Math 164

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Scientific Computing

Groundwater Flow through a confined aquifer

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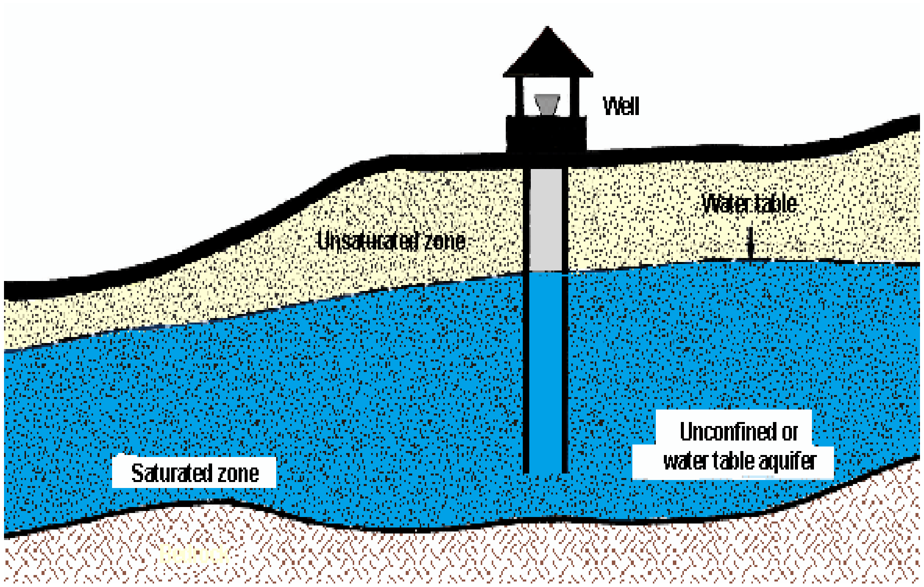
What is an aquifer?

- An aquifer is an underground layer of permeable material that allows a substantial amount of water to flow through it.
- Aquifers channel groundwater to wells. They can be made up of a variety of materials – gravel, sand, or clay.
- The local conductivity of an aquifer region dictates the flowrate of groundwater through it.

Unconfined aquifers

Unconfined aquifers consist of layers of permeable rock from the bottom of the aquifer to the surface of the earth.

- No pressure
- Recharge occurs from above the aquifer and through lateral groundwater flow.

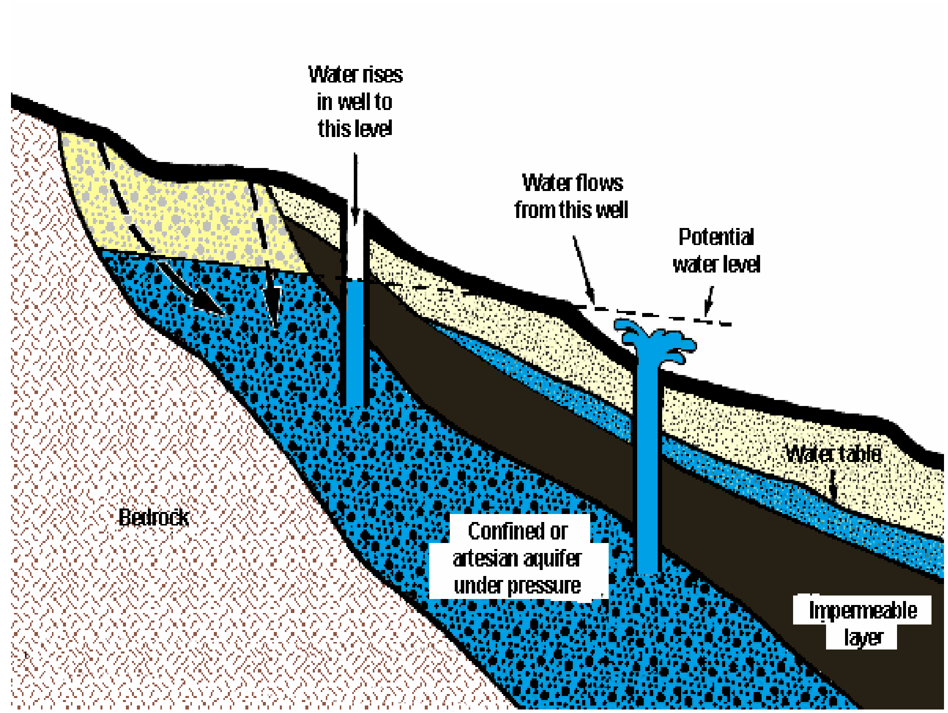


Source: http://groundwater.orst.edu/

Confined Aquifers

An aquifer is confined if it is surrounded by regions of impermeable rock.

- Under pressure.
- Recharge occurs primarily through lateral groundwater flow.



