Half Sine Shock Tests to Assure Machinery Survival in Explosive Environments

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ABSTRACT

Previous work has shown that traditional four coordinate pseudo velocity shock spectrum is the best format for violent machine transient foundation motion analysis. It emphasizes the motion severity. This paper applies these results to the theoretical half sine and other simple test pulses. It shows the simple pulses strongly related to actual explosive transients. The trick turned out to be inclusion of pre and post pulse motion in the calculation. The calculations applied to shaker shock pulse motions are also discussed.

INTRODUCTION

Previous work has shown that traditional four coordinate pseudo velocity shock spectrum is the best format for violent transient foundation motion analysis if one is trying to indicate severity [1, 2, 3]. It emphasizes the motion severity. This paper applies these results to the theoretical half sine and other simple test pulses, and in so doing, establishes three unappreciated characteristics of the simple shock pulses.

Inclusion of the drop and any rebound in the pseudo velocity shock spectrum plotted on four coordinate paper (PVSS on 4CP) shows the simple pulses similar, as observed by Gertel and Holland [4].

Shock spectra for explosive environments are shown similar to those of simple pulse tests produced on a drop table except for directionality. The trick turned out to be inclusion of pre and post pulse motion in the shock spectrum calculation.

Shock spectrum calculations applied to shaker generated shock pulse motions show that simple pulse tests conducted on a shaker with limited displacement do not match the low frequency severity of drop table shocks.

A difficulty with this paper is as follows. You can't confirm what I have done unless you have a shock spectrum program and the ability to plot the results on four coordinate paper. In that sense this is a "trust me" paper. Good shock spectrum programs are not common. I will give you my MATLAB program, [5, 6], and assure you that I have been using it for 20 years or more; I see no bugs in it. Only recently have I programmed good 4CP, but that correctness is easily verified.

PSEUDO VELOCITY AND FOUR COORDINATE PAPER.

The shock spectrum is a plot of an analysis of a shock motion (i.e., transient motions due to explosions, earthquakes, package drops, railroad car bumping, vehicle collisions, etc.) that calculates the maximum response of many different frequency damped single degree of freedom systems (SDOF) exposed to the motion. The plot is a graph of maximum response versus frequency. Pseudo velocity is specifically the maximum relative displacement times frequency in radians. It is surprising and not trivial, but this is the important quantity to plot for shock spectra when you want to indicate severity or capacity to cause damage. The best way to plot these is on four coordinate paper (4CP) (also called tripartite paper). Four coordinate paper is a logarithmic graph paper that has four sets of lines relating frequency, displacement, velocity, and acceleration of sinusoidal motions.

The shock spectrum algorithm finds the peak relative displacement for a base excited SDOF. That's the maximum energy stored in the elastic member during the transient event. At that instant of time the velocity was zero, thus there was no damping force, so the force in the spring at that instant times the mass was the acceleration at that instant too. If that maximum elastic energy stored in the spring during the shock were converted to velocity (kinetic energy) in the next or previous quarter cycle, that velocity would be (equating the kinetic and elastic energies):

$$\frac{kz^2}{2} = \frac{mv^2}{2}, \quad \text{or} \quad v^2 = \frac{k}{m} z^2 \tag{1}$$

Since $k/m = \omega^2$, we find:

$$v = \omega z \tag{1a}$$

That "v" is the pseudo velocity. We calculate z_{max} for each frequency, and we plot ωz_{max} on four coordinate paper.

The reasons why PV is so good are a little cloudy and difficult. The typical engineering structures explanation is given in Reference [7]. It needs more thought, but I'll give you my best now. Peak modal velocity in elastic structure is proportional to peak stress, and not acceleration as most still do not know [2, 8, 9, 10, 11, 12]. Say we have a steel chair, mounted on a shaker set to shake it horizontally and find its first mode. The top of the back of the chair will have the highest modal velocity (measure the peak acceleration and divide it by the frequency in radians). The peak stress is probably where one of the chair legs is welded to the seat. That peak stress in psi will be a constant between 1 and 5 times 146 times that velocity in ips. That is a fact [2, 9], and it is also true for second and third and all of the modes if we know for each, the peak velocity location and the peak stress location.

Pseudo velocity is the best quantity to predict the potential to generate modal velocities in structure. Relative velocity does not indicate modal velocity at low or high frequencies; it matches PV only in center severe section of the shock spectrum.

The asymptotic behavior of the PVSS is summarized as follows [2, 3]. When the PVSS is plotted on 4CP, the displacement is exact. One can expect to see two asymptotic values: at the high frequency end of the shock spectrum the curve should approach the peak pulse acceleration, and at the low frequency end of the spectrum it should approach the peak shock deflection. For intermediate values of the frequency, the peak pseudo velocity is often almost constant.

