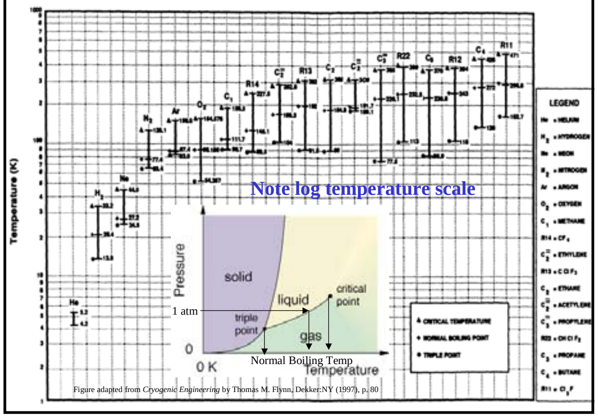
Characteristics of a cryogenic fluid

1. **Critical, normal boiling, and triple point temperatures of cryogenic fluids**
2. **Vapor pressure of liquids**
3. **Liquid Helium**
4. **Superfluids**

Critical, normal boiling, and triple point temperatures of cryogenic fluids



Note log temperature scale

1 atm

Normal Boiling Temp

Figure adapted from *Cryogenic Engineering* by Thomas M. Flynn, Dekker:NY (1997), p. 80

Vapor pressure of liquids

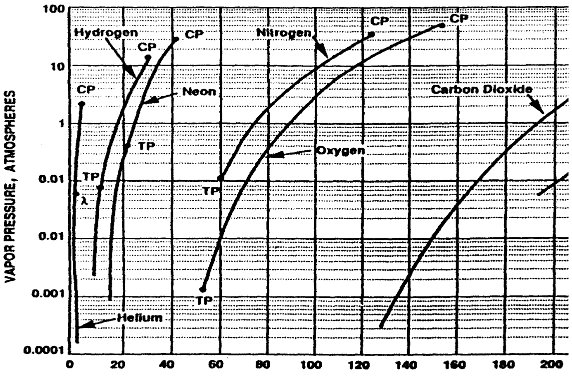


Figure adapted from *Cryogenic Engineering* by Thomas M. Flynn, Dekker:NY (1997), p. 81

Helium

- Spherical shape
- Two isotropic forms: ^{3}He and ^{4}He
- Low mass
- Van der Waals forces → low critical and boiling points
- Remains a liquid even at absolute zero (unless external pressure is applied)

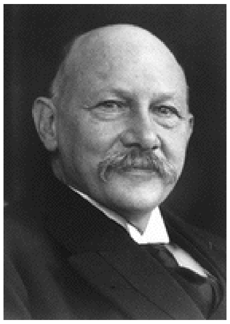
Name that man

In whose laboratory was helium first liquefied?

A. Sir James Dewar
B. Cailletet
C. Wroblewski
D. Onnes
E. Van der Waals

1882-Helium liquefied at Leiden University

H. Kamerlingh Onnes was one of the first professors in experimental physics at Leiden University. His lab first to liquefy helium (1908), for which he was awarded the Nobel prize in 1913, and he discovered superconductivity in 1911. He liquefied hydrogen to pre-cool the helium gas in his liquefier.

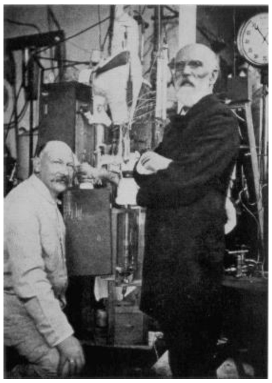


Spelling Bee

How do you spell the word for making a gas into a liquid?

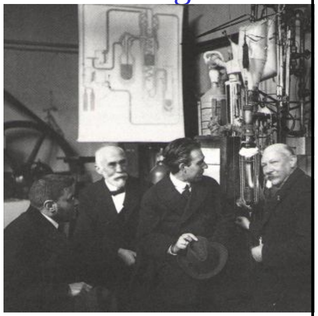
A. liquify
B. liquefy
C. liquafy
D. liquifi
E. liquiphy

- In 1882, Onnes was appointed Professor of Experimental Physics at Leiden University. In 1895, he established Leiden Laboratory
- His researches were mainly based on the theories of J.D. van der Waals and H.A. Lorentz
- Was able to bring the temperature of helium down to 0.9 °K, justifying the saying that the coldest spot on earth was situated at Leiden.



