Chapter 8

Potential Energy and Energy Conservation

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Learning Goals for Chapter 8

- Learn the following concepts: conservative forces, potential energy, and conservation of mechanical energy.
- Learn when and how to apply conservation of mechanical energy to solve physics problems.
- Examine situations when mechanical energy is not conserved, that is, when there are non-conservative forces such as kinetic friction and air resistance.
- Learn to relate the conservative force to the potential energy curve.

Introduction/Summary

- Start with W-KE theorem:

$$K_f - K_i = W = \int_i^f \vec{F} \bullet d\vec{\ell}$$

- There are two type of forces:

Conservative forces (such as gravity and spring force)

Non-conservative forces (such as kinetic friction and air resistance)

$$\Rightarrow K_f - K_i = W_{conservative} + W_{non-conservative}$$

- If there are only conservative forces then "mechanical energy" is conserved. Mechanical energy=kinetic energy (K) + potential energy (U)

$$K_f - K_i = W_{conservative} = U_i - U_f \Rightarrow (K_f + U_f) - (K_i + U_i) = 0$$

- If there are also non-conservative forces then "mechanical energy" is NOT conserved, but total energy is still conserved

$$\Rightarrow (K_f + U_f) - (K_i + U_i) = W_{non-conservative};$$

$$(W_{non-conservative}) = \text{other form of energy, e.g. "heat"}$$

Work done by a conservative force - e.g. gravity

- Work done by gravity:

$$W = \int_{y_i}^{y_f} F\,dy = \int_{y_i}^{y_f} (-mg)\,dy = mgy_i - mgy_f$$

Define gravitational potential energy:

$$\mathrm{U(y)} \equiv mgy$$

$$\Rightarrow W = mgy_i - mgy_f = U_i - U_f$$

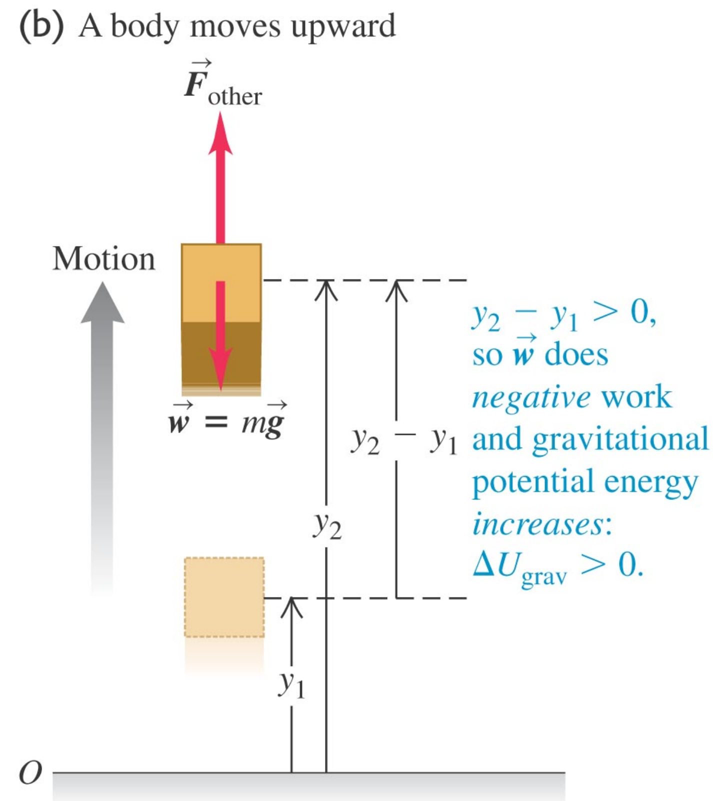
If only gravity is doing work, then by combining with the W - KE Theorem

