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Mass Difference between the Sigma Hyperons

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The problem of the mass differences of the triplet sigma is discussed within the general standpoint of the electromagnetic structures of elementary particles.

As the present data about these particles are very poor, we consider many possibilities for the values of the anomalous magnetic moments, especially those which according to meson theory have a reasonable magnitude and sign. Assuming the external distributions of charge and anomalous magnetic moments, as well as the magnitudes of these moments, the inner structures are guessed in order to explain the mass differences of the triplet.

Effects of higher order corrections due to strong interactions are also discussed.

§ 1. Introduction

In the Gell-Mann-Nishijima scheme, the hyperon Σ is considered as a triplet state which consists of Σ^+, Σ^-, Σ^0; these component particles have somewhat large mass differences as follows:*

$$\begin{aligned}\delta M_{-+}&=M_{\Sigma^-}-M_{\Sigma^+}=6.84\pm0.40\text{ Mev},\\ \delta M_{-0}&=M_{\Sigma^-}-M_{\Sigma^0}=4{,}45\pm0{,}63\text{ Mev}.\end{aligned}\tag{1}$$

In a similar way as for the neutron-proton mass difference, which was considered by Feynman and Speisman,[1] many authors tried to explain the above mass differences by assuming that they are due to electromagnetic interactions. In this way Marshak et al.[2], Katsumori[3], by the use of cut-off *à la* Feynman could explain the mass differences δM_{-+} and δM_{-0}. For doing this, they have used extremely high values for the anomalous magnetic moments (a.m.m.) of the sigmas, what does not seem reasonable. Indeed, Marshak et al.[2] has shown that if we do not want an a.m.m. of Σ^+ and Σ^- greater in absolute value than 4, in hyperon magnetons, then μ_0, the a.m.m. of Σ^0, must be greater than 1.5. On the other hand, as for the moment μ_0 the pions do not contribute, while the kaons do, we need a coupling constant $g_K^2/4\pi\sim3$, which is higher than the value generally accepted. And more, μ_- the a.m.m. of Σ^-, is always assumed positive, what is not clear from the standpoint of meson theory.

As is well known, perturbation theory gives wrong values for the a.m.m. of

* Latest data of sigma's mass-differences (July 1959) are
$\delta M_{-+}=6.76\pm0.33$ Mev, $\delta M_{-0}=4.45\pm0.4$ Mev.

the nucleons, but if we neglect the nucleon current and take into account only the pion current, then we can obtain good results. By analogy we expect that for the determination of the anomalous moments μ_+ and μ_- of Σ^+ and Σ^-, a good result can be obtained if we use a perturbation treatment where the hyperon current is neglected. Therefore we should get for μ_+ a positive value of the order of the proton's a.m.m. and for μ_- a negative one of the order of neutron's a.m.m. This situation has not been examined by the previous authors.

Similar considerations were made by Kato and Takeda.[4] They calculated δM_{-+} by supposing that Σ^- and Σ^+ have charge and a.m.m. distributions given by an exponential form factor, with a root mean square (r.m.s.) radius equal to 0.8 yukawa (1 yukawa$=10^{-13}$ cm), which is taken from Stanford experiments on $e-N$ scattering, and also assuming some reasonable a.m.m. values such as $\mu_0=0.08$ and $\mu_-=-1.74$. Their result was $\delta M_{-+}\sim 0.16\, m_e$ which is a rather small value compared with the experimental one.

Bransden and Moorhouse[5] suggested that the relatively high value of δM_{-+} could be due to the appreciable mass difference between K^0 and K^+. On these lines they have calculated δM_{-+} by perturbation theory, assuming $g^2_{\Sigma NK}/4\pi=4$. This assumption however is not consistent with the value $g^2_K/4\pi\sim 0.3$ deduced from the cross sections of K^+-p scattering at energies of 100 Mev.[6] For this reason we do not believe that the mass-difference of the intermediate kaons could give such an appreciable contribution to the δM_{-+} and δM_{-0} mass differences.

Recently Hiida-Sawamura[7] and Cini et al.[8] tried to explain the mass difference between neutron and proton by the use of exponential form factors, for the nucleons, obtained from Stanford experiments, and they got a negative result. Nevertheless, by making a critical analysis of the results of Stanford experiments, Katayama, Taketani and co-workers[9] (this paper is hereafter denoted by [A]) have shown that it is possible to obtain a correct neutron-proton mass difference by a convenient modification of the form factors in the region of high energies, where experimental data are not yet available. They have taken, as a possibility, form factors which are linear combinations of two Yukawa functions, which, within experimental errors, can explain the Stanford results in the range of energies used up to now. The form factors, however, differ appreciably from the generally accepted exponential form factor for higher energies.

