# 1 QCD

## 1.1 Quark Model

1. Isospin symmetry
In early studies of nuclear reactions, it was found that, to a good approximation, nuclear force is independent of the electromagnetic charge carried by the nucleons — charge independence. In other words, strong interaction has an $SU(2)$ symmetry which transforms $n$ into $p$ and vice versa. These $SU(2)$ generators $T_1, T_2, T_3$ satisfy the commutation similar to that of angular momenta,
$$[T_i, T_j] = i\epsilon_{ijk}T_k$$
Acting on $n$ or $p$ we have
$$T_3|p\rangle = \frac{1}{2}|p\rangle, \quad T_3|n\rangle = -\frac{1}{2}|n\rangle, \quad T_+|n\rangle = |p\rangle, \quad T_-|p\rangle = |n\rangle \ \cdots$$
This means that $n$ or $p$ form a doublet under isospin transformation. Isospin invariance simply means that
$$[T_i, H_s] = 0$$
where $H_s$ is the strong interation Hamiltonian.
We can extend the isospin assignments to other hadrons by assuming isospin invariant in their productions. For example we get the following isospin multiplets,
$$(\pi^+, \pi^0, \pi^-) \quad I = 1, \qquad (K^+, K^0), \ (\bar{K^0}, K^-) \quad I = \frac{1}{2}\ , \qquad \eta\ , \quad I = 0$$
$$(\Sigma^+, \Sigma^0, \Sigma^-) \quad I = 1, \qquad (\Xi^0, \Xi^-), \quad I = \frac{1}{2}\ , \qquad \Lambda, \qquad I = 0$$
$$(\rho^+, \rho^0, \rho^-) \quad I = 1, \ (K^{+*}, K^{0*}), \ (\bar{K^{0}}*, K^{*-}) \quad I = \frac{1}{2} \qquad \cdots$$
If isospin symmetry were exact, then all particles in the same multiplets have same masses, which is not the case in nature. But the mass difference within the isospin multiplets seems to be quite small.
$$\frac{m_n - m_p}{m_n + m_p} \sim 0.7 \times 10^{-3}, \qquad \frac{m_{\pi+} - m_{\pi 0}}{m_{\pi+} + m_{\pi 0}} \sim 1.7 \times 10^{-2} \qquad \cdots$$
Thus we can treat the isospin symmetry as approximate one and maybe it is good to few %.

2. <u>SU(3) symmetry and Quark Model</u>
When $\Lambda$ and $K$ particles were discovered, they were produced in pair (associated production) with longer life time. It was postulated that these new particles possessed a new additive quantum number, called **strangeness** $S$ , conserved by strong interaction but violated in decays,
$$S(\Lambda^0) = -1, \quad S(K^0) = 1 \ \cdots$$

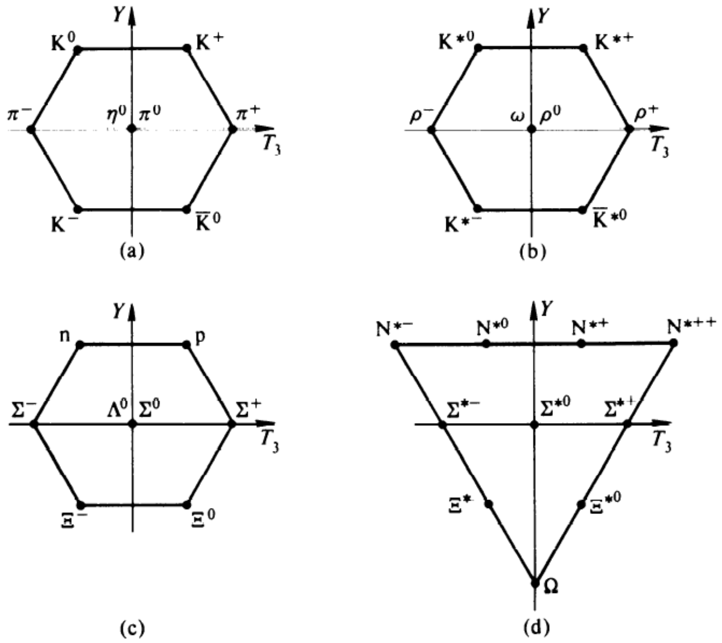
Extension to other hadrons systematically, we can get a general relation,

$$Q = T_3 + \frac{Y}{2}$$

where $Y = B + S$ is called hyperchargee, and $B$ is the baryon number. This is known as Gell-Mann-Nishijima relation.