$$K_f - K_i = W = U_i - U_f$$

$$\Rightarrow K_f + U_f = K_i + U_i$$

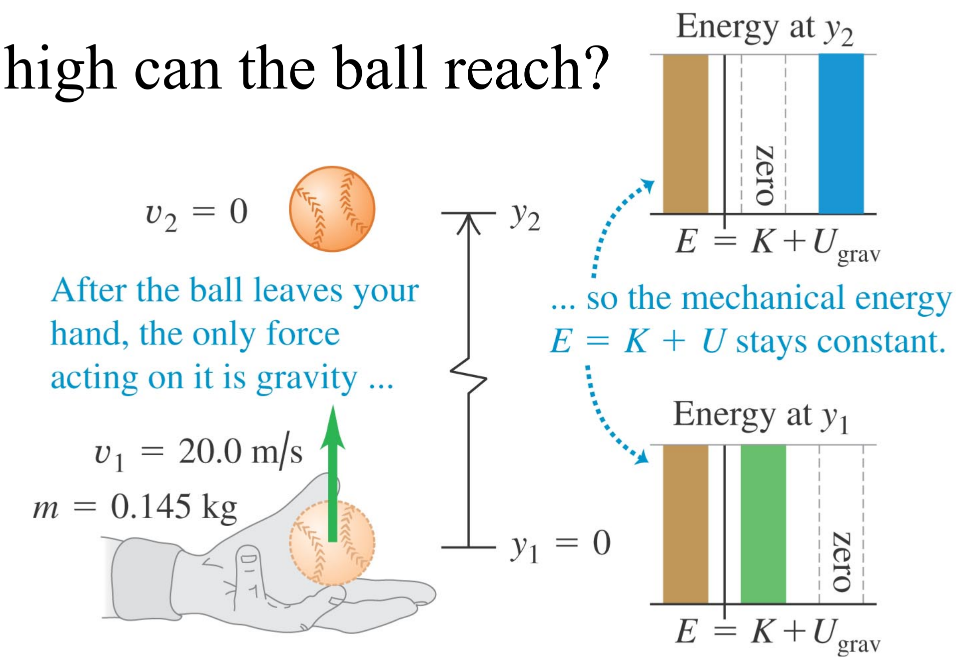
$\Rightarrow$"Conservation of mechanical energy"

Mechanical energy $\equiv$ K + U



Apply conservation of mechanical energy - Example 7.1

- Assume there is no air resistance. After the ball leave your hand, the only force acting on the ball is gravity.
- How high can the ball reach?



$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$$

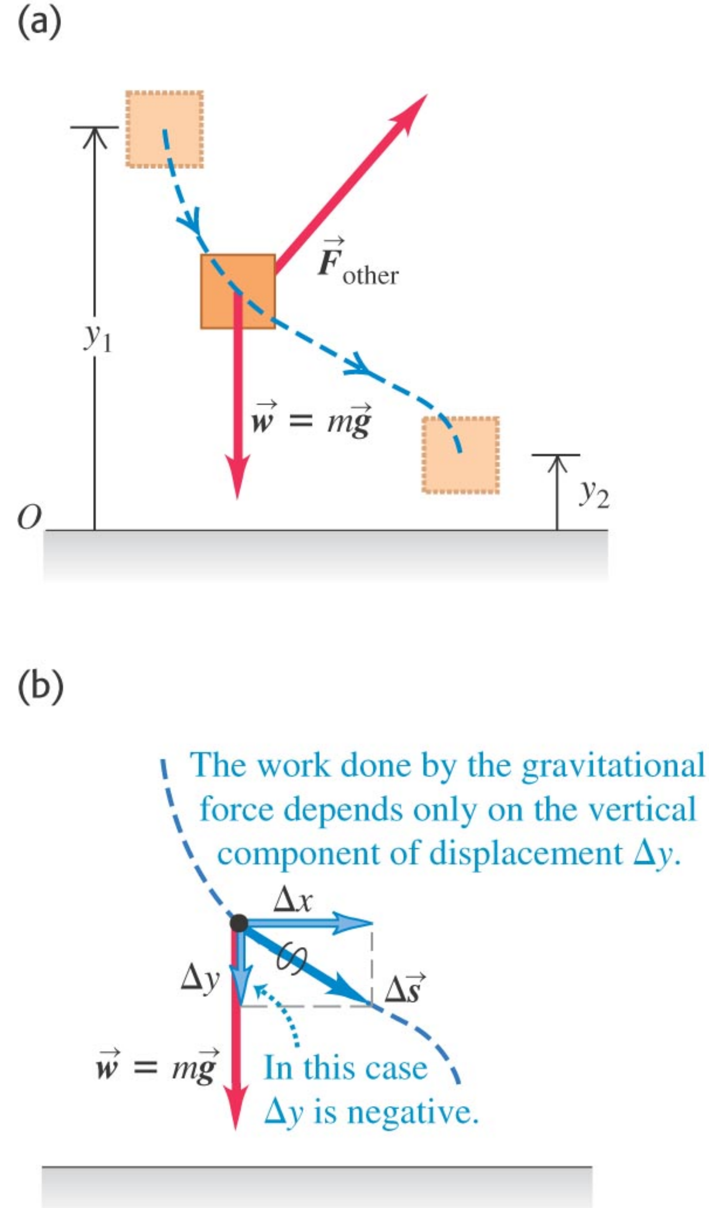
With the information given, find y_2.

