

Lecture 4

Fluid statics and dynamics

- Using pressure: Hydraulic Lift
- Archimedes principle (float or sink?)
- Continuity equation
- Bernoulli's equation
- Applications

Hydraulic Lift

- Use pressurized liquids for work (based on Pascal's principle): increase pressure at one point by pushing piston...at another point, piston can push upward

- Force multiplication:

$$p_1 = \frac{F_1}{A_1} + p_0$$

equal to $p_2 = \frac{F_2}{A_2} + p_0 + \rho gh$ $\frac{A_2}{A_1} > 1$

$$\Rightarrow F_2 = F_1 \frac{A_2}{A_1} - \rho gh A_2$$

- Relating distances moved by pistons:

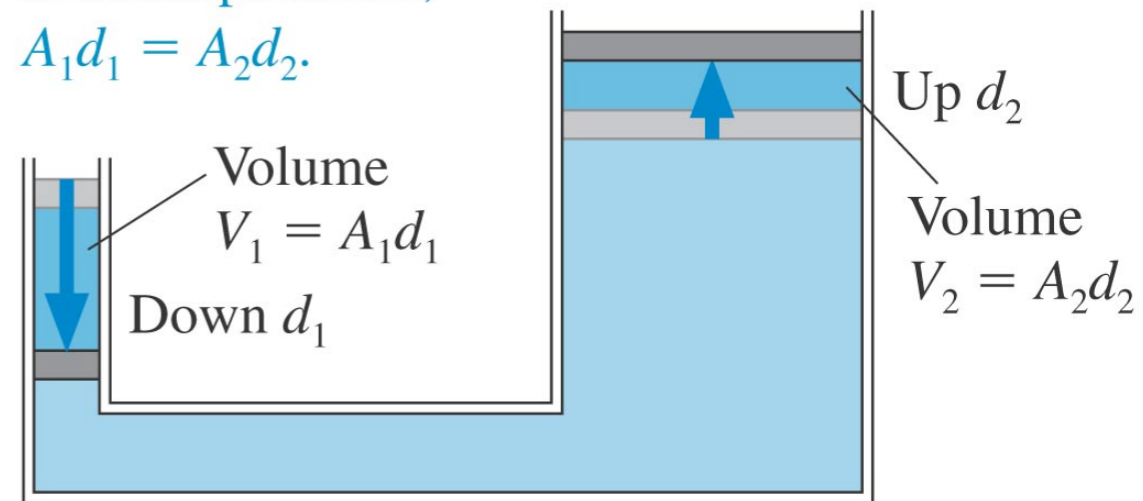
$$V_1 = A_1 d_1 \text{ equal to } V_2 = A_2 d_2$$

$$\Rightarrow d_2 = \frac{d_1}{A_2/A_1}$$

- **Additional** force to move heavy object thru' d_2 : $\Delta F = \rho g (A_1 + A_2) d_2$

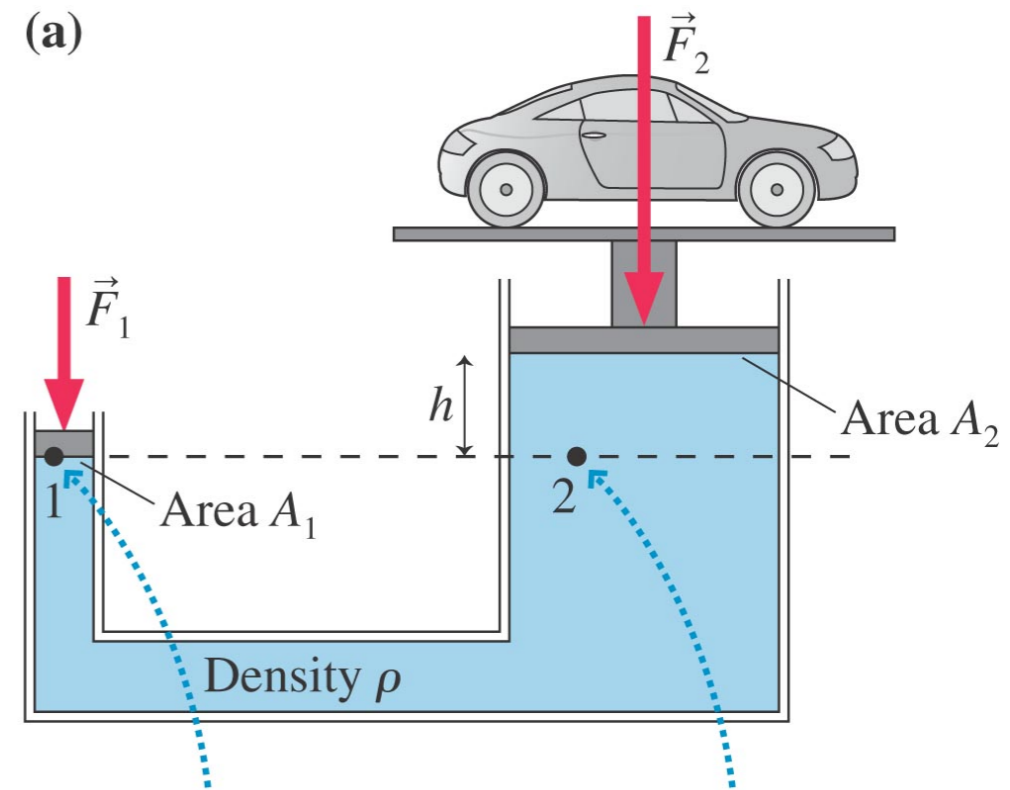
(b)