The purpose of the present article is to explain the mass differences between the hyperons sigma, by the introduction of suitable electromagnetic structures for these particles and consideration of different sets of values for the anomalous magnetic moments, especially those which, in analogy to the nucleon case, without having a very high absolute value, have a convenient sign. This possibility was not developed successfully by the previous authors.[2),3),4] Assuming that Σ^+ and Σ^- have similar structures as the nucleons, in the region of low momentum transfer studied in the Stanford experiments, and examining some possibilities for Σ^0, we can determine the inner structures in such a way as to explain the observed

δM_{-+} and δM_{-0} mass differences.

According to some authors,[10] the principle of charge independence in strong interactions which apparently holds in a certain level corresponding to great distances, could break down at short distances. Taketani suggested that this breakdown of the principle of charge independence at high energies could be responsible for the appreciable mass difference between the various components of each multiplet in the Gell-Mann-Nishijima scheme. However, even if there are some small fluctuations in the charge independence of strong interactions, these could be properly taken into account, in a phenomenological way, in the electromagnetic form factors which describe the inner structure of elementary particles.

§ 2. Effects of change of the form factors at high energy region

In what follows, we shall use the following hamiltonian for the interaction of the hyperons, $\bar{\psi}(x), \psi(x)$ with the electromagnetic field $A_\mu(x)$, $F_{\mu\nu}(x)$:

$$H=H_1+H_2+H_3+H_4+H_5 \tag{2}$$

where :

$$\begin{aligned}
H_1 &= -ie\int dx\, dy\, \bar{\psi}(x)\gamma_\mu\psi(x)\boldsymbol{F}_1(x-y)A_\mu(y),\\
H_2 &= -\mu\frac{e}{4M}\int dx\, dy\, \bar{\psi}(x)\sigma_{\mu\nu}\psi(x)\boldsymbol{F}_2(x-y)\boldsymbol{F}_{\mu\nu}(y),\\
H_3 &= \frac{e^2}{M^2}\int dx\, dy\, dz\, d\omega\bar{\psi}(x)\psi(y)\boldsymbol{F}_3(xyz\omega)\boldsymbol{F}_{\mu\nu}(z)\boldsymbol{F}_{\mu\nu}(\omega),\\
H_4 &= \frac{ie^2}{M^2}\int dx\, dy\, dz\, d\omega\, \bar{\psi}(x)\gamma_5[\boldsymbol{F}_{12}(z)\boldsymbol{F}_{34}(\omega)+\cdots]\psi(y)\boldsymbol{F}_4(xyz\omega),\\
H_5 &= \frac{ie^2}{M^2}\int dx\, dy\, dz\, d\omega\bar{\psi}(x)\gamma_i\gamma_m\gamma_n\psi(y)\boldsymbol{F}_{is}\frac{\partial \boldsymbol{F}_{mn}}{\partial\omega_s}\boldsymbol{F}_5(xyz\omega).
\end{aligned} \tag{3}$$

μ is the a.m.m. and M is the mass of the hyperon. Although the theory is based in a non-local interaction which does not satisfies the requirements of microcausality, it is possible that (2) could be derived from a more fundamental theory where the causality principle holds.

Neglecting H_3, H_4 and H_5, whose contributions we shall discuss in § 3, we obtain for the self-mass of the hyperon up to e^2:

$$\delta M=\delta M_{11}+\delta M_{12}+\delta M_{22} \tag{4}$$

where :

$$\delta M_{11}=\frac{2ie^2}{(2\pi)^4}\int \boldsymbol{F}_1^2(q^2)\,\frac{i\not{q}-M}{-2pq+q^2}\,\frac{1}{q^2}\,d^4q, \tag{5}$$

$$\delta M_{12}=\frac{3ie^2}{(2\pi)^4}\,\frac{\mu}{M}\int\frac{\boldsymbol{F}_1(q^2)\boldsymbol{F}_2(q^2)}{-2pq+q^2}d^4q, \tag{6}$$

$$\delta M_{22}=\frac{ie^2}{(2\pi)^4}\frac{\mu^2}{4M^2}\int\frac{F_2^2(q^2)[4ipq\!\!\!/+4Mq^2-3iq^2q\!\!\!/]}{[-2pq+q^2]q^2}d^4q \tag{7}$$

with $q^2=\vec{q}^2-q_0^2$ and $q\!\!\!/=\gamma\cdot q$.

Calculations of Hiida-Sawamura,[7] Cini et al.[8] have shown that with the exponential form factor we cannot explain the positive value of the mass difference between neutron and proton. However, according to [A] this would mean that, at high momemtum transfers the electromagnetic form factors should deviate appreciably form the exponential one, while for low momenta they would practically coincide. It is well known that, for example, the Villi-Clementel[11] model nicely explains the results of Stanford experiments up to now. In this model, the proton consists of a negative point charge at its center surrounded by a positively charged cloud of Yukawa type. Deviations from the exponential form factor would occur, to give an example, for electrons colliding with protons with an energy 1.1 Bev, for which energy we would have a zero diffraction at 140°.

