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THE NORMATIVE INTEREST RATE

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Introduction

The normative interest rate is defined as the discount rate the government ought to use in making its investment decisions. In the following sections various alternative ways of setting the level of the normative interest rate are examined. The return on the marginal private investment, the national time preference, and the long-term interest rate at which the government can borrow are all rejected on the basis that they are merely adaptive to the nation's interest rate structure which is determined in turn by the government itself. The concept of a positive national time preference is rejected for society as a whole, although it is considered appropriate for the individual because of his mortality. A zero normative interest rate is also rejected for society because of the declining marginal utility of a growing national product. The appropriate normative interest rate is then determined as a rate that is consistent with itself through the rate of growth of national product and the rate of decline in the marginal utility of national product that it implies.

The concept of investment used in the following analysis is Fisherian in the sense that it is a diversion of resources away from present welfare in anticipation of the return of a greater amount of future welfare.[1] It might thus be the purchase of an item of capital plant or equipment, but it might also be an investment of resources in education or basic research.

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1. See Irving Fisher, The Theory of Interest, the Macmillan Company, New York, 1930, p. 34.

I. Welfare Return and Investor's Return

In order to understand some of the difficulties involved in using various devices for setting the level of the normative interest rate, it would be useful to understand the distinction between the welfare return and the investor's return on an investment. These concepts may be defined as follows:

The welfare return on an investment is the total benefit accruing to the nation from the investment, suitably discounted.

The investor's return is that part of the welfare return that the investing individual, corporation, or government agency can claim for itself.[2]

There are three important elements in the welfare return which the investor may not be able to claim. These are: (1) the incremental taxation; (2) the risk premium; and (3) direct gains to the general welfare. We shall consider each of these in turn.

1. The Incremental Taxation. The incremental taxation is the increase in taxes the investor must expect to pay because of the investment, arising primarily out of the expected increase in income. The incremental taxation is clearly a part of the return from an investment that is diverted from the investor to the general welfare. There are three points about incremental taxation that should be mentioned:

First, the relevant incremental taxation is the expected incremental taxation before the act of investment, not the incremental taxation that actually turns out to be payable. In other words, it is the ex ante, not the ex post incremental taxation that concerns us.

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2. The term external economies has frequently been applied to the difference between the investor's and the welfare returns.

Second, the expected incremental taxation is discounted, using the same discount rate that is applicable to the investor's return and welfare return.

Third, it is inevitable that there be some expected incremental taxation from any investment project worth pursuing including a marginal one. One might think that the marginal private investment offers no ex ante income, and hence should avoid incremental taxation.[3] However, the combination of a dispersion in possible outcomes together with a progressive tax structure (if not for the corporation, at least for the owners thereof) brings about some incremental taxation even on the marginal investment. The incremental taxation on the marginal investment, which is the one of greatest interest to us, is thus an inevitable but minor factor.

2. The Risk Premium. The risk premium is a cost the investor has to pay for financing his investment, but it is not a cost to the general welfare. Thus it appears to be an element of divergence between the investor's return and the welfare return.[4] On analysis, however, most of the divergence proves to be illusory. To the extent that a risk premium diverts some of the profit of a favorable outcome from the investor to the financier, it merely converts the expected outcome of the investment made on borrowed funds to what it would have been if self-financed. Most of the risk premium allowed on a self-financed investment project is really a device for reducing the return expected if successful to a genuine expected return, and as such should not

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3. If the marginal project is financed by other than the investor the financier should absorb all of the return; if the project is self-financed, the return will not exceed what the investor could obtain by lending, and hence the incremental income of the decision to invest is still zero.

4. See Otto Eckstein, Water-Resource Development, The Economics of Project Evaluation, Harvard University Press, Cambridge, Mass., 1958, p. 45; and John V. Krutilla and Otto Eckstein, Multiple Purpose River Development, Johns Hopkins Press, Baltimore, 1958, pp. 123-124.

be part of the interest rate. Only to the extent that the risk premium represents the subjective or objective cost to the investor of a dispersion of outcomes *per se* (including the possibility of going through bankruptcy) is it a proper charge against the investor's return.

3. Direct Gains. The direct gains to the general welfare are those parts of the total welfare return which never pass through the investor's hands, but rather accrue directly to the general welfare. For example, if a man builds a very modern and elegant office building, he will probably enhance the real estate values in the immediate neighborhood, thereby increasing the general welfare. An even better example is the basic research project which offers large direct gains because (1) it tends to be undirected; the worker in basic research will usually pursue interesting tangents, some of which will lead to results that are profitable to someone other than the sponsor; (2) in addition, the payoff from a successful basic research project operates through an advance in some state-of-the-art which will be of general use to the nation.

The government and the private investor differ significantly in size, and hence differ significantly in their abilities to claim welfare return. The government as an investor cannot be considered to be large merely because the resources it commands are large relative even to the resources of a very large corporation. Since the government is responsible for the nation's general welfare it is as large as the whole country, and hence, for the government, the welfare and investor's returns are equal.

Projects which are high in welfare return but low in investor's return because of large direct gains will be rejected by the private investor. These projects should become the province of the government whatever other public characteristics they might or might not evidence. Other projects will tend to have a slightly higher welfare return than investor's return.

II. Three Traditional Devices for Setting the Level of the Normative Interest Rate

Three devices for setting the level of the normative interest rate come immediately to mind: (1) the rate of return which is earned on the marginal investment in the private sector of the economy; (2) the nation's time preference; and (3) the interest rate at which the government can borrow long-term funds.

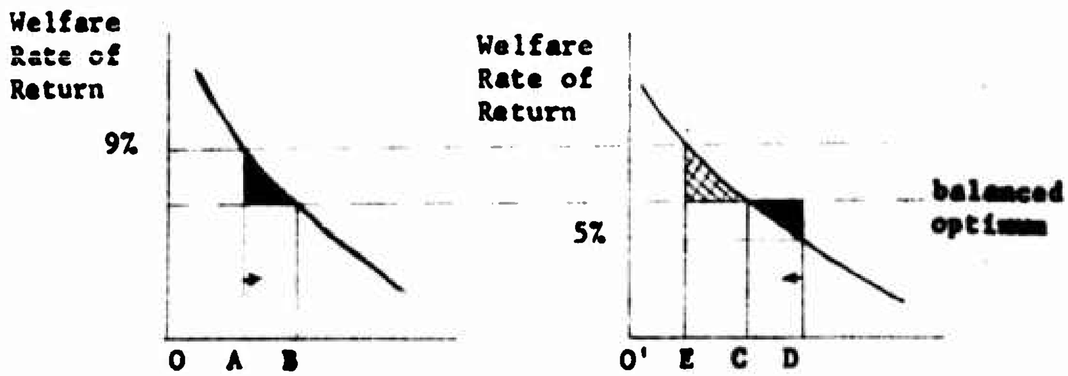
Each of these alternative normative interest rates appears on the surface to offer strong reasons for its use and has been strongly advocated. Let us examine each of them in turn.

a. The Rate of Return on the Marginal Private Investment. The rate of return earned on the marginal private investment offers the justification of balance. Other things being equal, it seems proper to have the marginal private and public investments earn the same rate of return. This idea grows out of our concept of investment which it should be remembered is the sacrifice of present welfare for the sake of a greater amount of future welfare. The nation should obtain as great a gain of future welfare as possible from any given level of present sacrifice. Figure 1 illustrates an example of an imbalance in the private and public marginal returns. In that figure, the marginal investment in the private sector of the economy earns a rate of return of 9%, while the marginal investment in the public sector earns a rate of return of 5%.[5] By diverting an amount

5. See the excellent analysis and summary of the literature on rate of return in J. Hirshleifer, "On the Theory of Optimal Investment Decision," Journal of Political Economy, Vol. LXVI, No. 4, August 1958.

