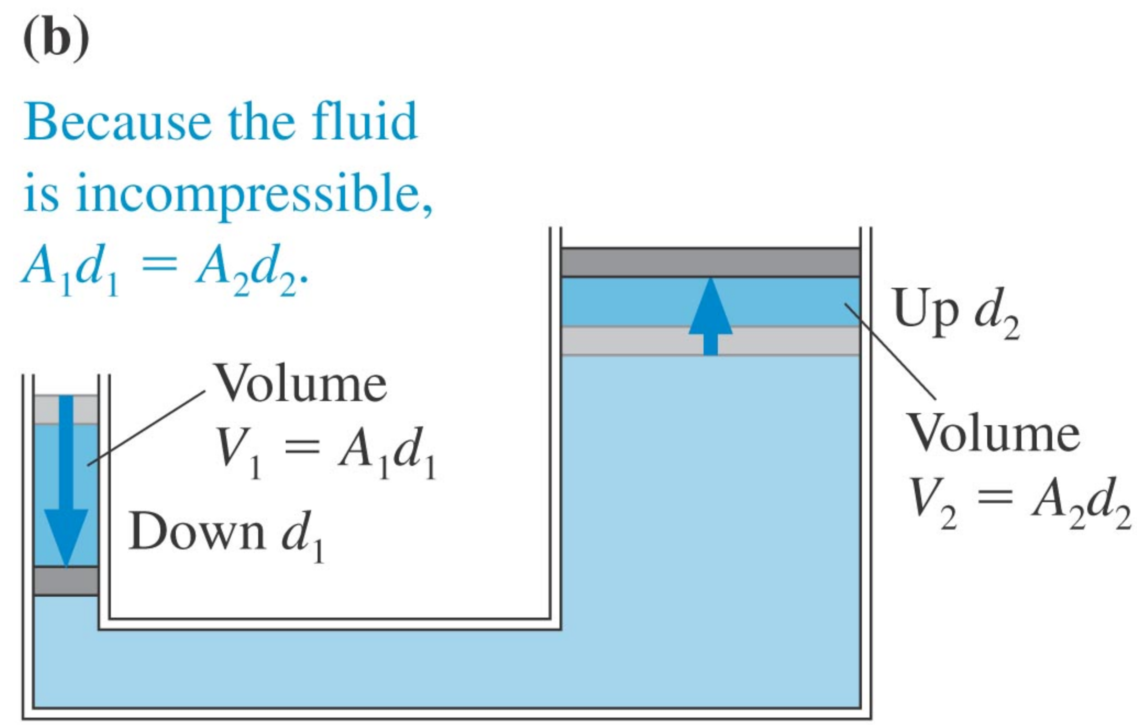
Lecture 4
Fluid statics and dynamics

- Using pressure: Hydraulic Lift
- Archimedes principle (float or sink?)
- Continuity equation
- Bernoulli's equation
- Applications

Hydraulic Lift

- Use pressurized liquids for work (based on Pascal's principle): increase pressure at one point by pushing piston...at another point, piston can push upward



- Force multiplication:

$$p_1 = \frac{F_1}{A_1} + p_0$$

$$\text{equal to } p_2 = \frac{F_2}{A_2} + p_0 + \rho g h \qquad \frac{A_2}{A_1} > 1$$

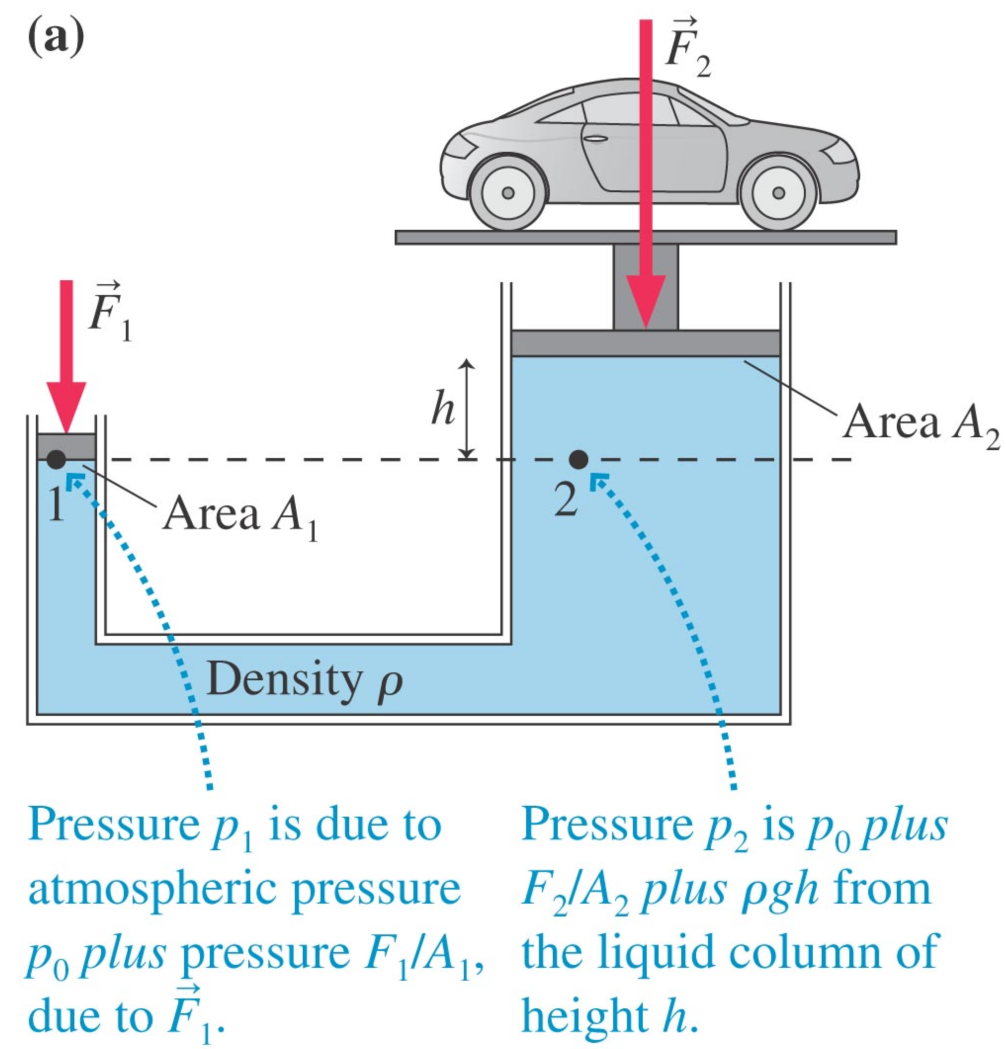
$$\Rightarrow F_2 = F_1 \frac{A_2}{A_1} - \rho g h A_2$$

