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Extensionalism: The Revolution in Logic

 Springer

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Motto:

*“... logic ... since Aristotle ... has been
unable to advance a step, and, thus, to all
appearances has reached its completion”*

Immanuel Kant,

The Critique of Pure Reason

Preface to the 2nd ed., 1787

*“Pure mathematics was discovered by Boole
... the fact being that Boole was too modest to
suppose his book the first ever written on
mathematics.... His book was in fact
concerned with formal logic, and this is the
same thing as mathematics”*

Bertrand Russell

Recent Work in the Philosophy of

Mathematics, 1901

Abstract

For a very long time, Aristotelian logic was accepted as a tool (*Organon*) for the generation of scientific theory. Yet, science did not quite take advantage of this tool: syllogistic terminology rarely appears in the scientific literature, and even Aristotle's own scientific theories seem to show almost no traces of his own formal tools. Does this discrepancy matter? Why was it so often claimed that Aristotle's logic was essential to science? And what, then, *was* the purpose of logic? These are the questions *Extensionalism: The Revolution in Logic* attempts to answer. The book argues that the discrepancy is indeed of great significance. It suggests that Aristotle's logic was, in fact, meant to serve as a means, not for the generation of scientific theory, but for judging it. This allowed, crucially, for the conflation of methodology, epistemology, and science. For philosophers, right up to Leibniz and Kant, this offered a means of overcoming major and seemingly insoluble philosophical problems – most notably of the skeptical sort. Logic, sustained by this conflation, functioned as a unifying principle in the attempt to make sense of the world. The problem, however, was that hence advancements in empirical science seemed to threaten the very consistency of traditional logic.

This book expands on the history of logic as the story of the undoing of the classic conflation of methodology, epistemology and science. As I attempt to show, the first stage in this undoing was the recognition of the conflation. This was achieved as early as Leibniz, who spelled it out so as to provide an explicit justification for it. Kant attempted to replace the conflation with his own system of so called transcendental logic. Others, such as Bolzano and De Morgan, achieved partial separation of logic and epistemology. But, as I will argue, it was Boole who finally managed to undo the conflation, thus setting in motion a process that reached its culmination a century later with Tarski's formal semantics. This process of recognizing the conflation and the successful attempt to undo it is the extensional revolution in logic.

My aim in this study was not the writing of history. Rather, I have tried to describe the intellectual background to the extensional revolution in logic and to understand some of its major turning points. I hope to explain how methodology, epistemology, and science were linked in a knot commonly known today by the name of Aristotelian essentialism. And I hope to analyze its overwhelming effect on modern philosophy and early modern logic. Consequently, this book attempts to

revisit one particular feature of George Boole's mathematical logic: its deliberately unsophisticated extensionalism. It was by reducing logic to the study of extensions that Boole was able to transcend, by default almost, some of Aristotle's most stubborn essentialist presuppositions, thereby inaugurating a new era in the study of formal logic. More specifically, Boole succeeded in separating the study of valid inferences from the Aristotelian endeavor to provide the complete taxonomy of all things (and of all things known) that he conflated as both logic and science.

By and large, Boole's extensionalism is taken for granted in current literature, which thus fails to acknowledge the novelty of this idea, and prefers to discuss his path-breaking technical contributions instead. My aim is to do the opposite. Without entering into the technicalities of Boole's theory, I want to focus on extensionalism and to analyze its intellectual background. The study of the technical, mathematical background to the rise of modern logic is undoubtedly an important part of the study of this rise. Yet, technical innovation does not occur in the void. It is thus surprising that the epistemological background to the rise of modern logic is hardly ever acknowledged. And it is, I suggest, precisely epistemological ideals, and above all the union of logic and empirical science, that held back the development of logic after Aristotle, right up to the early modern period. Gradual renunciation of this epistemological ideal allowed logicians the freedom to develop ever more formal logical systems, and thus to discover that this epistemological ideal is unattainable, and finally to denounce it. By discussing the philosophical impact of extensionalism and contrasting its effects with those of Aristotelian essentialism, I intend to explain the former's role within that extraordinary process of disillusionment that constitutes modern logic.

My study thus delimits the role of essentialism in the history of Western thought – positive and negative alike. I discuss the reluctance to render logic extensional and the difficulty and importance of doing so successfully.

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Chapter 2

Early Disagreements Concerning the Power of Proofs: The Uses and Misuses of Dialogues

As Aristotle tells us, and as standard histories of logic agree with him, its inception is in his reflections over a peculiar Platonic procedure known as *diaeresis*. *Diaeresis* is a refinement of a common argumentation procedure known as *dialektike*. Let me tell you briefly my opinion of what *dialektike* and *diaeresis* are and why refining them was so important.

Dialectics had sprung from reflections over the proper way of conducting the search for truth. Truth was sought by means of disputations, and already Plato's early dialogues are reports of such disputations and of the search for rules for conducting them fairly. Many interesting (then prevalent) procedures are mentioned there, tested, criticized and improved upon. A classic example is the *Protagoras*, where Socrates goes as far as to walk out on the search (335c–d) when the rules of conduct are bent by Protagoras who “sails to the ocean of speech” instead of restricting himself to a strict ping-pong of clear questions and brief and precise answers. Later on, when the offer is made to assign a moderator (338a) that would decide the results of the search he again refuses to play along until crowd and interlocutors alike agree to let the conclusion be decided naturally, jointly (338b–e). When attempting to describe *dialektike* we should remember that its exact character in Plato is understandably somewhat unfixed and perhaps even unclear as he could not formalize it.¹⁸ There is little doubt that he identified the highest form of Philosophizing with it, but he lacked variables, the need for which is apparent in his dialogues, where Socrates repeats a sentence about artisans with the change of the art thus saying the same thing over and over again, once about carpenters and once about shoemakers and once about doctors. Aristotle, as far as we know, invented the variable. This revolutionary tool enabled him to easily do away with such repetitions, to generalize them, and, for the first time in history, to study them as such. It is the explicit use of variables that made it possible to study not only particular examples of *dialektike* but also what is known today as its logical form. Since Aristotle's formulation of dialectics is the first known attempt at presenting its structure, all reconstructions of previous examples of the dialectical method are speculative. We do not even know if dialectics was a distinct method before Aristotle had formalized it (though Plato often refers to it as such). Speculations as to the nature of *dialektike* do cluster around a certain debate procedure, and this debate procedure, though rarely sanitized of what we see in hindsight as irrelevant

riders, repeatedly occurs in the discussions of Parmenides, Zeno, Socrates, Plato and the Sophists.

The meaning of “*dialektike*” is, more or less, ‘the skill of thinking together’, or ‘the ‘know-how’ of discoursing in two’. It seems to have sprung gradually as a distinct procedure of interrogation, a proper conduct of a debate, up to, and including, its very end. It was a prevalent debate procedure. Apparently it was taken to be very effective. It was employed not only in private, as a game or as part of oratory contests, but also, and more seriously so, in public debates and in law-courts. It constituted a primary tool for conducting them and concluding them. The more or less formal procedure that is most commonly identified as the core of *dialektike* goes roughly so: suppose alpha and beta are debating over some legal matter, and suppose alpha claims that P. Beta (accepting P for the sake of the game) leads alpha to admit that if P is true, then Q must also be true, and then to the admission that Q is false. They then conclude that P must inevitably be false since it leads to plain absurdities. Beta, thus, led Alpha to admit that the initial assumption P was an error, by *dialektike*.

In mathematics, as we have noted (preliminary iv), the refinement of *dialektike* into a rigorous procedure is known as indirect proof or a *reductio ad absurdum*. But the *reductio* is recognized today as distinct from ordinary refutation: it is a rigorous proof procedure practiced in formal systems – one in which a contradiction is formally derived from an inconsistent premise *thereby proving its negation*. And so we allow the identification of refutation with proof by *reductio ad absurdum* today only in restricted formal contexts. In other more loose contexts the identification is recognized as far from trivial, and in the empirical study of nature it is rightly deemed invalid since to refute a hypothesis is no proof (final and un-revisable) of its alternative. Strictly speaking, it is not even proof of its negation. We must stress this point since otherwise refutations might be taken as final, and they are certainly not: they are revisable, they can be refuted, and quite often they are.

How much of this was known to Plato and his contemporaries? Has it influenced their view of the methodological power of *dialektike*?

Before answering these questions, let us complicate things a little more by stressing yet again a point that we already stressed in the preliminaries. Let us remember that there are intermediate levels of formality and that the less formal the context, the more problematic the coupling of refutation to proof. And we are not always clear today where we should draw the line. Intuitions can go both ways. For example: the books of Euclid are written in Greek, which is far from being a formal language, yet many would maintain that they contain obvious cases of *reductio ad absurdum* (and they certainly comprise a semi-successful, and possibly conscious, attempt to slice out parts of the Greek language and to turn them into a formal language). There are, to be sure, some formal representations of Euclid’s geometry and also empirical interpretations of those representations, but the distinction between the two stems from developments remote from the world of Euclid. (What we do in non-formal and semi-formal contexts is to be judged, from a strictly logical point of view, as somewhat nebulous, even though mathematics often, and perhaps even

normally, is to be found in such contexts, as Lakatos maintained, since it is hardly ever quite formal.)¹⁹

We are thus in a delicate spot: in hindsight we deem almost trivial difficulties that the advocates of ancient logic may not even have realized properly. Plato's examples of dialectics are typically more confusing than Euclid's semi-formal proofs by *reductio ad absurdum*, because they concern metaphysics and ethics and politics, all of which notoriously less yielding to formality than geometry. Bearing this in mind we should nevertheless try to examine the following major question: assuming *dialektike* was recognized more or less as a fixed debate procedure akin to the procedure described above, what knowledge can it bring? It seems that it was considered the chief method for pursuing the truth and so it seems that deciding its scope is crucial for deciding the status of science.

