INTRODUCTION TO TRANSMISSION LINES

PART II

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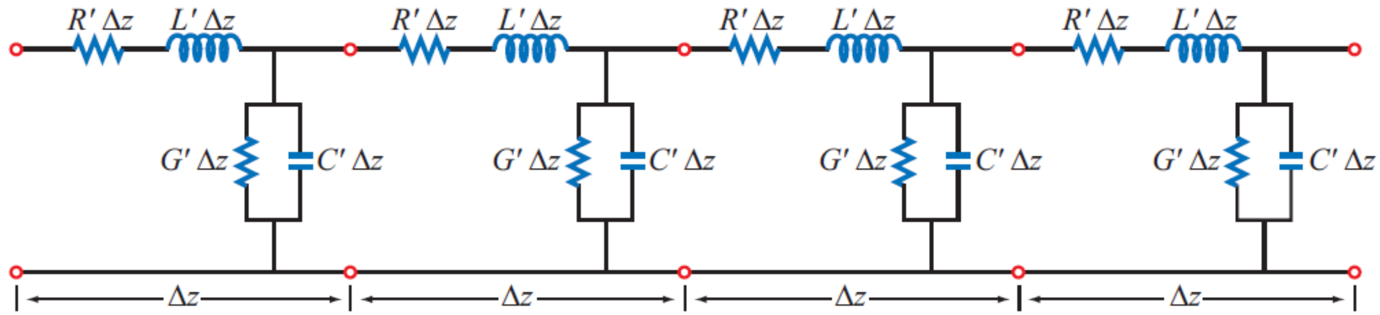
Transmission Line Model

(a) Parallel-wire representation

(b) Differential sections each Δz long

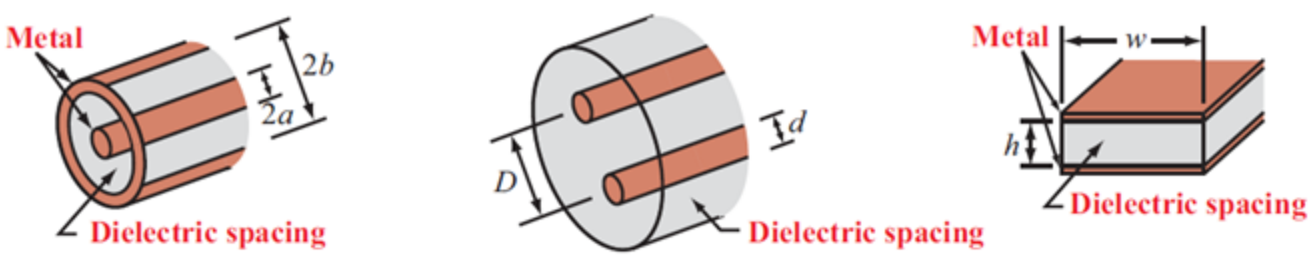


(c) Each section is represented by an equivalent circuit

Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

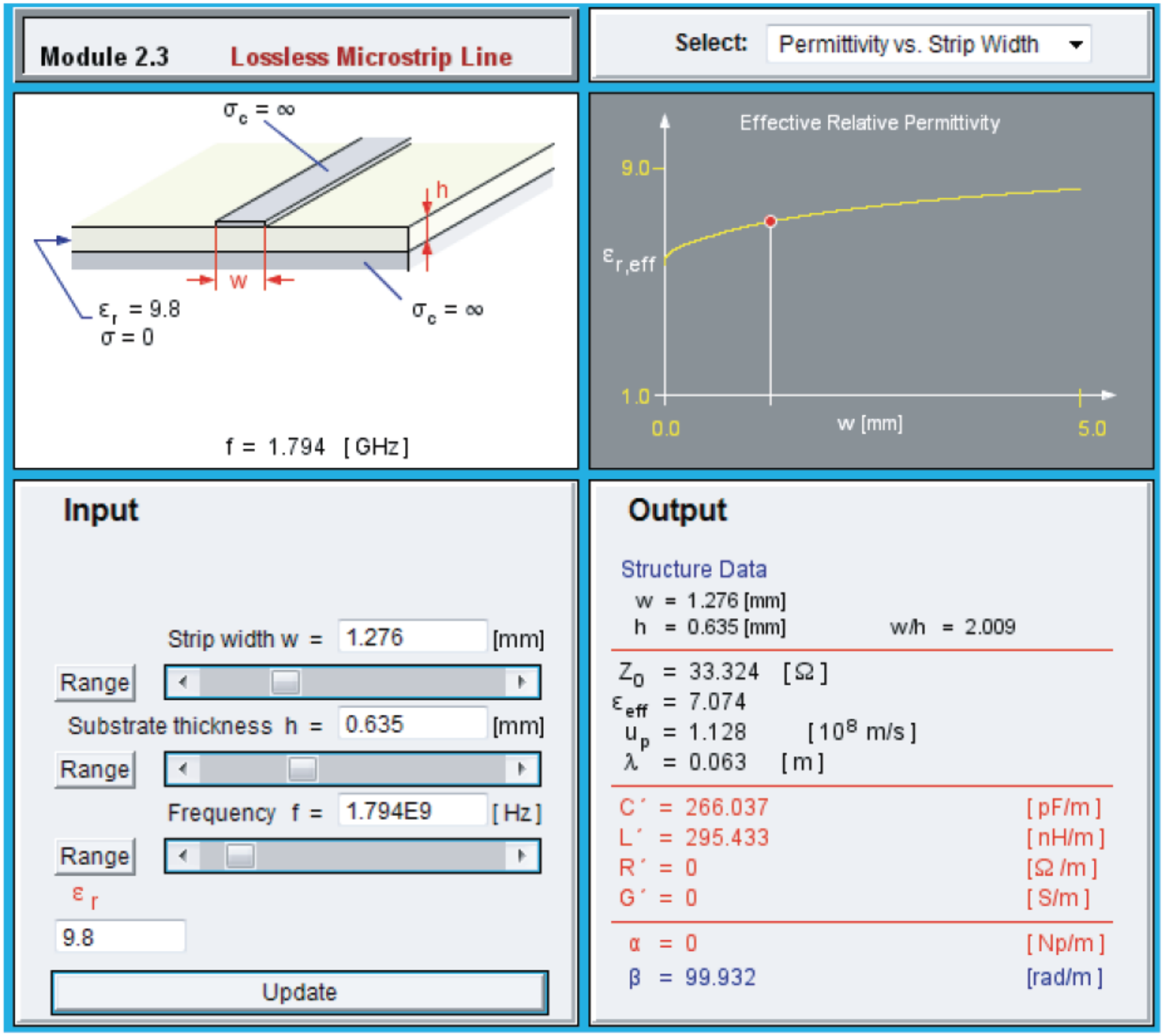
Notes: . (2) μ, ε, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right] \simeq \ln(2D/d)$.

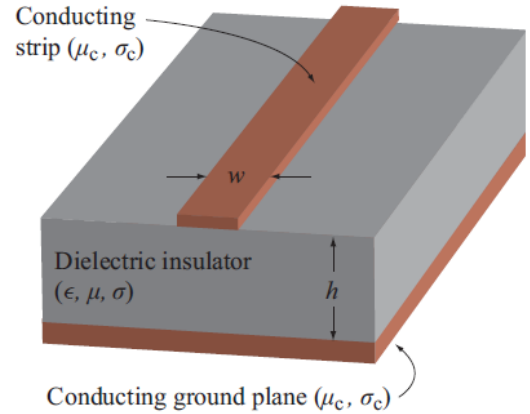


(a) Coaxial line (b) Two-wire line (c) Parallel-plate line

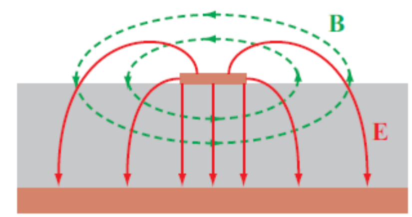
Perfect Conductor and Perfect Dielectric (notes)

