN $\mathbb{R}^3$

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Consider
$$\begin{aligned} L_m u : &= u_{x_1x_1} + u_{x_2x_2} - (x_3^m u_{x_3})_{x_3} \\ &= u^1_{\varrho\varrho} + \frac{1}{\varrho}\, u^1_{\varrho} + \frac{1}{\varrho}\, 2u^1_{\varphi\varphi} - (x_3^m u_{x_3}) x_3 = 0 \end{aligned}$$
$0 \le m < 2$ in
$$G := \{\varrho, \varphi, x_3 : x_3 > 0,\ 0 \le \varphi < 2\pi\,;\ \frac{2}{2-m}\, x_3^{\frac{2-m}{2}} < \varrho < 1 - \frac{2}{2-m}\, x_3^{\frac{2-m}{2}}\}\ .$$
G is bounded by the characteristic surfaces
$$\sum_1 := \{\varrho, \varphi, x_3 : 0 \le \varphi < 2\pi,\ \varrho = 1 - \frac{2}{2-m}\, x_3^{\frac{2-m}{2}}\}$$
$$\sum_2 := \{\varrho, \varphi, x_3 : 0 \le \varphi < 2\pi,\ \varrho = \frac{2}{2-m}\, x_3^{\frac{2-m}{2}}$$
and the sphere
$$\sum_0 : \{\varrho, \varphi, x_3 : x_3 = 0,\ \varrho < 1\}\ .$$
For $m > 0$, we have an equation of mixed type of second kind (or Keldysh type).

It is shown that there exist an infinite number of nontrivial solutions of
$$L_m u = 0 \quad \text{with} \quad P1 : u|_{\sum_0 \cup \sum_2} = 0 \quad \text{and} \quad P2 : u_{x_3}|_{\sum_0} = 0\,, u|_{\sum_2} = 0\ .$$
We then show for example that equation
$$L_m u + r(x)u = f\ ,\ 0 < m < 2\ ,\ r \in C^1(G)$$
has at most one quasi–regular solution satisfying the condition $u|_{\sum_1} = 0$ if $2r + \text{grad}\ r \cdot \bar{\alpha} > 0$ in G where $\vec{\alpha}^1 = (x_1, x_2, \frac{2}{2-m}\, x_3)$.

THREE ALGEBRAS RELATED TO RANDOM GRAPHS AND QUANTUM PROBABILITY

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Key Words and Phrases: random graphs, cycles, fermions, quantum random variables
AMS Subject Classification: 15A33, 05A15, 05C50, 05C80

Consider a real 2^n-dimensional commutative algebra $\mathcal{A}$ generated by a collection $\{x_i\}$, $(1 \leq i \leq n)$, along with the unit scalar $x_\emptyset = 1 \in \mathbb{R}$. Commutativity ensures $x_i x_j = x_j x_i$ for all $1 \leq i, j \leq n$. Three squaring rules leading to cases of particular interest are

$$
{x_i}^2 = \begin{cases} 0 & x_i \text{ is nilpotent,} \\ 1 & x_i \text{ is unipotent,} \\ x_i & x_i \text{ is idempotent.} \end{cases} \tag{1}
$$

The three cases generate algebras denoted $\mathcal{N}$, $\mathcal{U}$, and $\mathcal{I}$, respectively. Given a finite graph G, constructing the associated adjacency matrix using generators of $\mathcal{N}$, $\mathcal{U}$, or $\mathcal{I}$ results in quantum random variables whose m^{th} moments reveal information about m-paths, m-circuits, and m-cycles within the graph.

Applications of these algebras to problems in graph theory, enumerative combinatorics, and computer science will be discussed. In addition, realizations of all three algebras within fermion creator/annihilator algebras will be discussed.

AN APPROACH TO FUZZY LOGIC AND DEFAULT REASONING VIA CONDITIONAL PROBABILITY

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Key Words and Phrases: conditional probability, fuzzy logic, default reasoning
AMS Subject Classification: 60A05, 60A99, 03E72, 03B99

What is usually emphasized in the literature – when a conditional probability $P(E|H)$ is taken into account – is only the fact that $P(\cdot|H)$ *is a probability for any given* H: this is a very restrictive (and misleading) view of conditional probability, corresponding trivially to just a modification of the "world" Ω. It is instead essential to regard the conditioning event H as a "variable", *i.e.* the "status" of H in $E|H$ is not just that of something representing a given *fact*, but that of an (uncertain) *event* (like E) for which the knowledge of its truth value is not required. In this way we get, through a *direct* assignment of conditional probability, a general theory of probabilistic reasoning able to encompass other approaches to uncertain reasoning. A coherent conditional probability is looked on as a general non-additive "uncertainty" measure $m(\cdot) = P(E|\cdot)$ of the conditioning events.

This gives rise to a clear, precise and rigorous mathematical frame, which allows to define – among other things – fuzzy subsets and to introduce in a very natural way the counterparts of the basic continuous T-norms and the corresponding dual T-conorms, bound to the former by *coherence*.

Moreover, a suitable interpretation of the extreme values 0 and 1 of $P(E|H)$ for situations which are different, respectively, from the trivial ones $E \wedge H = \emptyset$ and $H \subseteq E$, leads to a "natural" treatment of the *default reasoning*. As it is well-known, a default rule is a sort of "weak implication". Given a *coherent* conditional probability P on a family $\mathcal{C}$ of conditional events, a *default rule*, denoted by $H \longmapsto E$, is defined as any conditional event $E|H \in \mathcal{C}$ such that $P(E|H) = 1$ (clearly, any logical implication $A \subseteq B$ between events can be seen as a trivial default rule). We consider also the problem of "new" default rules that are entailed by a given set of conditional events of probability 1, and we discuss the relevant inferential processes.

SOME EINSTEIN METRICS ON ORTHONORMAL FRAME BUNDLES

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We study the geometry of orthonormal frame bundles OM over Riemannian manifolds (M, g). The former are equipped with some modifications $\mathbf{tg}_c$ of the Sasaki-Mok metric $\mathbf{tg}$ depending on one real parameter $c \neq 0$. The metrics $\mathbf{tg}_c$ are "strongly invariant" in some special sense. In particular, we consider the case when (M, g) is a space of constant sectional curvature K. Then, for $\dim M > 2$, we find always, among the metrics $\mathbf{tg}_c$, two strongly invariant Einstein metrics on OM which are Riemannian for $K > 0$ and pseudo-Riemannian for $K < 0$. At least one of them is not locally symmetric.

KALMAN FILTER BASED SPEECH-LIKE SIGNAL DETECTION WITHIN A NOISY ENVIRONMENT

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Key Words and Phrases: quasi-periodic sound signals, signal segmentation, change point detection

Globally, communication systems play an increasingly important role in all fields. In aeronautics, the necessity to maintain a high level of flight safety has heightened the need for reliable Human-Computer Dialogue Systems, HCDS, able to recognize some standardized speech commands within a very noisy pilot cabin. In the development of such HCDSs, a primary problem can be stated as segmenting the acoustic signal into speech and speech-free portions.

In this paper we propose a solution based on only one Kalman filter. We start from the known Habibi's autoregressive model of images, then we convert it into a spiral-like grid set on a cylinder surface and use so constructed model to describe quasi-periodic sound signals that are close in sounding to speech or music tones. This method enables us to get a workable solution to the stated problem.

ON SOME INVERSE PROBLEM LEADING TO A SECOND-ORDER LINEAR FUNCTIONALS

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Key Words and Phrases: orthogonal polynomials, Rodrigues' formula, recurrence relations, semi-classical linear functionals, second-order linear functionals, Laguerre polynomials, integral representations
AMS Subject Classification: 33C45, 42C05

A linear functional $\mathfrak{L}$ is called positive-definite, if and only if $\langle \mathfrak{L}, p^2 \rangle > 0$, for all non-zero polynomial with real coefficients p. On certain regularity condition, it is well-known that the product of a positive-definite linear functional by a polynomial is still a positive-definite linear functional. This tool was used by Christoffel in 1858 and is considered a famous construction process. In this paper, we provide another construction process of a positive-definite linear functional from a positive-definite linear functional data. Indeed, for any $\epsilon \in \mathbb{R}^*$ and any positive-definite linear functional $\mathfrak{L}$, we show that the linear functional $\mathfrak{L}_\epsilon$ satisfying $\mathfrak{L}_\epsilon - \epsilon \mathfrak{L}_\epsilon^{'} = \mathfrak{L}$ is also positive-definite. This process allows us to construct a second-order positive definite linear functionals from semiclassical positive-definite linear functionals. However, we apply the above result to an example where we establish the Rodrigues' formula.

A QUADRATURE FREE CONVERGENT METHOD FOR THE NUMERICAL SOLUTION OF LINEAR FREDHOLM INTEGRAL EQUATIONS BASED ON HERMIT-SPLINE INTERPOLATION

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AMS Subject Classification: Hermit-spline interpolation, Ferdholm integral equation

In this paper we use the Hermit-spline interpolation in a special form for the numerical solution of linear Fredholm integral equations.We prove the convergence of this method as a main part of the paper and give an error bound for the error controlling of numerical results. Finally some numerical results are given to certify convergence and error bound of the method.

FUZZY EQUIVALENCE RELATIONS AND PARTITIONS

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The paper presents an introduction to the theory of fuzzy equivalence relations and fuzzy partitions, we give a new definition of fuzzy equivalence relation to construct a new fuzzy partition and we prove some theorems related these concepts.

INTUITIONISTIC FUZZY METRIC SPACES AND ITS APPLICATIONS

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Key Words and Phrases: intuitionistic fuzzy metric spaces, coincidence point, common fixed point, triangular norms, triangular conorms weakly compatible maps
AMS Subject Classification: 54A40, 54H25

We will present properties and examples of fuzzy metric spaces and multivalued functions defined by I. Kubiaczyk and S. Sharma [Common fixed point in fuzzy metric space, J. Fuzzy Math. Vol.11, No.1(2003),1-5].

Recently, J.H. Park [Intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals 22(2004),1039-1046] introduced the intuitionistic fuzzy metric space using the concept of intuitionistic fuzzy sets, introduced and studied by K. Atanassov [Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986), 87-96] . We will present properties ,examples and multivalued mappings in intuitionistic fuzzy metric spaces.

A number of observations motivated us to prove common fixed point theorems for finite non-compatible, discontinuous mappings in non-complete intuitionistic fuzzy metric spaces.

We will give examples to validate our results. To prove existence of common fixed point for finite number of mappings some commutativity conditions are required. How many commutativity conditions are necessary? We will give answer of this question by giving formulas.

We will discuss some recent results of workers of this line and some of our new results will be presented in the talk. We will discuss uses and applications of intuitionistic fuzzy metric spaces and fuzzy metric spaces.

