Review problem

Chapter 17 page 575

#13.

A 1.0cm wide diffraction grating has 1000 slits.

It is illuminated by light of wavelength 550nm. What are the angles of the first two diffraction orders?

Review problem

Chapter 17 page 576. Problem 25

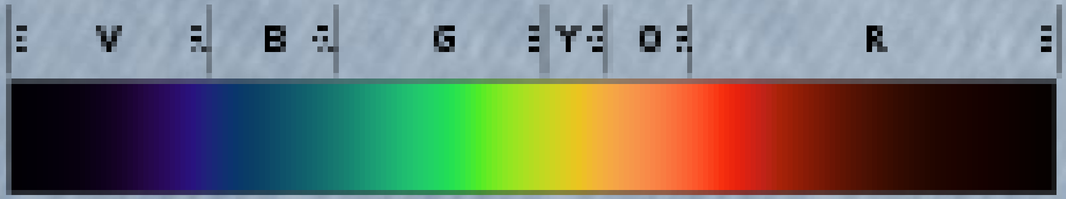
Solar cells are given antireflection coatings to maximize their efficiency…..

Wave Optics

- The wave model
- Diffraction and interference
- Double slit and grating interference
- Index of refraction
- Thin-film interference
- Huygens’ principle
- Single-slit and circular diffraction

What is light?

- We know light is a small part of the Electromagnetic spectrum, λ=400nm-750nm
- Three models for light
  - Wave model – light acts as waves
  - Ray model – light travels in straight lines
  - Photon model – light is made of quanta of energy

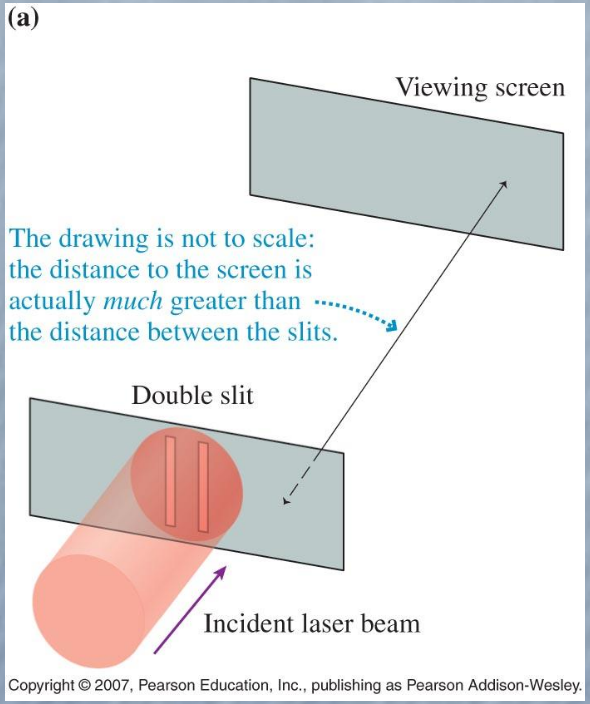


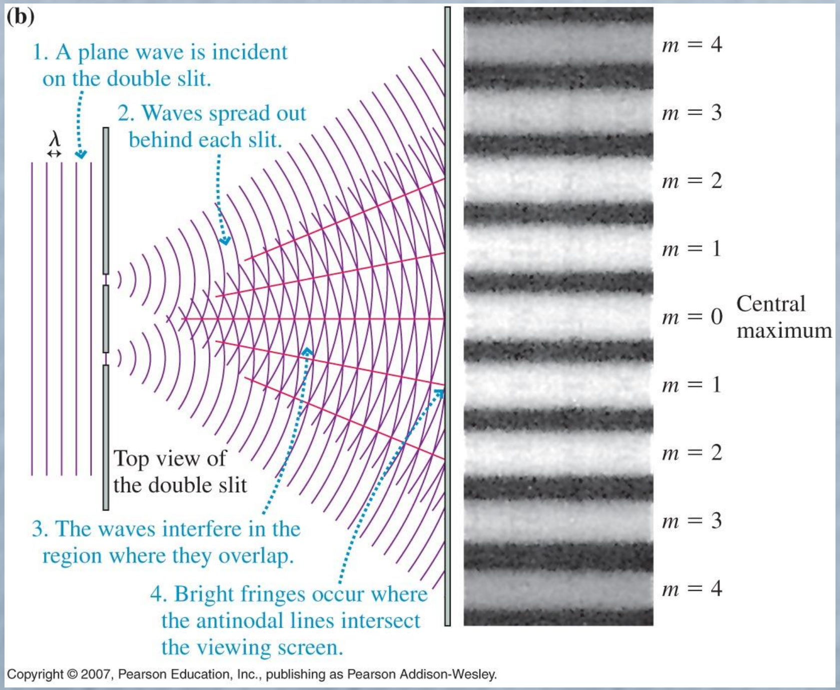
Interference of light

- Wave effects can be seen with light when we look at the them with “small” probes.
- Light wavelength is 500nm. We can just see the aspects of waves with instruments of dimensions of 0.5mm (1000 wavelengths)
- At larger dimensions, light acts as rays.

Young's double slit experiment

In 1801, Young showed light diffracted like water waves. Difficult to do with sunlight and cards in a darkened room.

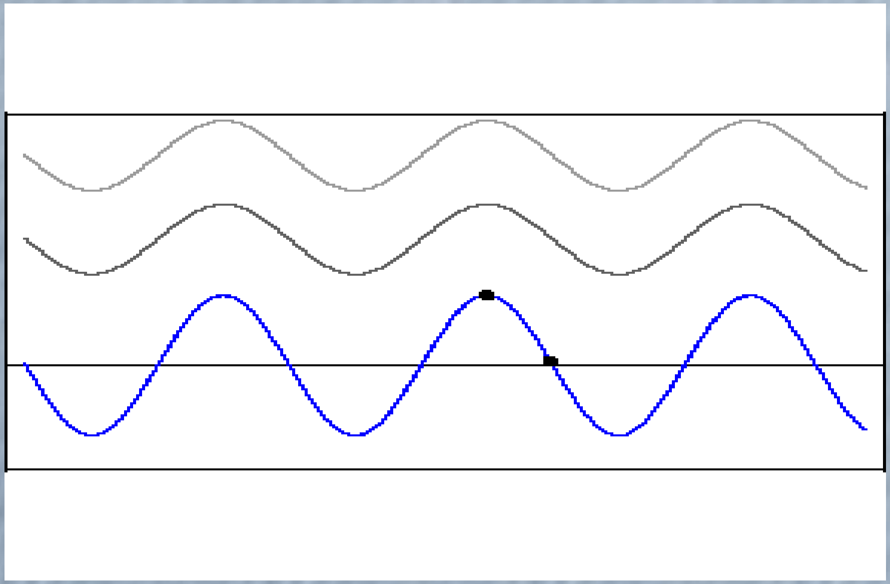




Fringes from the double slit

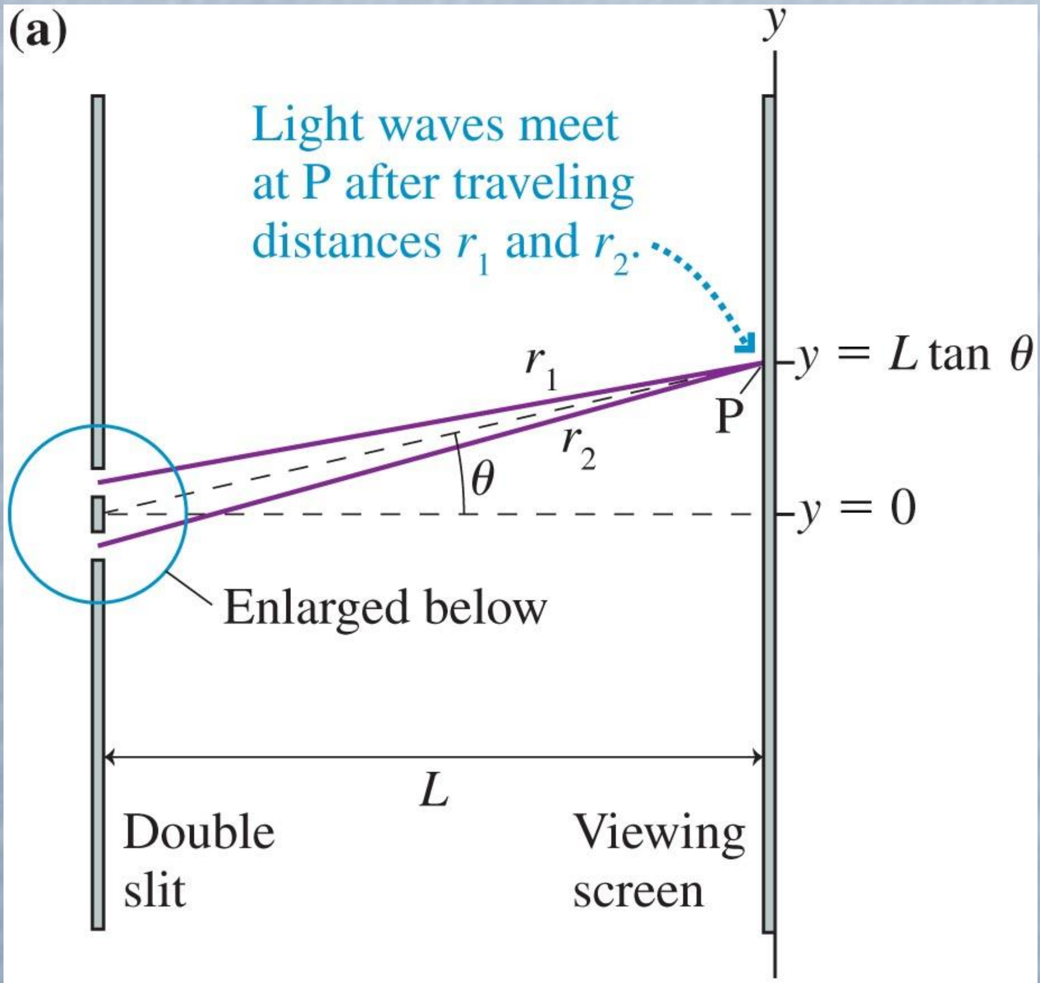
In Ch 16, we saw that

- constructive interference occurs when the waves align, or in phase.
- destructive interference occurs when the waves are out of phase



Fringe spacing

- The light fringes occur when the path difference between the 2 slits and the screen are a whole number of wavelengths

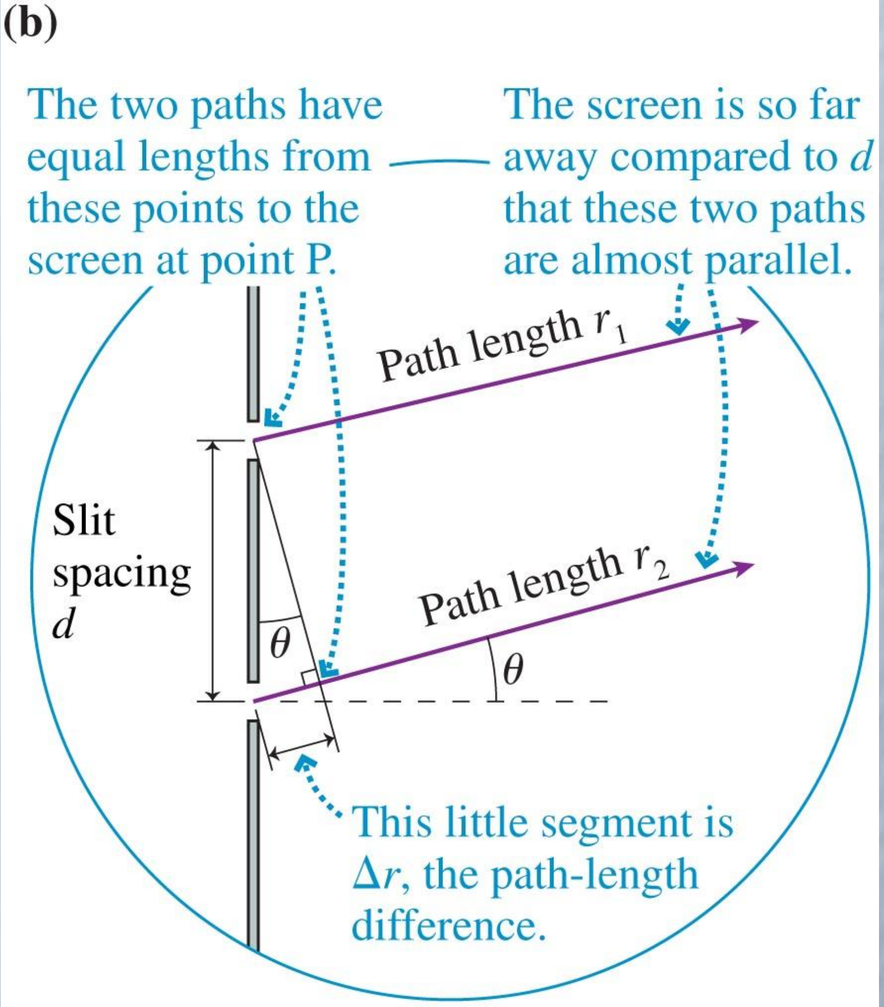


Fringe spacing

Path difference, Δr, at an angle, θ, must be a whole number of wavelengths:

$$\Delta r = m\lambda$$

$$m = 0,1,2,3\ldots$$



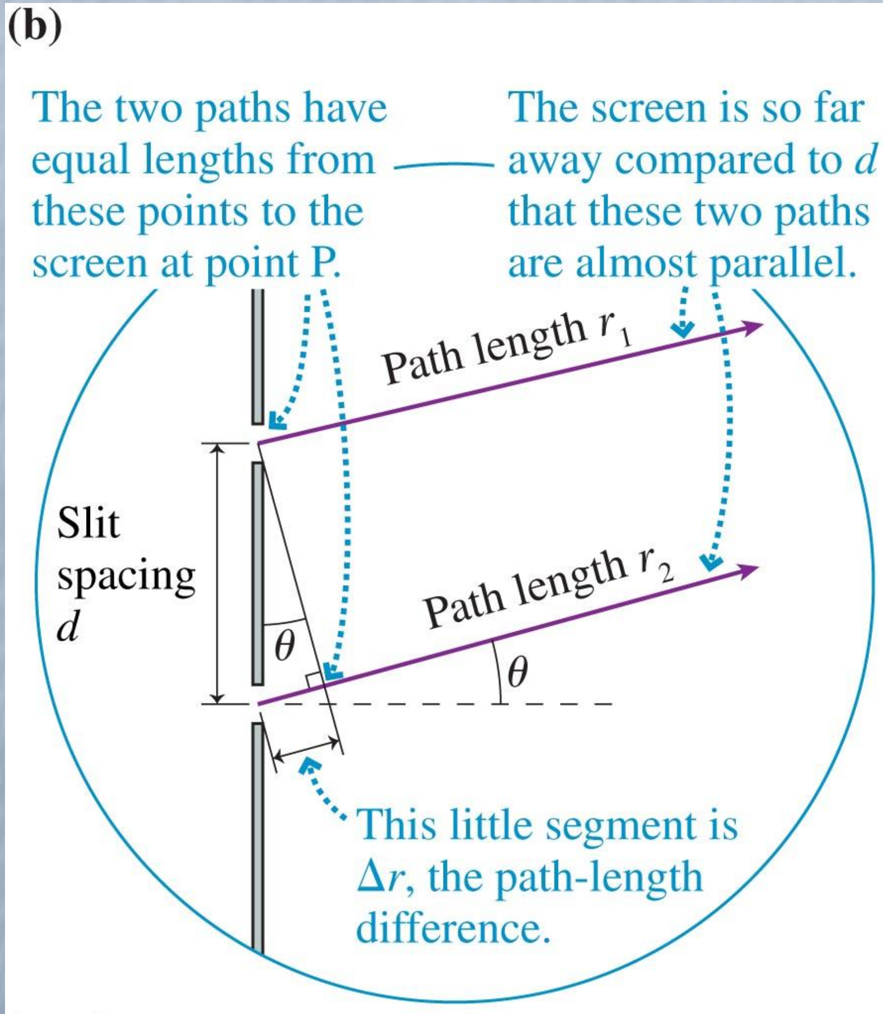
Fringe spacing

For a slit distance, d, and angle, θ:

$$\Delta r = d \sin\theta$$

$$d \sin\theta = m\lambda$$

$$m = 0,1,2,3\ldots$$



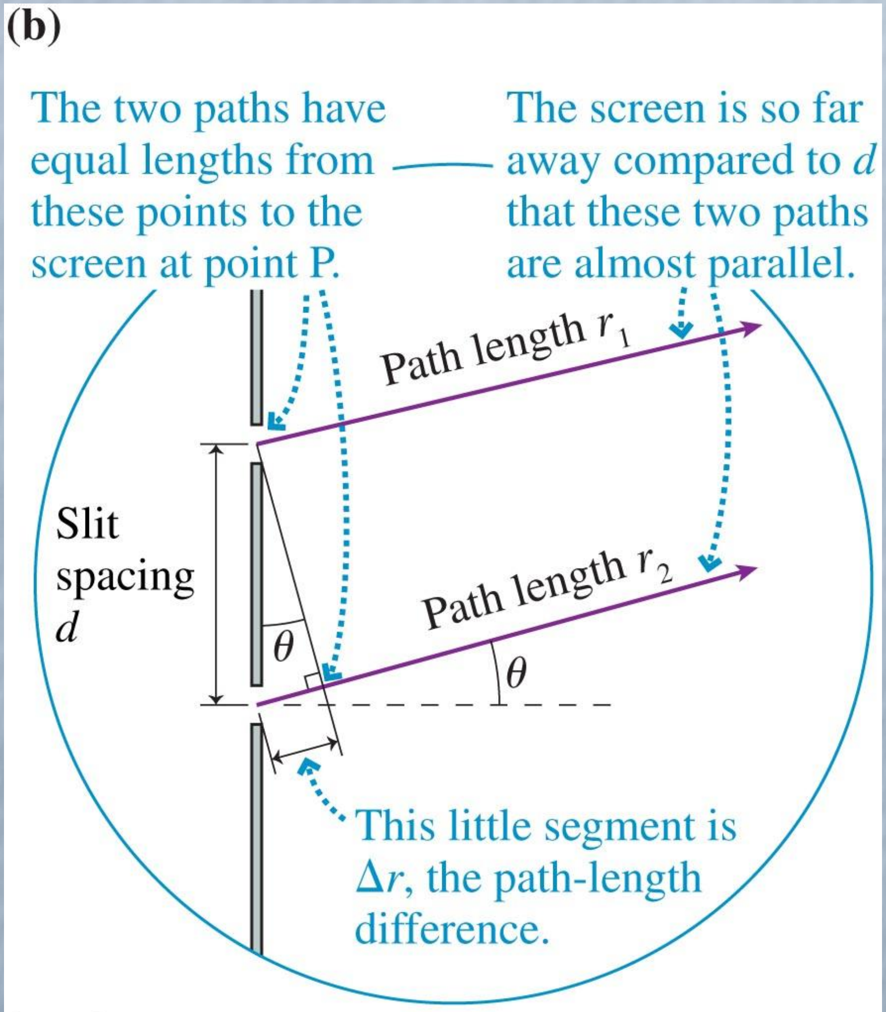
Fringe spacing

For small angles, θ<<1

$$d\theta = m\lambda$$

$$\theta_m = m\frac{\lambda}{d}$$

$$m = 0,1,2,3\ldots$$



Fringe spacing

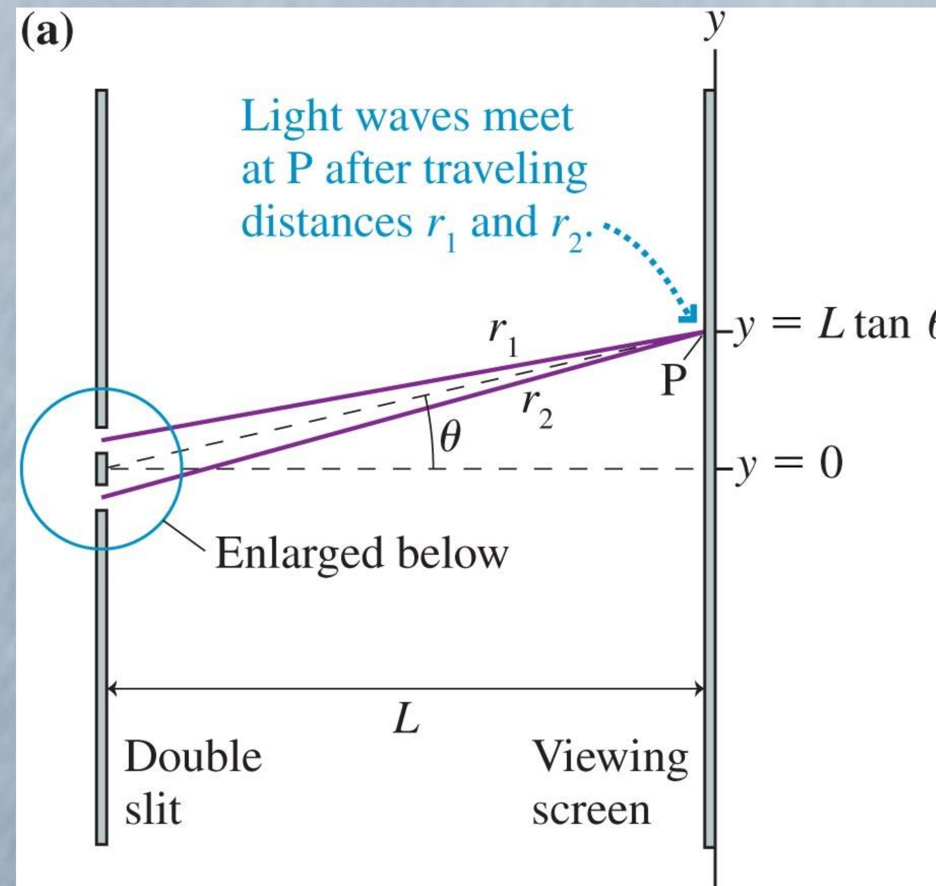
If the screen is a distance L away:

$$y = L\tan\theta$$

$$\theta = \sin\theta = \tan\theta$$

$$y_m = \frac{m\lambda L}{d}$$

$$m = 0,1,2,3\ldots$$

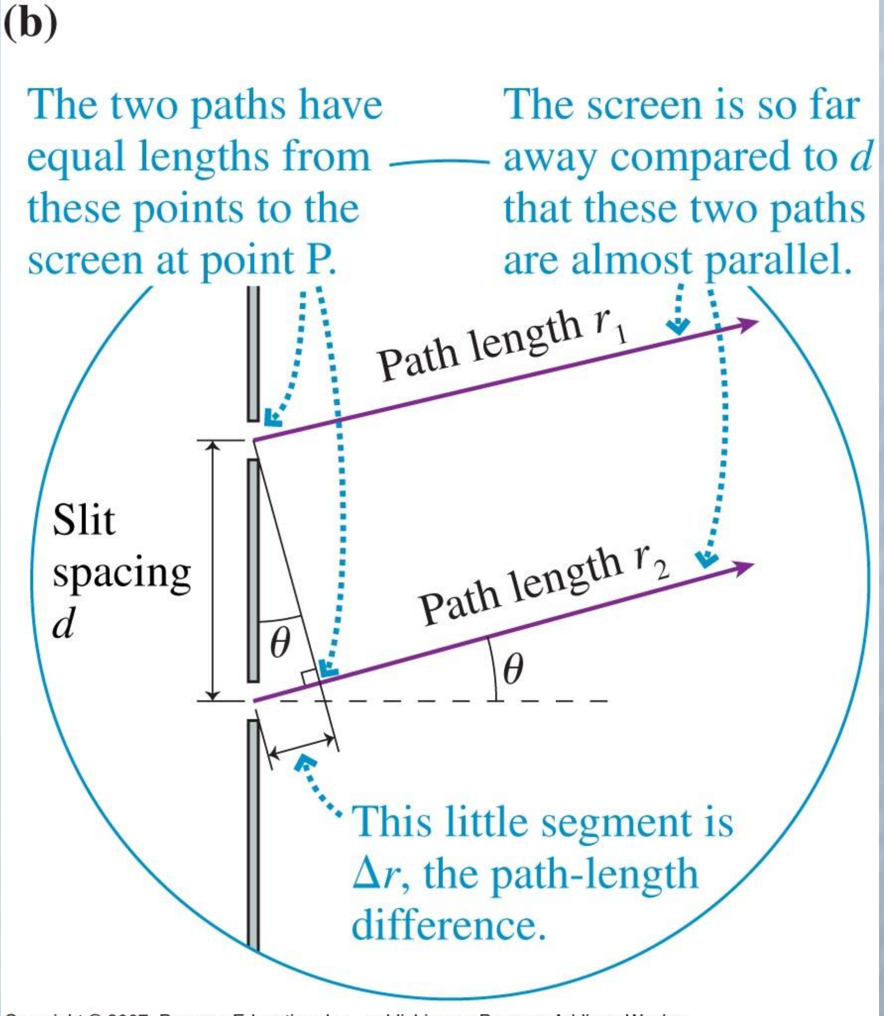


Dark fringe spacing

Path difference, Δr, at an angle, θ, must be a whole number + 1/2 of wavelengths for destructive interference:

$$\Delta r = \left(m + \frac{1}{2} \right) \lambda$$

$$m = 0,1,2,3...$$



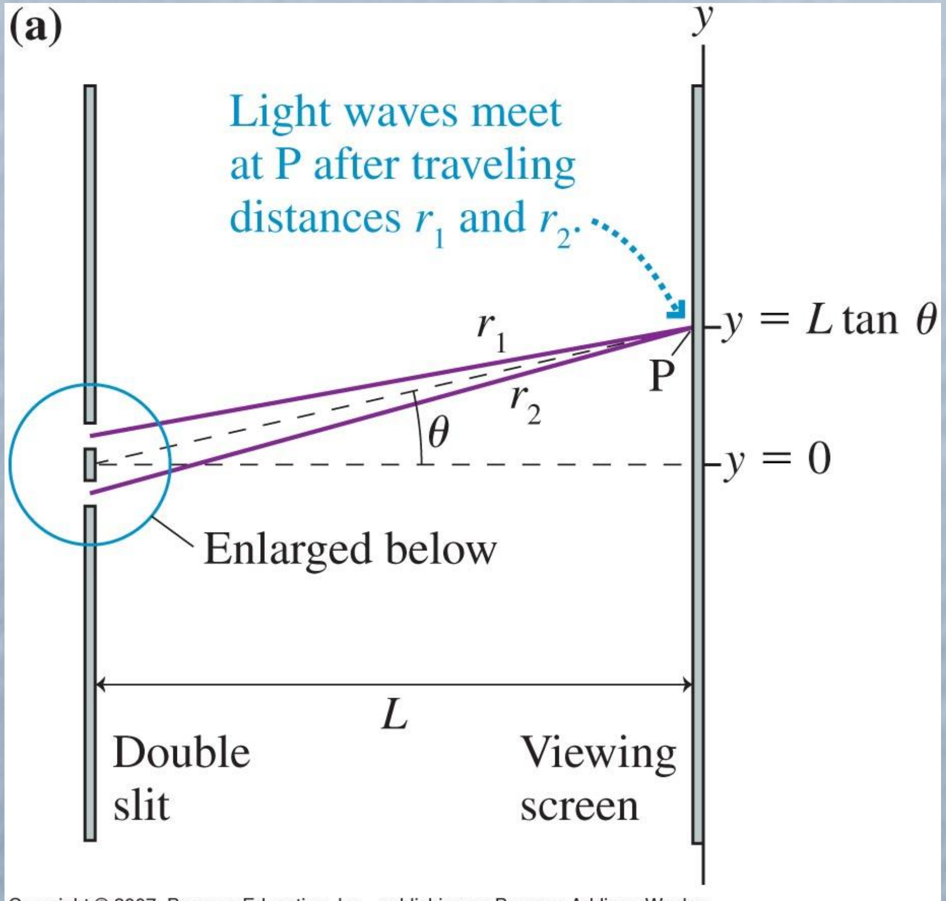
Dark fringe spacing

If the screen is a distance L away:

$$\theta = \sin\theta = \tan\theta$$

$$y_m = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d}$$

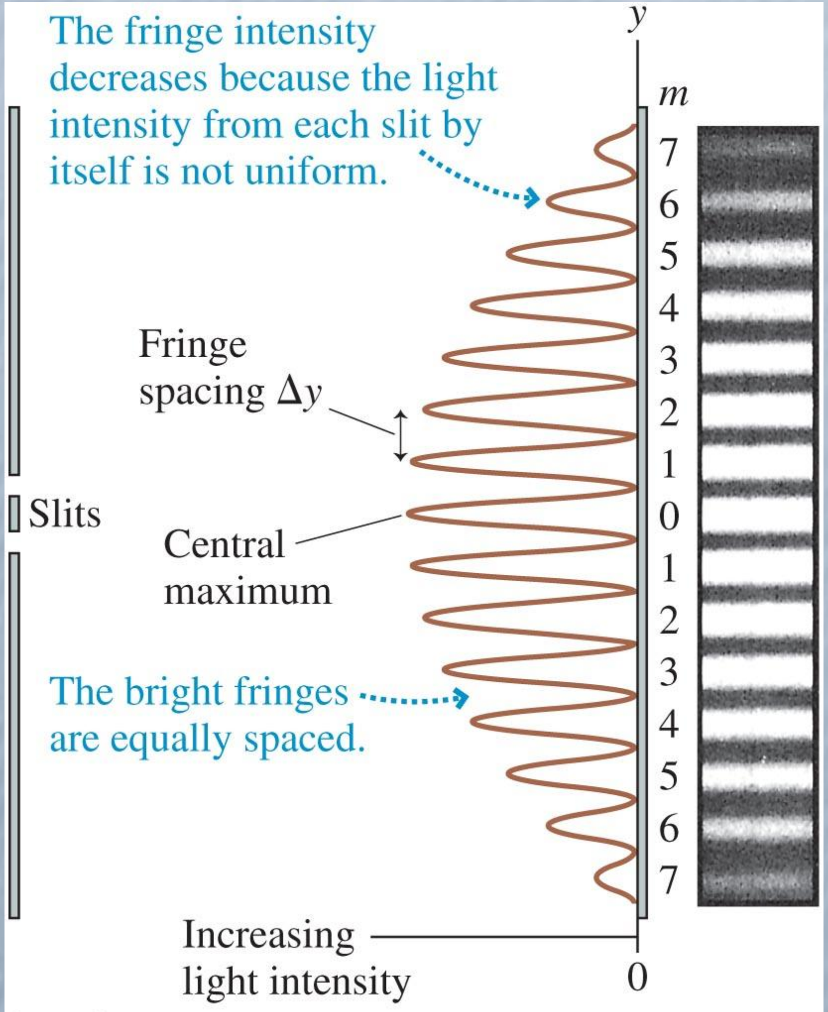
$$m = 0,1,2,3...$$



Distance between fringes

Adjacent fringe spacings, y_m and y_{m+1}

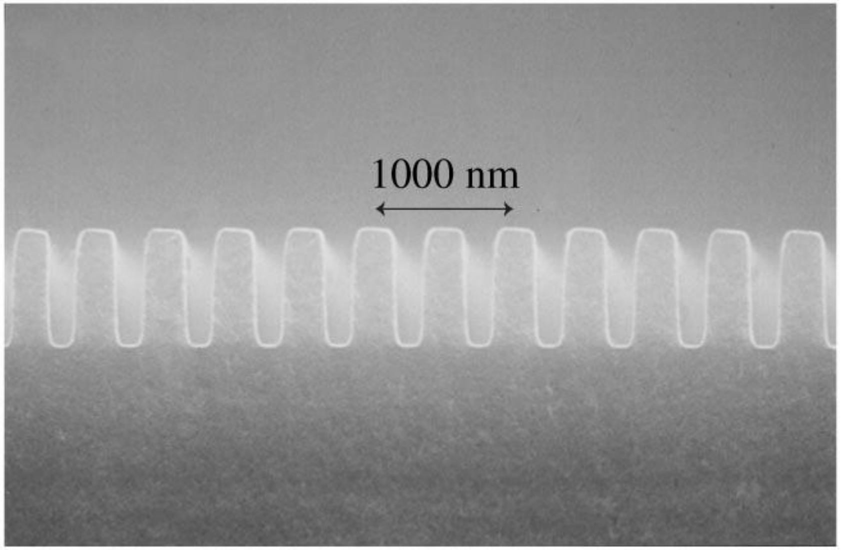
$$y_{m+1} - y_m = \frac{\lambda L}{d}(m+1-m) = \frac{\lambda L}{d}$$



- Dark fringes are exactly half way between the light fringes.
- We can measure the wavelength of light

Diffraction grating

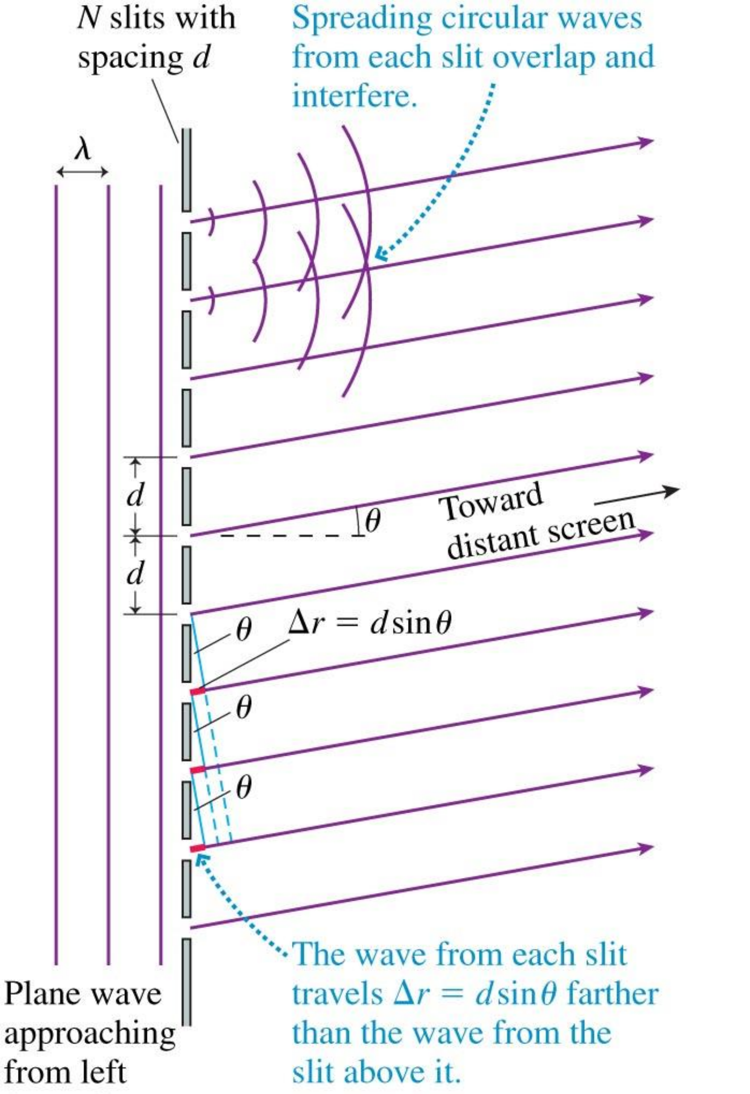
We can expand the 2 slit experiment to a diffraction grating which has a lines ruled every 1μm:



Diffraction grating

The light-path difference between adjacent slits is still

$$\Delta r = d\sin\theta$$

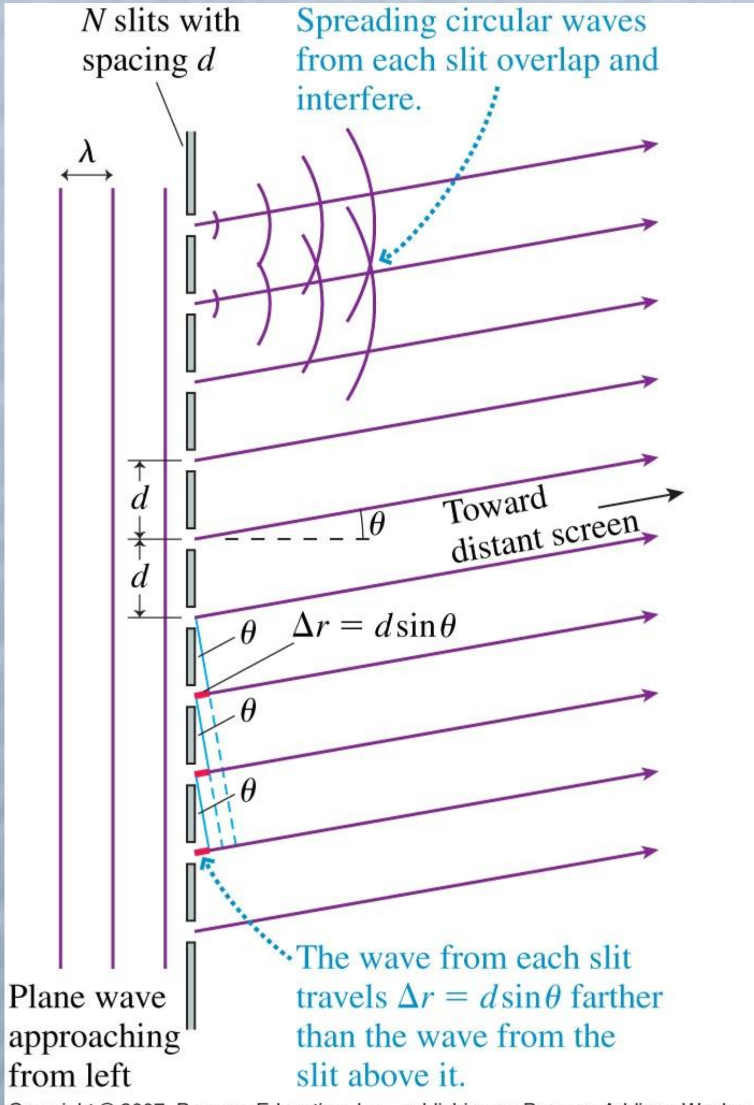


Diffraction grating

For constructive interference, we need the light path difference to be a whole number of wavelengths

$$d \sin\theta = m\lambda$$

$$m = 0, 1, 2, 3, \ldots$$



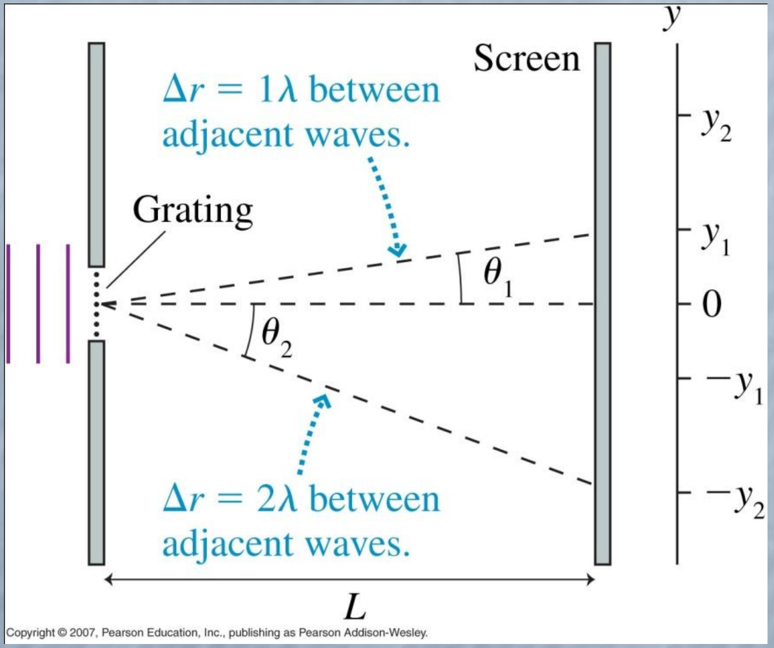
Diffraction grating

We define the central fringe as the zero order, and subsequent fringes as the m'th **order**