Katayama, Taketani and co-workers[9] have considered the problem of the inner structure of the nucleons from a more general standpoint. They have taken a certain class of form factors which explain all available experimental data for $e-N$ scattering (which correspond to distances greater than $0.5y$) and tried to determine the inner structure by attacking the problem of neutron-proton mass difference.

The form factor of the nucleon must satisfy the following conditions:

1) The total charge must be equal to one, i.e. $F(o)=1$.
2) The r.m.s. radius must be given by $\langle r^2\rangle=(0.80y)^2$.
3) For $r\geq 0.5y$ it should give more or less 60% of the total charge. This means that the Stanford experiments give only information about the amount of charge for $r\geq 0.5y$, but nothing cannot say about the details of inner parts.

A sufficiently general choice, satisfying 1) and 2), is the superposition of two Yukawa form factors:

$$F(q^2)=\frac{1-\lambda_1}{\lambda_0-\lambda_1}\frac{1}{1+(\lambda_0/\Lambda)(q^2/M^2)}-\frac{1-\lambda_0}{\lambda_0-\lambda_1}\frac{1}{1+(\lambda_1/\Lambda)(q^2/M^2)} \tag{8}$$

where $\Lambda=6/\langle r^2\rangle M^2$, λ_0 and λ_1 being dimensionless parameters. Condition 3) reads:

$$\lambda_0\lambda_1-\tfrac{3}{2}(\lambda_0+\lambda_1)+\tfrac{5}{4}=0, \tag{9}$$

λ_0 lying between 1/2 and 5/6; for $\lambda_0=\lambda_1=1/2$ (8) reduces to the exponential form factor and $\lambda_0=5/6$, $\lambda_1=0$ to the Villi-Clementel's.

As, according to the pion theory, Σ^+ and Σ^- have external charge and a.m.m. distributions very similar to the nucleons, it is natural to extend to the sigmas, in the low energy region, the form factors which were obtained in Stanford experiments for the nucleons, with a r.m.s. radius of the same order, i.e. $0.8y$.

The internal region could then be guessed by the consideration of the mass differences of the sigmas.

Therefore, describing the charge and a.m.m. distributions of Σ^- and Σ^+ by the form factor (8) with the requirement (9), Σ^- will correspond to a positively charged core surrounded by a negatively charged cloud, while the opposite would occur for Σ^+. The parameter λ_0 (or λ_1) gives the amount of charge in the core, as well as its radius, and therefore informs us about the inner region. For the Σ^0 we can suppose it to have only an a.m.m. distribution. No charge distribution is considered for this particle.

We shall restrict ourselves to the consideration of 3-dimensional form factors, i.e. $F(q^2)=F(\vec{q}^2)$. Under such circumstances, using Eqs. (4) ~ (7), we get the following expressions for the mass differences:

$$\delta M_{-+}=\frac{e^2}{4\pi}\frac{M}{\pi}\Big[\tfrac{1}{2}L_-^{(0)}-I_-^{(0)}-I_-^{(1)}-\tfrac{1}{2}L_+^{(0)}+I_+^{(0)}++I_+^{(1)}$$
$$+3(\mu_-I_-^{(1)}+\mu_+I_+^{(1)})--\tfrac{5}{4}(\mu_-^2I_-^{(1)}-\mu_+^2I_+^{(1)})\Big], \qquad (10)$$

$$\delta M_{-0}=\frac{e^2}{4\pi}\frac{M}{\pi}\Big[\tfrac{1}{2}L_-^{(0)}-I_-^{(0)}+(3\mu_--1.25\mu^2-1)I_-^{(1)}+1.25\mu_0^2I_0^{(1)}\Big], \qquad (11)$$

where $L^{(0)}$, $I^{(0)}$, $I^{(1)}$ for the form factor (8) with condition (9) are well defined functions of λ_0; They are given in the Appendix. We also give the expression for $I^{(1)}$ which corresponds to the exponential form factor. The lower indices $-$, $+$ and o refer to the Σ^-, Σ^+ and Σ^0 particles respectively.