FIXED POINT, COINCIDENCE POINT IN FUZZY METRIC SPACES AND INTUITIONISTIC FUZZY METRIC SPACES AND APPLICATIONS

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The notion of fuzzy sets was introduced by Zadeh in 1965. As a generalization of fuzzy sets introduced by Zadeh , Atanassov introduced the concept of intuitionistic fuzzy sets. Recently , using the idea of intuitionistic fuzzy sets, Park introduced the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric spaces due to George and Veeramani . Jungck and Rhoades gave more generalized concept weak compatibility then compatibility. Recently, many authors have studied fixed point theory in intuitionistic fuzzy metric spaces .

In this lecture we will discuss fixed point and coincidence point in fuzzy metric space and intuitionistic fuzzy metric space. We will point out that the continuity and compatibility for the existence of fixed point is not required.

We will give some applications of fuzzy metric space and intuitionistic fuzzy metric space.

UNIVERSAL FUZZY PARTITIONS

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In this talk, we study fuzzy partition on $[a, b]$. We introduced new type of fuzzy partition, namely universal fuzzy partition. We study the properties of our definition, as well as, we characterize the universal fuzzy partition in many way.

A TOPOLOGICAL CODE FOR PLANE IMAGES

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A topological code for contours of black-white plane images will be presented. This code was successfully applied for optical character recognition of printed and handwritten characters. One can apply it to recognition and compressing of any visual images.

ELIMINATION APPROACHES IN DISCRETE OPTIMIZATION

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Key Words and Phrases: discrete optimization, decomposition, elimination, nonserial dynamic programming, tree decomposition
AMS Subject Classification: 90C10, 90C39, 90C06

Discrete optimization problems (DOPs) arise in various applications such as planning, scheduling, computer aided design (CAD), robotics and artificial intelligence. Application areas include also supply chain design and management, telecommunications, manufacturing, transportation, scheduling, and finance.

Solving discrete optimization problems (DOP) can be a rather hard task. Many real discrete optimization (DO) problems contain a huge number of variables and/or constraints that make the models intractable for currently available solvers. There are few approaches for solving DOPs: tree search approaches (e.g., branch and bound), relaxation and decomposition methods. One of the promising ways to exploit sparsity in the interaction graph of an DO problem is nonserial dynamic programming (NSDP) (Bertele & Brioschi, Hooker), which allows to compute a solution in stages such that each of them uses results from previous stages.

Classification of various DP formulations on the basis of the dependencies between subproblems of a directed acyclic graph (DAG) of DP computational procedure is described.

Recursive procedure for nonserial DOP and nonserial DOPs with constraints are considered. Elimination game and elimination process in NSDP are described.

General elimination scheme in DO and its different realizations such as the elimination of variables one by one, block elimination scheme, bucket elimination scheme and tree decomposition scheme are analyzed.

DISCOVERY THROUGH NUMERICAL SIMULATION: EXAMPLES OF INELASTIC SYSTEM

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Key Words and Phrases: inelastic collapse, ringlet formation, computer simulation
AMS Subject Classification: 82-02 Research exposition

Computer simulation can discover new physical phenomena as well as reproduce and analyze them. Though elastic collision increase a system's entropy and disturb the order, inelastic collision dissipates the internal energy to environment as heat. As a result, inelastic system tends to make order automatically. The phenomenon known as "inelastic collapse" is the prototypical example. This tendency is found by computer simulation in late 1980s, through our study for planetary ringlet formation.

Assume some particles stand in a line in one-dimensional space, and a particle are driving into the line. No boundary nor external field. If the particles are elastic, all particle are scattered at last. But if they are inelastic, and the nuber of particles are more than a threshold which is determined by the restitition of coefficient, vast majority particle cannot escape and collapse into a few clusters in which both the relative velocities and distance between particle is becoming into infinitecimal. If the particles are completely inelastic, the expected number of final cluster made of n particles is $1/n$ which was proved mathmatically after the numerical discovery.

Such clusters are also found in two- and three- space dimension with and without boundary or external field. Planetary ringlets is an application of "inelastic collapse" and formed in computers. They might be formed by themself even with no shepherding satellites nor gravitational resonance.

OPTIMIZATION TECHNIQUES IN INTERPOLATION

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Key Words and Phrases: interpolation, genetic algorithms, ant colony model, optimization

The aim of this paper is to present many optimization technique on interpolation using temporal dependent conditions. In the first part of the article we introduce and analyze some interpolation schemes with temporal dependent conditions. Two dimensional spatial and one dimensional temporal space were considered to compose a 3D spatial-temporal space for a pixels based interpolation problem and an one dimensional spatial and one dimensional temporal space was considered to compose a 2D spatial-temporal space for an interpolation problem in a network of learning objects, from a virtual learning environment, having different weights in function of their relevance to a specific educational goal. For these problems we use a genetic algorithm based technique and an ant colony model based technique. The changes of spatial coordinates in time, requires a class variable interpolation and lead us to a classification problem.

ON LAMBDA APPROXIMATION FOR ANALYTIC FUNCTIONS

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We introduce a notion of best λ-approximation for analytic functions on the unit disk. Let $f(z) := \sum_{i=0}^{\infty} a_i z^i$, $|z| < 1$, be an analytic function and $S_n(z) := \sum_{i \le n} a_i z^i$ its partial sums. Define also the maximum modulus $M_f(r) := \max_{|z|=r} |f(z)| = |f(re^{i\phi_0})| = |f(z_0)|$; it increases with r and we suppose that $M_f(r) \to \infty \ \ (r \to 1^-)$. The comparison between $f(z)$ and its partial sums is a subject of many classical theorems. We shall find here the "shortest" partial sum which is well approximating $f(z)$ at the points of maximal growth in the following sense: we determine an integer-valued, monotone increasing with λ, function $n := n(r, \lambda) \to \infty \ \ (r \to 1^-)$, in such a way that for $\lambda > 1$ the partial sums $S_{n(r,\lambda)}(z_0)$ are well approximating $f(z_0)$, but for $0 < \lambda < 1$ it is not the case. We call such partial sums $S_{n(r,\lambda)}(z_0)$ the best λ-approximating sums (BLAS). Analogously to Valiron's proximate order in the theory of entire functions, we apply Karamata's class of regularly varying functions for measuring the growth of a given analytic function on the unit disk. A solution of the BLAS problem for a class of functions of rapid growth inside unit disk is given by the following:

Theorem 1. *If*

$$\log M_f(r) \sim \left(\frac{1}{1-r}\right)^{\rho} \ell\,(r)\,, \rho > 0 \qquad (r \to 1^-),$$

where $\ell\,(\cdot)$ is a slowly varying function in Karamata's sense, then

$$n(r, \lambda) \sim \frac{C_\rho\,(\lambda)}{1-r} \log M_f\,(r) \qquad (r \to 1^-),$$

where

$$C_\rho\,(\lambda) = (\rho\lambda^\rho, \rho > 1; \lambda^2, \rho = 1; \rho\lambda, 0 < \rho < 1).$$

Generating function for Hardy-Ramanujan partition problem is a good example for the above theorem.

A FAMILY OF FINITE-DIFFERENCE SCHEMES FOR SHALLOW-WATER EQUATIONS CONSERVING THE MASS AND TOTAL ENERGY

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It is considered the classical nonlinear shallow-water model (SWM) of an ideal fluid. It is well known that the model conserves several integral characteristics such as the mass, total energy and potential enstrophy. It is extremely desirable to conserve the same characteristics in a fully discrete SWM (discrete both in space and time), because they guarantee the stability of calculations and correct description of the energy cascades in the discrete model. However, the full discretization usually destroys some, if not all, of the conservation laws, and therefore, the construction of conservative fully discrete SWMs is a non-trivial and actual scientific problem.

For the last forty years there have been suggested various semi-discrete SWMs (discrete in space, but still continuous in time), which conserve one or all of the three above-mentioned integral characteristics. In particular, the model by Ringler and Randall (2002) uses rather complicated geodesic grids on a sphere, while that by Salmon (2004) applies a sophisticated stencil (containing 25 nodes) in a doubly periodic domain on the f-plane. Nevertheless, explicit time discretization used in both works resulted in the loss of all the conservation laws except the mass conservation.

In this work, new fully discrete SWMs are suggested, which exactly conserve the mass and total energy. The splitting of the SWM operator in geometric coordinates provides substantial benefits in the computational cost of the solution, as well as in the applicability to a doubly periodic domain on the plane, in a periodic channel on a rotating sphere, and on the whole sphere. Each split one-dimensional fully discrete system conserves the mass and total energy, too. In fact, a family of finite-difference schemes of different approximation order is suggested, either linear or nonlinear, depending on the choice of certain parameters. Note that on a sphere and in a doubly periodic domain, our approach allows constructing various linear conservative schemes of arbitrary approximation order in space. Results of numerical experiments are discussed.

MULTIVARIATE WAVELET FRAMES

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Key Words and Phrases: tight frames, compactly supported wavelets, multiresolution analysis, approximation order
AMS Subject Classification: 42C40

Tight wavelet frames are discussed. A system $\{f_n\}$ is said to be a tight frame for a Hilbert space H if any $f \in H$ may be decomposed as follows $f = \sum\limits_n \langle f, f_n \rangle f_n$. Wavelet tight frame is a tight frame for $L_2(\mathbb{R}^d)$ consisting of functions $\psi^{(\nu)}(M^j \cdot + n)$, $n \in \mathbb{Z}^d$, $j \in \mathbb{Z}$, $\nu = 1, \dots, r$, where $\psi^{(\nu)} \in L_2(\mathbb{R}^d)$, M is a matrix dilation, i.e. a $d \times d$ integer matrix whose eigenvalues are strictly bigger than 1 in module. We proved that for any matrix dilation M and for any positive integer n there exists a tight wavelet frame generated by compactly supported wavelet functions $\psi^{(\nu)}$ and providing approximation order n.

CONSTRUCTING A RETRACTION IN CONVEX MULTI-OBJECTIVE OPTIMIZATION

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Key Words and Phrases: Pareto-optimal, homeomorphic, fixed point, retraction, contractible
AMS Subject Classification: 90C29

In this paper we consider the Pareto-optimal solutions in convex multi-objective optimization with compact feasible domain. One of the most important problems in multi-objective optimization is the investigation of the geometrical and topological structure of the Pareto-optimal set. We present the problem of construction of a retraction function of the feasible domain onto Pareto-optimal set, if the objective functions are concave and one of them is strictly quasi-concave on convex feasible domain. Using this result it is also shown that the Pareto-optimal and Pareto-front sets are homeomorphic and contractible, and they have the fixed point property.