D-C (=B-A) from the public sector to the private sector, the nation could gain an addition in national welfare equal to the two shaded areas in Figure 1 without changing the level of present sacrifice.

FIGURE 1



Private Investment Opportunity Function

Public Investment Opportunity Function

Balancing the Returns on Private and Public Investment

It is clear that the returns on the marginal private and public investments should be equated. Yet it is easy to go astray in trying to derive a normative interest rate from this statement. Does this statement mean for example that the normative interest rate and hence the return on the marginal government investment should be set equal to whatever the (welfare) rate of return on private investment happens to be? In Figure 1, that would imply halting government investment at point E. If we were to assume for the moment that the correct normative interest rate is represented by the dotted line labeled "balanced optimum," such a policy would entail a loss in national welfare represented by the crosshatched area. Simply because the private sector is in error, as it is before adjustment in the figure, is no reason for the public sector to be similarly in error and thus compound the loss to the general welfare.

The technique of setting the normative interest rate at whatever the welfare return on marginal private investment happens to be is wrong because it does not balance a fixed level of present sacrifice. Instead it holds the level of investment in the private sector constant, and allows investment in the public sector to adjust to whatever rate of return is forthcoming from the private sector.

What about deriving the normative interest rate from a process of balancing the returns in the private and public sectors, as in Figure 1, with the total level of investment held constant? While clearly correct, the procedure of balancing returns does not really have the status of an independent technique. If the level of investment is given, as it must be, the statement that private and public returns should be equal is a truism, and the level of the normative interest rate is actually set by whatever it is that determines the given level of investment.

b. The National Time Preference. We shall next consider the national time preference as a basis for the normative interest rate.

The argument for the use of a national time preference for setting the level of the normative interest rate is a compelling one; for if a national time preference could be identified it would represent the nation's willingness to trade off consumption between the present and the future. Our search for a basis for the normative interest rate would end here if only it were possible to identify and adopt a national time preference. Several difficulties stand in the way of the use of a national time preference however; they are:

1. Can a single time preference represent the whole nation? There is a significant range in individual time preferences, particularly between the interest rate the individual receives as a lender and the interest rate he pays as a borrower. Can these heterogeneous time preferences be reconciled?

2. Assuming a unique time preference is identifiable, is it an independent force or the result of an interaction of independent forces? If the national time preference is found to result from a process in which each individual adjusts to an underlying interest rate established (either consciously or unconsciously) by the monetary and/or fiscal policy of the government, the government could then hardly consider that national time preference to be justifiable as an independent criterion for investment.

3. Can individual time preferences be a reliable source for a national time preference? This question breaks down into two parts.

(1) Can the individual be trusted to have a sufficiently matured time preference to make his own intertemporal allocations properly? Intertemporal judgment comes at a much later age than intratemporal judgment; perhaps a large part of the population never develops a mature intertemporal judgment. In fact a properly matured time preference is probably the chief criterion which distinguishes the wise man from the fool.

(2) Even if individuals are capable of making their own intertemporal allocations properly is it proper for society to rely solely on the time preferences of the current generation at the expense of future generations.[6]

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6. See Julius Margolis, "The Economic Evaluation of Federal Water Resource Development" (a review article) AER, March 1959.

Each of these three questions will be examined in turn, starting with the heterogeneity of time preferences.

Let us first see if it is possible to prove that the time preference of the marginal individual lender can be identified as the national time preference and that other individuals with higher time preferences are merely not in the market. If so the heterogeneity of individual time preferences would cause no difficulty. The case for this point becomes identical to the case for the use of the long-term rate at which the government can borrow as the normative interest rate.

Krutilla and Eckstein reject the use of the interest rate at which the government can borrow for their social cost of federal financing (our normative interest rate) for two reasons, one of which is accepted, and the other of which is rejected. They note that the use of the interest rate on long-term government bonds would require (1) that the entire cost of projects be financed out of voluntary bond purchases and (2) that risks created by the project be borne by the buyers (of the bonds).[7] The former implication they consider unreasonable in a government committed to contracyclical fiscal and monetary policies, which must therefore finance at least part of the cost of a project through taxation in order to compensate for its impact on aggregate demand.[8] This argument is acceptable. The second reason seems to arise out of a confusion of risk and uncertainty. The benefits of an investment project are uncertain, but risk as such is eliminated, not redistributed, by the intervention of the government between the investment

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7. See John V. Krutilla and Otto Eckstein, op. cit., p. 91.

8. Ibid., p. 90.

project and the bondholder. The possibility of unfavorable outcomes should lower the expected return and not becloud the normative interest rate, especially since such possibilities will vary in effect from project to project.

Either of the arguments put forth by Krutilla and Eckstein is sufficient to reject the use of the marginal lender's time preference, or the interest rate on long-term government bonds, as a basis for the normative interest rate.

Using another approach, Krutilla and Eckstein have overcome the problem of heterogeneity by weighting the time preferences of different groups in the economy by the impacts of marginal taxation on these groups. The result is a kind of average individual time preference which satisfies the heterogeneity problem, but remains subject to the other three questions.[9]

Let us turn to the second question: whether individual time preferences are the result of independent forces or are determined as the result of the government's fiscal and monetary policies. This question represents the long-standing dispute between the Keynesians and the loanable fund theorists on how the nation's (underlying) interest rate is determined.[10] According to the Keynesian theory, the interest rate is determined as a result of the interactions of the nation's liquidity preference, not time preference,

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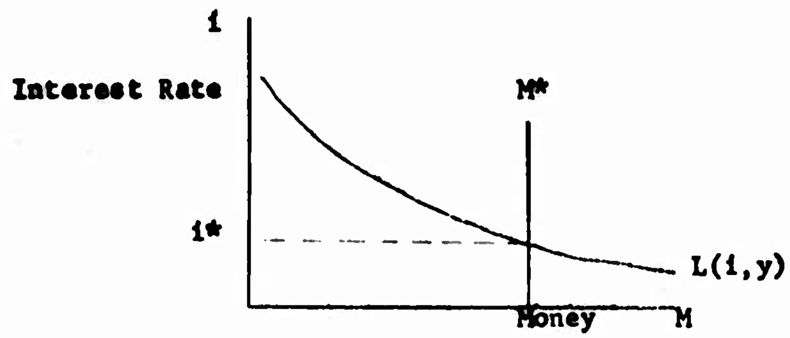
9. See Krutilla and Eckstein, op. cit., Chapter IV, pp. 78ff. Krutilla and Eckstein have included the risk premium in the investor's time preference. Their resulting social cost of federal financing is therefore somewhat overstated.

10. For a recent review of this dispute, see Hans Neisser, "A Pyrrhic Victory," The Economic Journal, Vol. LXVIII, No. 272, December 1958, pp. 699-706.

and the available supply of liquidity. Figure 2 illustrates the Keynesian process of interest rate determination.

FIGURE 2



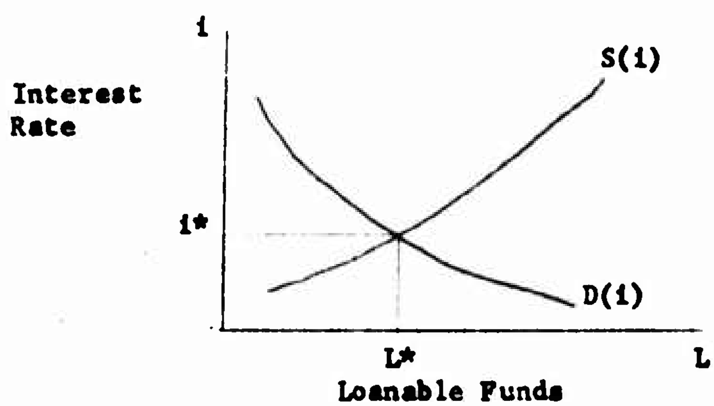
Keynesian Liquidity Preference Function

The L (i,y) function is the liquidity preference function which in this form states that the demand for the stock of money or liquidity is a function of i, the interest rate, with y, income, held constant. M, the stock of money or liquidity, is determined exogenously by the government directly and through its agent the Federal Reserve System. If the quantity of money is set at M* by the government, the (underlying) interest rate will be i*.

