P violation, *C* violation and *CP* conservation

In 1957 Wu and co-workers made an experiment on beta decay of Co^{60} . She was testing the bold hypothesis of Li and Yang that parity may be not conserved in weak interactions. They placed a sample of cobalt-60 inside a solenoid and cooled it to a temperature of 0.01 K. At such temperatures, the interaction of the magnetic moments of the nuclei with the magnetic field overcomes the tendency to thermal disorder, and more of the nuclear spins align parallel to the field direction. A polarized cobalt-60 nucleus produced in this way decays to an excited state of nickel-60 by the process:

$$^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni}^{*} + e^{-} + \bar{\nu}_e . \qquad (11.1)$$

Parity violation is established by the observation of a ‘forward–backward decay asymmetry’, that is the fact that fewer electrons are emitted in the forward hemisphere than in the backward hemisphere, with respect to the spins of the decaying nuclei. This is happening because the parity transformation reverses all particle momenta **p** while leaving their orbital angular momenta **r** × **p** (and hence their spin angular momenta), unchanged, as shown in Figure 11.1 on the next slide:

P violation, *C* violation and *CP* conservation (2)

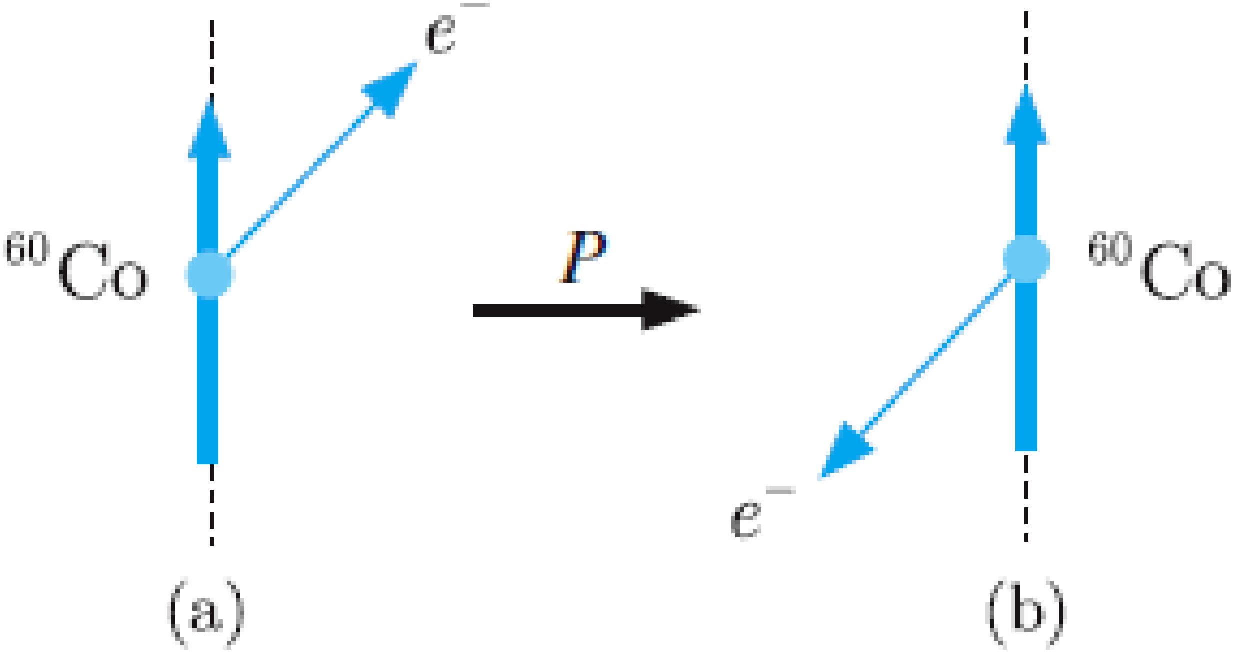

Figure 11.1 Effect of a parity transformation on ^{60}Co decay (11.1). The thick arrows indicate the direction of the spin of the ^{60}Co nucleus, while the thinner arrows show the direction of the electron's momentum.