Four coordinate paper (4CP) (tripartite paper) is important to the PVSS presentation. For a sinusoidal motion, the displacement, velocity, and acceleration are:

$$z = z_{max} \sin \omega t, \quad \dot{z} = \omega z_{max} \cos \omega t, \quad \ddot{z} = -\omega^2 z_{max} \sin \omega t \tag{2a}$$

By equating the maximum values we have:

$$\omega z_{max} = \dot{z}_{max}, \quad \omega^2 z_{max} = \ddot{z}_{max}, \quad \ddot{z}_{max} = \omega \dot{z}_{max} \tag{2b}$$

The four quantities: $z_{max}, \dot{z}_{max}, \ddot{z}_{max}, and \omega\ or\ 2\pi f$, are related by Eq. 2b. Knowing any two, you can calculate the other two. By taking logs the lines of constant $z_{max}, \dot{z}_{max}, \ddot{z}_{max}$, versus ω, or $2\pi f$, are all straight lines. That is why we can compute the plot of Figure 1.

THE SIMPLE PULSES HAVE SIMILAR SHOCK SPECTRA WHEN PLOTTED AS PVSS ON 4CP

To make the point of simple pulse similarity, I have to calculate the PVSS on 4CP for all the simple shocks and there is not space to present individual plots for each shock. The obvious way to show them similar is to superpose all of their spectra on a single composite plot and that is done in Figures 2 and 3. Below I will describe the characteristics of each shock and the compromises I had to make to prove this point.

a. Half Sine Shock

Half sine shocks are usually specified by a peak acceleration and a duration, such as an 11 millisecond 30g half sine. The shock has peak acceleration, $\ddot{x}_m$, and duration t_d ; there is a frequency associated with this duration since it is half of one period of a sine wave with frequency, $f_d = 1/2t_d$.

$$\ddot{x} = \ddot{x}_{max} \sin 2\pi f_d t \,, \quad \text{where} \quad f_d = \frac{1}{2t_d} \tag{1}$$

The velocity change during the half sine pulse is very important. Assume zero initial velocity and integrate over the half cycle to get the final velocity, which is the velocity change.

$$\Delta\dot{x} = \frac{\ddot{x}_{max}}{\pi f_d} = \frac{2\ddot{x}_{max} t_d}{\pi} \tag{2}$$

This result is important. It relates three important pulse properties: velocity change, peak acceleration, and duration. Two of these, $\Delta\dot{x}, \ddot{x}_{max}$, are apparent on the pseudo velocity shock spectrum. The shock spectrum of this pulse by itself is what is usually computed. A package is sitting on the table and somebody belts the table upward with a rubber mallet in a zero “g” situation, and the package continues going off into space at constant velocity. It is unrealistic, and leaves the low frequency asymptote (content or limit) out. Yet we find this in every book on shock that presents the half sine shock spectrum.

I am going to plot many shock spectra in this paper and I am going to select fairly severe parameters so you can get familiar with severe shock spectra. I will use a velocity change of 100 ips and peak “g” level of 200gs. Let's plot the above shock along with its two integrals, and then calculate the shock spectrum for this nude half sine shock. Using Eq. (2) we find the duration to be 2.034 ms. Figure 4 shows the acceleration shock and its two integrals.

This is what I advise you to learn to expect. The velocity change of 100 ips shows up as it should and the acceleration level is assympotic to 200gs, as expected. The fact that the pulse we analyzed indicates constant velocity of the shock table for ever, means that an SDOF system with a natural frequency of 0.1 Hz would have a peak deflection of about 140 inches, or about 12 ft. It is an excellent plot of an analysis of a dumb pulse.

Now we consider a realistic half sine shock. We drop it through a distance, such that a shock programmer delivering our half sine, just brings it to rest. The velocity begins and ends at zero, thus the shock will have a zero mean. A 1g fall through for a time t_{drop} so that the area (g x t_{drop}) equals the velocity change, and since $v^2 = 2gh$, the drop height is also found from Eq. (3). The time history plot and the integrals are shown in Figure 5.

$$g t_{drop} = \Delta\dot{x}, \quad \text{or} \quad t_{drop} = \frac{\Delta\dot{x}}{g}, \quad \text{and} \quad h = \frac{\Delta\dot{x}^2}{2g} \tag{3}$$

This is a shock that could occur Its shock spectra are shown as the red curves on Figures 2 and 3. Notice now we see the 13-inch assymptote at the low frequencies. We also see the 100 ips velocity change and the 200g peak assymptote clearly.

b. Trapezoidal Shock

We next consider a trapazoidal pulse of duration t_d, and maximum acceleration amplitude, A_m, expressed in "gs." It has a rise time, T_r , and a fall time of T_f. These definitions are taken from Reference [13], Figure 516.5-11, on page 516.5-16. For the ideal case we are calculating we will take the rise and fall times to be ϕ times the pulse duration, t_d. The duration includes the ramps up and down. The area of this pulse is equal to the velocity change it causes. The linear ramp trapazoidal equation relating velocity change, peak acceleration, and duration is given in Eq. (4).

$$t_d = \frac{\Delta \dot{x}}{(1-\phi)\,\ddot{x}_{max}} \quad (4)$$

I used MATLAB to calculate the time history preceeded by 200 zeros, the 1g drop to attain the 100 ips, the ramp up, the flat portion, and the ramp down followed by 200 zeros. That time history for ϕ = 0.1 and its two integrals are given in Figure 6. The damped and undamped shock spectra are given in Figure 7.

The 100 ips flat portion is severe from about 1.5 to 200 Hz. Notice that the high frequency acceleration starts as asymptotic to about 400gs, and then starts heading for the 200g at 5,000 Hz. This is due to the impulsive rise time of the trapazoidal acceleration.