In the previous chapter we considered Parmenides' proof as a case of *dialektike*, a kind of *reductio ad absurdum* of the possibility of void. Presumably this was noticed by Parmenides' disciple, Zeno of Elea, who is commonly regarded as the father of dialectics. It is, of course, reasonable to assume that at least some Pythagoreans have used it before him, as many have noted. (Popper has added that the followers of Thales did so too a generation or two earlier, when they refuted each other, though it seems that their practice of it was not as explicit, and perhaps it was not explicit at all.)²⁰ In any case, already Plato's Socrates calls Zeno the father of the method of indirect proof (*Parmenides* 128a) and of disproof (*Phaedrus* 261d) and Aristotle confirms this verdict.²¹ To be sure Zeno's work includes astounding examples of refutations in the mode later to be generally recognized as *reductio ad absurdum*: he not only refuted the critics of his mentor (who claimed that motion and diversity exist); he clearly alleged that by doing so he had proved his mentor's theory. He studied the premises of his mentor's critics and derived absurd assertions from them. Since the distinction between refutation and *reductio ad absurdum* was not born yet, it is likely that he was taken to have proved his master's theory (that the universe is a motionless dense unity) and to have refuted the claims of his critics (that the universe is plurality in flux) by the same argument.

Socrates, however, had a radically different view of what can be achieved by means of *dialektike*: in his practice, he said, dialectics served him as a tool for the search of truth, yet almost never as proof *techné*. This point is crucial. Unlike Parmenides and Zeno, he repeatedly denied that his dialectical quests are final and non-revisable. This fact is well known as a philosophical anecdote and as a *Weltanschauung* but is rarely noted from the strictly logical point of view. It is well known that Socrates self-proclaimed aim was to facilitate doubt in those deeming themselves in the know. But it is less noted that this entailed a crucial difference of opinion between him and the Eleatics regarding the power of (what would later become) Logic: while the Eleatics seem to conflate refutation with proof of the negation of the refuted proposition, Socrates never views refutation as more than just that, a mere refutation. Therefore he has claimed to have no knowledge but that of his own ignorance. Zeno, then, saw dialectics as methodology-in-the-rigid-sense (that leads to knowledge of the universe) and as part of epistemology, whereas Socrates considered it as methodology-in-the-loose-sense (that does not): he saw

dialectics as a way of pursuing the truth, perhaps as a distinct procedure for pursuing the truth, but as one that does not guarantee final positive results.

Sometimes, Socrates fascinatingly emphasizes that his dialectic refutations are the result of an impersonal force. He describes them as a *logos* with an independent will that frequently takes over him, disregarding his own will as well as that of the person being refuted. This rational infatuation (a deliberate oxymoron) is an obvious reverse allusion to his forefathers *ate* (divine infatuation).²² Tactless and indifferent to anything but the rational of argument, blind as fate herself, Socratic *logos* shatters the claims it encounters. Tactless and indifferent, blind as fate herself, it draws absurd conclusions from seemingly down-to-earth observations. Richard Robinson, who notes this in his famous study of Socratic *elenchus*,²³ claims it to be a mere manifestation of Socratic irony, for, so he contends, how could an indifferent (and hence impartial) skill continuously lead to refutations, never to positive results? This question is very sensible, of course, and yet, it should be observed that, it is guided by Robinson's own conflation of refutations with indirect proofs. Clearly, if refutations are not proofs, direct or indirect, it is not at all odd that Socrates is continuously refuting propositions without (or hardly) ever proving one. Indeed, this is perfectly coherent with Socrates' overall philosophical outlook (as well as with the modern understanding concerning the limits of logic, and the limits of knowledge in empirical science). There is certainly an irony here, but much of it is self-irony for it is clear that not even the great Socrates (and not even the great Socrates with the aid of his never-erring-whispering-daemon), could bend his dialectical skill so as to yield a proof of a comprehensible definition of some virtue, as some sophists claimed to have done. He seems to have had a solid philosophical ground to believing that this was impossible. No less important, is the fact that Socrates' ironical confession constitutes a rare testimony to *dialektike* being recognized as a more or less definite technique. It is recognized as a distinct tool, separate from the will and aims of its users, and from given cases that exemplify it. This is a ground-breaking declaration, for Aristotle had yet to introduced his logical variables and consequently the abstract consideration of *dialektike*, apart from given cases that exemplify it, was still very difficult. Socrates' insistence that his technique is an impersonal procedure is, then, the closest he gets to deeming it an abstract logical technique.

Some of Plato's early dialogues clearly achieve results that are not entirely negative despite being perplexing, of no real practical value or even downright paradoxical. Such, for example, is the final definition of courage achieved in *Laches* according to which courage is knowledge of all goods and evils (199c), and the final formulations of the definition of temperance in *Charmides* as knowledge of knowledge (166c), knowledge of ones' self (167a), knowledge of what one knows and does not know (167a), and knowledge *that* one knows and does not know (170d). In these fascinating peak moments, Socratic dialectics is subtly confronted with its own (logical) limits, as it yields what seem to be dead-ends: useless and even paradoxical conclusions (169b–c). What are we to do when our logic yields illogical conclusions? The Socrates that we meet in these moments is acutely self-aware of the graveness, helplessness and humor of the situation. He is clearly careful

not to derive any nihilistic conclusions of the sort that the sophists embraced as the grounds for their relativism. Even his divine daemon, he repeatedly stresses, has never offered him “positive” definitions, knowledge, only refutations.

Of a different category, perhaps, is Socrates’ famous geometrical inquiry with the slave in *Meno*. It seems to be an exception, for they certainly achieve some form of proof of a very limited case of Pythagoras’ theorem. However it is generally agreed that this part of the dialogue (as well as the discussion of the theory of Ideas which it serves) may be less genuinely Socratic than other parts of it, and was possibly a result of a later revision and or insertion (to the extent that such subtle matters can be decided).

Self-irony aside, it is clear that Socrates shared with Zeno the conviction that dialectics is the best means we have for the pursuit of truth. At least some sophists seem to have confronted both and laid down the first important challenge to rational speculative philosophy: they claimed (and demonstrated) that it was possible to refute any sentence as well as its negation. They boasted of the ability to refute any thesis whatsoever, regardless of whether it was true. They thus undermined the very notion of dialectics as a means of pursuing truth (with or without guarantees for success). If it is possible to refute any sentence as well as its negation, then dialectics is completely worthless as a tool of searching for the truth and it certainly isn’t a tool that guarantees final positive results. How, then, can a rational debate be conducted and decided? Is it at all possible? Is science possible? Is the use of reason as a tool for the pursuing of ultimate truth a mere misuse?

Gorgias, for example, aimed his attacks directly against Parmenides’ philosophy. He ridiculed him sarcastically in his “On that which is not, or on Nature”.²⁴ Assuming that his listeners were acquainted with Parmenides’ Philosophy, he eloquently demonstrated its negation. He argued that nothing exists, that if something does exist, then it cannot be known, and that even if it can be known, its knowledge cannot be communicated. Nature, then, is the Nothing, he showed. It is easy to mistake this semblance of a proof for a mere poignant parody. However, when taken, as it should be, together with the actual Parmenidean counterpart that proves the exact opposite thesis, it is one of the most profound philosophical attacks ever produced against rational debate as a means for pursuing the truth. For it is a demonstration that proofs are useless, that anything can be proved.²⁵ Gorgias’ anti-philosophical dialectic was generalized and systematized by his followers prominent among whom was his student Isocrates, who may have started the tradition of viewing dialectics as intellectual gymnastics, a tool guaranteeing superiority in public debates, regardless of the truth of the matter at hand and in frank contempt for the arrogance of philosophical quests as opposed to commonsense.

(It is not unlikely that some such sophistic attack on the very logic of Parmenides’ proof and its compelling power was the license that the atomists needed to invert it. For where Parmenides assumes that the void is nothing (and hence) that it does not exist (and hence) that the universe is dense (and hence) that movement is impossible, the atomists freely take license to start by negating his conclusion. Thus they freely assume that movement exists and therefore the universe cannot be dense, from which it follows that the void, must exist.)

Plato justly found this startling: if the sophists are right and everything can be proved, then perhaps the love of wisdom is a mere fantasy. He launched a propaganda campaign against them. It was one of the most successful propaganda campaigns in history. He is largely responsible for the derogatory connotations of the very word “sophist”.²⁶ He dismissed them as troublemakers, spreading corruption and confusion by empty rhetoric (another word he may have either invented or re-defined as derogatory in the course of his attacks).²⁷ He portrayed them all as moral relativists, political pessimists, and even shameless opportunists motivated by sheer selfish objectives. Sometimes he went as far as portraying them as hopelessly arrogant nihilistic fools (as in the *Euthydemus*). He refused to acknowledge that their relativism, however morally deplorable, was grounded in a serious challenge to rational philosophy. And yet he accepted their challenge all the same, for it was, to repeat, a serious challenge. He was clearly dissatisfied even with Socrates’ solution – the unending quest for truth by refutations of the common opinions – and hoped to reinstate refutations as indirect proofs, in Eleatic fashion. Accordingly, in some of his late dialogues, a cautious attempt to formulate Eleatic *dialektike* takes over. A striking example is the dialogue *Sophist*, where a stranger (the “stranger from Elea”) proves axioms of Plato’s theory of ideas, by dialectical means, while Socrates silently steps aside and pays heed.

This brings us to the idea of *diaeresis* (literally: splitting in two). The very idea of successful proof by *dialektike*, explains the mysterious stranger, assumes it. Proof requires the imposition of a strict dilemma (a strict ‘either or’ choice) on one’s opponent, so as to clinch the identification of refutations with indirect proofs. If we agree, for example, that Socrates is an animal and furthermore, that all animals should be divided exclusively into rational and non-rational, then the refutation of the assertion that Socrates is non-rational is also proof that he must be rational. This form of argumentation, Aristotle tells us, was a fundamental practice in Plato’s academy. Indeed, when Aristotle defines *dialektike* he incorporates Plato’s refinement into his definition by demanding that all dialectical quest begin with a reply to an ‘either-or’ question (*An. Pr.* 24a22–24a25).