Simulation Example

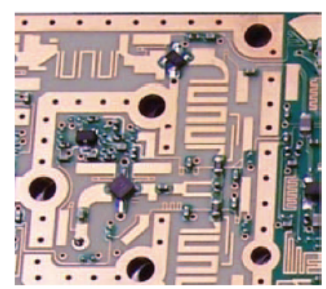




(a) Longitudinal view

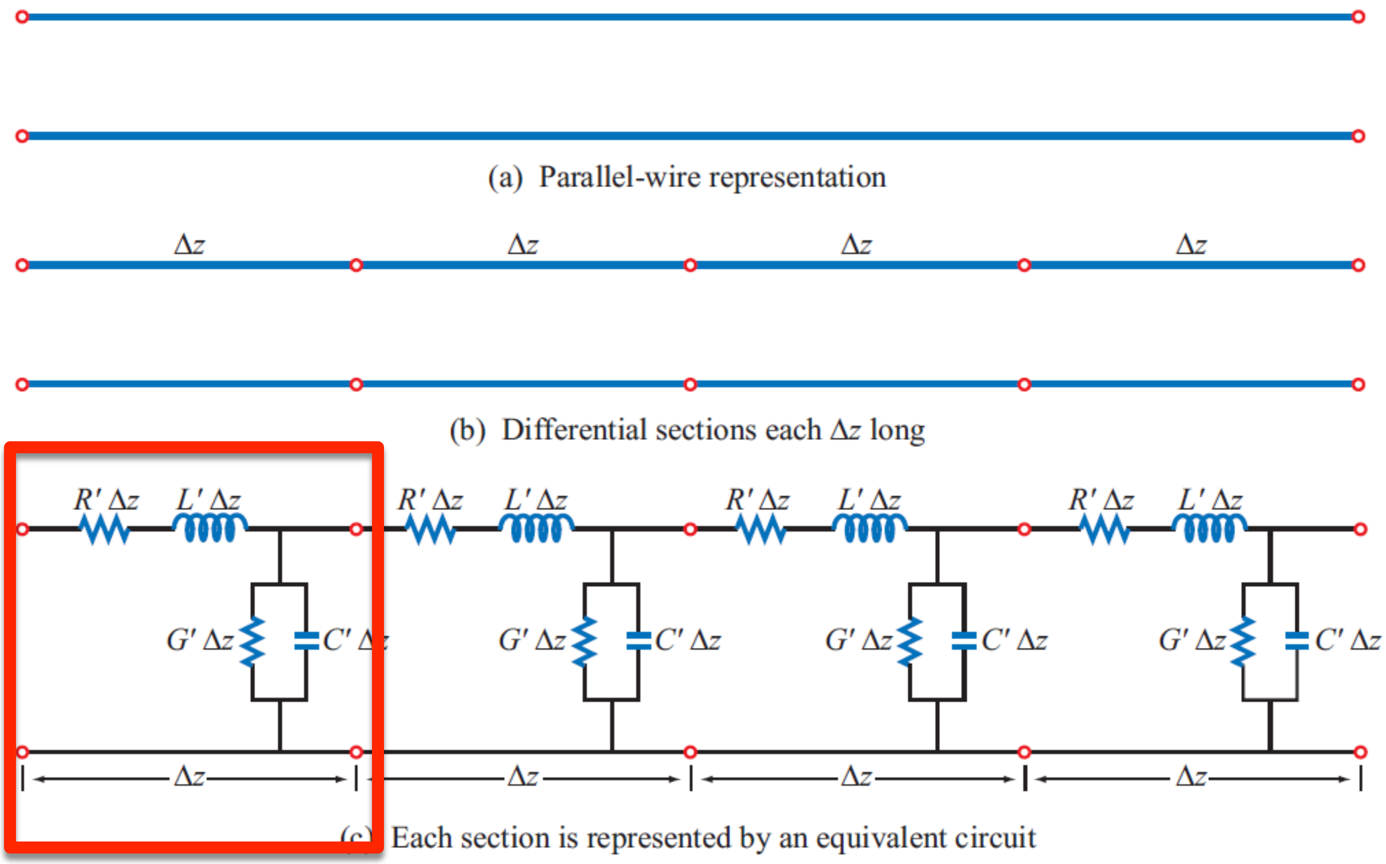


(b) Cross-sectional view with **E** and **B** field lines

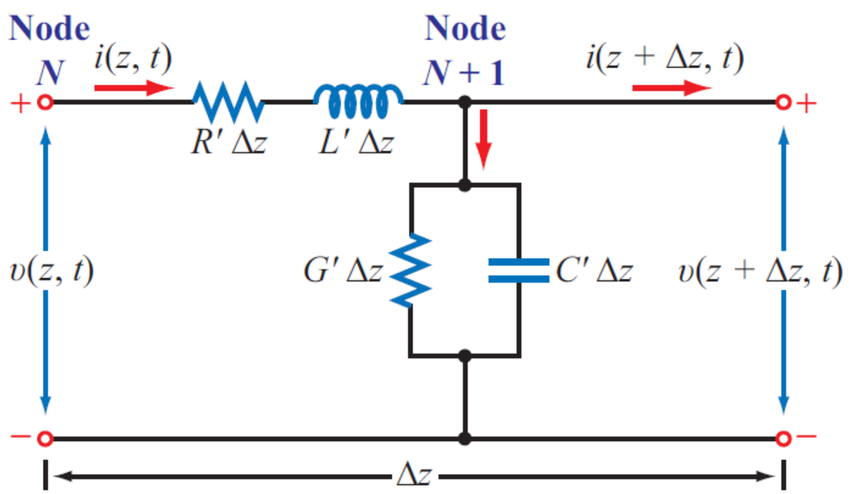


(c) Microwave circuit

Transmission Line Model



Transmission-Line Equations



$$Ae^{j\theta} = A\cos(\theta) + Aj\sin(\theta)$$

$$\cos(\theta) = A\,\mathrm{Re}[Ae^{j\theta}]$$

$$\sin(\theta) = A\,\mathrm{Im}[Ae^{j\theta}]$$

$$E(z) = |E(z)|\,e^{j\theta_z}$$

$$|e^{j\theta}| = 1$$

$$C = A + jB \rightarrow \theta = \tan\frac{B}{A};|C| = \sqrt{A^2 + B^2}$$

Remember:

Kirchhoff Voltage Law:

Vin-Vout – VR’ – VL’ =0

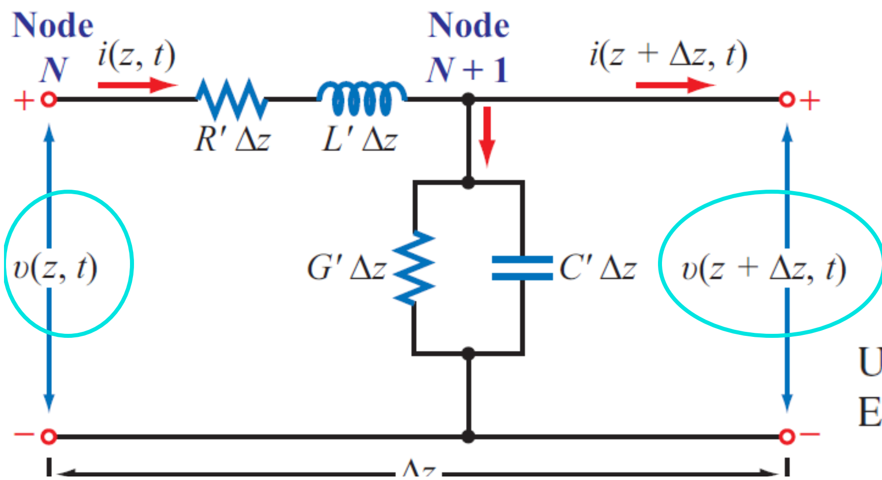
Kirchhoff Current Law:

Iin – Iout – Ic’ – IG’ =0

Note:

VL=L . di/dt
Ic=C . dv/dt

Transmission-Line Equations



$$i(z,t) - G'\Delta z\, \upsilon(z+\Delta z,t) - C'\Delta z \frac{\partial \upsilon(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0.$$

Upon dividing all terms by Δz and taking the limit $\Delta z \to 0$, Eq. (2.15) becomes a second-order differential equation:

$$-\frac{\partial i(z,t)}{\partial z} = G'\, \upsilon(z,t) + C'\frac{\partial \upsilon(z,t)}{\partial t}$$

$$\upsilon(z,t) - R'\Delta z\, i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - \upsilon(z+\Delta z,t) = 0.$$

Upon dividing all terms by Δz and rearranging them, we obtain

$$-\left[\frac{\upsilon(z+\Delta z,t) - \upsilon(z,t)}{\Delta z}\right] = R' i(z,t) + L'\frac{\partial i(z,t)}{\partial t}.$$

In the limit as $\Delta z \to 0$, Eq. (2.13) becomes a differential equation:

$$-\frac{\partial \upsilon(z,t)}{\partial z} = R'\, i(z,t) + L'\frac{\partial i(z,t)}{\partial t}$$

ac signals: use phasors

$$\upsilon(z,t) = \mathfrak{Re}[\widetilde{V}(z)\, e^{j\omega t}],$$

$$i(z,t) = \mathfrak{Re}[\widetilde{I}(z)\, e^{j\omega t}],$$

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\, \widetilde{I}(z),$$

$$-\frac{d\widetilde{I}(z)}{dz} = (G' + j\omega C')\, \widetilde{V}(z).$$

Transmission Line Equation in Phasor Form

Derivation of Wave Equations

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z),$$
$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\,\tilde{V}(z).$$

Transmission Line Equation
First Order Coupled Equations!
WE WANT UNCOUPLED FORM!

Combining the two equations leads to:

$$\frac{d^2\tilde{V}(z)}{dz^2} - (R' + j\omega L')(G' + j\omega C')\,\tilde{V}(z) = 0,$$

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\,\tilde{V}(z) = 0$$

Second-order differential equation

Wave Equations for Transmission Line

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

Impedance and Shunt Admittance of the line

complex propagation constant

attenuation constant (Neper/m)

Phase constant

$$\gamma = \alpha + j\beta$$

$$\alpha = \mathfrak{Re}(\gamma) = \mathfrak{Re}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(Np/m)},$$

$$\beta = \mathfrak{Im}(\gamma) = \mathfrak{Im}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(rad/m)}.$$

Pay Attention to UNITS!