Heike Kamerlingh Onnes (left) and Van der Waals in Leiden at the helium 'liquefactor' (1908)

Who would have ever thought…



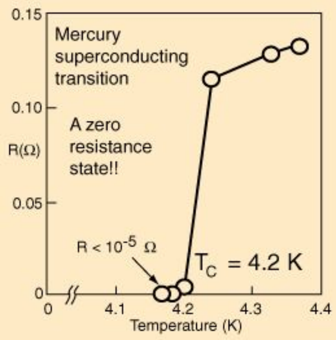
Heike Kamerlingh Onnes
His stamp, and (*right*) showing his helium liquefier to some casual passers-by: Niels Bohr (visiting from Kopenhagen), Hendrik Lorentz, and Paul Ehrenfest (*far left*).

Why Not A Solid?

- Zero-Point Energy
- E = (3h^2)/(8mV^(2/3)) energy of a free particle in a small box
- E decreases as V increases → the effect of the Zero-Point is to raise molar volume
- Kinetic energy exceeds the interaction potential energy

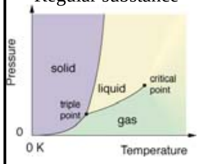
Superconductivity-1911

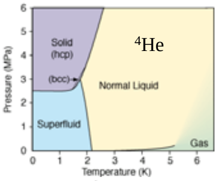
Heike Kamerlingh Onnes discovered superconductivity, the total lack of electrical resistance in certain materials when cooled to a temperature near absolute zero.

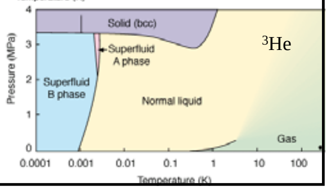


Phase Diagrams

Regular substance





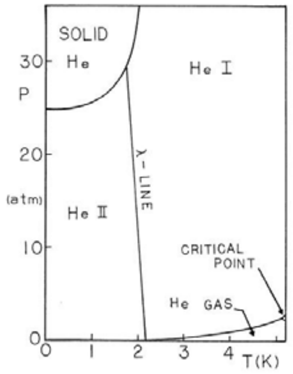


Why so low?

Superfluidity occurs in ^{4}He at about 4.2 K but only below about 0.002 K in ^{3}He. Why?

A. ^{3}He is rarer than ^{4}He in nature
B. ^{3}He is always in smaller containers than is ^{4}He
C. ^{3}He has different chemical properties than ^{4}He
D. ^{4}He superfluidity is an electronic process while ^{3}He superfluidity is a nuclear process
E. ^{3}He superfluidity is an electronic process while ^{4}He superfluidity is a nuclear process

Helium-4 Phase Diagram



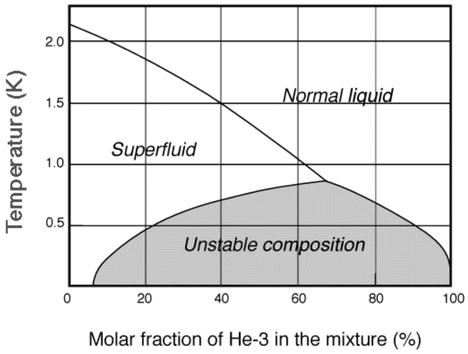
- At 2.17K ^{4}He undergoes a transition to the superfluid state
- The lambda line separates He I and He II
- ^{3}He does not become a superfluid until below 2mK

Do superfluids mix?

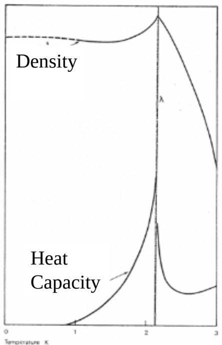
There are two isotopes of helium--under what circumstances do their liquid states mix?

A. They do not mix-it would violate thermodynamics to have a mixture at absolute zero.
B. They only mix when at absolute zero
C. ^{3}He can mix in ^{4}He but not the other way around at absolute zero

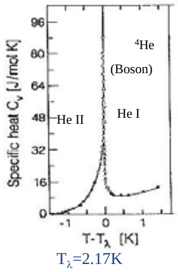
Helium Mixtures



1931: Keesom discovered lambda-specific heats in helium at Leiden



Allen and Misener and Kapitza (1939)



T_λ=2.17K

[Frank Pobell, 1992]

Superfluidity in Helium 4 in 1938

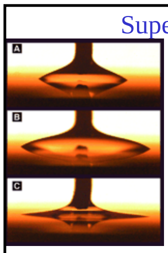


Fig. 2. (**A** through **C**) Microscope images showing an edge-on view of superfluid drops on a horizontal Cs substrate. The dark bar in the upper half of the image is the capillary tube. The pictures show the outline of the drop as well as its mirror image in the reflective substrate. As the volume of the drop increased from (A) to (B), the contact angle remained constant. When fluid was withdraw as in (C), the contact angle decreased but .
the diameter remained constant

Science 24 October 1997: Vol. 278. no. 5338, pp. 664 - 666

Figure 3. Microscope image of a superfluid drop on a Cs substrate inclined at 10° to the horizontal. A drop hanging off of the capillary is also seen in the upper right. The drop on the inclined substrate is stationary. The downhill edge of the drop has the same contact angle as shown in Fig. 2B, whereas the uphill edge has a vanishing contact angle.