In hindsight, it is easy to see that at most, Plato was reflecting and specifying in a clear and rigorous manner an assumption that Zeno has already made. We do not know whether or not Zeno had also assumed it explicitly but it is certainly reasonable that he was well aware of it (and Gorgias clearly is aware of it). And since this seems to be all that Plato’s response to the sophists challenge amounts to, the challenge remained. Plato’s dismissal of the sophists was, thus, more an expression of limitation, and personal disgust, than a suitable philosophical response (it bears some resemblance to his alleged dismissal of Antisthenes as slow-minded, and of the atomists, whom he never even mentions by name and always refers to by derogatory descriptions). He could not undo the sophists’ demonstration of the possibility to refute both thesis and anti-thesis, even when a strict dilemma is imposed. Their demonstration was that the imposition of a strict dilemma is useless because both thesis and anti-thesis can be refuted. Indeed the explicit imposition of the dilemma only strengthens their claim that proofs are impossible. To the best of our

knowledge, Plato never answered this challenge in a satisfying manner, indeed (in most contexts) no one has.

Let us sum up then. Searching for early confluences of the logical and the extra-logical, we have given a brief account of the scope of the argumentation technique known as *dialektike* before the inception of logic with Aristotle. We noted that the controversy regarding the scope of logical techniques was central to the development of philosophy even before logic was explicitly formulated, and before logical techniques were cleared of irrelevant riders. For the development of logic was the heart of attempts to answer the following questions: Can truth be discovered by reason? Is *dialektike* a mere loose method of refutation or is it the rigid proof method guaranteeing that truth will be found? Perhaps it is neither, since it is both, as anything can be proved and, or, disproved by its use? The arguments of Parmenides, Zeno and Plato, exhibits a tacit conflation of early notions refutation and indirect proof. They thus exhibit a tacit conflation of the two senses of methodology, the loose (refutations) and the rigid (indirect proofs) and even a conflation of both with science *per se* (by *diaeresis*) and hence, even with epistemology.

Chapter 4

The Beginning is the Term

Any discussion of Aristotle's logic should begin with an explanation of his extremely subtle notion of a term. Aristotle's logic is a logic of terms. Terms, not predicates, or sentences, or propositions (not even judgments) are its building blocks. I intend to show here that Aristotle's notion of a term is the source of his conflation of methodology and epistemology. It is the gist of his essentialism. Explaining this claim will be the focus of the next five chapters.

Literally "term" is "an end", a sentence end (*horos*, in Greek, *terminus* in Latin, means an edge, a limit or an end). This literal meaning suggests a structural definition: it suggests that the only thing necessary for identifying terms is pinpointing a position in a sentence. It tells us that a sentence is the composition of two 'ends' and a middle (the middle being a special binder, the copula). This definition is highly misleading, however, exactly because it seems to suggest that anything that can grammatically occupy a sentence's beginning or end is a legitimate term. This is not the case in Aristotle's logic.

Aristotle is the first that we know of to introduce variables explicitly. What is a variable, then? A variable is a sign replaceable by a term. The list of terms that can replace it is usually pre-determined. For example, in arithmetic the variable "x" in the formula " $2 + x = 5$ " is replaceable by any numeral (representing a number, in this or that mathematical field; and it is true only if that number is 3). In Boyle's (original) law "pressure is proportional to density" the words "pressure" and "density" are variables and both have to apply to some given quantity of air. Even the term "someone" in the sentence "Someone pinched my grapes" can be viewed as a variable, especially if it is replaceable by any of the names of my possible suspects.³⁷ Variables do not have to be replaced by any of the terms that they stand for and this is their most cherished feature: they constitute an apparatus which grants us the freedom to discuss such things as sets of numbers, volumes of air and suspects. We can do so in a loose, abstract setting committing to neither specific terms, nor to abstract ontology regarding sets and other non-material objects.

It is sometimes mentioned that a common practice of Greek mathematicians preceded and heralded Aristotle's groundbreaking discovery: when writing proofs of various sorts they habitually used shorthand; they wrote only the first letter of some terms and/or used the letters of the alphabet for numerals. (One dominant such method had alpha standing for 1, beta for 2, and so on). The practice, so it is

sometimes suggested, heralds Aristotle's discovery. This claim is off the point, and it is misleading. Using the letter S, as shorthand for "Socrates", or for "Square", or for "Seven" does not increase the level of abstraction of our discussion. Shorthand may lead to abstraction but not by itself, as is made evident by the very fact that something like it is almost universal without eliciting the use of variables. Thus, to take one notable example, we are told that, strictly speaking, only a few specific instances of Pythagoras' theorem were proved in Plato's days, at his Academy, not the general formula which, though certainly apprehended, implied the existence of irrational numbers and thus could neither be proved nor admitted. The value of a variable, then, is not in it being shorthand for a given term, but rather in it being an apparatus for handling abstraction, replaceable in principle by any of a list of terms, a possibly infinite list of terms (such as in the case of Pythagoras theorem).

Aristotle's aim was far and above that of enabling us to write shorter computations, of course. It was, at least in part, to examine the observed similarities, symmetries and isomorphism between inferences that made them all valid or invalid. Consider, for example, the rubber stamp example of an inferences "All Greeks are human, All humans are mortal, Therefore all Greeks are mortal". Let me repeat myself and say that Aristotle was not the first to observe such an inference, and possibly he was not the first to produce a shorthand version of it. (Since writing was so cumbersome those days, shorthand seems to have been almost inevitable.) He was also not the first to notice that other inferences are structurally similar to it. But he was the first that we know of to have constituted an apparatus which enabled to formulate and study these shared structures (and thus to declare them isomorphic, as opposed to merely similar). Thus, when Socrates defends the validity of an inference, he intuitively invents similar ones. Aristotle does better: he formulates their shared structure and proves it to be valid. In this manner, strictly speaking, he is not dependent on intuition when providing similar ones and can rest assured that no counter-examples ever arise. The logical form expressed by "All A's are B's, All B's are C's, therefore All A's are C's" is clearly shared by the above inference and by the following: "All unicorns are white animals, All white animals are good-natured, therefore All unicorns are good-natured". It is the formulation and discussion of the laws that apply to such symmetries that truly signify the birth of logic.

However, the above presentation is a considerable idealization of Aristotle's logic. The question before us is this: what terms can replace variables in Aristotle's syllogisms? The answer to it is short and yet devious and subtle: in Aristotle's logic the only legitimate terms are names of essences (and the genera and species that are required for defining them). How can we tell which terms are names of essences (and which are genera or species)? How can we depict, out of the pool of possible candidate terms, those that are genuine names of essences? (How can we tell genuine genera from apparent genera, and genuine species from apparent species?) Modern logicians explicitly ignore such questions: they declare that, if they are meaningful and answerable at all, they are certainly extra-logical. This is exactly why we say, today, that logic is methodology-in-the-loose-sense. How did Aristotle handle these questions? Did he deem the answer to them as extra-logical too?

Aristotle does make it very clear that his logic is not a formal study of all possible terms. Despite his admirably free use of variables (most notably in his *Prior Analytics*) his logic was not intended as a content-insensitive theory of inference. It is certainly not a predicate calculus without quantifiers, as it is sometimes presented in anachronistic contexts. In order to demonstrate this claim we must first note a few clear-cut restrictions which he makes explicitly. He divides terms roughly into three groups: proper names, such as “Socrates”, abstract names, such as “Men”, and super-abstract names, or categories, such as “substance”, “quantity” and “quality”.³⁸ In an attempt to delineate agreeable terms, Aristotle accepts into his logic *only* terms of the second group: abstract terms that do not designate categories. So, as many have noted, following Lukasiewicz, even the classical inference “All men are mortal, Socrates is a man, therefore Socrates is a mortal” is, strictly speaking, not a proper subject matter of Aristotle’s logic, since it contains the proper name “Socrates”. Similarly, the inference “Quantity is a category, categories are exceptionally abstract names, therefore quantity is an exceptionally abstract name” has no place in the realm of Aristotle’s logic: it refers to levels of abstraction that, according to Aristotle, are beyond the province of logic.³⁹

As a matter of fact this level of abstraction seems to be beyond the realm of Aristotle’s metaphysics as well: for he declares that the 10 categories are the most abstract of terms, and in the above syllogism the term “Category” is potentially more abstract than any of these 10 terms (for it includes all of them, all particular categories; thus, the observation that there are exactly 10 categories is an Aristotelian oxymoron) and the term “an exceptionally abstract name” is potentially even more problematic than the term “Category” (for it implies that some categories are more abstract than others, an implication that is unacceptable in Aristotle). More importantly, however, as many have noted (e.g. G. Patzig 1968, p.7), the above restrictions imply Aristotle’s refusal to admit into the province of logic all terms that represent the Universe of discourse (the Universal class), which is forbidden as it is more abstract than any of the categories, as well as all those terms that depict the empty-class (the complement class of the Universe of discourse). Thus, even the abovementioned syllogism about unicorns (“All unicorns are white animals, All white animals are good-natured, therefore All unicorns are good-natured”) was not admitted into Aristotle’s logic. “All unicorns” is neither a name of an existing genus nor a name of an existing species and so it is not a name for an essence: there are no unicorns.⁴⁰ In Aristotle’s logic, terms that depict impossible entities, non-existent entities, hypothetical entities, and even arbitrarily grouped together entities, are, thus, considered highly problematic and deemed logically undesirable.