Because the fluid is incompressible,
 $A_1 d_1 = A_2 d_2$.



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(a)



Pressure p_1 is due to atmospheric pressure p_0 plus pressure F_1/A_1 , due to \vec{F}_1 .

Pressure p_2 is p_0 plus F_2/A_2 plus ρgh from the liquid column of height h .

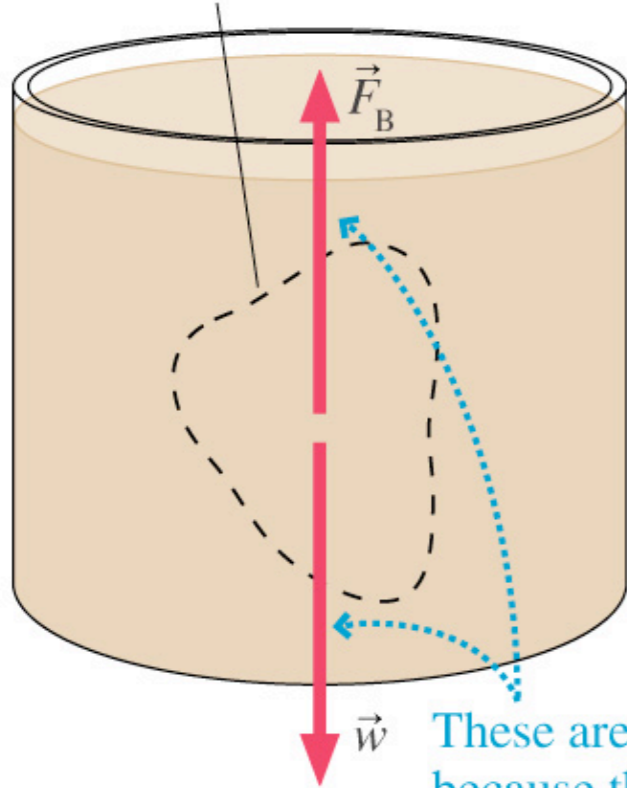
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Buoyancy: Archimedes' principle

- Buoyant force: upward force of a fluid
- Buoyant force, $F_B = \rho_f V_f g$
weight of displaced fluid,

(a)

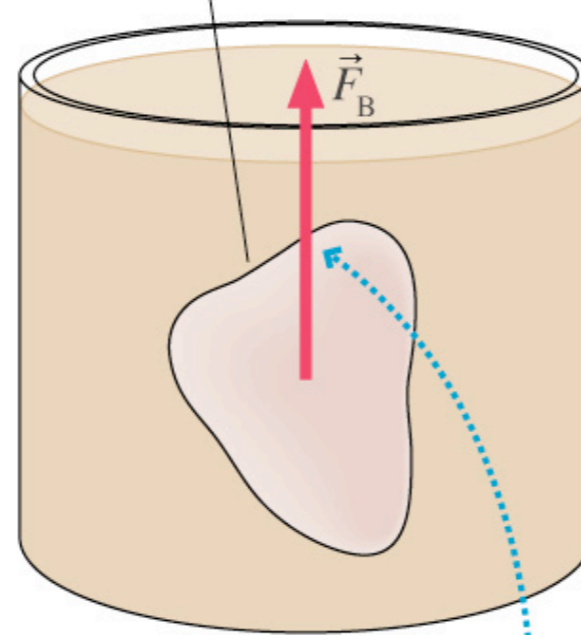
Imaginary boundary around a parcel of fluid



These are equal because the parcel is in static equilibrium.