There is no room in this system for individual time preferences. If individual time preferences are found to be equated to the interest rate (with appropriate risk premiums added), the Keynesian would explain that the individual has adjusted his time preference to conform to the interest rate rather than the other way around. Since the Keynesian also believes that the consumption-saving decision is very insensitive to the interest rate, the amount of adjustment in the consumption-saving distribution needed to effect conformity would be small if not non-existent.

The loanable fund theorists explain the derivation of the interest rate as the interaction of the supply and demand for loanable funds, rather than for the stock of money. Figure 3 illustrates the loanable fund explanation of interest rate determination.

FIGURE 3

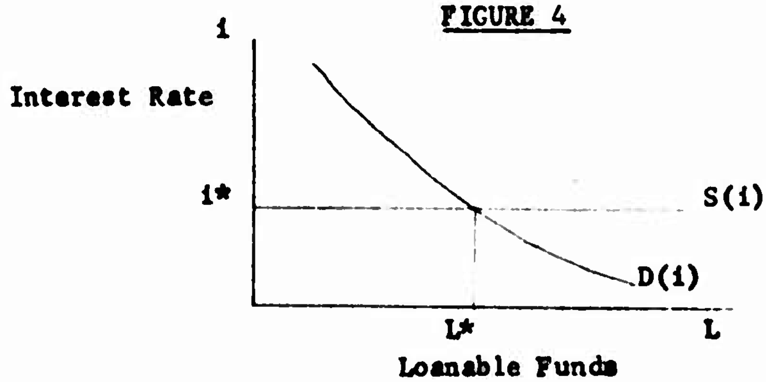


The Determination of Interest Rate in the Loanable Funds Market

In Figure 3, the D(i) function is the demand for loanable funds as a function of the interest rate, i. This demand arises through the existence of investment opportunities which offer returns great enough to justify the payment of an interest charge. The S(i) function is the supply of loanable funds made available by individuals willing to save from current income in order to make available loans which will earn returns in excess of or equal to their time preferences. The D(i) curve declines to the right because more and more investment opportunities become feasible as the interest rate falls. The S(i) curve rises to the right because individuals are willing to forego more and more of their current consumption as the return on savings rises. The equilibrium interest rate i*, and the equilibrium volume of loanable funds, L*, are both determined at the intersection of the D(i) and S(i) functions.

The Keynesian theory of interest rate determination is inconsistent with the existence of a national time preference independent of government fiscal and/or monetary policy. One might also question whether the supply of loanable funds, as represented by the S(i) curve of Figure 3 is really determined by the consumption-savings decisions of individuals (and/or corporations). We do not subscribe to the theory that savings are necessary as an enabling device preceding investment but propose rather that loanable funds are made available by the banking system. We prefer the Keynesian system in which savings are brought into balance with investment through either an increase in real income or through the forced savings of inflation. The Supply curve of the loanable funds market is therefore redefined to be the supply of loanable funds made available by the banking system, as in Figure 4.

FIGURE 4



Alternative View of the Loanable Funds Market

In Figure 4, the monetary authority determines the banking system will be flexible enough to supply whatever loanable funds are needed for investment at an interest rate set by the monetary authority: in this country, the Treasury and the Federal Reserve System.

It is our belief that flexibility in the supply of loanable funds, represented by the flat supply curve of Figure 4, has been one of the principal motives underlying the creation of central banking systems. The Figure 4 concept of the market for loanable funds is also consistent with the fact that the level of investment has varied widely in the past with only narrow movements of the interest rate.

We have now effectively killed the dog! All theories of interest rate determination have led us to the same conclusion; that the interest rate is set by the government, and not by individual time preferences, and hence cannot provide a justifiable criterion for the government itself. In the Keynesian system, the government determines the interest rate by way of control over the supply of money.[11] In the loanable fund theory, as it has been revised, the government merely sets a rate of interest and insures a flexible supply of loanable funds at that interest rate.[12] There is nothing left for individuals to do but to adjust their time preferences to the interest rate they find as well as they can considering the risk premiums they must pay. The private investor must also be considered to play a passive role in the determination of the interest rate; he merely adjusts his investment decisions to the interest rate he finds and, like the individual consumer, plays no part in setting that rate. The interest rate

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11. See Figure 2.

12. See Figure 4.

at which the government borrows long-term capital is also self-determined and hence not justifiable as a criterion.

At the risk of killing the dog again, let us turn to the last of the three questions we raised pertaining to the national time preference: Can individual time preferences be a reliable source for a national time preference?[13] As noted above, this question breaks down into two parts: (1) Can the individual be trusted to have a sufficiently matured time preference to make his own intertemporal allocations properly? (2) Even if individuals are capable of making their own intertemporal allocations properly, is it proper for society to rely solely on the time preferences of the current generation at the expense of future generations?

The first question turns on whether we consider a properly matured time preference to be a common attribute of the economic man, or whether that attribute is thought to be rare among men. It is quite clear that a child develops a capacity for intratemporal choice at an age at which his time preference is still quite undeveloped. A child of six can usually make simple choices among alternatives applying to the same period of time; perhaps between one chocolate and two caramels. But what usurious rate of interest in chocolates would that same child require to surrender one chocolate for a month! How certain are we that that child ever develops a mature time preference merely because he becomes an adult? The spendthrift is proof that some adults at least do not have a mature time preference;

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13. There is fairly extensive literature on this subject. See Krutilla and Eckstein, _op. cit._, p. 92, for a bibliography on the subject. See also _Ibid_., pp. 125-127; and Julius Margolis, "The Economic Evaluation of Federal Water Resource Development" (a review article), _American Economic Review_, Vol. XLIX, No. 1, March 1959.

and perhaps the spendthrift would be more common if more adults were wealthy enough to be spendthrifts. There is another more prevalent adult; the one who spends all his income; borrows to the limit of his credit; and is even short of cash as payday approaches. Such a person is a spendthrift sans wealth.[14]

One might answer that the individual's time preference is a subjective matter; that if a man's time preference is unusually strong, it is proper for him to spend heavily in the present at the expense of the future. But an intertemporal allocation of consumption has more than one reference point. The spendthrift may be maximizing the sum of his utility, now and next year, by spending all of his wealth this year, but that is true only from the point of view of the present. If the same spendthrift were to review his expenditures from the point of view of next year, he might very well rue his excesses of this year. Professor Pigou noted that whereas the anticipation of future pleasure is not as strong as the enjoyment of present pleasure, so too the memory of past pleasure is not as strong as the enjoyment of present pleasure.[15] It is this multiplicity of reference points that differentiates the intertemporal allocation of income from the intratemporal allocation of income. Only the person with a mature time preference is able to predict now how he will feel about his intertemporal allocation next year. Perhaps we

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14. For a more elegant description of the spendthrift, see R. H. Strotz, "Myopia and Inconsistency in Dynamic Utility Maximization," <u>Review of Economic Studies</u>, Vol. XXIII (1956), pp. 165-180.