THE APPLICATION OF H-ADAPTIVE FINITE ELEMENT METHOD FOR COUPLED MAGNETO-THERMO-MECHANICAL PROBLEMS WITH STRONG SINGULARITIES

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Key Words and Phrases: Strong singularities, Galerkin procedure, triangular finite elements, adaptive h-refinement procedure
AMS Subject Classification: 74S05, 65M60, 74F15

This paper deals with the numerical simulation of coupled magneto-thermo-mechanical problems with strong singularities. These problems involve modelling compressive and shear stress deformation created by thermal expansion. The thermal field is created by passing an electric current throw electrically conductive concentrated layer. This layer covers very small part of the computation domain and leads to strong singularities. The method of weighed residuals (the Galerkin procedure) is used for week form formulation of appropriate differential equations. The discretization procedure employs linear and quadratic polynomial approximations on triangular finite elements. Special adaptive h-refinement procedure with aposteriori error estimation is applied to the computation domain with strong singularities. A number of reported numerical results show efficiency of proposed numerical technique. As an example, numerical results that represent the coupled magneto-thermo-mechanical processes in electrically conductive solids caused by concentrated sources of current are reported.

INTEGRAL IDENTITIES AND SOME ESTIMATE INVOLVING SUMS OF RECIPROCALS OF SINGLE AND DOUBLE BINOMIAL COEFFICIENTS

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Key Words and Phrases: single and double binomial coefficients, combinatorial identities, integral representations
AMS Subject Classification: 11B65, 05A10, 33C20

Abstract: In this paper we will establish some identities involving sums of reciprocals of single binomial coefficients. We will subsume and extend some results obtained by Sury Wang and Zhao; furthermore we shall give bounds on some resulting series which depend on several parameters. We then extend the results to obtain identities for sums of reciprocals of double binomial coefficients.

POINT ESTIMATIONS IN ECOLOGICAL, ENVIRONMENTAL AND HUMAN STATISTICS WITH THE AID OF DEVELOPMENT OF TCHEBYCHEFF'S AND KOLMOGOROV'S IDEAS

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We must estimate a probable state for the environmental and human system under different initial conditions. Thus, there is a real problem with calculations under the case of unknown environmental, ecological and human distribution functions. The general property of the environmental distribution function has been found, with the aid of two new generalized inequalities. These inequalities have been stated as a result of generalization of Kolmogorov's and Tchebysheff's (Chebyshev's) inequalities. A new algorithm for the estimation of parameters of processes in environmental systems under the condition of unknown environmental distribution functions has been de- signed with the aid of this generalization for Environmental, Ecological and human Statistics.

ON PERTURBATION OF NONLINEAR MAPPINGS AND THEOREMS OF SOLVABILITY

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AMS Subject Classification: 47H15, 46E35, 35G15

In this article we will consider mappings acting on Banach spaces, and investigate the image of a mapping which is some perturbation of certain continuous mapping under various conditions. In particular, we investigate solvability of some class of equations and inclusions. Questions of such type are investigated under the different conditions (see, for example, references of [1, 2]).

Here for investigation of the considered mapping we will conduct of local comparing between considered mapping with defined mappings which are sufficiently smooth. The obtained here results can be applied to study different mixed problems for differential equations. In particular, we study such mappings which are founded between two monotone mappings in some sense and also some modification of Navier-Stokes equation. For investigation of the considered problem we will use such perturbation method which is based to results of [1, 2].

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NUMERICAL SIMULATION OF THE DYNAMICAL PROPERTIES OF THE HUMAN TYMPANUM

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A numerical simulation of the dynamical properties of the tympanic membrane is presented. A simple and different simulation of the vibratory patterns of the coupled system tympanum–malleus have been assessed by proposing the modeling of the tympanum through the vibrations of a forced elastic membrane, whereas the effect of the manubrium is introduced through a forced semi–membrane. We propose the superposition of these waveforms as a model for describing the vibrations of the coupled system tympanum–malleus. Both waveforms have analytical representations leading to simple computations. The results of the simulation for the vibrational mode (1,1) show an amplitude for the membrane larger than those for the handle of malleus. The maximum amplitude obtained was around 1 μm, at a test frequency of 2 kHz. Also, level curves corresponding to the simulated vibrational modes were obtained. The numerical model presented can be easily handled to change input parameters, such ! as sound pressure and frequency. Also, other situations such as the conical shape of the tympanum or some asymmetries could be considered.

ON THE STRUCTURE OF VARIABLE METRIC UPDATES

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Key Words and Phrases: unconstrained nonlinear optimization, variable metric updates, quasi-Newton methods, affine projections

AMS Subject Classification: 90C53, 15A99

The aim of the paper is to present the structure of the exisiting updates of the inverse Hessian approximations in variable metric methods for unconstrained nonlinear optimization. Those approximations should be symmetric and strictly positive definite and should verify the so-called quasi-Newton condition

$$\bar{H}r = s$$

where s – is the difference between two consecutive solution approximations and r – the corresponding difference between the derivatives at those points.

It is shown in the paper that the famous BFGS and DFP variable metrics are the sums of two parts: the projection part setting to zero vector components parallel to r and the second part – the same in all existing updates, yielding the quasi-Newton condition. The nprojection part is different in BFGS and DFP updates.

In the paper the possibility of using other affine projections than those used in the BFGS and DFP methods is shown. A new class of updates ensuring inheritance of the conjugacy property (when applied to a strictly convex QP problem with exact directional minimization) is introduced. It's properties are analysed and some preliminary computational results are reported.

EXISTENCE AND GLOBAL ASYMPTOTIC STABILITY OF POSITIVE PERIODIC SOLUTIONS OF N-SPECIES DELAY IMPULSIVE LOTKA-VOLTERRA TYPE SYSTEMS

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In this paper, the existence and global asymptotic stability of positive periodic solutions of periodic $n-$ species Lotka-Volterra impulsive systems with several deviating arguments are studied. By using the continuation theorem of coincidence degree theory and Lyapunov -Razumikhin method sufficient conditions are obtained. Some known results are improved and generalized.

ALMOST PERIODIC PROCESSES IN ECOLOGICAL SYSTEMS WITH IMPULSIVE PERTURBATIONS

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In the present paper we investigate the existence of almost periodic solutions of an nonautonomous N-dimensional impulsive Lotka Volterra competitive system with dispersion and fixed moments of impulsive perturbations. By means of piecewise continuous functions which are modifications of classical Lyapunov's functions we give new sufficient conditions for the global exponential stability of the unique positive almost periodic solutions of the system.

SOFTWARE MODELING FOR SUPPLY NETWORK SIMULATION

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Key Words and Phrases: supply network, modeling, simulation

Supply chains fall among the most complex business systems. More often the term supply network is used, which should reflect the need for design, analysis, and implementation of these distributed systems composed of many locations (nodes) among which there are multitude relationships regarding the material and information exchange. The most common approach that is applied is the simulation of the supply network as a discrete stochastic system. Simulation model, besides general requirements, should enable model construction of any complexity regarding the number of locations, products, and connections. Also, it should provide implementation of the processes at different locations and with desired level of detail, both from the theory and practice. The technology, organization, constrains and resources need to be taken into account as well. In other words, the simulation model should be as close to a real system as possible. The simulation model, described in this paper, is based on the specific production expert system, the model database, and the database. The expert system is consisted of the knowledge base and the conclusion engine in which the process interaction is managed using the three-phase rule. Model database store data about simulation models such as: Model structure with all the locations and their characteristics relationships. Process library, which includes process characteristics and different implementations. Best practice, applied technology, organization, and constrains. The database represents the current state of the systems components, and it is based on the essential system characteristics that should enable flexibility of the system. In order to model the supply network with such complex structures, information flows, dynamics, uncertainty and demands, and in which space and time dimensions are expressed, the new system modeling and software design methods and tools are required. In this paper, the methodology for supply network modeling and simulation software design is presented. It uses the Unified Modeling Language (UML) notation combined with the object- oriented approach. Also, the experimental results derived from the original simulation software are showed.

ILLUSIONARY MATHEMATICS

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Key Words and Phrases: illusion, magic, algebra, counting
AMS Subject Classification: 11Z05, 20D99

All of us have seen some of the mathematical illusions making the rounds on the internet with the query: "How does this work?" In this talk, we discuss several of these, look at the mathematics behind the phenomenon, and sometimes generalize the scope of the original presentation. We will discuss some of the "tricks" presented over the telephone or by TV magicians in which the trick is performed on live TV but the mark or scapegoat is at home, in many different homes across TV land. We will also look at some card tricks which are really based in mathematics, some of which is relatively trivial and on up to some of the more complex connections to mathematics. This is intended to be a light-hearted fun presentation.

INFINITE DIMENSIONAL LIE ALGEBRAS AND HIROTA BILINEAR DIFFERENTIAL EQUATIONS

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Key Words and Phrases: affine Lie algebras, vertex operators, Hirota bilinear differential equations, Solitons
AMS Subject Classification: 17B67, 58F07

A connection between soliton theory and classical affine Lie algebras was developed by Date, Jimbo, Kashiwara and Miwa using Boson - Fermion correspondence in 2 dimensional Q.F.T in [1]. Homogeneous vertex operators and the corresponding Hirota bilinear equations were constructed in [3] for $B_2^{(1)}$,using the construction of homogeneous vertex operators for $B_l^{(1)}$ in [2]. In [4], principal vertex operators for $B_3^{(1)}$, considering it as a subalgebra of $D_4^{(1)}$, were explicitly written and the corresponding hierarchy of super Hirota bilinear equations were constructed. The aim of this talk is to describe the connection between vertex operator representation of affine Kac-Moody Lie algebras and Hirota bilinear differential equations.Moreover, the construction of principal vertex operators for $A_3^{(1)}$ will be shown and the corresponding soliton equations will be derived.

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MODEL FORMULATION AND INTERPRETATION FOR CHEMICAL REACTIONS MECHANISMS — FROM EXPERIMENT TO THEORY

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Key Words and Phrases: mathematical modeling, chemical reaction mechanism, model assessment

AMS Subject Classification: 03H05 (Nonstandard models in mathematics), 60K35 (Interacting random processes; statistical mechanics type models; percolation theory), 65C60 (Computational problems in statistics), 68Q17 (Computational difficulty of problems), 93A30 (Mathematical modeling)

From kinetics study of a given pair of reactants different types of mechanisms were investigated. A mathematical model was formulated. Model parameters were evaluated and assessed. Obtained results from the optimization procedure opened an interesting discussion about the limits of parameters obtained from experiments data for imposed conditions, such as mechanism type and collecting procedure.

By using of a least squares method, obtained models as best fits correlates with experimental measurements; the results shown an average of 96.6% for a sample size average of 2967 pairs of data.

Comparing the results obtained by different experiments, all obtained parameters, which were not related with the experimental conditions, were in same range of 95% confidence interval. These results validates experimental data and as well as data obtained by model.

FRAME OPERATOR IN THE BANACH CASE

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Key Words and Phrases: X_d-frame, frame operator

As it is well known, frames for Hilbert spaces play an increasing role in Pure and Applied Mathematics. During the last years, some generalizations of frames to Banach spaces were introduced and became topic for investigation (Banach frames, X_d-frames).

