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    EFFECT OF APERTURE ILLUMINATION
ON THE ON-AXIS POWER DENSITY OF A
    CIRCULAR MICROWAVE ANTENNA
by
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T. R. LaSalle

Weapons Development and Evaluation Laboratory


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Effect of Aperture Illumination on the On-axis Power Density of a Circular Microwave Antenna<br>by<br>T. R. LaSalle<br>Weapons Development and Evaluation Laboratory

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#### Abstract

This report describes the development of two methods of determining the on-axis microwave power density in both the Fresnel and Fraunhofer regions, for circular-aperture antennas with any arbitrary, axially symmetric aperture illumination. Both methods have been programmed for digital computation. These method.s are described and are applied to a number of different aperture distributions which could conceivably occur in practice. The resultant on-axis fields are compared and some conclusions relating the aperture distribution and the resulting on-axis fields are made.


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## FOREWORD

The work reported herein was conducted for the Bureau of Naval Weapons under WEPTASK No. RM1500001/210/F009-12-001, which provides for the conduct of supporting research studies in the field of guided missile safety.

This report was reviewed by the following persons of the Weapons Development and Evaluation Laboratory:
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## INTRODUCTION

The microwave fields associated with the high power radar systems of today, and the even higher power radars expected in the future, present definite hazards to personnel, ordnance and fuels unless appropriate safety precautions are incorporated. It is, therefore, necessary to determine which regions of the various radar fields exceed established safe working levels of power density. The hazardous areas should preferably be examined through actual measurements with the radar in operation. However, such measurements are time consuming, costly and susceptible to many variables. It thus becomes essential that an accurate and reliable method of computing the theoretical power density on the axis of the beam be available both to facilitate optimum test planning and to aid in interpretation of experimental results.

In general, resort to a highly sophisticated theoretical approach is not necessary in order to estimate the maximum values of power density which may be encountered along the beam axis, which is the usual requirement when safety measures are the primary consideration. For such purposes, it is permissible to base calcula.tions on the simplifying assumption that the antenna aperture is uniformly illuminated by the feed. This assumption yields calculated values of power density in the Fraunhofer (far) field which are at least as high as produced by any other distribution of the aperture field, hence the error introduced, if any, is on the side of safety throughout the far field. Moreover, this assumption results in predicted values of the power density maxima in the Fresnel field which, except for a few unusual cases, are not significantly lower than those based on other distributions. For these reasons, the calculation of "safe" distances for circular microwave antennas in connection with personnel hazards studies at the Naval Weapons Laboratory has been based on the assumption of a uniformly illuminated aperture. Under this assumption, far field computations are reduced to a straightforward application of the inverse square law, and the prediction of maximum hazards in the Fresnel region is made possible through formulas which approximate the upper envelope of the power density contour in this region. References (a) and (b) describe these methods in detail.

While methods based on uniform illumination are useful for establishing safe distances and other cases when only the Fresnel field maxima are of interest, they are not sufficient for a comprehensive evaluation of measurements obtained in the Fresnel field of
non-uniformly illuminated antennas since instrumentation might be located at points of minimum or intermediate intensity. While uniform illumination theoretically gives maximum antenna gain, practical antenna design usually necessitates reduction of the power at the aperture edge to decrease the loss from "spill-over" and decrease the power in the sidelobes. Thus the aperture amplitude distribution in practical antennas is usually designed so that the power is concentrated at the aperture center and is tapered to lower values at the edge.

The effect of these tapered illuminations on the axial field has been calculated by other investigators for a few distributions expressible as analytic mathematical functions. Not all distributions of interest can be adequately approximated by simple functions, however. Even for those distributions which can be so expressed, the manual solution of the Fresnel equation for a sufficient number of points is time-consuming and difficult. It was thus desirable to develop a method by which the axial field could be conveniently calculated by computer for any arbitrary aperture distribution. Two somewhat similar methods, each having certain advantages, have been developed to fulfill this requirement. This report describes these methods and their application to the common distributions generally used to approximate the aperture illumination of practical antennas. The methods have also been applied to a number of arbitrary distributions for which the on-axis field could not readily be calculated by the usual means.