- Relating distances moved by pistons:

$$V_1 = A_1 d_1 \text{ equal to } V_2 = A_2 d_2$$

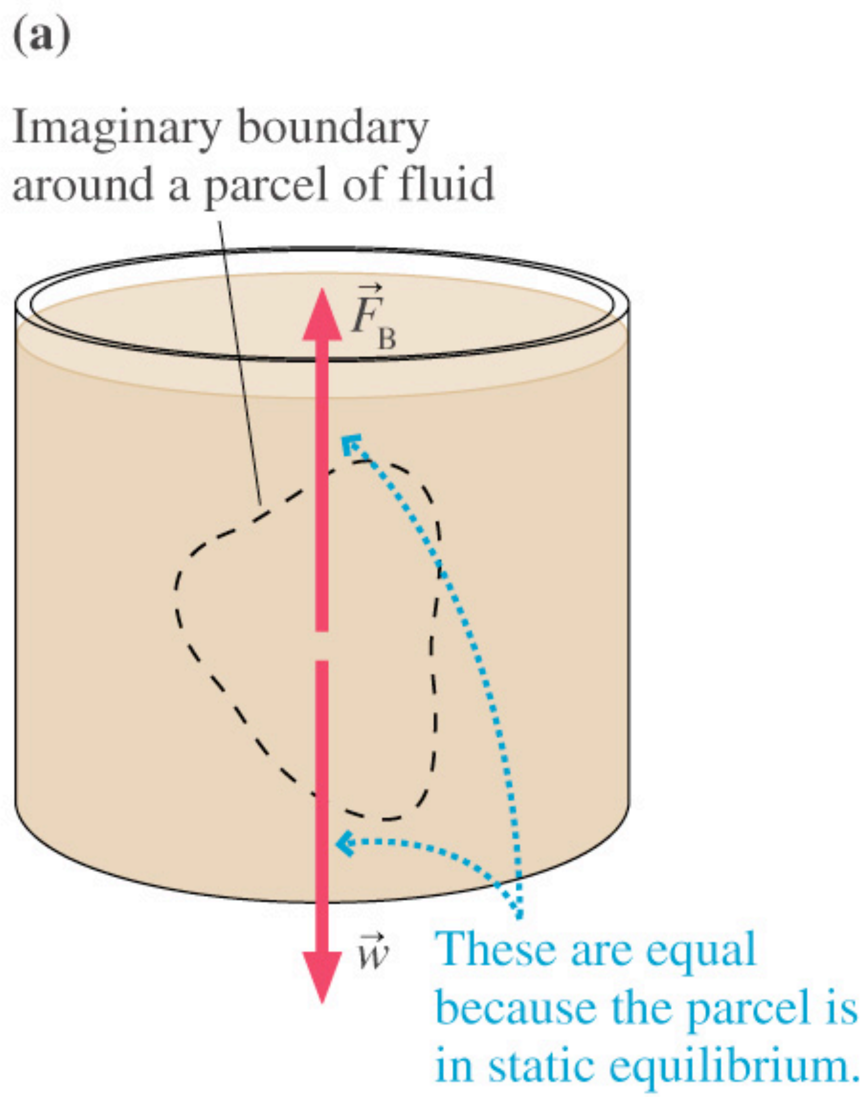
$$\Rightarrow d_2 = \frac{d_1}{A_2/A_1}$$