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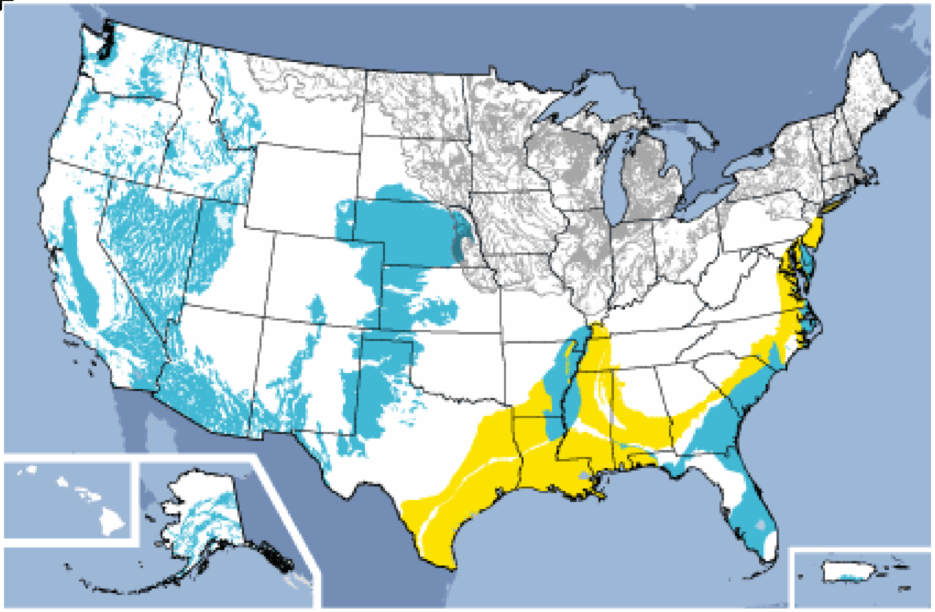
What's the big deal about aquifers?



http://capp.water.usgs.gov/aquiferBasics/images/us_uncon-semi.gif

- Important sources of freshwater. The Edwards aquifer in Texas supplies water for 2 million people.
- Aquifer depletion is a global problem.
- Solution - artificial recharge methods.

Our project

We model the flow of water through a 2-D confined aquifer. Specific cases include:

- regions of differing conducitivity
- Various initial flow situations

The governing equations for groundwater flow through an aquifer are:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{1}{K}\frac{\partial h}{\partial t}$$

