Rings around the sun and moon: coronae and diffraction

Les Cowley[1], Philip Laven[2] and Michael Vollmer[3]

[1] Norfolk, UK
[2] Chemin de l'Avanchet 20, 1216 Cointrin, Geneva, Switzerland
[3] University of Applied Sciences Brandenburg, Brandenburg, Germany

E-mail: atopt@lycos.co.uk (LC), philip@philiplaven.com, vollmer@fh-brandenburg.de

Abstract
Atmospheric optical effects can teach much about physics and especially optics. Coronae—coloured rings around the sun or moon—are large-scale consequences of diffraction, which is often thought of as only a small effect confined to the laboratory. We describe coronae, how they are formed and experiments that can be conducted on ones in the sky. Recognizing that this is not always convenient, we show how students can also learn about coronae and thus diffraction from experiments with accurate full-colour computer simulations and laboratory demonstrations.

M This article features online multimedia enhancements

Introduction
Sunlight or moonlight playing on raindrops, cloud droplets, dust or ice crystals in the atmosphere produces a host of visual spectacles. The rainbow is the best known but there are many others: fogbows, glories, coronae, ice halos and more. All are accessible to sky-watchers and there are several semi-popular descriptions [1–4]. Each phenomenon is a manifestation of physical principles at work, and their observation and study provide a context for teaching physics from the elementary to research levels. Here we look at opportunities provided by coronae.

What is a corona?
A corona is a number of delicate and softly coloured rings seen when thin clouds partially cover the sun or moon. Look for one around the moon when it is near to full and the sky is dark. If searching for a solar corona, shield the sun *and* reduce the light intensity to safer levels by looking at the sky reflected in a pool of water or a mirror of plain glass. Staring directly at or near to the sun can permanently damage eyesight, and we therefore do not recommend it and write here about coronae around the moon.

Figures 1 and 2 show the features of a corona. At the centre is a very bright *aureole*, almost white and fringed with yellow and red. Sometimes that

Figure 1. A multi-ringed lunar corona. The brightness of the central aureole has hidden the moon. Image by Laurie Kangas.

Figure 2. Coronae vary in size and can be up to $\sim 15^\circ$ in diameter. They often shrink and swell as different clouds scud across the moon. Comparison with the moon's disk of 0.5° diameter is a good way to estimate their size. Image by Eva Seidenfaden.

is all there is to be seen, but the better coronae have one or more successively fainter and gently coloured rings surrounding the aureole. The first is bluish on the inside, grading through greens and yellows to red outermost. The following rings lack the blue, but otherwise have similar colours. All the colours are subtle mixtures rather than the more direct hues of the rainbow.

A corona should not be confused with the larger *halo* of 22° radius produced by hexagonal prism-shaped ice crystals, nor is it related to the extended outer atmosphere of the sun that is visible during a total eclipse and given the same name.

Theory of the corona: diffraction at work

Diffraction produces the corona. Diffraction is often thought of as a small effect and a purely laboratory one at that. Coronae show diffraction operating in the sky for all to see, and we can use them to demonstrate some of its principles.

Diffraction occurs when light is obstructed by an obstacle. Whilst geometric optics predicts that the shadow cast by the obstacle will be well defined, wave optics shows that light passes into the shadow zone. The obstacles that produce coronae are cloud droplets. These typically have mean diameters of 10–15 μm and an overall size range of 1–100 μm. The wavelengths of visible light such as 0.45 μm (blue) and 0.70 μm (red) are smaller but sufficiently close to droplet dimensions that significant diffraction occurs when sunlight or moonlight shines through thin clouds. In contrast, raindrops with sizes in the mm range do not produce observable coronae.

Cloud droplets are so far from us compared with their diameters that all corona effects are from Fraunhofer far-field diffraction. Figure 3 illustrates a wave interacting with a droplet to produce a coronal diffraction pattern.

