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This study was conducted to develop finite element computer programs to calculate stresses and deflections in rigid pavements with cracks and joints subjected to loads and temperature warping, as well as in the supporting subgrade soil. This report is presented as a user's manual for the WESLIQID program, which deals with pavements on a liquid foundation. The program allows for analysis of pavements with full or partial loss of subgrade support over (Continued)

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## 20. ABSTRACT (Continued).

designated regions of the pavements. Variable slab thickness and modulus of subgrade reaction $k$ are incorporated and any number of slabs arranged in an arbitrary pattern can be handled. Also, multiple-wheel loads can be used, and the number of wheels is not limited.

The nature of the computer program and its programming logic are first delineated, followed by a general discussion on the efficient and correct usage of the program, e.g., the efficient way of arranging nodal numbers to minimize the bandwidth. The input guide to the computer program is presented with a detailed explanation for each input variable. Five example problems with input data are presented and the computer printouts of three problems are included with detailed explanations.

The study described herein was sponsored by the Office, Chief of Engineers, U. S. Army (OCE), as a part of the Mobility and Weapons Effects Technology RDT\&E Project No. 4A762719AT40, Work Unit 001, "Airfield Pavement Design and Parametric Sensitivity Analysis," and Work Unit 003, "Rigid Airfield Pavement Load-Deformation Response Analysis."

This report is Report 2 of a three-report series concerning the computer programs WESLIQID and WESLAYER, which provide for analysis of rigid multicomponent pavements with discontinuities on liquid foundations (WESLIQID) and on linear layered elastic solids (WESLAYER). This report is a user's manual for WESLIQTD. Report 1 provided a theoretical background and numerical results and discussed the capability and logic of the two programs. Report 3 will be a user's manual for WESLAYER.

The study was conducted by the U. S. Army Engineer Waterways Experiment Station (WES), Geotechnical Laboratory (GL), under the general supervision of Dr. Don C. Banks, Acting Chief, GL; Dr. Paul F. Hadala, Assistant Chief, GL; and Mr. Alfred H. Joseph, Chief, Pavement Systems Division (PSD), GL. Dr. Yu T. Chou, PSD, was in charge of the study and is the author of the report. Professor Y. H. Huang of the University of Kentucky, who originally developed the computer programs, assisted in the study.

COL John L. Cannon, CE, and COL Nelson P. Conover, CE, were Commanders and Directors of WES during this study and the preparation of this report. Mr. Fred R. Brown was Technical Director.


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U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

| Multiply | By | To Obtain |
| :---: | :---: | :---: |
| Fahrenheit degrees | 0.555 | Celsius degrees or Kelvins* |
| feet | 0.3048 | metres |
| inches | 2.54 | centimetres |
| pounds (force) | 4.448222 | newtons |
| pounds (force) per inch | 175.1268 | newtons per metre |
| pounds (mass) per cubic inch | 27,679.9 | kilograms per cubic metre |
| pounds (force) per square inch | 6,894.757 | pascals |
| square inches | 6.4516 | square centimetres |

[^0]
# STRUCTURAL ANALYSIS COMPUTER PROGRAMS FOR RIGID MULTICOMPONENT PAVEMENT STRUCTURES WITH DISCONTINUITIES-WESLIQID AND WESLAYER 

MANUAL FOR THE WESLIQID FINITE ELEMENT PROGRAM

PART I: INTRODUCTION

## Background

1. The U. S. Army Corps of Engineers (CE) has realized for many years that much of the maintenance of rigid pavements is associated with cracks and joints. The current CE rigid pavement design procedures have certain limitations that were imposed by the state of the art at the particular stage of development. During the development of the procedure, it was necessary to make simplifying assumptions and, in many instances, to ignore the effects of cracks and joints. Since the advent of high-speed computers and the development of the finite element method, a more comprehensive investigation than previously possible of the state of stress at pavement joints, cracks, and other locations in multicomponent pavement structures can now be achieved. Consequently, a better and more reasonable design procedure may be developed for rigid pavements.