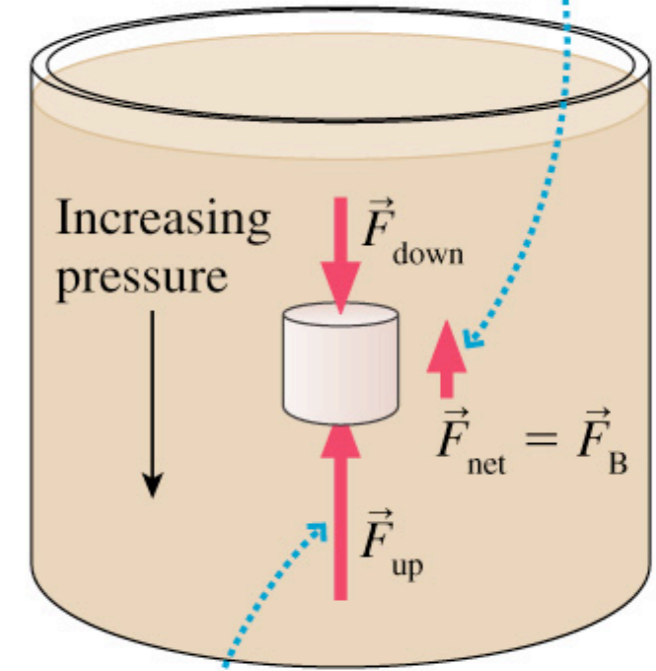
(b)

Real object with same size and shape as the parcel of fluid



The buoyant force on the object is the same as on the parcel of fluid because the surrounding fluid has not changed.

The net force of the fluid on the cylinder is the buoyant force \vec{F}_B .



$F_{up} > F_{down}$ because the pressure is greater at the bottom. Hence the fluid exerts a net upward force.

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To float or sink?

- Net force: $F_B - w$
 $\rho_f V_f g$ $\rho_{avg.} V_0 g$

- Float or sink or static equilibrium for
 $\rho_{avg.} < \rho_f$ OR $\rho_{avg.} > \rho_f$ OR $\rho_{avg.} = \rho_f$
 master formula

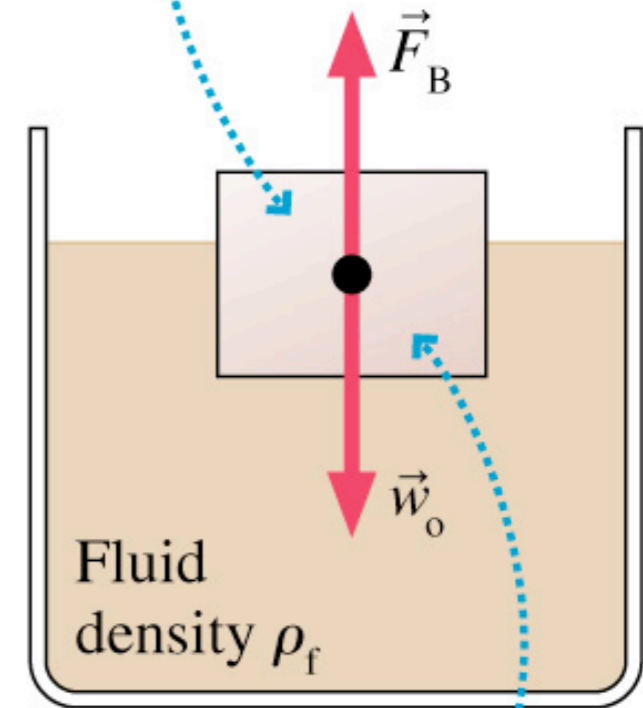
- ...rather for 1st case pushed up till only partly submerged:

$$F_B = \rho_f V_f g = w = \rho_0 V_0 g$$

$$\Rightarrow V_f < V_0$$

- 90% of ice underwater...

An object of density ρ_0 and volume V_0 is floating on a fluid of density ρ_f .