## THEORY

Consider the antenna aperture of Figure 1. The scalar diffraction field at any point $P$ located at a distance $R$ along the axis of a circular aperture of constant phase is given by the Fresnel approximation equation, (Reference (b), Page 198).

$$
\begin{equation*}
U_{p}=j \frac{e^{-j k R}}{\lambda R} \int_{0}^{2 \pi} \int_{0}^{a} F\left(\rho, \phi^{\prime}\right) e^{-j \pi} \rho^{2} / \lambda R \cdot \rho d \rho d \phi^{\prime} \tag{1}
\end{equation*}
$$

$$
b_{1}
$$

where
$U_{p}$ is the field at a point $P$ on the axis of the aperture
$R$ is the axial distance from the aperture to the point $P$
$\lambda$ is the wave length of the radiation
a is the radius of the aperture
$\rho$ is the radial coordinate in the plane of the aperture
$\phi^{\prime}$ is the angular coordinate of a point in the plane of the aperture
$F\left(\dot{\rho}, \phi^{\prime}\right)$ is the field strength or amplitude taper in the plane of the aperture
k is the wave number $2 \pi / \lambda$
For the case of axially symmetric aperture illumination, equation (1) reduces to

$$
\begin{equation*}
U_{p}=j \frac{k e^{-j k R}}{R} \int_{0}^{a} F(\rho) e^{-j \pi \rho^{2} / \lambda R} \rho d \rho \tag{2}
\end{equation*}
$$

If we now define the dimensionless parameters

$$
x \equiv \frac{R \lambda}{a^{2}} \quad \text { and } \quad r \equiv \frac{\rho}{a}
$$

we have

$$
\begin{equation*}
U_{p}=j \frac{2 \pi e^{-j k R}}{x} \int_{0}^{1} F(r) e^{-j \pi r^{2} / x} r d r \tag{3}
\end{equation*}
$$

If the function $F(r)$ is normalized with respect to its value at the center of the aperture, the magnitude of $U_{p}$ will be the ratio of the field at the point $P$ to that at the centes of the aperture.

For the special case of uniform aperture ,illumination, $F(r)=1.0$ and equation (3) reduces to

$$
\begin{align*}
U_{p} & =j \frac{2 \pi e^{-j k R}}{x} \int_{0}^{1} e^{-j \pi r^{2} / x} r d r  \tag{4}\\
& =j e^{-j k R}\left[2 \sin \frac{\pi}{2 x}\right] e^{-j \pi / 2 x} \tag{5}
\end{align*}
$$

## Method of Finite Differences

One approach to obtaining a digital computer solution for the case of axially symmetrical illumination of the aperture lies in converting equation (3) directly into a finite difference equation.

Equation (3) may be rewritten to eliminate the exponential under the integral sign:
$U_{p}=j \frac{2 \pi e^{-j k R}}{x}\left[\int_{0}^{1} F(r) \cos \left(\frac{\pi r^{2}}{x}\right) r d r-j \int_{0}^{1} F(r) \sin \left(\frac{\pi r^{2}}{x}\right) r d r\right]$.

By taiking the absolute value of both sides of equation (3a) and squaring, we obtain an expression for the ratio of the power density $W_{p}$ at the point on the aperture axis to the illumination power density $W_{0}$ at the center of the aperture. Thus

$$
\begin{align*}
\frac{W_{p}}{W_{0}} & =\left|U_{p}\right|^{2}=\left(\frac{2 \pi}{x}\right)^{2}\left\{\left[\int_{0}^{1} F(r) \cos \left(\frac{\pi r^{2}}{x}\right) r d r\right]^{2}\right. \\
& \left.+\left[\int_{0}^{1} F(r) \sin \left(\frac{\pi r^{2}}{x}\right) r d r\right]^{2}\right\} . \tag{6}
\end{align*}
$$

If we divide the aperture radius a into $N$ equal increments $\Delta r_{i}$, where $\Delta r_{i}$ designates the $i^{\text {th }}$ increment from the periphery of the aperture (see Figure 2), we have

$$
r_{i}=\frac{N+1 / 2-i}{N} \text { and } d r \cong \Delta r_{i}=\frac{1}{N}
$$

Further, letting $e_{i}$ represent the ordinate of the normalized aperture field intensity distribution at the center of the interval $\Delta r_{i}$, we have

$$
\begin{align*}
\frac{W_{p}}{W_{0}} & \cong\left(\frac{2 \pi}{x N}\right)^{2}\left\{\left[\sum_{i=1}^{N} e_{i}\left(\frac{N+1 / 2-i}{N}\right) \cos \frac{\pi}{x}\left(\frac{N+1 / 2-i}{N}\right)^{2}\right]^{2}\right. \\
& \left.+\left[\sum_{i=1}^{N} e_{i}\left(\frac{N+1 / 2-i}{N}\right) \sin \frac{\pi}{x}\left(\frac{N+1 / 2-i}{N}\right)^{2}\right]^{2}\right\} \tag{7}
\end{align*}
$$