$$q_x = -K\frac{\partial h}{\partial x}$$

$$q_y = -K\frac{\partial h}{\partial y}$$

$$h \rightarrow hydraulic\ head$$

$$K \rightarrow conductivity$$

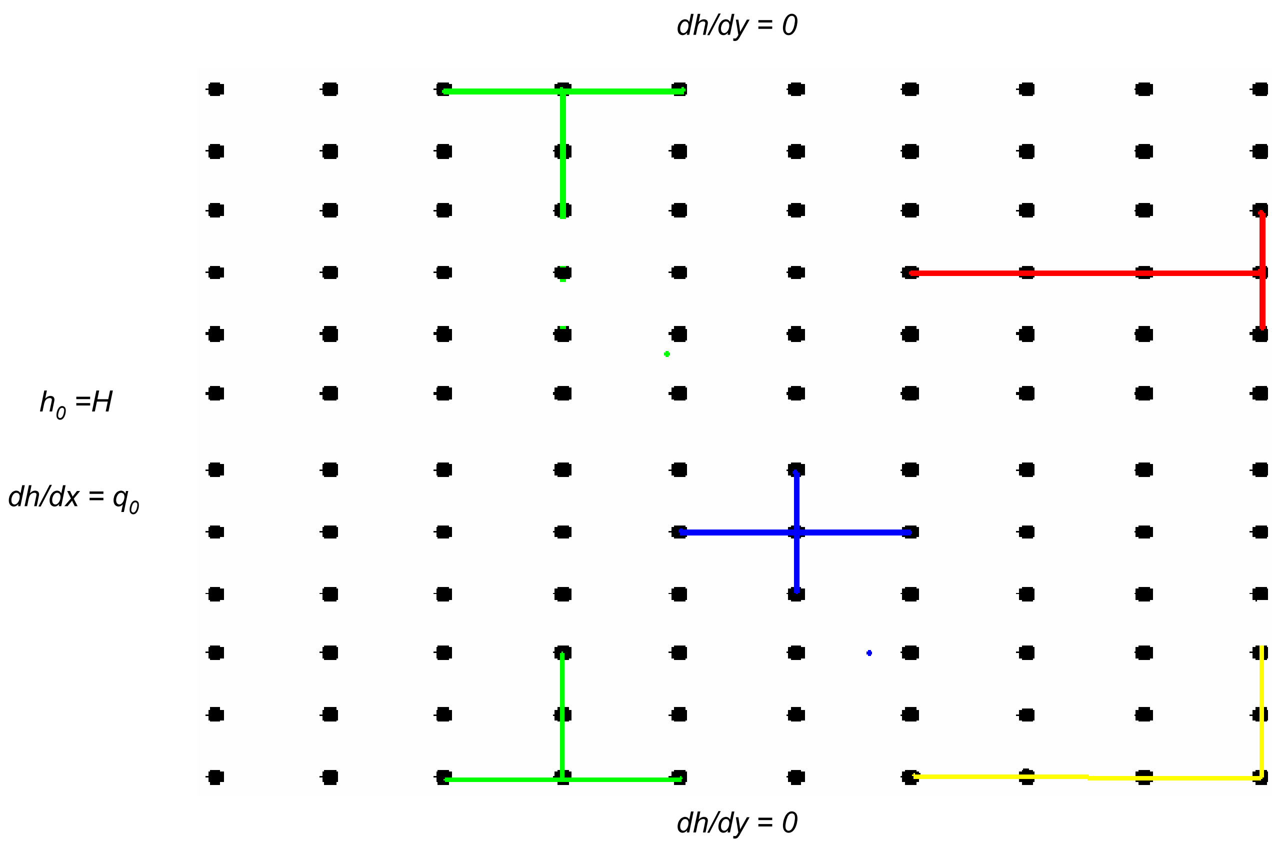
$$q_x \rightarrow flow\ velocity$$

Hydraulic head

Hydraulic head is a measure of the pressure of the water in the aquifer. This pressure depends on the depth of the aquifer and the inherent water pressure.

$$h = z + \frac{P}{\rho g}$$

Solution Strategy

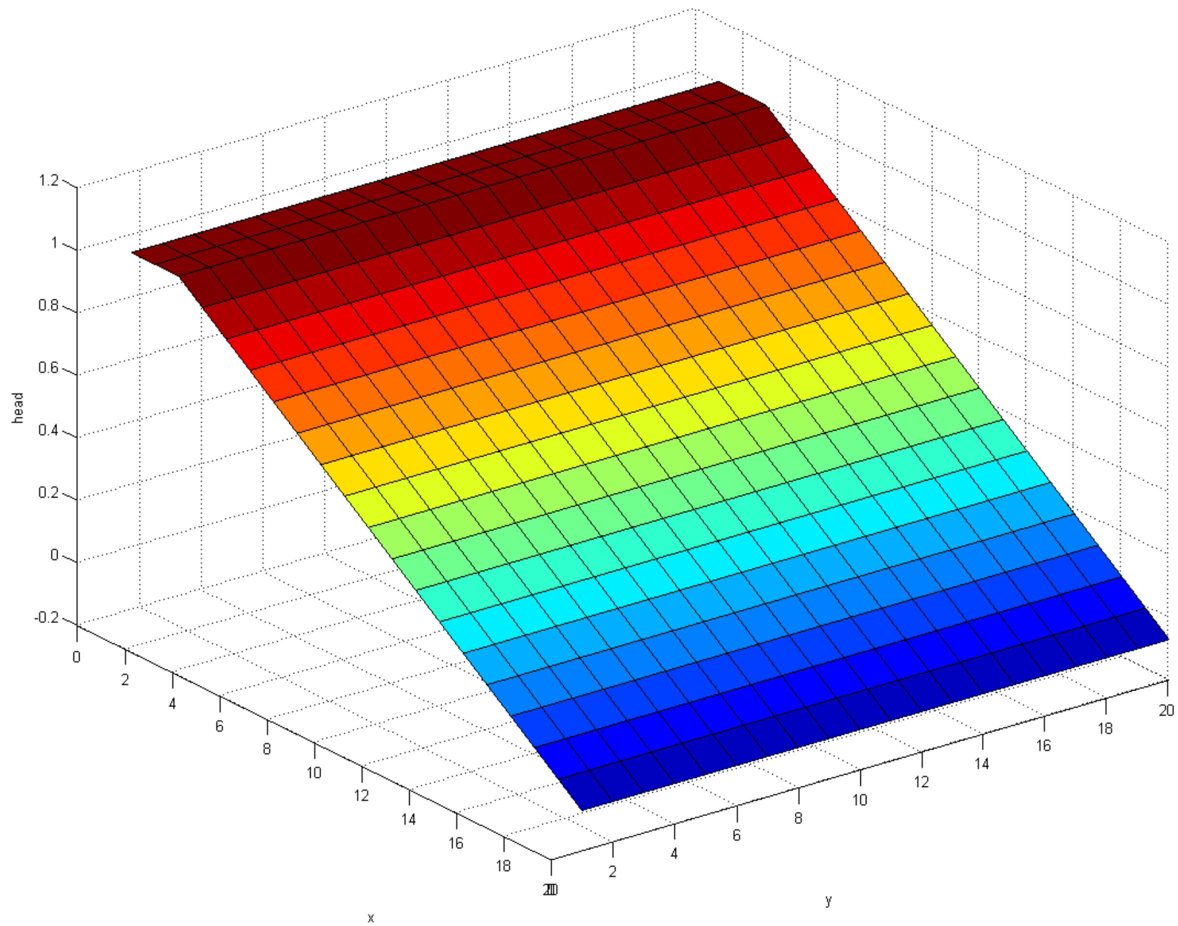


Solve the main PDE for the head as a function of x and y using a finite difference grid.