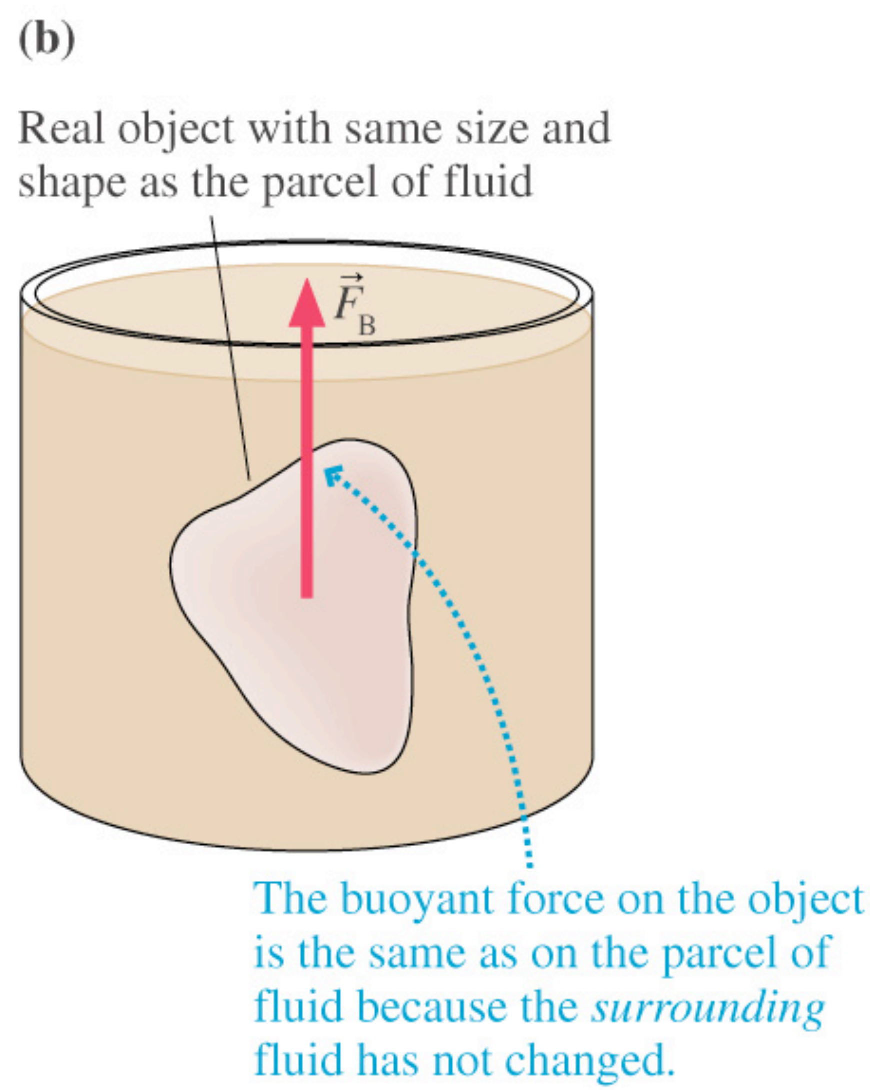
- Additional force to move heavy object thru' d_2: $\Delta F = \rho g \left(A_1 + A_2\right) d_2$

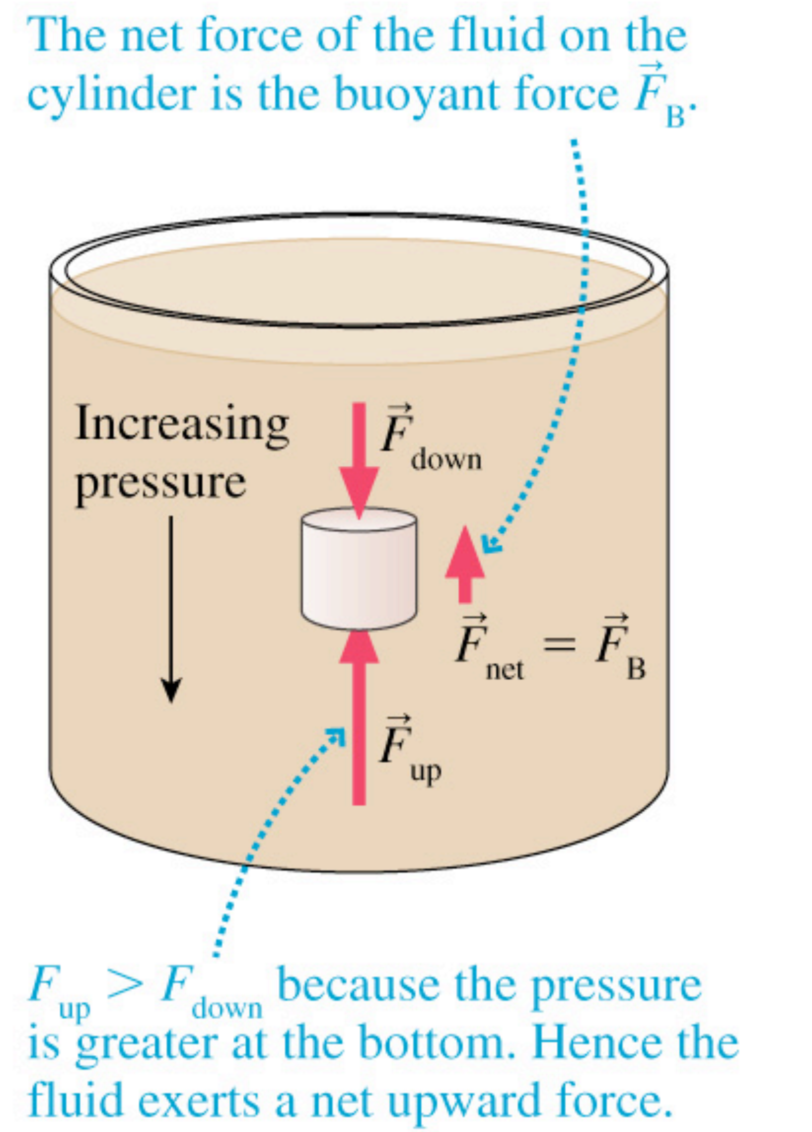


Buoyancy: Archimedes' principle

- Buoyant force: upward force of a fluid
- Buoyant force, $F_B = \rho_f V_f g$ weight of displaced fluid,







To float or sink?