Simple diffraction theory usually addresses the far-field diffraction from slits or circular apertures. The latter have obvious practical applications as lenses and mirrors. We can examine the diffraction from cloud droplets by referring to Babinet's principle of diffraction theory [5, 6]. This predicts that the diffraction from a circular aperture is identical to that produced by a disc-shaped obstruction, the only practical difference being the brightness in the exact forward direction. If we approximate spherical droplets as opaque discs then we have a basis for predicting their diffraction: the diffraction of a single cloud droplet is approximately that from a disc, which in turn is mostly that from a circular aperture. In this respect, the physics of the corona is the same as that describing the delicate diffraction rings surrounding a star when seen through a high power telescope.

Quantitatively, the Fraunhofer diffraction pattern is then calculated using Huygens' principle: each point of the aperture is assumed to be the source of outgoing spherical waves which interfere to produce the observed pattern. Mathematically, the intensity distribution as a function of scattering angle θ for an occulting disc is given by a Bessel function $J_1(x)$ where $x = 2\pi R\sin(\theta)/\lambda$, λ is the wavelength of the light, and R the radius of the aperture or obstructing disc:

$$I(\theta) = I(0)\left(\frac{2J_1(2\pi R\sin(\theta)/\lambda)}{2\pi R\sin(\theta)/\lambda}\right)^2. \qquad (1)$$

The intensity $I(0)$ in the exact forward direction is proportional to the area of the aperture. The Bessel function oscillates like a sine function but its amplitude decays rapidly with increasing angle. The first and second maxima are only 1.75% and 0.42% respectively of the forward scattered light intensity. These maxima occur at $x = 2\pi R\sin(\theta)/\lambda = 5.14$ and 8.42. The first two minima are at $x = 3.83$ and 7.02.

The scattering angle for the first minimum θ_1 at a given wavelength λ and radius R is therefore

$$\sin\theta_1 = \frac{3.83}{\pi}\frac{\lambda}{2R} = 1.22\frac{\lambda}{2R}. \qquad (2)$$

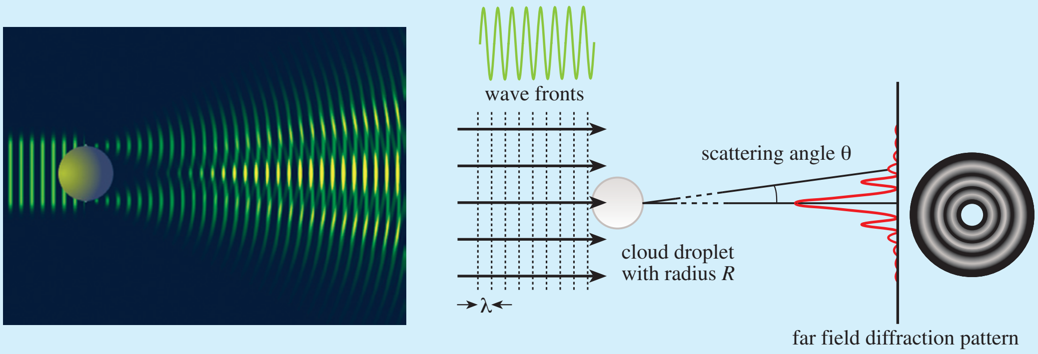

Figure 3. Single isolated droplets each produce a coronal diffraction pattern. Outgoing waves from scattering of the incoming plane wave by the droplet superpose and interfere. Typical school experiments with water tanks reveal the near-field pattern (left). In the far field an intensity pattern with circular rings is produced (right).

The smaller the droplet the greater the angular size of the corona, and in principle we can use corona scattering angles to measure the radii of cloud droplets. Equation (1) is for monochromatic light; we give more details later on for evaluating droplet sizes from white light scattering.

One obvious difference between the diffraction pattern from a single aperture and a corona is that the latter is produced by many droplets. Nevertheless, the light of the corona as seen by an observer is not due to multiple scattering: the cloud droplets are widely spaced compared with their diameters and it is *single scattering* by individual droplets that produces coronae. This is also implied by the fact that we can see through the clouds that produce coronae. Thicker clouds that obscure the disc of the sun or moon are characterized by multiple scattering. The corona we see, as shown in figure 4, is the collective light from the diffraction by each of millions of separate droplets.