As the present experimental information about the sigmas is very poor, many possibilities exist as far as the structure and values of anomalous magnetic moments are concerned. We shall consider here only a few instances of structures which can explain the observed mass differences δM_{-+} and δM_{-0}, and limiting ourselves to the case $|\mu_0|<1$. This last choice is connected to the restrictions imposed by meson theory. We shall then stress the possibility of a negative μ_-, with a value of the order of -2, which is the interesting case from the standpoint of pion theory, as was discussed in the introduction. On the other hand, we shall assume that Σ^+ and Σ^- have the same structure as the nucleons given by Eqs. (8) and (9), with the same r.m.s. radius.*

Case I. Same structure for Σ^+, Σ^- and Σ^0

The structure will be described by the form factors (8) and (9) with the same λ_0 for all the three Σ. Therefore the charge distributions of Σ^- and Σ^+ are perfectly symmetrical. In this case, δM_{-+} is given by

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* Really in this article we have used a value for the r.m.s. radius of Σ somewhat smaller than 0.8*y*. But the general conclusions will not be affected. We would like to express our thanks to Professors Y. Katayama, M. Taketani and Mssrs. D. R. de Oliveira, S. Ragusa for putting at our disposal their numerical data on the neutron-proton mass difference.

$$\delta M_{-+} = (e^2/4\pi)\ 2\mu_0\ I_-^{(1)} [3-2{,}5(\mu_- - \mu_0)], \tag{12}$$

using (10), where we have taken $2\mu_0 = \mu_+ + \mu_-$, the functions $L^{(0)}$, $I^{(0)}$ and $I^{(1)}$ having the same value for all the three particles.

Now, as $I_-^{(1)}$ is a positive definite quantity, in order to obtain $\delta M_{-+} > 0$ we must have $\mu_- > 1.\ 2 + \mu_0$ for $\mu_0 < 0$ and $\mu_- < 1.\ 2 + \mu_0$ for $\mu_0 > 0$. We have studied

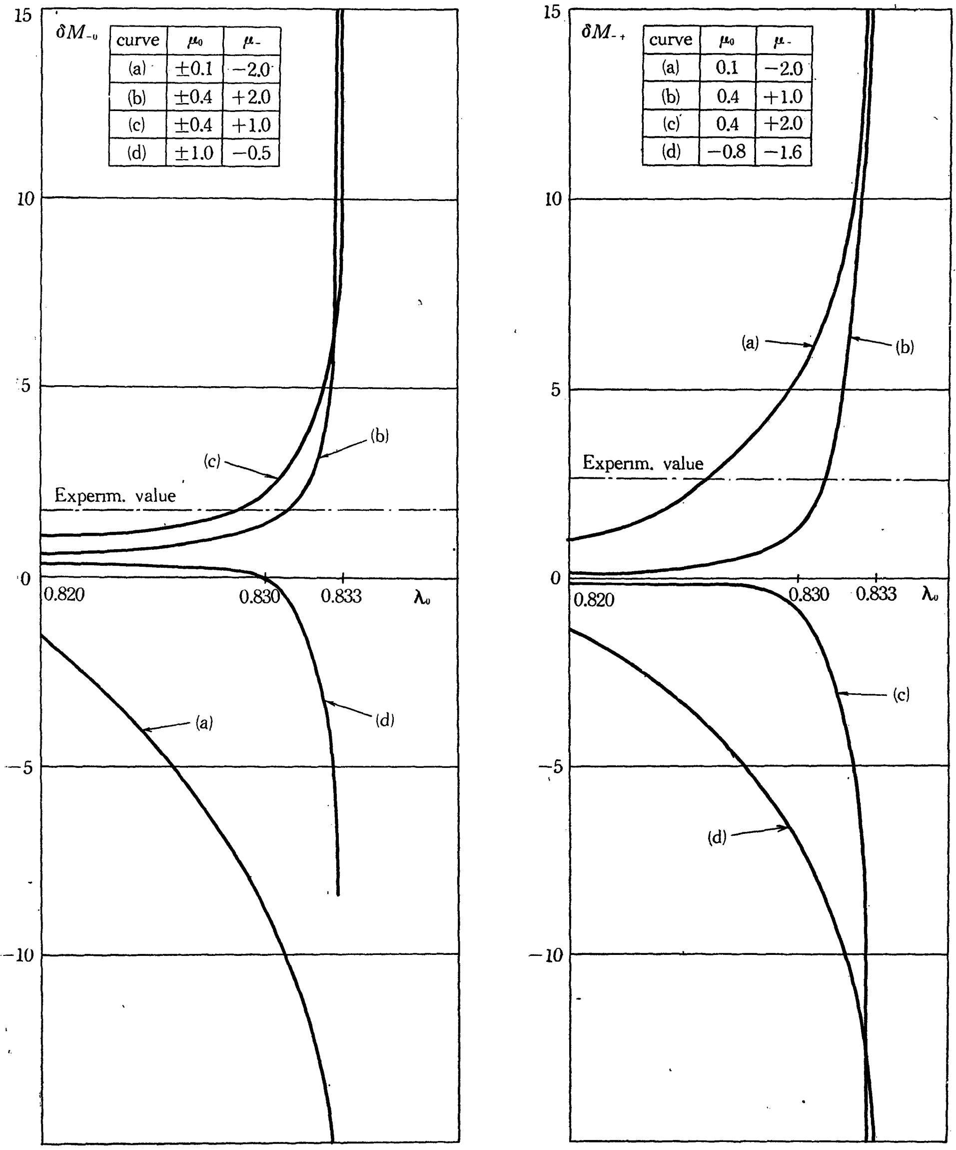

Fig. 1. Plotting of δM_{-+} and δM_{-0} as functions of the internal parameter λ_0, in units of $e^2/4\pi \times M/\pi = 1/137\ M/\pi$. Fog the δM_{-+} case, the choice $\mu_0 = -0{,}8$ $\mu_- = 2$ gives a curve very similar to (a).