It is convenient to express the dimensionless power density ratio in terms of the average power density over the aperture $W_{a}$, rather than in terms of $W_{0}$.

Now

$$
\begin{align*}
\frac{W_{2}}{W_{0}} & =\frac{1}{\pi} \sum_{i=1}^{N} e_{i}^{2} 2 \pi r_{i} \Delta r_{i} \\
& =\frac{2}{N} \sum_{i=1}^{N} e_{i}^{2}\left(\frac{N+1 / 2-i}{N}\right) \tag{8}
\end{align*}
$$

Substitution of this into equation (7) yields the following $\frac{W_{p}}{W_{a}}=\frac{2 \pi^{2}}{N x^{2} \sum_{i=1}^{N} e_{1}^{2}\left(\frac{\mathbb{N}+1 / 2-i}{N}\right)}\left\{\left[e_{i}\left(\frac{N+1 / 2-i}{N}\right) \cos \frac{\pi}{x}\left(\frac{N+1 / 2-i}{N}\right)^{2}\right]^{2}\right.$

$$
\begin{equation*}
\left.+\left[e_{i}\left(\frac{\mathbb{N}+1 / 2-i}{\mathbb{N}}\right) \sin \frac{\pi}{x}\left(\frac{\mathbb{N}+1 / 2-i}{\mathbb{N}}\right)^{2}\right]\right\} \tag{9}
\end{equation*}
$$

## Method of Superposition

An alternative approach follows from the concept that the aperture amplitude distribution may be approximated by a number of uniform fields distributed over coaxial disks or "sub-apertures" of varying radius. At each point on the aperture axis, the solutions for each of the uniformly illuminated sub-apertures may then be combined, by the principle of superposition, to yield the net power density at the field point. This concept is illustrated in Figure 3.

For each of the sub-apertures, equation (5) takes the form

$$
\begin{equation*}
\left(u_{p}\right)_{i}=2 \sin \frac{\pi r_{i}^{2}}{2 x}\left(\cos \frac{\pi r_{i}^{2}}{2 x}-j \sin \frac{\pi r_{i}^{2}}{2 x}\right) \Delta e_{i} \tag{10}
\end{equation*}
$$

where the subscript $i$ pertains to the $i^{\text {th }}$ sub-aperture, and $\left(U_{p}\right)_{i}$ is the field at any point $P$ on the aperture axis due to the $i^{\text {th }}$ sub-aperture, $\Delta e_{i}=\left(e_{i}-e_{i-1}\right)$ is the amplitude of the field associated with the $i^{\text {th }}$ uniformly illuminated sub-aperture, and $r_{1}$ is the radius of the $i^{\text {th }}$ sub-aperture.

Note that the phase angle component $j e^{-j k R}$ has been dropped. Since this angle is independent of $r_{i}$, it will not alter the relative phase of the contributions of the various sub-apertures and hence serves no useful purpose in this analysis.

If the difference in radius between successive sub-apertures is chosen to be aconstant, $\frac{l}{N}$, and if the i's progress from the periphery toward the center, we have $r_{i}=\frac{\mathbb{N}+1-i}{\mathbb{N}}$.

Substitution of the above quantity in equation (8) yields

$$
\begin{align*}
\left(U_{p}\right)_{i} & =2 \Delta e_{i} \sin \frac{\pi}{2 x}\left(\frac{N+1-i}{\mathbb{N}}\right)^{2}\left[\cos \frac{\pi}{2 x}\left(\frac{N+1-i}{\mathbb{N}}\right)^{2}\right. \\
& \left.-j \sin \frac{\pi}{2 x}\left(\frac{N+1-i}{\mathbb{N}}\right)^{2}\right]  \tag{11}\\
& =2\left[\Delta e_{i} \sin \frac{\pi}{2 x}\left(\frac{N+1-i}{N}\right)^{2} \cos \frac{\pi}{2 x}\left(\frac{N+1-i}{N}\right)^{2}\right. \\
& \left.-j \Delta e_{i} \sin ^{2} \frac{\pi}{2 x}\left(\frac{N+1-i}{\mathbb{N}}\right)^{2}\right]
\end{align*}
$$

METHOD OF FINITE DIFFERENCES


Figure 2
Approximation of aperture distribution by finite increments with $N=10$.