- Net force: $F_B - w$

$$F_B = \rho_f V_f g \qquad w = \rho_{avg.} V_0 g$$

- Float or sink or static equilibrium for

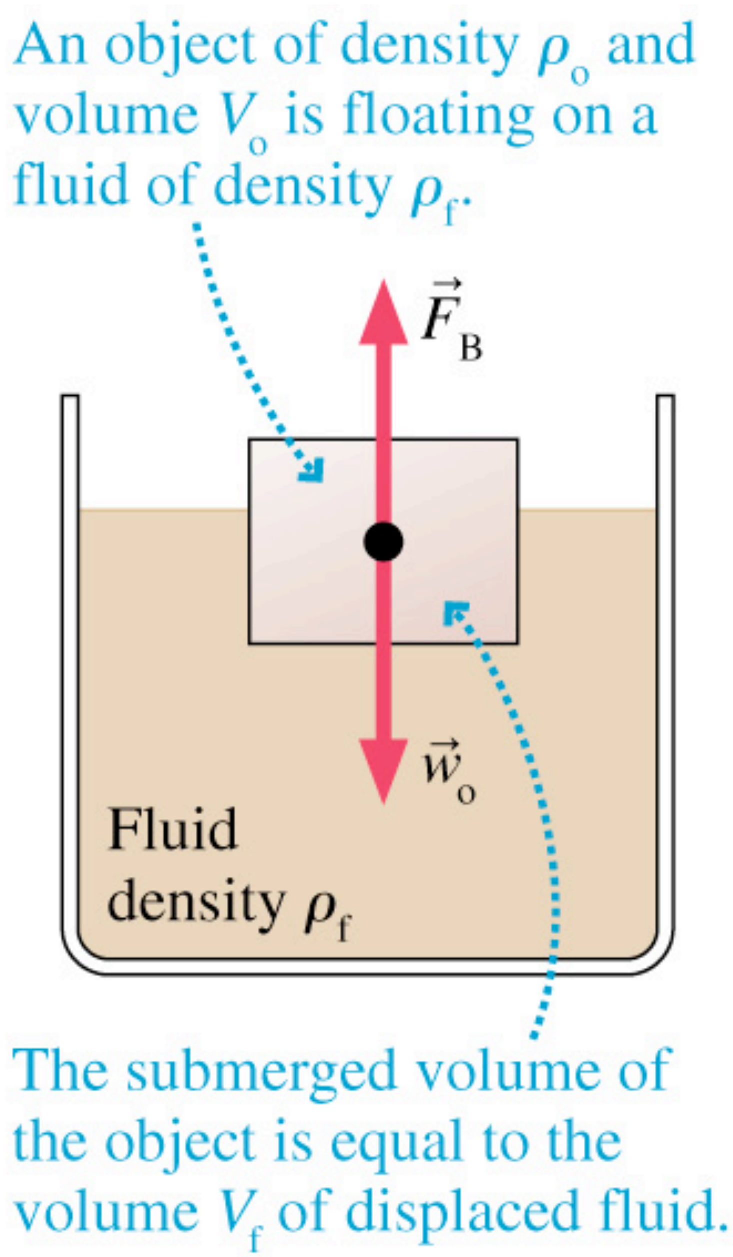
$$\rho_{avg.} < \rho_f \text{ or } \rho_{avg.} > \rho_f \text{ or } \rho_{avg.} = \rho_f$$

master formula

- ...rather for 1st case pushed up till only partly submerged:

$$F_B = \rho_f V_f g = w = \rho_0 V_0 g$$
$$\Rightarrow V_f < V_0$$

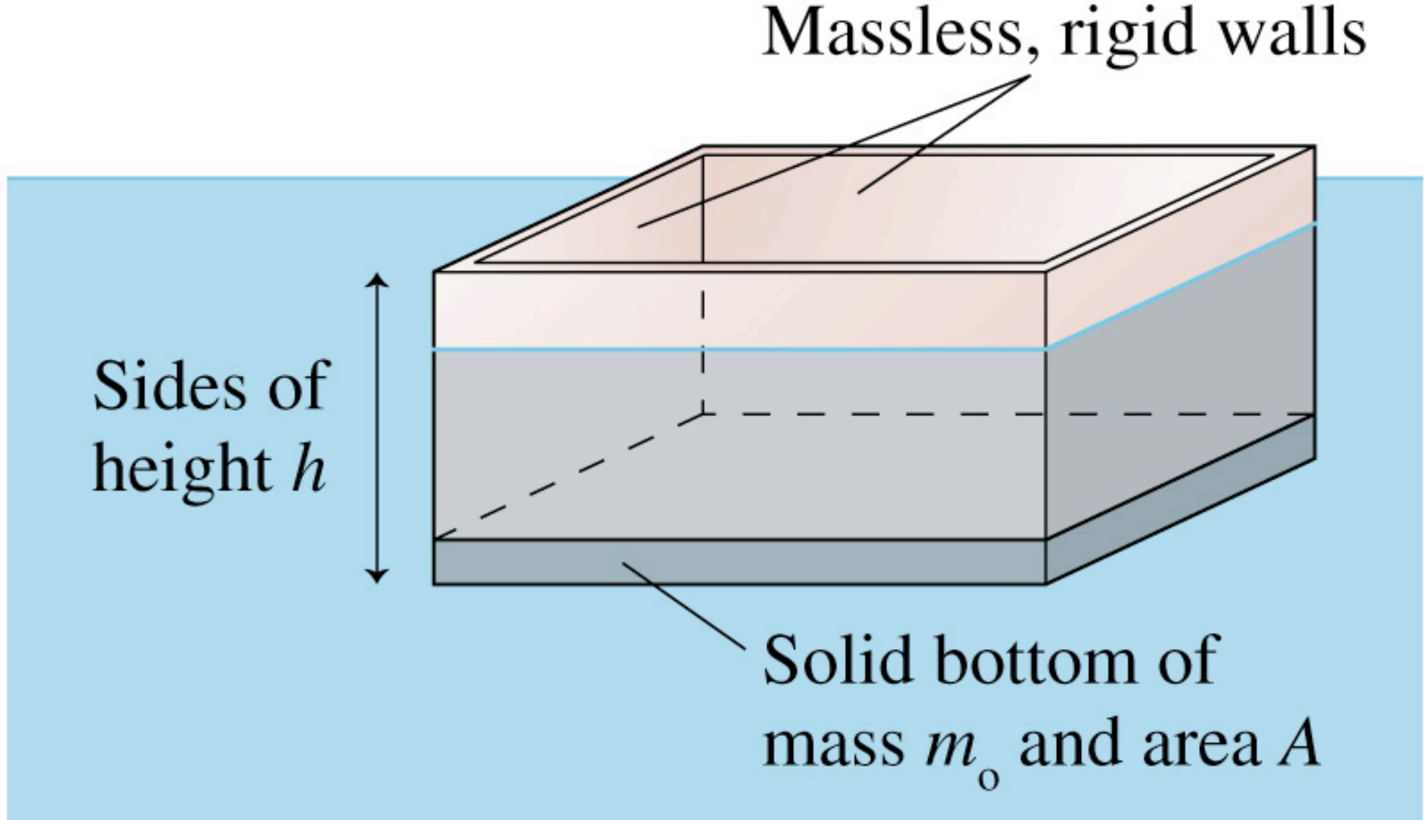
- 90% of ice underwater...



Example

- A 6.0 cm.-tall cylinder floats in water with its axis perpendicular to the surface. The length of the cylinder above water is 2.0 cm. What is the cylinder's mass density?

Boats



- steel plate sinks, but geometry (sides) allows it to dissipate more fluid than actual steel volume:

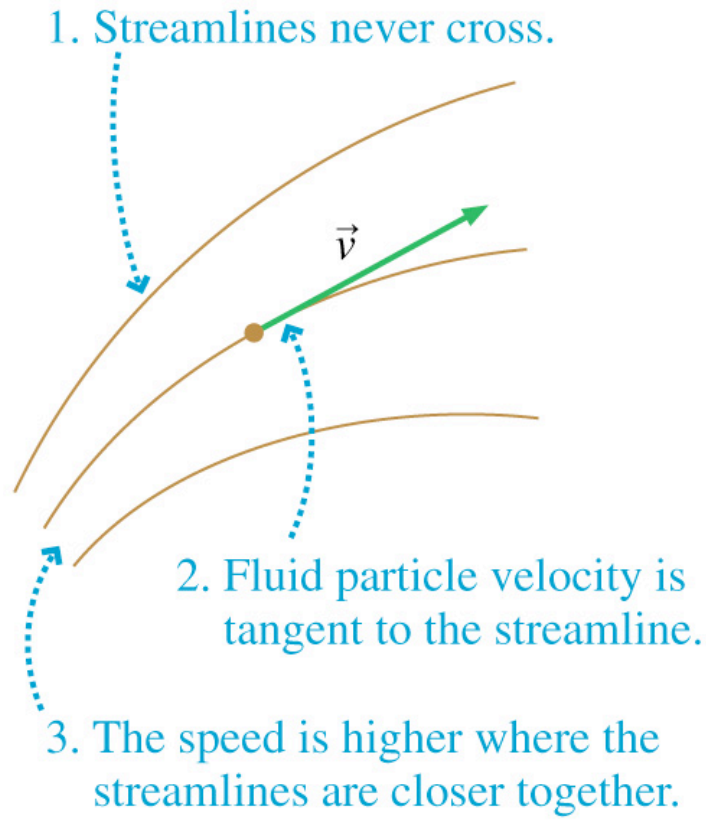
$$\rho_{avg.} = \frac{m_0}{Ah} < \rho_f$$