graphically the behaviour of δM_{-+} and δM_{-0} as a function of the internal parameter λ_0, for different sets of values of μ_0 and μ_-, especially those which correspond to $|\mu_0|\lesssim 1$. In Fig. 1 we give some of them. It is not possible to realize $\delta M_{-0}>0$ with $|\mu_0|\leq 1$ and μ_- negative with an absolute value greater than 0.5. For instance, the possibility $|\mu_0|\lesssim 1$ and $\mu_-\sim -2$ which seems interesting from the point of view of meson theory, cannot explain the positive value of δM_{-0}, while for realizing the experimental δM_{-+} we must have $\mu_0>0$.

Case II. Different structure for Σ^0

As in the case *I*, we could not explain δM_{-0} by assuming the possibility $|\mu_0|\leq 1$ and μ_-negative with an absolute value $\gtrsim 0.5$, we shall now consider the situation in which Σ^0 has a different a.m.m. structure from Σ^+ and Σ^-.

The simplest modification would be to assume the same radius and external structure, given by (8) and (9), for the three particles, but making the internal structure of the Σ^0 different from the other two.

As a possibility we can take $\mu_0=0.2$ and $\mu_-=-1.8$. To explain the experimental value of δM_{-+}, we then need a common structure for Σ^- and Σ^+, characterized by $\lambda_0=0.83$, and in order to explain the value of δM_{-0} we shall need to ascribe to Σ^0 a value of λ_0 slightly greater than 0.833.

Another possibility would be to suppose that Σ^0 has not the same r.m.s. radius than Σ^+ and Σ^-. We can describe the a.m.m. distribution of Σ^0 by an exponential form factor for example; confining ourselves only to values of $\mu_-\sim -2$ and $|\mu_0|\leq 1$, we have obtained the following values (see Table I):

Table I.

μ_0	μ_-	λ_0	δM_{-+}	δM_{-0}	$r^{(o)}/r^{(n)}$
0.2	−1.8	0.83	15	experimental value	1/35
0.75	−2.2	0.82	14.8	〃	1/6
0.75	−2.5	0.82	16	〃	1/6.5
0.8	−4.3	0.81	14	〃	1/7
0.8	−2.0	0.82	15	〃	1/8

δM_{-+} given in electron mass unit, $r^{(n)}=0.8y=$r.m.s. of nucleon, $r^{(0)}=$r.m.s. of Σ^0 (exponentiat form factor).

The last four rows are especially interesting, because there $r^{(0)}$ is not very small. Its order may be the right one, as for the a.m.m. form factor of Σ^0 the pions do not contribute. We should therefore expect for Σ^0 an r.m.s. radius smaller than the corresponding one for the nucleon, as a consequence of the higher mass of the kaon relatively to the pion.

Case III. Different structures for Σ^+, Σ^- and Σ^0

As for $\mu_0>0$ and $\mu_-<0$, with $|\mu_-|>\mu_0$ we cannot explain δM_{-+}, under the hypothesis that Σ^- and Σ^+ have the same structure, we shall now suppose that the triplet Σ has different structures.

This can be realized, for instance, by supposing that the hyperons Σ have the same radius and external structure, differing only in their internal structures. The corresponding form factor could be taken as (8) and (9), and then we should have a different λ_0 for each component of the multiplet. In this model the charge distributions of Σ^+ and Σ^- are not perfectly symmetric.

As an example with $\mu_0=0.1$ and $\mu_-=-2$, the experimental value of δM_{-+} can be explained by taking $\lambda_{0-}=0.81$ for Σ^- and λ_{0+} between 0.82 and 0.83 for Σ^+. To explain the observed δM_{-0} mass difference we should need a value λ_0, of the hyperon Σ^0, slightly greater than 0.833.

Another possibility would be to suppose that Σ^0 has not the same radius and external distribution than Σ^+ and Σ^-. Taking an exponential form factor, as an example, we obtain the following Table:

Table II.

