FORECAST OF SCHEDULE/COST STATUS UTILIZING COST PERFORMANCE REPORTS OF THE COST/SCHEDULE CONTROL SYSTEMS CRITERIA: A BAYESIAN APPROACH

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13 ABSTRACT

This report presents a Bayesian approach to a forecasting technique useful in projecting the future cost and schedule of work breakdown structure (WBS) items in Department of Defense contracts. The technique utilizes the data supplied in the cost performance reports (CPR) of the cost/schedule control systems criteria (C/SCSC). The forecast data are invaluable to the project manager supervising the contract, who might thereby avert costly schedule/cost program overruns. The advantages of this method are discussed in the present report and a solved example is hereby given.

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FOREWORD

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I. INTRODUCTION

The adoption of the Laird-Packard principle of closer ties with the contractor and more efficient supervision of the project operation throughout the life of the contract, created a new concept - that of establishing standards or criteria which enunciate the capabilities of a good cost/schedule management system, but leave the details of how to achieve these capabilities to the contractor. These standards are known as the cost/schedule control systems criteria (C/SCSC) and are contained in the Department of Defense Instruction 7000.2 (DODI 7000.2) . Generally, the C/SCSC require that the contractor's activities be integrated and performed in a formal, disciplined fashion, which will allow a follow-up of his contractual progress. The contractor is also required to periodically provide the program manager with work breakdown structure (WBS) summary data that allows assessing the contractual adherence to the approved cost and schedule plan of action for each of the items in the WBS, especially the cost and schedule of the project at completion. The cost performance report (CPR), furnished about once a month by the contractor for this purpose, relates the costs incurred to date to the budgeted cost of the work actually performed, as well as to that of the work originally scheduled. Variance of the cost and schedule of each item from those originally planned, are also reported. The data reported in the CPR are very valuable to the project manager because it keeps him updated about the progress of the contract. What is more important, however, is to be able to use these data to gain insight in the future status of the program. Stated otherwise, the project manager needs a forecasting tool. Such tool would enable him to predict cost/schedule problem areas that might require his immediate attention. Such vital information might be so valuable as to make it possible to avert costly schedule/cost program overruns. There are several possible forecasting techniques. Of these, two familiar ones are the discounted least squares technique and the time series. The former technique would involve extrapolation of the least squares equation, far beyond the available data range, a very risky procedure that might well lead to erroneous conclusions. The time series method is technically superior and dependable but much more elaborate.

The continuous influx of the monthly status reports (CPR) prompted the investigation of the Bayesian statistical approach,

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[1] Department of Defense Instruction DODI 7000.2, as Appendix E to Army Regualtion AR37-200, August 1968.

which calculates a posterior probability from an assumed prior probability. The present developed Bayes technique forecasts the expected cost/schedule at a future point in terms of the current data, as well as the variances to those forecast values.

II. METHODOLOGY

Bayes' theorem states that[2]:

$$p(\theta|\theta_0) = \frac{p(\theta)\, p(\theta_0|\theta)}{p(\theta_0)}$$

$$\propto p(\theta)\, p(\theta_0|\theta)$$

(since $p(\theta_0)$ is independent of θ and equals $\int p(\theta)p(\theta_0|\theta)\, d\theta$) $p(\theta|\theta_0)$ is the posterior pdf for the parameter vector θ, given the sample information θ_0; $p(\theta)$ is the prior pdf, for the parameter vector θ; and $p(\theta_0|\theta)$, viewed as a function of θ is the likelihood function. The following is a possible manipulation of Bayes' theorem to apply to the C/SCSC problem:

Assume that an item cost/schedule at some point "o" along the project be normally distributed with mean μ and variance σ_0^2. If the estimated cost/schedule at this point is μ_0, then the likelihood function is

$$p(\mu_0|\mu) = \frac{1}{\sqrt{2\pi}\,\sigma_0} \exp\left[\frac{-1}{2\sigma_0^2}(\mu_0 - \mu)^2\right]$$