To prove this point I re-did the trapezoidal shock with a half cosine ramp with a ramp duration of $\phi t_{d.}$ The total velocity change is (with a cosine ramp on the beginning and the end of the trapezoid) the same as given in Eq. (4). To figure out how much of a cosine ramp woulld be reasonable within the confines of the IEC Specification, [14] according to Figure 3., on p40, I plotted cosine ramps inside the tolerances given in their Figure 3. This easily permitted use of a a ϕ = 0.3. The resulting time history is shown in Figure 8.

This is sufficient rise time amelioration to be able to see the 200g asymptote of the shock spectra. The spectrum is shown as the green curve on Figures 1 and 4. Notice that this smooths it enough to permit it to dive down to 200gs at about 2500 Hz. The 5% damped spectrum is also as expected. All one can say is that an abrupt rise doubles the peak "g" level over a range before the peak "g" asymptote appears.

c. Saw Tooth Shocks

In the case of these triangular shocks, the velocity change during the pulse will be its area, one half of the height times the duration which yields:

$$t_d = \frac{2\Delta \dot{x}}{\ddot{x}_{max}} \quad (5)$$

This can not be exact because when you digitally draw it, there has to be an additional area of the little triangle from the peak down to the first zero sample. We will get out of this clumsy complication by adding a cosine ramp as was done with the trapezoidal shock. Notice that the initial peak sawtooth will have the same velocity change. In these pulses we assume that shock programmers on the shock machines bring the table to zero velocity so that the velocity change is equal to the velocity from Eq. (5). The cosine fall off will occur in a time interval ϕt_d. Here t_d means the total pulse duration including the linear ramp up from 0 to max acceleration and the cosine fall off back down to zero. Carrying out the integration yields the same velocity change equation, Eq. (5), independent of ϕ. A linear ramp has the same area as a cosine ramp. Graphically checking the tolerance levels of MIL STD 810F, Figure 516.5-10 [13], shows that a ϕ = 0.1 cosine is easily acceptable.

I will do the analysis of terminal peak and initial peaks shocks with this ϕ = 0.1, cosine ramp to save trouble. Now using Eq. (5) to define the t_d for our 100 ips velocity change, 200g shock, we get the time history, and it is two integrals shown in Figure 9.

This also requires a 12.95-inch drop. The shock spectrum for two damping values, 0 and 5%, are given as the blue curves of Figures 2 and 3. The 13-inch drop and the 100 ips severe region are right where they should be.

The curves come down to the 200g asymptote at the lowest frequency of any of the simple pulses. This is because of the gradual rise in acceleration.

Trouble develops with the initial peak saw tooth and its abrupt rise. Let's look at the initial peak time history with a 10% cosine ramp up to the initial peak shown in Figure 10.

The 10% cosine is difficult to see but it is there. We have trouble with the shock spectrum, shown in Figure 11, because of the steep initial rise.

Notice an initial doubling of the acceleration assymptote in most of the high frequency region. At 5,000 Hz it is definitelly heading for the 200g line, but does not get there. If I increase ϕ to 0.2, we will solve the problem. The time history with the 20% cosine ramp is illustrated in Figure 12.

Comparing this to the terminal peak tolerance figure in Reference [13], it is clear that a 20% rise is within the specification. The resulting shock spectra are shown as the black curves in Figure 2 and 3. This is more gradual rise is adequate to permit the 200 Hz asymptote to appear.

d. Composite plots and the similarities of the shock spectra

Now we have completed the examination of the PVSS of the simple pulses: the half sine, the trapezoid, the initial peak and the terminal peak saw tooth. We observed that both damped and undamped, the PVSS's are similar. The reason that we can see that they are similar is that I scaled them to have equal velocity change and peak acceleration. The undamped and 5% damped composite plots are shown in Figures 11 and 12.

This is very interesting and important. After a huge amount of computing and plotting, the shock spectra reveal the similarity. Only in the high frequency region, where the velocity levels are becoming less severe do they diverge. Gertel's [4] off the cuff comment that that all the simple pulses are similar is confirmed if not proven. The terminal peak saw tooth comes down to the 200g asymptote faster than the other three because it has a more gradual rise. The velocity change during the pulse was the similar thing about them. They all required the same drop height so they all have the same low frequency asymptote.

All of the simple pulses developed on a drop table shock machine by a programmer that results in zero velocity when the pulse is over will have a velocity change of the square root of 2gh. They will all have the same drop height or maximum displacement, hence the same low frequency asymptote. Since they all have the same velocity change, they all have the same central region. Since I adjusted the pulses to have the same peak acceleration, they all must have the same high frequency asymptote. The only way their shock spectra can differ are at the two corners, and this can be seen in Figures 2 and 3. I had trouble getting the acceleration asymptotes to appear in the trapezoid and the initial peak saw tooth. These have abrupt rise times that cause an initial doubling of the peak accelerations. I had to decrease the rise time abruptness by using a half cosine ramp rise of 30% in the trapezoid and 20% in the initial peak wave form.

Two other important conclusions have to be established. The half sine, and hence the other simple pulses are related and in a sense similar to the explosive shocks, the drop is very important. Shock simulation on shaker where the drop is not attainable weakens if not ruins the frequency range over which the shock is severe.

