# ON BOUNDEDLY SIMPLE GROUPS ACTING ON TREES

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A group $G$ is called $N$-*boundedly simple* if for every two nontrivial elements $g, h \in G$, the element $h$ is the product of $N$ or fewer conjugates of $g^{\pm 1}$, i.e.

$$G = \left(g^G \cup {g^{-1}}^G\right)^{\leq N}.$$

$G$ is boundedly simple if it is $N$-boundedly simple, for some natural $N$.

Tits in [2] proved that if the full automorphism group of a tree leaves no nonempty proper subtree nor an end of the tree invariant, then a subgroup generated by stabilizers of edges is simple. We show that for (almost) bi-regular trees such groups are 32-boundedly simple. We prove also the converse: if the full automorphism group of a tree leaves no nonempty proper subtree invariant and the subgroup generated by stabilizers of edges is boundedly simple, then our tree is almost bi-regular and this group is 32-boundedly simple. Hence, almost bi-regular trees are distinguished in this sense. We do not expect that the bound 32 is sharp.

We generalize this result also to the group action on a tree, i.e. if a boundedly simple group $G$ acts on a tree $A$ in such a way, that some element of $G$ stabilizes some edge and $G$ leaves no nonempty proper subtree invariant, then $A$ is almost bi-regular (so, if $A$ is not almost bi-regular, then $G$ leaves invariant some nonempty proper subtree of $A$).

Our motivation for study bounded simplicity of automorphism groups of trees comes from Bruhat-Tits building for $\mathrm{PSL}_2(K)$, where $K$ is a field with discrete valuation. That is, $\mathrm{PSL}_2(K)$ acts faithfully on a $n$-regular tree, where $n$ is the cardinality of the residue field. In fact, $\mathrm{PSL}_2(K)$ is a subgroup of an automorphism group of a regular tree generated by stabilizers of edges. On the other hand it is well known that for an arbitrary infinite field $K$, projective special linear groups $\mathrm{PSL}_n(K)$ and projective sympletic groups $\mathrm{PSp}_n(K)$ $(n \geq 2)$ are boundedly simple (as well as many other simple linear groups).

## References


[1] J. Gismatullin, *Boundedly simple groups of automorphisms of trees*, preprint 2009.

[2] J. Tits, *Sur le groupe des automorphismes d'un arbre.* (French) Essays on topology and related topics (Mémoires dédiés a Georges de Rham), 188–211, Springer 1970.



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