

## Who is Thabit ibn Qurra?

- Thabit ibn Qurra al-Harrani (826-901) is a brilliant Arabian scientist who was born in "Harran" in the northern of Mesopotamia.
- He was born in a rich family.
- He go to Bagdad to study mathematics and medicines.


## Some of His Contributions:

- He translated many of Greek works in mathematics, like Euclid, Apollonius, Archimedes and Ptolemy.
- He criticize and reform the Ptolemy's model of solar system.
- He expanded the concept of "Traditional Geometry" to "Geometrical Algebra".


## Some of His Contributions:

- He developed non - Euclidean geometry, spherical geometry, integral calculus and real numbers.
- He proposed a formula to find amicable numbers.

We decided to focus on one of these contributions, which is "amicable numbers"

## A Brief History about Amicable Numbers:

- In the $6^{\text {th }}$ century, Pythagoras discovered the first amicable numbers, which are 220 and 284.
- In the dark ages, amicable numbers were considered as numbers that have mystical powers.
- These numbers were used in religion and magic (especially in love and friendship).


## A Brief History about Amicable Numbers:

- Greece used amicable numbers in horoscopes, talismans and charms.
- By the time, Thabit ibn Qurra proposed a formula to find amicable numbers.


## The algorithm works in the following way:

1. WRITE THE POWERS OF 2 TO THE POWER OF N IN THE FIRST ROW, STARTING WITH $\mathrm{N}=1$
2. write the triple of the numbers of the first row in the second row

| $2^{n}$ | 2 | 4 | 8 | 16 | 32 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\times 3$ | 6 | 12 | 24 | 48 | 96 |

3. ADD ANOTHER ROW - IN IT WRITE THE NUMBER FROM THE ŞECOND ROW MINUSS 1, LIKE THIS

$$
\begin{array}{cccccc} 
& 2 & 4 & 8 & 16 & 32 \\
-1 & 6 & 12 & 24 & 48 & 96 \\
5 & 11 & 23 & 47 & 95
\end{array}
$$

4. ADD YET ANOTHER ROW AND WRITE THE PRODUCT OF THE NUMBERS IN THE SECOND ROW OF THE COLUMN YOU ARE IN AND THE LEFT NEIGHBOR OF THIS NUMBER

| 2 | 4 | 8 | 16 | 32 |
| :---: | :---: | :---: | :---: | :---: |
| 6 | 12 | 24 | 48 | 96 |
| 5 | 11 | 23 | 47 | 95 |
|  | $12 \times 6-1=$ <br> 71 | $24 \times 12-1=$ <br> 287 | $48 \times 24-1=$ <br> 1151 | $96 \times 48-1=$ <br> 4607 |

look and try to find primes which are neighbouring numbers in the third row and have a corresponding prime in the column above the greater one from the third row in the fourth row. The final number you need is number in the top row, exactly above the prime in the fourth.

| 2 | 4 | 8 | 16 | 32 |
| :---: | :---: | :---: | :---: | :---: |
| 6 | 12 | 24 | 48 | 96 |
| 5 | 11 | 23 | 47 | 95 |
|  | 71 | 287 | 1151 | 4607 |

5 and 11 are neighbouring and primes
corresponding to them is 71
corresponding to 71 is 4 .
You can now 'make' the smallest pair of
friendly numbers:
$4 \times 5 \times 11=220$ and $4 \times 71=284$

|  | x |  |  |  |
| :---: | :---: | :--- | :--- | :--- |
|  |  |  |  |  |
| y | z |  |  |  |
|  | p |  |  |  |

So, friendly pair will be the product of them in such a way that $\mathrm{A}=x^{*} y * z$ and $\mathrm{B}=x^{*} p$

## THABIT IBN QURRA'Ş RULE

$$
\begin{gathered}
H=3 * 2^{n}-1 \\
T=3 * 2^{n-1}-1 \\
S=9 * 2^{2 n-1}-1
\end{gathered}
$$

If they are ALL primes, then the pair $\left(\underline{2}^{n} \times h \times t, \underline{2}^{n} \times s\right)$ is an amicable pair.

## EXERCISE:

$\times N=2$
$\times N=3$

$$
\begin{gathered}
H=3 * 2^{n}-1 \\
T=3 * 2^{n-1}-1 \\
S=9 * 2^{2 n-1}-1
\end{gathered}
$$

$$
\left(2^{n} \times h \times t, \underline{2}^{n} \times s\right)
$$

## HOW MANY AMICABLE NUMBERS ARE THERE?

There is in fact no proof supporting either that there is finite quantity of amicable numbers.

## AMICABLE NUMBERS VS. PERFECT NUMBERS

## Amicable Numbers

\{220,284\}
$220=1,2,4,5,10,11,20,22,44,55,110$
Sum of proper divisor:
$1+2+4+5+10+11+20+22+44+55+110=284$
$284=1,2,4,71,142$
Sum of proper divisor:
$1+2+4+71+142=220$

## Perfect Numbers <br> $6=1,2,3$ <br> Sum of proper divisor:

$1+2+3=6$

## AMICABLE NUMBERS VS. PERFECT NUMBERS

In amicable numbers, the pair of numbers is different, while in perfect number the pair of numbers is the same.

## References

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