

The Theory of Belaying

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THE protection afforded by a rope in mountaineering depends primarily upon the ability of belayer and rope to stop a fall adequately. In a recent article in the *Sierra Club Bulletin*, some aspects of the art of belaying were examined critically, especially the problem of belaying the leader on high-angle climbing.¹ The dynamic belay, a relatively new idea for "safelying" a climber, was advanced, and its remarkable superiority over the traditional methods of holding a fall was demonstrated. After a number of the problems discussed in that article were investigated further, it seemed possible that the art of belaying could and should be formulated into a physical theory from which quantitative answers could be predicted about the loads and failures in rope and personnel, and these latter hazards minimized or eliminated. It also seemed possible and desirable to determine the loads experienced during the process of a belay under simulated and real climbing conditions. This report, therefore, is a theoretical and experimental study of what transpires in a climbing fall. It is hoped that a basic understanding of the factors involved in belaying will lead to the use of more effective and safer techniques in holding a fall.

Arresting a fall is fundamentally a problem of absorbing energy. This simple principle is the basis of all techniques, procedures and devices employed in belaying. The knowledge of this principle is the key to understanding the difference between an adequate belay and an unsafe belay and emphasizes why some of the practices in belaying have often resulted in snapped ropes, with tragic consequences.

When a man falls, his acquired velocity will develop kinetic energy which must be completely absorbed to arrest the fall. At any instant, this kinetic energy is equivalent to the product of the weight of the man and the distance through which he has fallen,

¹ R. M. Leonard and A. Wexler, "Belaying the Leader," *Sierra Club Bulletin*, XXXI (Dec. 1946).

that is, the potential energy he possessed due to his position before the fall started. The primary function of a belay is to absorb this energy, and to absorb it in such a fashion that no serious loads are imparted to either the climber or the belayer. The methods of rope-handling that serve to absorb this energy can be used to classify the belay into three fundamental types: the rigid or static belay, the resilient or indirect belay, and the dynamic or sliding belay. The static belay is one in which the kinetic energy of a fall is absorbed by the rope alone, one end of which is fixed to a rigid support, such as a tree or horn of rock. In the resilient belay, the support as well as the rope absorbs energy by yielding or "giving" under load, as in the case of the belayer snubbing the rope around his body. In the dynamic belay, the rope is allowed to slide over the support so that the friction of the sliding rope absorbs energy in addition to the energy absorbed by the rope in stretching. It will be shown that the dynamic belay is the most efficacious for it is capable of fully absorbing the energy under loads easily tolerated by men and equipment.

We shall now proceed to develop a mathematical theory of belaying, but in order to do so we shall make several simplifying assumptions. First, it will be assumed that climbing rope is elastic, that is, obeys Hook's law.² Second, it will be assumed that the climber is concentrated at the end of the rope and that the weight of the rope is negligible in comparison with the weight of the climber. Third, the reduction in strength of the rope due to sharp bends, age, wear, dampness and so forth, will be ignored. By making these assumptions any theory thereby derived will be a first approximation to the actual phenomena occurring in belaying. However, it will serve as a useful guide in indicating the nature of the forces and the limitations and best methods of rope control that underlie the art of belaying.

STATIC BELAY

The static belay readily lends itself to analysis and yields results

² Unfortunately, this is not the case. The relation between tension and elongation may more closely be expressed as a polynomial of the second or third degree. To utilize such an expression in a mathematical treatment would be unnecessarily complicated. For the sake of simplicity, the load elongation curves will be approximated by straight lines.

which are pertinent and important in understanding what occurs in the process of arresting a fall. In this belay, one end of the rope is tightly snubbed around an immovable object to prevent the rope from sliding. The strength of the support, which may be a rock, tree, or karabiner, is relied on to withstand the forces arising from the impact of the climber on the rope.

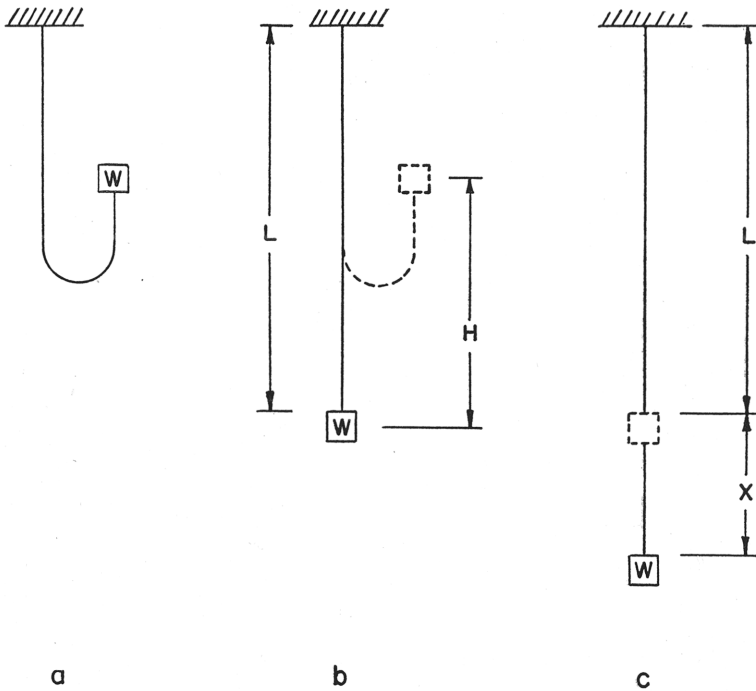


FIG. 1. Schematic representation of the static belay: (a) position of climber with slack rope; (b) position of climber after falling, just before the rope begins to stretch; (c) final position of climber after the rope has stretched and the climber's downward motion has stopped.

Consider a man, weighing W pounds, climbing either vertically above or below his belayer. (See Fig. 1.) Let the length of rope between the climber and his belayer be L feet. He slips and falls through a distance of H feet before all the slack in his rope is taken up. The rope then starts to stretch and will elongate x feet until his downward motion is stopped or the rope breaks.

Only by absorbing the kinetic energy of the climber can the plunge be arrested. In the static belay the rope is the sole medium available for this. By applying the law of conservation of energy and assuming that the kinetic energy of the fall is completely absorbed by the strain in the rope, this basic equation follows:

$$W(H + x) = \frac{1}{2} Px \quad (1)$$

where P is the maximum tension developed in the rope. Since the rope is assumed to be elastic, then

$$P = \frac{kx}{L} \quad (2)$$

where k is a proportionality constant that depends upon the cross-sectional area of the rope, the fiber content, the "lay," and so on. Substituting equation (2) into (1) yields the quadratic equation

$$x^2 - \frac{2WLx}{k} - \frac{2WHL}{k} = 0 \quad (3)$$

whose solution is

$$x = \frac{WL}{k} + \frac{WL}{k} \sqrt{1 + \frac{2kH}{WL}} \quad (4)$$

The tension developed in the rope follows from substituting in equation (2) the value for x given in equation (4):

$$P = W + W \sqrt{1 + \frac{2kH}{WL}} \quad (5)$$

It is necessary to emphasize that the tension P is the maximum produced in the rope. It starts at zero and in some finite time builds up to P , the peak load, then drops back to zero. This point will be discussed more fully later.