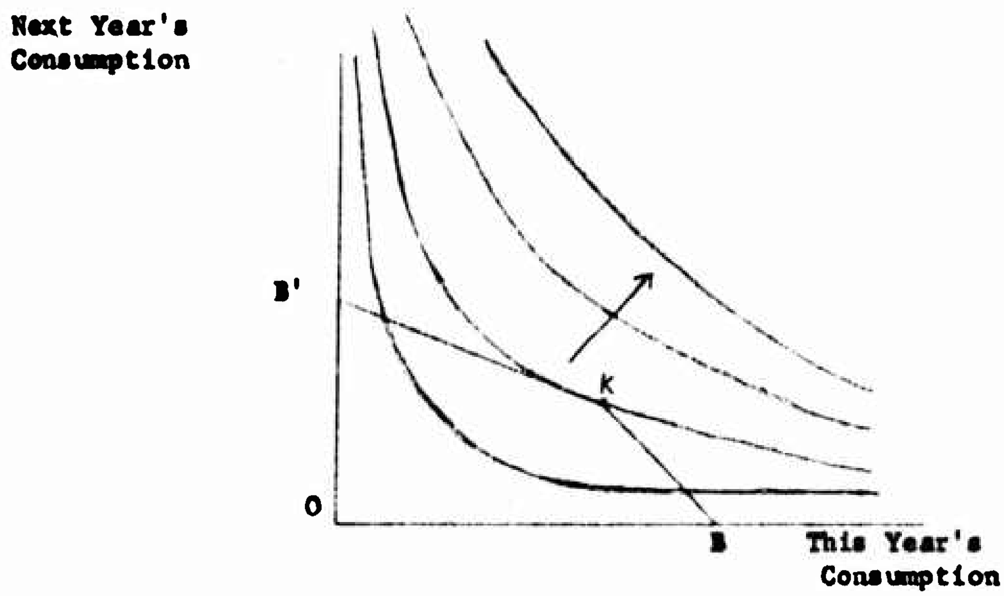
15. A. C. Pigou, <u>The Economics of Welfare</u>, Macmillan & Co., Ltd., London, 1920, pp. 23<u>ff</u>.

can use this aspect of the mature economic man to define a proper time preference as follows:

> A proper time preference is a preference for consumption in year x over consumption in year y which will be as strong when viewed in year x as it will be when viewed in year y.[16]

This concept of a proper time preference still leaves room for a legitimate preference for current consumption. For example, the young man with an expection of growing income and with the heavy current expenses of a new family, should properly divert consumption toward the present. Yet it is clear that individual time preferences, as a whole, are an unreliable source for the normative interest rate.

FIGURE 5



Individual's Intertemporal Utility Map

16. We abstract for the moment from the problem of uncertainty. It is actually the expected time preferences as viewed in future years which should be equated to time preferences as viewed in the present.

The reader might question whether a bias in time preference is consistent with our previous determination that the marginal time preference is adaptive to a national interest rate structure set by the government. In order to make this point clear, let us examine the familiar intertemporal utility map, as illustrated in Figure 5.

Figure 5 represents a two-period allocation of consumption. The BKB' line is a budget line representing the different combinations of consumption this year and next year that the individual can attain. Point K represents the spending of each year's income in the year received. Movement from K towards B^1 represents trading off some of this year's consumption for consumption next year proceeding at the lender's interest rate. Movement from K towards B represents trading off some of next year's consumption for consumption this year proceeding at the borrower's interest rate.

The curved lines are iso-utility bars, with movement in the direction of the arrow representing a gain in utility. The individual maximizes his utility by moving along the BKB^1 line until he reaches the highest iso-utility bar attainable.

The marginal time preference (the slope of the iso-utility bar at the individual's optimum) will always conform to the existing interest rate to the extent that it will lie between the limits of the individual lender's interest rate and the individual borrower's interest rate. The effect of an immature overly strong time preference is not to be found in the marginal time preference however. We must instead look to the very structure of the iso-utility bars to detect the immature time preference. The iso-utility bars of an individual with an immature time preference will be biased towards the axis representing this year's consumption. The immature time preference thus need not cause a high interest rate, but it will cause an overspending in the present at any interest rate.

The second part of our investigation of the reliability of time preferences has a somewhat longer horizon than the first. It is rational for a man to divert consumption somewhat towards the present because of his mortality. A man may rationally prefer to enjoy the fruits of his labor himself rather than to leave them to his heirs. Certainly a man without heirs is entitled to adopt an "après moi le déluge" attitude. The nation in contrast is at least potentially immortal. A proper national time preference may therefore very well be lower than a time preference built up from individual time preferences, even assuming that all individuals had proper time preferences. A. C. Pigou noted that the conservation movement in the United States is in recognition of just such a fact; it is an intervention by the government in the intertemporal allocations of society in the interest of future generations.[17]

In summary, the national time preference built up from individual time preferences has failed each of the three tests we established, with the failure of any one of the three sufficient for us to reject this basis for a normative interest rate:

1. Individual time preferences are heterogeneous; no single rate can be found to represent them adequately;

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17. Ibid., p. 28. Krutilla and Eckstein agree with Pigou on this point (op. cit., pp. 125-127). Because of this they recommend using a lower social cost of federal financing than the tax-impact average time preference they derived. They do not however scrap their tax-impact interest rate, but recommend that the government invest more than their tax-impact interest rate would imply. This seems incorrect. If the tax-impact interest rate is wrong (which it is for more reasons than Krutilla and Eckstein acknowledge) it is wrong to use it at all. The effect of preserving it except as a cut-off criterion for government investment is a misordering of investment projects with a bias towards projects with a short-run return.

2. Individual time preferences are not independent of government monetary and/or fiscal policy;

3. Individual time preferences are an unreliable source for a national time preference.

The Krutilla-Eckstein system of weighting time preferences of various groups by the relative tax impacts of these groups answers the problem of heterogeneity; but it remains subject to the other two failures and must hence be rejected.

Furthermore, the return on the marginal private investment and the long-term government bond rate both fail because of their lack of independence from government fiscal and monetary policy.

III. Should the Normative Interest Rate be Zero? A zero normative interest rate is the natural implication of Professor Pigou's comment that both the anticipation of future pleasure and the memory of past pleasure are weaker than the enjoyment of present pleasure.[18] Pigou favored the government intervening in the matter of the interest rate by investing itself. In fact he faced the question of whether a zero interest rate is appropriate as a criterion and concluded that it was not. To the extent that resources were diverted from consumption, Pigou thought, the zero interest rate was appropriate; to the extent however that resources were diverted from other investment yielding a normal return, he thought a higher interest rate would be appropriate.[19]

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18. Op. cit., pp. 23ff.

19. Ibid., pp. 28-29.

Pigou's rationale for rejecting the zero normative interest rate is not acceptable. His reasoning reflects the thinking of the pre-Keynesian era in which his book was written. Modern fiscal and monetary tools are versatile enough to permit the government to invest while diverting resources from consumption only. In fact, the truism that the marginal returns on private and public investment should be equal would force the government to lower the nation's underlying interest rate as close to zero as possible in order to raise private investment pari passu with public investment, and all with resources diverted from consumption.

If Pigou's reasoning in rejecting the zero normative interest rate is unacceptable, is there some other ground for rejecting it? Branko Horvat presents another reason.[20] He notes that the optimum rate of investment (assuming the objective is to maximize output) is not that rate of investment which drives the return on investment to zero. Beyond a certain point, additional investment detracts from national output more than it adds by decreasing consumption with a consequent decrease in the future productivity of the population. Thus at the optimum, the marginal efficiency of investment is zero, but the interest rate is still positive because the individual marginal investment still earns a positive product. Although Horvat's argument is logical and convincing, it is not applicable to the present case for two reasons:

1. Horvat's definition of investment is restricted to the purchase of physical capital. A less restrictive definition, such as we use, permits us to consider the investment in people just as much as the investment

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20. "The Optimum Rate of Investment," Economic Journal, Vol. LXVIII, No. 272, December 1958, pp. 748-756.

in physical capital as a claimant on national resources. Thus part of what Horvat would consider consumption is considered investment here.