So far we have assumed that the composition of the corona forming droplets is immaterial. This is not quite true, especially for the smallest particles, because simple diffraction theory then becomes inadequate and the more complex Mie theory has to be applied. Nonetheless, coronae are indeed produced by materials as varied as stratospheric volcanic ash, ice crystals in high clouds and, nearer the ground, windborne pollen grains.

We have also spoken only of particles all with the same radius and the best coronae result when this is true. Similar sized droplets abound when all

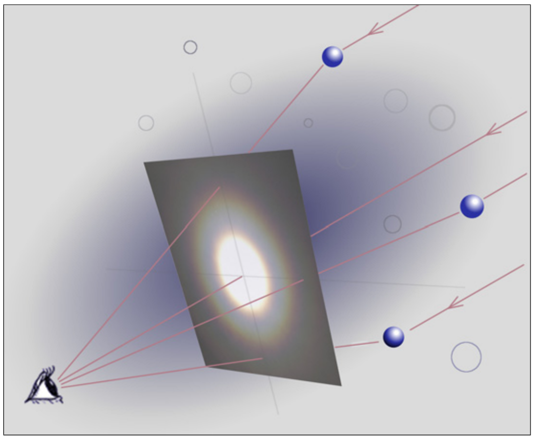

Figure 4. Coronae are the collective light from diffraction by millions of individual small water droplets or other particles.

the drops have the same history and were formed a short time previously. They are found most often in altocumulus, cirrocumulus and especially in lenticular clouds. If droplets have broad size distributions, the diffraction patterns of the various droplets overlap and the coronae are washed out. In this case, the aureole will be seen as a bright central disc around the sun or moon. When the size distribution is narrow but the mean diameters are different from point to point in the cloud the corona may be noncircular (figure 5). Sometimes it is really broken up and the jumbled patches of colour are referred to as *iridescence*.

Classically, coronae should not show any polarization features, because they are due to diffraction in the forward direction. Polarization,

Figure 5. Noncircular corona due to strong variations in droplet sizes from 7.6 to 16.6 μm in mountain wave clouds. (Photo courtesy Paul Neiman.)

as observed in other phenomena of atmospheric optics, such as rainbows and halos [4], usually results from a process where reflection is involved.

Computer simulations

Coronae are not always conveniently visible in the sky although they can always be reproduced in the laboratory. We discuss outdoor and laboratory experiments later on but here we describe another approach: experiments on a computer.

Small PC programs can easily run calculations using equation (1) to show ring intensities and positions for monochromatic light. However, they can do much more by making accurate simulations using Mie theory. The Fraunhofer diffraction theory described above is not wholly accurate, especially for very small particles or droplets, mainly because it does not take account of the scattering contributions from directly transmitted light. In contrast, Mie scattering theory[1] gives exact predictions for spherical droplets. Gustav Mie formulated it in 1908 from Maxwell's equations of electromagnetism and without recourse to approximations but its calculations were extremely laborious and complicated and had to await fast PCs to be done quickly and shown effectively.

Mie theory calculations may be performed with the freeware program MiePlot [7] for scattering of sunlight by droplets of variable size. The output gives intensity versus scattering angle as well as colour.

Another freeware program, IRIS [8], uses Mie theory to produce full-colour coronae simulations. The results are plotted as would be seen in the sky and can be directly compared with observations. Students can use it to experiment with changes in droplet size, drop size distribution and wavelength and see the results in seconds.

IRIS makes simulations by computing scattering intensities over a range of angles for a single wavelength. For white light simulations it repeats the calculations for hundreds of other wavelengths in the visible range. The results are summed together after weighting by factors to allow for the spectral power distribution of incident sunlight. An angular convolution has then to be made because the sun and moon are finite disks rather than point sources.