$$d \sin \theta = m\lambda$$

$$m = 0,1,2,3,\ldots$$

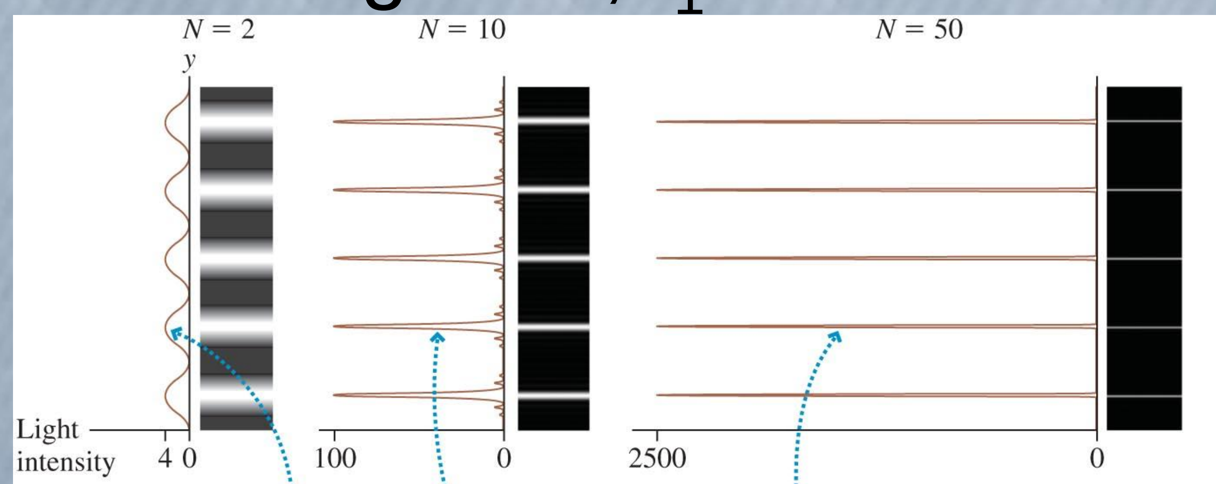
$$y_m = L \tan \theta_m$$



Intensity of fringes

- The fringes are better defined and brighter.
- The path length differences need to be closer to get constructive interference
- The maximum intensity of the fringes is related to the intensity from a single slit, I_1 as:

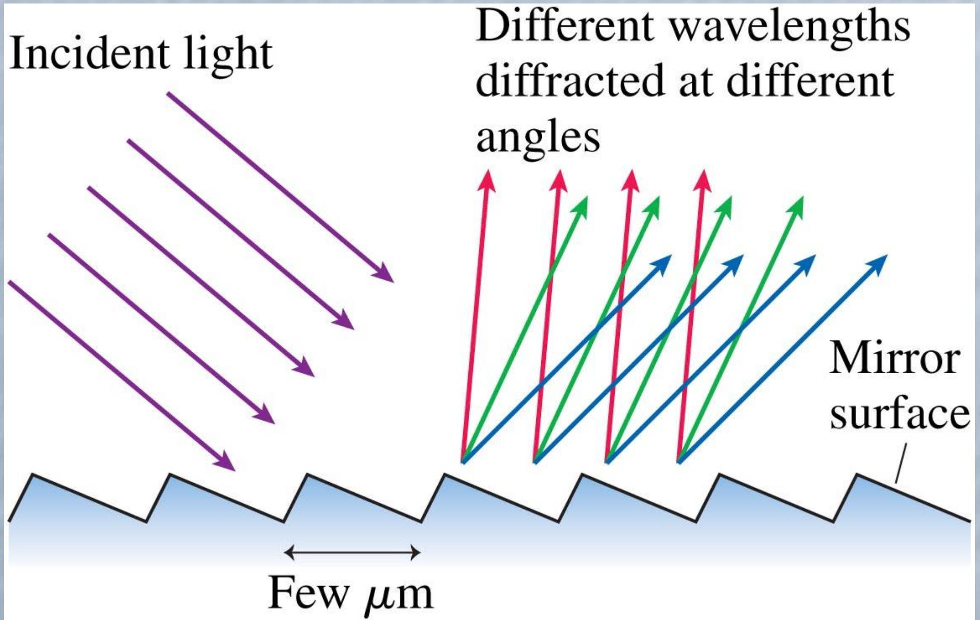
$$I_{\max} = N^2 I_1$$



As the number of slits in the grating increases, the fringes get narrower and brighter.

Reflection gratings

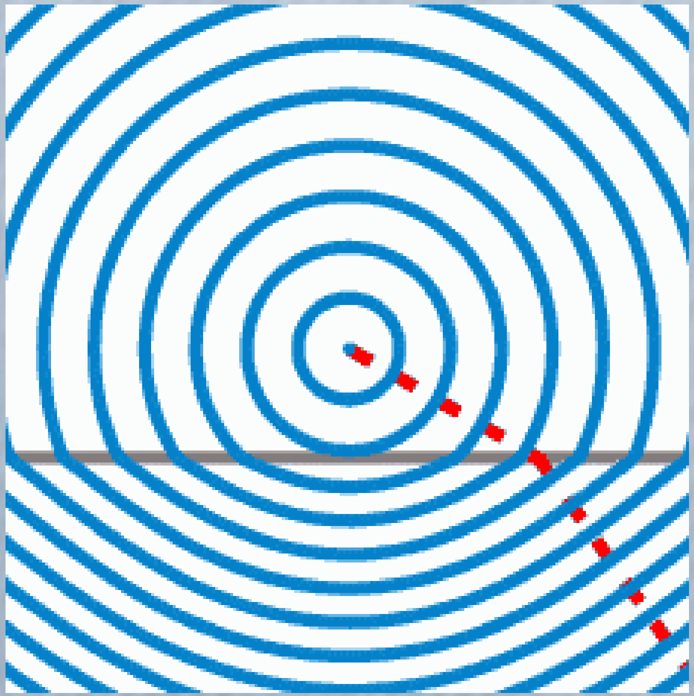
- Easier and cheaper to make.
- Same wavelength spacing laws
- Occur in nature - iridescence



A reflection grating can be made by cutting parallel grooves in a mirror surface. These can be very precise, for scientific use, or mass produced in plastic.

Index of refraction

- Light slows down in transparent materials
- We define the index of refraction as the ratio



$$n = \frac{c}{v}$$

Index of refraction is the ratio of the speed of light in a vacuum to speed of light in the material. It is 1 for a vacuum, and greater than 1 for materials.

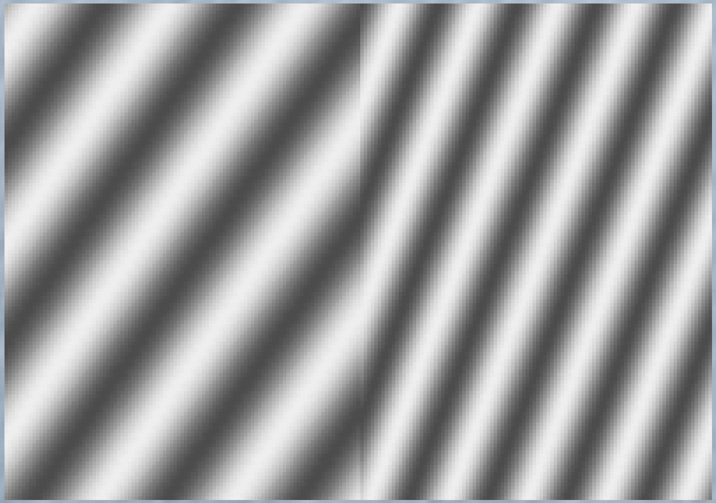
Index of refraction

The frequency must stay the same, but as the velocity changes, then wavelength must change

$$f_{material} = f_{vacuum}$$

$$\lambda_{material} = \frac{v_{material}}{f} = \frac{c}{nf}$$

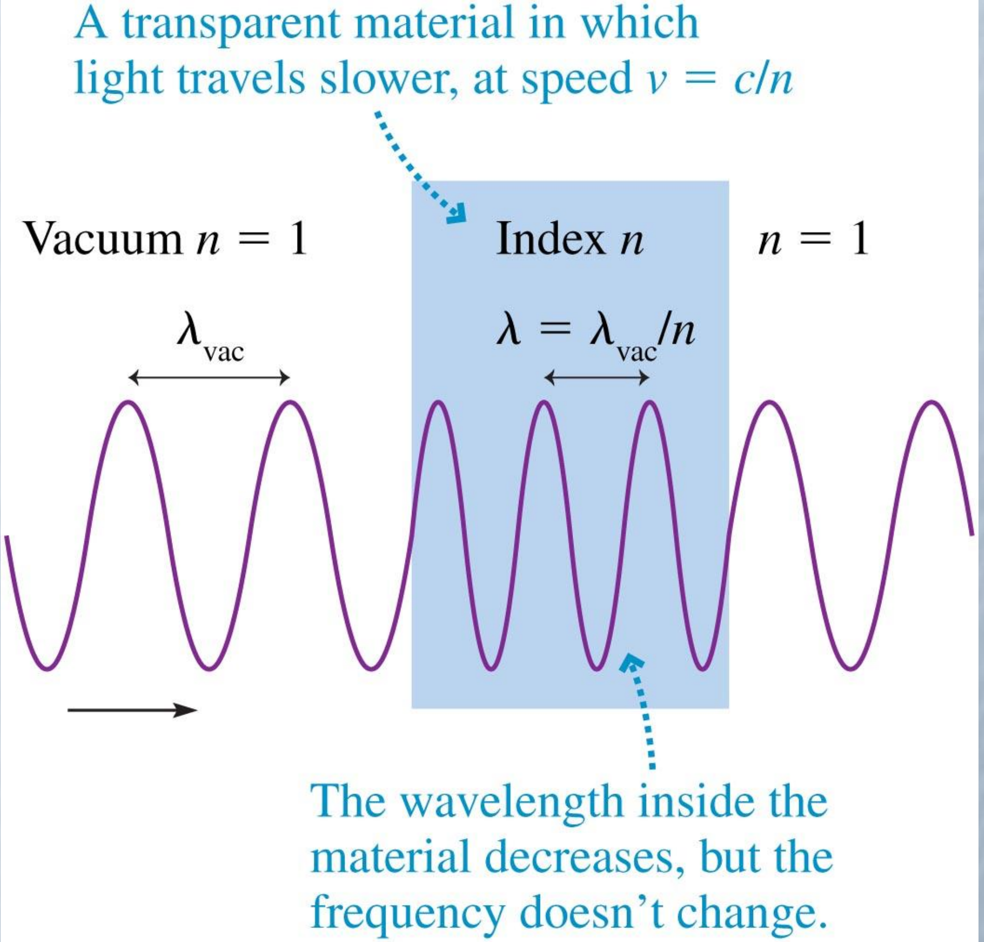
$$\lambda_{material} = \frac{\lambda_{vacuum}}{n}$$



Index of refraction

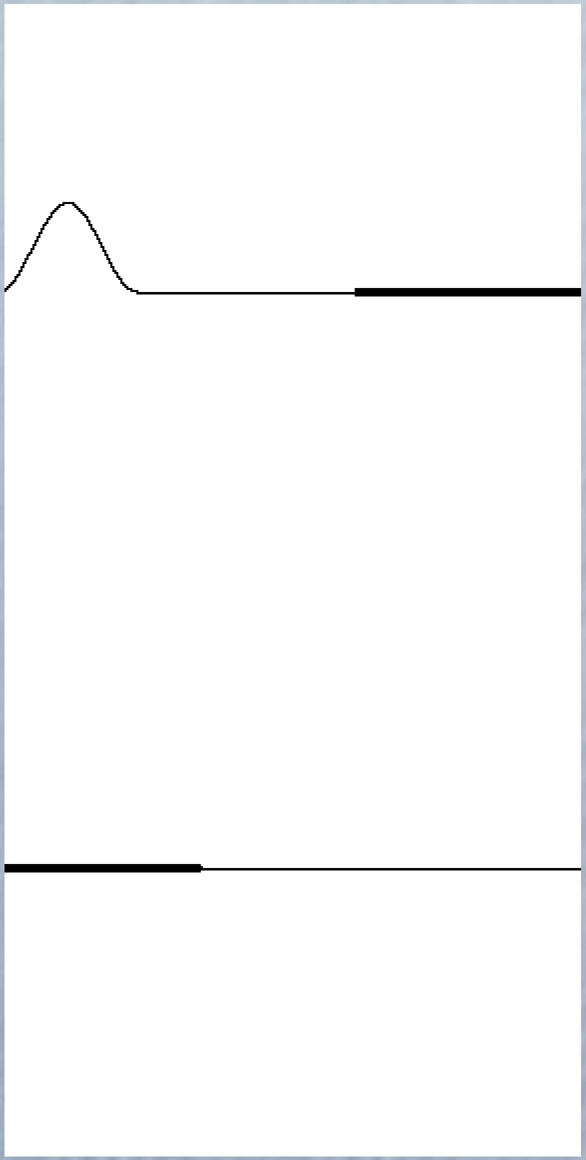
- Vacuum n=1.
- Air = 1.0003
- Water = 1.33
- Glass = 1.5
- Diamond = 2.42