Now we need to know the prior pdf for the parameter μ. Let the estimated cost/schedule value of the same item at an earlier point "α" along the project be μ'_α, where $\mu_0 = c\mu'_\alpha$ Let μ_α be the actual value of the cost/schedule of the item at this point as reported in the cost performance report (CPR). Now, we can reasonably construct the prior distribution for the parameter μ, by assuming that it is a normal distribution with an expected value of $c\mu_\alpha$, and a standard deviation of $c\sigma_\alpha$, where σ_α is the standard deviation of the item distribution at point "α". Such distribution will thus have

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2 Zellner, Arnold, An Introduction to Bayesian Inference in Econometrics, John Wiley & Sons, Inc., N. Y. 1971.

the form:

$$p(\mu) = \frac{1}{\sqrt{2\pi}\,\sigma_a c} \exp\left[-\frac{1}{2c^2\sigma_a^2}(\mu - c\mu_a)^2\right]$$

= prior pdf of μ

Combining this prior pdf with the likelihood function, the posterior pdf for the parameter μ becomes:

$$p(\mu \mid \mu_0) \propto p(\mu_0 \mid \mu)\, p(\mu)$$

$$\propto \exp\left[-\frac{1}{2c^2\sigma_a^2}(\mu - c\mu_a)^2 - \frac{1}{2\sigma_0^2}(\mu - \mu_0)^2\right]$$

$$\propto \exp\left[-\frac{1}{2}\left\{\frac{1}{c^2\sigma_c^2}(\mu - c\mu_a)^2 + \frac{1}{\sigma_0^2}(\mu - \mu_0)^2\right\}\right]$$

After some manipulation of the right hand side, we get:

$$p(\mu \mid \mu_0) \propto \exp\left[-\frac{1}{2}\left(\frac{\sigma_0^2 + c^2\sigma_a^2}{c^2\sigma_0^2\sigma_a^2}\right)\left\{\mu - \left(\frac{c\mu_a\sigma_0^2 + \mu_0 c^2\sigma_a^2}{\sigma_0^2 + c^2\sigma_a^2}\right)\right\}^2\right]$$

which shows that μ is normally distributed, a posteriori, with mean:

$$E(\mu) = \frac{c\,\mu_a\,\sigma_0^2 + \mu_0 c^2 \sigma_a^2}{\sigma_0^2 + c^2 \sigma_a^2}$$

and variance:

$$V(\mu) = \frac{c^2\,\sigma_0^2 \sigma_a^2}{\sigma_0^2 + c^2 \sigma_a^2}$$

Note that the posterior mean and posterior variance could be written in the forms:

$$E(\mu) = \frac{c\,\mu_a\,\sigma_0^2 + \mu_0 c^2 \sigma_a^2}{\sigma_0^2 + c^2 \sigma_a^2} = \frac{c\,\mu_a\,(c\sigma_a)^{-2} + \mu_0 (\sigma_0)^{-2}}{(c\sigma_a)^{-2} + (\sigma_0)^{-2}}$$

and

$$V(\mu) = \frac{c^2 \sigma_0^2 \sigma_a^2}{\sigma_0^2 + c^2 \sigma_a^2} = \frac{1}{(c\sigma_a)^{-2} + (\sigma_0)^{-2}}$$

which shows that the posterior mean is a weighted average of the prior mean $c\,\mu_a$ and the sample mean μ_0. with weights being the reciprocals

of $(c\sigma_a)^2$ and σ_0^2 respectively. If we let $(c\sigma_a)^{-2} = h_a$ and $\sigma_0^{-2} = h_0$ then:

$$E(\mu) = \frac{c\mu_a h_a + \mu_0 h_0}{h_a + h_0}$$

and:

$$V(\mu) = \frac{1}{h_a + h_0}$$

h_a and h_0 being the corresponding precision parameters. Hence the precision parameter associated with the posterior mean is just $\left[V(\mu)\right]^{-1} = h_a + h_0$, the sum of the prior and sample precision parameters.

III. DISCUSSION

This Bayesian approach to the problem of forecasting the cost and schedule of items involved in a Department of Defense contract, is simple and convenient.