Property of conservative force

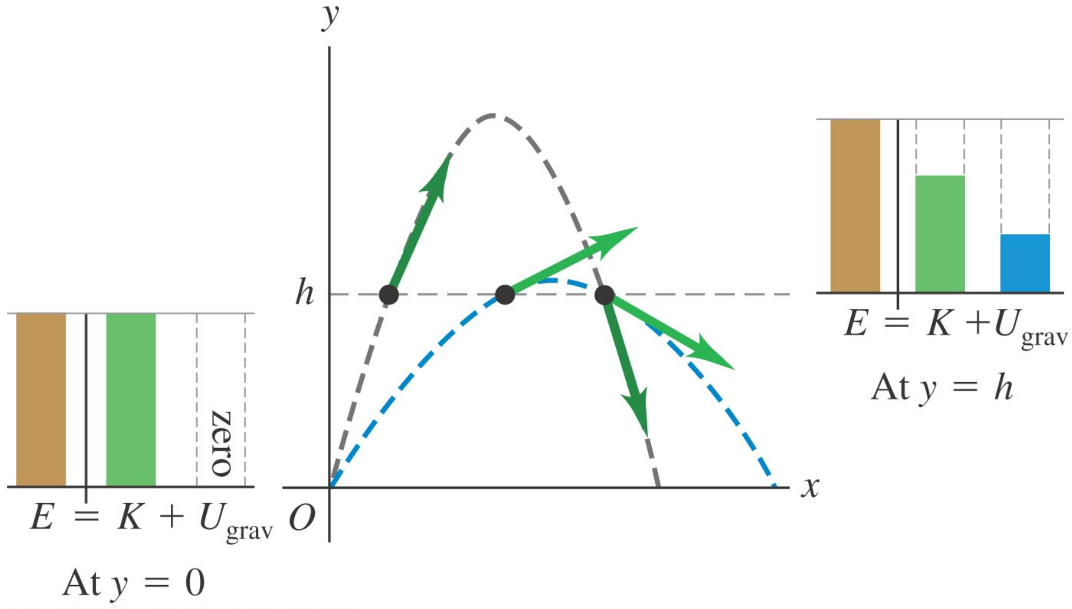
- **Work done by conservative force depends only on the initial and final position, independent of the path.**

