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GERGONNE'S POINT AND OUTSTANDING DISTANCES

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Abstract: *In this article is proved a metric relationship for the distance between Gergonne's point in a fixed triangle and any point in the plane of triangle.*

Keywords: Gergonne's Point, Stewart's Theorem, Gergonne's cevians.

1. Introduction.

We call Gergonne point of the triangle to the meeting point of the lines containing the vertex of a triangle and the point of tangency with the inscribed circle. The Gergonne's Point was discovered by Joseph Diaz Gergonne (1771-1859) French mathematician. The identity described here, gives us the distance between the Gergonne's point of a triangle and any point on the plane that contains the triangle.

2. Notations.

Let ABC be an acute triangle. We denote its side-lengths by $BC = a, AC = b, AB = c$, it's semi perimeter by $s = \frac{1}{2}(a + b + c)$, it's area by F, it's circumradius by R and inradius by r. Its classical centres are the Centroid G, the Incenter I, the Circumcentre O, the Orthocentre H and Nagel's point N_a.

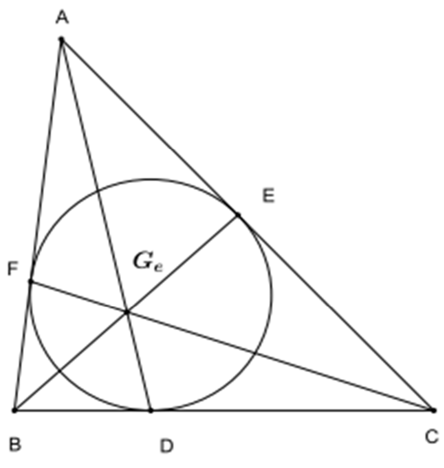

Figure 1.

3. Propositions.

Proposition 3.1

If AD, BE, CF are the Gergonne's cevians, then $AE = AF = s - a, BF = BD = s - b, CE = CD = s - c$.

Proof. Denoting $AF = AE = x, BF = BD = y$ and $CE = CD = z$. We have $y + z = a$, $z + x = b$ and $x + y = c$. Solving the system we find $x = s - a, y = s - b$ and $z = s - c$.

Proposition 3.2

If AD, BE and CF are the Gergonne's cevians of the triangle ABC then they are concurrent and the point of concurrence is the Gergonne's point G_e of the triangle ABC.

Proof. By Proposition 3.1, we have: $AF = AE = s - a, BF = BD = s - b$ and $CE = CD = s - c$. Using Ceva's theorem and replacing AF with AE, BD with F and C with CE, we soon find that

$$\frac{AE}{CE} \cdot \frac{CD}{BD} \cdot \frac{BF}{AF} = 1$$

Then

$$\frac{AE}{CE} \cdot \frac{CE}{BF} \cdot \frac{BF}{AE} = 1$$

Hence the three cevians that connect the vertices to the point of tangency of the circumference intersect at a point called the point of Gergonne G_e.

Proposition 3.3

If AD, BE and CF are the Gergonne's cevians of the triangle ABC then the length of each cevian is given by $AD = \sqrt{(s-a)^2 + \frac{4F^2}{as}}, BE = \sqrt{(s-b)^2 + \frac{4F^2}{bs}}$ and $CF = \sqrt{(s-c)^2 + \frac{4F^2}{cs}}$

Proof. We apply Stewart's theorem to triangle ABC in which AD is a cevian. See Figure 4.

We get:

$$BC \cdot AD^2 = BD \cdot AC^2 + CD \cdot AB^2 = BC \cdot BD \cdot CD$$

$$a \cdot AD^2 = c^2(s-c) + b^2(s-b) - a(s-b)(s-c)$$

$$AD^2 = \frac{c^2(s-c)}{a} + \frac{b^2(s-b)}{a} - (s-b)(s-c); (1)$$

After simplifying a few steps, we obtain $AD = \sqrt{(s-a)^2 + \frac{4F^2}{as}}$. Similarly, we can prove that:

$BE = \sqrt{(s-b)^2 + \frac{4F^2}{bs}}$ and $CF = \sqrt{(s-c)^2 + \frac{4F^2}{cs}}$.