Summing the contributions of the $N$ sub-apertures,

$$
\begin{align*}
\sum_{i=1}^{N}\left(U_{p}\right)_{i} & =2\left[\sum_{i=1}^{N} \Delta e_{i} \sin \frac{\pi}{2 x}\left(\frac{N+1-i}{N}\right)^{2} \cos \frac{\pi}{2 x}\left(\frac{N+1-i}{N}\right)^{2}\right. \\
& \left.=j \sum_{i=1}^{N} \Delta e_{i} \sin ^{2} \frac{\pi}{2 x}\left(\frac{N+1-i}{N}\right)^{2}\right] . \tag{12}
\end{align*}
$$

Taking the absolute value and squaring gives

$$
\begin{align*}
\frac{W_{p}}{W_{0}} & =\left|\sum_{i=1}^{N}\left(U_{p}\right)_{i}\right|^{2}=4\left\{\left[\sum_{i=1}^{N} \Delta e_{i} \sin \frac{\pi}{2 x}\left(\frac{N+1-i}{N}\right)^{2} \cos \frac{\pi}{2 x}\left(\frac{N+1-i}{N}\right)^{2}\right]^{2}\right. \\
& \left.+\left[\sum_{i=1}^{N} \Delta e_{i} \sin ^{2} \frac{\pi}{2 x}\left(\frac{N+1-i}{N}\right)^{2}\right]^{2}\right\} \tag{13}
\end{align*}
$$

In this method

$$
\begin{equation*}
\frac{W_{a}}{W_{0}}=\frac{1}{\pi} \sum_{i=1}^{N} \Delta e_{i}^{2} \pi(N+1-i)^{2}=\sum_{i=1}^{N} \Delta e_{i}^{2}\left(\frac{N+1-i}{\mathbb{N}}\right)^{2} \tag{14}
\end{equation*}
$$

where

$$
\Delta e_{i}^{2}=e_{i}^{2}-e_{i-1}^{2}
$$

Finally, upon substituting equation (14) into equation (13), we have

$$
\begin{align*}
\frac{W_{p}}{W_{a}} & =\frac{4}{\sum_{1=1}^{N}} \frac{4}{\left(e_{1}^{2}-e_{1-1}^{2}\right)\left(\frac{N+1-i}{N}\right)^{2}}\left\{\left[\sum_{i=1}^{N}\left(e_{1}-e_{1-1}\right) \sin \frac{\pi}{2 x}\left(\frac{N+1-1}{N}\right)^{2} \cos \frac{\pi}{2 x}\left(\frac{N+1-1}{N}\right)^{2}\right]^{2}\right. \\
& \left.+\left[\sum_{i=1}^{N}\left(e_{1}-e_{1-1}\right) \sin ^{2} \frac{\pi}{2 x}\left(\frac{N+1-1}{N}\right)^{2}\right]^{2}\right\} \tag{15}
\end{align*}
$$

Equations (9) and (15) were programmed for the IBM 7090 and STRETCH (IBM 7030) computers and represent the two methods of obtaining the on-axis fields of arbitrary aperture distributions.

While the method of superposition was found to give more accurate results than the finite differences method for a given value of $N$, either method will give results as precise as desired by choosing a large enough value for $\mathbb{N}$. It is desirable, however, to use as low a value of $\mathbb{N}$ as is consistent with the necessary accuracy, since each $e_{1}$ must be taken either fram a graph of the aperture distribution (if it is irregular) or calculated from the mathematical formula for the distribution (if it can be represented by a simple mathematical function). In the following calculations a value of $N=50$ was used.

While the method of superposition is more accurate, it does not lend itself well (although it could be modified to do so) to calculations of the field when the aperture distribution is not single valued or monotonic, e.g. Taylor distributions, distributions with blocking, etc. For distributions of this type, the method of finite differences is especially useful.