- Superfluidity is a dramatic visible manifestation of quantum mechanics, being the result of Bose–Einstein condensation in which a macroscopic number of ^{4}He atoms occupy the same, single-particle quantum state. It was discovered simultaneously by Kapitza, Allen and Misener working separately, though only Kapitza received the Nobel prize. It is also amusing to note that Allen was a “classical physicist” at heart, who didn’t much care for the subatomic world. He discovered superfluidity with a pen light.

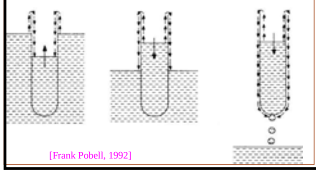
Superfluidity of the Quantum Fluid, ^{4}He

Thermal de Broglie wavelength of ^{4}He at 2K:

$$\lambda_T = \frac{h}{\sqrt{3mkT}} \approx 8.9Å$$

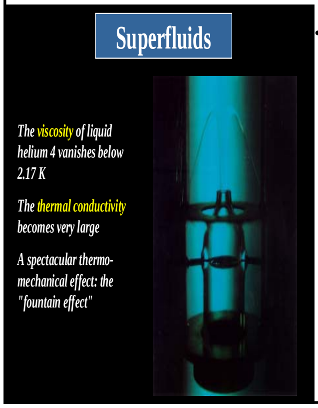
≥ mean interparticle distance of ^{4}He ≈ 3.6 Å

Breaker experiment:



[Frank Pobell, 1992]

The Fountain Effect-1938



Superfluids

The viscosity of liquid helium 4 vanishes below 2.17 K

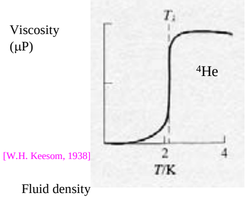
The thermal conductivity becomes very large

A spectacular thermo-mechanical effect: the "fountain effect"

- In February 1938 J.F. Allen and H. Jones had found that when they heated superfluid helium on one side of a porous medium or a thin capillary, the pressure increased sufficiently to produce a fountain effect at the end of the tube which contained the liquid. The "fountain effect" was a spectacular phenomenon that was impossible to understand within classical thermodynamics.

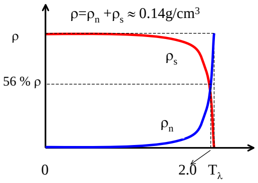
Two Fluid Model– Landau in 1941

(oscillating disc viscometer)



[W.H. Keesom, 1938]

Fluid density



Two Fluid model		density	viscosity	entropy
Two fluids	Super-fluid	ρ_s	$\eta_s = 0$	$S_s = 0$
	normal fluid	ρ_n	$\eta = \eta_n$	$S_n = S_{He}$

Two-fluid equations for He II:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{\upsilon} = 0 \text{ (mass conservation)}$$

$$\frac{\partial \rho s}{\partial t} + \vec{\nabla} \cdot \rho s \vec{\upsilon}_n = 0 \text{ (entropy conservation)}$$

$$\frac{D_s \vec{\upsilon}_s}{Dt} \equiv \frac{\partial \vec{\upsilon}_s}{\partial t} + (\vec{\upsilon}_s \cdot \vec{\nabla}) \vec{\upsilon}_s = -\vec{\nabla} \mu$$

$$\frac{\partial J_i}{\partial t} + \frac{\partial P_{i\alpha}}{\partial r_\alpha} = 0 \text{ (momentum conservation)}$$

stress tensor $P_{ij} = p\delta_{ij} + \rho_n \upsilon_{n,i} \upsilon_{n,j} + \rho_s \upsilon_{s,i} \upsilon_{s,j}$

total mass flow $\vec{J} \equiv \rho \vec{\upsilon} = \rho_n \vec{\upsilon}_n + \rho_s \vec{\upsilon}_s$

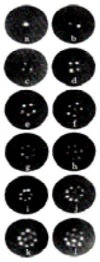
[S.J. Putterman, 1974]