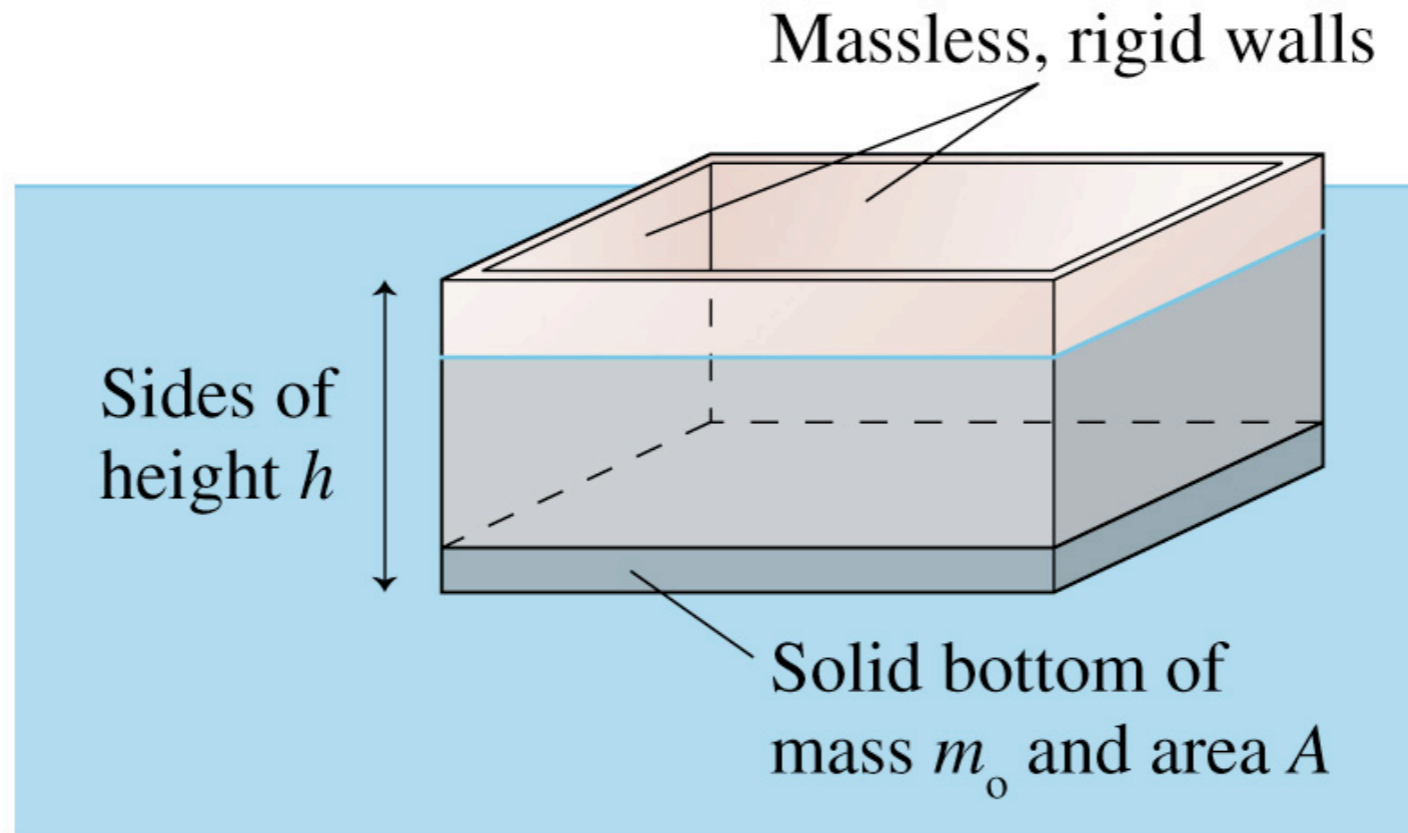


The submerged volume of the object is equal to the volume V_f of displaced fluid.

Example

- A 6.0 cm.-tall cylinder floats in water with its axis perpendicular to the surface. The length of the cylinder above water is 2.0 cm. What is the cylinder's mass density?

Boats



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- steel plate sinks, but geometry (sides) allows it to displace more fluid than actual steel volume:

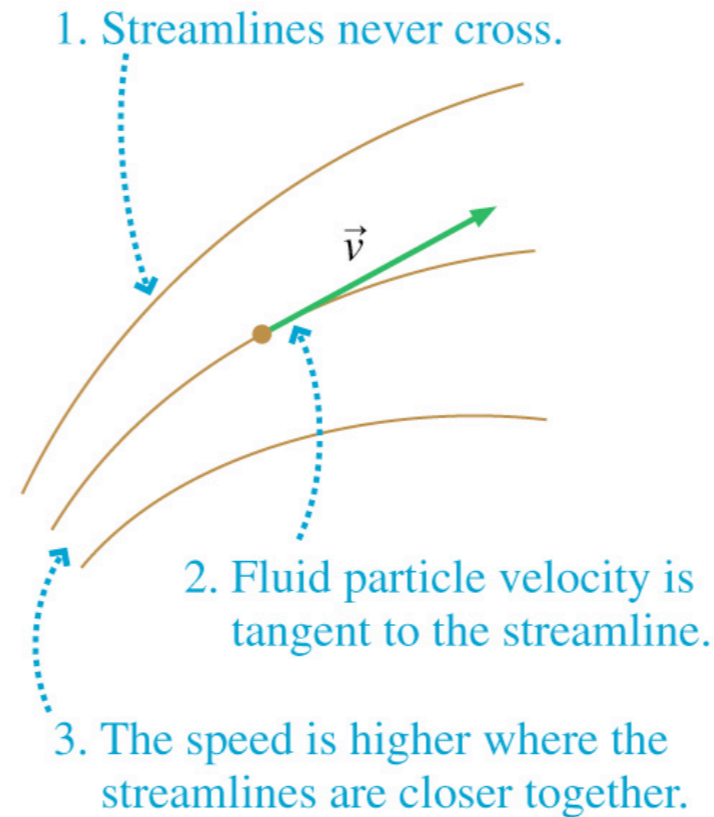
$$\rho_{avg.} = \frac{m_0}{Ah} < \rho_f$$