Equation (5) leads to a significant deduction. Consider the case of a climber vertically below his belay with no slack in his rope. Upon falling, he immediately begins to stretch the rope. Without slack, $H = 0$, and equation (5) reduces to

$$P = 2W \quad (6)$$

Thus, even without slack, a free fall into the rope produces a maximum load equal to twice the weight of the climber.

The tension in the rope depends on the ratio of the free fall to the length of rope, H/L . One cannot talk about a free fall, therefore, without specifying the length of rope involved. In fact, if H/L is constant, then the tension is constant, assuming the same climber and rope. This often occurs in leading. For instance, if the climber were directly above the belay with no slack, then should he fall he would drop a distance twice the length of the rope, that is, $H/L=2$, irrespective of how near or far he was above his belay. The tension developed in the rope under this condition is

$$P = W + W \sqrt{1 + \frac{4k}{W}} \tag{7}$$

This amounts to the worst possible fall, for H/L can never exceed 2.

What are the magnitudes of the forces involved in a static belay? What safety does the static belay offer a climber? The answers to these questions disclose the startling fact that, except under limited conditions, the use of the static belay is fraught with great danger. First, consider the situation of a climber falling from vertically above his belay ($H/L=2$). Will the climbing rope withstand this impact? More precisely, how heavy does the climber have to be in order to break the rope? A solution of equation (1)

$$W(H + x) = \frac{1}{2} Px \tag{1}$$

gives the desired information. Substituting $2L=H$ into the equation, we have

$$W\left(2 + \frac{x}{L}\right) = \frac{Px}{2L} \equiv E$$

in which x/L is the unit elongation of the rope at failure and $\frac{Px}{2L} \equiv E$ is the unit strain energy of the rope at failure. Since a knot is used invariably in climbing, reducing the strength 50 per cent, the tension at failure is $\frac{1}{2} P$. Accordingly, if a knot is used the equation becomes

$$W\left(2 + \frac{x}{L}\right) = \frac{Px}{4L} \equiv \frac{E}{2}$$

Substituting characteristic values of x/L and E for rope commonly employed in mountaineering, and solving for W , give the alarm-

ing results of 35 pounds and 132 pounds for $\frac{1}{2}$ -inch manila and $\frac{7}{16}$ -inch nylon respectively, as shown in the following table:

ROPE	ELONGATION x/L	STRAIN ENERGY	WEIGHT
		E <i>Ft.-Lbs./Ft.</i>	W <i>Lbs.</i>
$\frac{1}{2}$ -in. manila	0.15	150	35
$\frac{7}{16}$ -in. nylon	0.55	675	132

A comparison of the results of this computation with the data obtained in laboratory drop tests conducted by the National Bureau of Standards shows good agreement.³ Thus, after reducing the values by 50 percent because of the knot, the NBS data for $\frac{7}{16}$ -inch nylon show that a 139-pound weight, falling 10 feet on a 5-foot specimen of rope, caused failure, and a 149-pound weight, falling 20 feet on a 10-foot specimen of rope, caused failure. It is at once obvious that, even with nylon rope, a purely static belay will not support an average weight climber on a free fall.

The forces due to a fall, even for small values of H/L , are considerable. Equation (5) yields the magnitudes of these forces. The tensions developed in $\frac{1}{2}$ -inch manila and $\frac{7}{16}$ -inch nylon for vertical falls of a 150-pound man have been computed and are shown graphically in Figure 2.⁴ The tension in the rope is plotted as a function of the ratio of free fall to length of fall, H/L . These curves show the marked superiority of nylon over manila. For identical falls, the tension in the manila rope is almost double that in the nylon, owing to their different elasticities. Reducing the breaking strength of the rope 50 percent because of the knot, then manila will fail at about 1400 pounds and nylon at about 1800 pounds. Those portions of the curves above these breaking strengths are drawn dotted. Figure 2 shows that a ratio of $H/L = 0.24$ will snap a manila rope while a ratio of 1.34 will snap a nylon rope. Naturally,

³ "Impact Strength of Nylon and Sisal Ropes," *J. of Res. of N.B.S.*, XXXV (Nov. 1945), 417, RP 1679.

⁴ To make this computation, it was necessary to arrive at some value for the proportionality constant k . This constant k is an approximation, for, as previously explained, rope does not obey Hook's law. A value may be chosen by assuming a straight line from the origin to the point of failure for the relation between load and elongation. The value of k for $\frac{1}{2}$ -in. manila rope is 22,500 pounds and for $\frac{7}{16}$ -in. nylon rope is 6700 pounds.

the situation will be more serious if the weight of the climber is greater than 150 pounds. The curves also show that high loads develop with even relatively small values of H/L . For example, if $H/L = 0.1$, the tension in manila will be 920 pounds and in nylon 620 pounds; and if $H/L = 0.2$, the tension in manila and nylon will be 1300 and 800 pounds, respectively.

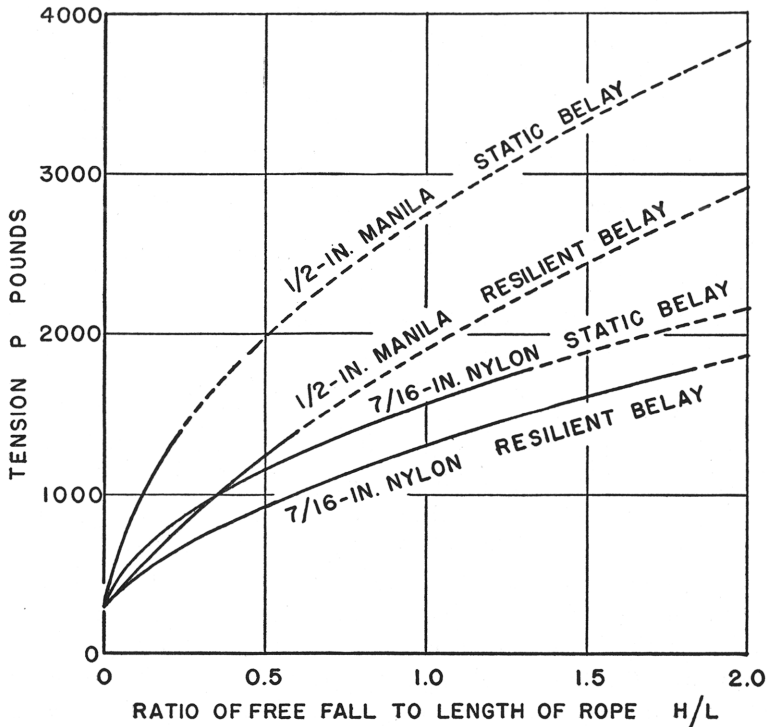


FIG. 2. The tension developed in rope due to the free fall of a 150-pound man for various ratios of free fall to length of rope, H/L , for the static and resilient belays.