Figure 6 shows IRIS simulations of coronae from three different droplet diameters; the screen can display sectors of four different simulations

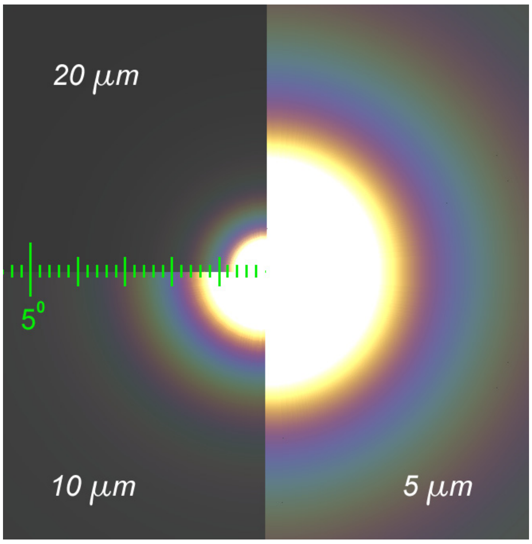

Figure 6. Mie simulations by IRIS showing the effect of changing the cloud droplet size. The droplets were monodisperse, the condition that produces the best rings. The numbers are droplet radii.

[1] Mie theory is derived from Maxwell's equations written for a periodic field. The resulting wave equation is then solved for the case of a plane wave interacting with a homogeneous sphere. The numerical solutions automatically take account of all the optical processes and interactions, e.g. diffraction around the drop, reflections from the drop surface, transmission through the drop, internal reflections and so on. The answers are exact but disappointing in the sense that the only physics they contain is that of the Maxwell equations and so, on their own, they give few clues into what is actually happening at the level of physical optics. Mie calculations for instance predict corona colours perfectly but do not tell us how they arise or why a bright ring is where it is—for those questions, simpler and approximate theories provide more insight.

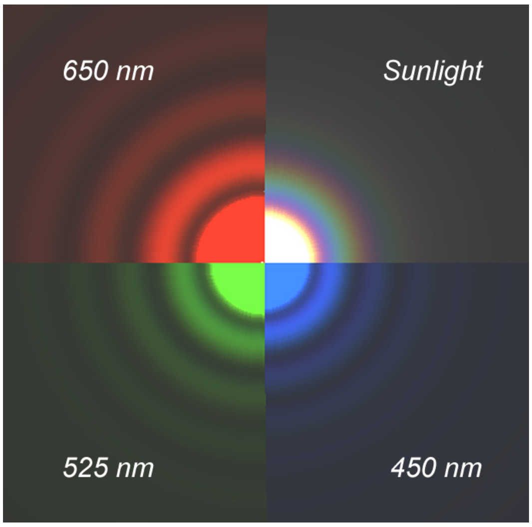

Figure 7. IRIS simulations of how a corona would be seen for sunlight and through narrow band filters for three colours. All plots are for the same droplet size, showing how the corona size changes with wavelength, that many more rings are visible for narrow band light than for white light and how the colours of white light corona arise: most ring colours are mixtures.

and students can easily see that the smallest droplets produce the largest coronae. The simulations in figure 6 also illustrate the faintness of the rings compared with the aureole and how successive rings become less distinct. These effects result from the rapid decrease in intensity predicted by equation (1) combined with smearing caused by the overlap of rings of different colour.

Students can simulate monochromatic coronae and compare them with the laser experiments described later. Monochromatic simulations can be a surprise because Mie calculations for pure monochromatic light show extra oscillations in ring intensities that, although physically faithful, are rarely seen from laboratory light sources. Better looking simulations are those for a narrow band of wavelengths, typically 10–40 nm.

Narrow band coronae reveal how the colours of the white light corona arise. Figure 7 illustrates how the central aureole swells as the wavelength increases, resulting in the red fringe seen in white light. The gap between the aureole and the first white light ring is not completely dark but is lit by an overlap of the edge of the red aureole and the inner edge of the first violet and blue rings, giving an overall reddish-violet hue. The rainbow-like order of blue through green to red of the first ring is produced by the increase in the monochromatic ring radius with wavelength as predicted by equation (2). However, the broad rings of each colour overlap considerably and the resulting colours of white light corona are all mixtures.

Figure 7 also shows how the overlap of the broad outer rings of different colour considerably lowers the contrast of the white light rings such that even for monosized cloud droplets more than one or two rings are rarely seen.

The simulations in figures 6 and 7 are for monosized droplets. Usually cloud droplets have a range of sizes which can, for convenience, be represented by a normal distribution with a mean and percentage standard deviation. How does this nonuniformity affect the appearance of the corona? Figure 8 shows simulations by the IRIS program. For 10 μm radius droplets, the overlap of different sized coronae resulting from a standard deviation of 10% leads to lower contrast and a barely visible second ring. A 20% standard deviation renders even the innermost ring invisible. Good coronae as opposed to a mere glow around the moon are a sign of surprisingly uniform cloud droplets.