Muon decay symmetries

Now we look for the evidence for the C violation in muon decays. The existence of *C* violation and its close relationship to *P* violation in leptonic weak interactions are both illustrated by considering the angular distributions of the electrons and positrons emitted in the decays of polarized muons:

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \quad \text{and} \quad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \qquad (11.2)$$

$$\Gamma_{\mu^\pm}(\cos\theta) = \frac{1}{2}\Gamma_\pm\left(1 - \frac{\xi_\pm}{3}\cos\theta\right), \qquad (11.3)$$

Here *ϑ* is the angle between the muon spin direction and the direction of the outgoing electron or positron, as shown in Figure 11.2(a) on the next slide.

The quantities *ξ±* are called the *asymmetry parameters* and Γ*±* are equal to the total decay rates, or equivalently the inverse lifetimes

$$\tau_\pm^{-1} \equiv \int_{-1}^{+1} \mathrm{d}\cos\theta\, \Gamma_{\mu^\pm}(\cos\theta) = \Gamma_\pm, \qquad (11.4)$$

Muon decay symmetries (2)

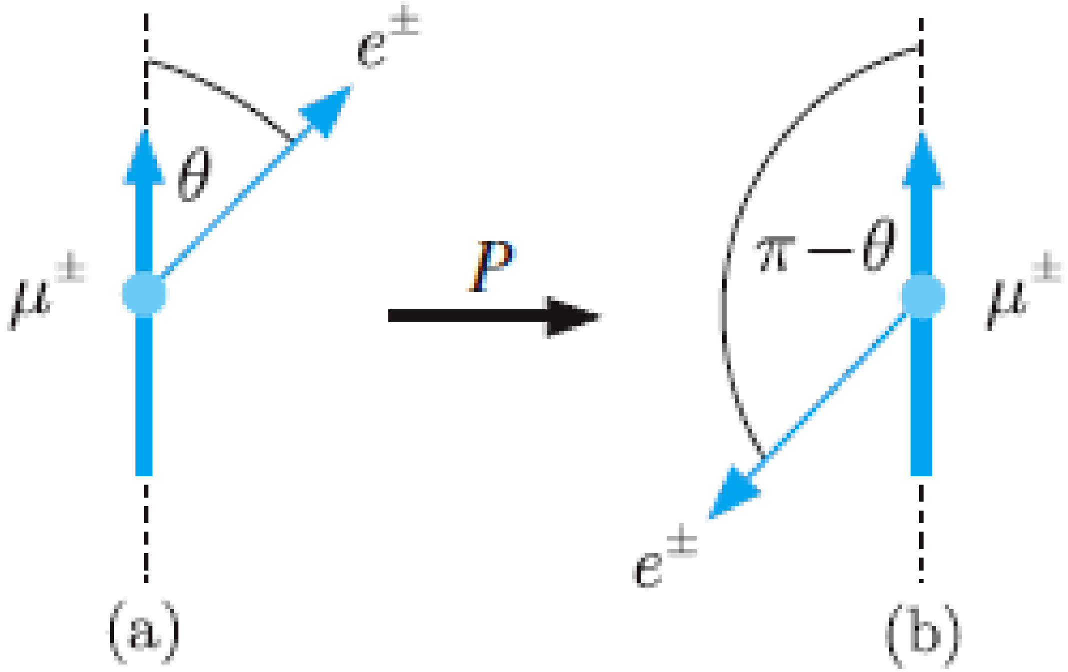

Figure 11.2 Effect of a parity transformation on the muon decays (11.2). The thick arrows indicate the direction of the muon spin, while the thinner arrows show the direction of the electron's momentum.