It is of interest to examine the loads that would be produced for the worst possible fall ($H/L = 2$) for climbers of various weights, if the rope did not break, and to compare these loads with the actual breaking strengths given above, that is, 1400 pounds for

½-inch manila and 1800 pounds for 7/16-inch nylon. The calculations were easily made using equation (7) and are given below:

WEIGHT OF CLIMBER	TENSION DUE TO A FREE FALL THROUGH TWICE THE LENGTH OF ROPE FOR	
	1/2-IN. MANILA	7/16-IN. NYLON
<i>Lbs.</i>	<i>Lbs.</i>	<i>Lbs.</i>
50	2175	1210
75	2665	1495
100	3105	1740
125	3485	1960
150	3830	2165
175	4145	2345
200	4450	2530
225	4715	2695
BREAKING STRENGTH	1400	1800

Field tests fully confirm the predictions of the static belay theory. With a concrete dummy, weighing 147 pounds, to simulate a climber, and a special dynamometer⁵ to measure impact loads, standard ½-inch manila climbing rope was subjected to drops in which one end of the rope was rigidly fixed as in a static belay. The rope had a specified breaking strength of 2650 pounds. First, a series of tests was made with 5-foot specimens. The dummy was dropped a distance of 10 feet ($H/L = 2$). Each specimen was attached to the dummy and to a fixed point (piton and karabiner) by means of eye splices. (Properly made eye splices will develop the full strength of the rope.) The tests were repeated with 10-foot specimens with the dummy falling 20 feet. In each case, failure occurred in either the rope, karabiner, or piton at loads less than the designated breaking strength of the rope. In other words, even without the knot invariably used in climbing, a fall of a 147-pound weight through twice the length of the rope produced a failure.

To duplicate climbing conditions more closely, another series

⁵ This instrument was smaller than a fist, weighed about one pound, and could measure impact loads of 0.005 second duration. By means of karabiners, it could be attached between dummy and rope or between piton and rope. It was capable of recording loads from 400 to 4600 pounds. Its size and simplicity made it a convenient as well as accurate instrument for field and laboratory use.

of tests was made with ½-inch manila rope but with the ends fastened to the dummy and fixed point (karabiner) by means of a bow-line knot. Under these conditions an impact load between 1345 and 1550 pounds was sufficient to break the rope. The data are given below:

TEST NO.	LENGTH OF SPECIMEN	FREE FALL	FREE FALL LENGTH OF ROPE	LOAD	FAILURE
	L Ft.	H Ft.	H/L	P Lbs.	
21	10	20	2.0	1515	In knot; all strands
22	10	20	2.0	1388	“ “ “
23	9	9	1.0	1345	“ “ “
24	9	9	1.0	1430	“ “ “
25	9.25	4.66	0.5	1550	“ “ “
26	9	2.25	0.25	1060	None
27	8.62	2.17	0.25	1370	“
28	9.5	2.29	0.24	824	“
29	8.83	1.10	0.125	1040	“
30	5	1.25	0.25	875	“
31	5	1.25	0.25	1030	“
32	5	1.25	0.25	1075	“
33	20	5	0.25	1380	“
34	20	5	0.25	1225	“
35	20	5	0.25	1315	“

It is important to observe that a fall through a distance of half the length of rope (Test No. 25) was sufficient to rupture the rope. Only when H/L was reduced to 0.25 or less did the rope consistently withstand the impact.

Both theory and experiment conclusively show the total inadequacy of the static belay for arresting a fall. Therefore, the static belay must not be used for protecting a climber, especially a leader.

RESILIENT BELAY

It has long been recognized that, if a belayer snubbed the rope around his body, the resiliency of his anatomy would aid in lessening

the severity of a fall. For this reason, many a mountaineer has followed the practice of running the rope over hips or shoulder in preference to an unyielding horn of rock or piton and karabiner combination. In such a belay, the impact of a fall is absorbed not only by the rope but also by the "give" in the support.

Again, as in the static belay, the resilient belay can be studied to good advantage by resolving the problem mathematically. Consider a man, weighing W pounds, climbing vertically either above or below his belayer. Let the length of rope between him and his belayer be L feet. He slips and falls a distance H feet before all the slack in the rope is taken up. The rope starts to stretch and will continue to elongate x feet until his downward motion is arrested or the rope breaks. At the same time, the support will deflect vertically by an amount d feet before the fall is stopped. The total travel is $H + x + d$. It will be assumed that the deflection of the support is elastic and therefore absorbs $\frac{1}{2} Pd$ foot-pounds of energy. The energy balance is

$$W(H + x + d) = \frac{1}{2} Px + \frac{1}{2} Pd \quad (8)$$

The value of x in equation (2) is substituted into equation (8) to give an expression for the maximum tension developed in the rope:

$$P = \left(W - \frac{kd}{2L} \right) + W \sqrt{1 + \frac{2kH}{WL} + \frac{kd}{WL} \left(1 + \frac{kd}{4WL} \right)} \quad (9)$$

A comparison of equation (9) with equation (5) shows that the two are similar in form and that if $d = 0$ they become identical. The resiliency of the support acts to decrease the load in the rope as compared to a static belay. If equation (9) is subtracted from equation (5), the result is the decrease in tension, ΔP , due to the resiliency of the support

$$\Delta P = W \left[\sqrt{1 + \frac{2kH}{WL}} - \sqrt{1 + \frac{2kH}{WL} + \frac{kd}{WL} \left(1 + \frac{kd}{4WL} \right)} \right] + \frac{kd}{2L} \quad (10)$$

Usually d is very small⁶ compared to H so that kd/WL and 1 are negligible in comparison to $2kH/WL$ and may be neglected, leaving

$$\Delta P = \frac{kd}{2L} \quad (11)$$

The factors contributing to the superiority of the resilient be-

lay over the static belay are apparent immediately. The rope tension decreases with increasing deflection of the support. As the length of rope between belayer and climber increases, the resiliency of the support has less and less effect. With rope that has great elongation, such as nylon, the effect of the resiliency of the support is small compared with rope that has little elongation, such as manila.

The magnitude of the loads involved in a resilient belay are computed from equation (9). For purposes of this calculation, it is assumed that the deflection occurring in a hip or shoulder belay is one foot⁶, and that the length of rope between climber and belayer is ten feet. The tension due to a fall of a 150-pound man on $\frac{1}{2}$ -inch manila and $\frac{7}{16}$ -inch nylon for these conditions is plotted on Figure 2 so that a comparison with the tension arising from a static belay may be made. Those portions of the curves above the breaking strength of the rope are drawn dotted. With manila, for a ratio of $H/L = 0.2$, the tension developed in the rope due to a fall on a resilient belay is 720 pounds, while the tension due to a fall on a static belay is 1280 pounds; and for a ratio of $H/L = 0.5$, the tension due to a fall on a resilient belay is 1240 pounds, while the tension due to a fall on a static belay is 1980 pounds, and the latter would break the rope. With nylon, for a ratio of $H/L = 0.2$, the tension developed in the rope due to a fall on a resilient belay is 620 pounds, while the tension due to a fall on a static belay is 800 pounds; for a ratio of $H/L = 0.5$, the tension due to a fall on a resilient belay is 930 pounds, while the tension due to a fall on a static belay is 1150 pounds; and for a ratio of $H/L = 1.5$, the tension due to a fall on a resilient belay is 1620 pounds, while the tension due to a fall on a static belay is 1890 pounds, and the latter would break the rope.