$$W = U_i - U_f \quad \text{(independent of path)}$$

$$\text{e.g. } W = mgy_i - mgy_f$$



Consider projectile motion

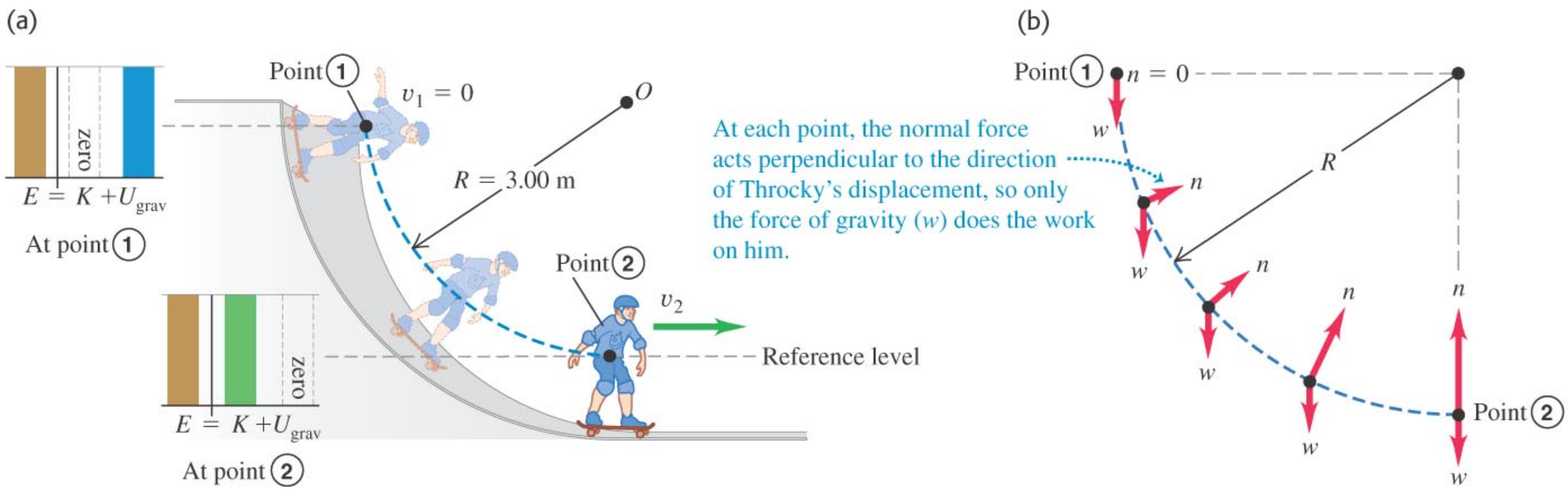


- If the initial SPEED and initial height are the same, then the SPEED at a given height will be the same even though they have different path and the velocity vectors point at different directions.

Application: What's the speed in a vertical circle?

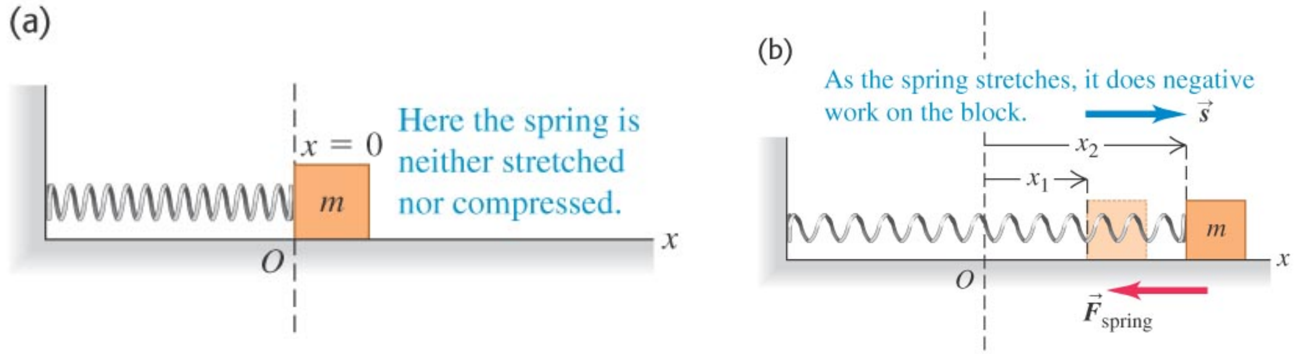
Given: R=3 m, find the skater's speed at the bottom

(assume no friction or air-resistance)



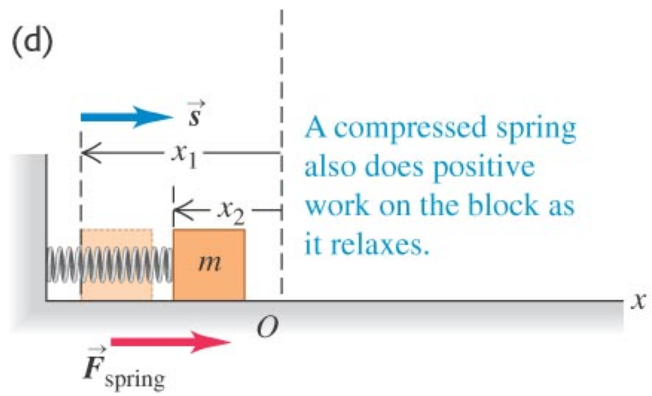
What about the "normal" force, does it do any work on the skater?

Another conservative force - idealize spring



$$W = \int_{x_i}^{x_f} -kx\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = U_i - U_f$$