Muon decay symmetries (3)

We consider now the consequences of assuming parity and charge conjugation for these decays, starting with the charge conjugation. Charge conjugation transforms all particles into their antiparticles, so that μ^- decay converts into μ^+ decay. If we assume *C* invariance, then that the rates and angular distributions for these decays should be the same, that is

$$\Gamma^+ = \Gamma^- \quad (C \text{ invariance}) \qquad (11.5a)$$

and

$$\xi^+ = \xi^- \quad (C \text{ invariance}) \qquad (11.5b)$$

Parity transformation *P* as we saw changes the angle ϑ to $\pi - \vartheta$ so that $\cos\vartheta$ changes sign. Hence *P* invariance implies

$$\Gamma_{\mu^\pm}(\cos\vartheta) = \Gamma_{\mu^\pm}(-\cos\vartheta) \qquad (P \text{ invariance}) \qquad (11.6a)$$

and substituting (11.3) into (11.6a) then gives that the asymmetry parameters vanish,

$$\xi^\pm = 0 \qquad (P \text{ invariance}) \qquad (11.6b)$$

Experimentally, the $\mu^\pm$ lifetimes are equal to very high precision, so that (11.5a) is satisfied, but the measured values of the asymmetry parameters are

$$\xi^- = -\xi^+ = 1.00 \pm 0.04, \qquad (11.7)$$

which shows that both *C* invariance (11.5b) and *P* invariance (11.6b) are violated

Muon decay symmetries (4)

In view of these results, a question that arises is: why do the $\mu+$ and $\mu-$ have the same lifetime if *C* invariance is violated? The suggested answer lies in the principle of *CP* conservation, which states that the weak interaction is invariant under the combined operation *CP*, even though both *C* and *P* are separately violated. The *CP* operator transforms particles at rest to their corresponding antiparticles at rest, and *CP* invariance requires that these states should have identical properties. Thus, in particular, the masses of particles and antiparticles are predicted to be the same. More specifically, if we apply the *CP* operator to muon decays, the parity operator changes ϑ to $\pi - \vartheta$ as before, while the *C* operator changes particles to antiparticles.

Thus *CP* invariance retains the symmetry between particles and antiparticles as observed by experiment in muon decays. *CP* invariance later has been verified in many experiments involving weak interactions, and the data are consistent with exact *CP* conservation in the weak interactions of leptons. As we will see later, the weak interactions of quarks do not respect *CP* invariance, although it is often a very good approximation, and the observed violations are so far confined to the decays of neutral *K* mesons and of *B* mesons. This will be discussed in Sections 11.2 and 11.3. For the moment, we will concentrate on the properties of the leptons and assume that *CP* conservation is exact.

Left-handed neutrinos and right-handed antineutrinos

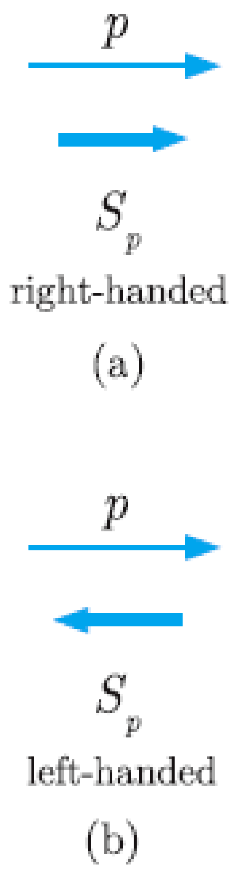

Figure 11.3 Helicity states of a spin-1/2 particle. The thin arrows represent the momenta of the particles and the thick arrows their spins.

Now we define *helicity states*, in which the spin is quantized along the direction of motion of the particle, rather than along some arbitrarily chosen ‘*z* direction’. For a spin-1/2 particle, the spin component along the direction of its motion can be either +1/2 or −1/2, as illustrated in Figure 11.3 on the left, corresponding to positive or negative helicity respectively. These states are called *right-handed* or *left-handed* respectively, since the spin direction corresponds to rotational motion in a right-handed or left-handed sense when viewed along the momentum direction. We will denote these states by subscripts *R* or *L*, so that ν_L means a left-handed neutrino, e^-_R a right-handed electron and so on.