Solution of Wave Equations (cont.)

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2 \tilde{V}(z) = 0,$$

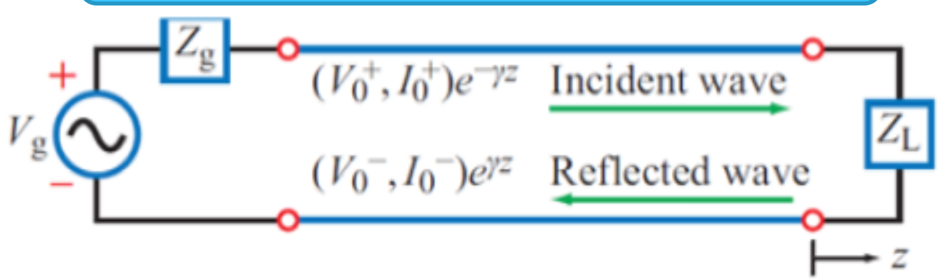
$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2 \tilde{I}(z) = 0.$$

Proposed form of solution:

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}.$$

So What does V+ and V- Represent?



Characteristic Impedance of the Line (ohm)

Note that Zo is NOT V(z)/I(z)

$$\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-},$$

Using:

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z),$$

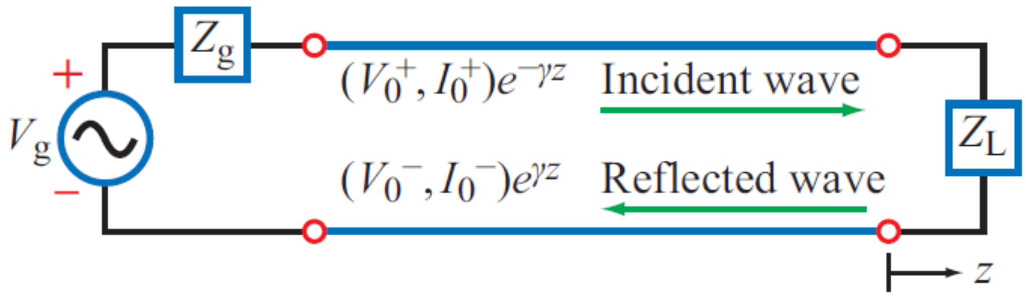
It follows that:

$$\tilde{I}(z) = \frac{\gamma}{R' + j\omega L'}\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right]$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \qquad (\Omega),$$

Pay att. To Direction

Solution of Wave Equations (cont.)



The presence of two waves on the line propagating in opposite directions produces a ***standing wave****.*

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad \text{(V)},$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad \text{(A)}.$$

So, V(z) and I(z) have two parts:

Applet for standing wave:
http://www.physics.smu.edu/~olness/www/05fall1320/applet/pipe-waves.html

Example: Air-Line

Assume the following waves:

$$V(z,t) = 10\cos(2\pi \cdot 700 \cdot 10^6 - 20z + 5)$$

$$I(z,t) = 0.2\cos(2\pi \cdot 700 \cdot 10^6 - 20z + 5)$$

Assume having perfect dielectric insulator and the wire have perfect conductivity with no loss

Perfect Conductor→ Rs=0→R' = 0
Perfect Dielec → COND=0 → G'=0

Draw the transmission line model and Find C' and L'; Assume perfect conductor and perfect dielectric materials are used!

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad (\Omega),$$

$$\beta = \mathfrak{Im}(\gamma) = \mathfrak{Im}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(rad/m)}.$$

Note: If atten. Is zero →real part MUST be zero!

With $R' = G' = 0$,

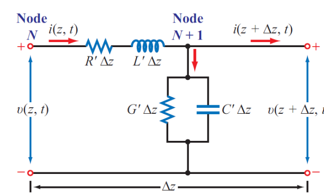
$$\beta = \mathfrak{Im}\left[\sqrt{(j\omega L')(j\omega C')}\right] = \mathfrak{Im}\left(j\omega\sqrt{L'C'}\right) = \omega\sqrt{L'C'},$$

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}}.$$

The ratio of β to Z_0 is

$$\frac{\beta}{Z_0} = \omega C',$$

or



$$C' = \frac{\beta}{\omega Z_0} = \frac{20}{2\pi \times 7 \times 10^8 \times 50} = 9.09 \times 10^{-11} \text{ (F/m)} = 90.9 \text{ (pF/m)}.$$

From $Z_0 = \sqrt{L'/C'}$, it follows that

$$L' = Z_0^2 C' = (50)^2 \times 90.9 \times 10^{-12} = 2.27 \times 10^{-7} \text{ (H/m)} = 227 \text{ (nH/m)}.$$

Transmission Line Characteristics

- Line characterization
  - Propagation Constant (function of frequency)
  - Impedance (function of frequency)
    - Lossy or Losless
- If lossless (low ohmic losses)
  - Very high conductivity for the insulator
  - Negligible conductivity for the dielectric

Lossless Transmission Line

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,.$$

If $R' \ll \omega L'$ and $G' \ll \omega C'$

Then:

$$\gamma = \alpha + j\beta = j\omega\sqrt{L'C'}\,,$$

$$\alpha = 0 \qquad \text{(lossless line)},$$
$$\beta = \omega\sqrt{L'C'} \qquad \text{(lossless line)}.$$

What is Zo? $Z_0 = \sqrt{\dfrac{(R' + j\omega L')}{(G' + j\omega C')}}$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{L'C'}}\,,$$

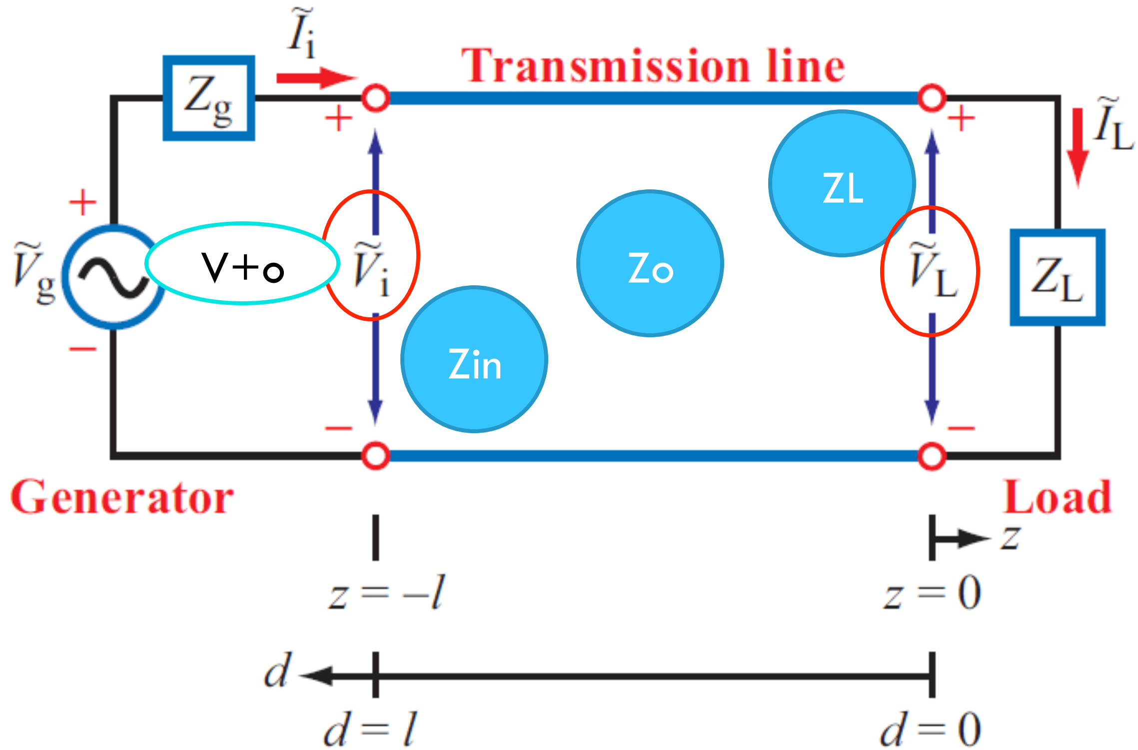
$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}\,.$$

$$\beta = \omega\sqrt{\mu\varepsilon} \qquad \text{(rad/m)},$$
$$u_p = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \text{(m/s)},$$

Non-dispersive line:
All frequency components have the same speed!

The Big Idea....

Impedance is measured at difference points in the circuit!