And that's why diamonds are so special



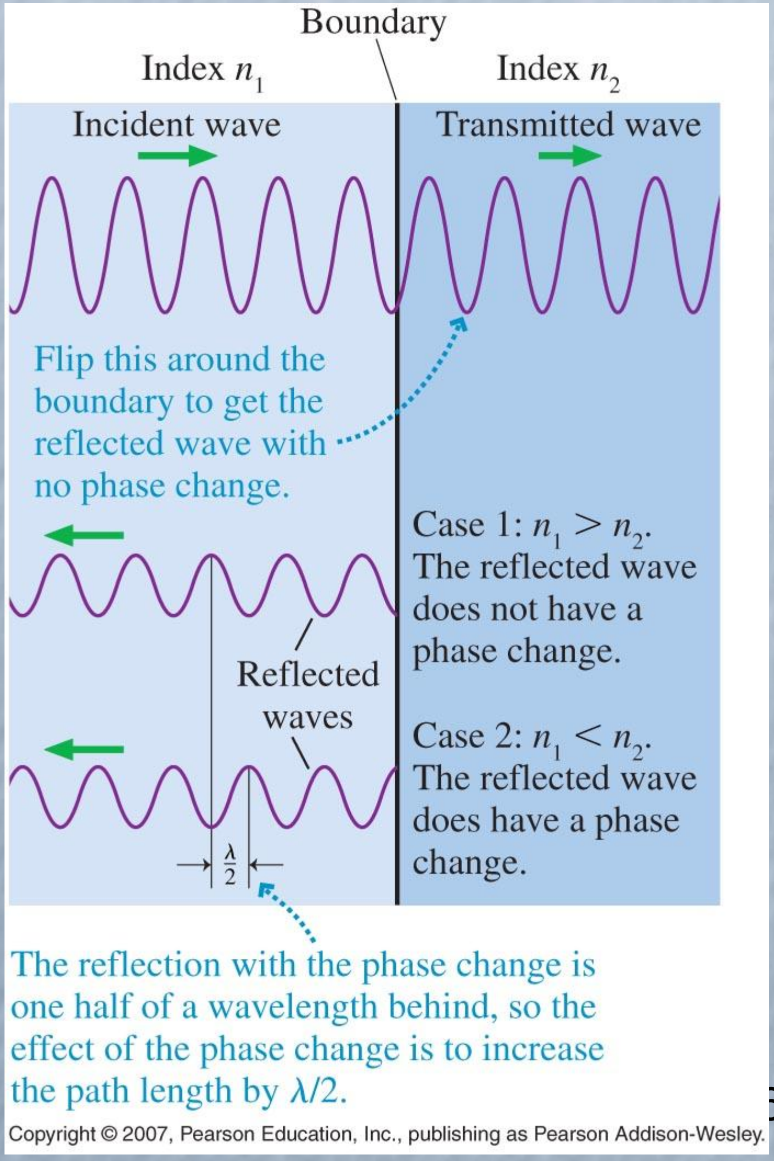
Thin-film interference

- Remember - waves get reflected at a medium change



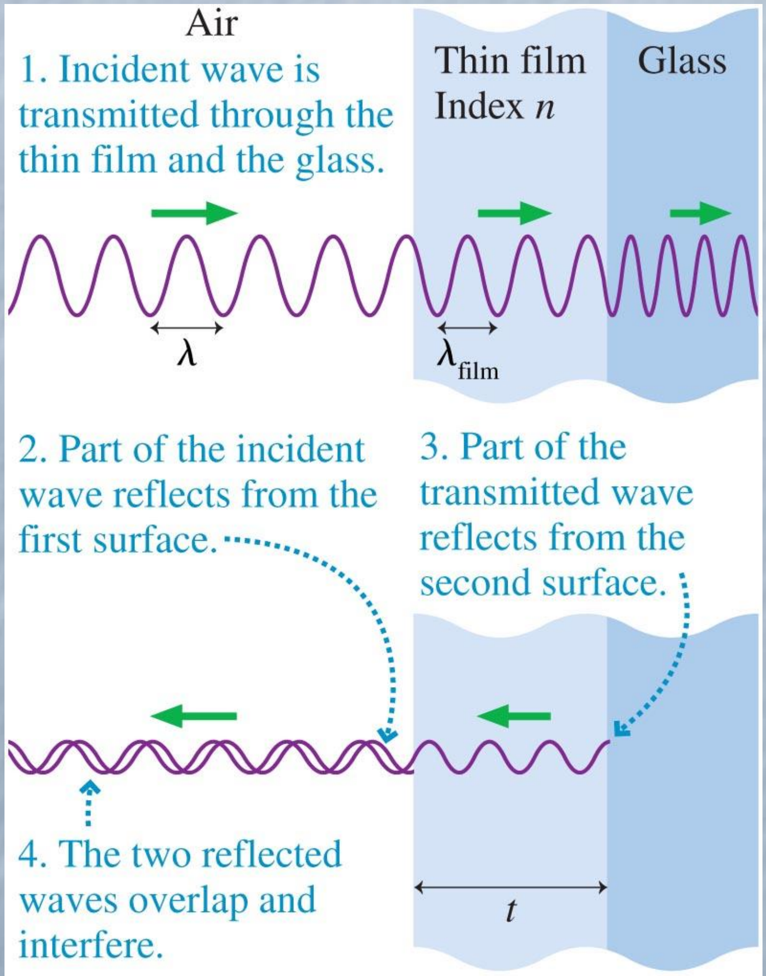
Thin film interference

- Light will get reflected at a boundary between two transparent media
- When $n_1 > n_2$ we do not have a phase change
- When $n_1 < n_2$ we do have a phase change
- Remember n is larger for slower materials



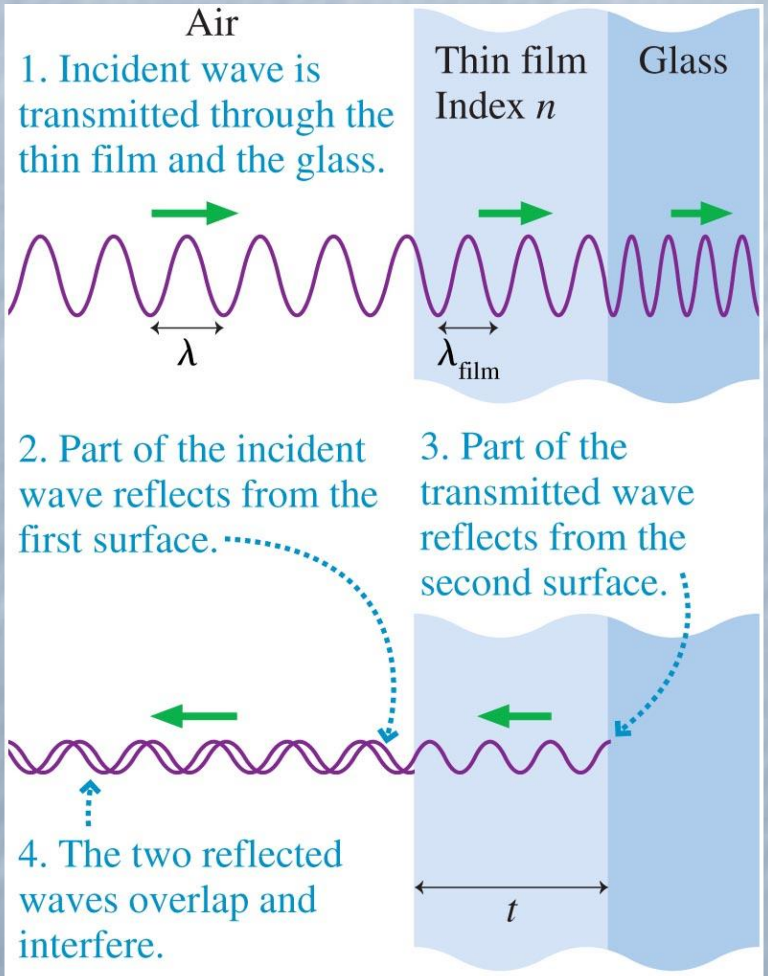
Thin film interference

- Consider the boundaries for a thin film, width t, on a medium
- Part of the light will get reflected at the air-film, and part reflected at the film-medium
- We can pick the film thickness so that the 2 reflections cancel



Thin film interference

- Due to the refractive indices being different, we can have either 0, 1 or 2 inversions at the 2 reflections.
  - If there are 0 or 2 inversions, the effective path-length change is 2t
  - If there is 1 inversion, the change is $2t+1/2\ \lambda_{film}$



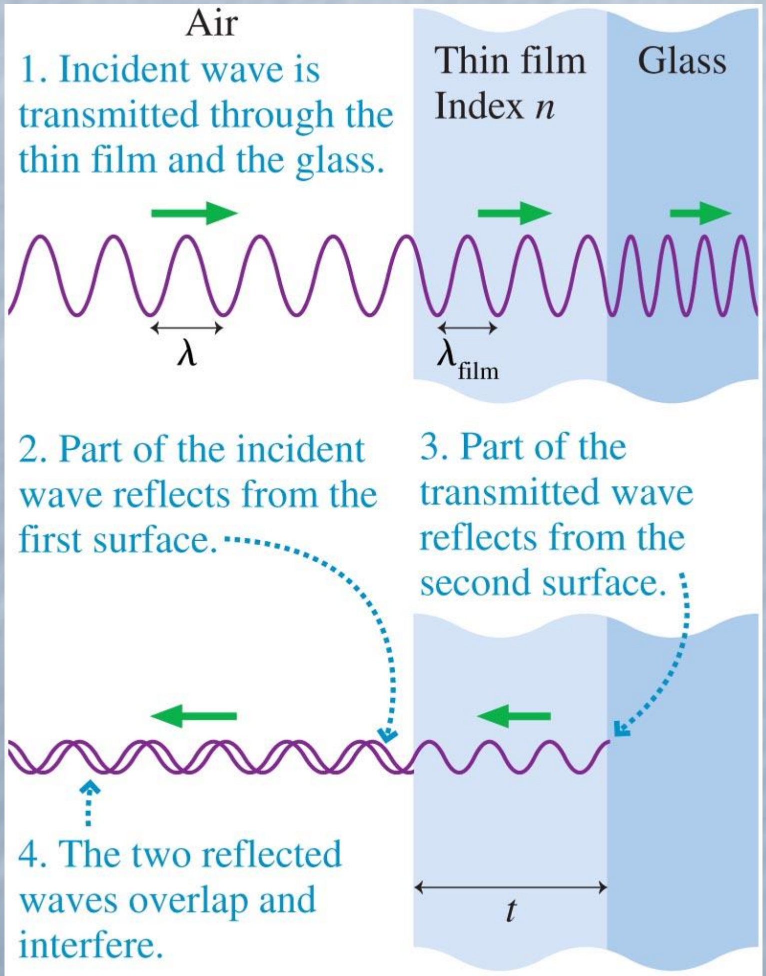
Thin film interference

- To get the reflections from the 2 surfaces to cancel, we need the path difference to be a whole + ½ number of wavelengths
- For an 0 or 2 reflections

$$2t = \left(m + \frac{1}{2}\right)\lambda_{film}$$

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$$

$$m = 0,1,2,3...$$



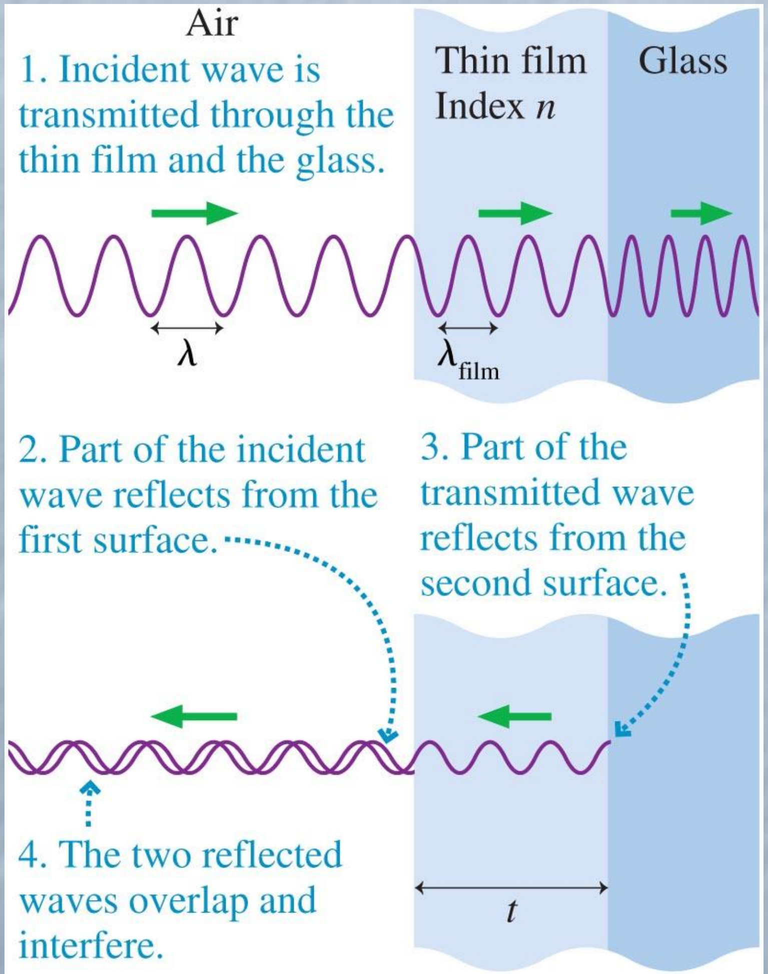
Thin film interference

- To get the reflections from the 2 surfaces to cancel, we need the path difference to be a whole + ½ number of wavelengths
- For 1 reflection

$$2t = m\lambda_{film}$$

$$2t = m\frac{\lambda}{n_{film}}$$

$$m = 0,1,2,3...$$



Application of thin films

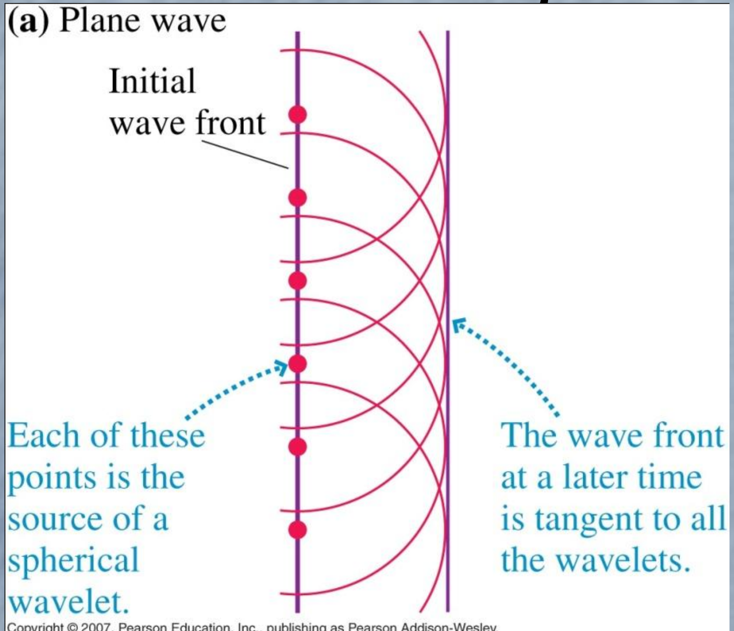
Many optical applications.