Proposition 3.4

The Gergonne's Point G_e of the triangle ABC divides each cevian in the ratio given by

$$\frac{AG_e}{G_eD} = \frac{a(s-a)}{(s-b)(s-c)}, \frac{BG_e}{G_eE} = \frac{b(s-b)}{(s-a)(s-c)} \text{ and } \frac{CG_e}{G_eF} = \frac{c(s-c)}{(s-a)(s-b)}.$$

Proof. We have by Proposition 3.1, $BD = s-b$ and $CD = s-c$.

Now in the triangle ABD, the line CF as transversal. Applying Menelaus' Theorem we have

$$\frac{AF}{FB} \cdot \frac{BC}{CD} \cdot \frac{DG_e}{G_eA} = 1$$

$$\frac{s-a}{s-b} \cdot \frac{a}{s-c} \cdot \frac{G_eD}{AG_e} = 1$$

$$\frac{AG_e}{G_eD} = \frac{a(s-a)}{(s-b)(s-c)}; (2)$$

Similarly we can prove that $\frac{BG_e}{G_eE} = \frac{b(s-b)}{(s-a)(s-c)}$ and $\frac{CG_e}{G_eF} = \frac{c(s-c)}{(s-a)(s-b)}$.

From expression (2) and the fact that $AD = AG_e + G_eD$, we will have

$$\frac{AG_e}{AD} = \frac{a(s-a)}{a(s-a)+(s-b)(s-c)} = \frac{a(s-a)}{bc-(s-a)^2}; (3)$$

$$\frac{G_eD}{AD} = \frac{(s-b)(s-c)}{a(s-a)+(s-b)(s-c)} = \frac{(s-b)(s-c)}{bc-(s-a)^2}; (4)$$

Proposition 3.5

Let a, b and c the sides of an triangle ABC, and s, r, R and F are, respectively, its semi-perimeter, inradius, circumradius and are of that triangle, then

1. $\boldsymbol{ab + bc + ca = s^2 + r^2 + 4Rr}$
2. $\boldsymbol{a^2 + b^2 + c^2 = 2s^2 - 2r^2 - 8Rr}$
3. $\boldsymbol{a^3 + b^3 + c^3 = 2s^3 - 6r^2s - 12Rrs}$

Proof. Using Heron's formula for the area of the triangle and the fact that $abc = 4RF = 4Rrs$, we have $F^2 = s(s-a)(s-b)(s-c)$.

$$s^2r^2 = s^2(-s^2 + ab + bc + ca - 4Rr)$$

Hence, $a^2 + b^2 + c^2 = 2s^2 - 2r^2 - 8Rr$.

Now, we know that

$$a^3 + b^3 + c^3 = (a+b+c)(a^2+b^2+c^2-ab-bc-ca) + 3abc$$

$$a^3 + b^3 + c^3 = 2s[(2s^2 - 2r^2 - 8Rr) - (s^2 + r^2 + 4Rr)] + 12Rrs$$

So, $a^3 + b^3 + c^3 = 2s^3 - 6r^2s - 12Rrs$.

Theorem 3.6

Let M be any point in the plane of an acute triangle ABC which Gergonne's point G_e.

Then

$$\boldsymbol{MG_e^2 = \frac{1}{bc-(s-a)^2}\cdot\left[(s-b)(s-c)\cdot MA^2 + (s-a)(s-c)\cdot MB^2 + (s-a)(s-b)\cdot MC^2\right] - \frac{4rs^2(R+r)}{(4R+r)^2}}$$

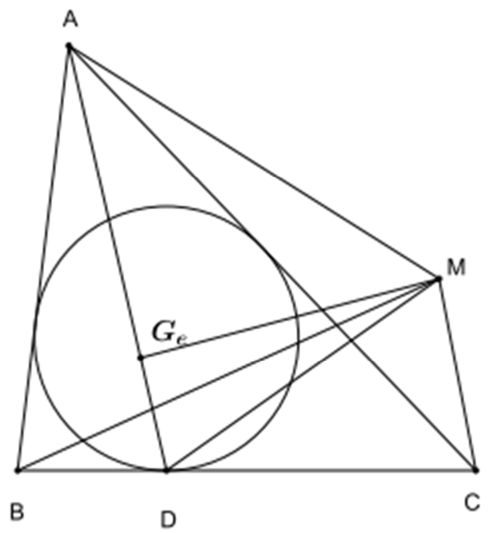

Figure 2: Gergonne's Point.