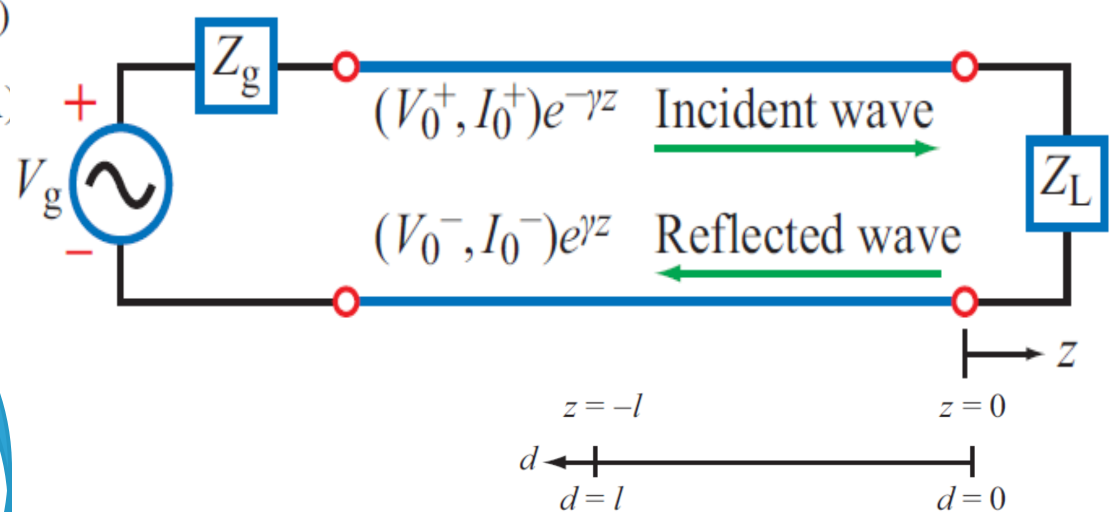
What is the voltage/current magnitude at different points on the line in the presence of load??

Voltage Reflection Coefficient

Consider looking from the Load point of view

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)}$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}$$



$$\tilde{V}_L = \tilde{V}(z{=}0) = V_0^+ + V_0^-,$$

$$\tilde{I}_L = \tilde{I}(z{=}0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}.$$

At the load (z = 0):

$$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L}$$

$$Z_L = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0.$$

The smaller the better!

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Reflection coefficient

$$= \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$

$$= \frac{z_L - 1}{z_L + 1} \quad \text{(dimensionless)},$$

$$z_L = \frac{Z_L}{Z_0}$$

Normalized load impedance

Expressing wave in phasor form:

- Remember:

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\widetilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}.$$

- If lossless
  - no attenuation constant

$$\widetilde{V}(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\widetilde{I}(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{j\beta z}).$$

$$V_0^- = \Gamma V_0^+$$

All of these wave representations
are **along** the
Transmission Line

Special Line Conditions (Lossless)

- Matching line
  - $Z_L = Z_o \rightarrow \Gamma = 0$; Vref=0
- Open Circuit
  - $Z_L = INF \rightarrow \Gamma = 1$; Vref=Vinc
- Short Circuit
  - $Z_L = 0 \rightarrow \Gamma = -1$; Vref=-Vinc

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1} \quad \text{(dimensionless)},$$

Remember:
Everything is with respect to the load so far!

$$\frac{I_0^-}{I_0^+} = -\frac{V_0^-}{V_0^+} = -\Gamma.$$

Vref= Vreflected ; Vinc = Vincident

Voltage Reflection Coefficient

$$\Gamma = |\Gamma| e^{j\theta_r}$$

Reflection Coefficient $\Gamma = |\Gamma| e^{j\theta_r}$

Load	$\lvert\Gamma\rvert$	θ_r
Z_0 — $Z_L = (r + jx)Z_0$	$\left[\frac{(r-1)^2 + x^2}{(r+1)^2 + x^2}\right]^{1/2}$	$\tan^{-1}\left(\frac{x}{r-1}\right) - \tan^{-1}\left(\frac{x}{r+1}\right)$
Z_0 — Z_0	0 (no reflection)	irrelevant
Z_0 — (short)	1	$\pm 180^\circ$ (phase opposition)
Z_0 — (open)	1	0 (in-phase)
Z_0 — $jX = j\omega L$	1	$\pm 180^\circ - 2\tan^{-1} x$
Z_0 — $jX = \frac{-j}{\omega C}$	1	$\pm 180^\circ + 2\tan^{-1} x$

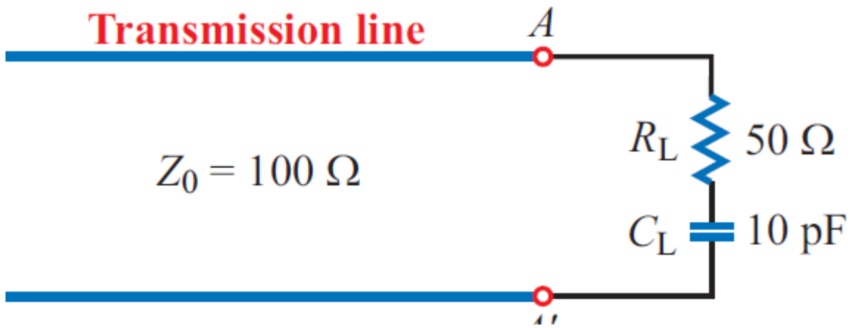
Pay attention!

Normalized load impedance

$$z_L = Z_L / Z_0 = (R + jX)/Z_0 = r + jx$$

Example

A 100-Ω transmission line is connected to a load consisting of a 50-Ω resistor in series with a 10-pF capacitor. Find the reflection coefficient at the load for a 100-MHz signal.



Notes

Standing Waves

Finding Voltage Magnitude

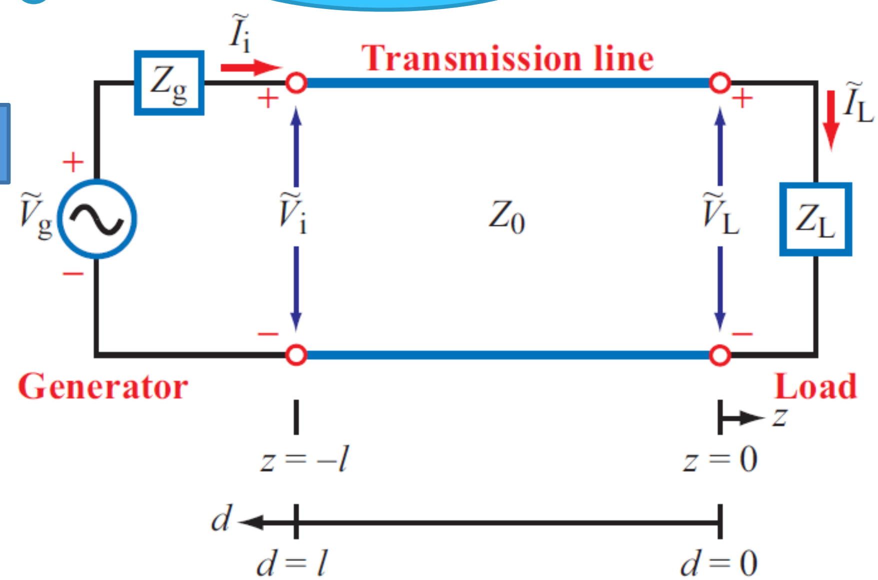
We are interested to know what happens to the magnitude of the |V| as such interference is created!

$$V_0^- = \Gamma V_0^+$$

When lossless!

$$\widetilde{V}(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\widetilde{I}(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{j\beta z}).$$