The present talk concerns generalization of the concept *Hilbert frame operator* and some of its properties to Banach spaces.

A DISTRIBUTION THEORY

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Key Words and Phrases: generating system, statistical estimation, differential equation
AMS Subject Classification: probability

The continuous improvement in information systems and data collection techniques allows the automatic generation of sizeable data-based collection, calling for sophisticated statistical investigations, aimed both at initially uncovering structure and proposing models in different application contexts. It is of particular interest to have, apart from classical generating systems (Pearson, 1895; Burr, 1942; Johson, 1949; D'Addario, 1949 and Dagum, 1980) a classification system that yields, starting from a few basic principles, a framework for models that could be useful, not only for the main estimators, but also for the modelling of size phenomena (income, assets, actuarial loss, etc.).

A systematic development of a distribution theory (Kotz and Kleiber, 2003, p. 53) presents a compendium that gives a summary of the properties and uses of old and new distributions at present time. A huge variety of size continous univariate distributions supported on positive half-line, including the well-known models, have been placed in an order that develops in a very natural way. In order to appreciate the usefulness of that compendium, this note shows the versatility of the above approach with examples and alternative models for providing models to a large variety of observed distributions.

Moreover, the aim of the present lecture is to present and study some new distributions with a view to properties and characterizations.

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PROJECTION–NET WEAK METRIC ERROR ESTIMATES

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Key Words and Phrases: projection–net methods, bilinear form, Hilbert spaces, error estimates
AMS Subject Classification: 65J99

Let V be Hilbert space and let $a(\cdot,\cdot)$ be a bilinear form on $V \times V$ such that

$$|a(v_1, v_2)| \leq B\|v_1\|_V \|v_2\|_V, \quad a(v_1, v_1)| \geq \delta \|v_1\|_V^2$$

for all $v_1, v_2 \in V$. For every $f \in V^*$ let $u \in V$ and $w \in V$ be solutions of the following dual problems:

$$a(u, v) = f(v) \ \text{ for all } \ v \in V, \tag{1}$$

$$a(v, w) = f(v) \ \text{ for all } \ v \in V. \tag{2}$$

Let $\{V^h\}$ be a family of subspaces $V^h \subset V$ and let $u^h \in V^h$ be a solution of the following problem:

$$a(u^h, v^h) = f(v^h) \ \text{ for all } \ v^h \in V^h. \tag{3}$$

Let W and H be Hilbert spaces such that the following conditions are satisfied:

(i) for every $v \in V \bigcap W$ there exists $v^h \in V^h$ such that

$$\|v - v^h\|_V \leq \alpha(h)\|v\|_W,$$

where $\alpha(h)$ does not depend on v;

(ii) for all $f \in H^*$ problems (1) and (2) are solved in $W \bigcap V$ and

$$\|u\|_W \leq D\|f\|_{H^*}, \quad \|w\|_W \leq D\|f\|_{H^*}.$$

Then the following estimate of difference between solutions of problems (1) and (3) is valid:

$$\|u - u^h\|_H \leq B^2 D^2 \delta^{-1} [\alpha(h)]^2 \|f\|_{H^*}.$$

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WEIGHTED NORM BOUNDS FOR A LOCAL HÖLDER NORM OF ELLIPTIC AND OF PARABOLIC FUNCTIONS ON A NON-SMOOTH DOMAIN IN EUCLIDEAN SPACE

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Key Words and Phrases: elliptic and parabolic equations, Lipschitz domains, Borel measures, Green's functions, semi-discrete Littlewood-Paley type inequalities

AMS Subject Classification: 35J25, 35J15, 42B25

Knowing rate of change of a temperature function or of a potential function in a limited environment is of fundamental importance in applications of mathematics. For solutions to the heat equation and for harmonic functions in the upper half space, Wheeden and Wilson proved necessary and sufficient conditions on two Borel measures, μ defined in $\mathbb{R}^{d+1}_+$, and $\nu(x')dx'$ defined on $\mathbb{R}^d$, so that

$$\left(\int_{\mathbb{R}^{d+1}_+} |\nabla u(x)|^q \, d\mu(x)\right)^{1/q} \leq C \left(\int_{\mathbb{R}^d} |f(x')|^p \, \nu(x')dx'\right)^{1/p}$$

for any solution to $Lu = 0$ in $\mathbb{R}^{d+1}_+$, $u = f$ on $\mathbb{R}^d$. Here $L = \Delta$ or $\partial/\partial t - \Delta$, and $1 < p \leq q < \infty$ with $q \geq 2$. Later work by Sweezy and Wilson established analogous results for solutions to more general second order operator equations on bounded Lipschitz and Lip(1,1/2) domains.

My talk will focus on recent work in which sufficient conditions on two Borel measures, μ and η, both defined on a bounded domain Ω, are stated and shown to imply that

$$\left(\int_{\not\leq} (\|u\|_{H^\alpha}(x))^q \, d\mu(x)\right)^{1/q} \leq C \left(\int_{\not\leq} \left(\left|\overrightarrow{f}(x)\right|^p + \left|div\,\overrightarrow{f}(x)\right|^p\right) d\eta(x)\right)^{1/p},$$

for solutions to $Lu = div\,\overrightarrow{f}$ on Ω, $u = 0$ on $\partial\Omega$. Here $L = \sum_{i,j=1}^{d} \frac{\partial}{\partial x_i}(a_{i,j}(x)\frac{\partial}{\partial x_j})$ and is strictly elliptic. $\|u\|_{H^\alpha}(x) = \sup_{y \in B(x)} \frac{|u(y)-u(x)|}{|y-x|^\alpha}$; $B(x)$ is a small box centered at x. Originally I proved such an inequality with $|\nabla u|$ instead of the Hölder norm of u when L is a strictly elliptic divergence form operator with coefficients bounded and measurable. Replacing $|\nabla u(x)|$ by $\|u\|_{H^\alpha}(x)$ allows one to prove two different kinds of sufficient measure conditions; the first involves bounding a singular potential of μ, the second utilizes the longer method that was used with $|\nabla u(x)|$. The second can be proved by establishing a Littlewood-Paley type inequality and using this inequality in a dual operator argument. One can use measure conditions that do not involve a singular kernel; the conditions are weaker than the ones needed for $|\nabla u|$.

For parabolic functions, to prove a norm inequality for the appropriate Holder norm, there is a condition on the measures similar to the first condition for elliptic functions. Here $Lu = div\overrightarrow{f}$ on Ω, $u = 0$ on $\partial_p\Omega_T$ with $L = \partial/\partial t - \sum\limits_{i,j=1}^{d} \frac{\partial}{\partial x_i}(a_{i,j}(x,t)\frac{\partial}{\partial x_j})$. To prove sufficient conditions on μ and η, for parabolic u, along the lines of the second kind of measure condition, one must establish estimates for the Green's function. These estimates are proved in Gruter and Widman for the elliptic Green's function on a non-smooth domain. It is well-known that the capacity arguments used by Gruter and Widman are not valid in the case of parabolic operators of the type considered here. One can however obtain geometric estimates on the parabolic Green's function that are needed for a Littlewood-Paley type inequality from results proved by Kaj Nystrom; so it is probable that a similar result can be established for parabolic Hölder norms on non-smooth domains. This is work in progress.

HAMILTON-JACOBI-BELLMAN EQUATIONS AND LARGE DEVIATIONS FOR STOCHASTIC PDE

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Key Words and Phrases: Hamilton-Jacobi-Bellman equations, viscosity solutions, large deviations
AMS Subject Classification: 35R15, 49L25, 60F10, 60H15

We will discuss how recent results on Hamilton-Jacobi-Bellman equations in Hilbert spaces can be used to provide an easy way to prove large deviation principle for a class of stochastic PDE with small noise intensities of the form

$$\begin{cases} dX_n(t) = (-AX_n(t) + b(t, X_n(t))dt + \frac{1}{\sqrt{n}}\sigma(t, X_n(t))Q^{\frac{1}{2}}dW(t) \quad t > 0, \\ X_n(0) = x \in H, \end{cases}$$

where A is a linear, densely defined maximal monotone operator in a real, separable Hilbert space H, Q is a bounded, nonnegative, self-adjoint operator of trace class in H, and W is a cylindrical Wiener process in H. The key ingredient in this procedure is the use of viscosity solutions to abtain the so called Laplace limit for the large deviation problem at single times. The approach borrows several ideas recently developed by Feng and Kurtz in their work *Large deviations for stochastic processes*, Mathematical Surveys and Monographs, vol. 131, American Mathematical Society, Providence, RI, 2006.

GOODNESS–OF–FIT TESTS VIA CHARACTERIZING CONDITIONS IN TERMS OF EXPECTED VALUES OF TWO FUNCTIONS OF ORDER STATISTICS

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Key Words and Phrases: order statistics, record values, moments, exponential and normal distributions, characterizations, goodness–of–fit tests, powers
AMS Subject Classification: 62E10, 62G10

There are many methods to construct goodness–of–fit tests. We show that the method using characterizations of continuous distributions via expected values of two functions of order statistics and record values appears to be very effective. Such characterizing conditions were first given by Lin and next they were extended (see [1] for references). This method allows to construct goodness–of–fit tests for a large class of continuous distributions. Furthemore, it gives families of tests depending on some constants (cf. [2]–[4] and references there). They have simple form and one can find, among other things, tests whose power is greater than powers of recommended tests for exponentiality and normality. Here we derive goodness–of–fit tests from characterizing conditions given by moments of order statistics. A comparison for the performence of the presented omnibus tests with selected tests of exponentiality, widely recommended by most authors is included.

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OPTIMIZATION DELAY CONTROL SYSTEMS WITH NON-FIXED INITIAL MOMENT

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Real controlled systems as a rule consist information about their behavior beforehand and this information greatly influences the dynamic of the system. Differential equations with delays in the phase coordinates and controls are well known mathematical models of such controlled systems.

In the present paper optimal control problems for dynamical control systems governed by delay differential equations with discontinuous, continuous and mixed initial conditions are investigated [1,2].

The discontinuous initial condition means that at the initial moment the values of the initial function and the trajectory, generally spiking, do not coincide.

The continuous initial condition means that at the initial moment the values of the initial function and the trajectory always coincide.

The mixed initial condition means that at initial moment, some coordinates of the trajectory do not coincide with the corresponding coordinates of the initial function.

Moreover, we consider optimal control problems for systems with variable structure[3] and incommensurable delays in controls.

For above discussed problems necessary conditions of optimality are obtained: for the optimal initial moment and finally moment in the form of equality and inequality; for the controls and initial functions in the form of point-wise maximum principle.