Aristotle’s restrictions upon terms are not all as explicit as the ones that we just discussed. Here the fine (modern) line between epistemology and logic is blurred. It is useful to distinguish here between terms that he admitted into the province of his logic, and tag them collectively “legitimate terms”, and contrast them, with the terms that he barred, for various subtle reasons, problematic terms, that we will tag here “non-legitimate terms”. The reason for introducing here the new technical term – “legitimate/non-legitimate term” (which will accompany us throughout this

book) – is due to my wish to lump together several groups of barred terms, while avoiding exegetical discussions of Aristotle’s “true” intensions as far as possible, resorting instead to a more general analysis of the logical province that he was studying, thus arriving, in as far as this is possible at all, at a fair description of the options he faced and the rationale of his system. The criteria by which Aristotle distinguished between what I call here legitimate and non-legitimate terms are, to repeat, diverse, and are not always explicit. Often they come as a scientific or metaphysical afterthought. Full-fledged formal rules do not come close to comprising them. This is a highly problematical situation. We do not have strict Aristotelian rules for determining whether or not a term depicts an essence and whether or not it is admitted into his province of logic. (We do not know exactly what an essence is.) Is it the case, then, that in Aristotle we can only pursue our logical inquiries once our scientific inquiries have ended and all that exists was found and listed in proper taxonomies (whatever “proper” means here exactly)? Must the essences that comprise our cosmos be fully inventoried before we can make our first steps into logic? Should it not be the opposite, that logic is valid irrespective of our knowledge of the cosmos? Should it not be the case that logic is an inseparable part of the tool kit we use in our search for knowledge of the cosmos, including (if we so wish) knowledge of essences? It is not easy to extract a single unequivocal answer to these questions, in Aristotle. Sometimes knowledge of essences seems to be, for him, a prerequisite for the very possibility of logical enquiries. On other times, it seems that it is not a prerequisite at all (since this would ultimately mean that the sophist was right all along, that their challenge is unanswerable) but rather an inevitable result of logical inquiries. The conflation of prerequisites and results is, in hindsight, the gist of all circular justification. Important hints as to Aristotle’s plan do exist, however. They have been studied since antiquity and will be inspected here. (The reference to science and metaphysics is unhelpful, of course, but it does complicate things in a manner that had a tremendous influence upon the development of logic.)

According to Aristotle, the terms ‘man’ and ‘goat’ are legitimate since they denote species, natural kinds, that is, groupings of particulars that share an essence; not so the terms “goat-man”, “man or goat”, “man and goat” “not-man”, “not-goat-man”, “not-not goat”, etc., Aristotle’s metaphysics attempts to make the division hold. The reason for the illegitimacy of all complement classes is that there is no essence common to all of them. For example, the term “not-man” is problematic because there is no essence common to all those (stones, plants, and brutes) which are not-man.⁴¹ Likewise goat-men do not exist, and hence their essence does not exist. Consequently “goat-man” as a term designating a species is meaningless, and the term is rendered illegitimate. Not-goat-men are in an even worse predicament since they are a complement class of an empty existent class. The problems with Aristotle’s notion of a term do not end here. In fact they just begin: for it is simple to show that an infinite list of illegitimate terms can always be produced from any list of legitimate terms (that allegedly refer to genuine essences) and the logical operations we now call elementary set operations. (This is precisely what was demonstrated above by means of the terms “man” and “goat”.) Aristotle’s logic is

usually presented as some formal theory of deductive derivations that is limited to a closed list of pre-chosen terms. However, it is impossible to provide that list, the list of the “legitimate” terms that are allowed in it, both because some of the criteria for “legitimacy” are unclear (as the very notion of essence is unclear) and because it is impossible to keep any list of allegedly legitimate terms closed to non-legitimate terms without thereby disallowing the most basic logical operations between these terms.⁴²

To make things even more complicated, sometimes Aristotle further divided the second group of terms into two distinct groups: he distinguished plural names such as “Men” on the one hand, and still more abstract terms such as “Man” – meaning the idea of a human being – on the other hand. To a large extent, these had already been identified by him as extension and intension, respectively. (His understanding of extension and intension is discussed in chapter 10, in which his theory of induction is explained.) In general, terms can be read as extensional (i.e. as designating an object or a collection of objects) and as intensional (i.e. as designating a property or an idea). Yet there is no explicit differentiation between extensional and, intensional syllogisms in Aristotle since, as we will see (in chapter 10), the fusion of extension and intension by induction is a pillar of his system. (It plays a crucial role in his epistemology which is practically founded on the claim that logic is limited to intensional relations, expressed by definitions, and that these are established by observation of particulars and the comparison of the extensional relations that observable classes of particulars form.) This subtle view created a fundamental problem in Aristotle’s logic. The inference “Humans are primates, Greeks are human, therefore Greeks are primates” is valid. Yet the inference “Humans are numerous, Greeks are human, therefore Greeks are numerous” seems invalid. The inference “Human is a form of behavior, Greeks are human, therefore Greeks are a form of behavior” too seems to be invalid. But all three inferences, on the face of it, seem to share a logical form, as long as extension and intension are not explicitly differentiated. Aristotle’s logic, then, rests not only on a putatively intuitive choice of admitted and barred terms, it rests on a putatively intuitive choice of admitted and barred senses of these terms. The problem, however, is that Aristotle could not make such a division fully explicit: he could not simply dismiss the last two inferences as extra logical, or as invalid, on the ground that they illegitimately fuse intension and extension. This is because, to repeat, he resorted to that very fusion in his theory of inductive inferences (which he regards as a special type of syllogisms), and which justifies, for him, the feasibility of the split between legitimate and non-legitimate terms. The fusion, then, is both a threat to the validity of his logic and a manner of securing it. A strong inner conflict in Aristotle’s theory is present here, one that neatly exemplifies its problematic character. It is, to the best of my judgment, nowhere to be resolved in his known texts. Rather, Aristotle disregards it when context allows for disregard. He seems to declare logic the study of intensions only when context allows, but then, clearly, this is not always the case, and if it is so, it seems that his claim to have secured the status of science by induction collapses, as we will observe later on. (With it, let me stress again, the very distinction between legitimate and non-legitimate terms collapses, and so the whole

of the magnificent Aristotelian system.) The point to observe, however, is not that Aristotle's epistemology may be problematic. It is that his logic presupposes a strict choice of terms (and of senses of terms) that his epistemology cannot allow.

The distinction between legitimate and non-legitimate terms is basic to classical logic and is ignored in modern logic. Therefore, readers of this book who are somewhat familiar with modern logic or mathematics or computer science may fail to see the historical impact of this classic and important distinction, whereas readers who are trained in classical biology or in the arts and humanities may fail to see the importance of consciously ignoring the distinction in modern logic. Allow me, then, a few explanatory remarks.

The curious and influential idea that successful discourse presupposes a list of all proper taxonomical terms is, essentially, Plato's. In such dialogues as *Sophist* and *Statesman* the stranger (who comes from Elea) explains the division of terms which underlies the search for definitions by *diaeresis*. It is, he says, a serious metaphysical affair, subtle and delicate. Improper splits give rise to unnatural terms leading to false classifications. And then the validity of *diaeresis* which relies on these splits, and the truth of the definition obtained by it are threatened. Socrates of the *Phaedrus* (265d) explicates this perplexing warning: dialecticians, he tells us, dissect the cosmos into forms as a carver of meat, a butcher, dissects the body of an animal: a perfunctory cut, made carelessly, not along the natural joints of the animal results in a useless piece. (The Greeks, alas, must have disliked T-bones). It is as if the world of forms is a giant goat, or –to be less carnivorous– a giant cake that can be cut in slices by means of our terms.⁴³ Some cuts produce authentic parts, slices, glorious slices corresponding to real platonic forms. Others do not. Some terms designate carefully delineated eternal ideas. Others do not. The former will serve us well in our dialectical pursuit of definitions; they will not threaten the validity of *diaeresis*. The latter will spoil our pursuits for true knowledge and mislead us. We must, then, use only terms that depict real, authentic, slices of reality.

Let us observe an example. "Animal" can be properly split into "Brute" and "Man", says the stranger, yet "Man" is improperly split into "Greeks" and "Barbarians". Why? Modern philosophers say that Man is a natural kind and Barbarian is not. Why? Perhaps the term Man seems well defined and sufficiently uncontroversial. Some may wish to define "Barbarian" so as to make it as clear and uncontroversial as Man. This, Plato seems to caution us, is dangerous. It is possibly doomed to fail. Barbarians are humans whose discourse makes no sense to us (to Greeks). This is its definition. It is a bad definition: clearly some Greeks are mute, some are deaf, some are poor speakers of Greek and some were born in foreign lands and do not speak Greek at all. Some non-Greeks may speak Greek (even if poorly), and so on. The borders of Greece are conventional and changing (and pureblooded nations are fictitious illusions anyway). So the split of all men into Greeks and Barbarians is an artificial (not a natural) one: it allows for confusing borderline cases (Greek speaking Persians, for example). Borderline case cannot mirror the heavenly world of forms, for they do not represent authentic slice of reality.

Borderline cases seemed to threaten the validity of *diaeresis* then: if two terms (A and B) do not sharply split our universe, if x can be a borderline case between them, then we cannot derive “x is B” from “x is either A or B” and, “x is not A”. A Greek speaking Persian seems neither Greek nor Barbarian, and then showing that he is not a Greek is not the same as showing that he is a Barbarian. This is the source of the stranger’s warning against the use of careless divisions: if they are allowed, he suggests, *diaeresis* cannot guarantee for the attainment of the truth.

This point is extremely subtle and confusing, but it is also highly important. I regard it as the corner stone of Aristotle’s logic. If terms are regarded as some kind of ontological hooks on proper parts of the world, then using the wrong terms may seem to result in unsuccessful fishing. This is clearly undesirable. The attainment of the truth, suggests the stranger, is possible only if discussion is limited to “true” terms, i.e. correct splits (that we called “legitimate terms” above).⁴⁴ Socrates is no brute; therefore, he is a man. This inference is valid, on the understanding that (in the universe of discourse) everything is either man or brute and nothing, absolutely nothing, is both.