Droplet size affects the visibility of coronae in another way. When the droplets are very small ($R < 5$ μm), the scattered light is spread over a large angular range and hence the corona is relatively dim. When the water drops are large ($R > 40$ μm), the 0.5° apparent angular diameter of the sun or moon smoothes the narrow coloured rings that would be visible if the incident light from the sun were truly parallel. Thus, natural coronae are seen at their best when R is between 5 and 20 μm.

Experiments with natural coronae

Simulations are all very well but students are encouraged to look for a real corona around a nearly full moon, count its rings, note the aureole and ring colours and check that it is unpolarized. They can take photographs with a digital camera and use them to measure the diameters of cloud droplets. An alternative is to use a simple cross staff made from two sticks of wood and marker pins to measure the ring diameters directly.

Images of coronae will need angular calibration. The focal length of the camera lens is needed and this is given for digital cameras

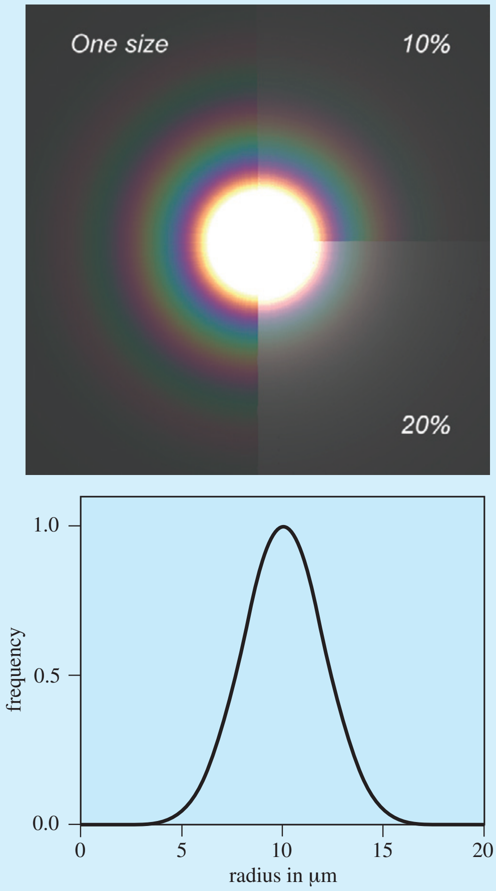

Figure 8. Good coronae need droplets of uniform size. Top: IRIS simulations illustrating the effects of broadening the drop size distribution. The numbers are % standard deviations about the mean radius of 10 μm. Bottom: Size distribution for a standard deviation of 20%.

in the EXIF data stored with each image. The equivalent focal length of a 35 mm camera must then be calculated from a conversion factor found in digital camera manuals or a knowledge of the CCD dimensions. The *angular* distance α of a point x pixels from the image centre is then

$$\alpha = \arctan\left(\frac{x}{f_{35}}\frac{w}{w_{\mathrm{p}}}\right) \tag{3}$$

where w_{p} is the full width of the image in pixels, $w = 36$ (or the measured long dimension in mm for 35 mm slides) and f_{35} is the equivalent focal length of a 35 mm camera lens.

Alternatively, take separate, correctly exposed, pictures of the moon and calibrate the corona images, knowing that the moon is approximately 0.5° in diameter.

In the absence of photographs for student exercises, calibrated versions of the images in this article are available online.

Analysis of cloud droplet sizes

Students can estimate the size of the droplets creating a corona by measuring the ring diameters for one or two colours and comparing them with the maxima of equation (1). This approach is not entirely accurate because a given colour in the white light corona does not correspond *exactly* to a maximum in the monochromatic light intensity. As a check, IRIS can give the droplet sizes by iterative comparison of its simulations with measured ring diameters.