SHOCK SPECTRA FROM EXPLOSIVE EVENTS ARE SIMILAR TO SIMPLE PULSE TESTS

One example will be given to explain the similarity. Figure 13 is an acceleration time history of an explosive event and its two integrals. The mean has been removed from the acceleration time history to assure that the velocity ends at zero.

The fact the extreme values are all minima makes no difference. Figure 14 gives its shock spectra for three dampings.

Notice on the left all three curves are asymptotic to just under 9 inches, that the center severe region is hovering near 300 ips, although the total velocity change on Figure 13 is close to 400 ips. In the initial part of the shock there is 300 ips more or less rapid change. At the highest frequencies the damped curves are at least heading for about 900gs. I would have to calculate much higher frequencies to see the asymptote on the undamped spectrum. (To do this, one has to interpolate the time history using MATLABs "Interp" function to increase the sampling rate) The point here, is that a simple pulse with a peak "g" level of 918gs, and a velocity change of 303 ips, would have a shock spectrum matching the severe and high frequency regions. To attain the velocity change with only a 9-inch drop would require a bungee or spring assist, but it could be done, if a "g" level of 13.8gs could be obtained. This would give the 9-inch displacement low frequency asymptote. In this sense I make the statement that the simple pulse tests are similar to explosive shock spectra.

SHAKER GENERATED HALF SINE SHOCK AND REBOUND REDUCE LOW FREQUENCY CONTENT

a. The Half Sine Shock with a Rebound

In the case of the half sine shock, there is often a coefficient of restitution to deal with because it can be formed by impact with a rubber like pad. In examining past data, I have found values from 0.3 to 0.5. In this case the required drop height is reduced because the velocity change is the sum of the falling velocity and the rebound velocity. The velocity change during the half sine shock is still given by Eq. (2), but now the falling and rebound velocity are given in Eq. (6).

$$\begin{aligned} \Delta\dot{x} &= v_f + v_r \\ v_r &= e v_f \\ \Delta\dot{x} &= v_f(1+e) \\ v_f &= \frac{\Delta\dot{x}}{1+e} \end{aligned} \qquad (6)$$

The drop and rebound heights, assuming the table is caught when the velocity goes to zero are given by Eq. (7).

$$\begin{aligned} v_f &= \sqrt{2gh_f} \\ v_r &= \sqrt{2gh_r} \end{aligned} \qquad (7)$$

The time history for our 200g, 100 ips half sine shock impacting with a coefficient of restitution of 0.33, as an example is given in Figure 15. This reduces the drop height from about 13 inches to 7.32 inches.

The undamped and 5% damped shock spectra are given in Figures 16 and 17 as the black curves. The red curves on Figures 18 and 19 are for a half sine shock formed by a programmer with no rebound. The displacement asymptote is now at 7.3 inches, and the low frequency severe velocity range is decreased or the lowest severe velocity increased from about 1.5 Hz to about 3 Hz.

b. Shaker Generated Half Sine Shocks

Half sine shock tests are also conducted on an electrically driven shaker and these have a limited displacement capability. Therefore, the shock spectra of shaker generated shock will reflect this with a reduced low frequency capability of shaker generated shock. Shaker generated shocks will have inadequate low frequency severity.

Lang [15] considers a host of pre and post pulses that allow the shaker armature to start from its center position, perform the half sine shock, and return the armature to its center position. Let's consider a half sine with rectangular pre and post pulses with a magnitude of θ times the maximum acceleration of the half sine to zero the mean. The area of the pre and post pulses must equal the velocity change from Eq. (2), which yields the pre and post pulse duration, t_p in Eq. (8). The time history and its integrals are given in Figure 18.

$$2\theta\ddot{x}_m t_p = \frac{\ddot{x}_m}{\pi f_d} = \frac{2\ddot{x}_m t_d}{\pi}$$

$$t_p = \frac{t_d}{\pi\theta} \tag{8}$$

From the shaker owner's point of view, this is not good. The shaker armature motion is all in one direction, however the peak displacement is only about 0.2 inch, and this limits the low frequency severe portion of its shock spectrum. The undamped and 5% damped shock spectra are shown as the green curves on Figures 16 and 17. The shock now is only severe from about 60 to 200 Hz.

In Figure 7, of Reference [15], Lang shows a rectanguar pre-pre positive pulse of about 1/3 the duration of the pre pulse, to center shaker. Let's say the pre-pre, the pre, the post, and the post post pulses are rectangular and have magnitudes of θ times the maximum acceleration of the half sine. For symmetry make the duration of the pulses t_p, and $t_p/3$. The pre and post pulses must again accomplish the same velocity change as the half sine and t_p is given by Eq. (9). The time history and its two integrals are given in Figure 19.

$$2\theta\ddot{x}_m t_p - \frac{2}{3}\theta\ddot{x}_m t_p = \frac{2\ddot{x}_m t_d}{\pi}$$

$$t_p = \frac{3}{2}\frac{t_d}{\pi\theta} \tag{9}$$

The one-third estimate was not correct since I did not achieve equal positive and negative displacement. It makes the point that we can reduce the shaker excursion; see Reference [15] for the exact result. Since the displacement is further reduced, the shock spectrum must show a low frequency asymptote of 0.11-inch which can be seen in the blue curves on Figures 16 and 17.