We should emphasize, however, a somewhat obvious yet crucial point: Plato’s dialectics was not really threatened by arbitrary splits. Barbarians and Greeks can be mutually exclusive with or without mirroring some authentic slice of reality. Many commentators and logicians throughout history came to acknowledge this point. But not until Boole was it fully internalized as part of logic proper, by means of neglecting altogether the notion of essence and the search, by means of logic, of essential definitions. Certainly Plato and Aristotle do not seem to fully appreciate it.

Paradoxically (and most interestingly) for the sake of illustration, it is best to forget Plato’s world of Forms and Aristotle’s world of essences for a brief moment, and use a Democritian model of the universe to visualize the conception of reality implied by the ancient view of terms.⁴⁵ Let us assume that reality is a gigantic Lego structure, containing pieces of diverse lengths and colors. Assume that the only parameters for comparing these pieces are their size and color. Then, some splits of that world are clearly legitimate and some not. An authentic slice of reality is a distinct Lego piece, or a distinct set of Lego pieces of equal length and/or color. Furthermore, we may wish that terms of our language would designate only those sets of elements that are chosen by legitimate splits. A term is unnatural, illegitimate, or false, if it designates none of the above but rather designates a part of a piece or a partial set of pieces, or a mixture of such partial sets. A term is unnatural, illegitimate, or false, if and only if it makes us break the true structure of reality in search for its reference. That such improper breaking up of the real world is possible, and that it is to be feared, is made very clear by both Plato and Aristotle.

This, by the way, is the rational behind Aristotle’s famous definition of truth. “To say of that which is, that it is, and of that which is not, that it is not, is to say the truth”. To read his texts with Tarski’s modern alternative to it in mind is problematic and misleading, for Aristotle seems to have in mind the act of pairing attributes and objects, tagging terms to entities, not the act of tagging truth conditions to propositions.⁴⁶

Aristotle – who had considerable criticism of the Platonic practice of dialectics – followed Plato’s main assumption closely: he took the legitimacy of certain terms, as well as the non-legitimacy of certain other terms, as both intuitive and crucial for the very possibility of logic. Humans beget Humans, he repeatedly reminds us (and brutes beget brutes). No fear of borderline cases can arise here. We can single out all Humans neatly and sharply, we can even imagine the world without them, but if we try to single out all barbarians, or to imagine a world without them, we immediately get entangled in fuzzy borderline cases depicted by dubious terms, resulting in pseudo-definitions. A successful term for Aristotle is a legitimate term applied correctly to the right entity, and this presupposes knowledge of the right entities (whatever that means). Terms are anchored in reality, they catch a *substance*: either a “primary substance” (a real authentic slice, a genuine particular, whatever that means), or a “secondary substance” (an essence, or a genuine attribute that resides in that genuine particular, whatever that means). An unsuccessful anchor term catches nothing, or (which is more or less the same, for Aristotle) it catches an alleged attribute that does not actually reside within an existing object, or it catches an artificial attribute, an arbitrary set of objects that do not constitute a genuine species and cannot feature in a genuine definition. According to Aristotle, then, to give a primary substance (its proper) proper name or to give a species, a (real) natural kind (its proper) universal name is to use legitimate terms correctly. Other than legitimate terms, used rightly or wrongly, other terms are illegitimate or false.

In Plato it often seems that the role of *diaeresis* is not only to assume correct splits, so as to formulate correct axioms, but also to discover correct splits (and thereby correct definitions). This is a classic conflation of epistemology and methodology: episteme (knowledge of correct splits) dictates the tools (correct splits) for the method of searching for correct splits. On chapter 10 I show that Aristotle’s theory of induction and intuition commits a more elaborate but similar conflation. In hindsight we may observe that Plato and Aristotle alike wish both to assume and discover intimate acquaintance of the structure of reality and that this intimate acquaintance both dictates and facilitates discovery of correct definitions. This is the bootstrap effect of the conflation of methodology and epistemology.

Famously, a crucial difference exists here between Plato and Aristotle: Plato’s heaven is beyond space and time (and quite possibly beyond the reach of our intellects) whereas Aristotle’s essences exist here and now, within the particulars of this world. Thus, in Aristotle, legitimate terms are also existence-pointers. They indicate not only essences, but also the existence of distinct particular objects that exhibit these essences. This fact will be studied in detail in the next chapter.

To sum up, following Plato, and as a direct result of reflections on Platonic *diaeresis* Aristotle assumed that certain terms are more legitimate than others and that they (and only they) should be found and sorted out, from the pool of all possible terms even before proper (logical) investigation can begin. Only legitimate terms are allowed to partake in the dialectical game (in Plato) and in the syllogism proper (in Aristotle). Legitimate terms, for Plato, are those that have their hooks into a Platonic idea. For Aristotle the legitimate terms are those that have their hooks into an essence. Thus, terms that depict the universal class, the empty class,

hypothetical entities, non-existent entities, or even arbitrarily grouped together entities, are to be avoided, in Aristotle's logic, as a matter of course, for using them will (so it was assumed) result in problematic and useless pseudo-definitions. However, as we stressed, undesirable non-legitimate terms are impossible to prevent. The problem is not merely that the notion of an essence is not adequately defined, though this surely is the case. It is that any list of allegedly legitimate terms turns into an infinite list of manifestly illegitimate terms by the most elementary logical operations that we can conceive of. Thus, commonsense knowledge of kinds plays a crucial role in Aristotle's logic: it is the keeper of the border-gate to the province of logic, which becomes an ideal realm by the very insistence that such a border-gate can exist.

The ancient view of terms clearly enmeshes our semantic knowledge with our ontological knowledge. Sometimes it bluntly assumes (as Aristotle had done) that a rough ontological theory, suggested by our language, is already presupposed by our very perceptions of reality, and that because of this, it is true. (For example that, observably, Humans beget Humans, and that, observably, they are rational animals, and that therefore rational animals must exist). This is circular justification, of course. At other times a dialectical detour is needed, and such matters are decided, once and for all, by careful dialectical quest. This too, as we noted earlier, is circular: dialectics is founded on correct splits of terms and establishes them. Either way, the assumption is made that *episteme* awaits us nearby on the crossroad of commonsense, perception, and careful rational thinking.

Noting the conflation of semantics and ontology that cradles the ancient view of terms is, I maintain, indispensable for a proper historical understanding of Aristotle's logic because the latter is founded on the former. Thus, the ancient view of terms cradles the conflation of epistemology and methodology that is Aristotle's logic. Plato's ironical and critical style may blur the fact that, like Aristotle, he conflated semantics and ontology, and consequently methodology and epistemology, in the hope to squeeze final positive results out of his *diaeresis*. Though he neither presented nor even attempted to present a rigorous logic the problems of the ancient view of terms are inevitably present in his dialogues, especially when he occasionally transcends his unmatched ability to smoothen difficulties through his unmatched poetical skills, and tries to form an explicit theory. Aristotle, on the other hand, was the first thinker we know of to explicitly provide a rigorous theory of logic. It is because of this great endeavor that the problems of the ancient view of terms loom large in his system. He was, in this respect, bolder and more direct than Plato. Alas, because of these merits, the deficiencies of his system are more apparent to us today. Plato elegantly and elusively conflates semantics and ontology while Aristotle attempts to rationalize and formulate this conflation as a more or less explicit theory.

Chapter 10

The Birth of Induction from Sea Foam

Dialectics in Aristotle is no mere display of rhetorical supremacy. It is a logical method burdened with a heavy epistemological cargo: searching, testing and possibly even establishing definitions.⁵⁹ In this sense, it echoes Zeno's method of semi-formal indirect proof and Plato's *diaeresis*, though it is not exactly identical with either. It may seem only natural, then, that induction, initially a tag name for a family of dialectical modes of argumentation, should share with dialectics that heavy epistemological overload.⁶⁰ In the *Posterior Analytics* induction takes over this very overload.

Here is the problem: Aristotle (justly) considers dialectics an inseparable part of logic (being a method of refutation, or of a semi-formal indirect proof). Induction, however, is not an inseparable part of logic. Nevertheless, Aristotle sometimes presents it as if it were, as if it is a peculiar aspect of the syllogism. This situation confuses readers, especially modern readers, since it seems to suggest that Aristotle's understanding of the situation does not resemble our own modern understanding of induction, and the syllogism. As we understand it today, induction is no deduction and no aspect of the syllogism. Aristotle, it seems, at times had it differently: at times he seems to stretch the notion of the syllogism so as to fit into it the inductive syllogism and at other times he seems to shrink the notion of induction so as to fit it in as an aspect of the syllogism. (Since he practically invented these terms, and as technical terms, this was not as difficult then as it may sound.) Let us now examine his view closely, since it is directly responsible for the fixating of logic as a part of unreachable epistemological ideals for over two millennia. Allow me to slow down a little.

Aristotle had opened his *Topics* as well as his *Prior Analytics* with a staggering observation: the syllogism, he said, is the form that dialectical syllogisms and apodeictic syllogisms share.⁶¹ We have observed already that this is one of the most decisive moments in the history of logic. Aristotle clearly recognizes here that his logic is a description of the *isomorphism* of dialectics and of apodeictic inferences. The observation not only makes perfect sense to us; it also justly demands to be regarded as the very moment of the inception of logic. For, we should understand it to mean that dialectical refutations and sound inferences share a logical form (the valid inference, the syllogism). I have stressed this point repeatedly in preliminary iv: valid inferences exhibit the isomorphism shared by sound inferences and

refutations, since they transmit truth from premises to conclusion (thus, rendering themselves sound inferences) and they likewise transmit falsity from conclusion to premises (thus, rendering themselves refutations). However, Aristotle's staggering and profound observation is not as neat and clean as it may seem at first sight. For Aristotle does not show interest in the neat logical isomorphism that we have just worded for him. For dialectics sometimes seems to be aimed at searching, testing, and possibly even finding and establishing definitions. And, as such, it often overlaps with induction within the Aristotelian corpus. We will soon observe the rationale behind this overlap. Before this we must notice the questions that this overlap invites: Can a purely deductive method lead to the discovery of definitions? Can it establish them? Is induction such a method? Is dialectics such a method? Is inductive inference isomorphic to sound inference? Is it isomorphic to dialectics? What then is the logical form of induction? Is it a valid form of inference? If yes, what is it? If no, how can it be isomorphic either to sound inferences or to dialectic refutations? Clearly, today we deny flatly that there exists an isomorphism between induction and any valid inference. But it looks as if Aristotle thought differently, at least some of the time. And then our task is to reconstruct his outlook and the rationale behind it.