In this report the method of superposition was used to calculate the fields of all single valued distributions, with the method of finite differences being incorporated for the remaining aperture illuminations. In all the calculations it is assumed that the field is in phase across the entire aperture.

## DISCUSSION

The aperture distributions for which the on-axis fields are calculated are uniform illumination; linear, square law, $\left(1-r^{2}\right)^{n}$ and (cosine) ${ }^{n}$ tapers; Taylor distributions; some irregular arbitrary distributions and distributions with "blocking", where the field strength or power density is zero near the center of the aperture to simulate the effect of the antenna primary feed blocking the center illumination. For the linear, square law ( $\left(1-r^{2}\right)^{n}$ taper with $n=1$ ) and (cosine) ${ }^{n}$ tapers, four variations of each taper are used, i.e., tapers in which the edge illumination is 5, 10,20 and $\infty \mathrm{db}$ down from the illumination at the center of the aperture. (It should be noted that these descriptive terms, linear tapers, square law tapers, etc. pertain to the field intensity distribution in the aperture and not to the power density distribution, which is proportional to the square of the field intensity.)

METHOD OF SUPERPOSITION


Figure 3
Approximation of aperture distribution by uniformly illuminated sub-apertures with $\mathrm{N}=10$.

While the on-axis fields have been determined in previous literature on this subject for many of the distributions contained herein (see, for example, reference (e)), these fields have been recalculated, along with a number of new distributions, using the methods of this report. This fulfills several functions. First, the resulting fields of these common distributions are presented, for a number of edge-to-center illumination ratios, in one paper, thus providing a convenient comparison of the resulting fields; second, it provides a basis for comparing the results of these methods with those obtained in previous literature, verifying that the results are identical for every distribution for which the on-axis field has been previously calculated; and finally, these distributions simulate a large number of the possible tapers which may occur in antennas, and by choosing the appropriate distribution in this report a good approximation of the on-axis field for the aperture distribution of interest may be obtained.

Linear tapers and square law tapers are two of the most commonly used aperture distributions in which the amplitude is increased from a low value at the aperture edge to a maximum at the center of the aperture. Linear amplitude distributions are given by the equation $\alpha(r)=1-(1-\alpha) r$ where $\alpha(r)$ is the normalized amplitude taper, $r$ is the normalized radial distance from the aperture center and $a$ is the ratio of the amplitude at the aperture edge to that at the center. Using similar notation, square law distributions may be represented by $\alpha(r)=1-(1-\alpha) r^{2}$.

The two families of amplitude tapers, $\left(1-r^{2}\right)^{n}$ and (cosine) ${ }^{n}$. cover a wide range of possible aperture distributions in which the amplitude is gradually decreased from a maximum at the center to a somewhat lower value at the aperture edge. The rate of decrease or slope may be increased by increasing the value of the parameter $n$. The on-axis fields have been calculated herein for $n=1 / 2,1,3 / 2$, 2 , 3 , and 4 at four different edge-to-center illumination ratios for the (cosine) ${ }^{n}$ tapers, and for $n=1,2,3$, and 4 with one illumination ratio $(\infty \mathrm{db})$ in the case of the tapers $\alpha(r)=\left(1-r^{2}\right)^{n}$. The $(\text { cosine })^{n}$ tapers are expressed by $\alpha(r)=(1-\alpha)\left(\cos \frac{\pi}{2} r\right)^{n}$.

Taylor aperture distributions for circular aperture antennas offer an appreciable improvement in beam width and sidelobe levels over conventional distributions, and are thus likely to be utilized in antenna design. One Taylor distribution is included to provide an example of the form of the on-axis field resulting from distributions of this type. This Taylor distribution, from reference (c),
produces a sidelobe level 25 db down from the intensity on the axis and has a transition integer of $\bar{n}=8$. Since this is not a monotonic distribution, the method of finite differences was used to calculate the on-axis field.

Reflector antennas frequently incorporate a primary feed at the aperture center which blocks out the center portion of the aperture illumination. To determine the effect blocking will have upon the field, 5 and $\infty \mathrm{db}$ cosine tapers with zero illumination for $\mathrm{r} \leq 0.1$ were included. For these distributions also, the method of finite differences was utilized.