## Purpose

2. The development of the finite element programs and the analysis of computed results are presented in Report 1 of this series. This report presents a user's manual for a computer program named WESLIQID. The program computes the state of stress in a linear elastic plate (approximating a rigid pavement) supported on a liquid foundaticr, as well as in the supporting subgrade soil.
3. The computer program is described in the report to give users a concise understanding of the program without reference to Report 1. The logic of the programming is explained by use of flowcharts. An input guide to the computer program is given, and five example problems are presented to illustrate the input procedures for using the computer program. The computer printouts for three example problems are also explained.

## PART II: PROGRAM DESCRIPTION

4. This report describes a finite element computer program named WESLIQID for the analysis of concrete pavements subjected to multiple-wheel loads. The program is developed for subgrade soil represented as a Winkler foundation (or a liquid foundation); i.e., only forces and deformations in the vertical direction are considered and the force is proportional to the deformation. The program can handle any number of rectangular-shaped slabs arranged in an arbitrary pattern. The slabs are connected to each other at joints by steel bars or other load transfer devices and can have cracks in directions parallel to or perpendicular to the joints.
5. The program determines stresses and displacements in the pavement and in the supporting subgrade soil due to loads and temperature warping. Part of the pavement can be out of contact with the supporting subgrade before applying the load and the temperature gradient, and the program determines the condition of contact at each nodal point after the application of loads and temperature gradient. Input data of the programs include (a) the physical properties and geometry of the pavement and subgrade soil, (b) the magnitude and distribution of the loads, (c) the temperature gradient, (d) gaps under the pavement at certain nodal points, if any, and (e) joint and crack conditions.
6. At a joint or a crack, the program considers both shear and moment transfer. Three options can be used for shear transfer: (a) the assumption of an efficiency of shear transfer at the joint, which is defined as a ratio between the deflection of the unloaded or less loaded slab and the deflection of the loaded slab; (b) the assumption of a spring constant at the joint, which is defined as the force in pounds per linear inch and which can be used for key joints or joints with aggregate interlock for shear transfer; and (c) consideration of the diameter and spacing of steel bars. The efficiency of moment transfer is not defined as the rotation ratio between the unloaded and loaded slabs, but as a fraction of the full moment, which is determined
by assuming that the rotations on both sides of the crack are the same. The theoretical development of the finite element model is presented in Report I of this series.
7. WESLIQID can analyze pavements with variable thicknesses. This option is useful for pavements with thickened edge joints or pavements adjacent to a cement-stabilized shoulder. Multiple-wheel loads can be input, and the number of wheels is not limited. The number of slabs is also not limited, but is subjected to the dimension and computer storage requirements. Also, the solution becomes more difficult to converge as the number of slabs and nodal points are increased. The slabs can have two layers with different physical properties. The interface of the layers can be either bonded or unbonded. The program is capable of considering variable subgrade reactive forces. This option is useful in dealing with nonuniform subgrade support.
8. The storage space required for the program depends on the total number of elements used in the problem. An iteration scheme is used in the program so that the computation is made only for one slab at each time. This scheme results in a great savings in computer time because the number of equations to be solved each time is reduced to only one slab. Two series of iterations are involved in the program: one is with respect to subgrade contact and the other is with respect to load transfer across the joint.
9. In the iteration with respect to subgrade contact, the contact condition at each node, i.e., whether the slab and subgrade are in contact or not, is first assumed; and the iteration with respect to load transfer proceeds until either the convergence criteria (DEL in Item 6 of Table 2,* the input guide) are satisfied or the maximum allowable number of iterations (ICL in Item 6 of Table 2) is reached. At this stage, the resulting contact condition is determined. If some nodes originally assumed in contact are found out of contact, or vice versa, the newly found contact condition is assumed, and the process is repeated until the same contact condition is obtained. This can usually be achieved in only a few iterations. The only control by the user is to specify the maximum number of iteration cycles NCYCLE . If $N C Y C L E=1$, the contact condition between the slab and subgrade is knuwn a priori, and no iterations are needed.
10. In the iteration with respect to load transfer across tio joint, the computation is made successively from the first slab to the last one. The reaction between two adjacent slabs can be either the superimposition of displacements or the transfer of shear forces along the joints. The rule to follow is that when the displacements for slab i are computed, the displacements along the joint will be superimposed to the adjacent slabs which have slab numbers greater than $i$, and the vertical shear forces will be transferred to the slabs that have

* Table 2 appears in Part IV where it is discussed in detail.
smaller slab numbers. The shear forces are computed from the deflections of elements adjacent to the joint through the stiffness matrix of the slab.
ll. In the iterations with respect to load transfer, the vertical shear forces are also used for checking convergence. If the shear forces at the joints are changed too much between two iterations, the solution may diverge and the shear forces will become unreasonably large. To ensure convergence, a self-adjusting relaxation factor is incorporated into the program.