It may be seen that, even though a decided reduction in tension is achieved, still the load developed in the rope for values of H/L

⁶ If the deflection of the support is excessive, very often the entire benefit of this deflection is vitiated. For example, in a body belay, if the belayer is jerked a considerable distance, his ability to continue handling the rope is greatly impaired, and in all probability the rope will be pulled out of his hands. If such a belay is attempted, the belayer should be well braced and securely tied to an anchor. Usually, body deflections of more than one foot result in loss of control of the rope. With a shoulder belay, the tendency is for the body to collapse under the impact. With a hip or buttocks belay, even with the feet well braced, the tendency is for the body to be lifted into the air.

$\cong 0.60$ for manila and $H/L \cong 1.85$ for nylon, under climbing conditions (that is, with a knot around the climber), will break the rope. The H/L ratios that produce failure will be smaller for climbers weighing more than 150 pounds. Since it is imperative that the rope withstand the most adverse fall ($H/L = 2$), the resilient belay must be ruled out as a means for arresting the fall of a leader.

DYNAMIC BELAY

The most significant advance in the art of belaying has been the introduction, in recent years, of the dynamic belay.⁷ This new technique of arresting a fall has overcome the limitations of rope and personnel and has reduced the hazards of climbing to the point where the fall of a leader no longer need be considered fatal.

The dynamic belay is simply a method of effectively absorbing kinetic energy. In the static belay, the strain energy of the rope is the only means available for absorbing energy and thereby stopping the motion of a falling man. In the resilient belay, the "give" in the support, as well as the rope, is utilized for absorbing energy. The dynamic belay goes one step further. The rope is permitted to slide, under control, over the support; and the friction generated between rope and support is the dominant factor in absorbing the energy of a fall.

Intuitively, one can infer that the dynamic belay should be considerably better than the other two belays. Mathematical analysis yields analytical proof. We approach the problem in a manner similar to that used before. Consider a man, weighing W pounds, climbing vertically either above or below his belayer. Let the length of rope between the climber and his belayer be L feet. He slips and falls through a distance H feet before all the slack in his rope is taken up. The rope starts to stretch and will elongate x feet, in which time the tension rises to P pounds. We shall now assume that the rope will start to slide when the tension reaches P . In actual practice, the belayer can control the value of P . There are two extremes to the possible values of tension. For instance, if the rope is held

⁷ Credit for the introduction of the dynamic belay goes to the Sierra Club. A discussion of the development of the dynamic belay, the method of rope management involved in the dynamic belay and examples of mountaineering falls stopped with the dynamic belay are given in the article listed in footnote 1.

loosely, or not at all, the rope will slide under no load, and the climber will continue to fall freely. If the rope is held tightly, then the rope will not slide at all, and the static or rigid belay is achieved. We shall further assume that the tension remains constant, that is, stays at P , as long as the rope slides. The rope will continue sliding s feet until the energy of the fall is absorbed, and then the motion of the man will stop.

In this description of the mechanism of the dynamic belay, the climber falls $H + x + s$ feet. The energy balance for arresting the fall is

$$W(H + x + s) = \frac{1}{2} P x + P s \tag{12}$$

in which $P s$ is the frictional energy of the rope over the support.

The value of x from equation (2) is substituted into equation (12) to give an expression for the tension P developed in the rope:

$$P = \left(W - \frac{ks}{L} \right) + W \sqrt{1 + \frac{2kH}{WL} + \left(\frac{ks}{WL} \right)^2} \tag{13}$$

If $s = 0$, then equation (13) reduces to equation (5), the case for the static belay. The maximum load will arise, for any constant value of s/L , when $H/L = 2$.

$$P = \left(W - \frac{ks}{L} \right) + W \sqrt{1 + \frac{4k}{W} + \left(\frac{ks}{WL} \right)^2} \tag{14}$$

We are now in a position where we can examine carefully the performance of the dynamic belay. For this purpose, we shall restrict ourselves to the most adverse fall ($H/L = 2$) and consider the loads resulting from the fall of a 150-pound man. Using equation (14), computations were made and the results plotted in Figure 3 for $\frac{1}{2}$ -inch manila rope and $\frac{7}{16}$ -inch nylon rope, respectively, for various ratios of amount of rope permitted to slide to the initial length of rope s/L . Those portions of the curves exceeding the breaking strength of the rope are drawn dotted.

Figure 3 gives a concise picture of the dynamic belay. For example, by consulting the curves, it is seen that when the rope is held rigid, that is, when $s/L = 0$, the maximum possible tension develops as the result of a fall. This tension for $\frac{1}{2}$ -inch manila is 3830 pounds and for $\frac{7}{16}$ -inch nylon 2165 pounds and corresponds to the tensions produced on a rigid belay. If some slip occurs, the

tension decreases appreciably. With a ratio of $s/L = 0.2$, the tension for manila rope is 1520 pounds and for nylon rope 1250 pounds. Thus, if the rope length between climber and belayer is 10 feet, then only 2 feet of rope need slide over the belay point (to produce $s/L = 2/10 = 0.2$) to effect the above reduction in tension. The same

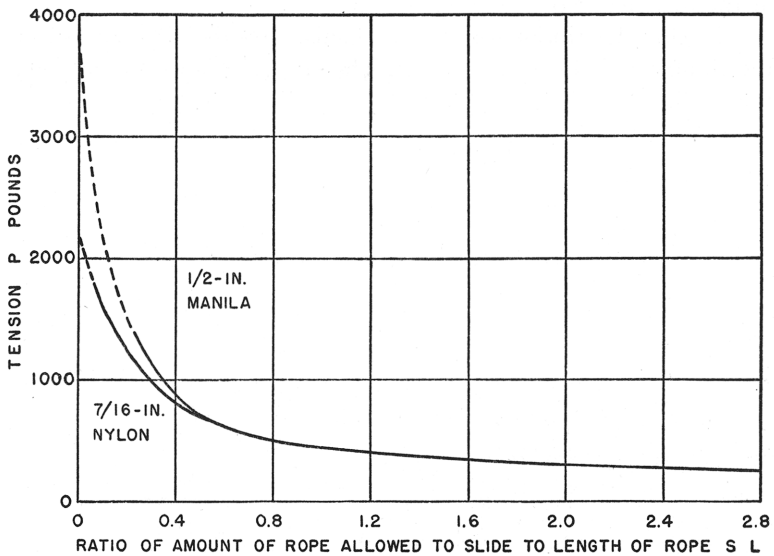


FIG. 3. Dynamic belay. The tension developed in rope for a free fall of a 150-pound man through a distance equal to twice the length of rope between climber and belay, $H = 2L$, for various ratios of s/L .

decrease in tension is achieved for 20 feet of rope between climber and belayer by allowing 4 feet of rope to slide. With a ratio of $s/L = 0.5$, the tension for manila rope is 720 pounds, and for nylon rope 710 pounds. In this case, with 10 feet of rope between climber and belayer, 5 additional feet must be used to arrest the fall, and with 20 feet between climber and belayer, 10 feet are required.