Find out the velocity of the water at different points in the grid by taking the derivative (numerically) of the head values with respect to x and y.

Results

Case 1: Solving the steady state equation (Laplace equation)



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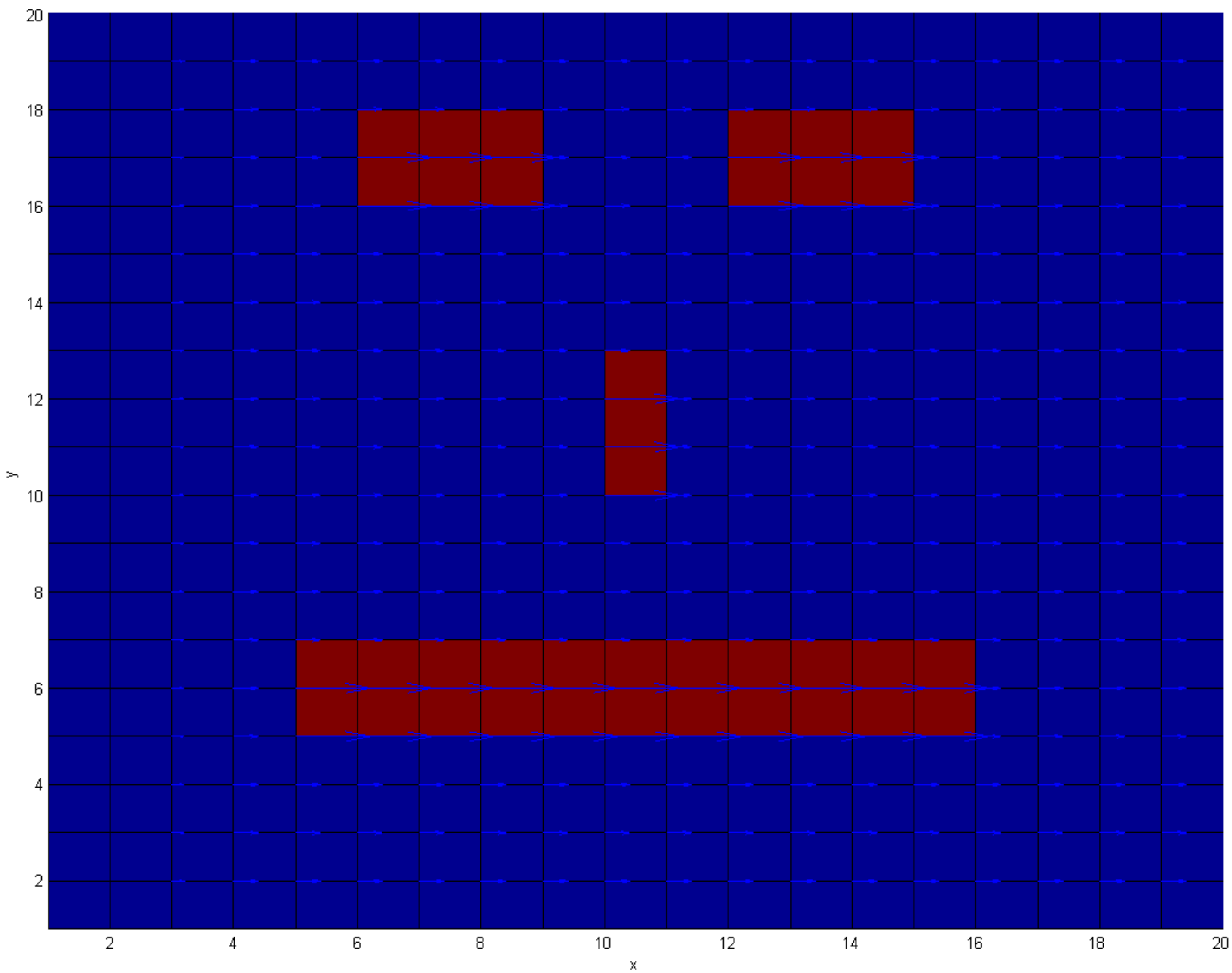
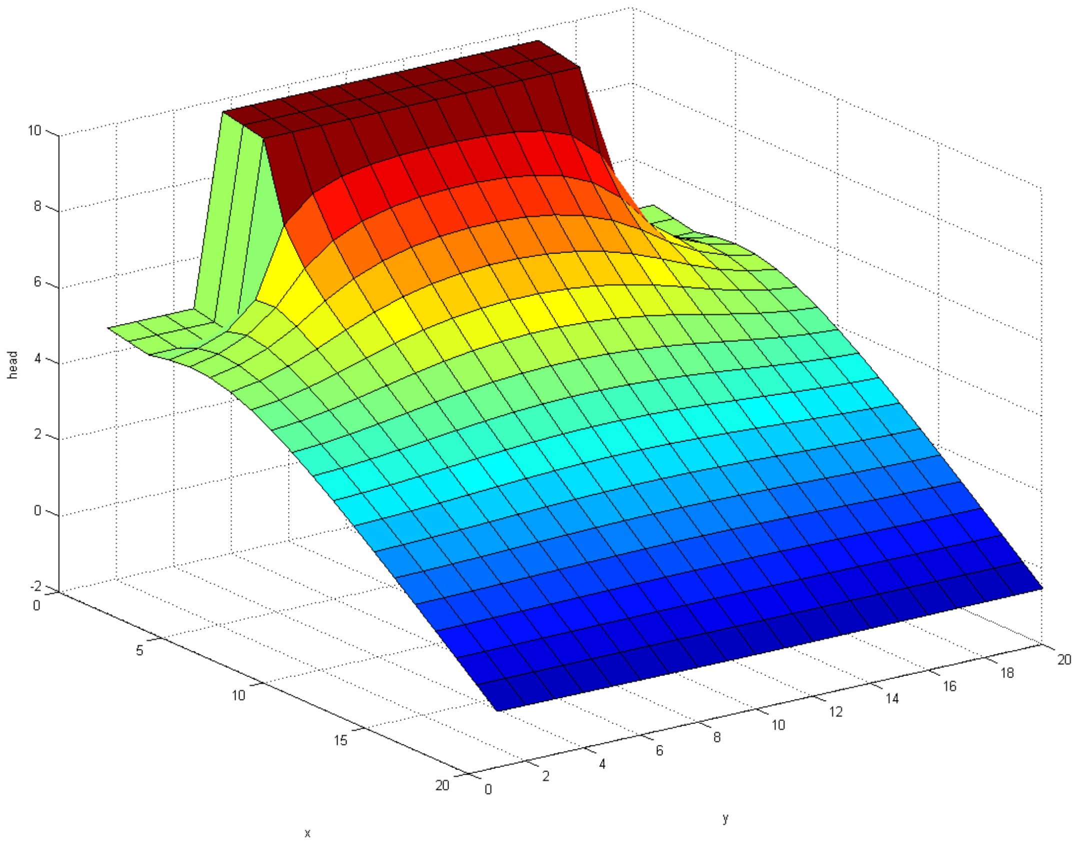