Ideal fluid

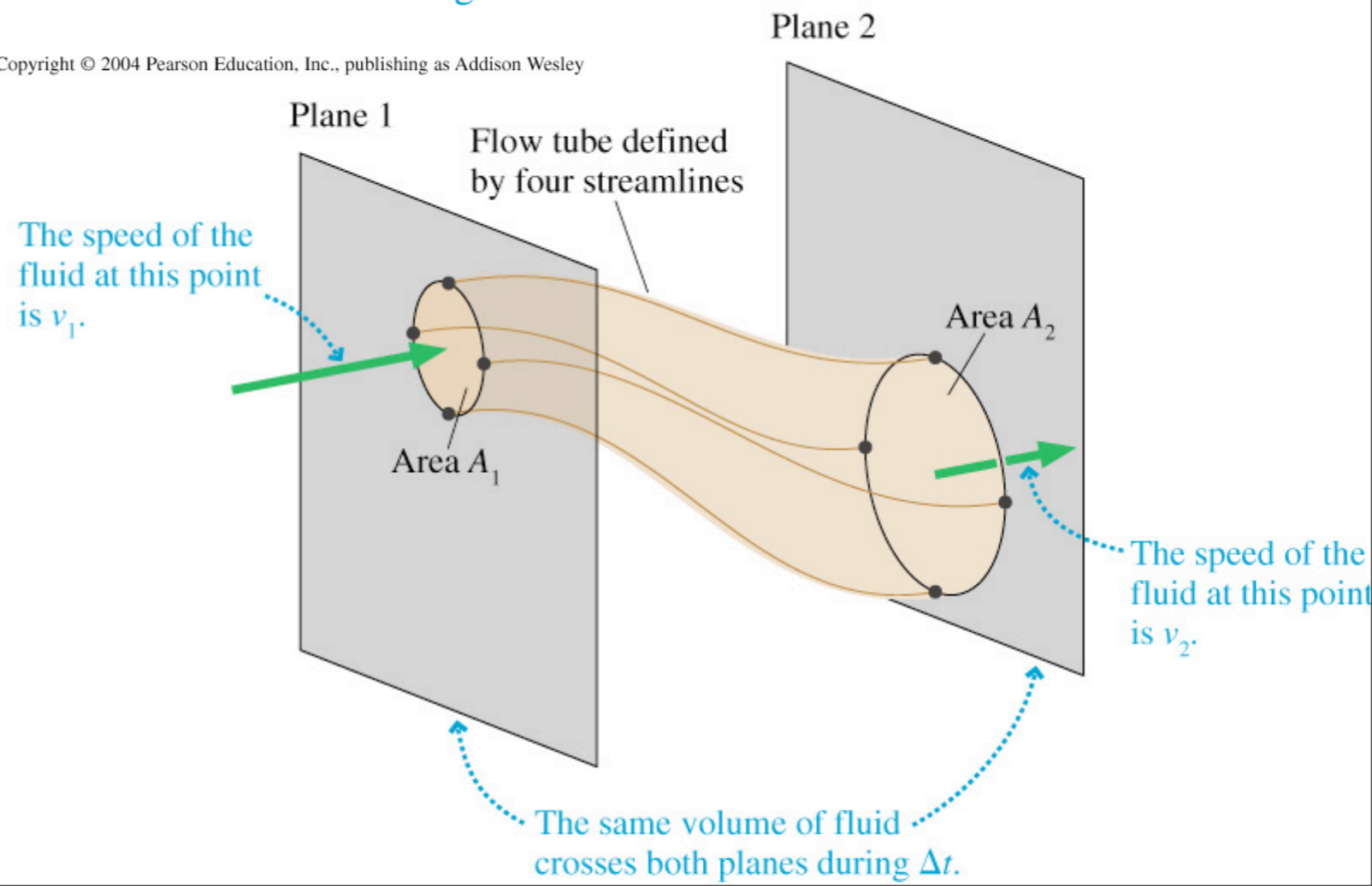
- incompressible (not so good approximation for gases)
- laminar (steady) flow (not turbulent): velocity at given point is constant with time
- non-viscous (no resistance to flow and no friction for solid object)
- irrotational (test paddle wheel won't rotate)