It is clear that a remarkable reduction in load is achieved with the dynamic belay. The tension due to a fall, for any given length of rope between climber and belayer, is basically a function of the length of rope that the belayer allows to slide, under control, over the support. The more rope that is employed in bringing a fall to

a stop, the lower the load experienced by climber, belayer and rope. The amount of rope permitted to slide may be chosen so that the load will be well below the maximum that any of the links in the belaying chain can withstand. The curves vividly demonstrate that no great slip is required to reduce appreciably the maximum tension. A value of $s/L = 0.50$ gives a tension, for both manila and nylon, of about 710 pounds. For values of s/L equal to or greater than 0.50, the tensions for manila and nylon are roughly the same.

Is it possible to set a criterion for holding the load at a value which can be withstood easily and safely by men and equipment? Let us set a load of four times the weight of a climber as the maximum tension to be developed in a rope under any circumstances. This is a load that can be tolerated by men and equipment. For a 150-pound man this is 600 pounds. What, now, is the length of rope a belayer should permit to slide so that $4W$ is not exceeded? The answer follows from equation (12). We substitute x from equation (2) into equation (12) and then substitute $4W$ for P to give

$$s = \frac{1}{3} \left(H - \frac{4WL}{k} \right) \quad (15)$$

Since $4WL/k$ is small compared to H , it may be neglected so that

$$s \approx \frac{1}{3} H \quad (16)$$

In other words, in order to insure that the tension developed in the rope during the dynamic belay will not exceed $4W$, the belayer must use a length of rope equal to one-third the distance through which the climber falls, i.e., $1/3 H$. For the case where $H = 2L$, then $s \approx 2/3 L$. *For example, if a leader advances 30 feet above his belayer, the latter should have an additional 20 feet of rope at his disposal for belaying.* Therefore, on a lead of 30 feet, the total available rope between climber and belayer should be 50 feet. If, on the other hand, the maximum tension is not to exceed $2W$, then by a similar reduction of equation (12) we get

$$s = H \quad (17)$$

and for the case where $H = 2L$, equation (17) becomes $s = 2L$. Here, on a lead, say, of 10 feet, 20 feet of rope are required to effect the sliding belay.

It is the usual practice for a leader to advance the entire length

of rope between the second man and himself. Thus, if three men are climbing with a 120-foot rope, the standard separation between the leader and the second man is 60 feet, and the leader can and often does advance this distance. Such a procedure is dangerous, for it prevents the application of the dynamic belay. The second man has no additional rope available with the result that, in case of a fall, only a static or, at best, a resilient belay can be employed. It is necessary, therefore, for the second man to retain enough rope so that the dynamic belay may be used effectively if the leader falls. With 60 feet of rope between the leader and the second man, the former should not advance more than 36 feet, thereby leaving 24 feet for belaying. A fall may then be handled adequately and the tension in the rope kept from exceeding $4W$.

To test the dynamic belay, the dummy and load dynamometer were employed again in a series of experiments. For this purpose, old, worn sisal that had deteriorated considerably was selected. The object was to employ rope with so little strength that only by proper handling could failure be prevented. It was analogous to catching a marlin on a trout line. The set-up shown in Figure 4 was adopted for these tests. The dummy could be raised a convenient height above a piton and the fall arrested by a belayer. First, a static belay was attempted. The dummy was raised $2\frac{1}{2}$ feet above the piton, and dropped through 5 feet, the lower end of the rope being rigidly fixed. The ratio of H/L was 0.145. The rope broke over the karabiner at a measured tension of 940 pounds. The test was repeated, except that a dynamic belay was attempted. In the first try the belayer allowed 7.5 feet of rope to slide, brought the dummy to a stop in midair and kept the maximum measured tension to 500 pounds. In the second try, 10 feet of rope were permitted to slide, the dummy was arrested, and the measured tension was less than 400 pounds. A third try was made with the free drop increased to 8 feet. Again, the dummy was stopped by allowing 20 feet of rope to slide, and the measured load was less than 400 pounds. Next, the free drop was increased to 12 feet. Once more the dummy was halted completely after a slide of 22 feet. The tension developed was 500 pounds. These tests confirm the predictions of the theory and demonstrate the remarkable effectiveness of the dynamic belay in arresting a fall.

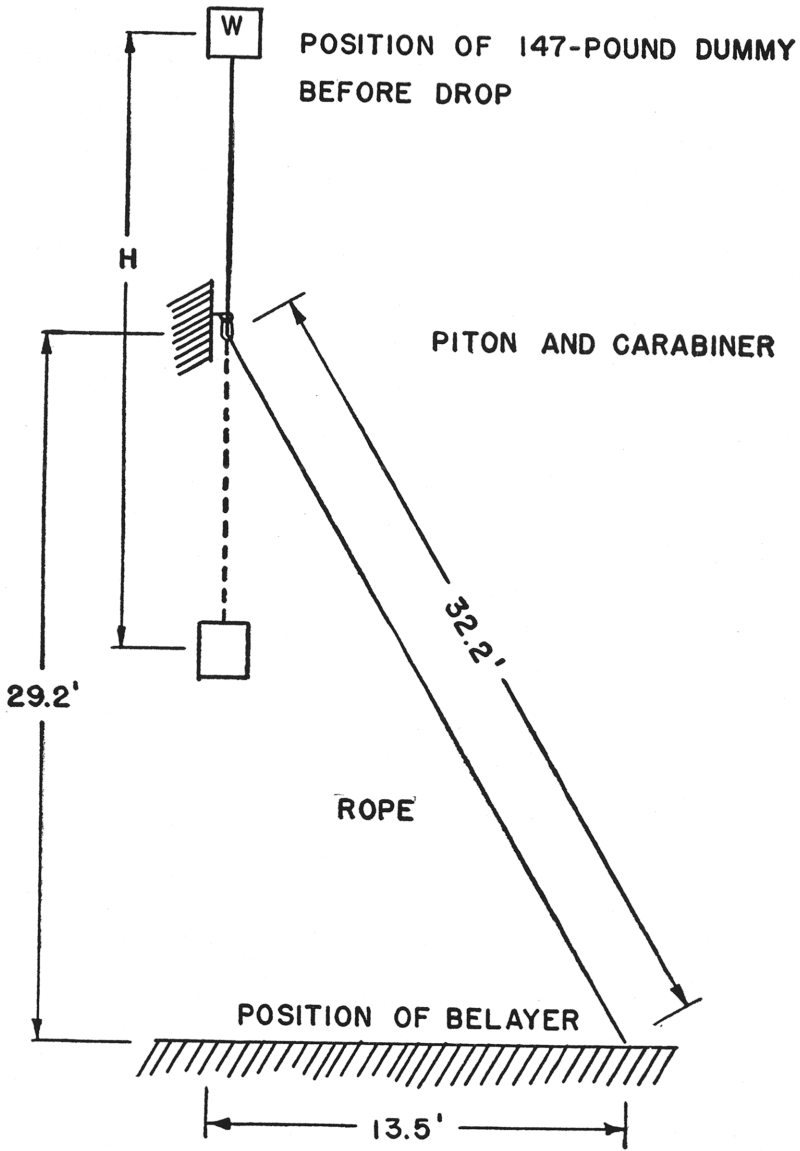


FIG. 4. Schematic representation of the set-up used in field tests.