Another approach is to compare digital images directly with simulations using widely available image editing software. We show as an example (figure 9(*a*)) a beautiful corona created by mist droplets and captured by Till Credner and Sven Kohle in a forest on the island of Tenerife. The 35 mm camera image was taken with a 51.5 mm focal length lens and the uncropped longest dimension was 35.04 mm. From equation (3) we compute that 5° is 12.86% of the uncropped long dimension from the centre. That calibration can be used to manually obtain the ring diameters, or alternatively, as in figure 9(*b*), an image editor can add 5° reference marks and equalize the angular scales of the image and an IRIS simulation. The match in figure 9(*b*) tells us that that the mist droplets had a radius of 6.75 μm and were essentially monosized, for otherwise fragments of the second and third rings would not have been visible.

Laboratory experiments

Single apertures

School diffraction experiments usually start with slits illuminated by HeNe lasers. For coronae, the slit can be exchanged for circular apertures obtainable from many suppliers (e.g. [9]) as iris diaphragms or precision pinholes with diameters down to 1 μm. Figure 10 shows the diffraction pattern from a pinhole of $R = 60$ μm illuminated by a red and a green HeNe laser and projected on a white screen. The quality of the result depends on the circularity of the pinhole. The diffraction

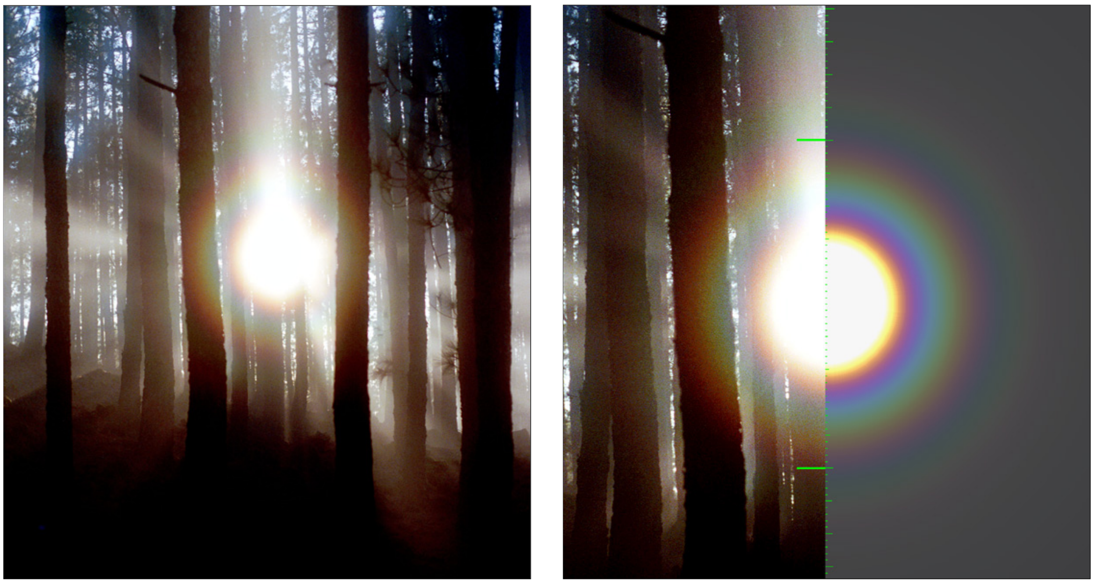

Figure 9. Left: Mist droplets in a Tenerife forest produced this delicate corona. Parts of three rings are visible above the central aureole. Perspective effects make the parallel rays from the sun appear to spread out radially. Image by T Credner and S Kohle, www.AlltheSky.com. Right: Comparison with a Mie theory simulation by IRIS. The best match for monodisperse droplets of radius 6.75 μm was obtained by iterative comparison of the first ring. The size of the central aureole is not a reliable indicator because it depends on camera exposure and simulation brightness settings.

angle is calculated from the geometry shown in figure 10(b). The precision of the value derived for the pinhole diameter increases if higher-order minima are measured.

Pinholes could be illuminated with white light but the much lower light intensity compared with a laser makes the rings hard to see even with dark-adapted vision in a totally darkened room. The corona is much more easily seen in another way. Use as a source an ordinary 100 W light bulb behind a screen pierced by a 1 or 2 mm diameter hole or one of the new very bright white light LED torches with the lens removed. Several metres away the light from the hole or torch is effectively parallel and students can hold up a second smaller pinhole to their eyes to see a corona. Do not use this method to look at a laser source!