Only left-handed neutrinos and right-handed antineutrinos have been observed in nature with the precision of experiments. This obviously violates *C* invariance, which requires neutrinos and antineutrinos to have identical weak interactions. It also violates *P* invariance, which requires the states ν_L and ν_R to have identical weak interactions, since the parity operator reverses the momentum while leaving the spin unchanged, and so converts a left-handed neutrino into a right-handed neutrino. It is, however, compatible with *CP* invariance, since the *CP* operator converts a left-handed neutrino to a right-handed antineutrino, as illustrated in Figure 11.4 on the right.

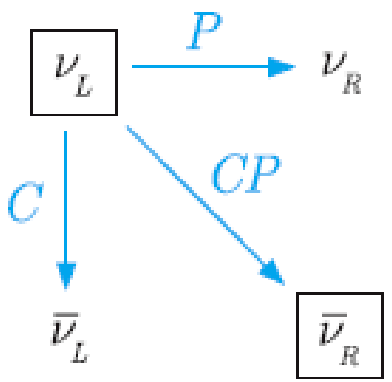

Figure 11.4 Effect of C, P and CP transformations on a left-handed neutrino ν_L. Only the states shown in boxes have been observed in nature.

Helicity of neutrino measurement

The helicity of the neutrino was first measured in an ingenious "tabletop" experiment by Goldhaber and co-workers in 1958. They studied electron capture in europium-152:

$$e^- + {}^{152}\mathrm{Eu}\,(J=0) \rightarrow {}^{152}\mathrm{Sm}^*\,(J=1) + \nu_e, \qquad (11.10)$$

where the spins of the nuclei are shown in brackets. The excited state of samarium that is formed decays to the ground state by γ-emission

$$^{152}\mathrm{Sm}^*(J=1) \rightarrow {}^{152}\mathrm{Sm}\,(J=0) + \gamma \qquad (11.11)$$

and it is these γ rays that were detected in the experiment. In the reaction (11.10) the electrons are captured from the *K*-shell and the initial state has zero momentum, so that the neutrino and the $^{152}\mathrm{Sm}^*$ nucleus recoil in opposite directions. This was done by resonant scattering from a second samarium target. It relies on the fact that those γ-rays travelling in the opposite direction to the neutrino have slightly more energy than those emitted in other directions, and only in this case the photons have enough energy to excite the resonance level. The experiment selected events in which the photon was emitted in the direction of motion of the decaying $^{152}\mathrm{Sm}^*$ nucleus, so that overall the observed reaction was

$$e^- + {}^{152}\mathrm{Eu}\,(J=0) \rightarrow {}^{152}\mathrm{Sm}\,(J=0) + \nu_e + \gamma, \qquad (11.12)$$

where the three final-state particles were collinear, and the neutrino and photon emerged in opposite directions, as shown in Figure 11.5. The helicity of the neutrino can then be deduced from the measured helicity of the photon by applying angular momentum conservation about the event axis to the overall reaction (11.12). In doing this, no orbital angular momentum is involved, because the initial electron is captured from the *K*-shell and the final-state particles all move along the event axis. Hence the spin components of the neutrino and photon, which can be ±1/2 and ±1, respectively, must add to give the spin component of the initial electron, which can be ±1/2. This gives two possible spin configurations, as shown in Figures 11.5(a), so the neutrino is lefthanded.

The polarization of the photons was determined by studying their absorption in magnetized iron, and the results obtained were consistent with the occurrence of left-handed neutrinos only, corresponding to Figure 11.5(a)

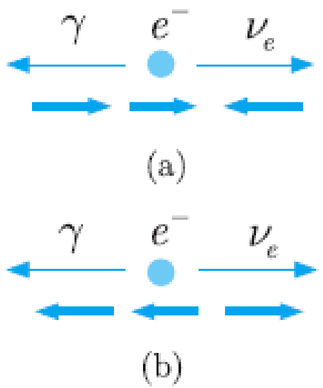

Figure 11.5 Possible helicities of the photons and neutrinos emitted in (11.12) for those events in which they are emitted in opposite directions. Measurements of the photon helicity show that only events corresponding to case (a), left-handed neutrinos, are observed.