MATH 712 – MATHEMATICAL LOGIC I: FORMAL LANGUAGES AND THEIR MODELS

TIME and ROOM: T/Th 12:30 in MATH 2300

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DESCRIPTION: MATH 712 is a self-contained introduction at the graduate level to contemporary model theory, which is a branch of mathematical logic. No previous study of logic is assumed, but an undergraduate course in abstract algebra (groups, rings, fields) would be helpful. Math 712 begins with an introduction to first-order structures languages and the family of definable sets of a structure. After some naive set theory, the method of ultraproducts is introduced and is used to prove the Compactness Theorem. Examples and applications in algebra, number theory and graph theory will be included. From there, the Stone topology on the space of complete types is introduced, leading to the Omitting Types Theorem. With these tools, we investigate the classes of countable models of complete theories in a countable language.

TEXT: There is no 'official textbook' for this course. Course notes written by Professor David Kueker available online and additional recommended texts will be discussed in the lectures.

EVALUATION: There will be multiple homework assignments, a take-home midterm exam, and a final exam on Friday, December 19, from 1:30 – 3:30 pm.

TOPICS: A. **Elementary logic**

1. Structures, first-order languages, and definable sets
2. Theories and their models
3. Ordinals, cardinals, and arithmetic
3. Ultraproducts and compactness
4. Completeness, Löwenheim-Skolem theorems
5. Lefschetz principles, Ax's theorem
6. Quantifier elimination and model completeness
7. Nonstandard analysis and applications to number theory

B. **Model Theory**

1. Realizing and omitting types
2. Stone spaces of complete types
3. Elementary extensions and chains
4. Prime and saturated models
5. Model construction techniques