Proof. We apply Stewart's theorem in to triangle MBC in which MD is a cevian, according to figure 2. We get

$$a\cdot MD^2 = BD\cdot MC^2 + CD\cdot MB^2 - a\cdot BD\cdot DC$$

$$a \cdot MD^2 = (s-b) \cdot MC^2 + (s-c) \cdot MB^2 - a(s-b)(s-c)$$

$$MD^2 = \frac{s-b}{a} \cdot MC^2 + \frac{s-c}{a} \cdot MB^2 - (s-b)(s-c); (5)$$

Now, we apply Steward's theorem in to triangle MAD in which MG_e is a cevian. We get

$$AD \cdot MG_e^2 = AG_e \cdot MD^2 + G_eD \cdot MA^2 - AD \cdot AG_e \cdot G_eD$$

$$MG_e^2 = \frac{AG_e}{AD} \cdot MD^2 + \frac{G_eD}{AD} \cdot MA^2 - AG_e \cdot G_eD$$

Using the expressions (3),(4) and (5), we obtain

$$MG_e^2 = \frac{1}{bc-(s-a)^2} \cdot [(s-b)(s-c) \cdot MA^2 + (s-a)(s-c) \cdot MB^2 + (s-a)(s-b) \cdot MC^2\,] -$$

$$= \frac{(s-a)(s-b)(s-c)}{[bc-(s-a)^2]^2} \cdot [4(s-a)(s-b)(s-c) + abc]$$

Now, observe that

$$a(s-a) + (s-b)(s-c) = bc - (s-a)^2 = \frac{1}{4}(-a^2 - b^2 - c^2 + 2ab + 2bc + 2ca) =$$

$$= \frac{1}{4}(4r^2 + 16Rr) = r^2 + 4Rr \text{ and } abc = 4RF = 4Rrs$$

After simplifying a few steps we obtain,

$$MG_e^2 = \frac{1}{bc-(s-a)^2} \cdot [(s-b)(s-c) \cdot MA^2 + (s-a)(s-c) \cdot MB^2 + (s-a)(s-b) \cdot MC^2] -$$

$$-\frac{4rs^2(R+r)}{(4R+r)^2}$$

4. Main result

Corollary 4.1

Be G the centroid of the triangle ABC and G_e the Gergonne's point, then

$$\boldsymbol{GG_e^2 = \frac{2}{9}\left(a^2 + b^2 + c^2\right) - \frac{8s^2\left(r^2 + 2R^2\right)}{3(r+4R)^2}}$$

Proof. In Theorem 3.6, replace M by the incenter G. We get

$$GG_e^2 = \frac{1}{bc-(s-a)^2} \cdot [(s-b)(s-c) \cdot GA^2 + (s-a)(s-c) \cdot GB^2 + (s-a)(s-b) \cdot GC^2] -$$

$$-\frac{4rs^2(R+r)}{(4R+r)^2}$$

We know that

$$GA^2=\frac{1}{9}(2b^2+2c^2-a^2), GB^2=\frac{1}{9}(2a^2+2c^2-b^2) \text{ and } GC^2=\frac{1}{9}(2a^2+2b^2-c^2)$$

Then

$$GG_e^2=\frac{1}{bc-(s-a)^2}\cdot\left[(s-b)(s-c)\cdot\frac{1}{9}(2b^2+2c^2-a^2)+(s-a)(s-c)\cdot\frac{1}{9}(2a^2+2c^2-b^2)+(s-a)(s-b)\cdot\frac{1}{9}(2a^2+2b^2-c^2)\right]-\frac{4rs^2(R+r)}{(4R+r)^2}$$

$$GG_e^2=\frac{1}{9(bc-(s-a)^2}[(s-b)(s-c)+(s-a)(s-c)+(s-a)(s-b)](2a^2+2b^2+2c^2)-$$

$$-\frac{1}{3(bc-(s-a)^2)}[a^2(s-b)(s-c)+b^2(s-a)(s-c)+c^2(s-a)(s-b)]-\frac{4rs^2(R+r)}{(4R+r)^2}$$

$$GG_e^2=\frac{2a^2+2b^2+2c^2}{9(bc-(s-a)^2)}[bc-(s-a)^2][a^2(-s^2+as+bc)+b^2(-s^2+bs+ac)+c^2(-s^2+cs+ab)]-\frac{4rs^2(R+r)}{(4R+r)^2}$$