12. In this method, the vertical shear force at each node along the joints obtained in a given iterations is not used directly in the next iteration. Instead, an underrelaxation factor $R_{f}$ is applied such that

$$
\begin{equation*}
F_{i+1}=F_{i-1}+R_{f}\left(F_{i}-F_{i-1}\right) \tag{1}
\end{equation*}
$$

in which $F_{i+1}$ is the vertical force to be used at $(i+1)^{\text {th }}$ iteration; and $F_{i}$ and $F_{i-l}$ are vertical forces obtained during the $i^{\text {th }}$ and $(i-1)^{t h}$ iterati $1 s$, respectively. When $R_{f}=1, F_{i+1}=F_{i}$, or the force obtained in iteration $i$ is used directly in the next iteration, $i+1$. It was found that for most problems, the solution could not converge when $R_{f}=1$. An initial relaxation factor $R F I$ must be specified by the user. An initial value of 0.5 can be arbitrarily assumed unless the user's experience indicates that a smaller value is more appropriate.
13. To adjust the relaxation factor automatically, a maximum shear force at a given node on a joint MAXFAJ must be specified by the user. If the shear force at the node exceeds MAXFAJ, the indication is that the solution is divergent and a smaller relaxation factor should be used. If it is desired, beginning from the sixth iteration, the program also checks the convergence of the specified vertical force after every five iterations. If the solution diverges or oscillates back and forth, the relaxation factor is reduced by one-half (or onequarter if desired), and the computation is restarted.
14. The program first computes the dimensions of certain important variables and checks them with the declared dimensions. If the computed value exceeds the declared value, the program will be stopped and unnecessary computations are avoided. Once the checks are performed, the program carries out the computations in the following sequence (see the flowchart, Figure 1):
a. Generate stiffness matrix for each element and then superimpose them to form an overall stiffness matrix.


Figure 1. Flowchart for computer program WESLIQID
b. Store stiffness matrix adjacent to joints for later use.
c. If it is known that gaps exist under certain nodes in the subgrade soil, the gaps are read into the program to combine them with the computed curls of the slabs due to temperature warping to form the initial subgrade contact condition.
d. Determine the nodal reactive condition based on the subgrade contact condition.
e. If externally applied loads are considered, the uniformly applied loads are distributed to the adjacent nodes using statics.
f. Compute the displacements of slab l, assuming that there is no shear and moment transfer along the joints; i.e., slab 1 has four free edges.
g. Impose deflections along the joints to the adjacent slabs that have greater slab numbers. For illustrative purposes, a four-slab pavement system is chosen, as shown in Figure 2. Displacements of slab lat nodes 1 , 2 , and 3 of joint 3 and nodes 1,4 , and 7 of joint 2 are superimposed to slabs 2 and 3 , respectively.
h. Compute the displacements of slab 2. This is done with a fixed boundary condition at joint 3 and reactive forces at nodes 10,13 , and 16 of joint 1 that are induced from the deflections of slab 4 computed in the previous iteration cycle. At the first cycle, the reactive forces at the nodes are zero because the deflections of slab 4 have not been computed and are thus assumed to be zeros. The nodal reactive forces are identical to the vertical shear forces mentioned earlier. It should be pointed out that reactive forces at nodes 16,17 , and 18 at joint 3 induced by the deflections of slab 1 exist but are of no importance in the computation of displacements of slab 2 because the boundary condition at joint 3 is arbitrarily fixed as the prescribed displacement imposed by slab 1 . Once the displacements of slab 2 are computed, the displacements at the joints are superimposed to the adjacent slabs which have greater slab numbers, such as slab 4 in Figure 2.
i. Compute the nodal reactive forces at the joints between slab 2 and adjacent slabs that have smaller slab numbers, such as joint 3 in Figure 2. The reactive forces acting at nodes 1,2 , and 3 of slab 1 are induced by the deflections computed at slab 2. It may be worth mentioning here that the relaxation factor is used in transferring the shear forces from slab 2 to slab 1.

note: number next to the nodes denotes nodal number. number inside the circle denotes element number. number inside the square along the joint denotes JOINT NUMBER.