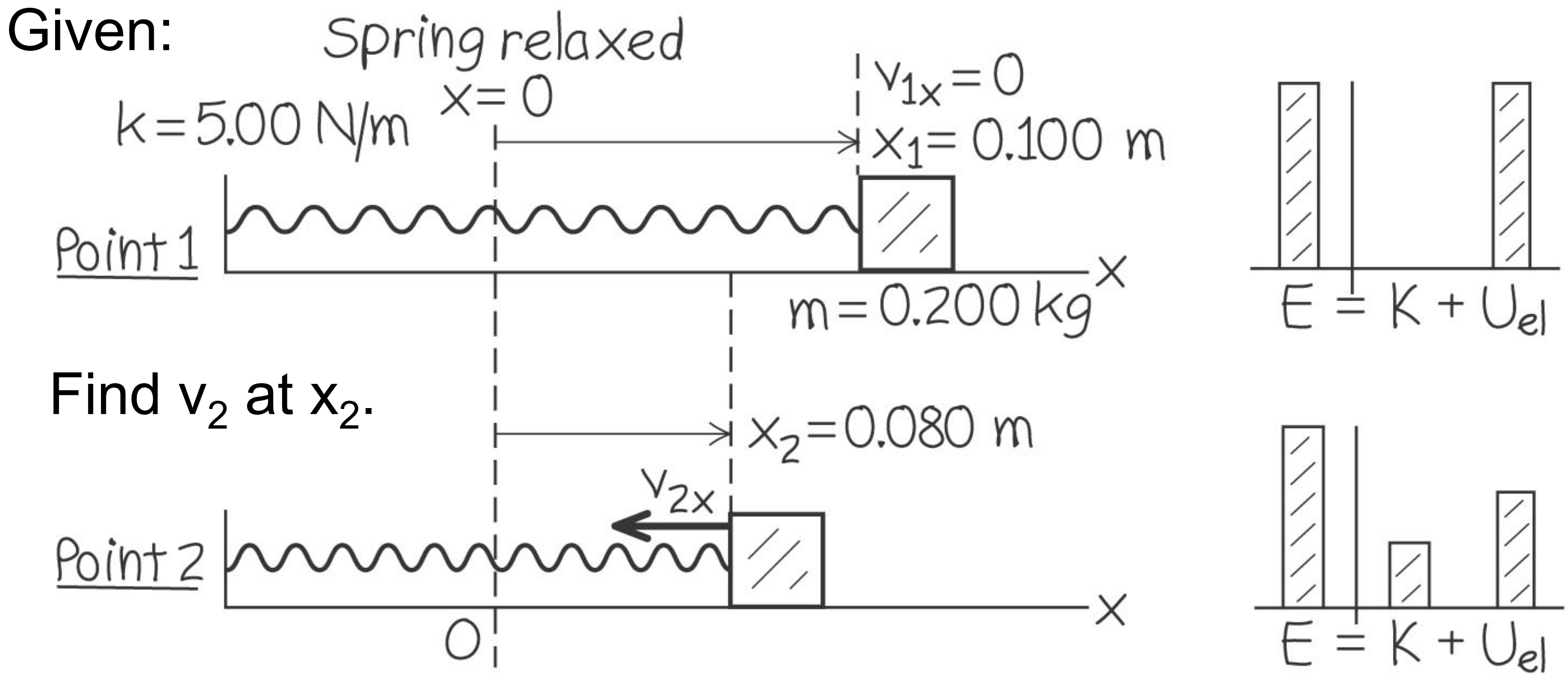
$$\Rightarrow \text{Spring potential energy U(x)} = \frac{1}{2}kx^2$$



Also true for compression

Example: Conservation of mechanical energy with spring

See figure below, find the speed at x_2.



$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2;$$ Subtitue values and find v_2.