2. Horvat's analysis applies only to a country with a fairly low national output. It is doubtful that the diversion of resources from consumption that would be needed to drive the marginal return on investment to zero in this country would have a significant deleterious effect on the future productivity of American manpower -- providing the remainder were reasonably distributed.

So far Pigou's argument against a zero normative interest rate was rejected on the grounds that it assumed an inflexibility of government fiscal policy which we know now not to be true; and Horvat's argument was rejected on the grounds of inapplicability.

Let us turn to a third argument. This one is based upon the fact that as the national product increases, successive increments of this product represent less and less national welfare. This phenomenon is an example, on the national scale, of the familiar economic fact that as the available quantity of any good is increased, the marginal utility of that good will fall.[21] Since the national product will increase from year to year in a well-run economy, the resources diverted to investment from this year's income should be weighted more heavily than the resources paid back next year by that investment. Thus the present cost of an investment should be weighted more heavily than its future savings, not because the savings are deferred in time, but becausé the savings are part of a larger national product per capita than was the initial cost, and hence represent a lower marginal utility.

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21. Professor J. K. Galbraith stresses the declining marginal utility of the national product. See <u>The Affluent Society</u>, Houghton Mifflin Co., Boston, 1950, p. 138<u>ff</u>.

Thus the zero normative interest rate must be rejected. In doing so, however, a lead into a wholly new approach to the problem has been developed, namely basing the criterion interest rate on the declining marginal utility of a growing national product.

A higher rate of growth implies a more rapidly declining marginal utility of national product and therefore a higher rate of growth implies a higher normative interest rate. At the same time a higher normative interest rate will effect a lower rate of growth. These statements may seem inconsistent, but actually they are merely two independent relationships between the rate of growth and the normative interest rate which enable us to determine a unique solution for both variables. Section IV will examine the rate of growth of national product as a function of an arbitrarily established normative interest rate. Section V will develop the normative interest rate implied by the rate of growth of national product, and place these two relationships into juxtaposition to determine a unique solution for the normative interest rate and the rate of growth.

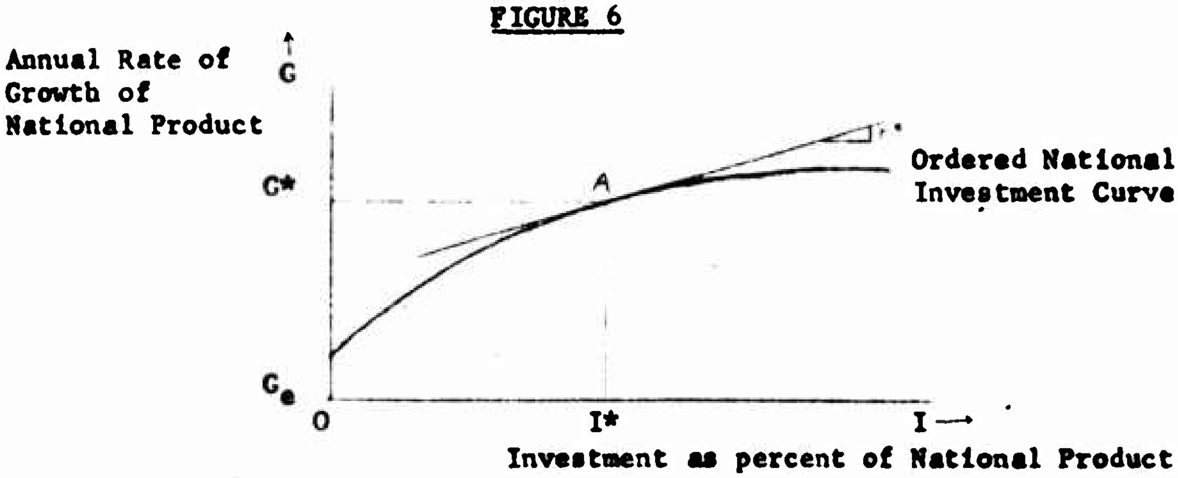
IV. The Rate of Growth of National Product Implied by an Arbitrarily Determined Interest Rate

For the purposes of this section two assumptions are made: (1) Full employment is maintained at all times and (2) The welfare returns on the marginal private and public investments are kept in balance. Thus both private and public investments are traded off against consumption, pivoting on full employment.

Figure 6 illustrates a hypothetical two-period example of the national investment decision as it would be made by public policy. The ordered national investment curve represents all private and public investment opportunities, taken in descending order of expected welfare rate of return, where the appropriate rate of return concept is the present value over present cost using a zero discount rate.[22] A different discount rate would not affect the ordering of the projects in a two-period case. All projects in which the return is less than the cost are rejected with the result that the rate of return can become zero but not negative. The horizontal axis measures I, the investment this year as a percent of this year's national product. The vertical axis measures G, the growth in national product next year as a percent of national product this year.

The slope at any point on the ordered national investment curve may now be identified as the welfare rate of return of the investment project represented by that point.

FIGURE 6



The National Investment Decision; Two-Period Case

In the two-period case pictured in Figure 6, the interrelationships between the rate of growth, the level of investment, and the normative interest rate are clear. Essentially, there is a single decision to be

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22. See Hirshleifer, op. cit., for an elaboration on rate of return concepts.

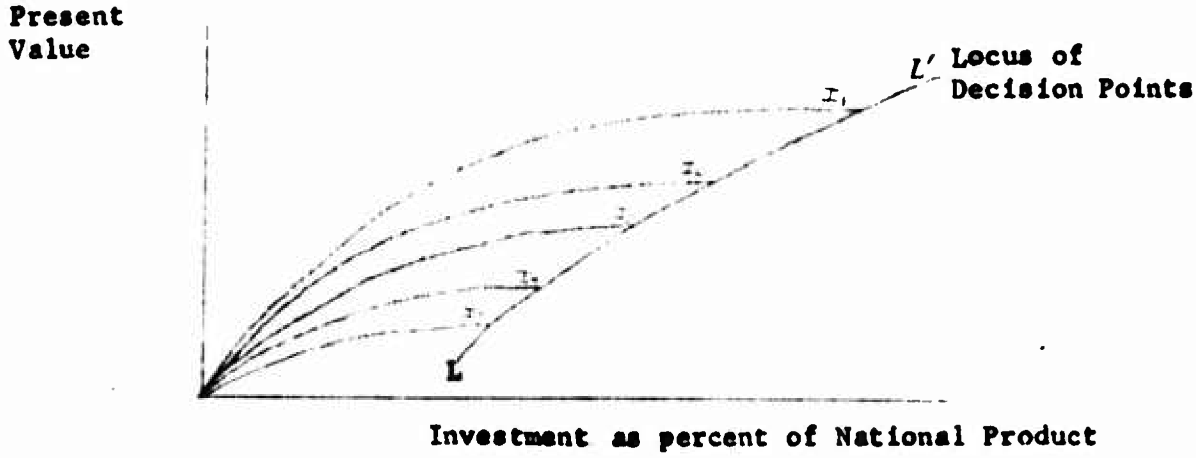
made, the selection of a stopping point on the ordered national investment curve, represented by point A. This point has three characteristics: (1) a horizontal displacement which is the level of investment, (2) a vertical displacement which is the growth in national product, and (3) a slope which is the normative interest rate. Whichever of these three characteristics is chosen to set point A, the other two characteristics become determinate, and I*, G* and r* are determined simultaneously. For our purposes we wish to consider r* to be set arbitrarily and I* and G* determined thereby.