Figure 2. A four-slab pavement system
Once the shear forces are transferred, they become nodal reaction forces at slab l. The adjusted nodal forces are computed from Equation 1.

1. Compute the difference in deflection between the two slabs, which is equal to $\Delta_{s}+2 \Delta_{c}$, where $\Delta_{s}$ is the shear deformation of the dowel bars and $\Delta_{c}$ is the deformation of concrete due to the shear force on the dowel bar. The values of $\Delta_{s}$ and $\Delta_{c}$ are computed from Equations 18 and 19a of Report 1 of this series.
k. Continue the process for slabs 3 and 4. The displacements in slab 3 are computed with fixed displacements at joint 2 superimposed from slab 1 and reactive nodal
forces at joint 4 induced by the deflections of slab 4. As explained earlier, the reactive forces are zero at the first cycle of iteration. Once the displacements at slab 3 are computed, the displacements are superimposed to slab 4 at joint 4 , and the reactive forces at nodes 1 , 4 , and 7 at slab 1 induced by the deflections of slab 3 are computed and the difference in deflection between slabs 3 and 1 is computed. With superimposed displacements at joints 1 and 4 , the displacements at slab 4 are computed.
2. With displacements of slab 4, the vertical nodal, or shear, forces at joints 1 and 4 are computed and the differences in deflections between slabs 2 and 4 and slabs 3 and 4 are computed. Assuming joint 1 is the joint designated for checking convergence, this completes the first cycle of iteration with respect to load transfer.
m. With reactive nodal forces at joints 2 and 3, the displacements at slab 1 are computed again.
n. Repeat steps $\underline{h}$ through $\underline{1}$ until the vertical forces along joint 1 converge to a specified tolerance. In step $h$ at this time the displacements of slab 2 are computed by setting the deflections along joint 3 equal to the deflections of slab 1 minus the difference in deflections between slabs 1 and 2 computed previously and the reactive forces at joint 1 induced from displacements of slab 4.

ㅇ. Once a convergent solution is obtained or the maximum allowable number of iterative cycles has been reached (ICL of Item 6 of Table 2), the signs of the deflections at each node are compared with those of the initial (or the previous) subgrade contact condition. A change of sign at any node indicates that the contact condition at these nodes has changed. Based on the renewed subgrade contact condition, the computational process from steps $g$ to $k$ is repeated. The iteration process stops when either the contact condition ceases changing or the maximum allowable number of iterations (NCYCLE , Item 6 of Table 2) has been reached.
p. Once the subgrade contact condition no longer changes, the computational process from steps $g$ through $k$ is repeated once more with a refined convergence criterion. The controlling variables in the program are ICLF and DELF in Item 6 of Table 2.
q. The stresses at selected nodal points are computed based on the curvature of the deflected slab, i.e., the nodal displacements.
$\underline{\text { r }}$. Compute stresses and deflections in the subgrade soil if so desired.
s. Note for a single slab, i.e., NSLAB $=1$, the steps from $g$ to $n$ are neglected.
15. In superimposing the displacements aiong the joint, both vertical deflection and rotations are involved. The amounts of vertical deflections superimposed are determined based on the three shear transfer methods. The rotations superimposed depend on the efficiency of moment transfer. For 100 percent moment transfer, the rotations are equal at both the loaded and unloaded slabs. For zero percent moment transfer, the moments are zeros at both slabs. For a percent moment transfer other than zero or 100 percent, the process becomes more complicated. In dealing with such cases, users should consult Part II of Report 1. Example Problem 5 in Part $V$ of this report presents such a case.