Ideal fluid

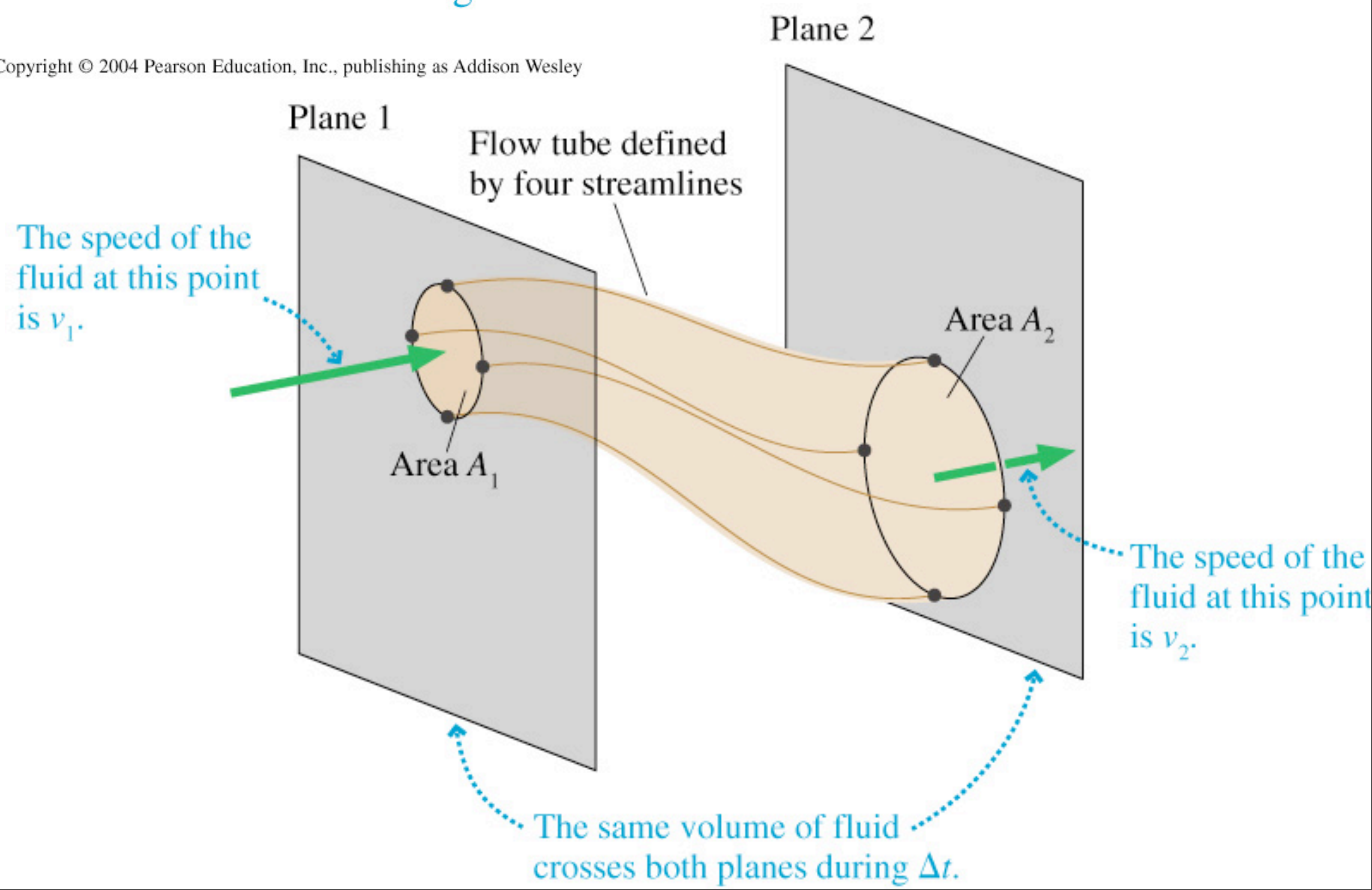
- incompressible (not so good approximation for gases)
- laminar (steady) flow (not turbulent): velocity at given point is constant with time
- non-viscous (no resistance to flow a la no friction for solid object)
- irrotational (test paddle wheel won't rotate)

Equation of continuity (I)

- Streamlines (path of “particle of fluid”:e.g. colored drop of water in stream)



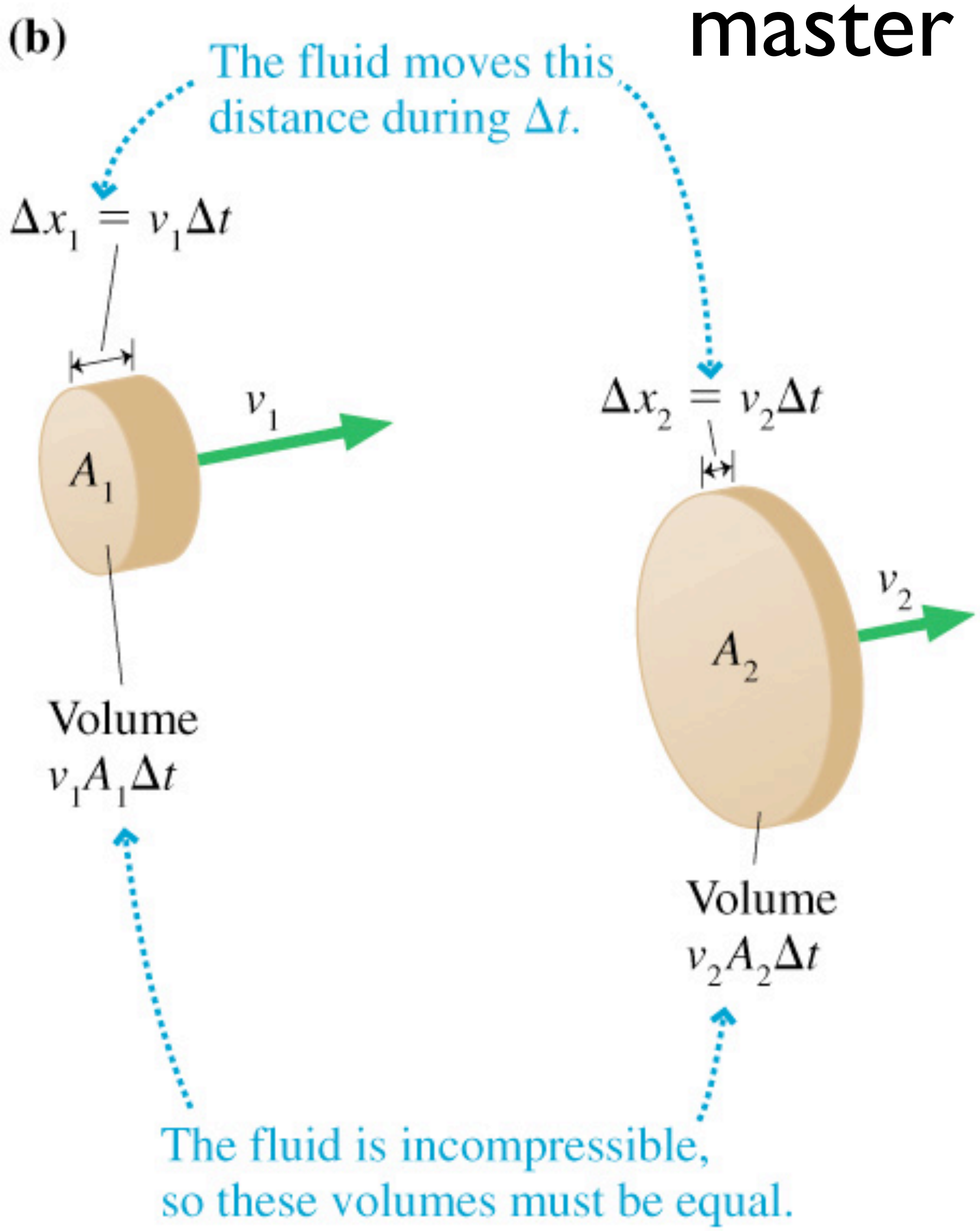
- Flow tube: bundle of streamlines (“invisible pipe”)