Four irregular distributions which could be approximately simulated by 5 db cosine ${ }^{1 / 2}, \infty \mathrm{db}$ cosine ${ }^{1 / 2}, 5 \mathrm{db}$ cosine ${ }^{4}$ and $\infty \mathrm{db}$ cosine ${ }^{4}$ are included to emphasize the usefulness of the methods of this report in determining the fields of irregular distributions and to determine the extent of the deviation occurring in the on-axis fields when these approximations are made.

## RESULTS

The on-axis fields were determined by computer for the aperture distributions discussed in the previous section and the results are presented in the figures of Appendix A in a manner believed to best facilitate comparison:

The on-axis fields for the aperture distributions (linear, square law and cosine ${ }^{n}$ tapers) which have four edge-to-center illumination ratios are presented with the four resultant fields for each taper on the same graph (Figures 5 through 13). In addition, Figures 14 through 22 present the (cosine) ${ }^{n}$ tapers with the edge-tocenter ratio held constant (at each of the values $5 \mathrm{db}, 10 \mathrm{db}, 20 \mathrm{db}$ and $\infty \mathrm{db}$ down from that at the aperture center) and the taper varied by letting $n$ take on values $1 / 2,1,3 / 2,2,3$ and 4 . Since for each edge-tomenter ratio there exist six distribution functions (one for each value of $n$ ), the fields for each $d b$ level are presented in two figures for clarity with $n=1 / 2,3 / 2$ and 3 on one graph and $n=1$, 2 and 4 on another. From these arrangements both the effect of varying the edge-to-center illumination ratio for a given distribution function, and the effect of varying the distribution function for a given edge-to-center illumination ratio may be observed.

The on-axis fields resulting from the $\left(1-r^{2}\right)^{n^{2}}$ tapers (with $\mathrm{n}=1,2,3$ and 4), the Taylor distribution and the irregular tapers are presented in Figures 22 through 27. All four fields from the $\left(1-r^{2}\right)^{n}$ tapers are presented on the same graph." Each of the fields from the irregular tapers is plotted with the regum lar taper that is being approximated.

The effect, on the on-axis field, of blocking due to a central feed horn was found to be so slight that it would not be observable if the fields were plotted. Thus these tapers were not presented in the figures of Appendix A; however, the computer tabum lations for these distributions are included in Appendix B, listing $W_{p} / W_{a}$ at various distances $x$.

All of the power density fields in the figures of Appendix A are plotted on a logarithmic scale using the dimensionless coordinates $W_{p} / W_{a}$ as ordinates and $x\left[=\frac{R \lambda}{a^{2}}\right]$ as abscissae. The corresponding aperture distributions are included in the upper right hand corner of each figure.

## ERROR ANALYSIS

To obtain an estimate of the error resulting from the use of these methods (with $\mathbb{N}=50$ ), the exact solutions of the Fresnel equation for the 5 db and $\infty \mathrm{db}$ square law and (cosine) ${ }^{4}$ tapers were computed. These represent the two extremes of deviation, with the 5 db square law taper deviating least fram unfform illumination and the $\infty \mathrm{db}$ (cosine) ${ }^{4}$ having the greatest deviation from uniform illumination. It is assumed that the error occurring in the calculations of the on-axis fields for other tapers will not exceed that of these extreme tapers. Both the 5 db and the $\infty \mathrm{db}$ illumination ratios were calculated for each of the two tapers to determine the effect of edge-to-center illumination ratios upon the error.

The accuracy of $W_{p} / W_{a}$ in the tables of Appendix $B$ is limited to three significant figures since the computer input data (the $e_{i} s$ ) were only given to three decimal places. Comparison of the exact values of $W_{p} / W_{a}$ with those given in Appendix $B$ at various values of $x$ from 0.1 to 100, showed that for the majority of cases there was no error occurring in the first three significant figures. In no instance did the superposition method give an error in excess of $1.0 \%$ and only in the most extreme distribution ( $\infty \mathrm{db}$ cosine ${ }^{4}$ ) did the method of finite differences have an error greater than this. This amounted to $1-3 \%$ and can be expected to occur when this method is used for the more extreme distributions ( $\infty d b$ cosine ${ }^{3}$, cosine ${ }^{4}$ and $\left.\left(1-r^{2}\right)^{4}\right)$. In both methods the error is greatest at low values of $x$.