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NUMERICAL INVESTIGATION OF PARTIAL DIFFERENTIALS EQUATIONS SYSTEM WITH VARIABLE COEFFICIENT AND APPLIED TO COMPOSITE MATERIAL

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Field equations of composite material with spatially locally or periodically curved structure in the framework of continuum approach proposed by Akbarov and Guz [1] are given below:

Motion Equations: $\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}$, x_1, x_2, $x_3 \in \Omega$, $i, j = 1$, 2, 3 (1)

Elasticity Relations: $\sigma = \mathbf{D}\varepsilon$ (2)

Relations between stains and displacement:$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$, $i, j = 1$, 2, 3 (3)

where

$$\sigma^T = \begin{pmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{23} & \sigma_{13} & \sigma_{12} \end{pmatrix}, \quad \varepsilon^T = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{22} & \varepsilon_{33} & \varepsilon_{23} & \varepsilon_{13} & \varepsilon_{12} \end{pmatrix}, \tag{1}$$

In this equation, $A_{ij}(x_1, x_3)$ show mechanical properties of material. These properties depend on the curving in the structure of the material characterized by the function$x_2 = F(x_1, x_3)$. Therefore it can be written the relations $A_{ij} = A_{ij}(F(x_1, x_3))$. The explicit from of these relations is given in [1]. Under consideration the concrete problems for structure elements the corresponding boundary and initial conditions are added to the equations (1)-(5). Namely these problems, .i.e. the boundary-value problems for system partial- differentials equations with variable coefficients, are investigated in our research by the use of the FEM. In the present report the review of these investigations is detailed.

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OPTIMIZATION OF PROCEDURES OF IMAGE INTERFRAME GEOMETRICAL DEFORMATIONS PSEUDOGRADIENT ESTIMATION

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Video information usage permanently increases. The studying of the observed objects time dynamic leads to necessity of the image sequence analysis. At that the observed scene dynamic, spatial displacements of signal sensors and other factors may be taken into account by means of image interframe geometrical deformations (IIGD) estimation. The usage of recurrent pseudogradient procedures (PGPs) applied to image processing in the conditions of a priory uncertainty is very promising. Such procedures assume small computational expenses and the estimates formed through them are immune to impulse interference and converge to optimal values under rather weak conditions. However PGPs also have disadvantages. In particular when real image processing local extremums of the estimate of the goal function (GF) characterizing the estimation quality are shown. It significantly reduces the estimates convergence speed. Furthermore PGPs have relatively not large operating range, therefore the optimization by convergence speed and computational expenses is very urgent. The estimates convergence character and computational expenses are determined in many respects by the local sample size (LSS) used to find the GF pseudogradient. However the possibilities of the LSS optimization are studied not well. This work is devoted to this problem where two basic aspect are considered

- the method of a priory optimization of the LSS at the given brightness probability distribution density (PDD) and an autocorrelation function of the image (criteria of computational expenses minimum, minimum number of iterations of estimation and ensuring of the given convergence speed of parameters estimates are considered);

- a posteriori optimization of the LSS, where the sample size is automatically specialized at each iteration to meet the condition that facilitates PGPs recovery from the local extremums of the GF.

As a value characterizing the estimates convergence quality and enabling to estimate the convergence of the parameters vector in general the PDD of distance between points of the deformed frame image included into the local sample and their estimates on the sought frame is used. The characteristic which has enabled to fully characterize the parameters of the analyzed images and interfering noise at the simulation is obtained.

A priory and a posteriori optimization of PGP may be directly applied in various fields of image processing and to guarantee high accuracy of estimation at small computational expenses. The developed algorithmic and programme software may be used to solve applied problems of space-time signal processing, where parameters recurrent estimation is employed.

COMPETITIVE LOGISTIC NETWORKS WITH ADAPTATION

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Key Words and Phrases: bifurcation theory, complex (chaotic) behavior
AMS Subject Classification: 34K18, 34K23

A general N-node network is considered for which, in absence of interactions, each node is governed by a logistic equation. Interactions among the nodes take place in the form of competition, which also includes adaptive abilities through a (short term) memory effect. As a consequence the dynamics of the network is governed by a system of NxN nonlinear ordinary differential equations. Existence of classes of invariant subspaces is proven, which allow the introduction of reduced models, where N appears as a parameter, giving full account of existence and stability for the equilibria in the network. Reduced models are found effective also in describing time-dependent regimes, both in the form of periodic oscillations and chaotic behavior.

DIGITAL TOPOLOGY AND THE DIGITAL LINE

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Azriel Rosenfeld initiated the study of Digital topology. Following this , Scientists and mathematicians like Khalimsky , Kopperman , Meyer, Saha, Kong, etc. worked on this area. They proved the Jordan curve theorem for digital curves and digital surfaces. A computer screen is a finite rectangular array of lattice points and hence it admits only one T_1 topology. This topology is the discrete topology, which has no nontrivial connected sets, and hence no Jordan curve theorem. There are two approaches to study digital images namely Graph theoretic approach and Topological approach. Rosenfeld initiated the first approach in 1979. Kong, Kopperman, Meyer, Khalimsky and others initiated the second approach in 1990s. The set Z of all integers together with the Khalimsky topology is called the Digital Line. In this paper we discuss the properties of the non-empty digital intervals [a, b]∩Z. We also characterize the cardinalities of the subspace topologies on the digital intervals.

NUMERICAL MODELLING OF DRIFTING FLAME BALLS

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Key Words and Phrases: flame balls, drift, numerical modelling
AMS Subject Classification: 80A25

Experimental results have shown that when more than one flame ball is established in a combustion chamber they drift apart. Whilst a simple explanation of this is that the flame balls tend to move away from competing for partially burnt gasses into fresher fuel mixtures there has been little attempt at modelling this behaviour.

In this presentation we shall describe the use of some simple chemical models to describe flame balls, some difficulties encountered in the numerical modelling and the results concerning the behaviour of one flame ball in the presence of a second one.

POSITIVE SOLUTIONS FOR NONLINEAR SINGULAR BOUNDARY VALUE PROBLEMS ON THE HALF LINE

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We discuss the existence of positive solutions for singular second-order boundary value problems $x'' = \mu f(t, x, x')$, $ax(0) - bx'(0) = k \geq 0$, $x'(\infty) = 0$, where f may be singular at $x = 0$ and $x' = 0$ and can change sign. Via fixed point theory, we establish the existence of positive solutions under some conditions on f. Our results deal with the situation where the solutions approach the singularities of the equation.

ANALYSIS AND APPROXIMATION OF THE VELOCITY TRACKING PROBLEM FOR MHD FLOWS WITH DISTRIBUTED MAGNETIC FIELD CONTROL

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We consider the mathematical formulation and the analysis of an optimal control problem associated with the tracking of the velocity and the magnetic field of a viscous, incompressible, electrically conducting fluid in a bounded two-dimensional domain through the adjustment of distributed controls. Existence of optimal solutions is proved and first-order necessary conditions for optimality are used to derive an optimality system of partial differential equations whose solutions provide optimal states and controls. Semidiscrete-in-time and fully discrete space-time approximations are defined and their convergence to the exact optimal solutions is shown.

The results of some computational experiments are provided.

INTELLIGENT QUANTUM SYSTEMS

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Key Words and Phrases: quantum computation, pattern recognition, associative memory
AMS Subject Classification: 81P68, 68T10

Associative pattern retrieval, one of the hallmarks of intelligence, can not only be realized by the traditional attractor dynamics of the Hopfield model but also by a reversible, unitary evolution of quantum bits (qubits). I will show that qubit networks with long-range interactions governed by the Hebb rule can be used as quantum associative memories. Starting from a uniform superposition, the unitary evolution generated by these interactions drives the network through a quantum phase transition at a critical computation time, after which ferromagnetic order guarantees that a measurement retrieves the stored patterns. The memory capacity of these qubit networks depends on the computation time: the maximum capacity is reached at a memory density $\alpha = p/n = 1$, after which a phase transition to a quantum spin glass state implies total amnesia. At these loading factors, however the retrieval quality is poor; admitting only a few percent of errors requires lower memory loading factors, comparable with the classical Hopfield model.

SUBMANIFOLDS WITH NONZERO MEAN CURVATURE IN A EUCLIDEAN SPHERE

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The purpose of this paper is to prove the following theorem:

Theorem. *Let M^n be a complete, connected and orientable submanifold with nonzero constant mean curvature H in $S^{n+2}(c)$. Moreover, we put $|\phi|^2 = S - nH^2$ and B_H the square of the positive root of the equation $x^2 + \frac{n(n-2)}{\sqrt{n(n-1)}}Hx - n(H^2+c) = 0$ with respect to x, where S denotes the squared norm of the second fundamental form. If $|\phi|^2$ satisfies*

$$|\phi|^2 \leq B_H \qquad \text{for all } x \in M^n,$$

then M^n lies in a totally geodesic hypersurface $S^{n+1}(c)$ of $S^{n+2}(c)$ and
(1) *either $|\phi|^2 \equiv 0$ and M^n is totally umbilic or $|\phi|^2 \equiv B_H$.*
(2) $|\phi|^2 \equiv B_H$ *if and only if*

(B) $n \geq 3$ *and* $M^n = S^{n-1}(r_1) \times S^1(r_2) \subset S^{n+1}(c)$ *where* $r_1^2 + r_2^2 = \dfrac{1}{c}$ *and* $r_1^2 < \dfrac{n-1}{nc}$

or

(C) $n = 2$ *and* $M^2 = S^1(r_1) \times S^1(r_2) \subset S^3(c)$ *where* $r_1^2 + r_2^2 = \dfrac{1}{c}$ *and* $r_1^2 \neq \dfrac{1}{2c}$.

The following generalized maximum principle due to Omori and Yau will be used in order to prove our theorem.

Generalized Maximum Principle (Omori and Yau). *Let M^n be a complete Riemannian manifold whose Ricci curvature is bounded from below and $f \in C^2(M)$ a function bounded from above on M^n. Then, for any $\epsilon > 0$, there exists a point $p \in M^n$ such that*

$$f(p) \geq \sup f - \epsilon, \ \|grad\ f\| < \epsilon, \ \Delta f(p) < \epsilon.$$

PROBABILITY THEORY ON TIME SCALES

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The study of dynamic equations on time scales was introduced by Stefan Hilger in his Ph.D. thesis in 1988 in order to unify continuous and discrete analysis. Since Hilger formed the definition of derivatives and integrals on time scales, several authors have expended on various aspects of the new theory.

Since time scales approach will unify the standard discrete and continuous random variables. Probability is a discipline in which appears to be many applications of time scales. We present measure theory on time scales, probability functions on time scales, and some basic random variables on a time scale in this paper.