Equation of continuity (I)

- Streamlines (path of “particle of fluid”: e.g. colored drop of water in stream)
- Flow tube: bundle of streamlines (“invisible pipe”)



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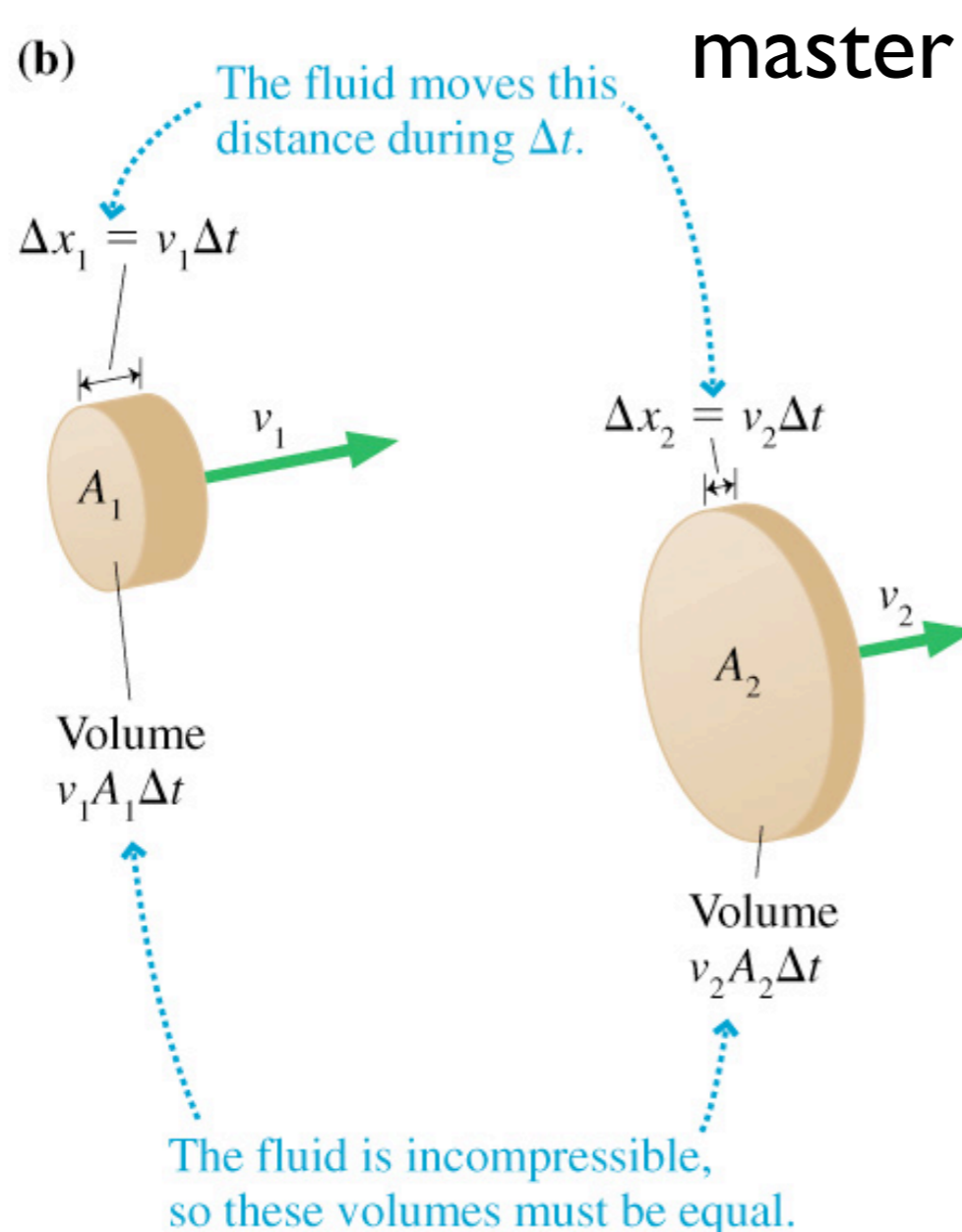


Equation of continuity (II)