Randomly distributed circular discs

To imitate more closely the formation of a corona from millions of cloud droplets we can study diffraction from a large number of small black disks on a two-dimensional 'target'. We generate the discs on a computer screen [10] (see the program that is available online) and then photograph a printout like figure 11 onto black and white slide film with low sensitivity but high resolution. For diffraction experiments this works much better than colour film. If the whole width of figure 11(a) has, for example, a length of 15 mm on the slide, then the black discs will be about 100 μm in diameter. The disc size can be reduced or enlarged by altering the camera distance. In the example, the mean centre-to-centre distance between the discs is of the order of two disc diameters so that many discs are illuminated by the laser beam. This has no influence on the experiment because we are dealing with a planar sample and multiple scattering is not possible but students should be told that cloud droplets are much further apart.

Shine an expanded laser beam through the slide to produce the diffraction pattern as in figure 11(b). Expand the beam by passing it through two lenses of different focal length separated by the sum of their focal lengths. The shorter focus lens is closest to the laser. The central spot of the pattern is very bright because much laser light passes through the transparent parts of the target and is unscattered.

Make more slides with discs of different

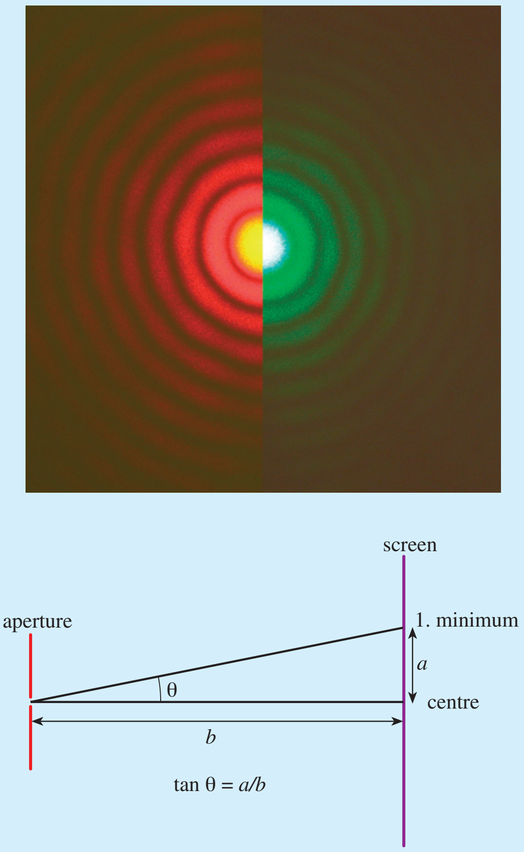

Figure 10. Top: Diffraction by a pinhole of radius 60 μm illuminated by a HeNe laser of λ=632.8 nm and 543 nm. The separately obtained images are arranged side by side. The central false colours are caused by overexposure. Bottom: Geometry for measuring the diffraction angle. The angle of first red light minimum was 0.37°, yielding a pinhole radius of 59.8 μm.