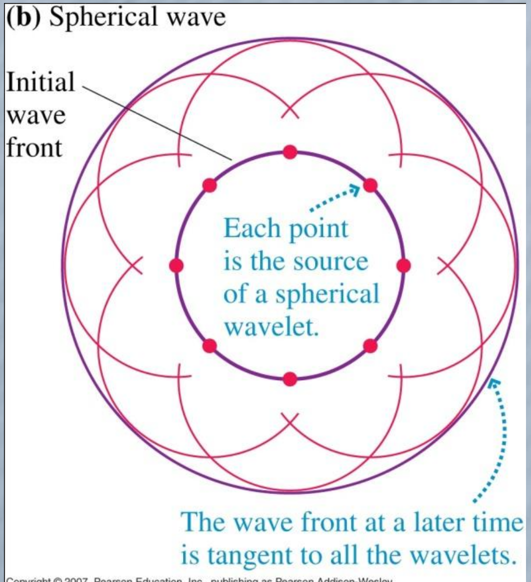
Only works for one wavelength, but reflections for nearby wavelengths are reduced. (We see < 1 octave)

Depends on the n_{film}, but we need to know if its > or < n_{medium}

Huygen’s principle

- Each point on a wave front is the source of a spherical wavelet that spreads out at the wave speed.
- At a later time, the shape of the wave front is the line tangent to all the wavelets.



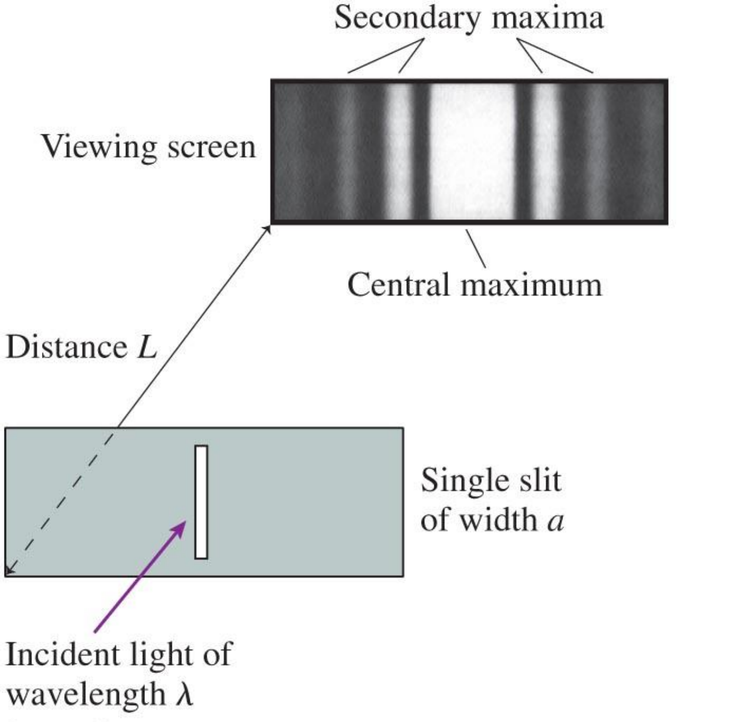


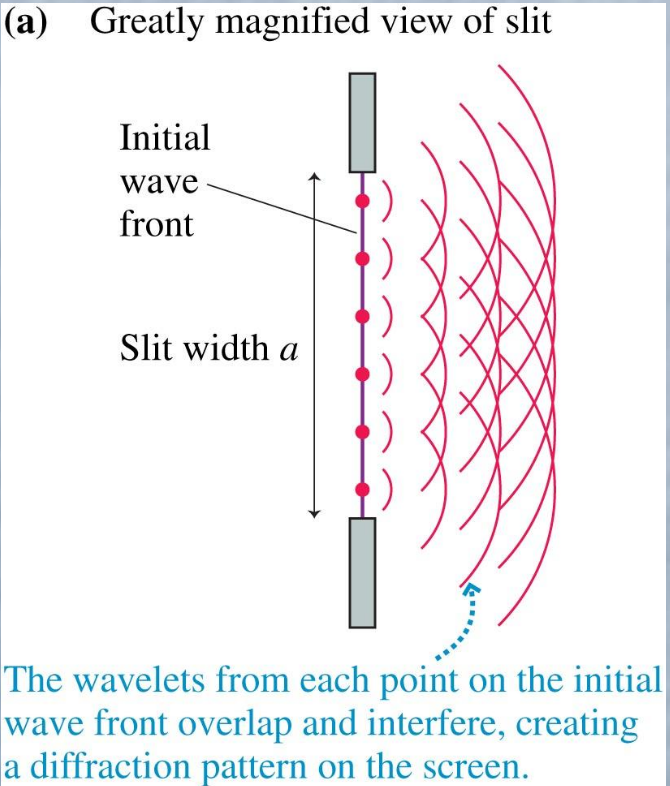
History of Huygens' principle

- Also called Huygens-Fresnel principle
- Developed in the 19th century to help understand waves (not quantitative)
- Developed when Newton's ideas that light is corpuscles, not waves
- The wavelets interfere, and the wave front is the result of the constructive interference.

Single slit interference

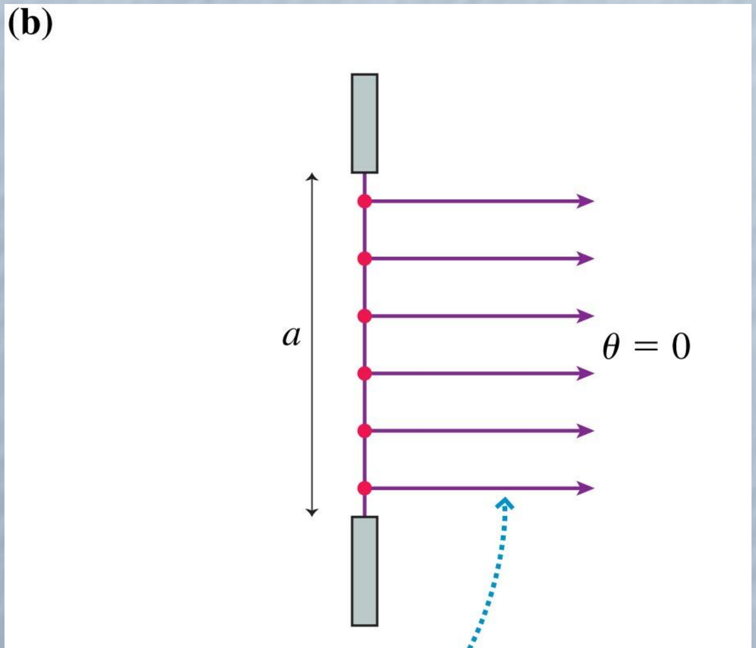
We also get interference from a single slit – where are the light sources ?





Single slit diffraction

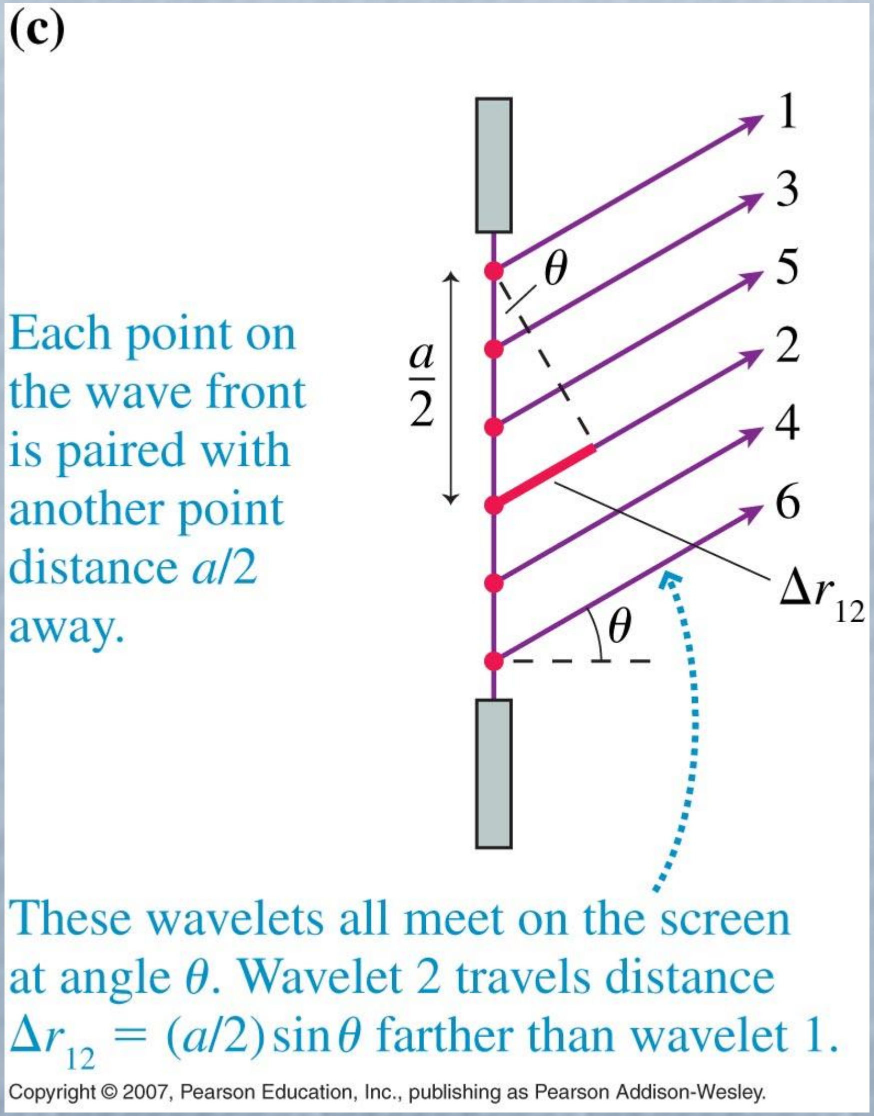
- The individual wavelets can be thought of as separate sources
- At θ=0, the light from each wavelet adds constructively to give a bright central fringe.



The wavelets going straight forward all travel the same distance to the screen. Thus they arrive in phase and interfere constructively to produce the central maximum.