ON CHARACTERISTIC PROBLEMS FOR MULTIDIMENSIONAL PSEUDOPARABOLIC EQUATIONS

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AMS Subject Classification: 35B, 35J

Work devoted to investigation characteristic problems for equation

$$L(u) \equiv \sum_{i_1=0}^{m_1}\sum_{i_2=0}^{m_2}\dots\sum_{i_n=0}^{m_n} a_{i_1 i_2 \dots i_n}(x_1,x_2,...,x_n)\frac{\partial^{i_1+i_2+\dots+i_n}u}{\partial x_1^{i_1}\partial x_2^{i_2}...\partial x_n^{i_n}} = F(x_1,x_2,...,x_n), \quad (1)$$

in $D=\{x_{10}<x_1<x_{11}, x_{20}<x_2<x_{21},...,x_{n0}<x_n<x_{n1}\}$- parallelepiped in n- dimensional Euclid space. We suppose $a_{m_1 m_2 \dots m_n} \equiv 1$, smoothness of other coefficients is defined by the inclusions $a_{i_1 i_2 \dots i_n} \in C^{\sum_{\alpha=1}^{n} i_\alpha}\left(\overline{D}\right)$, $F \in C^{0+0\dots+0}\left(\overline{D}\right)$. Here, $C^{\alpha_1+\alpha_2+\dots+\alpha_n}$- class of continuous in $\overline{D}$ together with derivatives

$$\partial^{r_1+r_2+\dots+r_n}/\partial x_1^{r_1}\partial x_2^{r_2}...\partial x_n^{r_n}(r_1=0,\ .\ .\ .,\alpha_1,$$

$r_2=0,\ .\ .\ .,\alpha_2,\dots r_n=0,\ .\ .\ .,\alpha_n)$, functions. We use meaning [1]: D_z^0- unit operator; $D_z^i=\left(\frac{\partial}{\partial z}\right)^i$, i=1,2,...; $D_z^{-i}=\left(\int\limits_{z_0}^{z}\right)^{-i}$, i=-1, -2,... For solving put up problem we use the development of the Riemann method variant, which was builded for all $m_k=1$ and any n by V.I.Zhegalov and his pupils [2]. The main for equation (1) is

Goursat problem: find in D the solution of equation (1) in class $C^{\sum_{i=1}^{n} m_i}(D)\cap C\left(\overline{D}\right)$, satisfying for conditions:

$$D_{x_1}^{i_1}u(x_{10},x_2,...,x_n)=\varphi_{1\ i_1}(x_2,...,x_n), \quad \left(i_1=\overline{0,m_1-1}\right),$$
$$D_{x_2}^{i_2}u(x_1,x_{20},...,x_n)=\varphi_{2\ i_2}(x_1,x_3...,x_n), \quad \left(i_2=\overline{0,m_2-1}\right), \quad (2)$$
$$.\ .\ .\ .\ .$$
$$D_{x_n}^{i_n}u(x_1,x_2,...,x_{n0})=\varphi_{n\ i_n}(x_1,...,x_{n-1}), \quad \left(i_n=\overline{0,m_n-1}\right),$$

$\varphi_{1\ i_1} \in C^{\sum_{\alpha=2}^{n} m_\alpha}\left(\overline{X_1}\right)$, $\varphi_{2\ i_2} \in C^{\sum_{\alpha=1,\alpha\neq 2}^{n} m_\alpha}\left(\overline{X_2}\right)$, ... , $\varphi_{n\ i_n} \in C^{\sum_{\alpha=1}^{n-1} m_\alpha}\left(\overline{X_n}\right)$. Let X_1, X_2,..., X_n - ribs D when $x_1=x_{10}$, $x_2=x_{20}$,... $x_n=x_{n0}$ accordingly. We suppose the satisfaction of the condition (2) in ribs D:

$$D_{x_2}^{i_2}\varphi_{1i_1}(x_{20},x_3,...,x_n)=D_{x_1}^{i_1}\varphi_{2i_2}(x_{10},x_3,...,x_n),$$
$$D_{x_3}^{i_3}\varphi_{1i_1}(x_2,x_{30},x_4...,x_n)=D_{x_1}^{i_1}\varphi_{3i_3}(x_{10},x_2,x_4...,x_n),\dots$$
$$D_{x_n}^{i_n}\varphi_{1i_1}(x_2,x_3,...,x_{n-1},x_{n0})=D_{x_1}^{i_1}\varphi_{ni_n}(x_{10},x_2,...,x_{n-1});$$
$$D_{x_3}^{i_3}\varphi_{2i_2}(x_1,x_{30},x_4,...,x_n)=D_{x_2}^{i_2}\varphi_{3i_3}(x_1,x_{20},x_4,...,x_n),\dots$$

$$D_{x_n}^{i_n}\varphi_{2i_2}\left(x_1,x_3,x_4,...,x_{n0}\right)=D_{x_2}^{i_2}\varphi_{ni_n}\left(x_1,x_{20},x_3,...,x_{n-1}\right),$$

$\ldots..,\ D_{x_n}^{i_n}\varphi_{n-1i_{n-1}}\left(x_2,x_3,x_4,...,x_{n0}\right)\ =\ D_{x_{n-1}}^{i_{n-1}}\varphi_{ni_n}\left(x_1,x_2,x_3,...,x_{n-10}\right)$, and agreed values continuously differentiated.

By author in functions Riemann terms builded formulas for deciding this problem. Then method was spreaded for more difficult problems.

Lets note, that equation of type (1) can be meet in supplements . For example, in book [3, 258] “main differential equation bending thin spherical cover”, Aller’s equation [4], [5, p.261-262] from theory studying process absorbing moisture by planets roots and Bussineska-Liav’s, describing longitudinal waves in thin elastic bar, registering effects diametrical inertia [1, (20)]. The same equation describe waves moving in periodical flaky environment [6]. Mangeron’s polivibration equation are belong to the same type, as (1) [7].

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CONDITIONAL INDEPENDENCE: AVOIDING INCONSISTENCIES

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Key Words and Phrases: stochastic independence, logical dependencies, graphical models, graphoid properties

AMS Subject Classification: 60A05, 62C05

It is well known that the classical definition of stochastic independence of two events A, B (*i.e.* $P(AB) = P(A)P(B)$) gives rise to counterintuitive situations when the events have probability 0 or 1: e.g. an event A with $P(A) = 0$ or 1 is stochastically independent of any other, in particular of itself (a situation of "maximum" *logical dependence*), while a minimum requirement for *any* event E should be its *dependence on itself* (due to the intuitive meaning of independence, a concept that should catch the idea that A being independent of B entails that assuming the occurrence of B would not change the probability of A). So a natural aim is a definition of stochastic independence of two events that implies their *logical independence*. Other formulations of the classical definition resorting to conditional probability show similar inconsistencies not only in the Kolmorogovian setting, but also in the more general approach by de Finetti to conditional probability. Then, to avoid these situations, a new definition of stochastic independence between two events has been introduced by Coletti and Scozzafava. This notion agrees with the classical one for probabilities strictly between 0 and 1 and behaves well in the extreme cases of probabilities 0 and 1. This notion can be extended to the conditional case for finite random variables. We provide a characterization of conditional independence structures (i.e. the set of conditional independence relations induced by a conditional probability) in terms of "graphoid properties" and we face the representation problem of such models by means of directed acyclic graphs. Since the induced models are not necessarily closed with respect to the symmetry, a new separation criterion, able to represent not symmetric statements, is introduced: it allows also to deal with other independence models under different uncertainty measures.

LAGRANGIAN APPROXIMATION FOR THE NAVIER-STOKES EQUATIONS

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Key Words and Phrases: Navier-Stokes equations, Lagrangian approximation
AMS Subject Classification: 35B65, 35D05, 76D05

For the description of fluid flow there are in principle two approaches, the Eulerian approach and the Lagrangian approach. The first one describes the flow by its velocity $v = (v_1(x), v_2(x), v_3(x)) = v(x)$ in every point $x = (x_1, x_2, x_3)$ of the domain G containing the fluid. The second one uses the trajectory $x(t) = X(t, x_0)$ of a single particle of fluid, which at time $t = 0$ is located at some point $x_0 \in G$.

We consider the steady motion of a viscous incompressible fluid in a bounded domain $G \subset \mathbf{R}^3$ with a smooth boundary S. Because for steady flow the streamlines and the trajectories of the fluid particles coincide, both approaches are correlated by the autonomous system

$$x'(t) = v(x(t)), \quad x(0) = x_0 \in G, \tag{1}$$

which is an initial value problem for $t \longrightarrow x(t) = X(t, x_0)$, if the velocity field $x \longrightarrow v(x)$ is known in G.

To determine the velocity, in the present case we have to solve the stationary nonlinear partial differential equations

$$-\nu\Delta v + v \cdot \nabla v + \nabla p = F \text{ in } G, \operatorname{div} v = 0 \text{ in } G, v = 0 \text{ on } S \tag{2}$$

of Navier-Stokes. Here p is an unknown kinematic pressure function. The constant $\nu > 0$ (kinematic viscosity) and the external force density F are given data. The incompressibility of the fluid is expressed by $\operatorname{div} v = 0$, and on S we require the no-slip condition $v = 0$.

It is the aim of this lecture to develop a new approximation method for (2) by coupling both the Lagrangian and the Eulerian approach. The method leads to a sequence of approximate systems, whose solution is unique and has a high degree of regularity. Moreover, we show that our method allows the construction of weak solutions of the Navier-Stokes equations: The sequence of approximate solutions has at least one accumulation point satisfying (2) in a weak sense.

MARKETS AND THE PRIMAL-DUAL PARADIGM

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Irwing Fisher, in a Ph.D. thesis submitted to Yale University in 1891, defined a fundamental market model. A remarkable convex program, given by Eisenberg and Gale in 1959, captures, as its optimal solutions, equilibrium allocations for the linear utilities case of this market. In recent joint work with Devanur, Papadimitriou and Saberi, we gave a combinatorial, polynomial time algorithm for computing equilibrium allocations, and hence optimal solutions to the Eisenberg-Gale program, for this case.

Our algorithm uses the classical primal-dual paradigm – not in its usual setting of LP-duality theory but in the enhanced setting of KKT conditions and nonlinear convex programs. In this talk, I will describe the difficulties raised by this new setting and show how our algorithm circumvents them. I will also present a generalization to spending constraint utilities and show how they can be useful in Google's AdWords market. Finally, I will allude to several basic algorithmic questions that arise from these two works.