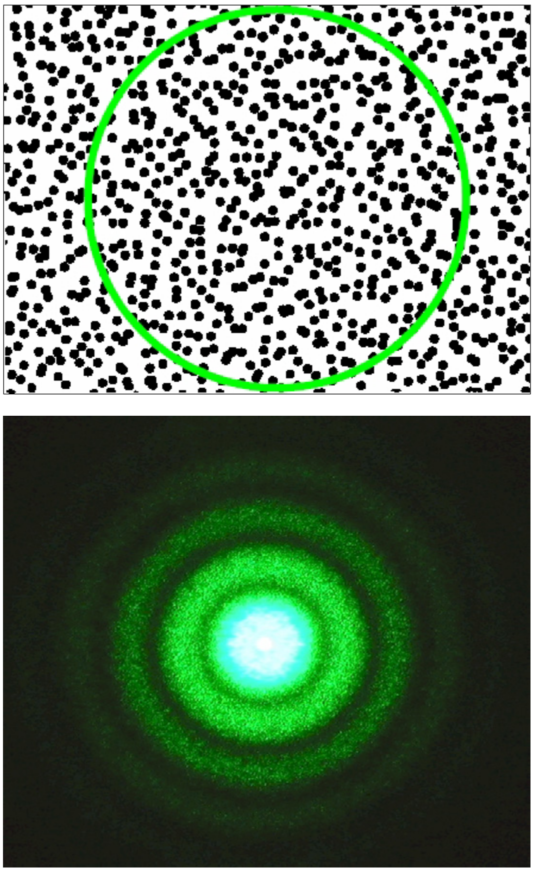

Figure 11. Randomly distributed circular discs. Top: Master for transfer to slides. The green circle indicates the area containing ~500 discs to be illuminated by an expanded laser beam. Bottom: Corresponding diffraction pattern from illumination by a green HeNe laser.

size or non-circular shapes. The latter [10], particularly if oriented, demonstrate coronae formed by non-spherical pollen grains. Students can see white light coronae by looking through the slides at a distant point source of white light. Again, do not do this with a laser!

Lycopodium spores

Rather than construct artificial scattering targets, dust a glass plate with Lycopodium powder. The powder is the spores of Lycopodium club moss, which are nearly spherical and have a very uniform average diameter of about 34 μm. Shine an expanded laser through a plate dusted with the powder to see monochromatic coronae similar to figures 10 and 11; the quality depends on the evenness of dusting. View white light corona in the same way as for the slide targets.

Water droplets condensed on glass

Perceptive students will ask about the rings seen around light sources viewed through glass misted by condensation. Superficially they look like coronae but there are differences! There is no aureole and the rings often have radial streaks.

The differences arise from the close spacing of the condensed droplets. The theory we have outlined assumes that the light diffracted from the different particles is incoherent, which in turn demands that the particles be separated from centre to centre by at least 2 to 2.5 particle diameters.

Cloud droplets are much further apart but those condensed on glass are very close together and interference occurs from neighbouring drops. The radial streaks result from non-randomness of the packing of the closely spaced droplets.

More atmospheric effects

We have presented concepts and experiments on coronae to help the teaching of diffraction. A great attraction is the use of a natural and easily seen atmospheric phenomenon. After awakening interest in atmospheric optics, other fascinating natural optical effects like halos, mirages and glories may be used to enhance school physics and optics.

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References

[1] Cowley L T, Atmospheric Optics website at www.sundog.clara.co.uk/atoptics/phenom.htm
[2] Naylor J 2002 *Out of the Blue: A 24-hour Skywatcher's Guide* (Cambridge: Cambridge University Press)
[3] Greenler R 1980 *Rainbows, Halos, and Glories* (originally Cambridge: Cambridge University Press, now obtainable from Milwaukee: Elton-Wolf Publishing)
[4] Lynch D K and Livingston W 2001 *Color and Light in Nature* (Cambridge: Cambridge University Press)
[5] Hecht E 1998 *Optics* 3rd edn (Reading, MA: Addison-Wesley)
[6] Guenther R 1990 *Modern Optics* (Chichester: Wiley)
[7] Laven P, MiePlot simulator is at www.philiplaven.com/mieplot.htm
[8] Cowley L T, IRIS Mie simulator at www.sundog.clara.co.uk/droplets/iris.htm
[9] Edmund Optics Ltd, www.edmundoptics.co.uk
[10] Schneider W and Vollmer M 2005 *Proc. 8th Int. Meeting on Meteorological Optics* (to be published in *Appl. Opt.*)

Les Cowley is a retired research scientist. He has a PhD in chemical physics from the University of Edinburgh. His interests have moved from the dynamics of chemical reactions, combustion theory and experiments to meteorological optics.

Philip Laven is Technical Director of the European Broadcasting Union. In his spare time, he plays with mathematical models of atmospheric optical phenomena.

Michael Vollmer is professor of experimental physics at the University of Applied Sciences in Brandenburg. He has a PhD and habilitation from the University of Heidelberg. His current activities include research in atmospheric optics and infrared imaging, experiments for physics education and teacher training seminars.