Comparing shaker shocks with the drop table shocks, one notes a reduced high velocity severe region. Shaker simulated half sines would be inadequate for machinery and equipment with lower modal frequencies. This is excluding the shocks synthesizing a shock spectrum with a collection of oscillatory motions. The beauty of shaker shock is that the direction of the shock on its polarity, can be reversed.

CONCLUSIONS

The PVSS on 4CP (pseudo velocity shock spectrum plotted on four coordinate paper) emphasizes a velocity change section of the spectrum, mentioned by Roberts [16] as severe section. The IEC Specification [14] calls the simple pulse velocity change the severity of the pulse, which I completely agree with. This view of the spectrum was the basis of equipment installation design without actually calling it the velocity change region. [1, 3]

The shock spectra of the simple pulses are similar and have similar damage potential. Reference [4] is the only reference I have found that actually makes the statement: (page 26)"...the maximax of the shock spectrum curve of all simple pulses have the same general shape." They are right. They make it so casually that one might assume it was common knowledge. Most authors [17, 18] emphasize the unimportant high frequency differences of the undamped spectra because they plot acceleration shock spectra.

The low frequency content inadequacy of shaker generated simple pulse shock tests for equipment fragility testing makes clear the need to include the drop height in half-sine shock calculations.

The general similarity of the PVSS on 4CP of the simple shock pulse tests and explosive tests argues for their usefulness in general fragility testing.

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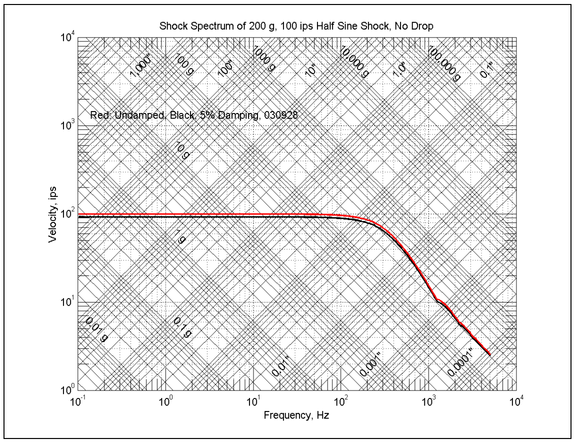

Figure 1. The PVSS os a 200-g, 100 ips half sine aceleration shock. In the lower right corner the spectra are asymptotic to 200gs. From a little under 200 Hz down to 0.1 Hz the spectra show a constant velocity of 100 ips. That proably can not be.

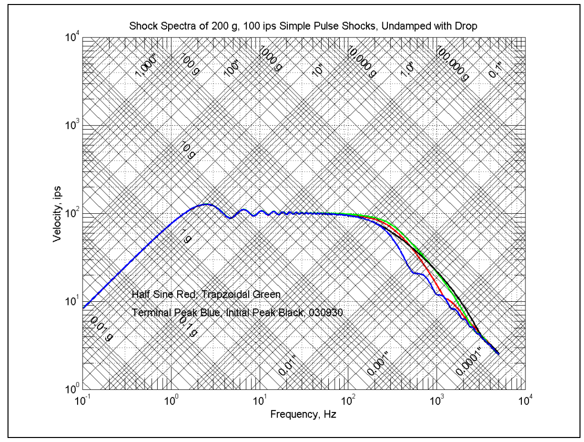

Figure 2. Composite undamped shock spectrum plots of the half sine, the trapezoid, the initial peak and the terminal peak saw tooth shocks.

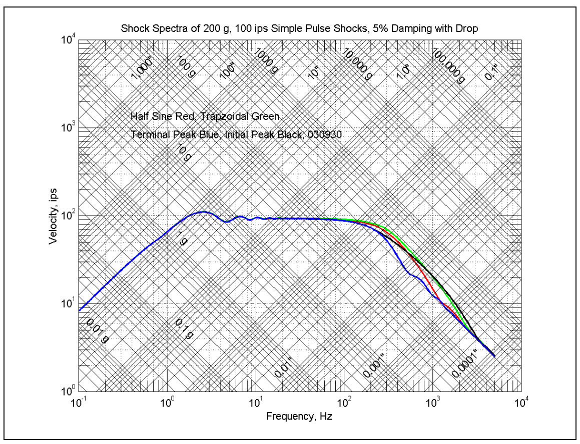

Figure 3. Composite 5% damped shock spectrum plots of the half sine, the trapezoid, the initial peak and the terminal peak saw tooth shocks.

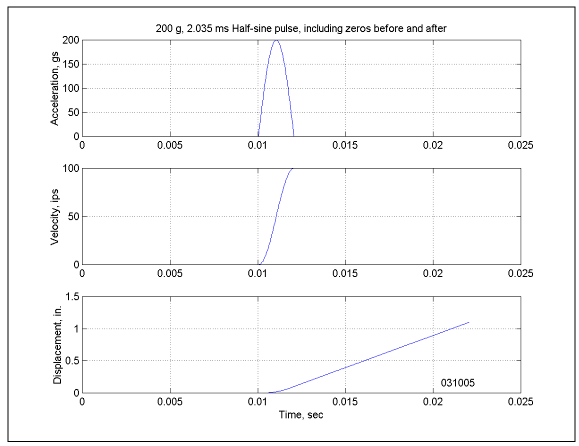

Figure 4. Acceleration, velocity, and displacement of a half sine acceleration shock preceded and followed by zeros. The shock has a velocity change of 100 ips; and continues at this velocity forever. This is an unrealizable shock; but the shock spectrum for it undamped and with damping of 5 %, is given in Figure 2.