Results

Case 2: Solution to Laplace's equation for artificial injection



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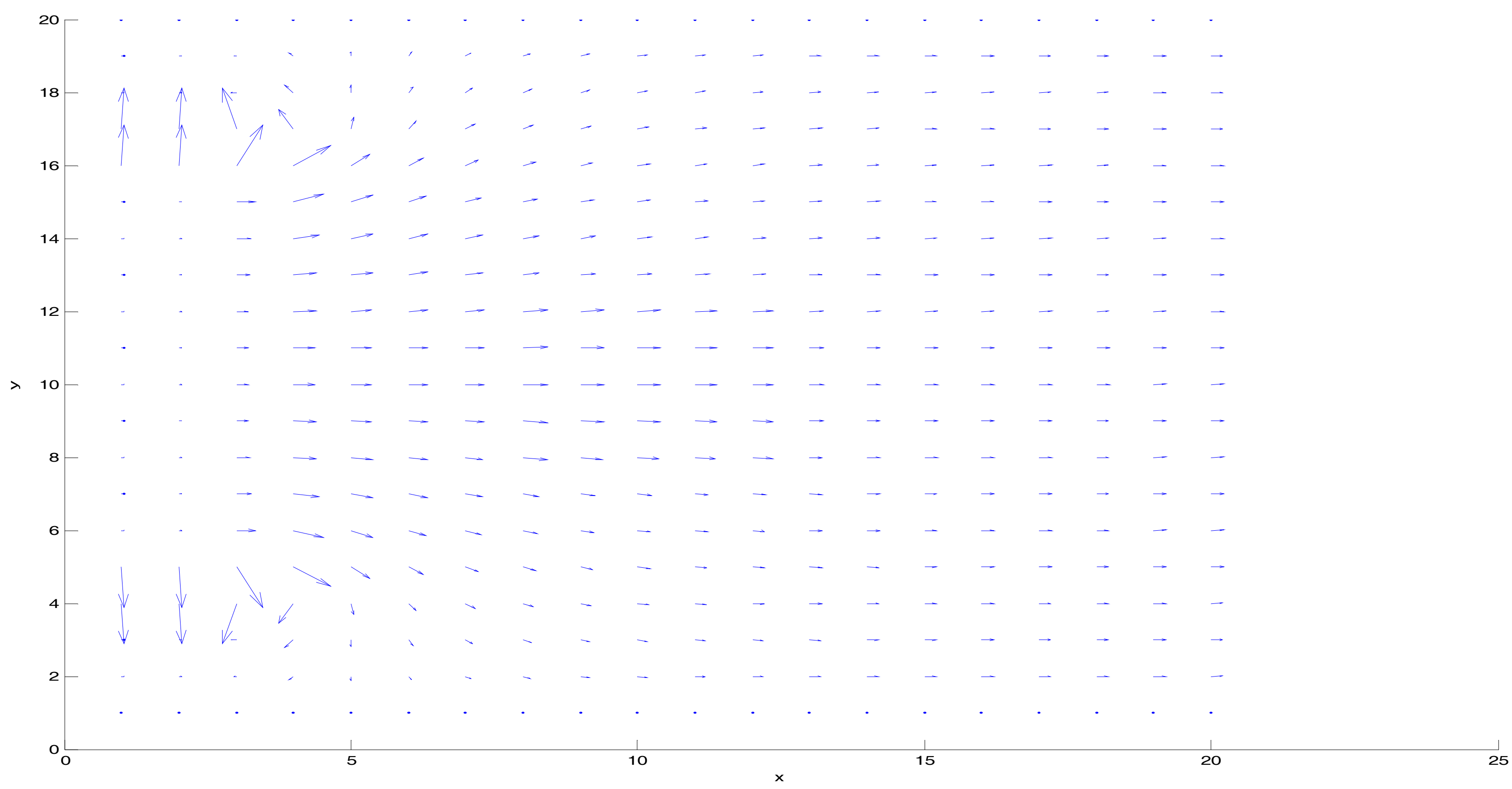