$$GG_e^2=\frac{2}{9}(a^2+b^2+c^2)-\frac{1}{3(bc-(s-a)^2)}[a^2(-s^2+as+bc)+b^2(-s^2+bs+ac)+c^2(-s^2+cs+ab)]-\frac{4rs^2(R+r)}{(4R+r)^2}$$

$$GG_e^2=\frac{2}{9}(a^2+b^2+c^2)-\frac{1}{3r(r+4R)[-s^2(a^2+b^2+c^2)+s(a^3+b^3+c^3)+abc(a+b+c)]}-\frac{4rs^2(R+r)}{(4R+r)^2}$$

$$GG_e^2=\frac{2}{9}(a^2+b^2+c^2)-\frac{1}{3r(r+4R)}[-s^2(2s^2-2r^2-8Rr)+s(2s^3-6r^2s-12Rrs)+8Rrs^2]-\frac{4rs^2(R+r)}{(4R+r)^2}$$

$$GG_e^2=\frac{2}{9}(a^2+b^2+c^2)-\frac{4s^2(r-R)}{3(r+4R)}-\frac{4rs^2(R+r)}{(4R+r)^2}$$

Therefore,

$$GG_e^2 = \frac{2}{9}(a^2+b^2+c^2) - \frac{8s^2(r^2+2R^2)}{3(r+4R)^2}$$

Corollary 4.2

Be I the incenter of the triangle ABC and G_e the Gergonne's point, then

$$\boldsymbol{IG_e^2 = r^2 - \frac{3r^2s^2}{(r+4R)^2}}$$

Proof. In Theorem 3.6, replace M by the incenter I. We get

$$IG_e^2 = \frac{1}{bc-(s-a)^2}\cdot[(s-b)(s-c)\cdot IA^2 + (s-a)(s-c)\cdot IB^2 + (s-a)(s-b)\cdot IC^2] -$$

$$-\frac{4rs^2(R+r)}{(4R+r)^2}$$

Of the triangle whose vertices are A, I and the point of contact of the incircle with side AB, we obtain

$$AI^2 = \frac{(s-a)^2}{\cos^2\frac{A}{2}}$$

Now, we know that

$$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}, \cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}} \text{ and } \cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}} \text{ then}$$

$$AI^2 = \frac{bc(s-a)}{s}$$

Similarly, we can prove $BI^2 = \frac{ac(s-b)}{s}$ and $CI^2 = \frac{ab(s-c)}{s}$.

$$IG_e^2 = \frac{1}{bc-(s-a)^2}\cdot[(s-b)(s-c)\cdot IA^2 + (s-a)(s-c)\cdot IB^2 + (s-a)(s-b)\cdot IC^2]$$

$$-$$

$$-\frac{4rs^2(R+r)}{(4R+r)^2}$$

$$IG_e^2 = \frac{1}{bc-(s-a)^2}\cdot\frac{F^2}{s^2}[bc+ca+ab] - \frac{4rs^2(R+r)}{(4R+r)^2}$$

$$IG_e^2 = \frac{1}{r(r+4R)}\cdot\frac{s^2r^2}{s^2}[s^2+r^2+4Rr] - \frac{4rs^2(R+r)}{(4R+r)^2}$$

$$IG_e^2=r^2+\frac{rs^2}{r+4R}-\frac{4rs^2(R+r)}{(4R+r)^2}$$

Hence,

$$IG_e^2=r^2-\frac{4rs^2(R+r)}{(4R+r)^2}$$

Corollary 4.3

For any acute triangle ABC

$$\boldsymbol{r+4R\geq s\sqrt{3}}$$

with equality when the triangle is equilateral.

Proof. This follows from Corollary 4.3. We know that $IG_e^2\geq 0$, then

$$IG_e^2=\frac{r^2(r+4R)^2-3r^2s^2}{(r+4R)^2}=\frac{r^2}{(r+4R)^2}\left[(r+4R)-s\sqrt{3}\right]\left[(r+4R)+s\sqrt{3}\right]$$

$$(r+4R)-s\sqrt{3}\geq 0$$

$$r+4R\geq s\sqrt{3}$$

Hence proved.