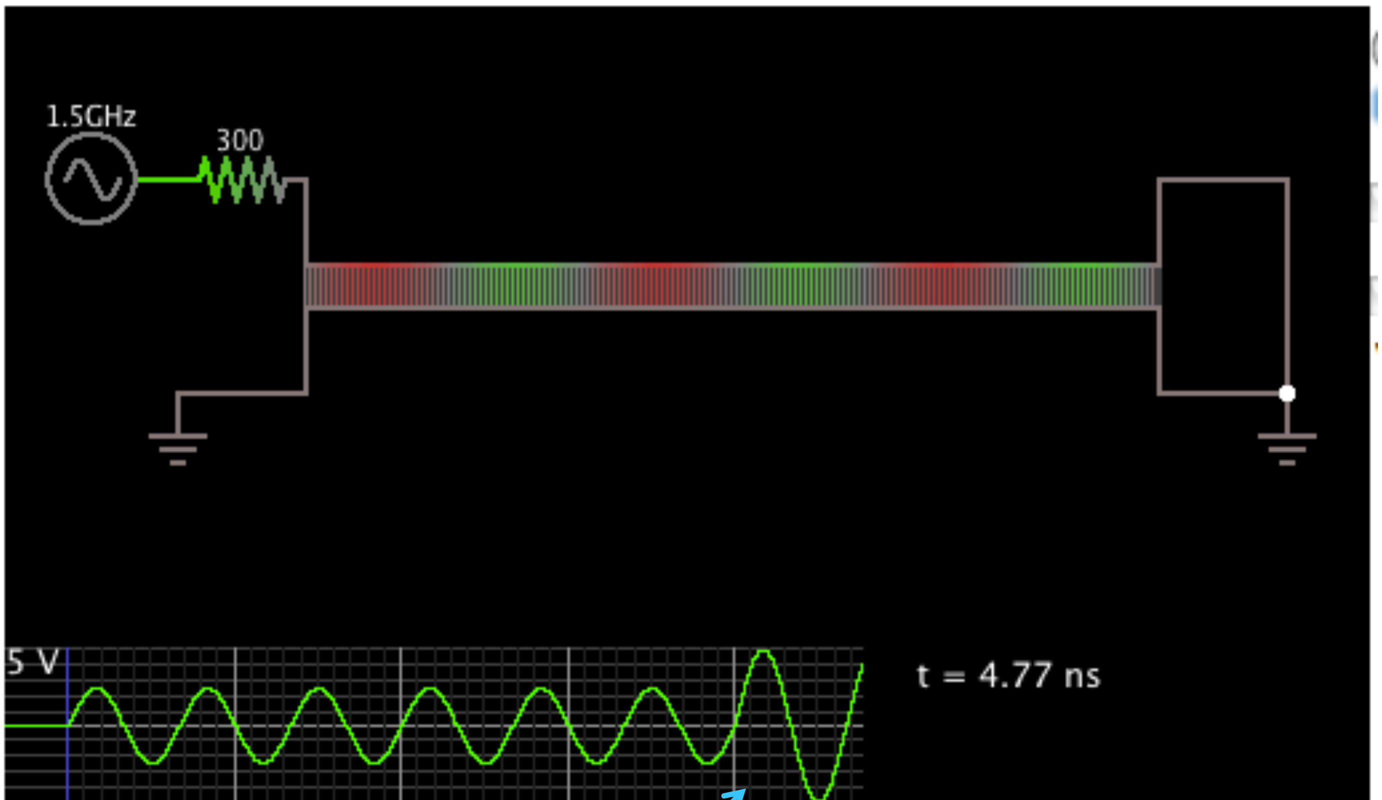


Remember: Standing wave is created due to interference between the traveling waves (incident & reflected)

Note: When there is no REFLECTION Coef. Of Ref. = 0 → No standing wave!

Standing Wave

http://www.falstad.com/circuit/e-tlstand.html



Due to standing wave the received wave at the load is now different

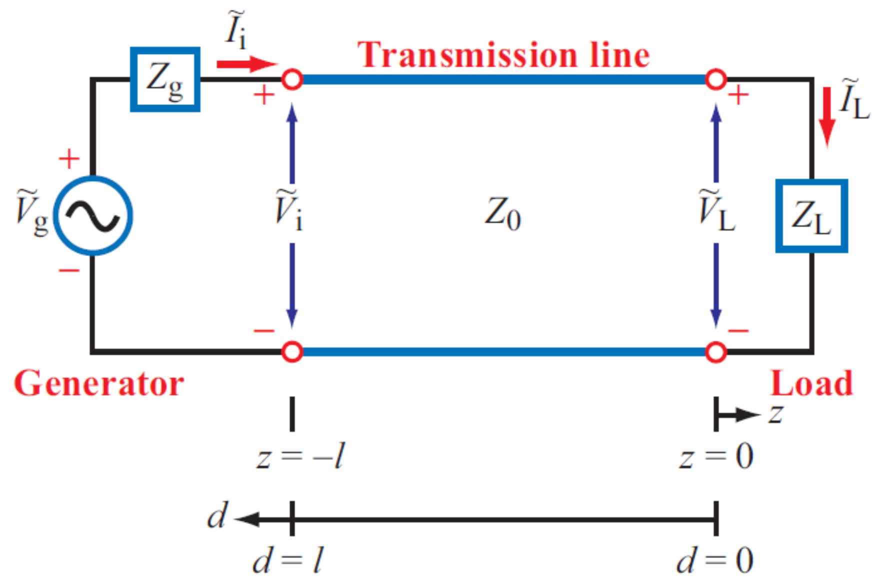
Standing Waves

Finding Voltage Magnitude

$$V_0^- = \Gamma V_0^+$$

$$\tilde{V}(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z}).$$



$$|\tilde{V}(d)| = |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}$$

voltage magnitude at z= -d

$$|\tilde{I}(d)| = \frac{|V_0^+|}{Z_0}[1 + |\Gamma|^2 - 2|\Gamma|\cos(2\beta d - \theta_r)]^{1/2}.$$

current magnitude at the source

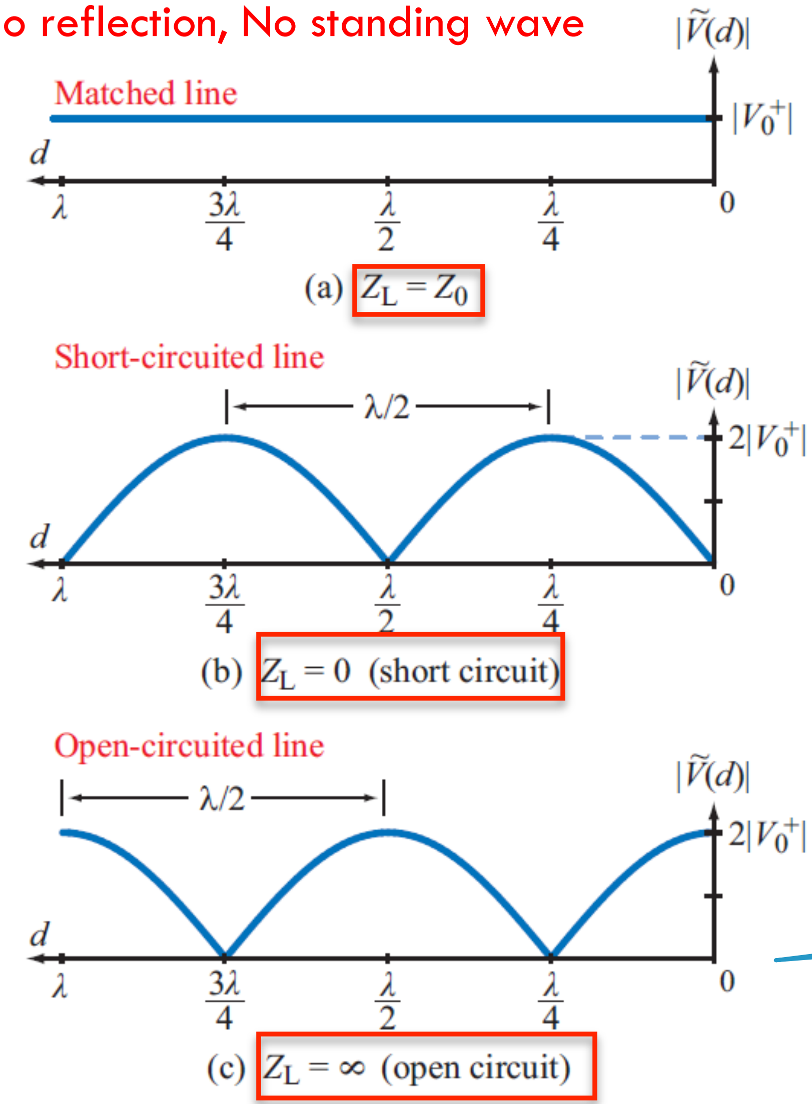
Let's see how the magnitude looks like at different z values!

Remember max current occurs where minimum voltage occurs (indicating the two waves are interfering destructively)!

Standing Wave Patterns for 3 Types of Loads

(Matched, Open, Short)

No reflection, No standing wave



$$|\tilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r)\right]^{1/2}.$$

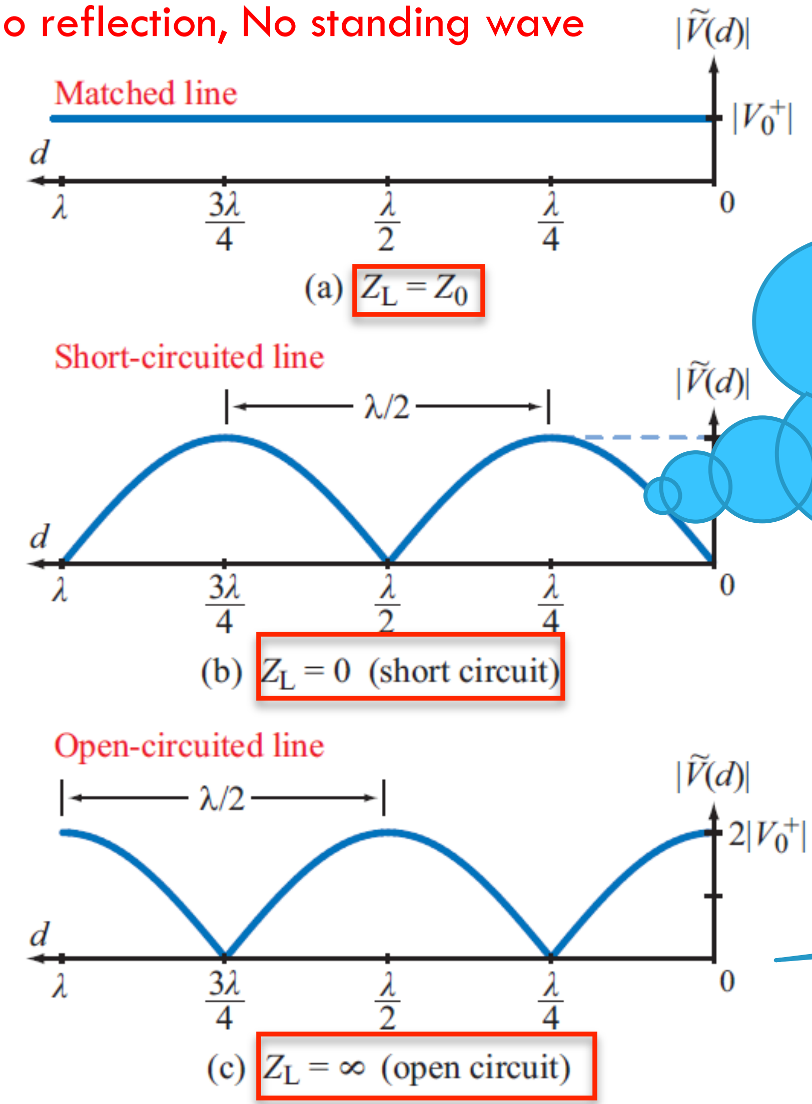
- Matching line
  - $Z_L = Z_o \rightarrow \Gamma = 0$; Vref=0
- Short Circuit
  - $Z_L = 0 \rightarrow \Gamma = -1$; Vref=-Vinc (angle $-/+\pi$)
- Open Circuit
  - $Z_L = \text{INF} \rightarrow \Gamma = 1$; Vref=Vinc (angle is 0)

Remember max current occurs where minimum voltage occurs!