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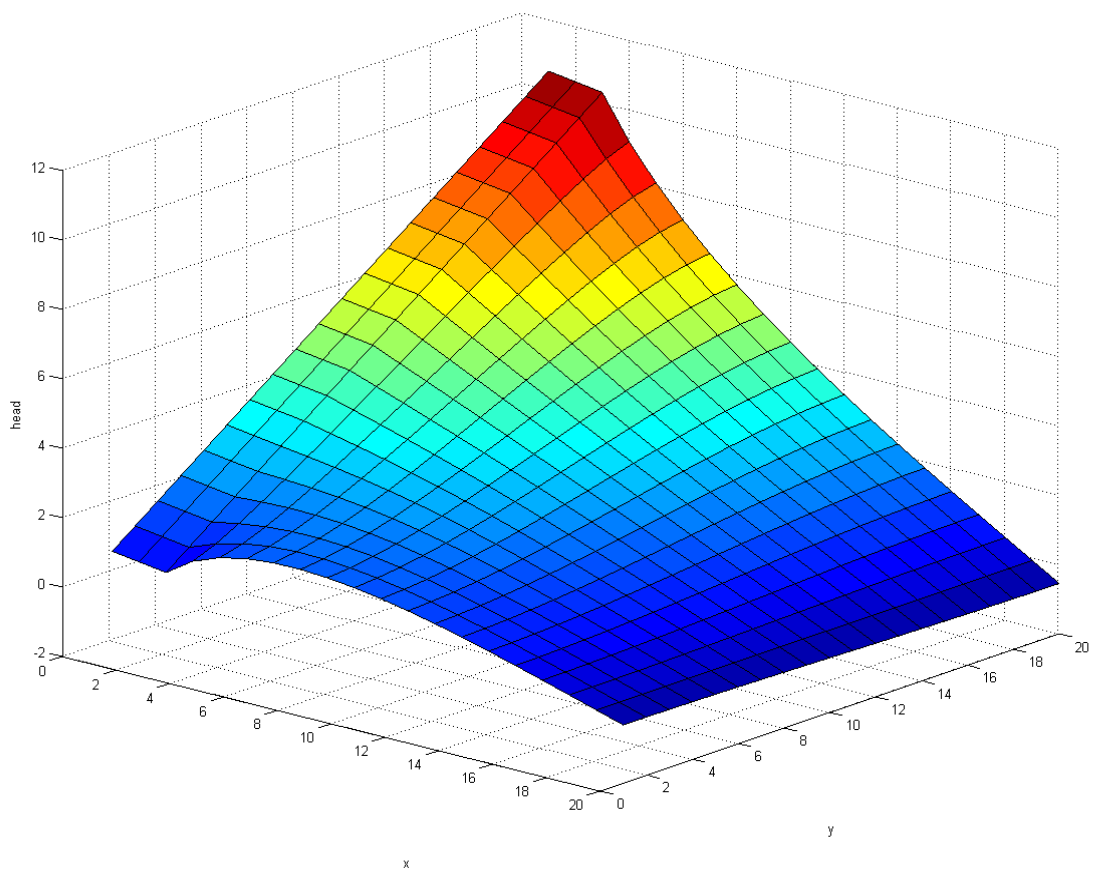
Case 3: Solution to Laplace's Equation for flow along a slope