**Eight-fold way :** Gell-Mann, Neeman

When we group mesons or baryons with same spin and parity, we see that




we see that they form a patterns of 8 or 10 as shown in the figure. These are the same as some irreducible representations of SU(3) group. This can be extended to other hadrons which are either in octet or decouplet representations of $SU(3)$. Thus the spectra of hadrons seem to show some pattern of $SU(3)$ symmetry. But this symmetry is lots worse than isospin symmetry of $SU(2)$ because the mass splitting within the $SU(3)$ multiplets is about 20% at best. Nevertheless, it is still useful to classify hadrons in terms of $SU(3)$ symmetry. This is known as the **eight-fold way**.

**Quark Model**

One peculiar feature of the eight fold way is that octet and decuplet are not the fundamental representation of $SU(3)$ group. In 1964, Gell-mann and Zweig independently propose the quark model, in which all hadrons are built out of spin $\frac{1}{2}$ quarks which transform as members of the fundamental representation of $SU(3)$, the triplet,

$$q_i = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

with the quantum numbers

| | $Q$ | $T$ | $T_3$ | $Y$ | $S$ | $B$ |
|---|---|---|---|---|---|---|
| $u$ | $2/3$ | $1/2$ | $+1/2$ | $1/3$ | $0$ | $1/3$ |
| $d$ | $-1/3$ | $1/2$ | $-1/2$ | $1/3$ | $0$ | $1/3$ |
| $s$ | $-1/3$ | $0$ | $0$ | $-2/3$ | $-1$ | $1/3$ |

In this scheme, mesons are $q\bar{q}$ bound states. For examples,

$$\begin{aligned}\pi^+ &\sim \bar{d}u \quad \pi^0 \sim \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d). \quad \pi^- \sim \bar{u}d \\ K^+ &\sim \bar{s}u \quad K^0 \sim \bar{s}d \ , \ K^- \sim \bar{u}s. \quad \eta^0 \sim \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)\end{aligned}$$

and baryons are $qqq$ bound states,

$$\begin{aligned}p &\sim uud \ , \ n \sim ddu \\ \Sigma^+ &\sim suu \ , \ \Sigma^0 \sim s\left(\frac{ud+du}{\sqrt{2}}\right) \ , \ \Sigma^- \sim sdd \\ \Xi^0 &\sim ssu \ , \ \Xi^- \sim ssd \ , \ \Lambda^0 \sim \frac{s(ud-du)}{\sqrt{2}}.\end{aligned}$$

It seems that the quantum numbers of the hadrons are all carried by the quarks. But we do not know the dynamics which bound the quarks into hadrons. Since quarks are the fundamental constituent of hadrons it is important to find these particles. But over the years none have been found.

**Paradoxes of simple quark model**

(a) Quarks have fractional charges while all observed particles have integer charges. At least one of the quarks is stable. None has been found.

(b) Hadrons are exclusively made out $q\bar{q}, qqq$ bound states. In other word, $qq, qqqq$ states are absent.

(c) The quark content of the baryon $N^{*++}$ is $uuu$. If we chose the spin state to be $\left|\frac{3}{2},\frac{3}{2}\right\rangle$ then all quarks are in spin-up state$\tilde{}$ $\alpha_1\alpha_2\alpha_3$ which is totally symmetric. If we assume that the ground state has $l=0$, then spatial wave function is also symmetric. This will leads to violation of Pauli exclusion principle.

**Color degree of freedom**

One way to get out of these problems, is to introduce color degrees of freedom for each quark and postulates that only color singlets are physical observables. 3 colors are needed to get antisymmetric wave function for $N^{*++}$ and remains a color singlet state. In other words each quark comes in 3 colors,

$$u_\alpha = (u_1, u_2, u_3) \ , \ d_\alpha = (d_1, d_2, d_3)\cdots$$

All hadrons form singlets under $SU(3)_{color}$ symmetry, e.g.

$$N^{*++} \sim u_\alpha(x_1)\alpha_\beta(x_2)u_\gamma(x_3)\varepsilon^{\alpha\beta\gamma}$$

Futhermore, color singlets can not be formed from the combination $qq$, $qqqq$ and they are absent from the observed specrum. Needless to say that a single quark is not observable.

**<u>Gell-Mann Okubo mass formula</u>**

Since $SU(3)$ is not an exact symmetry, we want to see whether we can understand the pattern of the $SU(3)$ breaking. Experimentally, $SU(2)$ seems to be a good symmetry, we will assume isospin symmetry to set $m_u = m_d$. We will assume that we can write the hadron masses as linear combinations of quark masses.