μ_0	μ_-	μ_+	λ_{0+}	λ_{0-}	δM_{-+}	$r^{(o)}/r^{(n)}$	δM_{-0}
0.1	−2	2.6	between 0.82∼0.83	0.81	experimental value	1/10	experimental value
−0.5	−2.2	1.2	0.83	0.80	13.5	1/8	12∼15
−0.8	−2.6	1	between 0.83∼0.833	0.78	experimental value	1/4	14

δM_{-+} and δM_{-0} given in electron mass unit.

By the arguments given above, it may be that the r.m.s. radius of Σ^0 is reasonable.

§ 3. Effects of the higher order effects corrections of the strong interactions

Our problem now is to discuss the influence of the terms like H_3, H_4 and H_5 which appear in the interaction (2) upon the hyperon mass differences δM_{-+} and δM_{-0}. These terms mean the consideration of electromagnetic self-energy correc-

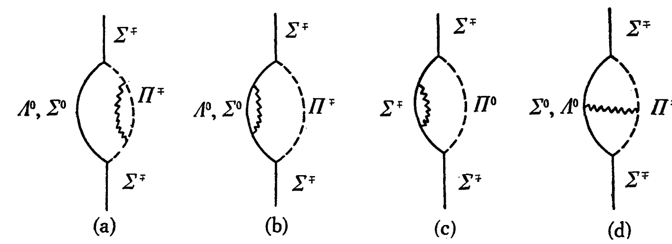

Fig. 2.

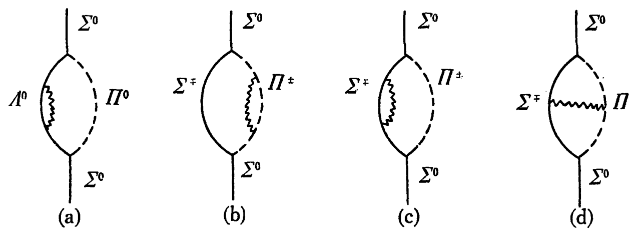

Fig. 3.

tions in the intermediate states in which the hyperon Σ dissociates [Figs. 2, 3 and 4].

Up to the order e^2 only H_3 contributes to the self-mass.

We shall study the contribution of these graphs to δM_{-+} and δM_{-0}, by using relativistic perturbation theory and assuming global symmetry.

a) *Influence of Pion interactions*

The corresponding graphs are raised in Figs. 2 and 3:
Figs. 2(a) and 2(b) give $\delta M_{-+}=0$, while 2(c) gives

$$(\delta M_{-+})_{(2c)} \sim \frac{g^2_{\Sigma\Sigma\pi}}{(4\pi)^2}(M_{\Sigma^-}-M_{\Sigma^+}) = \frac{15}{4\pi}(M_{\Sigma^-}-M_{\Sigma^+}), \tag{13}$$

where $\Sigma_{\Sigma^-}-\Sigma_{\Sigma^+}$ is the mass difference of the intermediary $\Sigma^{\pm}$.

Now Fig. 2(a) which contains only the intermediary Λ^0 and 3(a) gives

$$(\delta M_{-0})_{(2a)-(3a)} \sim \frac{g^2_{\Sigma\Lambda\pi}}{48\pi^2}\,\frac{M_\pi}{M}\,\delta M_\pi \sim \frac{1}{20}(M_{\Sigma^-}-M_{\Sigma^0}), \tag{14}$$

where δM_π is the mass difference between π^- and π^0. The quantity (14) is neglegible.

Fig. 2(b) which contains only the intermediary Σ^0 and 3(c) gives

$$(\delta M_{-0})_{(2b)-(3c)} \sim -\frac{g^2_{\Sigma\Sigma\pi}}{(4\pi)^2}(M_{\Sigma^-}-M_{\Sigma^0}) = -\frac{15}{4\pi}(M_{\Sigma^-}-M_{\Sigma^0}). \tag{15}$$

Considering Fig. 2(d) and 3(d), in which we keep only the terms of higher order divergence, and if we make a cut-off of the order of baryon mass, we obtain

$$(\delta M_{\Sigma^0})_{(2d)-(3d)} \sim 0.8\,(\mu(\Lambda^0)-\mu(\Sigma^0)) \tag{16}$$

in electron mass units. Taking $|\mu(\Lambda^0)| \sim |\mu(\Sigma^0)| \lesssim 1$, we see that the contribution of Fig. 2(d) and Fig. 3(d) gives at most 10% for δM_{-0}.