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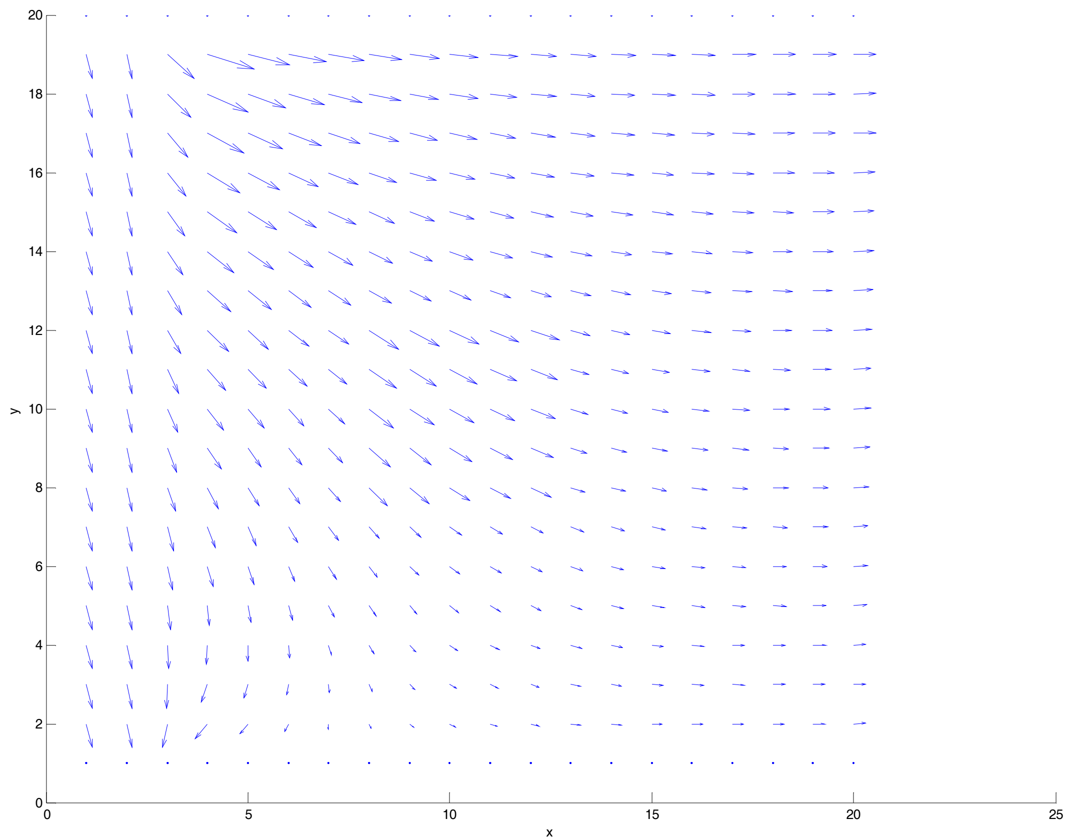
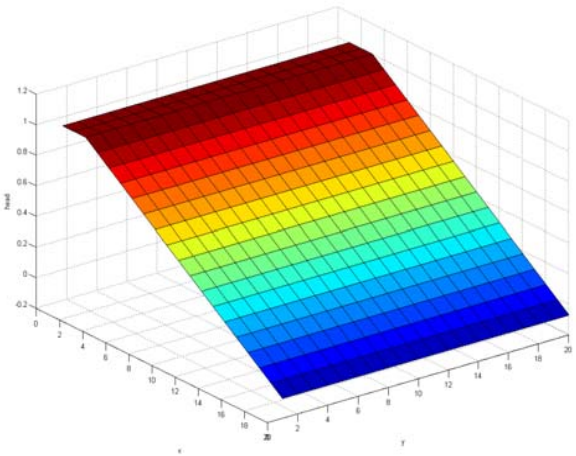
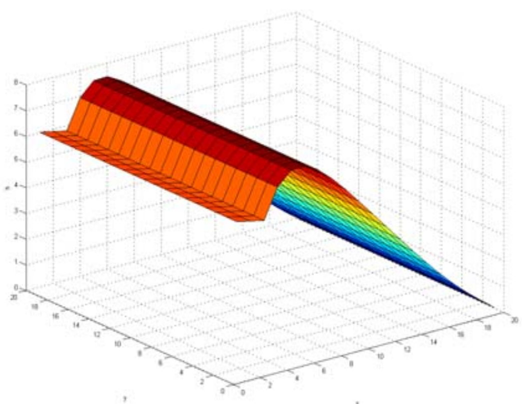
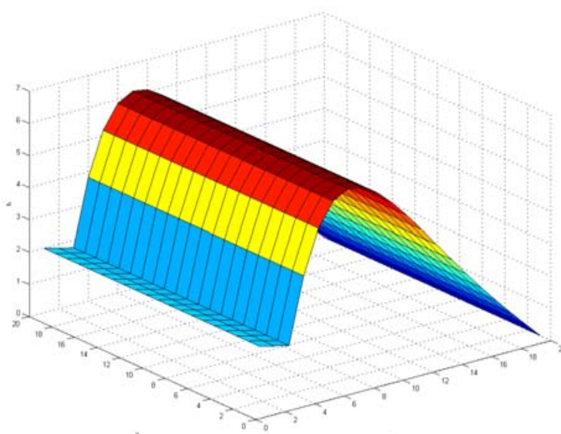