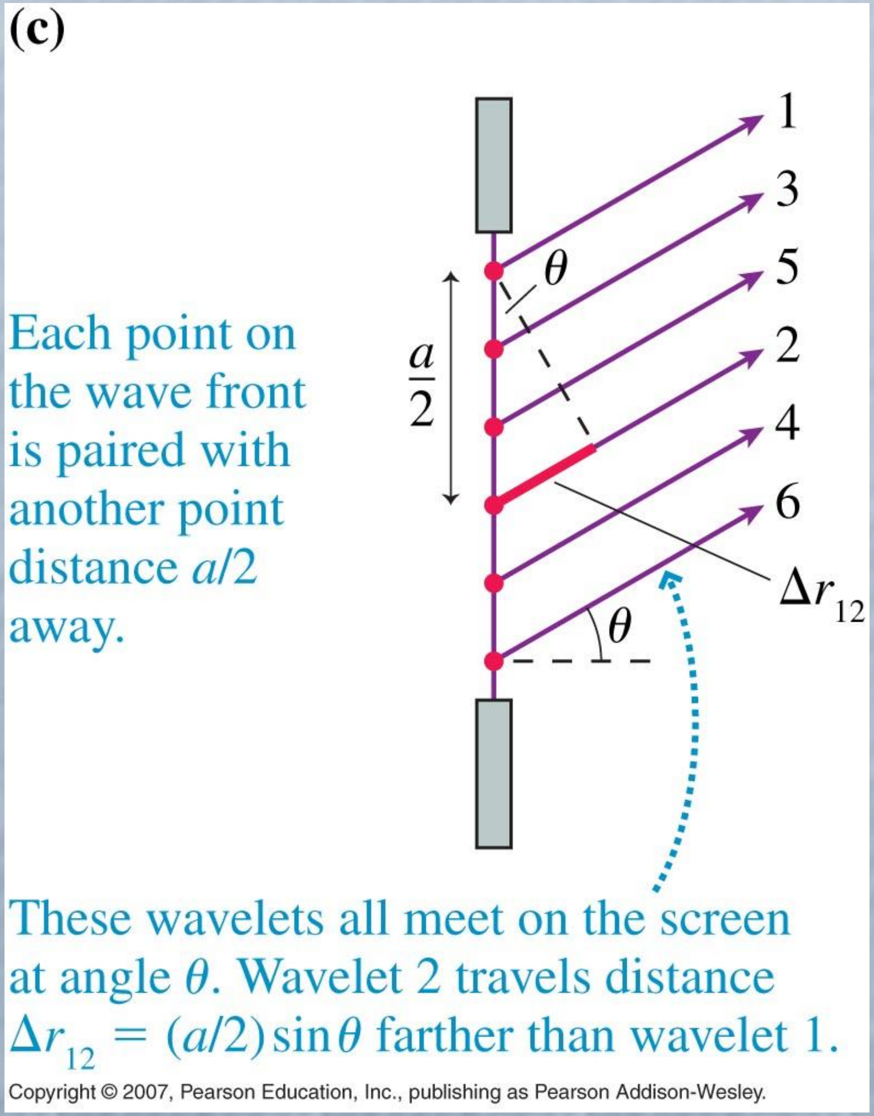
Single slit diffraction

- For situations where θ>0, we will calculate the conditions where we get destructive interference.
- Divide the slit in half, and consider the 2 rays from the edge and midway



Single slit diffraction

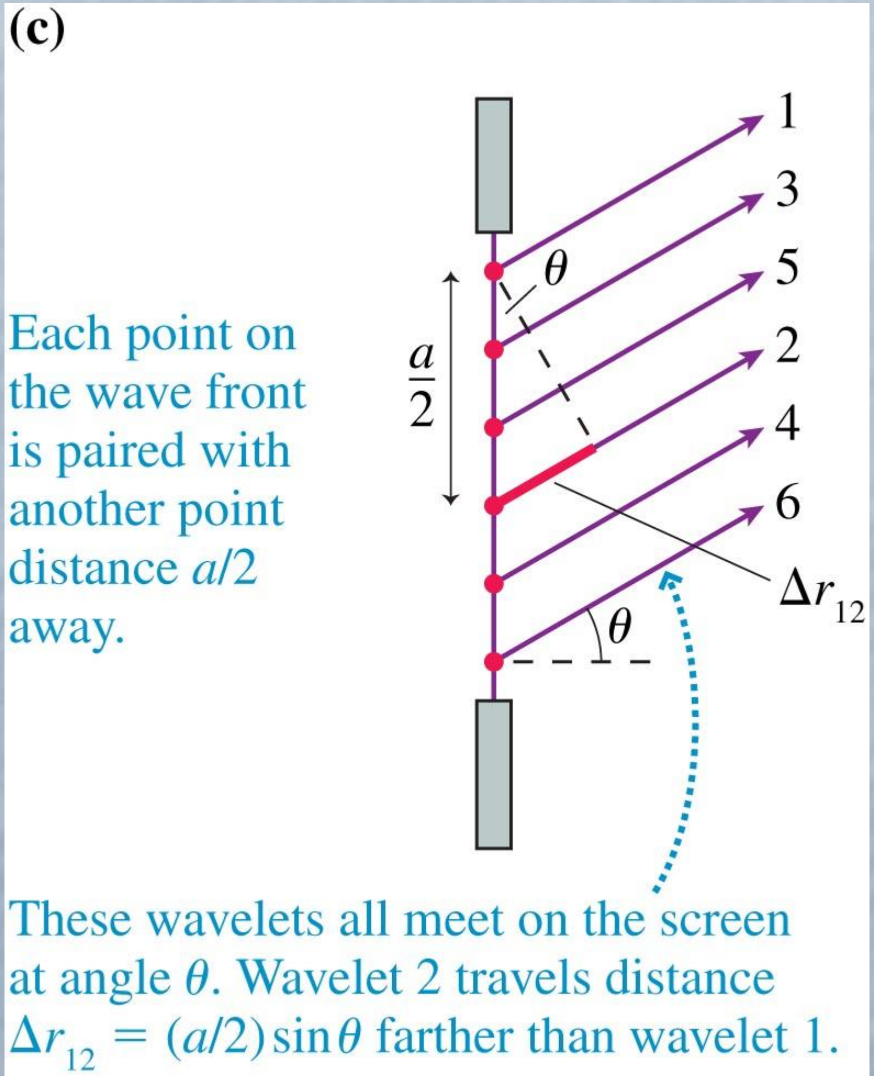
- For destructive interference, (dark fringe) the path length Δr will be one half wavelength between 1 and 2
- Similarly, all other pairs will be λ/2 – for 3 & 4, and 5 & 6



Single slit diffraction

- If we consider more wavelets along the front, then when Δr=λ/2, every ray from the top half of the slit will cancel with a corresponding wave from the bottom half of the slit.

$$\Delta r = \frac{a}{2}\sin\theta_{dark} = \frac{\lambda}{2}$$

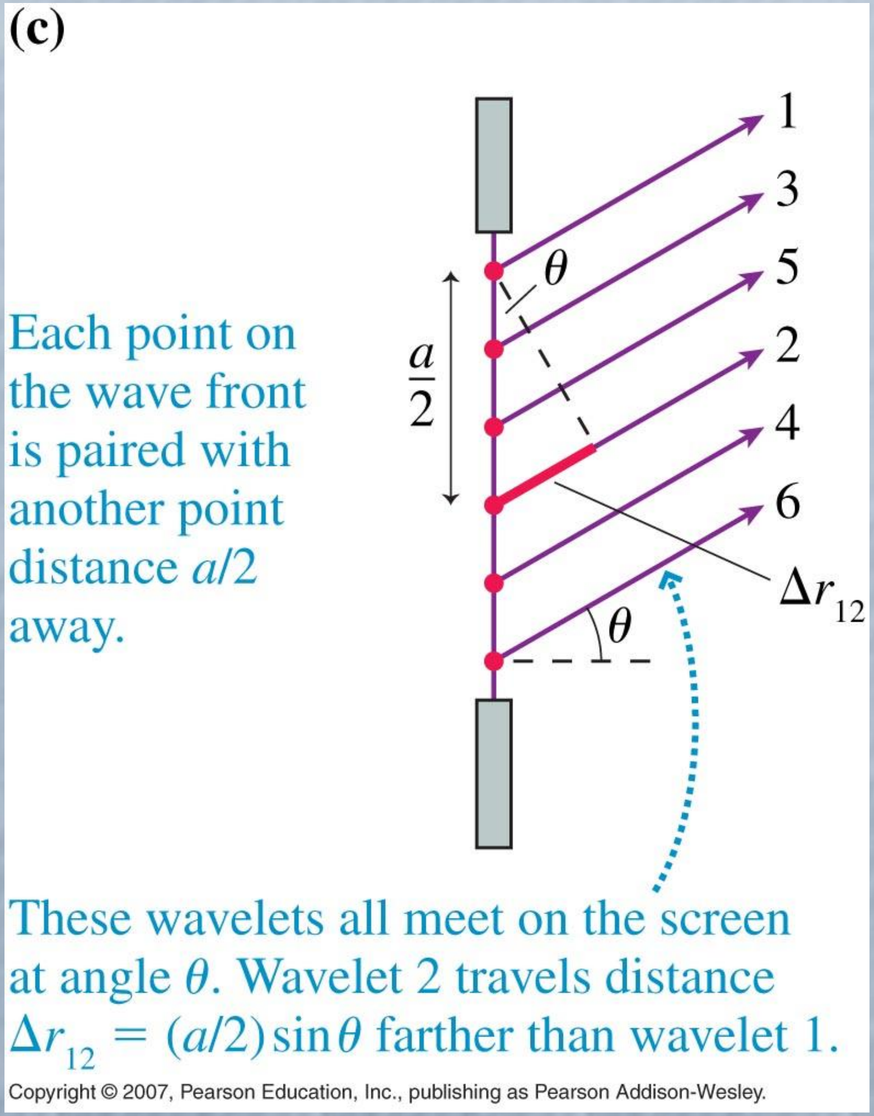


Single slit diffraction

- For the next fringes, divide the slit into 4, 6 segments

$$\Delta r_2 = \frac{a}{4}\sin\theta_{dark2} = \frac{\lambda}{2}$$

$$\Delta r_3 = \frac{a}{6}\sin\theta_{dark3} = \frac{\lambda}{2}$$



Single slit diffraction

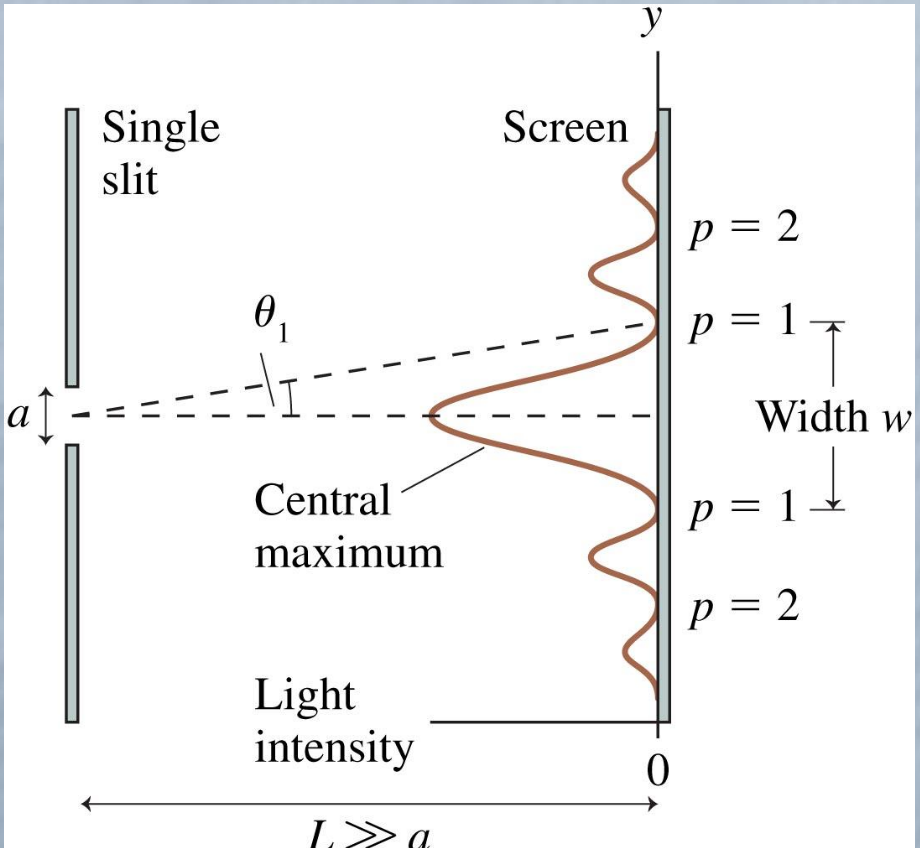
In general, the dark fringes occur when

$$a \sin \theta_{dark} = p\lambda$$

$$p = 1,2,3,...$$

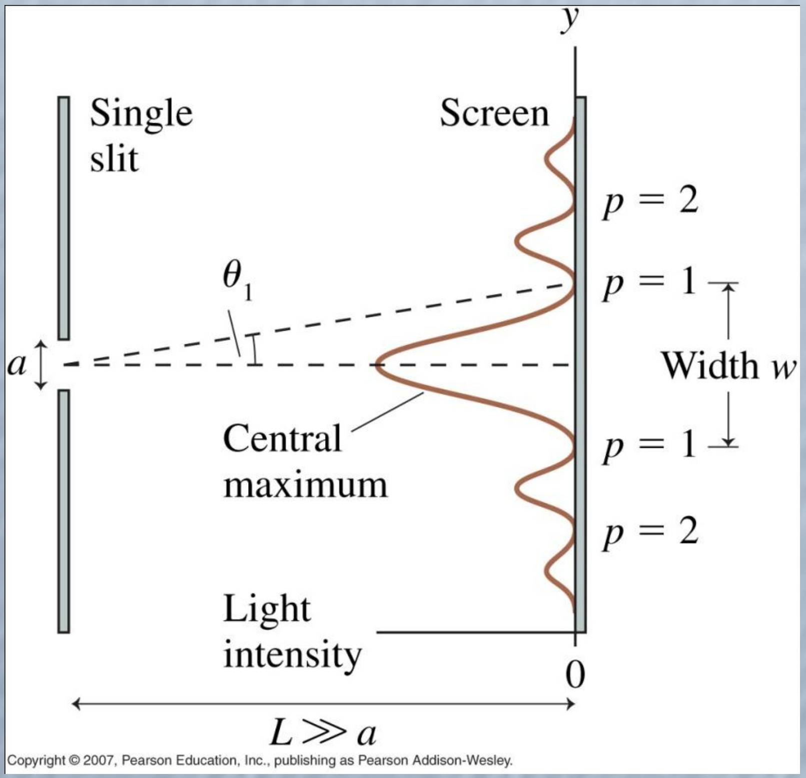
$$\theta << 1$$

$$\theta_{dark} = p \frac{\lambda}{a}$$



Single slit diffraction

- We’ve calculated where the **dark** fringes occur – the light fringes are almost, but not quite half way between the dark fringes
- The central fringe is twice the width off subsequent slits
- When a<λ, we do not get diffraction