Note that the ordered national investment curve of Figure 6 has a positive vertical intercept G_e. This represents exogenous growth, or growth in national product which would occur in the absence of investment.

Unfortunately, several complications are encountered in extending the two-period case into the multi-period case. For one thing, it is no longer legitimate to use a zero discount rate. In the case of projects defined over three or more periods, the ordering of projects is affected by the discount rate chosen, with a low discount rate favoring those projects with returns relatively far in the future. Thus, a family of investment curves exists in the multiple-period case, one for each possible discount rate. The family of ordered national investment curves is illustrated in Figure 7.

In moving from Figure 6 to Figure 7, it was necessary to redefine the vertical axis since the rate of growth in national product can no longer be identified. The vertical axis was therefore changed to present value, one of the several items it represented in the more restricted case of Figure 6.

FIGURE 7



Family of Ordered National Investment Curves

The I_i curves represent a family of ordered national investment curves representing several discrete discount rates, with the discount rates increasing with increasing i. Each curve in the family represents a single investment curve obtained with the use of a single discount rate. The present value measured on the vertical axis should be interpreted separately for each curve, with the vertical displacement of each of the curves measuring present value at the same discount rate as is assumed for the curve. This implies that one cannot properly compare the different curves with each other in the present form of the diagram. If a homogeneous measure of present value were used, however, (e.g., present value at a single discount rate for all curves) we would lose the right to draw the curves with monotone declining slopes, except for the one curve representing the discount rate used for the numeraire present value.[23]

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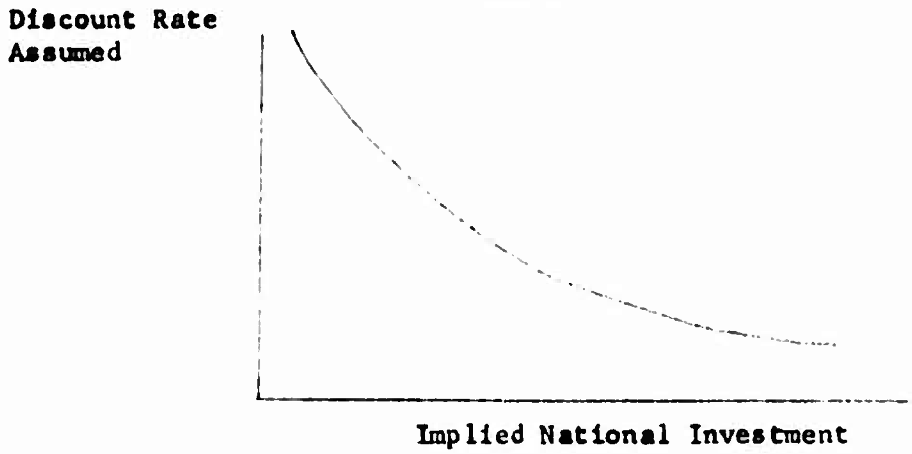
23. This loss in monotonicity of slope results from the fact that different discount rates change the ordering of investment projects. Only in terms of present value measured at the same discount rate used for ordering are we guaranteed the monotonicity of slope.

The only relevant point on each curve is the point at which it becomes flat, or in other words where the present value of the marginal project is zero with present value measured in the appropriate discount rate. The locus of points representing the tops of the ordered national investment curves forms a new curve LL' which represents the set of decision points consistent with the assumptions of full employment and equality in welfare rate of return between marginal private and marginal public investment. This locus is meaningful in terms of its horizontal displacement, but not in terms of its vertical displacement. The locus of decision points does represent however a correct relationship between the discount rate implied by each point and the horizontal displacement representing the national level of investment. Figure 8 represents a translation of the level of investment implied by each discount rate from the locus of decision points of Figure 7.

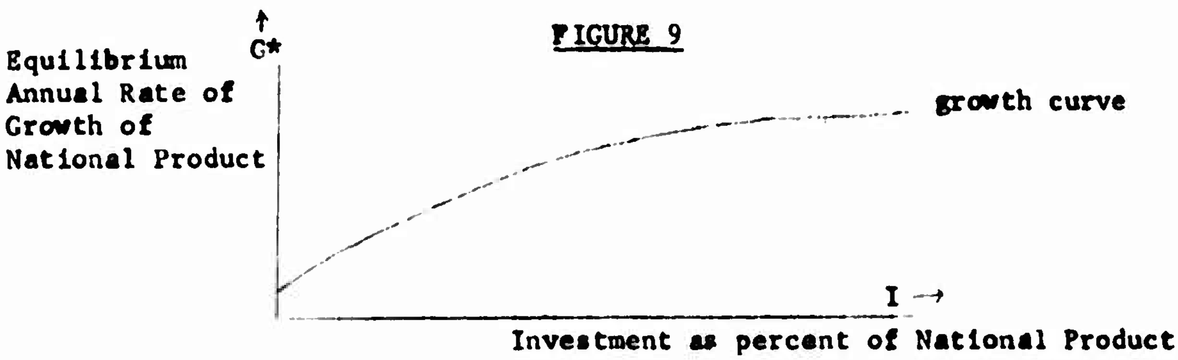
FIGURE 8



Relationship Between Discount Rate and Level of National Investment Provided by the Locus of Decision Points of Figure 7

In order to make the multi-period national investment decisions more meaningful, it will be necessary to bring back into the analysis the rate of growth of national product. This factor was lost in moving to the multi-period case because returns on investment projects no longer all accrue <u>next year</u>, but are spaced out into the future. Let us assume for the moment a static ordered national investment curve, or family thereof, such that the curve representing a given discount rate in Figure 7 would shift neither upward nor downward over time. Let us also assume that the time distribution of returns from investment is stable both over time and, for the moment, at alternative levels of investment, and that a single normative interest rate is to be selected to apply to all periods of time. This set of assumptions provides the function illustrated in Figure 9.

FIGURE 9



Growth Curve for Stable Ordered National Investment Curve

The growth curve pictured in Figure 9 is not the same as any of the family of ordered national investment curves shown in Figure 7, but it is related to the one representing a zero discount rate by a factor which depends only on the time distribution of returns from investment. A short mathematical note has been appended to explain the derivation of the translation factor from the time distribution of returns. Figure 9 relates a fixed level of investment as a per cent of national product to an equilibrium annual rate of growth of national product, and it achieves this balance at a normative interest rate which is stable over time. The normative interest rate cannot be read from the growth curve of Figure 9. A normative interest rate can be chosen arbitrarily, however, the implied level of national investment determined from Figure 8, and then the equilibrium rate of growth determined from Figure 9.

Let us now remove the assumption that the time distribution of returns is constant over alternative levels of investment, since the dispersion in the time distribution of returns is a function of the discount rate and thus also of the level of the investment program. In order to do this, the growth curve of Figure 9 is translated from the locus of decision points of Figure 7, with a separate time distribution of returns being applied to each discount rate. The result is a growth curve similar to the one in Figure 9, and applicable in the same way.