## General Discussion

16. The input guide for the program is presented in this Part of the report. Special features in the correct and efficient use of the program are presented and discussed in the following paragraphs. Element size and shape
17. As with many other numerical procedures for solving structural problems, the accuracy of the finite element method depends greatly on the correct use of the technique. While the computational cost and computer storage space increase drastically with an increasing number of elements, the program does have a required number of elements. The element size should be smaller near the loads (such as 10 to 12 in . in one dimension) and joints where stresses are transferred to another slab. In some cases, the minimum number of elements for a particular problem has to be determined by a trial-and-error procedure. It was found that an insufficient number of elements can cause the solution to diverge. This is particularly true when temperature warping is considered and gaps exist under the pavement. Also, users should be aware that the aspect ratio of an element, defined as the ratio of the larger dimension to the smaller dimension of a rectangular element, should not exceed four or five to one. It is always a good practice for the beginning user of this program to familiarize himself with the program by using different numbers of elements for a particular problem and then comparing the results.
Dimension requirements
18. The method developed in this program can be applied to any number of slabs. Based on the present dimensions declared in this program, it can be applied to 9 slabs, 12 joints, 200 nodes, and 130 elements. Each slab can have as many as 15 X -coordinates and 15 Y coordinates. A maximum of 75 nodal points may be out of contact from the subgrade support. If an axis of symmetry exists, each axis can have a maximum number of 50 nodes. If any of these dimensions is
exceeded, the corresponding dimensions should be increased accordingly. The variables whose dimensions are subject to increase are given in Table 1 for various conditions.
19. The dimensions of $C, G, C L$, and $C U$ vary with the number of elements and the half bandwidth. The required dimensions are explained in the input guide. The storage, and consequently the cost, required for a particular problem depends primarily on the dimensions of $C$ and $G$, and therefore the dimensions of $C$ and $G$ should be changed according to the requirement of the problem.
20. It should be noted that when the dimensions of certain variables are changed in the main program, they should also be changed accordingly in the subroutines when the dimensions of the same variables are declared.

## Arrangement of slabs

21. Although the slabs can be arranged in any manner, there are rules to be followed. Along a joint between two slabs, the rules are: (a) the number of nodes along the joint should be equal, and (b) for a node on one side of the joint, there is one and only one corresponding node on the other side and the distance between the two nodes is the joint width.
22. The arrangements shown in $\underline{a}$ and $\underline{b}$ of Figure 3 are allowable. Arrangement $\underset{c}{ }$ is not acceptable because at the intersection of the joints, the node in slab 3 corresponds not only to the node in slab 1 but also to the node in slab 2. This situation may be remedied by creating a fictitious joint in slab 3 as shown in Figure 3d. The efficiencies of moment and shear transfers are both 100 percent along the fictitious joint. In this way, when the stresses are transferred along the joint between slabs 3 and 4, the node in slab 4 near the intersection of the joints corresponds to the node in slab 3. The same node in slab 4 corresponds to another node in slab 2 when the stresses are transferred along the joint between slabs 2 and 4 , which is permissible. Similarly, the arrangement in e of Figure 3 is not acceptable because the number of nodes along the joint in slab 2 is greater than
Table 1. List of Variable Names, the Dimensions of Which Are Subject to Increase