PITON PROTECTION

The use of pitons and karabiners for protection can be studied to good advantage with the aid of the theory of belaying. How do these devices contribute to increased safety? The answer lies in the fact that their basic function is to shorten the possible fall without shortening the length of rope. Mathematically, this means that the H/L ratio is reduced from a possible maximum of 2 to some smaller value with a corresponding reduction in tension P . In the static belay, the tension P is given by equation (5), in the resilient belay by equation (9), and in the dynamic belay by equation (13). In all these equations, the tension is a function of H/L . If the ratio H/L decreases, the tension P decreases.

As a leader climbs, his margin of safety is increased every time he employs a piton. If a lead above a piton is not permitted to exceed, say, 10 feet, then the maximum free fall is 20 feet. Now, if a climber continues to ascend for greater distances, but drives in a piton every 10 feet, the length of rope increases, but the maximum free fall still remains the same. Consequently, the ratio H/L will decrease and, with it, the tension developed in the rope in case of a fall.

Thus, if a leader, weighing, say, 150 pounds, falls vertically on a static belay without the aid of pitons, the ratio H/L will be 2, and the maximum tension developed in the rope will be the maximum that is possible. For $\frac{1}{2}$ -inch manila, the tension will be 3830 pounds (see Fig. 2); and, since this exceeds the breaking strength (with a knot) of 1400 pounds, the rope will fail. However, if after climbing 10 feet the leader uses a piton, then a fall from 10 feet above the piton, for which the ratio H/L is $20/20 = 1$, now produces a tension of only 2750 pounds. A second piton gives rise to a ratio $H/L = 20/30 = 0.67$ and produces a tension of 2280 pounds. The two pitons have reduced the tension due to a fall on a static belay from 3830 to 2280 pounds. By using more pitons or confining the advance above a piton to less than 10 feet, a greater reduction in tension is achieved. In this example, even with the reduction in tension due to the use of pitons, it is evident that $\frac{1}{2}$ -inch manila will not sustain these falls. While the above computations have been made only for $\frac{1}{2}$ -inch manila, similar computations may be made for $\frac{3}{16}$ -inch nylon and also for the cases of the resilient and

dynamic belays with corresponding results. The conclusion is obvious. When long leads are attempted, the use of pitons can materially increase the climber's safety.

The amount of rope that a belayer must retain in order to apply the dynamic belay need not be as great when pitons are used for protection as would be required without pitons. We have shown that in order to keep the maximum tension in the rope from exceeding $4W$, it is necessary to let slide a length of rope equal to $\frac{1}{3} H$. As a climber advances above his belay point, the amount of rope that the belayer must retain increases proportionally to the distance the climber can fall. We have shown also that, with 60 feet of rope between the leader and the second man, the leader may not advance more than 36 feet unprotected. If pitons are used for protection, say every 10 feet, then the maximum fall cannot exceed 20 feet. Since the amount of rope that the belayer needs to apply a dynamic belay so that the tension does not exceed $4W$ is $\frac{1}{3} H$, the belayer now need not retain more than 7 feet of rope. The leader may advance 53 feet above the belayer. Thus, for a given length of rope, the use of pitons for protection considerably extends the lead distance without increasing the tension that would result in case of a fall.

Sometimes, however, the protection expected from a piton is not achieved because excessive friction or a twist or jamming of the rope occurs. Under these circumstances, the piton near the climber may act as a fixed end for the rope, so that the effective length of rope available for absorbing energy is reduced and the ratio H/L may closely if not actually reach the value of 2. It is desirable, therefore, in order to avoid this contingency, that the rope be removed from the lower pitons and karabiners as upper ones are used.

LOAD-TIME RELATIONSHIPS

It was previously emphasized that the tension developed in a rope due to a fall is the peak load. In some fashion the tension has to increase from zero to this maximum and then decrease. How rapidly does this occur? More precisely, what is the relation between tension and time? Consider a weight falling freely through a distance H feet on a length of rope L feet, rigidly fixed at one end (static belay). At some time t the rope will have a tension P and

elongation x . Apply Newton's laws of motion. The basic differential equation is

$$\frac{W}{g} \frac{d^2x}{dt^2} = W - P \quad (18)$$

where g is the acceleration due to gravity. The rope is again assumed to be elastic. Hence equation (2) is used in equation (18) to yield

$$\frac{d^2x}{dt^2} + \frac{kx}{WL} - g = 0 \quad (19)$$

whose solution is

$$x = \left(a_0 - \frac{WL}{k}\right) \cos\left(t\sqrt{\frac{kg}{WL}}\right) + \frac{a_1}{\sqrt{\frac{kg}{WL}}} \sin\left(t\sqrt{\frac{kg}{WL}}\right) + \frac{WL}{k} \quad (20)$$

in which a_0 and a_1 are constants of integration. When $t = 0$, then

$x = 0$ and therefore $a_0 = 0$. Likewise when $t = 0$, $\frac{dx}{dt} = \sqrt{2gH}$ and $a_1 = \sqrt{2gH}$. Equation (20) reduces to

$$x = \sqrt{\frac{2WHL}{k}} \sin\left(t\sqrt{\frac{kg}{WL}}\right) - \frac{WL}{k} \cos\left(t\sqrt{\frac{kg}{WL}}\right) + \frac{WL}{k} \quad (21)$$

The tension follows from inserting equation (21) into equation (2)

$$P = \sqrt{\frac{2WHk}{L}} \sin\left(t\sqrt{\frac{kg}{WL}}\right) - W \cos\left(t\sqrt{\frac{kg}{WL}}\right) + W \quad (22)$$

Another form of the equation is

$$P = W + W \sqrt{1 + \frac{2kH}{WL}} \sin\left[t\sqrt{\frac{kg}{WL}} - \arcsin \frac{1}{\sqrt{1 + \frac{2kH}{WL}}}\right] \quad (23)$$

Equations (22) and (23) show how the tension in the rope varies with time as a result of an impact load due to a free fall of a weight onto the rope. The practical significance of these equations may readily be understood by considering an actual fall. Let a man weighing 150 pounds fall 20 feet on a 10-foot length of rope. The H/L ratio is 2. Using equation (23), it is possible to compute what the tension in the rope will be at any instant after the rope has begun to stretch. If these values of tension and time are plotted, the

result will be an oscillating curve. By changing the H/L ratios, a family of curves is obtained. This has been done for 1/2-inch manila rope and is given in Figure 5, and for 7/16-inch nylon rope and is given in Figure 6.