Equation of continuity (II)

- Fluid not created/ destroyed/stored within flow tube

$$V_1 = A_1 \Delta x_1 = A_1 v_1 \Delta t$$
$$(\text{volume flowing across } A_1) = V_2 \ldots$$



master formula

$$v_1 A_1 = v_2 A_2$$

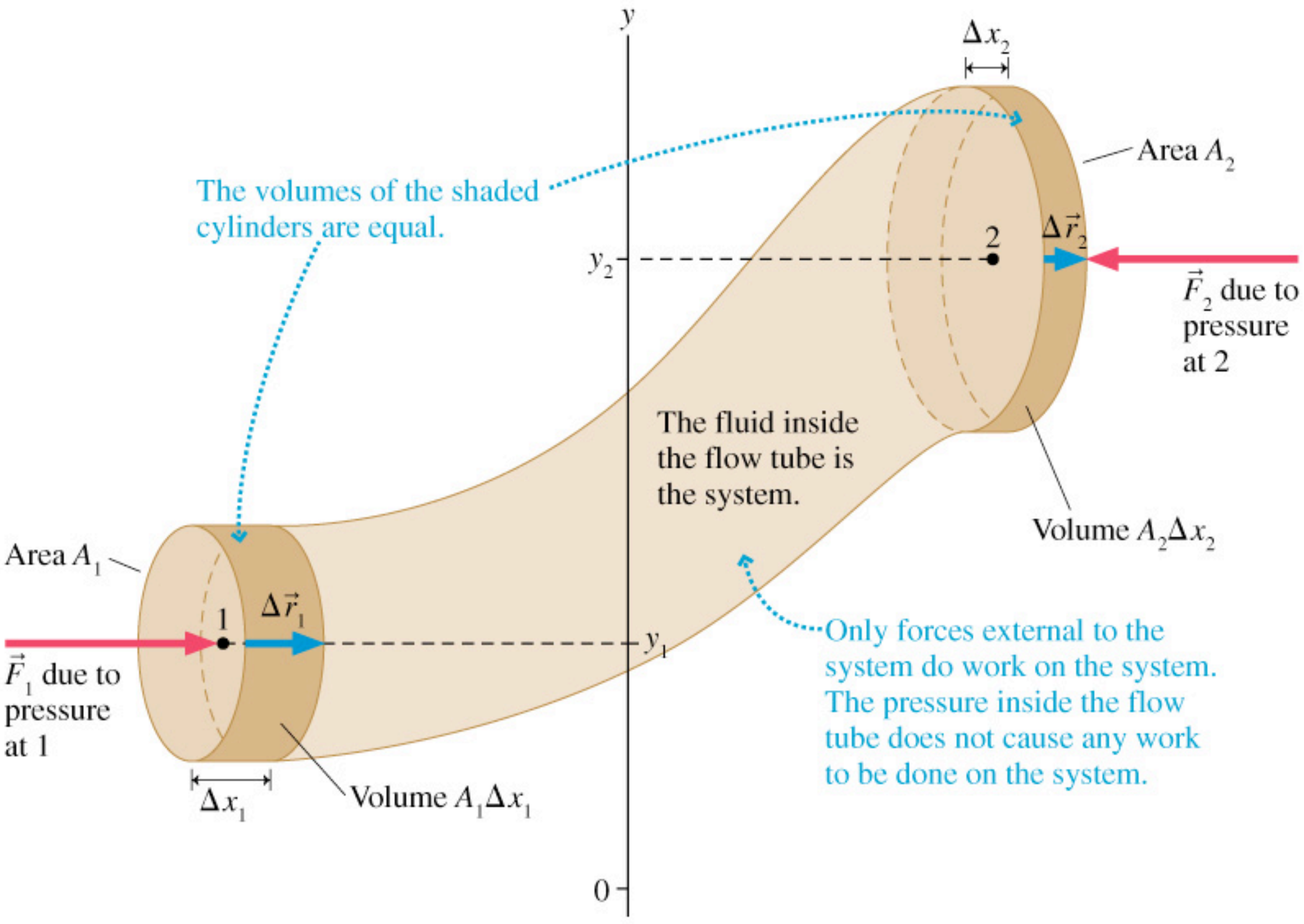
Flow faster in narrower part : e.g., water from tap