Width of the central peak

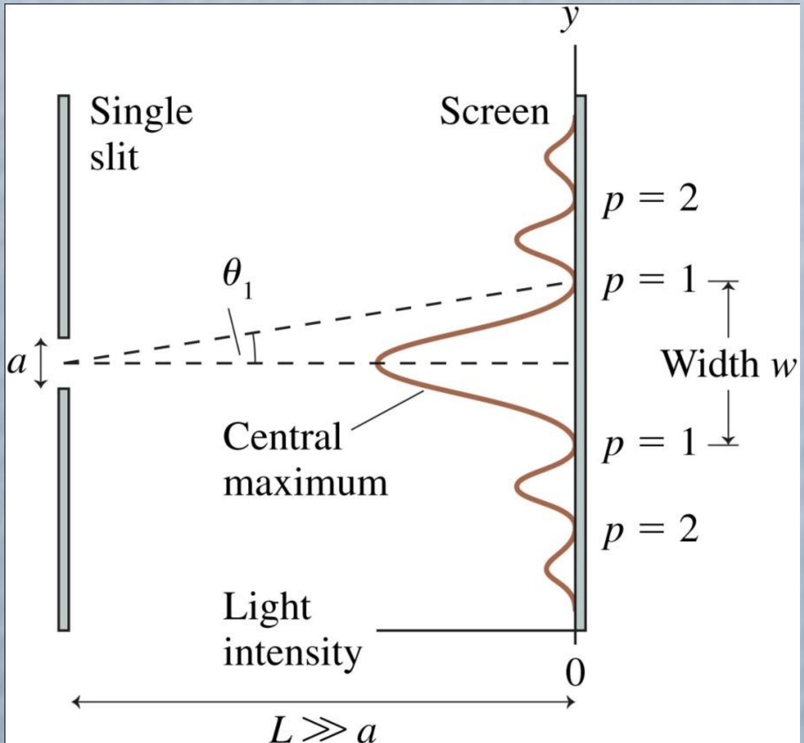
Using y=Ltanθ, we find the positions on the screen of the dark fringes, and the width of the central fringe:

$$y_{dark} = \frac{p\lambda L}{a}$$

$$p = 1,2,3,...$$

$$w = y_1 - y_{-1}$$

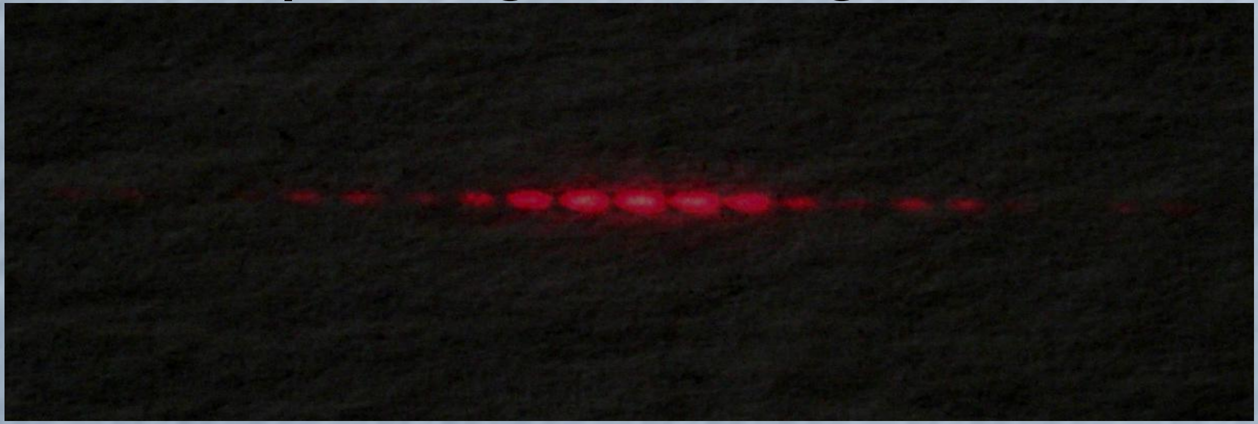
$$w = \frac{2\lambda L}{a}$$

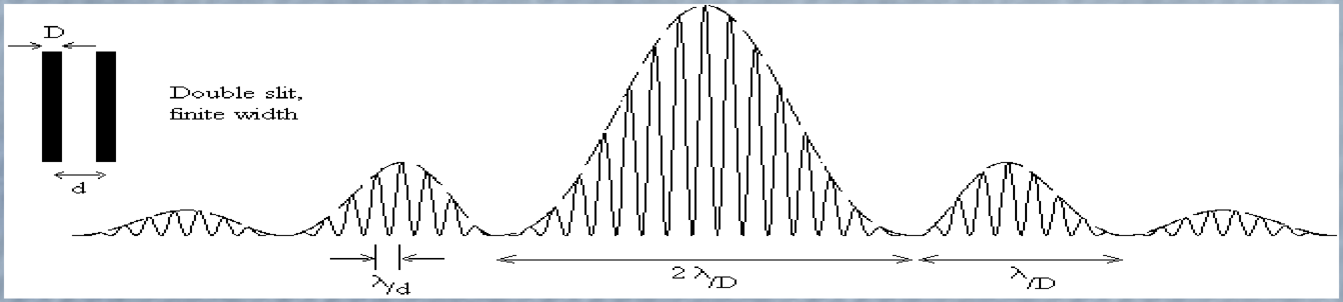


Structure in the double slit diffraction

In the double slit experiment, we were really seeing two diffraction patterns merged together.

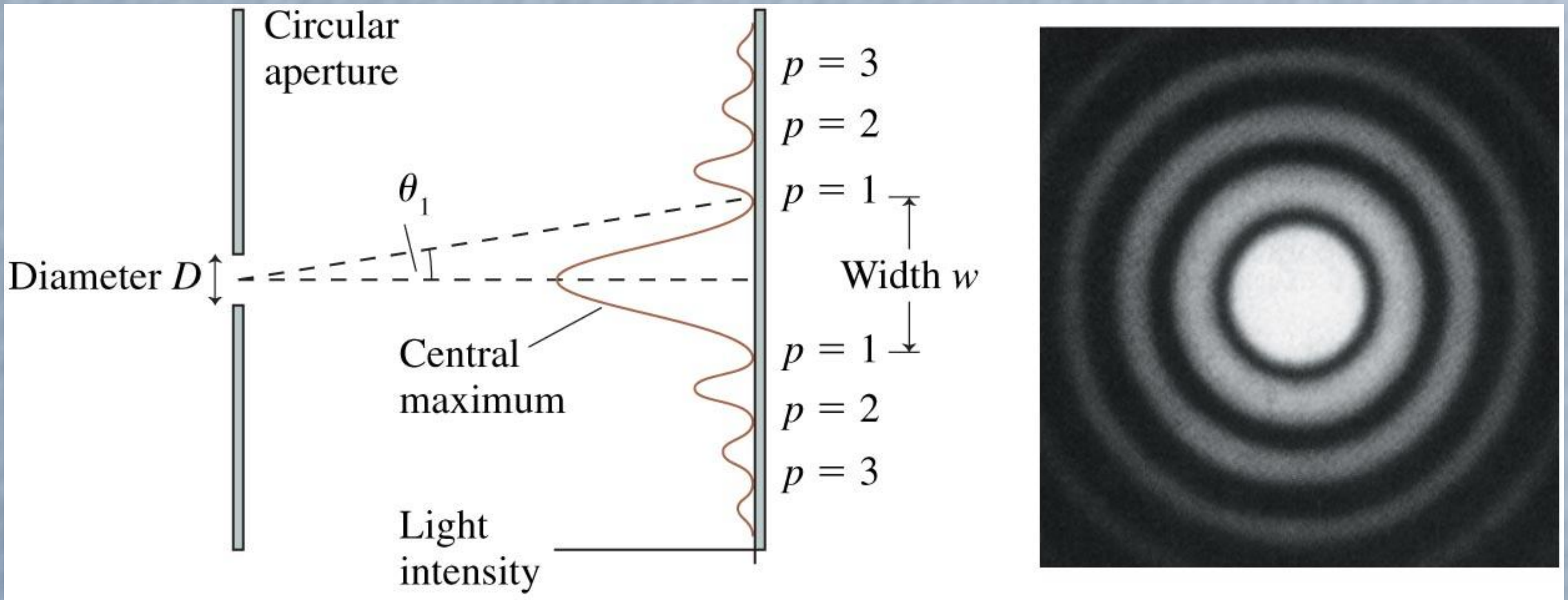
The smaller the opening, the larger the effect





Circular-Aperture Diffraction

For a pin-hole, we get diffraction in two dimensions, sometimes called the Airy disk:



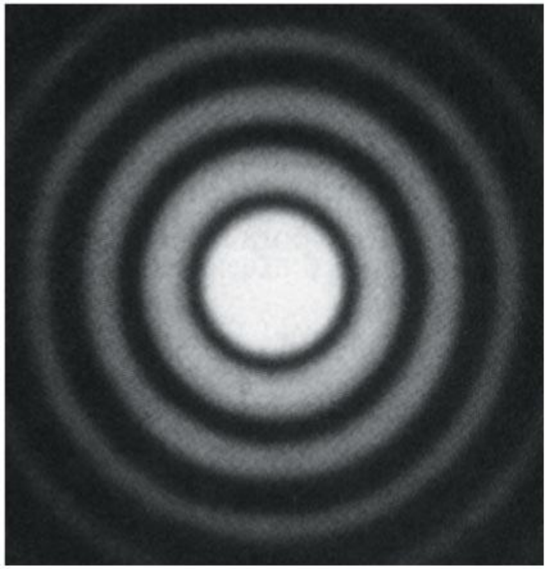
Circular-Aperture Diffraction

The dark fringes are now shifted from the single slit

$$\theta_{1slit} = \frac{\lambda}{a_{slit}}$$

For a pin-hole, diameter D, we get a new factor of 1.22 from the geometry

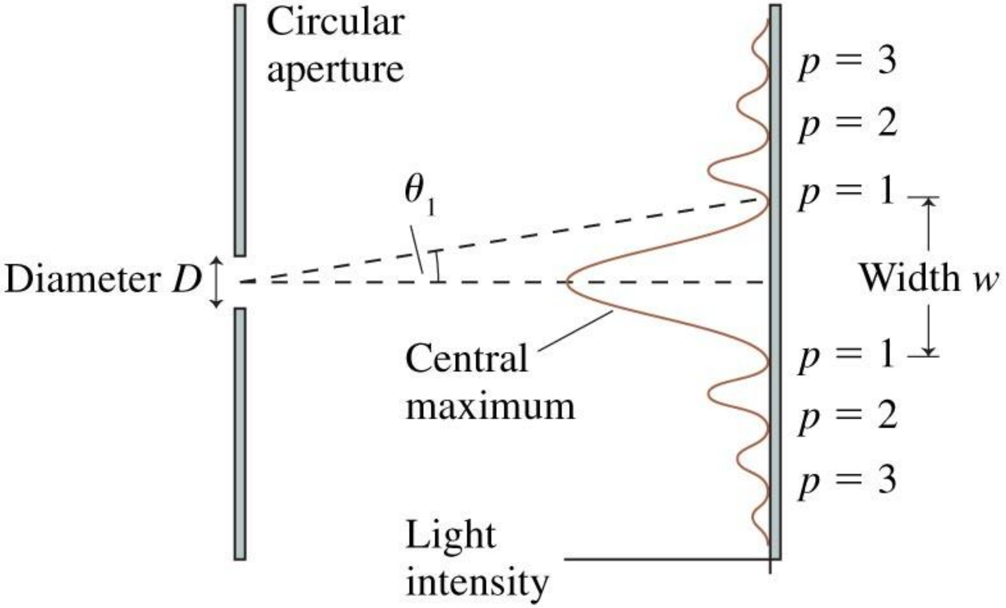
$$\theta_{circle} = \frac{1.22\lambda}{D}$$

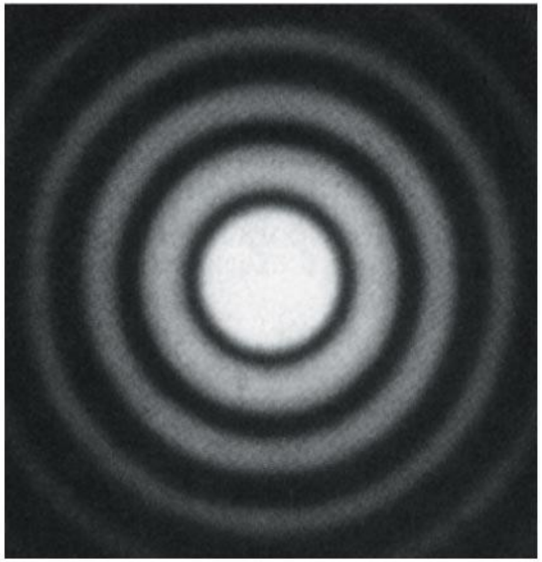


Circular-Aperture Diffraction

The width of the central maximum is then

$$w = 2y_1 = 2L\tan\theta_1 \approx \frac{2.44\lambda L}{D}$$





Summary

- The wave model
- Diffraction and interference
- Double slit and grating interference
- Index of refraction
- Thin-film interference
- Huygens' principle
- Single-slit and circular diffraction

Homework problems

Chapter 17 Problems

42,55,57,60,70,73