Standing Wave Patterns for 3 Types of Loads

(Matched, Open, Short)

No reflection, No standing wave



BUT WHEN DO MAX & MIN Voltages Occur?

=Vinc (angle is 0)

Remember max current occurs where minimum voltage occurs!

Finding Maxima & Minima Of Voltage Magnitude

$$|\widetilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r)\right]^{1/2}.$$

$$|\widetilde{V}|_{\min} = |V_0^+|[1 - |\Gamma|],$$

$$\text{when } (2\beta d_{\min} - \theta_r) = (2n+1)\pi$$

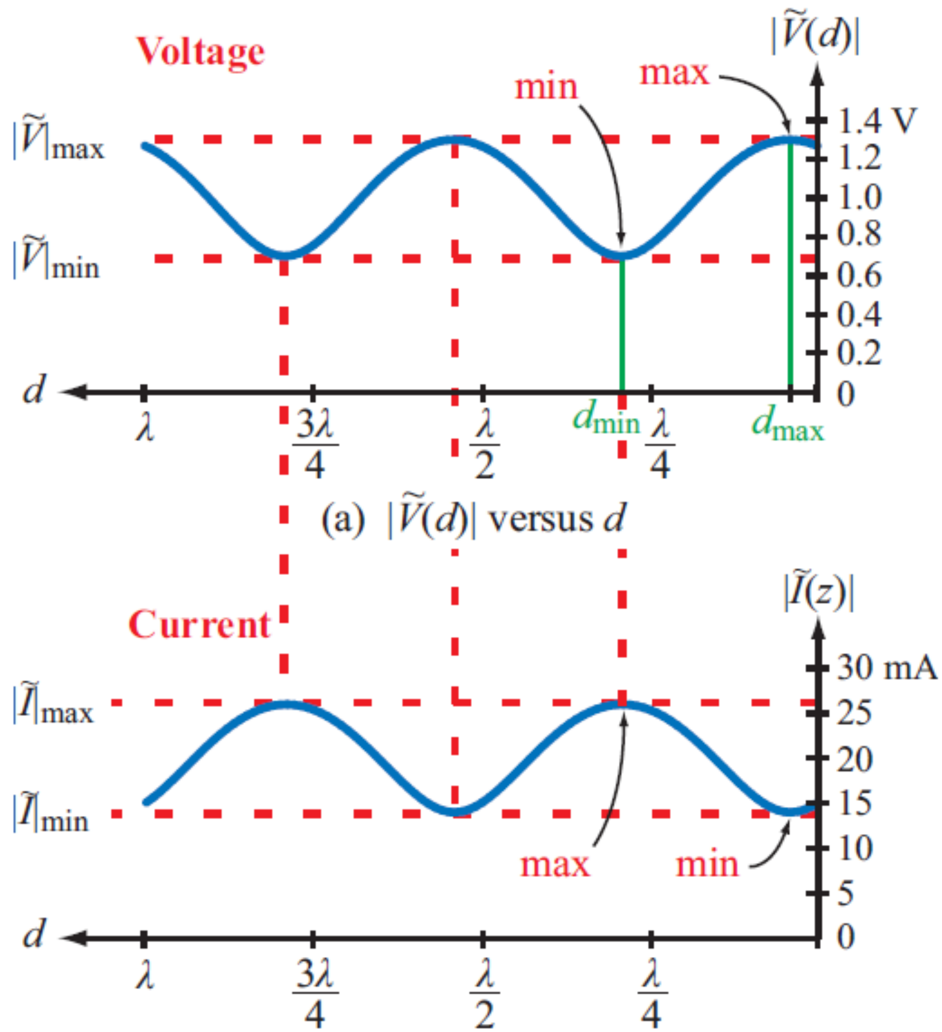
$$|\widetilde{V}(d)| = |\widetilde{V}|_{\max} = |V_0^+|[1 + |\Gamma|],$$

$$S = \frac{|\widetilde{V}|_{\max}}{|\widetilde{V}|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \text{(dimensionless)}$$

S = Voltage Standing Wave Ratio (VSWR)

For a matched load: *S* = 1

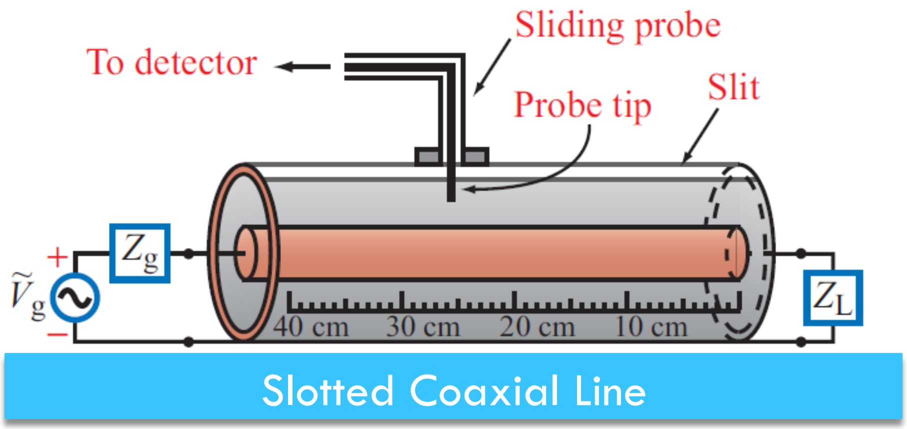
For a short, open, or purely reactive load:
S(open)=*S(short)* = INF where | Γ |=1;



(a) $|\widetilde{V}(d)|$ versus d

(b) $|\widetilde{I}(d)|$ versus d

Example Measuring Z_L with a Slotted Line



$$Z_0 = 50\ \Omega,$$
$$S = 3,$$
$$d_{min} = 12\text{ cm}.$$

Since the distance between successive voltage minima is $\lambda/2$,

$$\lambda = 2 \times 0.3 = 0.6\text{ m},$$

and

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.6} = \frac{10\pi}{3} \qquad \text{(rad/m)}.$$

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = 0.5.$$

$$2\beta d_{min} - \theta_r = \pi, \qquad \text{for } n = 0 \text{ (first minimum)},$$

which gives

$$\theta_r = 2\beta d_{min} - \pi = 2 \times \frac{10\pi}{3} \times 0.12 - \pi = -0.2\pi \text{ (rad)} = -36^\circ.$$

Hence,

$$\Gamma = |\Gamma| e^{j\theta_r} = 0.5e^{-j36^\circ} = 0.405 - j0.294.$$

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right] = 50 \left[\frac{1 + 0.405 - j0.294}{1 - 0.405 + j0.294}\right] = (85 - j67)\ \Omega.$$

What is the Reflection Coefficient (Γ d) at any point away from the load? (assume lossless line)

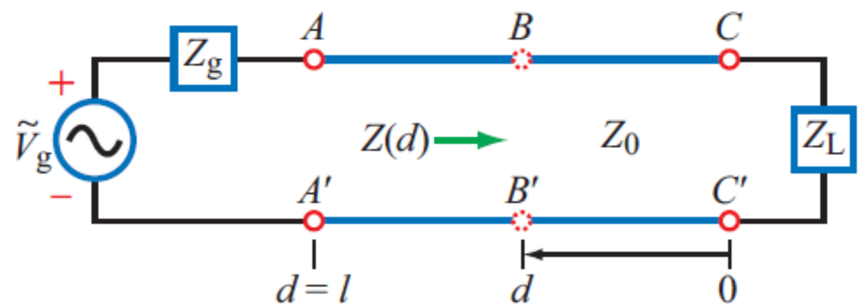
At a distance *d* from the load:

$$Z(d) = \frac{\tilde{V}(d)}{\tilde{I}(d)}$$

$$= \frac{V_0^+[e^{j\beta d} + \Gamma e^{-j\beta d}]}{V_0^+[e^{j\beta d} - \Gamma e^{-j\beta d}]} Z_0$$

$$= Z_0 \left[\frac{1 + \Gamma e^{-j2\beta d}}{1 - \Gamma e^{-j2\beta d}}\right]$$