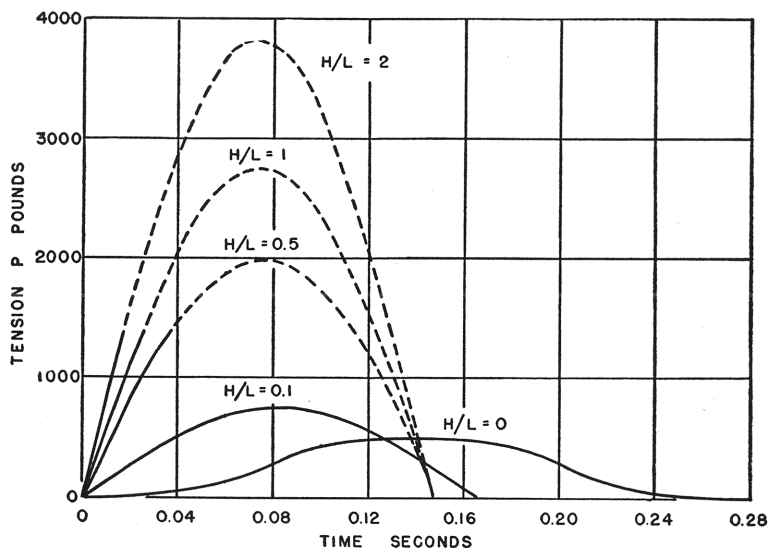


FIG. 5. Relation between tension and time in 1/2-inch manila rope for a fall of a 150-pound man on a static belay for various ratios of H/L.

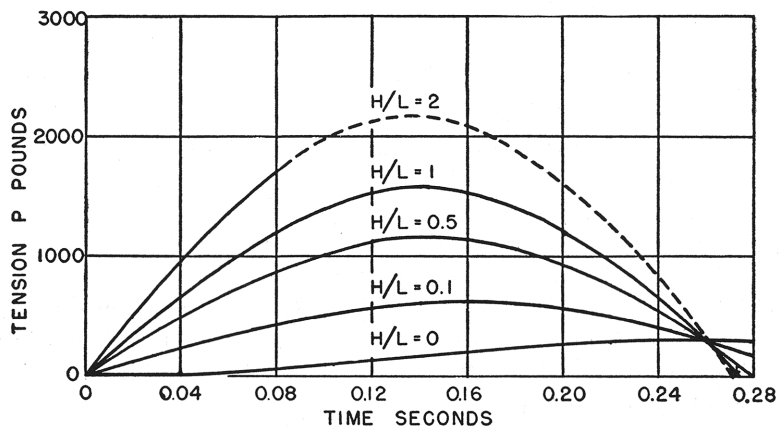


FIG. 6. Relation between tension and time in 7/16-inch nylon rope for a fall of a 150-pound man on a static belay for various ratios of H/L.

Theoretically, a weight dropped on a rope will continue to oscillate indefinitely, like a mass attached to a coil spring. Actually, the oscillations will be dampened rapidly, owing primarily to the absorption of energy by the friction generated by the relative motion of the fibers and strands of the rope. Furthermore, since rope cannot take a compressive load, the negative half of the cycle of oscillation has no physical meaning. In addition, for those situations in which the breaking strength of the rope is exceeded (1400 pounds for manila and 1800 pounds for nylon), the curves are drawn with a dotted line to indicate that the rope would have failed and that these parts of the curves are hypothetical.

Equation (23) and the curves of Figures 5 and 6 show that the tension P varies sinusoidally with time. The time it takes the tension to reach its maximum value from the instant the rope commences to stretch is given by

$$T = \frac{\pi}{2} \sqrt{\frac{WL}{kg}} + \sqrt{\frac{WL}{kg}} \arcsin \frac{1}{\sqrt{1 + \frac{2kH}{WL}}} \quad (24)$$

provided the rope does not break. For identical falls of 150 pounds on 10 feet of rope, the time required for the load to attain its maximum value is almost twice as great for nylon as for manila. The value for $\frac{7}{16}$ -inch nylon is approximately 0.14 second and for $\frac{1}{2}$ -inch manila 0.075 second. It is apparent that the duration of an impact is very small.

The relation between tension and time for the resilient belay may be derived in a fashion similar to that for the static belay, but it will not be treated here.

In the dynamic belay, the static belay solution is applicable up to the instant the rope begins to slide, for until then the rope acts as if it were rigidly fixed at the belay point. In the theory of the dynamic belay, it is assumed that, once the rope starts to slide, the tension P in the rope remains constant until all the energy of the fall is absorbed and the fall is arrested. We now inquire how long the tension remains at P . Since P is constant, the weight W is moving with constant velocity v . The tension P persists for time Θ and is given by the equation

$$\Theta = \frac{s}{v} \quad (25)$$

where s is the length of the rope that is allowed to slide. The velocity v is obtained by differentiating equation (21) to give

$$v = \frac{dx}{dt} = \sqrt{2Hg} \cos\left(t \sqrt{\frac{kg}{WL}}\right) - \sqrt{\frac{WLg}{k}} \sin\left(t \sqrt{\frac{kg}{WL}}\right) \quad (26)$$

Therefore

$$\theta = \frac{s}{\sqrt{2gH} \cos\left(t \sqrt{\frac{kg}{WL}}\right) - \sqrt{\frac{WLg}{k}} \sin\left(t \sqrt{\frac{kg}{WL}}\right)} \quad (27)$$

An alternate equation for v is obtained directly by integrating equation (18) to give

$$v = \frac{dx}{dt} = \sqrt{2gH + 2gx - \frac{kg}{WL} x^2} \quad (28)$$

The value for x from equation (2) is substituted into equation (28), yielding

$$v = \sqrt{2gH + \frac{2gPL}{k} - \frac{gP^2L}{Wk}} \quad (29)$$

Therefore

$$\theta = \frac{s}{\sqrt{2gH + \frac{2gPL}{k} - \frac{gP^2L}{Wk}}} \quad (30)$$

or

$$\theta = \frac{\frac{s}{L}}{\sqrt{\frac{2gH}{L^2} + \frac{2gP}{kL} - \frac{gP^2}{WkL}}} \quad (31)$$

In equation (31), P is not an independent variable but is related to both s/L and H/L by equation (13).

The value of θ may be calculated readily from equation (31). This was done for a man weighing 150 pounds falling 20 feet on a 10-foot length of rope. The results are shown graphically in Figure 7 for $\frac{1}{2}$ -inch manila rope and $\frac{7}{16}$ -inch nylon rope. The curves for both ropes coincide over most of the load range. Above the breaking strength of the rope, the curves are drawn with dotted lines.