It is also of interest to observe the effect that varying the value of $\mathbb{N}$ has upon the accuracy of the calculations. While the method of superposition gives the exact solution of the Fresnel equation for a uniformly illuminated aperture, there is a slight amount of deviation when the method of finite differences is used. The on-axis field for a uniformly illuminated aperture was calculated using values of $N=10,25,50,75,100,150,200$, 500 , and 1000 . The resulting values of $W_{p} / W_{a}$ were compared with the exact solution and the percentage of error was plotted as a function of $\mathbb{N}$ for various values of $x$ in Figure 4.

As can be seen from the graph, a linear relationship exists (on a logarithmic scale) between the error and $N$, and if it is assumed that other distributions will have a similar error for the same value of $N$, Figure 4 may be used to choose the appropriate value of N for whatever accuracy is desired.

The curves of Figure 4 are applicable only to the method of finite differences, since no error occurs when the method of superposition is used for uniform illumination.

## CONCLUSIONS

Examination of the resulting fields reveals a number of significant effects of non-uniform aperture distributions upon the on-axis field. These may be summarized as follows:

1. The deep nulls which occur in the Fresnel field of a uniformly illuminated aperture are filled in considerably when the distribution is tapered. This is most pronounced for the distributions which deviate most from uniform illumination.
2. There is a decrease in power density in the far field (a shifting of the curve to the left) although the slope of this far field (inverse square law) curve remains the same. Upon closer analysis it is found that the values of $W_{p} / W_{a}$ in the far field are precisely the values predicted by the far field formula

$$
\frac{W_{p}}{W_{a}}=\frac{G_{M} A}{4 \pi R^{2}} \text { where } G_{M}=\frac{\left|\int_{0}^{I} A(r) d r\right|}{\int_{0}^{2} A^{2}(r) d r} G_{0}
$$

$G_{M}$ and $G_{o}\left[=\frac{4 \pi A}{\lambda^{2}}\right]$ being the gain of the aperture for tapered and uniform illumination, respectively.


Error as a function of the number of increments $N$ for uniform illumination using the method of finite differences.
3. The envelope of the maxima in the Fresnel field of a uniformly illuminated aperture is a constant value, i.e, $W_{p} / W_{0}=4.0$, while for a tapered aperture distribution the peaks tend to either increase or decrease as $x$ decreases depending upon the particular distribution. From an analysis of the resulting curves and from theoretical considerations, a formula for this envelope may be found, namely that as x approaches zero the maxima approach the value

$$
\left(\frac{W_{p}}{W_{a}}\right)_{\text {max }}=\frac{W_{0}}{W_{a}}(1+\alpha)^{2}=\frac{(1+\alpha)^{2}}{2 \int_{0}^{1} A^{2}(r) r d r}
$$

For tapers which approach $W_{0}$ very rapidly from the value at the aperture edge (square law and (cosine) ${ }^{1 / 2}$ tapers), the maxima decrease in field strength as $x$ decreases, while for most other tapers the field strengths of the maxima increase as $x$ decreases.
4. As the edgento-center ratio is decreased, the values of $x$ at which the near field maxima and minima occur decreases, i.e., there is a slight shift of the peaks and nulls to the left.
5. The edge-to-center illumination ratio has more effect upon the on-axis field than the particular type of taper used.
6. "Blocking" of the center portion of an aperture (from center out to $r \leqq 0.1$ a) has only a small effect upon the on-axis field. The far field is decreased by about $1 \%$, while the near field peaks are increased on the order of $5 \%$.

## SUMMARY

1. Two equations which lend themselves well to computer solutions have been developed, predicting the on-axis power density for any arbitrary aperture distribution.
2. These equations have been programmed for computer solution and are readily available for use as a tool in the study of radar fields.
3. The on-axis fields were calculated for a number of aperture distributions which were felt to approximate many of the distributions occurring in practice. By choosing the distribution in this report which best simulates an actual distribution under study, a good approximation of the on-axis field may be obtained.
4. Analysis of the resulting onmaxis fields reveals a number of significant effects which tapered distributions have upon the field as compared to uniform distribution. These are:
a. The nulls occurring in the near field are partially or completely filled in.
b. The far field gain is reduced.
c. The envelope of the Fresnel field peaks is no longer constant but increases or decreases as $x$ decreases.
d. The values of $x$ at which the Fresnel field peaks occur decrease.
e. The edge-to-center illumination ratio has moxe effect upon the on-axis field than the particular type of taper used.
f. Blocking of a small center portion of the aperture ( $r \leq 0.1$ a) has little effect upon the on-axis field.