$$= Z_0 \left[\frac{1 + \Gamma_d}{1 - \Gamma_d}\right] \quad (\Omega),$$



(a) Actual circuit

Wave impedance

where we define

$$\Gamma_d = \Gamma e^{-j2\beta d} = |\Gamma| e^{j\theta_r} e^{-j2\beta d} = |\Gamma| e^{j(\theta_r - 2\beta d)}$$

as the ***phase-shifted voltage reflection coefficient***,

Example

http://www.bessernet.com/Ereflecto/tutorialFrameset.htm

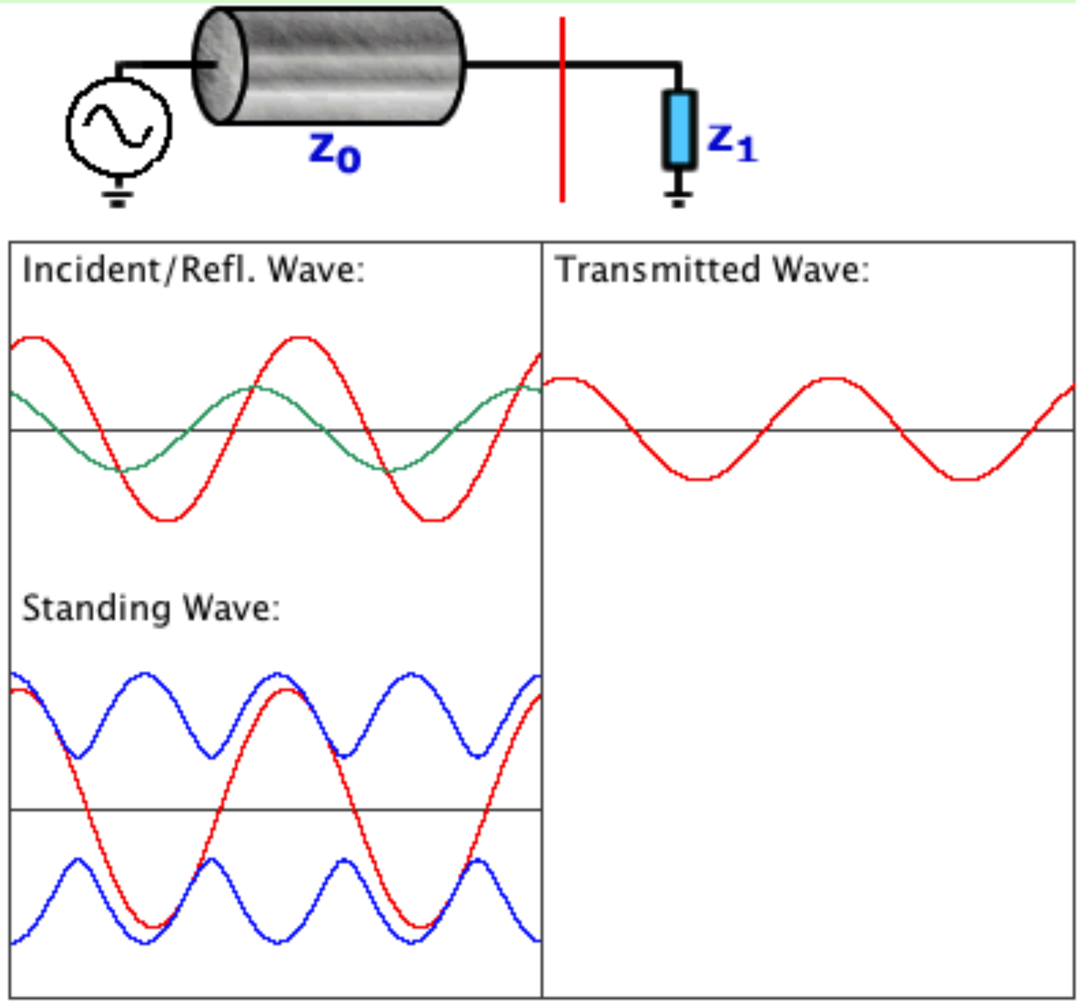
Reflectometer Calculator

Type a value in one of the fields below and hit 'enter':

Reflection Coefficient	0.444
SWR	2.6
Return Loss	7.044
Mismatch Loss	0.956
eR	1
Z1	130.00

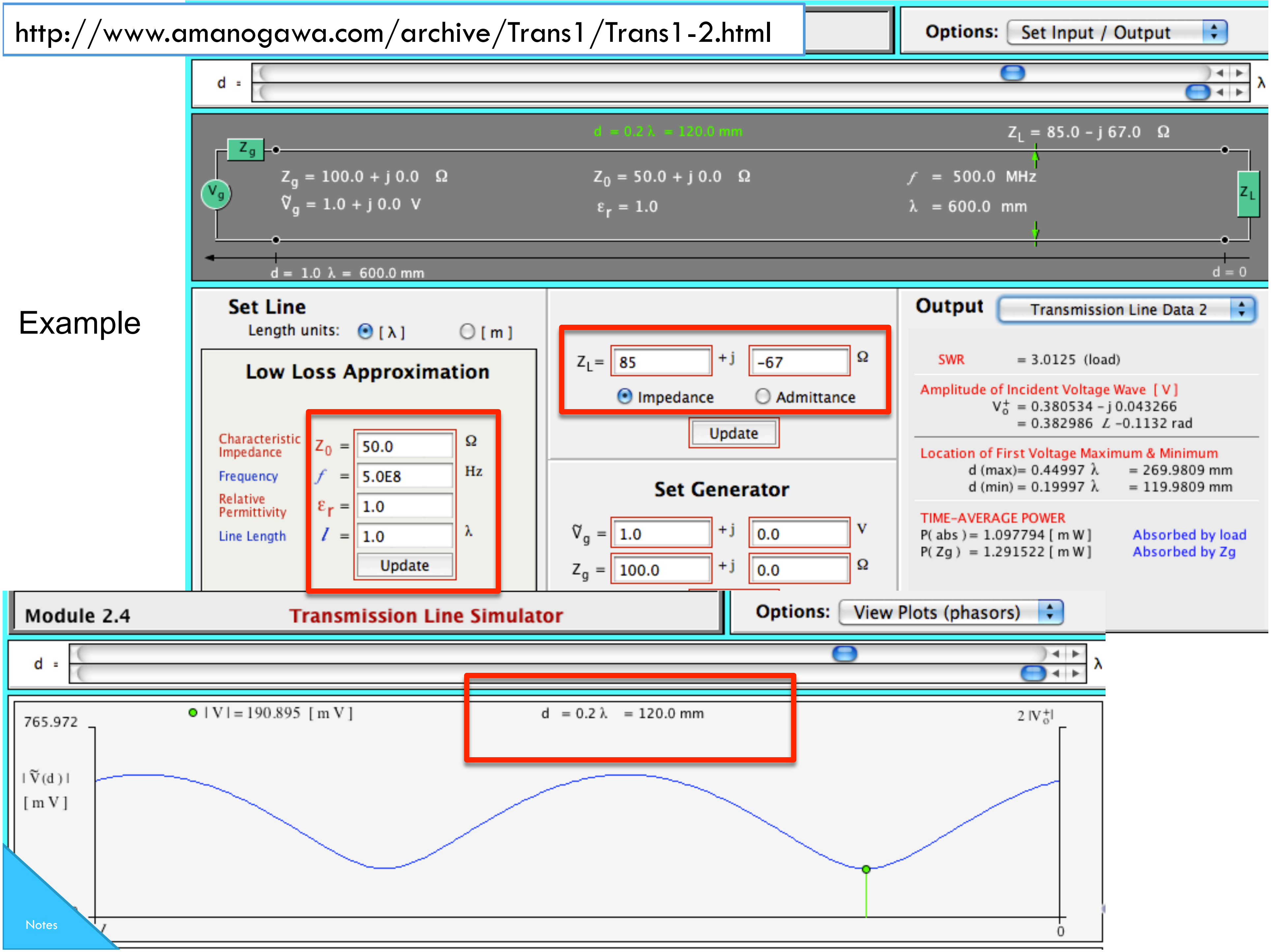
☑ Show two interfaces

Resume

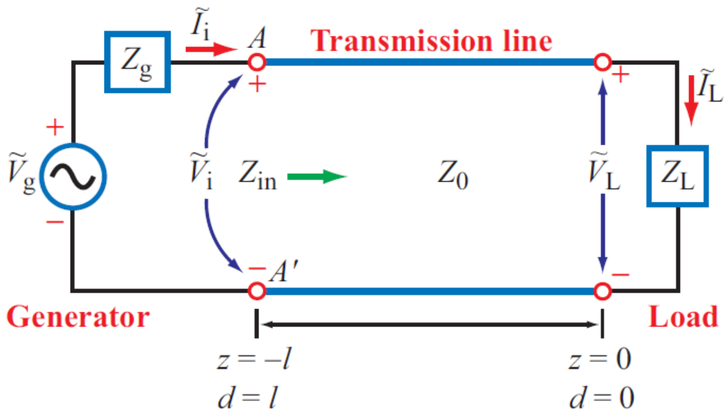


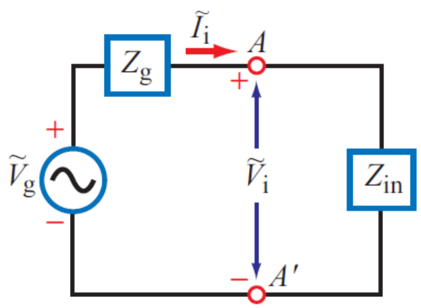
http://www.amanogawa.com/archive/Trans1/Trans1-2.html