Corollary 4.4

Be O the circumcenter of the triangle ABC and G_e the Gergonne's point, then

$$\boldsymbol{OG_e^2=R^2-\frac{4rs^2(r+R)}{(r+4R)^2}}$$

Proof. In Theorem 3.6, replace M by the circumcenter O, and consider that $OA=OB=OC=R$, we get

$$OG_e^2=\frac{R^2}{bc-(s-a)^2}\cdot[(s-b)(s-c)+(s-c)(s-a)+(s-a)(s-b)]-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$OG_e^2=\frac{R^2}{bc-(s-a)^2}\cdot[2ab+2bc+2ca-a^2-b^2-c^2]-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$OG_e^2=\frac{R^2}{bc-(s-a)^2}\cdot[bc-(s-a)^2]-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$OG_e^2=R^2-\frac{4rs^2(r+R)}{(r+4R)^2}$$

Corollary 4.5

Be H the orthocenter of the triangle ABC and G_e, the Gergonne's point, then

$$\boldsymbol{HG_e^2 = 4R^2 + \frac{8s^2(Rr - 2R^2)}{(r + 4R)^2}}$$

Proof. In Theorem 3.6, replace M by the orthocenter H, so

$$HG_e^2 = \frac{R^2}{bc - (s-a)^2} \cdot [(s-b)(s-c)HA^2 + (s-c)(s-a)HB^2 + (s-a)(s-b)HC^2] - \frac{4rs^2(r+R)}{(r+4R)^2}$$

Using the half angle formulas for the cosine function of an internal angle, we have

$$2\cos^2\frac{A}{2} = 1 + \cos A$$

$$\cos A = 2\cos^2\frac{A}{2} - 1$$

$$\cos A = \frac{2s(s-a)}{bc} - 1$$

Similarly, $\cos B = \frac{2s(s-b)}{ac} - 1$ and $\cos C = \frac{2s(s-c)}{ab} - 1$, then

$$HG_e^2 = \frac{R^2}{bc - (s-a)^2} \cdot \left[(s-b)(s-c)\left(\frac{2s(s-a)}{bc} - 1\right)^2 + (s-c)(s-a)\left(\frac{2s(s-b)}{ac} - 1\right)^2 + (s-a)(s-b)\left(\frac{2s(s-c)}{ab} - 1\right)^2\right] - \frac{4rs^2(r+R)}{(r+4R)^2}$$

After simplifying a few steps we obtain

$$HG_e^2 = 4R^2 + \frac{16sF^2R^2}{r(r+4R)}\left(\frac{s-a}{b^2c^2} + \frac{s-b}{a^2c^2} + \frac{s-c}{a^2b^2}\right) - \frac{16F^2R^2}{r(r+4R)}\left(\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}\right) - \frac{4rs^2(r+R)}{(r+4R)^2}$$

$$HG_e^2 = 4R^2 + \frac{16sF^2R^2}{r(r+4R)} \cdot \frac{a^2(s-a)+b^2(s-b)+c^2(s-c)}{a^2b^2c^2} - \frac{16F^2R^2}{r(r+R)} \cdot \frac{a+b+c}{abc} - \frac{4rs^2(r+R)}{(r+4R)^2}$$

$$HG_e^2 = 4R^2 + \frac{4s}{r+4R} \cdot [s(a^2+b^2+c^2)-(a^3+b^3+c^3)] - \frac{8Rs^2}{r+4R} - \frac{4rs^2(r+R)}{(r+4R)^2}$$

$$HG_e^2 = 4R^2 + \frac{4s}{r+4R} \cdot [(2s^3-2r^2s-8Rrs)-(2s^3-6r^2s-12Rrs)] - \frac{8Rs^2}{r+4R} - \frac{4rs^2(r+R)}{(r+4R)^2}$$

$$HG_e^2 = 4R^2 + \frac{4s^2(r+R)}{r+4R} - \frac{8Rs^2}{r+4R} - \frac{4rs^2(r+R)}{(r+4R)^2}$$

Further simplification gives

$$HG_e^2 = 4R^2 + \frac{8s^2(Rr-2R^2)}{(r+4R)^2}$$

Hence proved.