CONSTRUCTING ELLIPTIC CURVES WITH GIVEN WEIL PAIRING

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Key Words and Phrases: elliptic curve, Weil pairing
AMS Subject Classification: 14H52, 12E05

Let $\mathbb{K}$ be a field with a non-trivial l-th root of unity ζ_l, where $l = 3$, 5 or 7. Based on the result of our paper *Lagrange resolvents and torsion of elliptic curves*, Int. J. Pure Appl. Math., 33, No 1, 2006, 75–92, we will give a parametrization of triples (E, P, Q) where E is an elliptic curve defined over $\mathbb{K}$ with $E[l] \subset E(\mathbb{K})$, and P, Q are l-torsion points such that the Weil pairing $e_l(P, Q) = \zeta_l$.
We use MAGMA to compute the parametrization over the cyclotomic field $(Q)(\zeta_l)$, and show that the computation is made over $\mathbb{Z}[l^{-1}, \zeta_l]$, so that we can deduce that it is true for every field of characteristic different from l. For example, for $l = 5$, we find the following: the modular curve has equation

$$f(S,T) = T(S^5 - 1) - (S^5\beta_5 - \alpha_5) = 0$$

(α_5, β_5 are defined in the previous article) and the parametrization is the following: Given any point $M = (s, t)$ on the modular curve, with $s \neq 0$, we define the following triple (E_M, P_M, Q_M) by

$$E_M = y^2 + (1-t)xy - ty = x^3 - tx^2,$$
$$P_M = (0,0)$$

and

$$\begin{aligned}
Q_M = (&\frac{-3s^9 - 14s^8 - 14s^7 - 2s^6 + 3s^5 - 2s^4 - 6s^3 - 6s^2 - 3s - 3}{s^9 + s^8 + s^7 + s^6 + s^5 - s^4 - s^3 - s^2 - s - 1}\zeta_5^3 \\
&+ \frac{-3s^9 - 3s^8 + 5s^7 + 8s^6 - 2s^5 - 7s^4 - 7s^3 - 10s^2 - 3s - 3}{s^9 + s^8 + s^7 + s^6 + s^5 - s^4 - s^3 - s^2 - s - 1}\zeta_5^2 \\
&+ \frac{-7s^8 - 13s^7 - 10s^6 - 5s^5 - 5s^4 - 3s^3 - 7s^2}{s^9 + s^8 + s^7 + s^6 + s^5 - s^4 - s^3 - s^2 - s - 1}\zeta_5 \\
&+ \frac{-5s^8 - 17s^7 - 21s^6 - 13s^5 - 5s^4 - 5s^3 - 3s^2 - 4s - 2}{s^8 + 2s^7 + 3s^6 + 4s^5 + 5s^4 + 4s^3 + 3s^2 + 2s + 1}, \\
&\frac{-13s^{13} - 73s^{12} - 61s^{11} + 47s^{10} + 115s^9 + 66s^8 + 16s^7 - 13s^6 - 34s^5 - 40s^4 - 53s^3 - 43s^2 - 26s - 13}{s^{13} + 2s^{12} + 3s^{11} + 4s^10 + 5s^9 + 3s^8 + s^7 - s^6 - 3s^5 - 5s^4 - 4s^3 - 3s^2 - 2s - 1}\zeta_5^3 \\
&+ \frac{-13s^{13} - 8s^{12} + 54s^{11} + 92s^{10} + 55s^9 + s^8 - 24s^7 - 53s^6 - 79s^5 - 80s^4 - 88s^3 - 68s^2 - 26s - 13}{s^{13} + 2s^{12} + 3s^{11} + 4s^{10} + 5s^9 + 3s^8 + s^7 - s^6 - 3s^5 - 5s^4 - 4s^3 - 3s^2 - 2s - 1}\zeta_5^2 \\
&+ \frac{-40s^{12} - 70s^{11} - 25s^{10} + 40s^9 + 30s^8 - 20s^7 - 55s^6 - 75s^5 - 65s^4 - 55s^3 - 40s^2}{s^{13} + 2s^{12} + 3s^{11} + 4s^{10} + 5s^9 + 3s^8 + s^7 - s^6 - 3s^5 - 5s^4 - 4s^3 - 3s^2 - 2s - 1}\zeta_5 \\
&+ \frac{-21s^{13} - 53s^{12} + 16s^{11} + 118s^{10} + 115s^9 + 22s^8 - 24s^7 - 32s^6 - 26s^5 - 15s^4 - s^3 + 2s^2 + 16s + 8}{s^{13} + 2s^{12} + 3s^{11} + 4s^{10} + 5s^9 + 3s^8 + s^7 - s^6 - 3s^5 - 5s^4 - 4s^3 - 3s^2 - 2s - 1})
\end{aligned}$$

OPEN MAPS AND TIMED EQUIVALENCES

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Key Words and Phrases: real-time and concurrent systems, timed transition systems, behaviour equivalences, category theory, open maps, abstract bisimulation

AMS Subject Classification: 68Q85

In concurrency theory, a variety of equivalences has been promoted, and the relationships between them have been quite well-understood. In an attempt to explain and unify apparent differences between the extensive amount of research within the field of equivalences, several category-theoretic approaches to the matter have appeared. In particular, Joyal, Nielsen, and Winskel proposed an abstract way of capturing the notion of bisimulation through the so-called spans of open maps.

It is generally recognized that time plays an important role in many concurrent systems. This has motivated the development of several models and analysis methods to support the correctness of real-time systems. In the framework of timed transition systems — a widely studied model for real-time systems, Hune and Nielsen illustrated the use of open maps for providing an abstract characterization of timed bisimulation.

The intention of our work is to show the applicability of the open maps approach to the setting of strong and weak equivalences of linear time – branching time spectrum on timed transition systems. In particular, we define different categories of timed transition systems, whose morphisms are to be thought of as simulations, and accompanying (sub)categories of observations, to which the corresponding notions of open maps are developed. We then use the open maps framework to obtain abstract bisimilarities which are established to coincide with the timed equivalences under consideration.

A LIE-ALGEBRA VERSION OF CLASSICAL OR QUANTUM HAMILTONIAN PERTURBATION THEORY & CONTROL, WITH EXAMPLES IN PLASMA PHYSICS

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We consider a Hamiltonian which leaves globally invariant a sub-Lie-algebra $\mathcal{B}$ of the Lie algebra of "observables", which happens for instance when the Hamiltonian is "locally integrable", or has some "symmetry". For any perturbation of this Hamiltonian, we give an expression for the sub-Lie-algebra isomorphic to $\mathcal{B}$ which is globally invariant under the perturbed system. More precisely we give an expression for the automorphism ("change of variables") which conjugates the 2 sub-Lie-algebras. A simpler problem is to "slightly" modify the perturbed system (by an additive "control" term, for instance quadratic in the perturbation) such that the above automorphism is simple to compute. This theory generalizes a recent control of Hamiltonian systems that has been already applied in some physical examples. Here we give some examples, in Classical or Quantum Mechanics, mainly to control turbulence in Plasma Physics.

MULTIPLE STATE OPTIMAL DESIGN PROBLEMS AND HASHIN-SHTRIKMAN BOUNDS

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Key Words and Phrases: optimal design, homogenization, Hashin-Shtrikman bounds
AMS Subject Classification: 35B27, 49J45, 49K20, 65K10, 74Q20

In optimal design problems the goal is to determine the optimal distribution of given materials in a given domain. Optimality of a distribution is measured by an objective function, which is usually an integral functional depending on the distribution of materials and the state function, obtained as a solution of the associated boundary value problem for the corresponding partial differential equation. We are interested in thermal conductivity problem, where the state function is the temperature of considered body. Source term and boundary conditions are presumed fixed. Therefore, if we change them, optimal distribution changes.

This situation refers to multiple state optimal design problems, where the optimality takes into account temperatures obtained for different right-hand sides. Relaxing the original problem by homogenization method, we are able to write optimality conditions. It was proved that these optimality conditions are satisfied by sequential laminates. Using the fact that we can calculate precise order of optimal laminates, we can simplify numerical algorithms for optimal distribution of materials. We show the connection of this problem to Hashin-Shtrikman energy bounds, which are known to be saturated by sequential laminates, but we precisely determine the order of these laminates.

MORSE THEORY WIHOUT (PS) CONDITION AT ISOLATED VALUES FOR STRONGLY INDEFINITE FUNCTIONALS AND APPLICATION TO WAVE EQUATION WITH STRONG RESONANCE

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We develop a Morse theory for strongly indefinite functional, when it looses the (PS) condition at some isolated values. One of our purpose in developing this theory is to study multiplicity results for strongly indefinite functional with strong resonance, and we apply our results to the semi linear wave equation with strong resonance.

AN ALGEBRAIC COMPUTATION FOR THE JACOBI SYMBOL OF QUADRATIC RESIDUES, AND CONTINUED FRACTIONS

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Key Words and Phrases: quadratic residues, continued fractions
AMS Subject Classification: 11A15, 11A55, 11A63

The theory of quadratic residues has a rich influence in the pure and applied mathematics(see F.Lemmermeyer "Reciprocity Laws" Springer, 2000 and F.J.Macwilliams-N.J.A.Slone "The Theory of Error-Correcting Codes" North-Holland, 1978).

This talk gives an algebraic computation for the Jacobi symbol of quadratic residues, which is given by the coefficients of continued fraction.

MEROMORPHIC CONTINUATION OF THE SCATTERING MATRIX

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Key Words and Phrases: scattering matrix, resonance, stark, Schrödinger
AMS Subject Classification: 35P25

The scattering matrix will be considered in the context of non-relativistic quantum scattering of a single particle in $\mathbf{R}^n$ by a long or short range potential in the presence of a constant electric field (the "Stark" case). The scattering matrix is an operator on the Hilbert space $L^2(\mathbf{R}^{n-1})$ indexed by (free) energy λ. The main result discussed here is that, as a function of λ, the scattering matrix has a meromorphic continuation to the entire complex plane as a bounded operator on $L^2(\mathbf{R}^{n-1})$ even when the potential is long range. This is in marked contrast to the case of (Schrödinger) scattering, that is, when there is no electric field. The differences will be clarified. The meromorphic extension is of interest because any possible poles of the extension are resonances of the system.

STABILITY AND CHAOS IN A LASER WITH AN INTRACAVITY SATURABLE ABSORBER

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Key Words and Phrases: Statz-de Mars equations, stability, chaos, fixed points, intracavity saturable absorber
AMS Subject Classification: 37N20

An intracavity saturable absorber laser, described by the Statz-de Mars equations is studied, and the parameters characteristics are found in order to bring the laser into a chaotic state. This study is based on the analysis of the parameters effects on the system's critical points

FORMATION OF THE EQUATION OF A CURVE IN E^4 AND E_1^4 BY USING FRENET FORMULAS AND CHEBYSHEV SERIES SOLUTION OF THIS EQUATION

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In this study, our aim is to obtain that a position vector of a curve satisfies fifth order differential equations in E^4 and E_1^4 by using Frenet Formulas. Moreover, the other our goal is that solving these equations by using of Chebyshev-Matrix method, Chebyshev series of the curve is to obtain.

DIFFEOMORPHIC PATTERN MATCHING METHODS

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Key Words and Phrases: Groups of diffeomorphisms. Image registration. EPDiff.
AMS Subject Classification: 58E50

We describe a general framework under which diffeomorphic matching can be understood and implemented. Let Ω be an open subset of $\mathbb{R}^d$, and $Diff(\Omega)$ be the group of diffeomorphisms of Ω.