A similar contribution to δM_{-+} appears through Fig. 2(d) as

$$(\delta M_{-+})_{(2d)} \sim 1.6\,(\mu(\Lambda^0)+\mu(\Sigma^0)).$$

From (13) and (15) we see that the contribution to δM_{-+} and δM_{-0} could be of the same order as the electromagnetic mass differences. But, as is well known, the relativistic perturbation theory, when applied to the various processes as nuclear forces, scattering of pions and nucleons at low energies, anomalous magnetic moments of the nucleons, etc. gives wrong results. We can get reasonable results for the above phenomena if we neglect the nucleon current (static theory) and make a cut-off in the momenta of the intermediary virtual states. Extending the static model to the interactions baryon-pion, we would obtain values for δM_{-+} and δM_{-0} which are hundred times smaller than those given by (13) and (15). Then, it may be that the true value would be in between, that is, ten or twenty times smaller. Therefore, according to these arguments, the effects of higher order pion interactions would give a very small contribution to δM_{-+} and

δM_{-0}, of 20% at most. But to have a better answer we should have a good method of calculation for strong interactions.

b) *Influence of kaon interactions*

The corresponding graphs are raised in Fig. 4

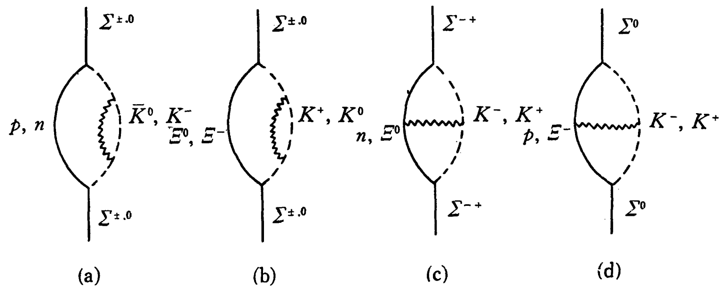

Fig. 4.

Fig. (4a)−(4b) were considered by Brahsden and Moorhouse,[5] who claimed that it would be possible to explain the experimental δM_{-+} and δM_{-0} through the mass difference of $\bar{K}^0$ and K^-. In the more favorable case, which corresponds to assume the coupling of the kaons with nucleons as pseudoscalar and to cascade particles as scalar, we have

$$\delta M_{-+}=[M(\bar{K}^0)-M(K^-)]\left[0.037\frac{g^2{}_{\Sigma NK}}{4\pi}+0.445\frac{g^2{}_{\Sigma\Xi K}}{4\pi}\right] \tag{17}$$

by neglecting the intermediate $N-P$ and $\Xi^- - \Xi^0$ mass differences. To explain the experimental value of δM_{-+} they have taken $g^2_{\Sigma NK}/4\pi=4$ and $g^2_{\Sigma\Xi K}/4\pi=3$; these values seem to be extremely high. In fact, in order to explain the observed cross sections for scattering at low energies of the order of 100 Mev, we need coupling constants ten times smaller than the value assumed by Brahsden and Moorhouse.

Similar considerations are valid for δM_{-0}. Fig. 4(c) gives, with $(g^2{}_{\Sigma NK}/4\pi)=g^2{}_{\Sigma\Xi K}/4\pi\sim 0.3$,

$$(\delta M_{-+})_{(4c)}\sim 2.10^{-2}[\mu(n)-\mu(\Xi^0)]. \tag{18}$$

whille Figs. 4(c) and 4(d) give

$$(\delta M_{-0})_{(4c)-(4d)}\sim 2.10^{-2}\left[\mu(n)-\frac{\mu(p)+\mu(\Xi^-)}{2}\right] \tag{19}$$

in electron mass units. We see, from (19) and (20), that even if the a.m.m. of the cascade particle is very high, let us say 10, even then δM_{-+} and δM_{-0} obtained from Figs. 4(c) and 4(d) are less than one electron mass.

§ 4. Conclusions

The point of view of this paper as well as of Marshak et al.,[2] Katsumori[3] and Kato and Takeda,[4] is that the mass differences δM_{-+} and δM_{-0} are due to electromagnetic interactions. The authors of references (2) and (3) have used a particular set of form factors to describe the electromagnetic structure of Σ, namely the Feynman form factors and the straight cut-off respectively. However, with these types of structures we cannot explain δM_{-+} and δM_{-0} if we do not assume also very high anomalous magnetic moments with convenient signs. In fact, an interesting case which has been studied in this paper is that one in which the anomalous magnetic moment is smaller than 1 for Σ^0 and of the order of -2 for Σ^-. This happens when we use a pertur bation treatment for the a.m.m. of the sigma, where in analogy to the nucleon case we neglect the hyperon current. This situation was also considered by Kato and Takeda[4] for the $\Sigma^- - \Sigma^+$ mass difference, by extending to these particles the Stanford form factors (exponential) obtained for the nucleons. They have got a very small value for δM_{-+} and so in order to obtain higher values we should have a very small r.m.s. radius.

In this paper we have shown that in order to get a right result for δM_{-+} and δM_{-0} with reasonable values for the a.m.m. of the sigma, we need not take very small r.m.s. radius, it is sufficient to modify the inner region of the electromagnetic structures, some external behaviour being assumed. That is, if we know the outer region of the electromagnetic form factor, we can then, by consideration of problems like the mass differences, get some knowledge about the inner structure. This is the procedure which the Japanese school of Nuclear Forces has followed in order to gain some information about the high energy region of nuclear forces,[12] after the low energy phenomena were reasonably understood. The same standpoint was followed in the problem of the nucleon mass difference in reference 9).