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APPENDIX A

## CONIENIS

Figure 5 On-axis field of a uniformly illuminated aperture
Figure 6 On-axis fields for linear aperture distributions
Figure 7 On-axis fields for square law aperture distributions
Figure $8 \quad$ On-axis fields for (cosine) $1 / 2$ aperture distributions
Figure 9 On-axis fields for cosine aperture distributions
Figure 10 On-axis fields for (cosine) ${ }^{3 / 2}$ aperture distributions
Figure $11 \quad$ On-axis fields for (cosine) ${ }^{2}$ aperture distributions
Figure $12 \quad$ On-axis fields for (cosine) ${ }^{3}$ aperture distributions Figure $13 \quad$ On-axis fields for (cosine) ${ }^{4}$ aperture distributions

Figure 14 On-axis fields for 5 db (cosine) $^{\mathrm{n}}$ aperture distributions

Figure 15 On-axis fields for $5 \mathrm{db}\left(\right.$ cosine) ${ }^{\text {n }}$ aperture distribum tions.

Figure 16 On-axis.fields for 10 db (cosine) ${ }^{\mathrm{n}}$ aperture distributions

Figure 17 On-axis fields for 10 db (cosine) $^{\mathrm{n}}$ aperture distributions

Figure 18 On-axis fields for 20 db (cosine) $^{\mathrm{n}}$ aperture distributions

Figure 19 On-axis fields for 20 db (cosine) $^{\mathrm{n}}$ aperture distributions

Figure $20 \quad$ On-axis fields for $\infty \mathrm{db}$ (cosine) ${ }^{n}$ aperture distributions

Figure $21 \quad$ On-axis fields for $\infty d b$ (cosine) ${ }^{n}$ aperture distributions

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## CONTENTS (Continued)

Figure 23 Comparison of the on-axis fields of an irregular distribution and a 5 db (cosine) ${ }^{1 / 2}$ approximation of the irregular distribution

Figure 24 Comparison of the on-axis fields of an irregular distribution and an $\infty d b$ (cosine) ${ }^{1 / 2}$ approximation of the irregular distribution

Figure 25 Comparison of the on-axis fields of an irregular distribution and a 5 db (cosine) ${ }^{4}$ approximation of the irregular distribution

Figure 26. Comparison of the onmaxis fields of an irregular distribution and an $\infty \mathrm{db}$ (cosine) ${ }^{4}$ approximation of the irregular distribution

Figure 27 Onmaxis field of a Taylor distribution (sidelobes 25 db down, $\overline{\mathrm{n}}=8$ )

## NOTATION FOR AIL FIGURES

$W_{p}=$ Power density at a field point on the aperture axis
$W_{a}=$ Average power density over the aperture
$=\frac{\text { Power Radiated }}{\text { Aperture Area }}$
$X$ = Dimensionless parameter proportional to the distance from aperture to field point, along the aperture axis

$$
=\frac{R \lambda}{a^{2}}
$$

$R=$ Distance from aperture to field point along the aperture axis
a = Radius of aperture
$\lambda=$ Wave length of radiation
$r=$ Normalized distance from center of aperture in the plane of the aperture
$A(r)=$ Normalized field intensity distribution across the aperture







On-axis fields for cosine ${ }^{2}$ aperture distributions.

















On-axis field of a Taylor distribution (side lobes $25 \mathrm{db}, \overline{\mathrm{n}}=\mathbf{8}$ ).

APPENDIX B

## CONTIENIS

Table $1 \quad W_{p} / W_{a}$ versus $x$ for uniform illumination
Table $2 \quad W_{p} / W_{a}$ versus $x$ for 5 db linear distribution
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Table $4 \quad W_{p} / W_{a}$ versus $x$ for 20 db linear distribution
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Table $21 \quad W_{p} / W_{a}$ versus $x$ for $\infty d b(\text { cosine })^{3 / 2}$ distribution


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APPENDIX C

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