Let me give a classic example to the complexity of the situation that we discuss now. Zeller's famous rewording of Aristotle's staggering isomorphism observation discussed above is the following clear-cut remark: "Aristotelian logic (in the "Second Analytics") deals with induction as well as proof; but both are preceded by the doctrine of the syllogism, which is the form common to both".⁶² We will now try to reconstruct the intellectual setting that made this extremely influential and problematic view possible.

The overlap between induction and dialectics in Aristotle has its roots in his understanding of Socratic dialectics. For he often made clear that Socratic dialectics somehow anticipated and even involved induction. Aristotle regularly presented Socrates as his one *bona fide* predecessor, who discovered (partially, and within the limited field of ethics) the method of induction, of tracing the universal within the particular, and formulating the finding by means of definitions.⁶³ A good Platonic source for this view seems to be the short excerpt from the *Phaedrus* which we have already mentioned with reference to the distinction between legitimate and non-legitimate terms; in it, Socrates states clearly the two principles that dialectics comprises of: "That of perceiving and bringing together in one idea the scattered particulars, that one may make clear by definition the particular thing which he wishes to explain...[and]... That of dividing things again by classes, where the natural joints are, and not trying to break any part, after the manner of a bad carver" (265d–e).⁶⁴ We have already explained in detail *diaeresis* but have identified it as Plato's version of Socratic dialectics. Here, however, *diaeresis* appears as only half-of-the-dialectical-story: its other half, its inversed mirror reflection, is the very method "of perceiving and bringing together in one idea the scattered particulars", namely, the method of induction. This makes induction and division isomorphic, of course, for induction is the inverse of division!

The isomorphism that we find here is not exactly the isomorphism that we seek in Aristotle. As far as Socrates is concerned, both dialectics and induction (division and integration) are pure methodology-in-the-weak-sense: for Socrates was not in the habit of offering definitions which he endorsed and defended. On the contrary, he observed the isomorphism of induction and division because both methods offer speculations, hypotheses, refutable conjectures. Induction for Socrates is an attempt to integrate in one idea scattered particulars, so as to explain them, and division is an attempt to explain scattered particulars by naming the ideas which (by the same hypothesis) they fall under. Aristotle tells a different story, which complicates things somewhat. He viewed the method of induction as establishing irrefutable definitions, not conjectures. Socratic dialectics, then, may very well be the source of Aristotle's view that induction and division go up and down the same ladder of abstraction, but we should not regard Socratic dialectics as the source of the view that we have found such a heavenly ladder by which to climb to legitimate abstractions. Aristotle presents the heavenly ladder as his logic. If we are to reconstruct his view, we must take a closer look at the structure of induction.

I am afraid that I cannot forego here introducing briefly yet another terminological distinction. In explaining the rationale of induction it is crucial to make significant use of the division of logical techniques into the analytic and the synthetic.⁶⁵ Please let me explain.

Synthesis is a common Greek word denoting putting together. In the present context, it is the putting together of truths with increased complexity, as in geometry, by deriving complex theorems from simpler ones, and ultimately from axioms. Analysis is the contrary movement: it is the decomposition of the complex to the simple. When the structure in question is a complex statement, its elements may be elementary truths or elementary errors. The need for the analysis of error rests on the supposition that to recognize elementary errors is easier than to recognize complex ones, that common ones are usually complex, and that this process takes place in a dialectical game (and in indirect proofs). Similar to the analysis of error is the analysis of truth: it is hard to see that an advanced theorem is true, but its analysis brings about conviction: in the very reduction of it to elementary, easily recognizable truths, it becomes obvious. The purported service of analysis then is this: it makes the complex into a complex of simples, so as to make its status evident.

What is induction, then? Is it analysis or synthesis? The answer is not as simple as it may seem. According to one traditional view, which will now be inspected, induction is synthetic. According to another traditional view, which will also be inspected now it is analytic. And, odd as it is, both views are right. For the answer depends on one's ontology and epistemology: it depends on what one deems particular, simple objects, and what one deems elementary, simple truths. Consequently, it depends on what one deems complex objects and truths. Aristotle had his own astounding answer to this question which reveals the astounding depth of his vision and which is closely connected to his compromise between the admission of the commonsense-concrete and the Platonist-abstract (as explained in detail in chapter 5). His answer, which actually sums up his conflation of semantics, ontology, methodology and epistemology, is that induction is *both* synthetic and analytic.

In order to explain this amazing contention let me first put it in Aristotle's favorite compromise formula: Induction, he says, is *in one sense* (which he calls the "prior in relation to us") a method of synthesis and in *another sense* (which he calls the "absolutely prior", or the "prior by nature") a method of analysis.⁶⁶ In Aristotle the two senses not only shift so as to accommodate different epistemological and ontological difficulties, *they also justify one another*. This, then, is perhaps the most clear-cut and intriguing case of circularity in Aristotle. Allow me to explain.

There are at least two very different notions of induction which complement each other, in Aristotle as well as in general. Together they form a kind of bootstrap theory of induction: they allegedly turn it from a fallible loose method for formulating conjectures into a seemingly rigid method which seemingly delivers infallible definitions. The two different notions correspond to the two distinct ontologies which Aristotle united by means of his existential import (as we know call it): the ontology of the commonsense-concrete and that of the Platonist-abstract. Let us recall: the two ontologies put forward opposite "things" as simples, as building blocks of the cosmos.

According to the view of the cosmos as commonsense-concrete, its elementary building blocks are you and me and particular sticks and stones. Abstract objects like Swan or White are then complex assortments or collections of particular objects (particular swans and white objects). Indeed, as we explained earlier, they are so complex that materialists and nominalists alike deny that they are genuine objects. These complexes, then, are the result of synthesis, of the putting together of many scattered simples under one roof. Induction, according to this view, is such a putting together of scattered particulars into complex assortments. (More accurately, induction is the synthesis of singular propositions lumped together in one universal proposition; but, we may remember, the difference between a universal concept and a universal proposition is blurred within the ancient view of terms; indeed, even for Hume the so called "problem of induction" and the so called "problem of universals" are parallel if not identical, although he does not seem to be fully aware of this interesting fact.) We can call this type of induction, then, induction by enumeration or simply induction by synthesis.⁶⁷ One particular (say a swan), and another, and another, together make up a general concept (Swan), so it is claimed. Nature's lullaby puts a spell on us: we observe that the first swan was white, and the second, and the third and suddenly, after this or that critical point, critical reason itself is lulled, and we fall into a dogmatic slumber, reckoning that all swans must be white.

We already noted that Induction by synthesis is no effective rigid methodology: it conflates the source of our concepts with the good, infallible source of our concepts, and it does not explain *why* we choose certain general concepts while ignoring others that are just as general and no less adequate. Already Bacon called that type of induction childish. We all tend to rely on such inductive generalizations, of course. (It was assumed, for example, as a matter of course, that all swans are white – until James Cook discovered Australia.) But habits are irrelevant from the rigid epistemological perspective, as Hume noted: we all have our good and bad habits, of course, but it does not follow that we should make up a philosophy out

of them. However, we certainly do not need to resort to Hume's famous critique of induction in order to stress this point: Aristotle's square of oppositions would do just fine. For it states clearly that all judgments of the form "some x are y" can be true, alongside "some x are not y"; the wrong conclusion that "all x are y" from "some x are y" would be sanctified if all induction by enumeration were allowed. And if not, which ones should be allowed? And why?

Aristotle, then, was well aware of the epistemological limits of induction by enumeration, by synthesis. This is why he also resorted to a second, complementary, type of induction. It is born out of the Abstract-Platonist aspect of his ontology. Recall that Platonism recognizes only abstract forms as genuine building blocks of the cosmos. Concrete objects, such as particular sticks and stones, then, are not real (not genuinely real). They are only real to the extent that they partake in this or that abstract Form. And they partake in many such abstract Forms at once, so that they are, in this sense, complex. According to Platonism, then, abstract ideas are the simple parts of our thought, as well as simple parts of the cosmos. And note this: Aristotle stresses that there is a strict hierarchy of such abstract universals: the more abstract a universal is, the simpler it is, for other ideas may participate in it, but not *vice versa*. For example, every white patch is also colored but not *vice versa*, and so Colored, according to Aristotle, is simpler, more general, than White. (The sketch of a single and unique hierarchy of abstractions is unworkable, of course, and is one of the main reasons that Aristotle's Grand matrix of essences was never assembled, but it must be allowed here as a part of the presentation of the rationale behind his theory of induction.)

Aristotle, then, presented his categories as the simplest, most abstract, building blocks of the cosmos. As he has put it, they are simpler and more basic than all else in the "absolutely prior" sense, for nothing is contained in them and everything else contains at least some of them. (You may recall that this is exactly why they are not allowed into his logic: they are too simple to take the place of a subject of a sentence). Slightly less simple objects are second in order: predicates such as (possibly) Colored or (possibly) Animal. (These are guesses, of course, for, to repeat, The Grand Matrix was never assembled.) Next in order are complex predicates such as White, or Mammal or Fish. (These, again, are guesses.) Next are even more complex predicates such as Human or Whale (and perhaps Greek). Finally, the most complex objects around are objects which cannot be predicated at all of anything else, but contain the most predicates in themselves. These are objects such as you and me, a certain bird or a certain tree. Each one of us individuals, then, is the most complex object in the universe from the "absolutely prior" perspective, exactly because each one of us is the simplest and most basic particular around from the point of view that considers the cosmos as comprising commonsense-concrete things. A similar (though not identical) idea exists in classic sensationalism and idealism: the complex idea of a particular apple, as Berkeley noted, is decomposable into the simple conceptual atoms of the idea of its shape, the idea of its color and the idea of its scent.