To reach more definite conclusions about the sigma mass differences it would be necessary to have experimental data for the anomalous magnetic moments and for the electromagnetic form factors. Although the experimental situation is very poor, we can raise some plausible hypothesis from meson theory, viz., to assume $|\mu_0| \lesssim 1$ as well as that Σ^- and Σ^+ have, in the low energy region, the same electromagetic structures as the nucleons. We then consider the following possibilties for explaining the mass differences δM_{-+} and δM_{-0}:

i) $\mu_- > 0,\ \mu_0 > 0,$ ii) $\mu_- > 0,\ \mu_0 < 0$;

iii) $\mu_- < 0,\ \mu_0 > 0,$ iv) $\mu_- < 0,\ \mu_0 < 0.$

The cases i) and ii) can be realised by assuming that the sigmas have the same structure. The case iii) for $|\mu_-| \gtrsim 0.5$ only works if we assume a different structure for Σ^0, while for case iv) we need to assume different structures for the three particles.

From the arguments given in this paper as well as in [A], the mass difference between the components of a baryon multiplet is essentially determined by the inner structure, i.e. the region of high energy phenomena. In this region the strong, intermediary and weak interactions have an important role, and they are supposed to be taken into account by convenient electromagnetic form factors. It would be interesting to see more clearly how these various interactions in the high energy, where the law of charge independence could not be valid, influence on the mass differences of the components of baryon and meson multiplets.

Evidently many other possibilities exist to explain the sigma mass differences. For instance, we could ascribe to Σ^0 *a* charged form factor, which could give an appreciable contribution to δM_{-+} and δM_{-0}, but this will not be considered here.

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Appendix

For the form factor (8) :

$$L^{(0)}=\frac{\Lambda}{(\lambda_0-\lambda_1)^2}\left[\frac{(1-\lambda_1)^2}{\lambda_0^2}+2\frac{(1-\lambda_1)(1-\lambda_0)}{(\lambda_0-\lambda_1)}\log\frac{\Lambda}{\lambda_0}+(\lambda_0\leftrightarrow\lambda_1)\right],$$

$$I^{(0)}=\frac{\Lambda}{(\lambda_0-\lambda_1)^2}\left[-\frac{1}{2}\frac{(1-\lambda_1)}{(\lambda_0-\Lambda)}+(1-\lambda_1)\left(2\frac{1-\lambda_0}{\lambda_0-\lambda_1}-\frac{1-\lambda_1}{2\lambda_0}\frac{\lambda_0-2\Lambda}{\lambda_0-\Lambda}\right)\Omega\left(\frac{\Lambda}{\lambda_0}\right)+(\lambda_0\leftrightarrow\lambda_1)\right],$$

$$I^{(1)}=\frac{\Lambda^2}{(\lambda_0-\lambda_1)^2}\left[-\frac{1}{2}\frac{(1-\lambda_1)^2}{\lambda_0(\lambda_0-\Lambda)}+\frac{(1-\lambda_1)}{\lambda_0}\left(2\frac{1-\lambda_0}{\lambda_0-\lambda_1}-\frac{1-\lambda_1}{2\lambda_0}\frac{\lambda_0-2\Lambda}{\lambda_0-\Lambda}+\frac{1-\lambda_1}{\lambda_0}\right)\Omega\left(\frac{\Lambda}{\lambda_0}\right)+\right.$$
$$\left.+(\lambda_0\leftrightarrow\lambda_1)\right],$$

where

$$\Omega\left(\frac{\Lambda}{\lambda_0}\right)=4\omega\left(\frac{\Lambda}{\lambda_0}\right),$$

with

$$\omega(\lambda)=\begin{cases}\dfrac{1}{\sqrt{\lambda(4-\lambda)}}\operatorname{arctg}\sqrt{\dfrac{4-\lambda}{\lambda}} & \text{for } \lambda<4,\\[2ex] \dfrac{1}{\sqrt{\lambda(\lambda-4)}}\log\frac{1}{2}(\sqrt{\lambda}+\sqrt{\lambda-4}) & \text{for } \lambda>4.\end{cases}$$

If we take an exponential form factor

$$F(q^2)=\frac{1}{(1+1/2\Lambda\ q^2/M^2)^2},$$

we have

$$I^{(1)}=\frac{\Lambda^2}{12(1-2\Lambda)^3}\Big[(3-20\Lambda-32\Lambda^2)+12(1-8\Lambda+32\Lambda^2)\,\omega(8\Lambda)\Big],\ \text{etc.}$$

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