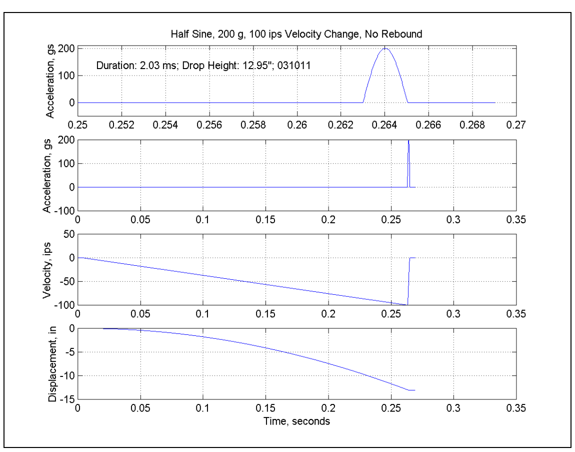

Figure 5. A 200-g, 100 ips half sine shock preceded by a 12.95-inch drop, and its two integrals.

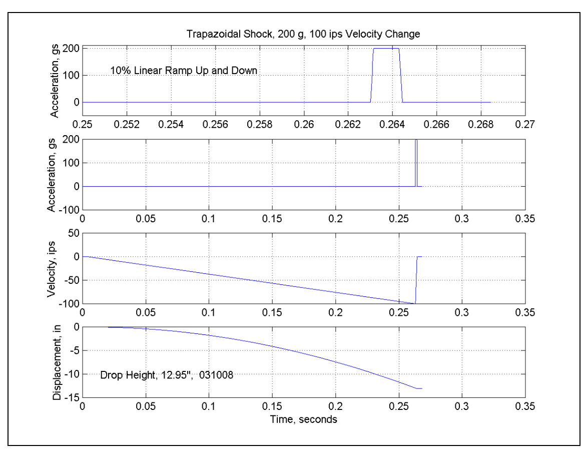

Figure 6. Ten percent linear ramp trapazoidal shock preceded by a 12.95-inch drop.. The pulse duration turned out to be 1.44 mili-seconds.

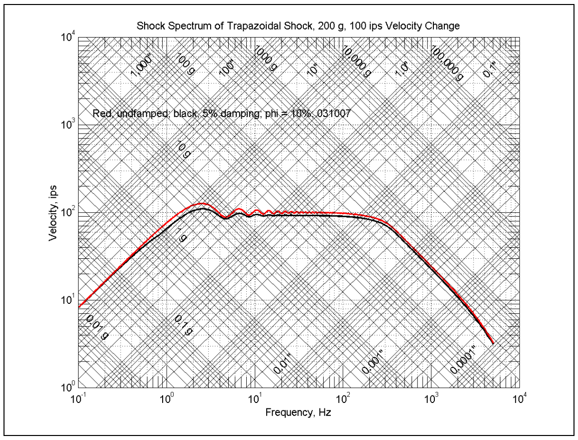

Figure 7. The 0 and 5 % damped shock spectra for the 10 percent linear ramp trapazoidal shock of Figure 4.

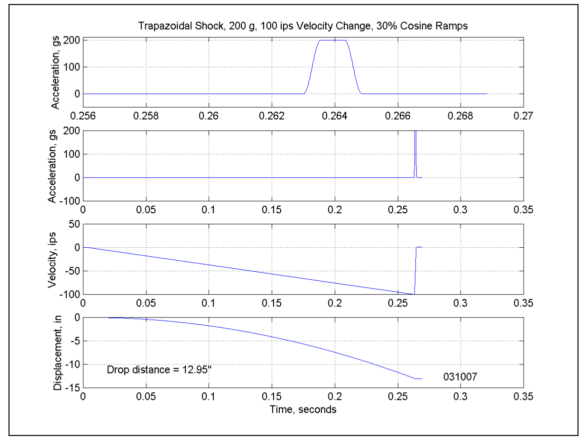

Figure 8. Trapezoidal shock with 30% cosine ramp preceded by a 12.95-inch drop. The time duration for this pulse came out to be 1.85 mili-seconds.

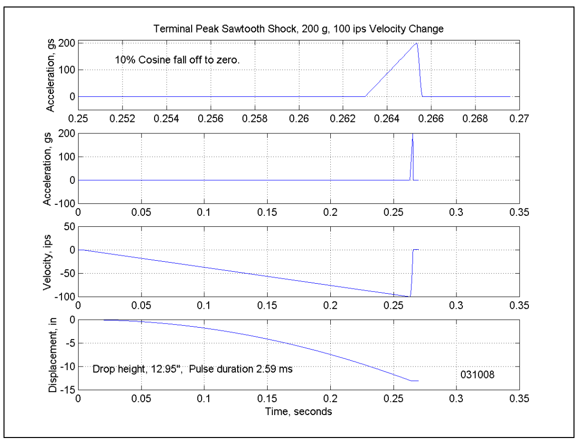

Figure 9. The time history of a 200-g, 100 ips, terminal peak saw tooth shock with a 10% cosine drop back to zero, and its itegrals to velocity and displacement. Drop height is 12.95 inches, and pulse duration is 2.59 ms.