| Conditions | $\begin{aligned} & \text { Variabl } \\ & \text { Main } \\ & \text { Program } \end{aligned}$ | Location Slabroutine | Dimensions of Variables Need to be Increased |
| :---: | :---: | :---: | :---: |
| When numbur of slabs exceeds 9 | X |  | $\operatorname{INITNP}(9), \operatorname{JONO}(2,4), \operatorname{LASTNP}(9), \mathrm{NB}(9), \mathrm{NO}(9), \mathrm{NOB}(9), \mathrm{NX}(9), \operatorname{NY}(9)$ |
|  |  | x | $\operatorname{INITNP}(9), \operatorname{JOMO}(9,4), \operatorname{LASTEN}(9), \operatorname{LASTNP}(9), \operatorname{NB}(9), \operatorname{NO}(9), \operatorname{NOB}(9), \operatorname{NX}(9)$, $\operatorname{NY}(9), X(\underline{2}, 15 i, \mathrm{XX}(9), \mathrm{Y}(\underline{9}, 15), \mathrm{YY}(9), \operatorname{AREA}(2,130)$ |
| When number of joints exceeds 12 | X |  | $\operatorname{EFF}(12,3), \operatorname{ICK}(12), \operatorname{IJOINT}(12), \operatorname{ISLAB}(12), \operatorname{ISNN}(12,2), \operatorname{IST}(12,2)$, $\operatorname{LFNN}(12,2), \operatorname{LLS}(12), \operatorname{LUS}(12), \operatorname{NJT}(12,2), \operatorname{NKT}(12,2), \operatorname{ISLABI}(12)$ |
|  |  | X | $\operatorname{BARNO}(12,15), \operatorname{BD}(12), \operatorname{BS}(12), \operatorname{DC}(12,15), \operatorname{DCGF}(12), \operatorname{DID}(12,15), \operatorname{DIDF}(12)$, $\operatorname{DS}(12), \operatorname{EFF}(12,3), \operatorname{FAJ}(12,15,3), \operatorname{FGF}(12), \operatorname{FOJ}(12,15,3), \operatorname{ICK}(12)$, IJOINT (12), $\overline{\operatorname{ISLAB}}(12), \overline{\operatorname{ISNN}}(12,2), \operatorname{IST}(12,2), \overline{\operatorname{LFNN}}(12,2), \operatorname{LLS}(12)$, $\operatorname{LTR}(12), \operatorname{LUS}(12), \operatorname{NJT}(12,2), \operatorname{NKT}(12,2), \operatorname{PFAJ}(12,15,3), \operatorname{SCKV}(12,2)$, $\operatorname{SPCON}(12), \operatorname{WI}(12), \operatorname{ISLABI}(12), \operatorname{CEF}(12,15)$ |
| When total number of nodes exceeds 200 |  | X | $\operatorname{AB}(200), \operatorname{CURL}(200), \operatorname{GAP}(200), \operatorname{NCC}(200), \operatorname{NCCP}(200), \operatorname{NG}(200), \operatorname{NP}(200)$, $\operatorname{NS}(200), \operatorname{NT}(200), \operatorname{SUBMOD}(200), \operatorname{STR}(200,6,2), T(200,2), X N(200), \operatorname{YN}(200)$, AREA E(200) |
| When total number of concentrated forces (moments included) exceeds 200 | X | x | NFF(200), NFI(200), NF(200) NNPD |
| When total number of elements exceeds 130 | $\mathrm{x}$ | $x$ | $\begin{aligned} & \operatorname{DN}(130,2), \operatorname{NL}(130), \operatorname{PC}(130), Q(130), \operatorname{RM}(130,2), \operatorname{XDA}(130,2), \operatorname{YDA}(130,2) \\ & \operatorname{NELD} \end{aligned}$ |
| When number of nodes at either X- or (-axis of one slab exceeds 15 |  | X | $\operatorname{BARNO}(12,15), \operatorname{DC}(12,15), \operatorname{DFAJ}(15,3), \operatorname{DID}(12,15), \operatorname{DSB}(15), \operatorname{FAJ}(12,15,3)$, $\operatorname{FOJ}(12,15,3), \operatorname{PDFAJ}(15,3), \operatorname{PFAJ}(12,15,3), X(9,15), Y(9,15), \operatorname{FAJPD}(15,3)$, $\operatorname{CEF}(12, \overline{15})$ |
| When the number of nodes at an axis of symmetry exceeds 50 |  | X | NODSX (50), NODSY(50) |
| When number of nodal points out of contact exceeds 75 |  | $\chi$ | NODNC(75) |
| When number of nodes exceeds 130 in any slab |  | $x$ | AREA ( 9,130 ) |



Figure 3. Arrangements of slabs
that in slab 1. Again this can be remedied by creating a fictitious joint, as shown in the arrangement of Figure 3.