Example



Input Impedance





At input, $d = l$:

$$Z_{in} = Z(l) = Z_0 \left[\frac{1 + \Gamma_l}{1 - \Gamma_l} \right].$$

$$\Gamma_l = \Gamma e^{-j2\beta l} = |\Gamma| e^{j(\theta_r - 2\beta l)}.$$

Wave Impedance

$$Zd = Z_0 \left[\frac{1 + \Gamma_d}{1 - \Gamma_d} \right]$$

$$Z_{in} = Z_0 \left(\frac{z_L \cos \beta l + j \sin \beta l}{\cos \beta l + j z_L \sin \beta l} \right)$$
$$= Z_0 \left(\frac{z_L + j \tan \beta l}{1 + j z_L \tan \beta l} \right).$$

Short-Circuit/Open-Circuit Method

- For a line of known length l, measurements of its input impedance, one when terminated in a short and another when terminated in an open, can be used to find its characteristic impedance Z_0 and electrical length βl

$$Z_{in}^{sc} = \frac{\widetilde{V}_{sc}(l)}{\widetilde{I}_{sc}(l)} = jZ_0 \tan\beta l.$$

$$Z_{in}^{oc} = \frac{\widetilde{V}_{oc}(l)}{\widetilde{I}_{oc}(l)} = -jZ_0 \cot\beta l.$$

$$Z_0 = +\sqrt{Z_{in}^{sc}\, Z_{in}^{oc}}\,,$$

$$\tan\beta l = \sqrt{\frac{-Z_{in}^{sc}}{Z_{in}^{oc}}}\,.$$

Standing Wave Properties

Voltage Maximum **Voltage Minimum**	$\lvert\widetilde{V}\rvert_{\max} = \lvert V_0^+\rvert[1 + \lvert\Gamma\rvert]$ $\lvert\widetilde{V}\rvert_{\min} = \lvert V_0^+\rvert[1 - \lvert\Gamma\rvert]$
Positions of voltage maxima (also positions of current minima) Position of first maximum (also position of first current minimum)	$d_{\max} = \frac{\theta_r\lambda}{4\pi} + \frac{n\lambda}{2}, \quad n = 0, 1, 2, \ldots$ $d_{\max} = \begin{cases} \frac{\theta_r\lambda}{4\pi}, & \text{if } 0 \le \theta_r \le \pi \\ \frac{\theta_r\lambda}{4\pi} + \frac{\lambda}{2}, & \text{if } -\pi \le \theta_r \le 0 \end{cases}$
Positions of voltage minima (also positions of current maxima) Position of first minimum (also position of first current maximum)	$d_{\min} = \frac{\theta_r\lambda}{4\pi} + \frac{(2n+1)\lambda}{4}, \quad n = 0, 1, 2, \ldots$ $d_{\min} = \frac{\lambda}{4}\left(1 + \frac{\theta_r}{\pi}\right)$
Input Impedance	$Z_{in} = Z_0\left(\frac{z_L + j\tan\beta l}{1 + jz_L\tan\beta l}\right) = Z_0\left(\frac{1+\Gamma_l}{1-\Gamma_l}\right)$
Positions at which Z_{in} is real	at voltage maxima and minima
Z_{in} at voltage maxima	$Z_{in} = Z_0\left(\frac{1+\lvert\Gamma\rvert}{1-\lvert\Gamma\rvert}\right)$
Z_{in} at voltage minima	$Z_{in} = Z_0\left(\frac{1-\lvert\Gamma\rvert}{1+\lvert\Gamma\rvert}\right)$
Z_{in} of short-circuited line	$Z_{in}^{sc} = jZ_0\tan\beta l$
Z_{in} of open-circuited line	$Z_{in}^{oc} = -jZ_0\cot\beta l$
Z_{in} of line of length $l = n\lambda/2$	$Z_{in} = Z_L, \quad n = 0, 1, 2, \ldots$
Z_{in} of line of length $l = \lambda/4 + n\lambda/2$ Z_{in} of matched line	$Z_{in} = Z_0^2/Z_L, \quad n = 0, 1, 2, \ldots$ $Z_{in} = Z_0$

$\lvert V_0^+\rvert$ = amplitude of incident wave; $\Gamma = \lvert\Gamma\rvert e^{j\theta_r}$ with $-\pi < \theta_r < \pi$; θ_r in radians; $\Gamma_l = \Gamma e^{-j2\beta l}$.

Power Flow

- How much power is flowing and reflected?
  - Instantaneous $P(d,t) = v(d,t).i(d,t)$
    - Incident
    - Reflected
  - Average power: $Pav = Pav^i + Pav^r$
    - Time-domain Approach
    - Phasor-domain Approach (z and t independent)
      - $\frac{1}{2} \mathrm{Re}\{I^*(z) . V(z)\}$

$$P^i(d,t) = \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)],$$

$$P^r(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_r)].$$

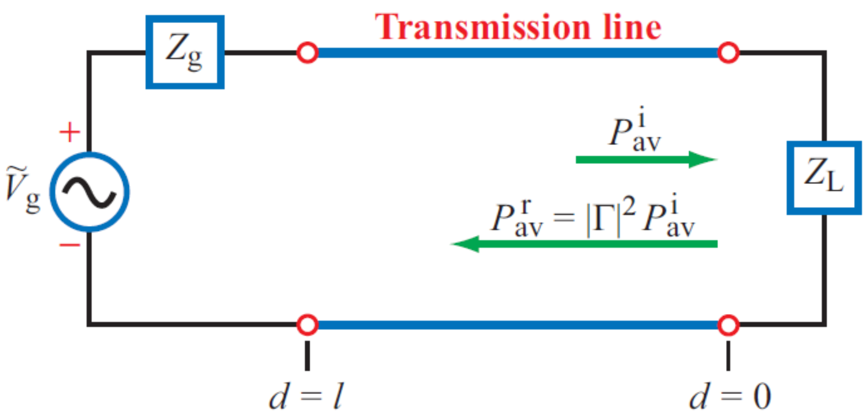
Average Power

(Phasor Approach)

Avg Power: ½ Re{I(z) * V_(z)}

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)]$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_{\mathrm{r}})].$$



$$V_0^+ = \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right).$$

Fraction of power reflected!

$$P^{\mathrm{i}}_{\mathrm{av}} = \frac{|V_0^+|^2}{2Z_0} \quad \text{(W)},$$

which is identical with the dc term of $P^{\mathrm{i}}(d,t)$ given by Eq. (2.102a). A similar treatment for the reflected wave gives

$$P^{\mathrm{r}}_{\mathrm{av}} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} = -|\Gamma|^2 P^{\mathrm{i}}_{\mathrm{av}}.$$

The average reflected power is equal to the average incident power, diminished by a multiplicative factor of $|\Gamma|^2$.

Summary

TEM Transmission Lines

$$L'C' = \mu\varepsilon$$

$$\frac{G'}{C'} = \frac{\sigma}{\varepsilon}$$

$$\alpha = \mathfrak{Re}(\gamma) = \mathfrak{Re}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(Np/m)}$$

$$\beta = \mathfrak{Im}(\gamma) = \mathfrak{Im}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad \text{(rad/m)}$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad (\Omega)$$

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

Step Function Transient Response

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}$$

$$V_\infty = \frac{V_g R_L}{R_g + R_L}$$

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

Lossless Line

$$\alpha = 0$$

$$\beta = \omega\sqrt{L'C'}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

$$u_p = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{(m/s)}$$

$$\lambda = \frac{u_p}{f} = \frac{c}{f}\frac{1}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}$$

$$d_{min} = \frac{\theta_r \lambda}{4\pi} + \frac{(2n+1)\lambda}{4}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$P_{av} = \frac{|V_0^+|^2}{2Z_0}\left[1 - |\Gamma|^2\right]$$

Practice

1- Assume the load is 100 + j50 connected to a 50 ohm line. Find coefficient of reflection (mag, & angle) and SWR. Is it matched well?

2- For a 50 ohm lossless transmission line terminated in a load impedance ZL=100 + j50 ohm, determine the fraction of the average incident power reflected by the load. Also, what is the magnitude of the average reflected power if |Vo|=1?

3- Make sure you understand the slotted line problem.

4- Complete the Simulation Lab answer the following questions:

- Remove the MLOC so the TEE will be open. How does the result change? Take a snapshot. Briefly explain.

- In the original circuit, what happen if we use paper as the dielectric (paper has er of 3.85). Take a snapshot. Briefly explain.

- For the obtained Zo in your Smith Chart <u>calculate</u> the admittance. You must show all your work.

- What exactly is mag(S11)? How is it different from coefficient of reflection? Is the reflection of coefficient measured at the source or load?

- What happens if the impedance of the source (TERM1) is changed to 25 ohm? How does the impedance on the smith chart change?

- How do you calculate the effective length?