(a) <u>$o^-$ mesons</u>

Here we assume that the meson masses are linear functions of quark masses,

$$\begin{aligned} m_\pi^2 &= \lambda(m_o + 2m_u) \\ m_k^2 &= \lambda\left(m_o + m_u + m_s\right) \\ m_\eta^2 &= \lambda\left[m_o + \frac{2}{3}(m_u + 2m_s)\right] \end{aligned}$$

where $\lambda$ and $m_0$ are some constants with mass dimension. Eliminate the quark masses we get

$$4m_k^2 = m_\pi^2 + 3m_\eta^2$$

This known as the Gell-Mann Okubo mass formula. Experimentally, we hav $LHS = 4m_k^2 \simeq 0.98(Gev)^2$ while $RHS = m_\pi^2 + 3m^2 \simeq 0.92(Gev)^2$ This seems to show that this formula works quite well.

(b) <u>$\frac{1}{2}^+$ baryon</u>

The masses of $\frac{1}{2}^+$ baryons can be written as,

$$\begin{aligned} m_N &= m_0 + 3m_u \\ m_\Sigma &= m_o + 2m_u + m_s \\ m_\Xi &= m_o + m_u + 2m_s \\ m_\Lambda &= m_o + 2m_u + m_s \end{aligned}$$

The Gell-Mann-Okubo mass formula takes the form,

$$\frac{m_\Sigma + 3m_\Lambda}{2} = m_N + m_\Xi$$

Expermentally, $\dfrac{m_\Sigma + 3m_\Lambda}{2} \simeq 2.23\ Gev$, and $m_N + m_\Xi \simeq 2.25\ Gev$ .

(c) <u>$\frac{3}{2}^+$ baryon</u>

The mass relation here is quite simple,

$$m_\Omega - m_{\Xi^*} = m_{\Xi^*} - m_{\Sigma^*} = m_{\Sigma^*} - m_{N^*}$$

This sometimes is referred to as equal spacing rule. In fact when this relation is derived the particle $\Omega$ has not yet been found and this relation is used to predicted the mass of $\Omega$ and subsequent discovery gives a very strong support to the idea of $SU\,(3)$ symmetry.

$\underline{\omega - \phi \text{ mixing}}$

For the $1^-$ mesons, the situation seems to be quite different. If we make an analogy with $o^-$ mesons, we would get the Gell-Mann Okubo mass relation in the form,

$$3m_\omega^2 = 4m_{K^*}^2 - m_\rho^2$$

Using $m_{K^*} = 890\ Mev$ and $m_\rho = 770\ Mev$ we would get $m_\omega = 926.5$ $Mev$ from this. But experimentally, we have $m_\omega = 783\ Mev$ which is qiute far away. On the other hand, there is a $\phi$ meson with mass $m_\phi = 1020 Mev$ and has same $SU(2)$ quatntum number as $\omega$. In principle, when $SU(3)$ symmetry is broken, $\omega - \phi$ mixing is possible. Suppose for some reason there is a significant $\omega - \phi$ mixing we want to see whether this can save the mass relation.

Denote the $SU(3)$ octet state by $V_8$ and singlet state by $V_1$

$$V_8 = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \quad , \quad V_1 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)$$

Write the mass matrix as

$$M = \begin{pmatrix} m_{88}^2 & m_{18}^2 \\ m_{18}^2 & m_{11}^2 \end{pmatrix}$$

Assume that the octet mass is that predicted by Gell-Mann Okubo mass relation, i.e.

$$3{m^2}_{88} = 4m_{K^*}^2 - m_\rho^2$$

After diagonalizing the mass matrix $M$, we get

$$R^+ M R = M_d = \begin{pmatrix} m_\omega^2 & 0 \\ 0 & m_\phi^2 \end{pmatrix}, \qquad \text{with} \ \ R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Thus

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} m_\omega^2 & 0 \\ 0 & m_\phi^2 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$
$$= \begin{pmatrix} m_\phi^2 \sin^2\theta + m_\omega^2 \cos^2\theta & m_\phi^2 \cos\theta\sin\theta - m_\omega^2 \cos\theta\sin\theta \\ m_\phi^2 \cos\theta\sin\theta - m_\omega^2 \cos\theta\sin\theta & m_\phi^2 \cos^2\theta + m_\omega^2 \sin^2\theta \end{pmatrix}$$

and we get

$$\sin\theta = \sqrt{\frac{(m_{88}^2 - m_\omega^2)}{(m_\phi^2 - m_\omega^2)}}$$

The mass eigenstates are

$$\begin{aligned} \omega &= \cos\theta V_8 - \sin\theta V_1 \\ \phi &= \sin\theta V_8 + \cos\theta V_1 \end{aligned}$$