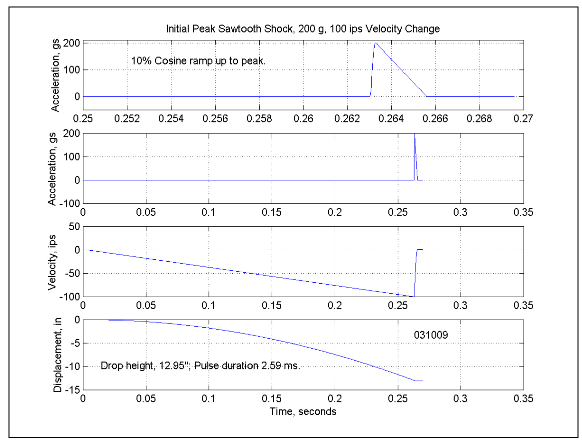

Figure 10. The time history of a 200g, 100 ips initial peak saw tooth shock with a 10% cosine ramp up to tha maximum acceleration, and its itegrals to velocity and displacement. Drop height is 12.95 inches, and pulse duration is 2.59 ms.

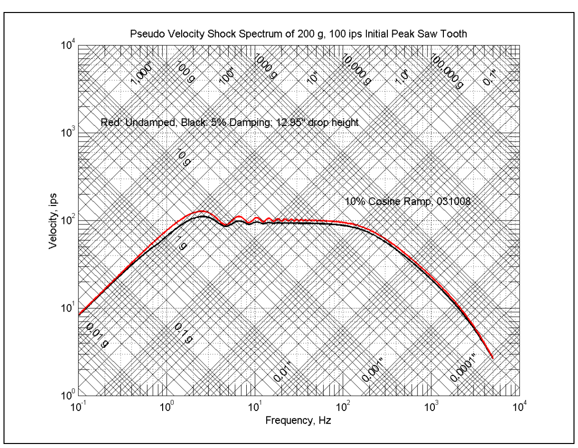

Figure 11. The shock spectrum of a 200-g, 100 ips initial peak saw tooth shock with a 10% cosine ramp up to the maximum acceleration. Drop height is 12.95 inches, and pulse duration is 2.59 ms.

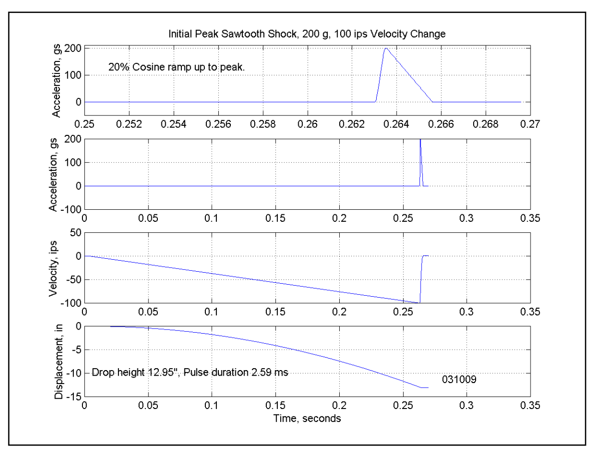

Figure 12. The time history of a 200-g, 100 ips initial peak saw tooth shock with a 20% cosine ramp up to the maximum acceleration, and its itegrals to velocity and displacement. Drop height is 12.95 inches, and pulse duration is 2.59 ms.

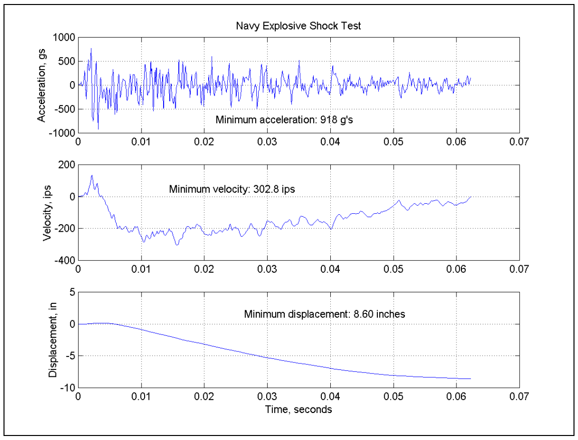

Figure 13. The acceleration time history and its two integrals for an example explosive shock.

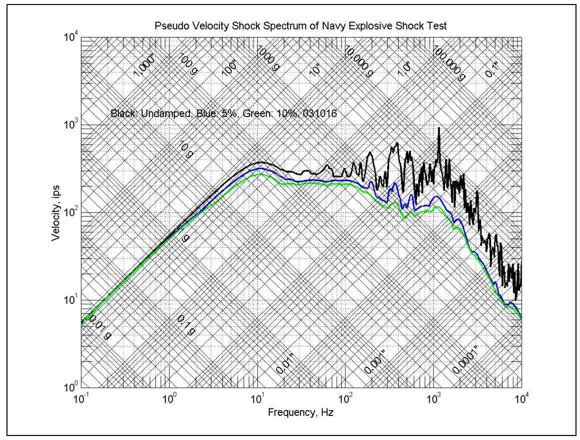

Figure 14. Shock spectra of an example explosive shock test.