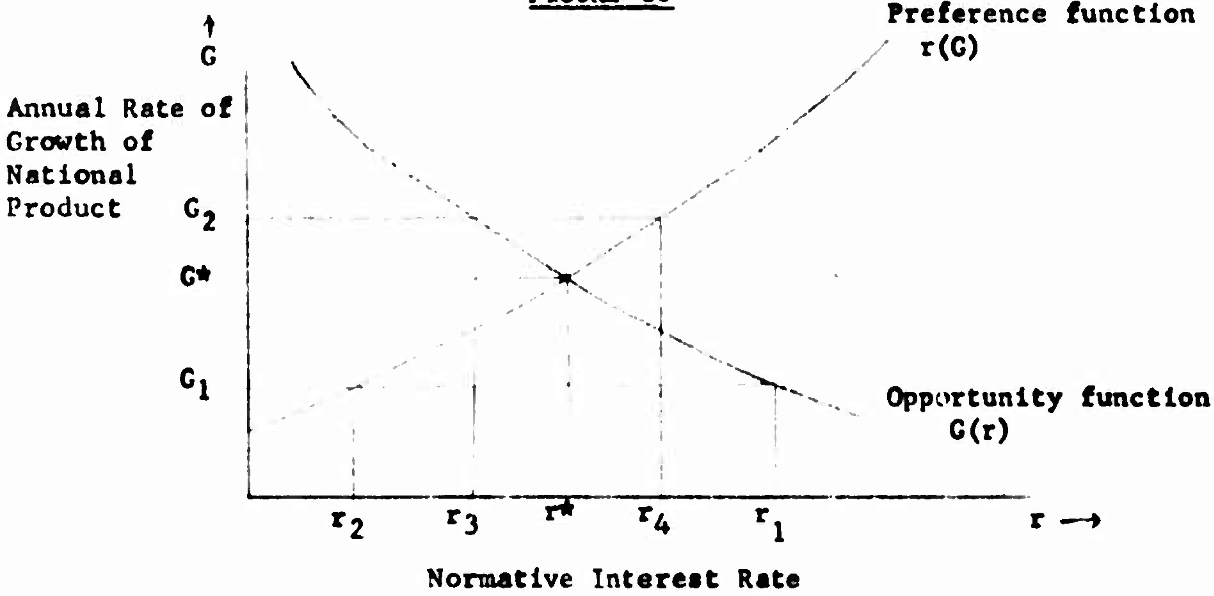
If the family of ordered national investment curves were to shift upwards or downwards over time, as a function of the level of investment chosen in earlier years, a somewhat more complicated kind of national decision is presented. For our present purposes, however, it would seem sufficient to assume that movements of the investment curves are of a fairly long-run nature, and that for the short and intermediate run investment decision it is sufficient to assume no trend movement in the curves. We shall proceed on that assumption for the moment. Section VI considers the problem of movements in the ordered national investment curves over time.

In summary, given an arbitrary normative interest rate, the level of investment is determined through Figure 8; given the level of investment, the rate of growth of national product is determined through Figure 9. The rate of growth thus determined as a function of an arbitrarily selected normative interest rate is represented as the opportunity curve of Figure 10.

V. The Normative Interest Rate as a Function of the Rate of Growth of National Product

The second relationship between the rate of growth of national product and the normative interest rate operates through the decline in the marginal utility of a growing national product. This second relationship is represented as the preference function of Figure 10.

FIGURE 10



Determination of the Normative Interest Rate and the Rate of Growth of National Product

The normative interest rate and the annual rate of growth of national product are negatively related in the opportunity function; a high normative interest rate implies a low investment program (see Figure 8); and a low investment program implies a low rate of growth (see Figure 9). This opportunity relationship represents the national rate of growth as a function of an arbitrarily determined normative interest rate.

The preference relationship between the normative interest rate and the rate of growth of national product represents the normative interest rate as a function of the rate of growth. Here the normative interest rate is defined as the rate of decline of the marginal utility of national product over time. Thus the relationship is positive; a larger rate of growth implies a faster rate of decline of the marginal utility of national product over time which in turn implies a higher normative interest rate.

Point A, where the opportunity and preference curves of Figure 10 meet, is the only point in which national opportunity and national preference are consistent; it is the end of our search for a normative interest rate. The intersection of the two curves determines both a preferred rate of growth G* and a preferred normative interest rate of r*. It also implies a preferred level of investment either from Figure 8 or from Figure 9.

A rate of growth less than G* (e.g., G_1 of Figure 10) would require the setting of an arbitrarily high normative interest rate r_1; but this low rate of growth would imply a slow decline in the marginal utility of national product and hence the low and wholly inconsistent normative interest rate r_2. Here the nation is underinvesting since the marginal investment earns a substantilly higher rate of return (r_1) than the rate properly required by the nation for the sacrifice of additional present resources (r_2).

A rate of growth higher than G* (e.g., G_2 of Figure 10) would require the setting of an arbitrarily low normative interest rate r_3, but this high rate of growth would imply a rapid decline in the marginal utility of national product and hence the high and equally inconsistent normative interest rate r_4. Here the nation is overinvesting since the marginal investment earns a lower rate of return (r_3) than the rate properly required by the nation for the sacrifice of additional present resources (r_4).

At G* the normative interest rate required by the opportunity curve is consistent with that implied by the preference curve; only at that rate of growth could the nation's investment program be considered proper.

VI. The Normative Interest Rate and the Equilibrium Rate of Growth

At the end of Section III we briefly noted the complexity of the decision process if one assumes that the movement of the ordered national investment curve in a later year is a function of the levels of investment chosen in earlier years; but this is a very possible kind of interrelationship.[24] Let us now examine briefly the interaction between such a phenomenon and the normative interest rate system of the preceding section.

There are many possible kinds of relationships between current investment and the level of future investment curves. The one which interests us is the equilibrium rate of investment concept in which a rate of investment in excess of the equilibrium rate is assumed to drive future investment opportunity curves downward, and hence to drive future investment downward towards the equilibrium rate. The reverse is assumed to result from a rate of investment less than the equilibrium rate. What then happens if a normative interest rate determined without consideration of such an equilibrium rate of investment is inconsistent with it? Let us assume that the normative interest rate is determined through the methodology of the preceding section, and that it is too low, and hence the rate of investment is too high, for the investment equilibrium rate of growth. Everything we say will apply in reverse for a rate of investment too high for the investment equilibrium rate of growth.

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24. See James S. Duesenberry, Business Cycles and Economic Growth, McGraw-Hill, N. Y., 1958, p. 205ff; N. Kaldor, "A Model of Economic Growth," The Economic Journal, Vol. LXVII, Dec. 1957, pp. 597-598. R. Harrod, Toward a Dynamic Economics, Macmillan & Co., Ltd., London, 1952.

The first impact of the excessive rate of investment and growth would be to force downward the ordered national investment curves of Figure 7. This would force downward the implied investment curve of Figure 8 since the same level of investment would now require a lower discount rate; and would force downward the growth curve of Figure 9, since the same level of investment would now provide less growth. Both of these phenomena would reinforce each other to lower the opportunity curve of Figure 10. This would provide a lower normative interest rate which would tend to inhibit but not stop the decline in the rate of growth of national product. Over time, the process would necessarily reach a new equilibrium rate of growth at which the G^* of Figure 10 was equal to the investment equilibrium rate of growth, at which point the process would stop. Our interest is in analyzing the relationship between the new equilibrium rate of growth and the two inconsistent equilibrium rates of growth with which we started. We shall maintain that the new equilibrium rate of growth will lie between the two inconsistent initial rates of growth. This necessarily implies that just as the equilibrium rate of growth of Figure 10 will fall towards the investment equilibrium rate of growth, so too will the investment equilibrium rate of growth rise towards the equilibrium rate of growth of Figure 10.

In order to understand why we should expect the investment equilibrium rate of growth to adjust let us consider why current investment is assumed to influence the future levels of the investment curves. There are several impacts which tend to offset each other but to do so poorly; hence they can cause either an upward or a downward shift in the future investment curves. Let us consider each of the impacts separately.