Using $m_{88} = 926.5 Mev$ from Gell-Mann Okubo mass formula, we get

$$\sin\theta = 0.76$$

This is very close to the ideal mixing $\sin\theta = \sqrt{\frac{2}{3}} = 0.81$ where mass eigenstates have a simple form,

$$\omega = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$
$$\phi = \bar{s}s$$

This means that the physical $\phi$ meson is mostly made out of $s$ quarks in this scheme.

**Zweig rule**

Since $\omega$ and $\phi$ have same quantum numbers under $SU(2)$, one expects they have similar decay widths. Experimentally, $\omega \to 3\pi$ mostly, but $\phi \to 3\pi$ is very suppressed relative to $\phi \to KK$ channel even though $\phi \to KK$ has very small phase space since $m_\phi = 1020\ Mev$ and $m_k \approx 494\ Mev$.

$$B(\phi \to KK) \approx 85\% \quad , \quad B(\phi \to \pi\pi\pi) \sim 28\%$$

Quark diagrams

In term quarks contents, the decays of $\phi$ meson proceed as following diagrams indicate $\phi \to K^+K^-$

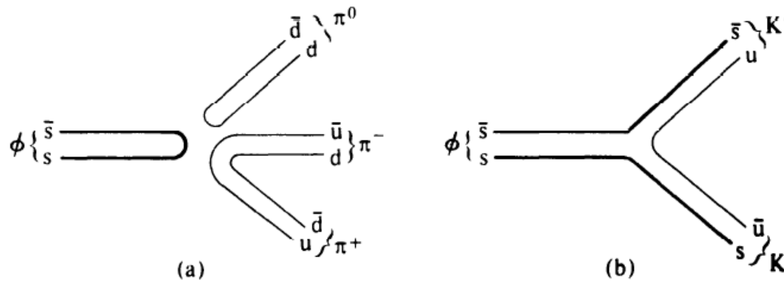



**FIG. 4.10. φ decays: (a) disallowed; (b) allowed by the Zweig rule.**

Zweig rule postulates that processes involving quark-antiquark annihilation are highly suppressed for some reason. This explains why $\phi$ has a width $\Gamma_\phi \approx 4.26\ Mev$ smaller than $\Gamma_\omega \approx 8.5\ Mev$.
Note that the Zweig rule is very qualitative and is hard to make it more quantitive.

$J/\psi$ and charm quark

In 1974 the $\psi/J(3100)$ particle was discovered with unusually narrow width, $\Gamma \sim 70\ kev$ as compared to $\Gamma_\rho \sim 150 Mev$,and $\Gamma_W \sim 10 Mev$. A simple explanation is that $\psi/J$ is a bound state of $\bar{c}c$ and is below the threshold of decaying into 2 mesons containing charm quark. Thus it can only decay by $c\bar{c}$ annihilation in the initial state. By Zweig rule, these decays are highly suppressed and have very narrow width.

## 2 Asymptotic freedom

1. $\lambda\phi^4$ theory
   The Lagrangian is
   $$\mathcal{L} = \frac{1}{2}[(\partial_\mu\phi)^2 - m^2\phi^2] - \frac{\lambda}{4!}\phi^4$$
   Effective coupling constant $\bar{\lambda}$ satisfies the differential equation
   $$\frac{d\bar{\lambda}}{dt} = \beta(\bar{\lambda}) \ , \quad \beta(\lambda) \approx \frac{3\lambda^2}{16\pi^2} + O(\lambda^3)$$
   It is not asymptotically free. The generalization to more than one scalar fields is the replacement,
   $$\lambda\phi^4 \to \lambda_{ijkl}\phi_i\phi_j\phi_k\phi_l \ , \qquad \lambda_{ijkl} \text{ is totally symmetric}$$
   Then the differential equations are of the form,
   $$\beta_{ijkl} = \frac{d\lambda_{ijkl}}{dt} = \frac{1}{16\pi^2}\left[\lambda_{ijmn}\lambda_{mnkl} + \lambda_{ikmn}\lambda_{mnjl} + \lambda_{ilmn}\lambda_{mnjk}\right]$$
   For the special case, $i = j = k = l = 1$, we wee that$\beta_{1111} = \frac{3}{16\pi^2}\lambda_{iimn}\lambda_{mn11} > 0$
   and theory is not asymptotically free.