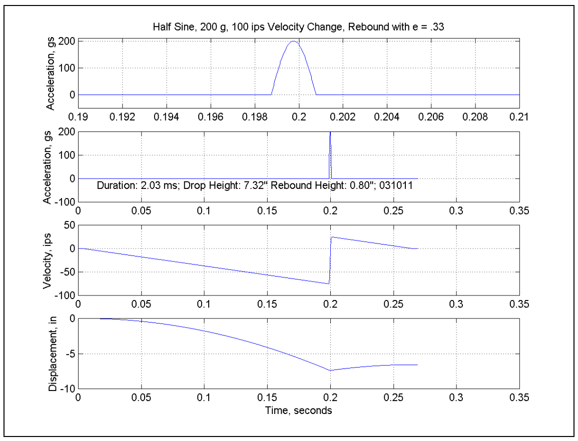

Figure 15. The acceleration time history and its two integrals for a 200g, 100 ips half sine shock rebounding with a coefficient of restitution of 0.33. The crop and rebound heights are 7.32 inches and 0.8 inch, respectively.

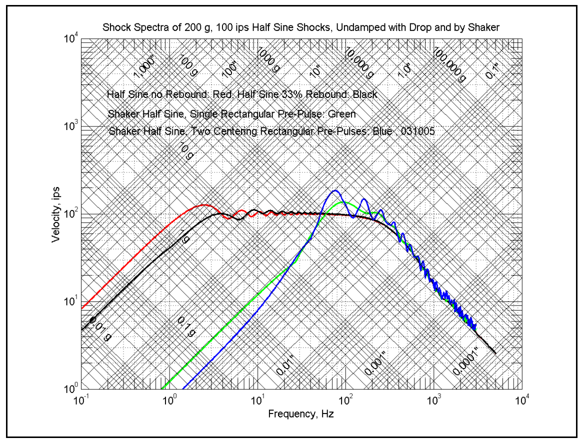

Figure 16. Shock spectrum comparison of shaker and drop table 200g, 100 ips, half sine shocks for the undamped case:

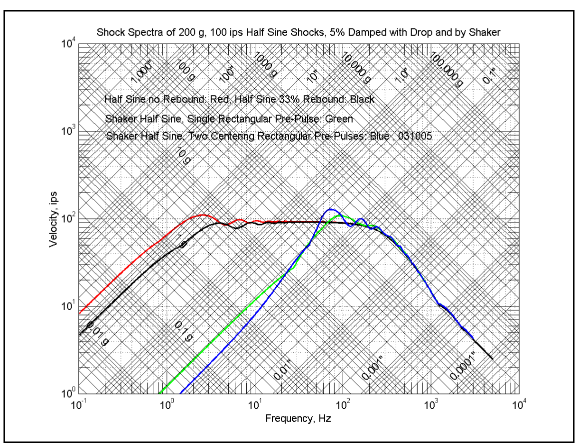

Figure 17. Shock spectrum comparison of shaker and drop table 200g, 100 ips, half sine shocks for the 5% damped case.

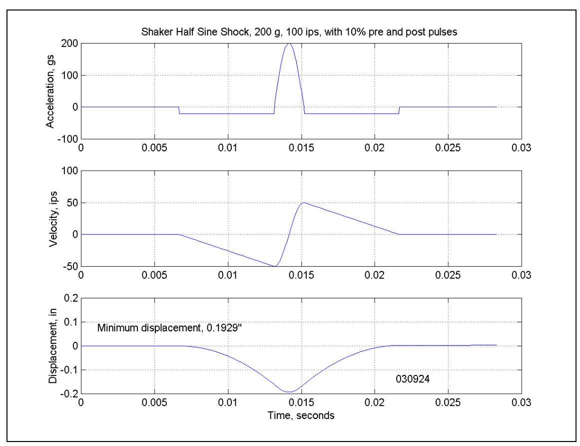

Figure 18. Acceleration time history with 10% rectangular pre and post pulses to accomplish the 200g, 100 ips half sine shock on a shaker.

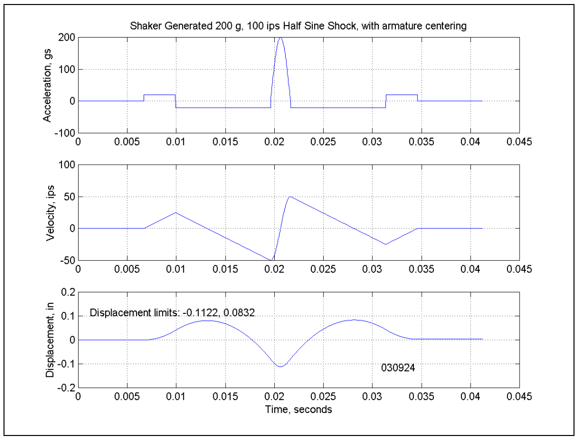

Figure 19. Acceleration time history with two 10% rectangular pre and post pulses to accomplish a 200g, 100 ips half sine shock on a shaker, and more nearly have equal positive and negative shaker displacement.