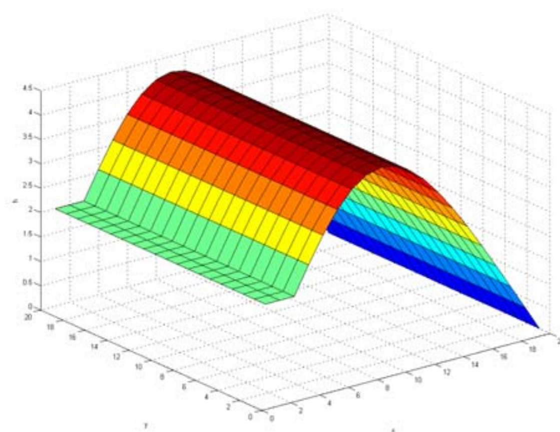
Results

Case 4: Solving the original time dependent equation









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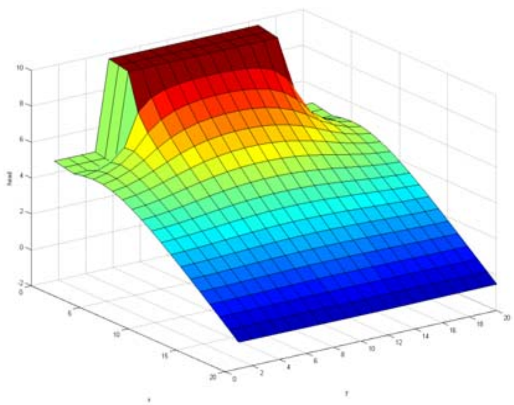

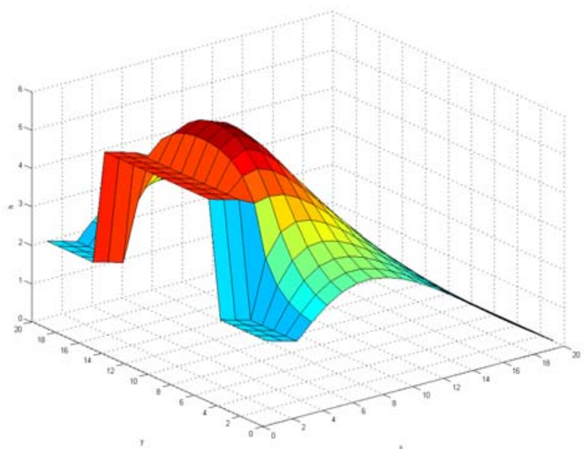

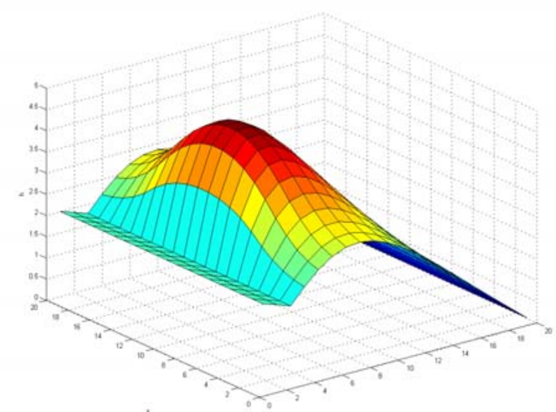

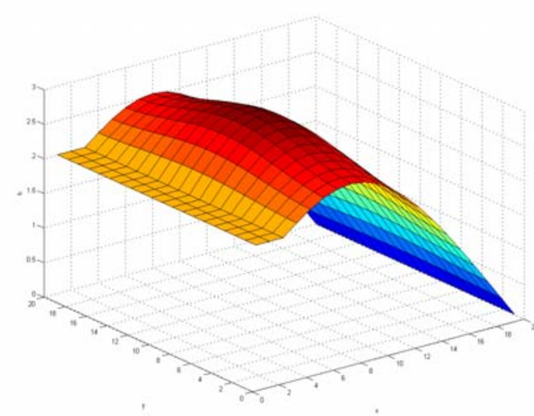

Further work

Find out the flow for an aquifer with gradually changing conductivity. The steady state equation for this case is:

$$K(x,y)\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right)+\frac{\partial K}{\partial y}\frac{\partial h}{\partial y}+\frac{\partial K}{\partial x}\frac{\partial h}{\partial x}=0$$

References

- C.W. Fetter, *Applied Hydrogeology,* Third Ed., Merill Publishing Company,1994
- Private Communication:
  - Dr. Richard Elderkin, Mathematics Dept., Pomona College
  - Dr. Linda Reinen, Geology Dept., Pomona College