2. Yukawa interaction
   Here we need to include the scalar self interaction $\lambda\phi^4$ in order to be renormalizable
   $$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \frac{1}{2}[(\partial_\mu\phi)^2 - \mu^2\phi^2] - \lambda\phi^4 + f\bar{\psi}\psi\phi$$
   Now we have a coupled differential equations,
   $$\beta_\lambda = \frac{d\lambda}{dt} = A\lambda^2 + B\lambda f^2 + Cf^4, \qquad A > 0$$
   $$\beta_f = \frac{df}{dt} = Df^3 + E\lambda^2 f, \qquad D > 0$$
   To get $\beta_\lambda < 0$ , with $A > 0$ , we need $f^2 \sim \lambda$. This means we can drop $E$ term in $\beta_f$. With $D > 0$, Yukawa coupling $f$ is not asymptotically free. Generalization to the cases of more than one fermion fields or more scalar fields will not change the situation.

3. Abelian gauge theory(QED)
   The Lagrangian is of the usual form,
   $$\mathcal{L} = \bar{\psi}i\gamma^\mu(\partial_\mu - ieA_\mu)\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
   The effective coupling constant $\bar{e}$ satisfies the equation,
   $$\frac{d\bar{e}}{dt} = \beta_e = \frac{\bar{e}^3}{12\pi^2} + O(e^5)$$

For the scalar $QED$ we have

$$\frac{d\bar{e}}{dt} = \beta'_e = \frac{\bar{e}^3}{48\pi^2} + O(e^5)$$

Both are not asymptotically free.

4. Non-Abelian gauge theories
It turns out that only non-Abelian gauge theories are asymptotically free. Write the Lagrangian as

$$\mathcal{L} = -\frac{1}{2}T_r(F_{\mu\nu}F^{\mu\nu})$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \qquad A_\mu = T_a A_\mu^a$$

and

$$[T_a, T_b] = if_{abc}T_c \ , \quad T_r(T_a, T_b) = \frac{1}{2}\delta_{ab}$$

The evolution of the effective coupling constant is governed by

$$\frac{dg}{dt} = \beta(g) = -\frac{g^3}{16\pi^2}(\frac{11}{3})t_2(V) < 0$$

The graphs which contribute to the $\beta-$function are listed below.

FIG. 10.5. Vector self-energy graphs (the dotted loop is that of the FP ghosts).

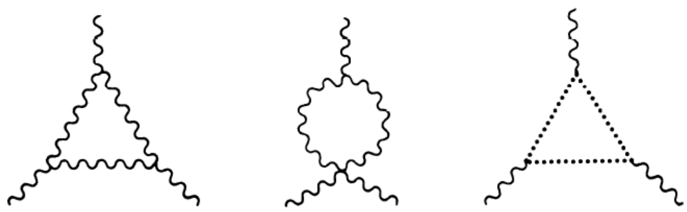


FIG. 10.6. Trilinear gauge-boson vertex correction.

Since $\beta(g) < 0$ for small $g$, this theory is asymptotically free. Here

$$t_2(V)\delta^{ab} = t_r[T_a(V)T_b(V)] \qquad\qquad t_2(V) = n \quad for \quad SU(n)$$

If gauge fields couple to fermions and scalars with representation matrices, $T^a(F)$ and $T^a(s)$ respectively, then

$$\beta_g = \frac{g^3}{16\pi^2}\left[-\frac{11}{3}t_2(V) + \frac{4}{3}t_2(F) + \frac{1}{3}t_2(s)\right]$$

where

$$t_2(F)\delta^{ab} = t_r(T^a(F)T^b(F))$$
$$t_2(S)\delta^{ab} = t_r(T^a(S)T^b(S))$$

# 3 QCD

Quark model needs colors symmetry to overcome paradoxes of simple quark model. On the other-hands, Bjroken scaling in deep inelastic scattering seems to require asymptotically free theory. Since only the non-Abelian gauge theories are asymptotically free, it is then natural to make this color symmetry of the quarks into a local symmetry. The resulting theory is the Quantum chromodynamics. It is straight forward to write down the Lagrangin for this theory

$$\mathcal{L}_{QCD} = -\frac{1}{2}t_r(G_{\mu\nu}G^{\mu\nu}) + \sum_k \bar{q}_k(i\gamma^\mu D_\mu - m_k)q_k$$

where

$$\begin{aligned} G_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \\ D_\mu q_k &= (\partial_\mu - igA_\mu)q_k \ , \qquad A_\mu = A_\mu^a \frac{\lambda^a}{2} \end{aligned}$$