Q = V A (volume flow rate) constant

Bernoulli's equation

- work and energy conservation applied to volume of fluid in flow tube: $\Delta K + \Delta U = W_{ext.}$

by pressure of <u>surrounding</u> fluid



$$\begin{aligned}
W_1 &= F_1 \Delta x_1 &&= p_1 V \\
W_2 &= -F_2 \Delta x_2 &&= -p_2 V \\
\Delta U &= mgy_2 - mgy_1 &&= \rho V g \left(y_2 - y_1\right) \\
\Delta K &= \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2 &&= \frac{1}{2} \rho V \left(v_2^2 - v_1^2\right)
\end{aligned}$$

master formulae

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Example

- Water flows at 5.0 L/s through a horizontal pipe that narrows smoothly from a 10 cm diameter to 5.0 cm. diameter. A pressure gauge in the narrow section reads 50 kPa. What is the reading of a pressure gauge in the wide section?

Applications I: Venturi tube

(Measuring speed of flowing) gas

- Combine master formulae: (i) continuity equation (ii) Bernoulli's equation (equal y's) and (iii) pressure vs. depth

$$v_1 = A_2\sqrt{\frac{2\rho_{liq}.gh}{\rho\left(A_1^2 - A_2^2\right)}}$$

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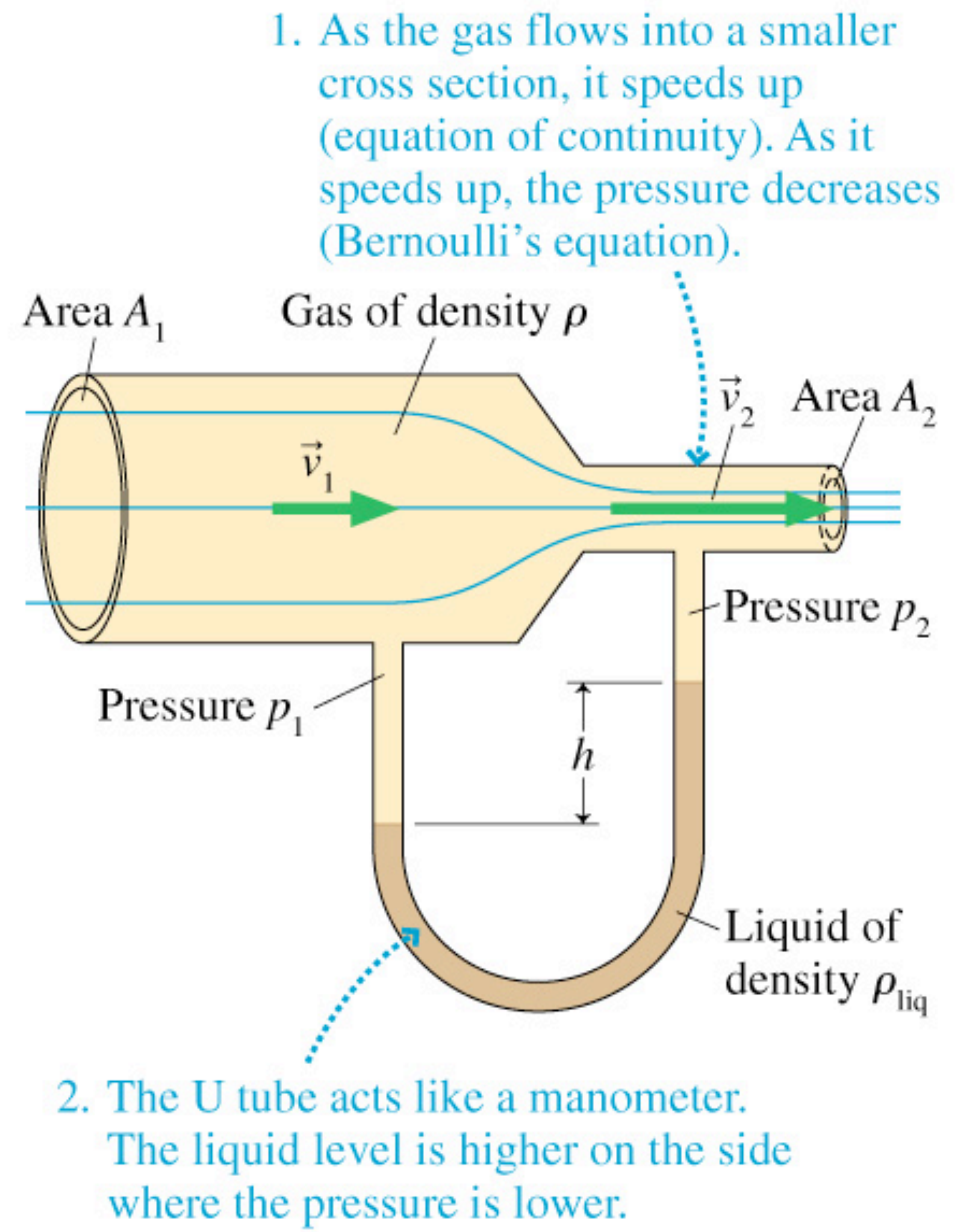

Airplane lift

- Continuity and Bernoulli's equations