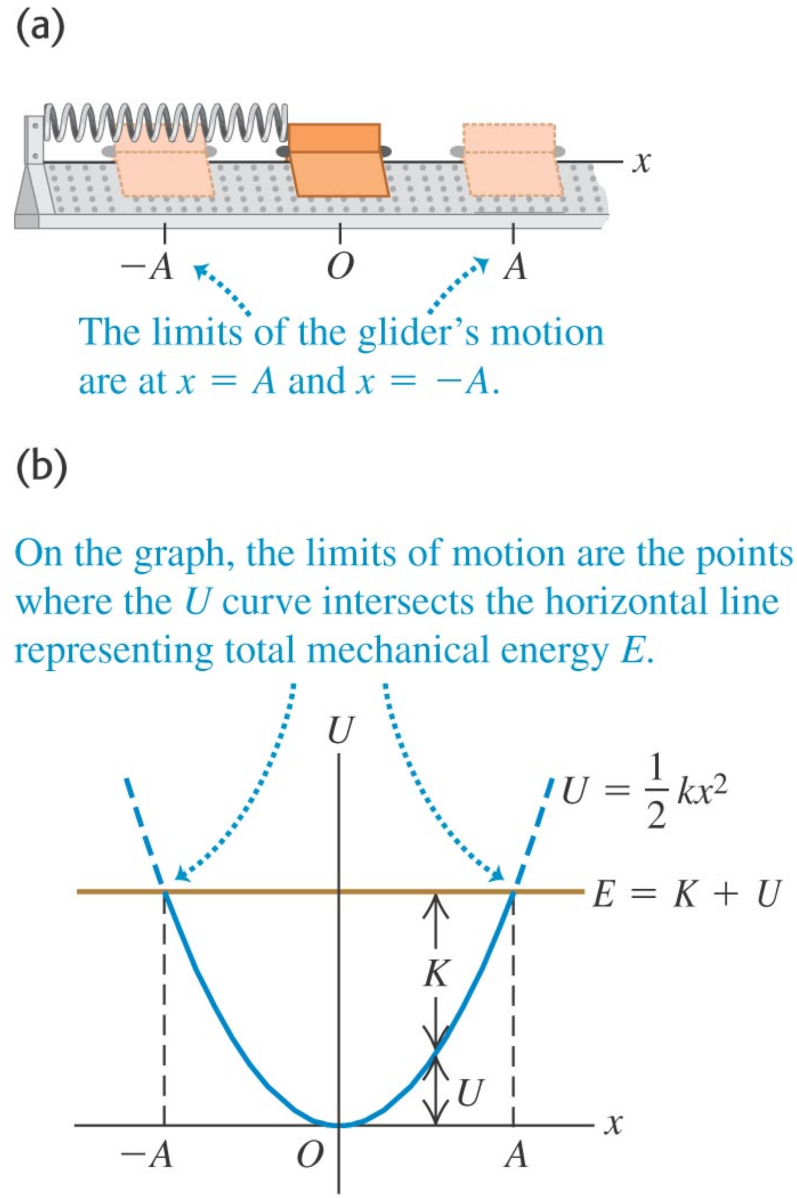
Visualize KE and PE in a potential energy graph

Suppose the glider is initially stretched to position A.

As the glider slides back and forth, potential energy is converted to kinetic energy and back to potential energy and so forth.

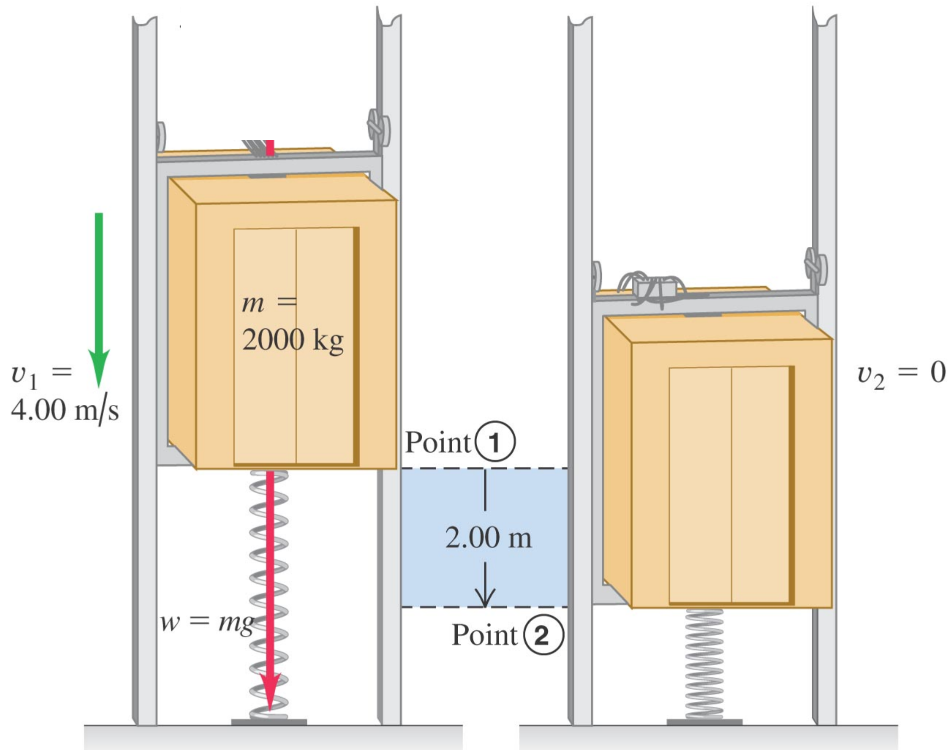
K+U=constant can be visualized in the potential energy graph.