Quantization of Superfluid Circulation

Quantization of superfluid circulation:

$$\kappa = \oint \vec{\upsilon}_s \cdot d\vec{\ell} = \frac{nh}{m} \approx 9.97 \times 10^{-4} n \ cm^2 / s$$

(postulated separately by Onsager and Feynman)



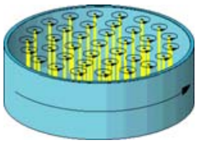
The angular velocity Ω is
(a) 0.30 /s, (b) 0.30 /s,
(c) 0.40 /s, (d) 0.37 /s,
(e) 0.45 /s, (f) 0.47 /s,
(g) 0.47 /s, (h) 0.45 /s,
(i) 0.86 /s, (j) 0.55 /s,
(k) 0.58 /s, (l) 0.59 /s.

[Yarmchuk, 1979]

All superfluid vortex lines align along the rotation axis with ordered array of areal density= length of quantized vortex line per unit volume= $\frac{2\Omega}{\kappa} = \frac{\vec{\nabla} \times \vec{\upsilon}_s}{\kappa}$ ≈ 2000Ω lines/cm^2

1. Circulation round any circular path of radius r concentric with the axis of rotation=$2\pi r^2 \Omega$
2. Total circulation=$\pi r^2 n_0 h/m$ (n_0: # of lines per unit area)
3. $\therefore n_0 = 2 \Omega m/h = 2 \Omega/\kappa$

Rotating bucket of Superfluid



Properties of Superfluids

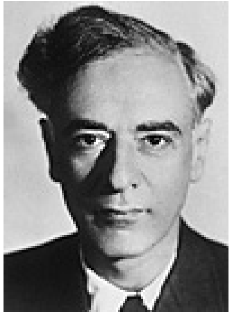
- All of their atoms are in the same quantum state → they have identical momentum; if one moves, they all move
- Ordinary Sound
- Second Sound (Temperature/Entropy Waves)
- Third Sound (Surface Waves)
- Fourth Sound

Second sound is a--

A. Echo of regular sound
B. Pressure/Density wave
C. Sound on the surface of liquid helium
D. Temperature/Entropy wave
E. Sound in a pure superfluid where there is no normal fluid motion

1962 Nobel – Lev Landau

- Constructed the complete theory of quantum liquids at very low temperatures
- He developed theories on both the Bose and Fermi type liquids



http://www.nobel.se/physics/laureates/1962/index.html

1971 – Superfluidity discovered in ^{3}He (US)

- Super fluidity was first discovered in helium-3 by American physicists David M. Lee, Douglas D. Osheroff, and Robert C. Richardson. It occurs at temperatures a few thousandths of a degree above absolute zero and is distinguished by either an A phase or a higher-pressure, lower-temperature B phase. Helium-3 is anisotropic, which means it displays different properties when measured in different directions, and as such, its study has become valuable to scientists in the fields of big-bang theory and superconductivity.

David M. Lee
Cornell University
Ithaca, NY, USA

Douglas D. Osheroff
Stanford University
Stanford, CA, USA

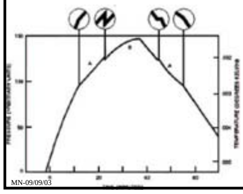
Robert C. Richardson
Cornell University
Ithaca, NY, USA

Discovery of superfluidity in helium-3

Douglas D. Osheroff, David M. Lee, and Robert C. Richardson (US)

In a cryostat, figure 1, a container of ^{3}He was cooled to about 2mK. While the ^{3}He was being slowly compressed at a constant rate, the inner pressure was measured and when 3.4 MPa was reached, the helium was allowed to expand. As the volume decreased and then increased, small changes in the slope of the pressure curve were observed, and also small kinks. These observations were the first evidences of transitions to superfluid phases in ^{3}He. Two superfluid phases were discovered, "A" and "B", figure 2.

Figure 2.
Pressure inside a sample containing a mixture of liquid ^{3}He and solid ^{3}He ice.



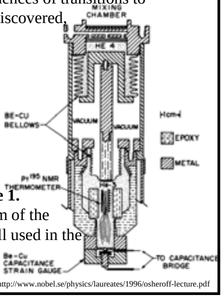


Figure 1.
Schematic diagram of the Pomeranchuck cell used in the discovery.