- Fluid not created/destroyed/stored within flow tube

$$V_1 = A_1 \Delta x_1 = A_1 v_1 \Delta t$$

(volume flowing across A_1) = $V_2 \dots$



master formula

$$v_1 A_1 = v_2 A_2$$

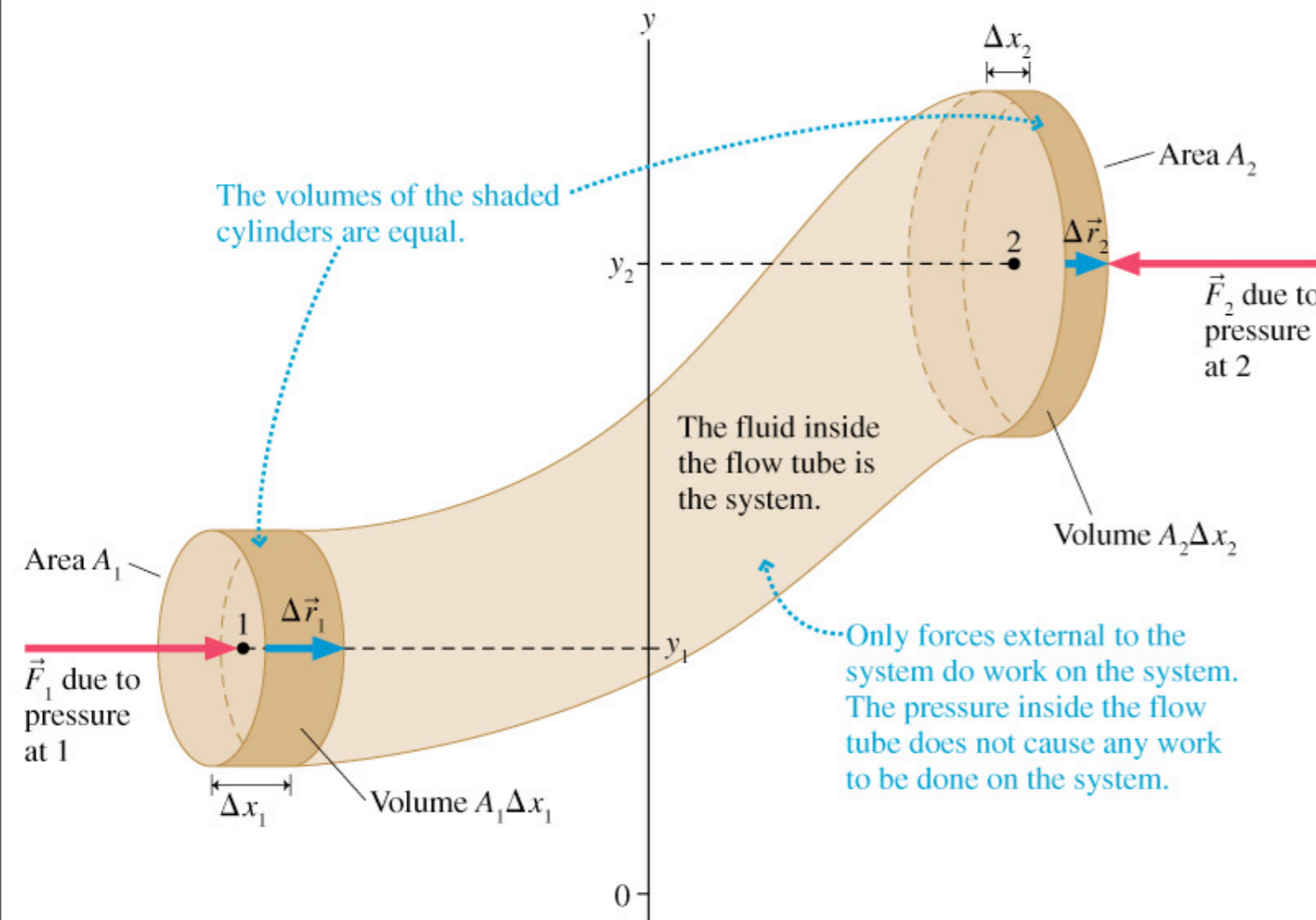
Flow faster in narrower part :
e.g., water from tap

$Q = V A$ (volume flow rate)
constant

Bernoulli's equation

- work and energy conservation applied to volume of fluid in flow tube: $\Delta K + \Delta U = W_{ext.}$

by pressure of surrounding fluid



W_1	$= F_1 \Delta x_1$	$= p_1 V$
W_2	$= -F_2 \Delta x_2$	$= -p_2 V$
ΔU	$= mgy_2 - mgy_1$	$= \rho V g (y_2 - y_1)$
ΔK	$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$	$= \frac{1}{2} \rho V (v_2^2 - v_1^2)$

master formulae

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

$$p + \frac{1}{2} \rho v^2 + \rho g y = \text{constant}$$

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Example

- Water flows at 5.0 L/s through a horizontal pipe that narrows smoothly from a 10 cm diameter to 5.0 cm diameter. A pressure gauge in the narrow section reads 50 kPa. What is the reading of a pressure gauge in the wide section?