By the Idealist, then, induction has nothing to do with synthesis, for it accents simplification and abstraction rather than collection and enumeration. It breaks up

a single apple into its basic components: Redness and Roundness. And it breaks up these components to simpler ones: that of being colored and that of being a closed curve. Thus, induction is, strictly speaking, *analysis* of a particular (such as an apple) into the universals which comprise it. To analyze particular (immediate) observations into the universals and universal regularities which explain them is science, if idealism is true. Similarly with Platonists: they seek to analyze the universal ideas (eternal forms) that are implicit within each and every particular. They seek the universals that make the very existence of common particulars possible. They too, then, view induction as analysis.

The difference between induction as analysis and induction as synthesis may seem structural. Induction as analysis may seem merely a new way of formulating the process of induction by synthesis, by enumeration. This is true, of course, from the epistemological point of view, but not from the ontological or the methodological points of view. For the ontological disagreement between concretists and abstractists implies a methodological disagreement about the *direction* of induction. What one group views as premises of induction the other group views as conclusions, and *vice versa*.

Commonsense-concretists enumerate observation statements as immediate premises and universal regularities as conclusions resulting from them. Abstractists, by contrast, stress that universal regularities are logically prior to observation statements: they are their presuppositions. Consequently, whereas concretists deny the existence of universal forms, abstractists observe that observation statements were impossible unless universal forms existed. It should not surprise us that this last formulation of induction as analysis sounded like a transcendental argument: it is. For abstractists stress that we would be unable to conceive of a single individual had we not implicitly assumed, beforehand, that it participates in some universal idea. Thus *what seems like an invalid attempt to reach a sound generalization by enumeration (for the concretist) is really a transcendental argument (according to the abstractist)*. This is the historical transition from Hume to Kant. It should not surprise us to find, then, that commonsense-concretists stress that they enumerate as many particular cases as they can in the hope of making their universal generalization stronger, better founded. They try to exhaust by enumeration the extension of a universal in an attempt to achieve a good grasp over its intension, and they try to exhaust the extensions of different universals in order to achieve a good grasp over their intensional relations. Abstractists, who perform induction as analysis, however, typically have no need of such an exhausting exercise: they need only one (perhaps two) concrete objects in order to dismantle the universals which these concrete objects (allegedly) pre-suppose. For example, idealists strip the universal Red out of a single observation of a given red apple or, at most they compare a single red apple to a single non-red apple (as Hume had done), and then infer Redness by comparison.⁶⁸ Thus, induction by analysis is not about exhausting extension so as to get a better grip of intension. Rather, it is about extracting intensions that are *presupposed* by the very existence of extensions.

What may, perhaps, surprise some modern readers is how well Aristotle noted all this, and how often, and how central he deemed it to his system. We need no go

as far as his grand opening chapter to his *Physics*. The grand opening chapter to his *Posterior Analytics* should do. Having established there that, "...inductive arguments...prove the universal from the self-evident nature of the particular" (71a 4–9), he explains that there are two senses in which previous knowledge is necessary (71a11–28): "Recognition of a fact may sometimes entail both previous knowledge and knowledge acquired in the act of recognition; viz., knowledge of the particulars which actually fall under the universal, which is known to us... that *this* figure inscribed in the semicircle is a triangle we recognize only as we are led to relate the particular to the universal (for some things, viz., such as are ultimate particulars not predictable of anything else as subject, are only learnt in this way ...)". Ignoring the geometrical example, which complicates things by sending us to the overly neat field of geometry, we may conclude and say that, according to Aristotle, the very recognition of ultimate particulars already presupposes knowledge of the universals within which they participate. Otherwise, we would not have recognized them as what they are. For example, when we perceive Socrates, by necessity we also perceive a human being, and likewise an animal. This, according to Aristotle, is part and parcel of our very ability to perceive Socrates (So that syllogisms are presupposed by observation statements!) A bit later on Aristotle resolves the priority controversy between concretists and abstractists as follows: "There are two senses in which things are prior and more knowable. That which is prior in nature is not the same as that which is prior in relation to us, and that which is < naturally > more knowable is not the same as that which is more knowable by us. By 'prior' or 'more knowable' in relation to us I mean that which is nearer to our perception and by 'prior' or 'more knowable' in the absolute sense I mean that which is further from it. The most universals concepts are furthest from our perception, and particulars are nearest to it. And these are opposite to one another." (71b32–72a5)

According to Aristotle, then, we are able to analyze the perception of an individual into the universals which make it up, and we are also able to do the exact opposite, to collect many scattered particulars into a single universal term. He calls the faculty which enables us to perform the first task, analysis, *Nous*. (*Nous* is normally translated as intuition, and, whether or not this translation is accurate, it is clearly the source of Kant's concept of the transcendental intuition; they share an epistemological function and structure: justifying and explaining universal propositions as presuppositions or prerequisites of our immediate perceptions of a particular fact). *Nous*, then, is Aristotle's name for the ability to perform induction-as-analysis: it is the faculty which enables us to perceive particulars, by presupposing the universals which comprise them.

Two things remain to be noted. The first is that by elucidating induction-as-analysis we have explained why Aristotle thought that it somehow overlaps with dialectics. For both methods are methods of analysis, of decomposing and simplifying the immediate (be it a proposition or an observation) into the more basic which it presupposes. The only important difference between these two methods seems to be that dialectics aims (mainly) at exposing these presuppositions as inconsistent, whereas induction as analysis aims at showing that the presuppositions are necessary, if the immediate proposition is to be admitted as true.

The second thing to be noted is that the method of induction-as-analysis and the method of induction-as-synthesis are not only inverse mirror images of the same process, in the philosophy of Aristotle (and elsewhere): They are used to *justify one another*. Thus, in the closing paragraphs of *Posterior Analytics* Aristotle explains that general concepts are formed (in our *Nous*) by repetition of reoccurring similar events. Yet he likewise repeatedly stresses the very opposite: recognition of the very similarity of any two objects already presupposes universal criteria for similarity. (Criteria that are imposed by our *Nous* on our perceptions). For example, different humans would not be viewed as human unless we previously attained the universal Human and applied it to them, and we would not have attained any such universal concept, Human, unless we were raided steadily with (similar) perceptions humans. The result of this circularity is peculiar: every observation (allegedly) sends epistemological aid to the two far ends of the province of our knowledge: it sets and strengthens the universal concepts which it exemplifies (induction as synthesis) and it performs the exact opposite, it deduces (by induction as analysis) the universal which the very perceptibility of the particular presupposes.

This guarantees the logical certitude of empirical science, according to Aristotle: empirical science becomes the logical result of experience. The cost is that of considering the project of science inherently complete and completely given by experience (and logic). To put it differently: the cost is that any advancement in empirical science entails the inconsistency of our logic.

Induction-by-analysis is, of course, as much a way of formulating our conjectures as induction-by-synthesis, and no more. The two methods are parallel in a strict manner and hence not complementary: the one cannot be used to justify the other, or *vice versa*, for both offer conjectures regarding abstraction; both are methodology in-the-loose-sense. To stipulate, for example, that the humans Socrates, Plato, and Aristotle presuppose a single and unique concept Rational, is the same as to observe that these humans add up to determine a single and unique concept Rational. Setting this unachievable ideal of a full-proof bootstrap theory of induction aside, however, the greatness and acuity of Aristotle immediately captures us. Kant, to note the most famous historical case of induction-as-analysis, had argued that Euclidian geometry and even Newtonian mechanics function as presuppositions of our perceptions. He deemed this staggering – later on refuted – conjecture, a novel type of proof, a transcendental proof, and a second Copernican revolution. Not wishing to belittle this enormous achievement, nor its claim for originality, we should note that already Aristotle recognized the gist of that proof and utilized it, at least in a rudimentary form, as the foundation of his epistemology.

Chapter 16

Extensionalism as Exorcism

This discussion is devoted to explaining the revolutionary character of extensionalism. We are thus slowly zeroing in on Boole: you should soon feel the domino effect that he impelled, the rapid unstoppable toppling of one traditional dogma after another. Our aim is to emphasize the radical alternative to Aristotelian essentialism that Boole's extensionalism comprises; that it thus exorcises Aristotle's essences, thereby setting logic free of its ancient, traditional constraints. Let me first offer in this chapter a description of extensionalism in the abstract. Then, in the following chapters, I will enrich that description with a discussion of a few decisive moments in that historical development. Revolutionary ideas rarely mature at the moment of their inception. Extensionalism is no exception, though it did mature at an astonishingly high rate. Since things will happen fast once they are set into motion it helps to take a good look at the fully fledged idea before surveying its chronicles so as to find its earliest sources and announce its precise moment of inception.

Extensionalism, like essentialism, is not a theory. It is an *attitude* toward logic. It is a logical point-of-view with radical, swiping, and distinctive implications. The fundamental concept at issue is that of a class. Whether classes come with their essences or not was, under various terminologies and theoretical disguises, the hottest traditional dispute about traditional logic. Classes in modern logic clearly and indisputably replace essences. A class is then any *arbitrary* collection of particular objects (whatever an object may be). Extensionalism stresses this and this is its gist: a class is nothing but any arbitrary collection of particular objects. I contend that Boole was the first to have embraced completely this seemingly paradoxical and somewhat simplistic view of logic and that he thereby inaugurated a new era in the history of logic. Since a class is nothing but any arbitrary collection of objects, as a matter of course, within logic all classes are indisputably equally legitimate. This is all that there is to it. Extensionalism asserts that within logic the very distinction between legitimate/correct and non-legitimate/incorrect terms is obsolete and useless. And since this distinction silently sustained the traditional conflation of logic and science, in its various forms, this conflation too is gone.