The $\beta$ function to lowest order in $g$ is of the form,

$$\beta_g = \frac{-1}{16\pi^2}(11 - \frac{2}{3}n_f) = -bg^3 \qquad n_f : \text{number of flavors}$$

and the equation for the effective coupling constant is

$$\frac{d\bar{g}}{dt} = -b\bar{g}^3, \qquad \text{with} \quad t = \ln\lambda$$

The solution is

$$\bar{g}^2(t) = \frac{g^2}{1+2bg^2t} \qquad \text{where} \quad g = \bar{g}(g,0)$$

For large momenta, $\lambda\phi_i, \lambda$ large, $\bar{g}^2(t)$ decreases like $\ln\lambda$

Conveient to define

$$\alpha_s(Q^2) = \frac{\bar{g}^2(t)}{4\pi}$$

then we can write

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + 4\pi b\alpha_s(\mu^2)\ln(Q^2/\mu^2)}$$

Introduce $\Lambda^2$ by the relation,

$$\ln\Lambda^2 = \ln\mu^2 - \frac{1}{4\pi b \ \alpha_s(\mu^2)}$$

then effective coupling constant can be written as

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2}{3}n_f)\ln Q^2/\Lambda^2}$$

Thus the effective coupling constant $\alpha_s(Q^2)$ decreases slowly $\sim \frac{1}{\ln Q^2}$. QCD can make prediction about scaling violation (small) in the forms of integral over structure functions.

Quark confinements

Since $\alpha_s(Q^2)$ is small for large $Q^2$ , it is reasonable to believe that $\alpha_s(Q^2)$ is large for small $Q^2$ . If $\alpha_s(Q^2)$ is large enough between quarks so that quarks will never get out of the hadrons. This is called quark confinement. It is most attractive way to "explain" why quarks cannot be detected as free particles.

QCD and Flavor symmetry

QCD Lagrangian is of the form

$$\mathcal{L}_{QCD} = -\frac{1}{2}t_r(G_{\mu\nu}G^{\mu\nu}) + \sum_k \bar{q}_k i\gamma^\mu D_\mu q_k + \sum_k \bar{q}_k m_k q_k$$

where

$$\begin{aligned} G_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \ , \qquad A_\mu = A_\mu^a \frac{\lambda^a}{2} \\ D_\mu q_k &= (\partial_\mu - igA_\mu)q_k \ , \quad q_k = (u, d, s \cdots) \end{aligned}$$

Consider the simple case of 3 flavors,

$$q_k = (u, d, s)$$

$$\mathcal{L}_{QCD} = -\frac{1}{2}t_r(G_{\mu\nu}G^{\mu\nu}) + (\bar{u}i\gamma^\mu D_\mu u + \bar{d}i\gamma^\mu D_\mu d + \bar{s}i\gamma^\mu D_\mu s) + m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s$$

In the limit $m_u = m_d = m_s = 0$, $\mathcal{L}_{QCD}$ , is invariant under $SU(3)_L \times SU(3)_R$ transformations

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}_L \to U_L \begin{pmatrix} u \\ d \\ s \end{pmatrix}_L \quad , \qquad \begin{pmatrix} u \\ d \\ s \end{pmatrix}_R \to U_R \begin{pmatrix} u \\ d \\ s \end{pmatrix}_R$$

where $U_L$ and $U_R$ are $3 \times 3$ unitary matrices. However, hadron spectra shows only approximate $SU(3)$ symmetry, not $SU(3) \times SU(3)$ symmetry. We can reconcile this by the scheme $SU(3) \times SU(3)$ is broken spontaneously to $SU(3)$ so that particles group into $SU(3)$ multiplet. This would require 8 Goldstone bosons, which are massless. However in real world quark masses are not zero, these Goldstone bosons are not exactly massless. But if this symmetry breaking makes sense at all, these Goldstone bosons should be light. Thus we can identify them as pseudoscalar mesons. In other worlds, pseudoscalar mesons are "almost" Goldstone bosons. We can identify them with the pseudoscalar octet mesons, $\pi, K$ and $\eta$.