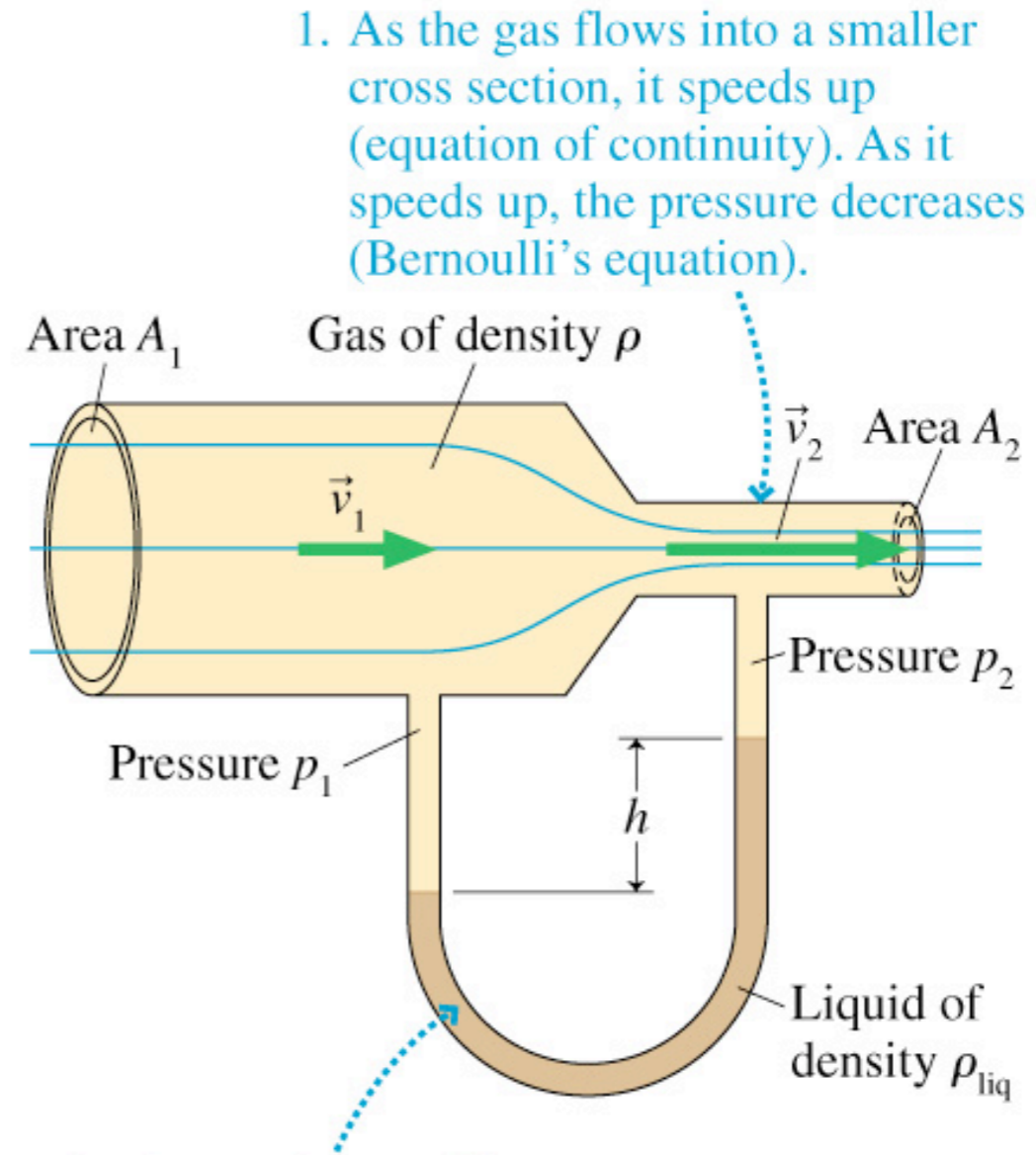
Applications I: Venturi tube

(Measuring speed of flowing) gas

- Combine master formulae: (i) continuity equation (ii) Bernoulli's equation (equal y's) and (iii) pressure vs. depth

$$v_1 = A_2 \sqrt{\frac{2\rho_{liq}.gh}{\rho (A_1^2 - A_2^2)}}$$

$$v_2 = A_1 \sqrt{\frac{2\rho_{liq}.gh}{\rho (A_1^2 - A_2^2)}}$$



Airplane lift

- Continuity and Bernoulli's equations

1. Flow tube decreases in size due to compression of streamlines. The higher speed lowers the pressure to $p < p_{\text{atmos}}$.

2. The pressure difference above and below the wing causes lift.