The magnitude of Θ has a practical importance for the belayer, for Θ is the time required to let enough rope slide, under control, so that the maximum tension will not exceed some predetermined value. If the maximum tension in the rope is not to exceed, say, $4W$, then, from previous considerations, a length of rope equal to $\frac{1}{3} H$ must be permitted to slide. The time Θ required for the rope to

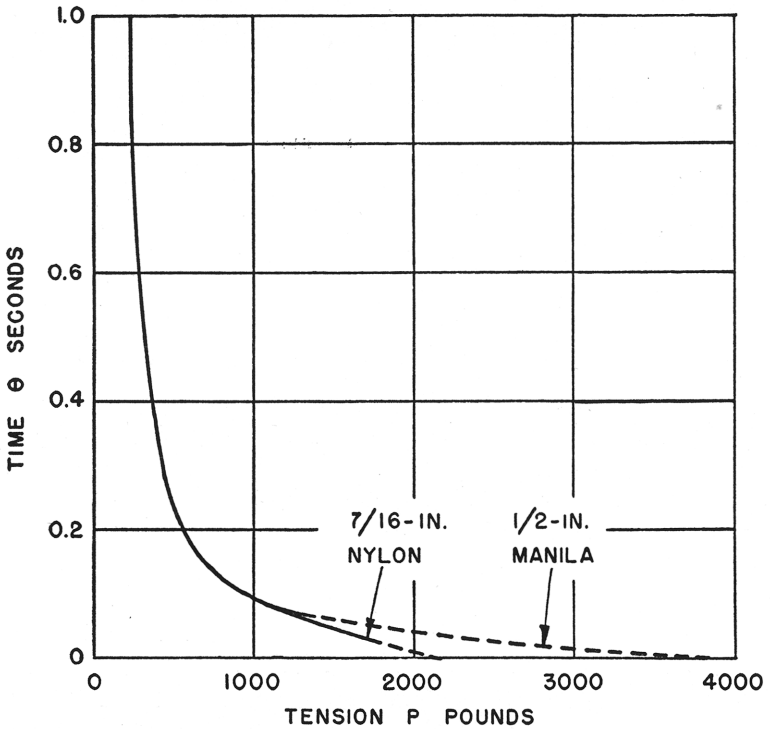


FIG. 7. Time duration of the peak tension in the dynamic belay.

slide $\frac{1}{3} H$ is 0.22 second, and during this time the tension will be 600 pounds ($4W$). If more rope is allowed to slide, the tension will be less but the time involved will be greater, whereas if less rope is allowed to slide, the tension will rise rapidly and the time involved will become small.

The discussion has been confined thus far to the physical phenomena occurring in the rope from the instant the rope begins to

stretch owing to the impact of a fall. Preceding the elongation and resultant absorption of energy by the rope is the time interval in which the climber is falling freely. This time interval may be denoted by τ and is given by the equation for a freely falling mass:

$$H = \frac{1}{2} g\tau^2 \tag{32}$$

where H is the distance through which the mass falls and g is the acceleration due to gravity. The duration of a free fall is

$$\tau = \sqrt{\frac{2H}{g}} \tag{33}$$

For the typical situation where three people are climbing with a 120-foot rope, the normal separation between the leader and second man is 60 feet. A fall of 120 feet is possible. The duration of this fall, from equation (33), is 2.73 seconds. The times involved in shorter falls are given in the following table.

FREE FALL	DURATION OF FALL
H	τ
<i>Feet</i>	<i>Seconds</i>
10	0.79
20	1.12
50	1.80
75	2.16
100	2.50
120	2.73

We now have a picture of what transpires in a fall. First, there is a time interval τ during which the climber falls freely a distance H , and during which neither the climber nor the rope is subjected to any load or strain. Then, at the end of distance H , with all the slack in the rope gone, the rope begins to stretch and the tension P builds up. In the case of the static belay, this tension increases sinusoidally to a peak load and immediately decreases. In the case of the dynamic belay, this tension increases to a peak load, and remains at this peak load until the fall is arrested. The load in the dynamic belay is less than that in the static belay.

The analysis of the times involved in a fall discloses a fact of major importance: a fall is more or less an instantaneous event. A

belayer must be always alert and prepared to arrest a fall and not expect or hope to assume a proper stance or position after the fall has commenced. This latter practice is dangerous and can result in a serious accident. Neither should a belayer attempt to take in slack during the fall. Even for a high fall of 120 feet, there are only 2.73 seconds available before the slack is gone and the rope begins to strain. In so short a time interval, very little, if any, slack can be taken in by the belayer. More harm than good would result, for the belayer would no longer be prepared to handle the rope properly.

SUMMARY AND CONCLUSIONS

In this study of belaying, an attempt has been made to present the principles involved in arresting a fall and to show the nature and magnitude of the phenomena occurring in the rope. A physical theory has been developed from which predictions can be made as to the forces and strains and load-time relationships produced in the rope. Experimental evidence has been used to substantiate the predictions of the theory.

The basic concept involved in belaying is the conservation of energy. Associated with every fall is a quantity of energy which must be absorbed in order to arrest the fall. The belay, therefore, is a mechanism whereby this energy is absorbed, partially or completely. Three types of belays are used commonly in climbing: the static or rigid belay, the resilient or indirect belay, and the dynamic or sliding belay. The effectiveness of these three types in arresting a fall has been investigated in the body of this report.

In the static belay, only the rope serves to absorb energy. It can be stated definitely that this belay is of little value in catching the leader. There is no chance of arresting a serious free fall with the static belay, for the tension produced in the rope will exceed the breaking strength and rupture the rope. A leader is inviting disaster if he allows his second man to employ a static belay. Only in belaying from above, that is, only if the climber is well below the point of support, and if the belayer prevents the accumulation of slack, is the static belay permissible.

In the resilient belay, the strain in the rope and the deflection of the belay point under load combine to absorb energy. The typical resilient belay is the body belay. Here the "give" of the body when

used for snubbing the rope lessens the impact force of a fall, but not enough to make the resilient belay safe. The rope will fail when this belay is employed for arresting a leader on a fall. Furthermore, with such a belay the precaution of tying the belayer to a piton must always be taken, for the force developed by a fall may jerk the belayer off his perch.

In the dynamic belay, the rope is allowed to slide, under control, over the belay point, thus absorbing the energy of a fall. The dynamic belay provides a positive technique for effectively catching a fall with a minimum of load on the rope, climber or belayer. By controlling the length of rope that is allowed to slide over the support, the forces can be held at values that are safe and easy to handle. In the case of a free fall, the tension in the rope can be kept to four times the weight of the climber if an additional length of rope equal to one-third the distance of the fall is allowed to slide in bringing the climber to rest. This value of $\frac{1}{3} H$ may be adopted as the desired length of rope to use under all conditions. It must be remembered that in serious, difficult or high-angle climbing the leader must therefore never extend himself the full length of the rope, for then his second man has no means of employing a dynamic belay. The leader should terminate his pitches so that the second man still has available a length of rope equal to $\frac{1}{3} H$. *This means that with 60 feet of rope between the leader and belayer, and without piton protection, the leader should not advance more than 36 feet.*

The time interval from the instant a man falls to the moment the rope begins to stretch is usually less than 2.75 seconds. The time consumed in the belay proper is invariably less than one second. A fall, therefore, may be considered an instantaneous occurrence. In order for a belayer to perform his function adequately, it is imperative that he be constantly alert and in a position to effect a belay. There is no time available for a belayer to assume an adequate stance after the leader has started to fall, nor is there time for the belayer to pull in slack.

In conclusion, it may be stated that the only safe method of arresting a fall is by means of the dynamic belay, and that preparedness is the only course open to a belayer.