Corollary 4.6

If N_a is Nagel's point of the triangle ABC and G_e is the Gergonne's point, then

$$\boldsymbol{N_aG_e^2 = -16Rr + \frac{16s^2(Rr+R^2)}{(r+4R)^2}}$$

Proof. We know that

$$N_aA = \frac{a}{s}\sqrt{s^2 - \frac{4F^2}{a(s-a)}} \Rightarrow N_aA^2 = a^2 - \frac{4a}{s}(s-b)(s-c)$$

Similarly $N_aB^2 = b^2 - \frac{4b}{s}(s-a)(s-c)$ and $N_aC^2 = c^2 - \frac{4c}{s}(s-a)(s-b)$.

Using the Theorem 3.6, replace M by Nagel's point N_a, then

$$N_aG_e^2 = \frac{1}{bc-(s-a)^2} \cdot [(s-b)(s-c)\cdot N_aA^2 + (s-a)(s-c)\cdot N_aB^2 + (s-a)(s-b)\cdot N_aC^2] - \frac{4rs^2(R+r)}{(4R+r)^2}$$

$$N_aG_e^2=\frac{1}{bc-(s-a)^2}\cdot\left[(s-b)(s-c)\cdot\left(a^2-\frac{4a}{s}(s-b)(s-c)\right)+(s-a)(s-c)\cdot\left(b^2-\frac{4b}{s}(s-a)(s-c)\right)+(s-a)(s-b)\cdot\left(c^2-\frac{4c}{s}(s-a)(s-b)\right)\right]-\frac{4rs^2(R+r)}{(4R+r)^2}$$

$$N_aG_e^2=\frac{1}{bc-(s-a)^2}[a^2(s-b)(s-c)+b^2(s-a)(s-c)+c^2(s-a)(s-b)]-$$

$$-\frac{4}{s}[a(s-b)^2(s-c)^2+b(s-a)^2(s-c)^2+c(s-a)^2(s-b)^2]-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$N_aG_e^2=\frac{1}{bc-(s-a)^2}[a^2(-s^2+as+bc)+b^2(-s^2+bs+ac)+c^2(-s^2+cs+ab)]-$$

$$-\frac{4}{s(bc-(s-a)^2)}[a(-s^2+as+bc)^2+b(-s^2+bs+ac)^2+c(-s^2+cs+ab)^2]-$$

$$-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$N_aG_e^2=\frac{1}{bc-(s-a)^2}[-s^2(a^2+b^2+c^2)+s(a^3+b^3+c^3)+abc(a+b+c)]-$$

$$-\frac{4}{s(bc-(s-a)^2)}[a(s^4+a^2s^2+b^2c^2-2as^3-2bcs^2+2abcs)$$

$$+b(s^4+b^2s^2+a^2c^2-2bs^3-2acs^2+2abcs)+$$

$$+c(s^4+c^2s^2+a^2b^2-2cs^3-2abs^2+2abcs)]-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$N_aG_e^2=\frac{1}{bc-(s-a)^2}[-s^2(2s^2-2r^2-8Rr)+s(2s^3-6r^2s-12Rrs)+4Rrs\cdot 2s]-$$

$$-\frac{4}{s(bc-(s-a)^2)}[s^4(a+b+c)+s^2(a^3+b^3+c^3)+abc(a+b+c)-$$

$$-2s^3(2s^3-6r^2s-12Rrs)-24Rrs^3+16Rr^3]-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$N_aG_e^2=-\frac{1}{r(r+4R)}(-4r^2s^2+4Rrs^2)-\frac{4}{sr(r+4R)}[2s^5+s^2(2s^3-6r^2s-12Rrs)+$$

$$+4Rrs(s^2+r^2+4Rr)-2s^3-6r^2s-12Rrs)-24Rrs^3+16Rr^3\,]-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$N_aG_e^2=\frac{4s^2(R-r)}{r+4R}-\frac{4(16R^2r+4Rr^2-2rs^2)}{r+4R}-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$N_aG_e^2=-16Rr+\frac{8rs^2}{r+4R}+\frac{4s^2(R-r)}{r+4R}-\frac{4rs^2(r+R)}{(r+4R)^2}$$

$$N_aG_e^2=-16Rr+\frac{4s^2}{(r+4R)^2}$$

Hence,

$$N_a G_e^2 = -16Rr + \frac{16Rs^2(r+R)}{(r+4R)^2}$$

5 Conclusion

In this article we find a metric relationship for the Gergonne’s point. With this relationship we can find the distance between the Gergonne’s point and other notable centers of the triangle. The proofs presented here only require basic knowledge of Geometry and its manipulation and application.

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