Extensionalism stands in contrast of sorts to traditional Aristotelian essentialism, the dominant meta-logical view which dominated the field ever since its inception. Extensionalism is a contrast of sorts in the sense that it need not be anti-essentialist;

suffice it that it postpones discussion of essences to a later stage of any inquiry. Essentialism portrays logic as the (true) taxonomy of reality. It thus obviously conflates taxonomy and taxonomy rules, and by the sheer insistence that only essences participate in syllogisms. By rendering this insistence logical it achieves, by a vicious circle, self-justification. Paradoxically, it achieves so much by the seemingly innocent act of limiting logic to terms that name essences. These terms are said to designate directly essences and indirectly particular, tangible, objects. It thus presupposes that an entire given ontology of entities exists, allegedly organized by “correct” groupings into “legitimate” or “natural” kinds or essences. By that stage the conflation of logic and science is already complete. Traditional logic, then, contains the semi-explicit assumption that all traditional problems of epistemology (and indeed of philosophy) have been solved by its very saturation with essentialism. By contrast, extensionalism has no ontological doctrine. It does not deny the existence of essences. It has, however, an explicit agenda with an astounding effect: to exorcise the essences out of logic proper, perhaps in order to reintroduce them later in a more appropriate way (as Boole had certainly hoped). Thus, extensionalism tears logic and ontology apart. And it also tears logic and epistemology apart, and in the very same manner. It takes essentialism to be an undesirable conflation of the extra-logical and the logical, of science and its method. All criteria – at least temporarily – lose their import for the determination of the legitimacy of terms that name classes: all terms are declared equally legitimate within logic and all classes are declared equally legitimate there. In other words, extensionalism suggests that from the strictly logical point of view, any criterion for assembling any collection of objects is as good as any other. The question of the legitimacy of classes or union of classes is, within logic, rejected as extra-logical. Hence, questions of legitimacy – epistemological, metaphysical, etc. – become irrelevant to the construction of any logical system.

In modern logic three classes, or three fundamentally or essentially different class types, are new and of the outmost importance. They find their immediate way into logic as a result of Boole’s radical new and permissive attitude: the universe of discourse (or the universal class), the complement class of any class, and the complement class of the universal class, the empty class (Boole called it “The class Nothing”). The extensionalist revolution in logic contains little more than these three classes, or class types. Historically, these three classes, or class types, were deemed problematic and undesirable. Consequently they were exiled from the realm of logic. They designated no essence, and hence had no place there. Tradition still allowed a universe of discourse: the entire cosmos, the sum of all things in the world. Since it was a sum, however, it could not have an essence of its own; it was too vast, too abstract an aggregate to be pinned down by a single term, or a single definition. It was supposedly too abstract to be the subject of predication, too abstract to be a substance and consequently too abstract to participate in syllogisms. Similar considerations, we should recall, caused the complement class to be rejected as unacceptable. For, consider the term Not-Goat under a traditional logical apparatus: it includes every single entity that is not a goat. Since the only admissible universe of discourse is the whole cosmos, Not-Goat indistinguishably includes

Humans, Trees, Minerals, Platonic ideas, Cities, Laws, Feelings, even Dreams and Reflections. Not a very useful term, from the traditional point of view, of course. For, the traditional point of view demanded that each and every term should designate an essence. And most important: traditional logicians refused to admit the empty class as legitimate. Ever since Parmenides, they cast a taboo on terms that had no reference. Indeed, traditionally the notion of a term with no reference was so problematic that traditional logicians regularly conflated (under this tag), meaningless terms, contradictions (“in terms”, as traditional terminology goes) and even terms that depicted entities that merely happened not to exist (whether golden mountains or mermaids, not to mention centaurs).

What truly strikes us, from the historical point of view, however, is not even the aforementioned novelties but rather the uninhibited manner by which they were suddenly introduced. Extensionalists are not disrespectful of these ancient taboos. They simply have no sense of veneration for essences. They fail to comprehend the ancient sense of dread that their essentialistic predecessors were coping with by means of their taboos. Every class is legitimate, extensionalists argue; therefore, clearly, even a class whose objects have been chosen by inconsistent criteria is legitimate. It is not very special, it is just empty. The empty class is legitimate, then, as are all the terms that name it (such as “square circle” or “largest prime number”). If every term is legitimate, then terms formed by the addition of the word “not” (or any other sign for negation) to any term are legitimate too, and the class of all objects that are not members of any other class is legitimate as well. It is its complement class, of course. If every class is legitimate, then the complement to the empty class is also legitimate, and this is the universal class or the universe of discourse. Indeed, for any two terms that an essentialist would not hesitate to endorse (‘man’ and ‘goat’, for example) the extensionalist immediately produces by means of elementary class operations an infinite list of terms that the essentialist would reject (such as “goat-man”, “man or goat”, “man and goat” “not-man”, “not-goat-man”, “not-not goat”, etc.)

These changes are not cosmetic. They lead to a metaphysically neutral logic. And this new openness to any metaphysics, the ability to suspend all metaphysical judgment while doing logic, resonates in every corner of the logical cosmos: it is the demand to replace judgments by propositions and sentences, and it is the demand to replace classic syllogisms by modern formal inferences. For, in traditional logic the ancient division (or aspiration for division) of terms into legitimate and non-legitimate has created the division (or call for division) of judgments into true and false and as this doesn’t work the true is further divided into the essential and the accidental: traditional essentialists, then, must divide judgments into essential or necessary (“Man is a rational animal”) on the one hand, and accidental (“Man is a featherless biped”) on the other. The task then forced itself to differentiate between these allegedly distinct sets of judgments. This task weighed heavily upon traditional logic. The reasons for this burden is that the propositions deemed necessary were empirical conjectures (some of them in error) just as those deemed accidental. The reason for the burden, then, was that a distinction was called for when there was no distinction to be made. Insisting upon making it (and further stressing that

it is logical, and hence necessary, and even trivially so) merely put logicians in an even more uncomfortable position when they failed to specify this distinction or at least criteria for making it. The distinction, then, was problematic first and foremost because there is no logical differentiation between the propositions traditionally described as necessary and those traditionally described as accidental: both are informative, and hence contingent, from the strictly logical point of view. These two groups should have been placed *together* in direct opposition to logically necessary propositions (tautologies), but tautologies were traditionally dismissed – as negligible and uninteresting, exactly because they are empty of content, because they offered no taxonomy of essences and thus of things, because they did not express anything like an essential definition. (Thus, the traditional underrating of tautologies was a disguised effort to avoid admission of the fact that generating informative tautologies is impossible. Only this way can we understand the titanic efforts of Leibniz and of Kant, and their inevitable doom.)

Extensionalism provided the basis on which to construct a new algebra. By means of that algebra the contents of any two propositions are comparable – in the sense that they do or do not express the same content. This was a tremendous advance over classical logic. Consider the following propositions: “All citizens are loyal” and “All traitors are foreigners”. Both express the same content and are therefore expressed by the same formula. However, within classical logic the issue is difficult to decide: intuitively, these propositions are obviously equivalent. Aristotelian metaphysics makes things awkward by its division of all expressions into positive and negative. The expression “loyal” is positive, whereas the expression “traitor” is negative. It is therefore considered inferior and therefore unsuitable for designating an essence. Thus, Aristotelian metaphysics ties the hands of traditional logic in order to ensure that only propositions of a certain type may be included in the results of Aristotelian epistemology. Indeed, it is very hard (perhaps impossible) to determine who really has the upper hand here, since traditional logic is saturated with Aristotelian metaphysics. Therefore the question – are the above propositions interchangeable? – gets out of hand.

Inevitably, extensionalism changes also our conception of inference. It is, as matter of course, permissive in the choice of conclusions derivable from any given class of premises, as long as it is valid, of course. Traditional logic determines for the user one conclusion for each single premise (eduction) or each pair of premises (syllogisms) by means of fixed schemes. It is not a neutral system of inference, but rather an instructional system of taxonomies. Ordinary logical intuition is not free to operate within such a pre-fixed taxonomy. All this is obliterated (offhandedly) by the concept of logic as the study of extensions: suddenly an inference can include any number of premises, and a set of premises can have any number of logical conclusions. Logic, then, turns into a garden of forking paths along which one may wander at will, as long as one takes care not to stray. Traditional logic, by contrast, is a strict tour guide who purports to lead one towards a sublime destination, when in truth this destination lies beyond the guide’s ken.

Extensionalism must not be confused with formalization, though the two are closely linked. Extensionalism invites formalization yet it is not identical with it.

(Essentialism does not challenge the very idea of formalization; it simply gets in its way by resorting to informal intuitions, informal judgments, that comprise extra-logical agenda.) Extensionalism and essentialism are mere attitudes to logic, whereas the question whether a system is formal or informal concerns its properties. A formal system is one open to manipulations in complete disregard for the meanings of its signs. Success in doing this is due to the presence of the complete list of permissible manipulations within it. Extensionalism, then, invites formalization by insuring us that such manipulation of signs is warranted since it allows all classes as legitimate. For example, Boole's system of logic is extensional without being formal (as yet).

To conclude, extensionalism amounts to a revolution within logic: it radically changed its most basic concepts, its most basic methods and its aims and scope, thus opening the road to modern, mathematical logic. It gave the notion of term a new meaning which in turn gave the notion of a proposition a new meaning which in turn gave the notion of inference a new meaning, more formal and more mathematical: extensionalism is permissive: all classes allowed by essentialists are allowed by extensionalists but not *vice versa*. And this completely erases the need to establish by logical means alone the theories of empirical science, since it permits empirical hypotheses such as that one class is a sub-class of another, while insisting that such empirical hypotheses are extra-logical. All this happened by fiat: by the mere declaration that a class is any arbitrary collection of particular objects, and that any collection of objects is as much a class as any other, at least within the province of logic.