1. Taking away an investment opportunity. The first impact of present investment on future investment curves is the fact that an investment opportunity taken this year is not available again next year. Thus investment this year tends to lower the future investment curves, and to lower it more the more investment there is this year. One might think that the marginal investments this year are not important for next year's investment curve except around the margin, but there is a tendency for all investment projects to increase in value as they are postponed. Thus the replacement project postponed this year grows in value next year because the old system is even more decrepit and the new system available next year is presumed better than that available now.[25] Similarly an expansion of steel capacity will have a higher return next year because the incremental capacity will offer a higher expected load factor next year than it does this year. Thus the investment opportunities taken this year will lower the investment curve next year throughout its length. This first impact taken together with an exogenous generation of new investment opportunities (not a function of this year's investment) is enough to explain the phenomenon of an investment equilibrium rate of growth. Clearly there will be some equilibrium rate of investment which will lower next year's investment curve just enough to compensate for the exogenous upward shift in the curve. Less investment than this will cause the curve to shift less than enough to compensate for the exogenous upward movement, and the curve will shift upward. More investment than this will cause the curve to shift more than enough to compensate for the exogenous upward movement, and the curve will shift downward.

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25. This does not mean it would have been a mistake to replace now since the growth in value by postponement would have been considered in determining the value of replacement now.

2. <u>The accelerator</u>. The accelerator principle relates certain induced investments to the rate of change of national product. For example, the expansion of steel capacity discussed previously will have a higher value next year by way of a higher expected load factor if a higher rate of growth this year has stimulated auto sales and the prospect of future auto sales. The accelerator however is not really relevant to our analysis since it influences the path by which we arrive at an eventual equilibrium rate of growth rather than the level of that equilibrium. The accelerator, as a first derivative, cannot permanently sustain the rate of investment over its equilibrium rate -- any tendency to do so would merely shift future investment curves downward and hence would lower investment and growth towards their equilibrium rates.

It should be understood that the accelerator in this context is not the usual accelerator since a full employment policy by the government is assumed. Thus the impact of investment operates, not through an increase in short-run aggregate demand, which is held constant, but rather through an increase in full employment national product. Thus this accelerator is quite small relative to the accelerator operating through aggregate demand, and is undoubtedly stable.

3. <u>The endogenous generation of investment opportunities</u>. Since the concept of investment specifically includes such diversions of present resources to the future as research and education, it seems reasonable to assume that investment will tend to speed up the generation of altogether new investment opportunities. This third factor unlike its two predecessors will have a favorable influence on the investment equilibrium rate of growth. In order to understand this, let us imagine an investment opportunity curve

cumulated over several years, perhaps over a decade. This will include investment opportunities that were available at any time during the decade, whether taken or not. Such a curve will remain stable from the first impact, and the second impact will have only a temporary effect on it. However, a forced acceleration of investment would tend to shift the cumulative curve upwards because of the third impact. Research and education projects would be ordered into the investment curve throughout its length; hence an increase in investment over the equilibrium rate would breed new investment opportunities and raise the 10-year cumulative investment curve. Since the equilibrium rates of investment and growth are related to the cumulative investment curve rather than to the curve of any one year, the third impact should represent a route by which a continued rate of growth greater than the investment equilibrium rate of growth would raise the latter rate of growth to a new higher and stable level. It is a way for a country to pull itself up by its bootstraps. Surely nothing could be more discouraging to those who would develop underdeveloped countries than the concept of a stable cumulative investment curve, and nothing could be more completely disproved by the examples of countries which have carried on forced-draft investment programs without exhausting investment opportunities (e.g., the U.S.S.R.).

Research and education are not the only kinds of investment that will breed new investment opportunities and thus shift the investment equilibrium rate of growth upward permanently. Railroads in the last century, roads in this century, the electrification of a new area, and the introduction of atomic energy in areas inaccessible to other power, are all examples of investments with a potential for breeding other investment opportunities above and beyond the accelerator effect. In fact, with the example before us of the long-continued forced-draft investment program in the U.S.S.R. without any seeming decline in the return on investment we are inclined to believe that the investment equilibrium rate of growth rather than the G* of Figure 10 would make most if not all of the adjustment needed for equilibrium. In other words, we believe that the rate of generation of investment projects would tend to rise and fall with the rate of investment and hence that the ordered national investment curves would tend to be fairly stable over time in relationship to national product.

VII. Note on the Normative Interest Rate and the Level of Employment

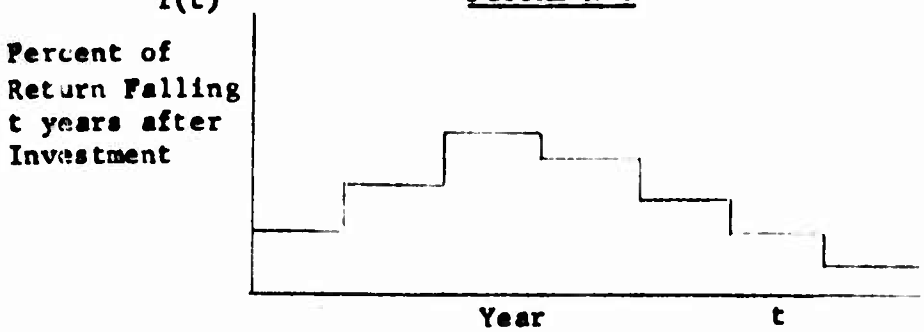
In our preceding analysis of the normative interest rate, it was assumed that it was one of the functions of the national government to maintain full employment. A stronger argument can be made for the assumption of full employment, however, for the normative interest rate as a criterion for government investment cannot exist except at full employment. This requirement for full employment in order to define a normative interest rate arises out of the very essence of investment as a diversion of present resources from the creation of present welfare to the creation of future welfare. Thus, if substantial unemployment exists in the nation, an increase in government investment will increase the use of resources for present consumption and investment through the Keynesian Multiplier and/or Accelerator. Thus there would be no present sacrifice; and an exchange rate between present and future welfare could not be defined.

APPENDIX

Note on Translation between Undiscounted Present Value and the Rate of Growth of National Product in the Multiperiod Case

Let us assume that a function exists describing the time distribution of investment returns, as in Figure A-1.

FIGURE A-1



Time Distribution Function for Investment Returns

This function need not be the same for each investment of course; it must however remain approximately stable from year to year, at a level of investment which is constant in percentage of national product.

Let us introduce some items of terminology:

g_t = growth of national product in year t; g_o = growth this year.

I_t = investment in year t; I_o is investment this year.

P_t = undiscounted present value of investment made in year t. Since the investment curves of Figure 7 are assumed stationary over time, and a single interest rate is being chosen, $P_t = cI_t$ where c is some constant.

The growth this year, g_0, is equal to the undiscounted present value for each past year, P_t, times the per cent of that present value accruing as a growth in welfare this year, f(t), summed for all years in the f(t) function:

1.) $$g_0 = \sum_{t=0}^{n} P_t \, f(t),$$ where n is the upper limit of the f(t) function.

But since P_t is a constant percent of National Product, we may define it in terms of this year's present value, P_o, merely by tracing backwards the growth in intervening years:

2.) $$P_t = \frac{P_o}{\prod_{x=1}^{t} (1+g_x)}.$$

Substituting equation 2 into equation 1, we obtain

3.) $$g_o = \sum_{t=0}^{n} \frac{P_o \, f(t)}{\prod_{x=1}^{t} (1+g_x)}$$

The equilibrium rate of growth, g*, will be that rate of growth which would enable the series of g_t's to be equal, and can be obtained by solving equation 4, which was obtained by substituting g* for all of the g_t.

4.) $$g^* = \sum_{t=0}^{n} \frac{P_o \, f(t)}{(1+g^*)^t}$$

The equilibrium rate of growth as a ratio of the undiscounted present value (assumed to be a constant percent of each year's national product) may be written:

$$\frac{g^*}{P_o} = \sum_{t=0}^{n} \frac{f(t)}{(1+g^*)^t}$$