When does the glider speed up? When does it slow down?



Combine gravity and spring

- Elevator's cable broke.
- Given: v_1=4 m/s; spring is compressed 2 m. Find the spring constant.



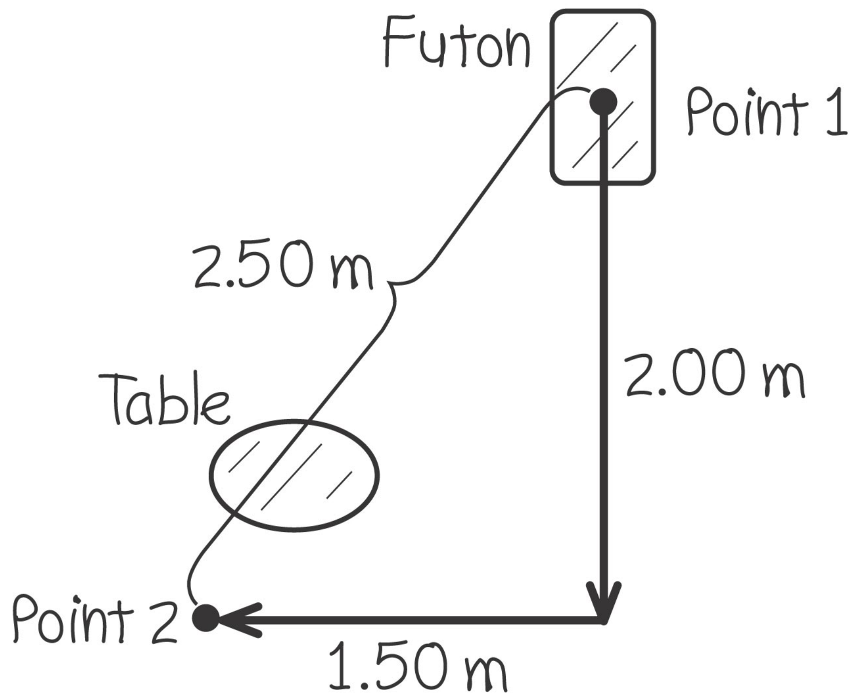
$$\frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}ky_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}ky_2^2$$

What are the appropriate values for y_1 and y_2? Solve for k.

Non-conservative force - e.g. kinetic friction

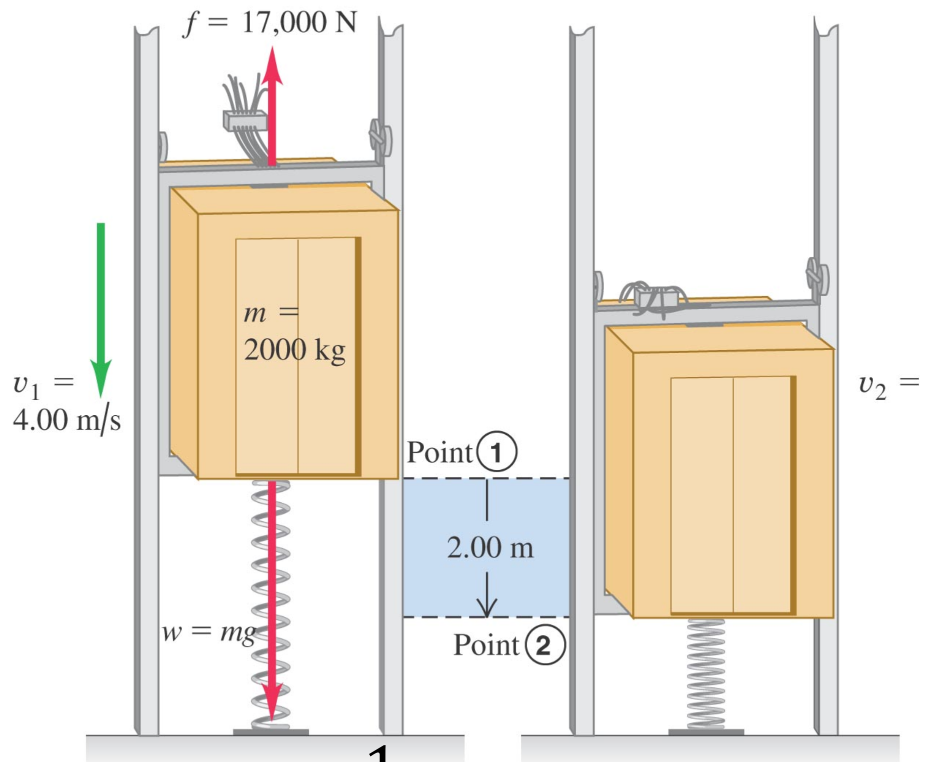
- Work done by the non-conservative force going from point 1 to point 2 depends on the path, hence

$$W_{non-conservative} \neq U_i - U_f$$



Combine gravity, spring force and friction

- DO NOT neglect friction (f), find the spring constant.