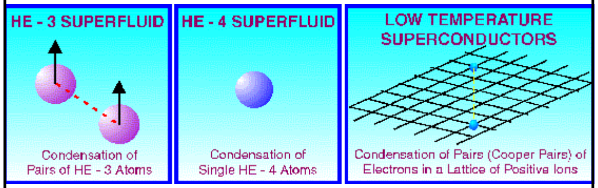
MN-09/09/03

Pictures taken from:http://www.nobel.se/physics/laureates/1996/osheroff-lecture.pdf

Ways to the Superfluid State

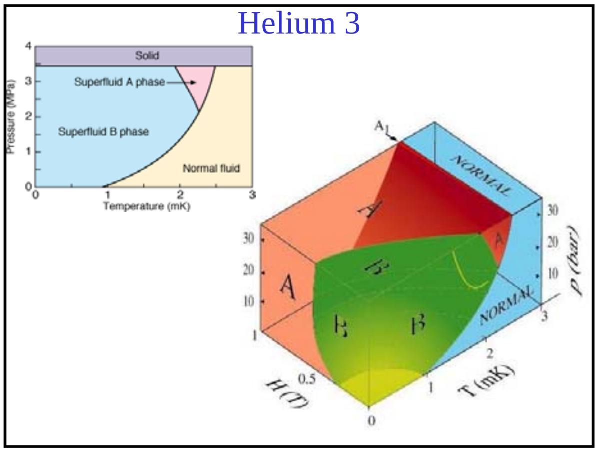
- ^{4}He (even number of elementary particles (6) each with intrinsic angular momentum ½ → integral angular momentum: BOSON; Bose Statistics
- 3He (odd number of elementary particles (5) → half-integral spin: FERMION; Fermi Statistics

Microscopic Picture of Superconductivity

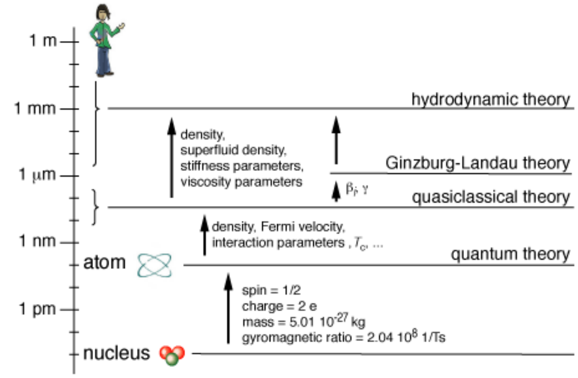


S = 1, L= 1 Hard core interaction	S = 0, L= 0 No pairs required	S = 0, L= 0

Helium 3



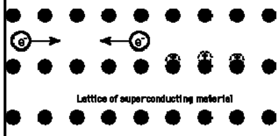
Length Scales in Superfluid ^{3}He

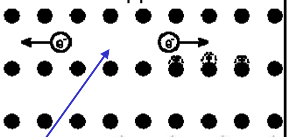


Superconductivity-How do electrons attract?

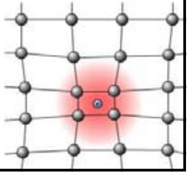
A passing electron attracts the lattice and causes a small ripple in its path

Another electron traveling by is attracted to the slightly positively charged ripple





Cooper Pair



Different view of lattice

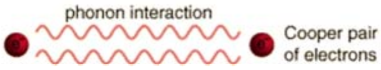
Pictures From
http://sina.sharif.edu/~varahram/hts-course/coop.htm

Cooper Pairs

- The “ripple” propagates as a wave down the lattice through which momentum can be carried
- A phonon has been emitted
- The second electron is attracted to this momentum
- The electron absorbs the phonon
- The electrons have interacted through the exchange of a phonon
- “Its like the following electron surfs on the virtual lattice wake of the leading electron.” *Stephen Godfrey*
- Result is a boson!

www.physics.carleton.ca/courses/ 75.364/mp-2html/node16.html

Cooper Pairs



- Weakly Bound
- Continuously breaking up and reforming
- Large in size-- ~100 nm or more
- Degenerate Energies
- Must have anti-parallel spins S = 0 if electrons
- Must have parallel spins S = 1 if ^{3}He
- Linear momentum must be equal in magnitude but opposite in direction

Picture Adapted From http://hyperphysics.phy-astr.gsu.edu

Properties of Cooper Pairs

Which of the following is true for Cooper pairs?

A. All Cooper pairs have total spin S = 0
B. All Cooper pairs are the same size
C. If the particles making up the Cooper pair have a hard core interaction, then total orbital angular momentum of the pair L = 0
D. If the particles making up the Cooper pair have spin 1/2, then total spin angular momentum of the pair S = 0
E. ^{3}He Cooper pairs have S = 1 and L = 1