Specific subgroups of $Diff(\Omega)$ can be defined constructively as solutions of flows of ordinary differential equations. We shall summarize the construction of a right-invariant Riemannian metric on this group, and how it can be used to define a well-posed variational framework that solve a large range of registration problems, related to medical imaging in particular. These variational problems optimize over differential equations, represented by time-dependent velocities. More precisely, they take the optimal control form

$$\text{minimize} \int_0^1 \|v(t)\|_V^2 + U(\phi_1^v)$$

where V is a Hilbert space of smooth vector fileds on Ω and $\phi_t^v, t \in [0,1]$ is defined by $\phi_0^v(x) = x$ and

$$\frac{d}{dt}\phi_t^v(x) = v(t, \phi_t^v(x)).$$

The Euler-Lagrange equations for this problem directly relate to what is commonly called the shape derivative of U in shape optimization, and leads to a particular form of the Pontryagin principle, expressed as a conservation of momentum equation called EPDiff. We will show how this equation can be used to provide a useful representation of deformations in template matching.

This work is partially supported by NSF DMS-0456253.

3-D NONHOMOGENEOUS DIFFUSION EQUATION WITH IRREGULAR DOMAINS BY THE FUNDAMENTAL SOLUTIONS, PARTICULAR SOLUTIONS AND EIGENFUNCTIONS METHODS

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Key Words and Phrases: method of fundamental solutions; method of particular solutions; eigenfunctions; 3-D nonhomogeneous diffusion equation; singular value decomposition
AMS Subject Classification: numerical analysis for partial differential equations

The method of fundamental solutions (MFS), the method of particular solutions (MPS), and the eigenfunctions expansion method (EEM) are adopted for the numerical solutions of 3-D nonhomogeneous diffusion equations in the complex geometries in this article. This is an extension work for a two-dimensional paper published in the Journal of Numerical Methods for Partial Differential Equations [1]. The nonhomogeneous diffusion equations with time-independent source terms and boundary conditions are analyzed by the proposed meshless numerical scheme through the MFS-MPS-EEM model. Nonhomogeneous diffusion equation in complex domains can be separated into a Poisson equation and a homogeneous diffusion equation by using present model. The Poisson equation is solved by the MFS-MPS model, in which the compactly-supported radial basis functions (RBF) are adopted for the MPS and the fundamental solution of Laplace operator is used for the MFS. On the other hand, utilizing the EEM, the homogeneous diffusion equation is first translated into a Helmholtz equation, which is then solved by the MFS together with the technique of singular value decomposition (SVD). Numerical test results obtained for 3-D homogeneous and nonhomogeneous diffusion problems show good agreements with the analytical and finite element solutions. Thus, the present numerical scheme has provided a promising mesh-free numerical tool to solve the 3-D nonhomogeneous diffusion equations with time-independent source terms and boundary conditions in regular or irregular domains.

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GLOBAL (IN TIME) SOLUTIONS TO THE 3D-NAVIER-STOKES EQUATIONS

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Key Words and Phrases: global (in time), 3d-Navier-Stokes equations
AMS Subject Classification: 35Q30, 47H20, 76DO3

A well-known unsolved problem in the classical theory of fluid mechanics is to identify a set of initial velocities, which will allow global in time solutions to the three-dimensional Navier-Stokes equations. A related problem is to provide conditions under which we can be assured that the numerical approximation of these equations, used in a variety of fields from weather prediction to submarine design, have only one solution (uniqueness of weak solutions). In this talk, I will discuss an approach based on a new method for including geometry into physical systems which allows us to prove that there exists a number $\mathbf{u}_+$ such that for all initial velocities in a ball of radius $\mathbf{u}_+$, the Navier-Stokes equations have unique global in time solutions and the weak solutions are unique. I will first consider the problem on a bounded domain, where our method is transparent. However, the true power and generality of our approach will reveal itself when we consider the problem on $\mathbb{R}^3$.

A HIERARCHICAL FRAMEWORKS OF SCAN ADAPTIVE IMAGE PROCESSING

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Key Words and Phrases: multi-dimensional space-filling curves; high-performance algorithms; efficient image-data-structures; content-based image-retreival; pseudo-inverses operators on hilbert-curves

The central theme of the paper is that certain operations on images interact with the spatial distribution, and in particular the orientation, of specific features in the image. For example, most data compression algorithms will perform differently on images of vertical features such as pillars, Stonehenges bar-codes, and sparse gathering of people etc, when presented in two different orientations. More generally, nonlinear filters such as median filters will behave much better if the neighborhood over which they consolidate the underlying statistics of certain decision from a consistent group pixels, which may or may not be spatially compact.

As a part of our studies on space filling curves and their clustering properties we have developed a framework to pre-permute the pixels, process the image, and post permute the result. In doing so, the emphasis has been placed on maintaining certain topological properties of the space filling curves. This leads to generalization of Z-trees, built out peano scanned images into *-trees built from a variety of space filling curves.

The paper will describe a detailed analysis of computational effort required to the same. The effectiveness of this framework will has been verified on perceptual grouping of textured patterns in images, and thus applied for content based image retrieval. The analysis, results and performance evaluation will be presented.

MODIFICATION OF PRONY ALGORITHM FOR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS OF THE FIRST ORDER

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Key Words and Phrases: prony algorithm, system of ordinary linear differential equations, uniform net, inverse Cauchy problem
AMS Subject Classification: 34A55, 41

Let

$$\frac{d}{dt}Y(t) = AY(t) + BF(t), \quad t \in \mathbb{R} \tag{1}$$

be a system of ordinary linear differential equations with constant complex coefficients, where $Y(t)$ is unknown vector-functions of n components, $F(t)$ is known vector-function of 2m components, A is unknown matrix $n \times n$ with constant complex coefficients, B is known matrix $n \times 2m$ with constant complex coefficients.
Consider the direct and inverse Cauchy task for system (1) satisfying condition:

$$Y_k = Y(t_0 + kd), \quad k = 0, 1, 2, \ldots, N, \quad d > 0, \quad N > n + 2m, \tag{2}$$

where t_0 is a fix time moment. There exist the Prony algorithm and its modifications for solving the above task (e.g., [1]). In the report we are going to present in detail the conditions under which the algorithm is well posed. The received results of the research are based on the methods of solving similar task for ordinary differential equation with unknown constant coefficients ([2, chapter 2]).

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DESIGN FOR POSITIVITY OF FUNCTIONS ADMITTING TAYLOR EXPANSION

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Key Words and Phrases: positivity, robust control, stability
AMS Subject Classification: 11T06

A large group of control problems can be casted as a problem of design for positivity. A new algorithm is presented for choosing a parameter which ensures the robust positiveness of any real function admitting Taylor expansion. Numerical examples are provided, which illustrate the applicability of the proposed method to real-life problems.

ACTIVE DAMPING OF PIEZO ACTUATING MICRO-BEAMS USING OPTIMAL BOUNDARY CONTROL

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Key Words and Phrases: Micro-beams, Piezoceramics, Maximum Principle, Optimal Boundary Control, Eigenfunction Expansions Technique

AMS Subject Classification: 49J20

Optimal control theory is formulated and applied to damp out the vibrations of micro-beams, where the control action is implemented using piezoceramic actuators. The use of piezoceramic actuators such as PZT in vibration control is preferable because of their large bandwidth, their mechanical simplicity and their mechanical power to produce controlling forces. In the present study, the objective function is specified as a weighted functional of the dynamic responses of the micro-beam which is to be minimized at a specified terminal time using continuous piezoelectric actuators. The expenditure of the control forces is included in the objective function as a penalty term. The optimal control law for the micro-beam is derived using a maximum principle developed by Sloss et al (1998) for one-dimensional structures where the control functions appear in the boundary conditions in the form of moments. The derived maximum principle involves a Hamiltonian expressed in terms of an adjoint variable as well as admissible control functions. The state and adjoint variables are linked by terminal conditions leading to a boundary-initial-terminal value problem. The explicit solution of the problem is developed for the micro-beam using eigenfunction expansions of the state and adjoint variables. The numerical results are given to assess the effectiveness and the capabilities of piezo actuation by means of moments to damp out the vibration of the micro-beam with a minimum level of voltage applied on the piezo-actuators.

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CARLEMAN AND OBSERVABILITY ESTIMATES FOR STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS

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Key Words and Phrases: Carleman estimate, singular weight function, observability estimate, stochastic heat equation, stochastic wave equation
AMS Subject Classification: 35B45, 93B07, 35R30

In this work, based on two fundamental identities for stochastic parabolic and hyperbolic-like operators, we derive respectively global Carleman estimates (with singular weight function) for stochastic heat and wave equations. This leads to observability estimates for stochastic heat and wave equations with non-smooth lower order terms. Moreover, the observability constants are estimated by explicit functions of the norms of the involved coefficients in the equations.

STABILITY OF ELECTRICALLY DRIVEN JETS

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Key Words and Phrases: flow stability, electrically driven jets
AMS Subject Classification: fluid mechanics

A one-dimensional momentum conservation equation for a straight jet driven by electric fields is developed. It is not associated with any constitutive relation of the jet fluid, with the only assumption that the fluid is incompressible. Our results indicate that both the axial and the radial constitutive relations are required to close the governing equations of the straight charged jet. A comparison with other developed momentum equations is made.

The axisymmetrical stability of a straight electrically driven jet is discussed. A one-dimensional Giesekus constitutive equation is developed and is introduced into the governing equations. By means of perturbation method a dispersion equation is derived. The dispersion equation is of degree 5 while it is a cubic when Newtonian viscosity is used. Based on the dispersion equation the axisymmetrical stability of the electrically driven jet is analyzed in terms of numerical methods. An equivalent conclusion is that the instability may occur for the straight electrically driven jet if the viscosity ratio of the polymer to the solvent is large.

DENSITY MATRIX THEORY FOR LONG-RANGE CORRELATION OF ORGANIC π ELECTRONIC SYSTEMS

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Key Words and Phrases: correlation, density matrix, organic π systems
AMS Subject Classification: quantum theory

The electronic correlations are important issues in discussing the electronic structures such as band gap, etc., of semiconductors. For conjugated polymers and other organic molecular structure, the on-site Coulomb interaction (Hubbard U) is not so strong as it be in the strong correlation systems, and the long-range Coulomb interactions are important interactions for excitations and the band gap. However, until now the calculation of the correlation energies is made by using complicated numerical computations, including *ab initio* with GW approximation and the density matrix renormalization group (DMRG) method. Here we report an simple and available approach to calculate the long-range correlation energies for the π electronic systems, and discuss relations between the density matrix of charge and the correlation energies and also try to discuss problems of the correlation and the entanglement.

GAMES AND SELECTIONS OF SEQUENCES

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We consider games and selections on the set of sequence of positive real numbers, which satisfy certain asymptotic conditions (slow variability, regular variability and so on).

Also, several kinds of convergence related to such sequences are defined and investigated.