$$\left(\frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}ky_2^2\right) - \left(\frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}ky_1^2\right) = W_{Friction}$$

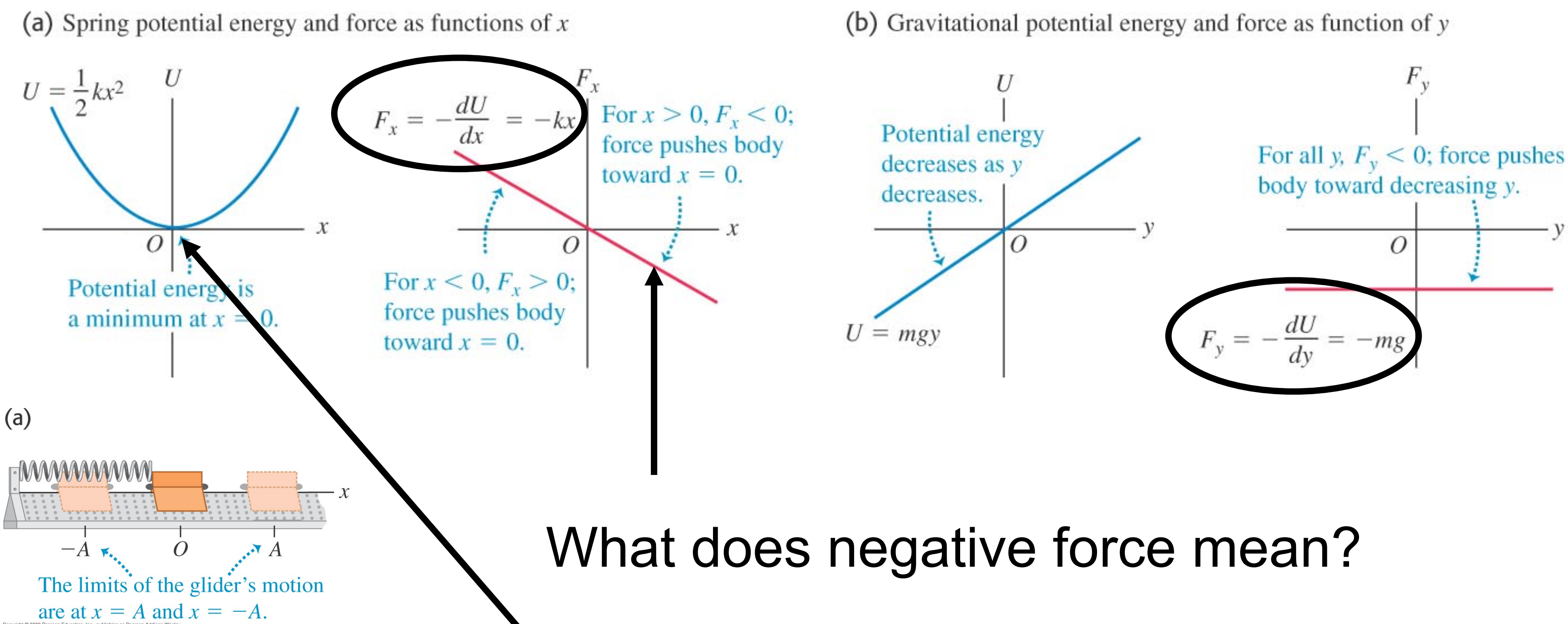
$$Given: W_{Friction} = -fs = -(17{,}000)(2)\,Joule$$

$$y_1 = 0, \quad y_2 = -2m$$

Solve for k.

Conservative Force as the derivative of potential energy

- Refer to Figure 7.22 as it examines spring and gravitational energies.



What does negative force mean?

Stable equilibrium position

The potential energy curve for motion of a particle

- Refer the potential energy function and its corresponding components of force.