Two main assumptions have been made during the course of development of this method: (1) that the cost and schedule at any point along the path of the project are normally distributed. Though not the most realistic, normal distributions are considered a fair approximation and are usually adopted for mathematical convenience; (2) that the prior distribution at point "o" has mean $c\mu_\alpha$ and standard deviation $c\sigma_\alpha$, c being the ratio between the planned cost/schedule at points "o" and "α", respectively. This is tantamount to assuming that the expected value of the cost/schedule at a particular point along the path of the project would relate proportionally with respect to the position of this point on the path. Such assumption is reasonable and logical.

To apply the formulas developed by the present method, it is needed to assign values to the standard deviations σ_α and σ_0. Fair estimates of these two quantities may be obtained by one of two methods: (1) subjective estimates through personnel that are knowledgeable about the particular contract; (2) using the cost/schedule variances reported in earlier cost performance reports (CPR), which are indicative of the dispersion, e.g., assuming they loosely follow a normal distribution, then calculating σ in the usual manner.

The advantages of this Bayesian Scheme are: (1) easy closed formulas are used, which can readily be handled with a desk calculator; (2) the formulas are equally valid at any point along the path of the program, with no extrapolation involved; (3) updating the forecast does not require a new elaborate smoothing or reiterative process, only a reapplication of the formulas by substituting the new data; (4) this is the only plausible method to use in the very early stage of the life of the contract, since then available information is too scanty for any other method to apply meaningfully.

APPENDIX: EXAMPLE

The adjoining table presents an actual contract cost performance report (CPR) of a particular item. The Bayesian Statistical technique will be used to forecast the cost at project completion of this item, projected from these reported data. For cost forecast, we need the following quantities: (1) Budgeted cost of work performed (BCWP), reported in column 8 in CPR; (2) Actual cost of work performed (ACWP) reported in column 9; (3) Budgeted cost at completion reported in column 12. The ACWP is μ_a, and the bugeted cost at completion is μ_0. The quantity c is μ_0/BCWP and the estimated values of σ are, $\sigma_a = 0.1\ \mu_a$ and $\sigma_0 = 0.05\ \mu_0$. Substituting the values of μ_a = 2416.0, μ_0 = 6716.6, σ_a =241.60, σ_0 =335.83, and c = 6716.6/2204.1 = 3.047. Hence:

$$E(\mu) = \frac{c\,\mu_a\sigma_0^2 + \mu_0 c^2 \sigma_a^2}{\sigma_0^2 + c^2\sigma_a^2}$$

$$= \frac{(3.047)(2416.0)(335.83)^2 + (6716.6)(3.047)^2(241.60)^2}{(335.83)^2 + (3.047)^2(241.60)^2}$$

$$= 6827.70 \text{ (in \$1000)}$$

and the statistical variance is

$$V(\mu) = \frac{c^2\sigma_0^2\sigma_a^2}{\sigma_0^2 + c^2\sigma_a^2} = 93353.61$$

$$\sigma = (V)^{1/2} = 305.5 \text{ (in \$1000)}$$

Therefore, based on the CPR reported data at point "a" of the project, the expected cost at completion, point "o", is $6,827,700, with a standard deviation of $305,500.

A similar procedure would be applied to the schedule problem. In this case, μ_a = 2,204.1, viz. the entry in column 8, whereas μ_j will remain the same, i.e., 6716.6 of column 12, and c = 2,286.4/6716.6, where the numerator is the entry in column 7. A value for σ_a will have to be estimated, and calculations will proceed as before.

COST PERFORMANCE REPORT (CPR)

ITEM WBS ELEMENT	CURRENT PERIOD					CUMULATIVE TO DATE					AT COMPLETION		
	BUDGETED COST		ACTUAL COST	VARIANCE		BUDGETED COST		ACTUAL COST	VARIANCE				
	WORK SCHED	WORK PERFORMED	WORK PERFORMED	SCHEDULE	COST	WORK SCHED	WORK PERFORMED	WORK PERFORMED	SCHEDULE	COST	BUDGETED	LATEST REVISED ESTIMATE	VARIANCE
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Rotor System	314.8	303.4	245.9	(11.4)	57.5	2,286.4	2,204.1	2,416.0	(82.3)	(211.9)	6,716.6	7,209.5	(492.9)

()Indicates Unfavorable variance (behind schedule or over cost)