Symmetries
23. The application of the finite element method for analyzing rigid pavements invovles solving a large set of simultaneous equations. However, because of symmetry, the number of simultaneous equations could be greatly reduced by considering only one quarter or one half of the slab. The symmetry is with respect to the load, the pavement geometry and property, the finite element grid space, and the load transfer device along the joint. The users are strongly urged to take advantage
of the symmetry option provided by the program to arrange the loadings in such a way that the problem becomes symmetrical. A coded data input for a symmetrical example problem is presented in Part V. It should be pointed out that symmetry should not be placed at a joint, unless the joint is 100 percent rigid, i.e., 100 percent shear and moment transfers.
24. When the effects due to temperature and loadings are considered separately, the computed results due to temperature alone are expected to be symmetrical with respect to the pavement geometry. For instance, the stresses and deflections are the same at the four corner nodes in a square slab subjected to a temperature warping. This may not be the case, however, if the finite element grid lines are not divided symmetrically. In practical cases, smaller elements can be used around the applied loads, which may result in a nonsymmetrical finite element grid pattern. If this is the case, the computed results due to temperature alone may not be symmetrical as they ought to be and consequently may affect to a certain extent the final results when the temperature effect is combined with the effect of the load. The error in most cases is insignificant because the load effect usually outshadows that of the temperature. Nevertheless, users should be aware of this possible discrepancy. The finite element grid pattern shown in Figure 4 can be used to illustrate this point.
25. In Figure 4, the loads are placed at the pavement's center next to the joint. Smaller elements are used around the loads and larger elements are used elsewhere. Although the finite element pattern is symmetrical with respect to the pavement center line and symmetrical with respect to the joint, the element sizes are not identical. Consequently, if there is no moment transfer along the joint, the computed results due to the temperature's effect at nodes 1 and 57 are not equal as they are theoretically supposed to be. Consequently, the final computed results are not strictly correct. However, the error is believed to be insignificant when the effect of applied loads is combined. It should be pointed out that the solutions obtained from the finite element application are by no means completely correct; they are only close, acceptable approximations. It is the correctness of the computed larger

b. HALF SLABS USED IN COMPUTATIONS
note number next to nodes oenotes
NODAL NUMBER.
NUMBER INSIOE THE CIRCLE DENOTES
NUMBER INSIOE TH

Figure 4. Finite element layout for Example Problem 3
values that is important. The smaller values computed at insignificant locations of the pavement, such as at places far away from the load, are of no significance in engineering problems.
Slab numbering system
26. The iterative scheme developed in this program provides the computation of displacements for one slab at a time; the computations
are then carried out for other slabs in a sequential order until the shear forces converge to a prescribed limit. The relationships among the slabs are (a) the superimposition of displacements to an adjacent slab through the joint and (b) the nodal reactive forces at the joints, which are induced by the deflections of adjacent slabs. When the displacements are superimposed from one slab to its adjacent slab, it makes sense only when the displacements of the imposing slab are greater than those of the slab being imposed upon; otherwise, the solution will either diverge or converge slowly. The rule of thumb in the numbering system is that the slabs are numbered such that the deflections in a slab are superimposed to the adjacent slab that has smaller deflections. Therefore, the slab with greater deflections should be numbered earlier than the neighboring slab that has lesser deflections. Accordingly, the slab subjected to the largest load is numbered first. Slabs that do not carry loads should be numbered based on the anticipated magnitude of deflections. For instance, in Figure 5a, slab 1 is subjected to the largest load and slab 2 to the smallest load. Since the deflections in slab 3 are anticipated to be greater than those in slab 4, slab 3 is numbered before slab 4. The numbering system shown in Figure 5a ensures proper convergence of the solution. If the loads on two slabs are nearly the same or it is difficult to judge which is greater, either order may be used. Figure 5 b shows the proper slab numbering system for a five-slab pavement. For illustrative purposes, the slab numbers for the same five-slab pavement are changed as shown in Figure 5c. The load transfer mechanism along joint 1 will have a problem since the deflection in slab 5 is greater than that of slab 4, and thus the deflection should transfer from slab 5 to slab 4 along joint 1 . According to the slab numbering system shown in Figure 5c, the deflections in slab 4 are transferred to slab 5, as the slab number of slab 4 is smaller than that of slab 5. However, it is not logical to transfer the deflections from slab 4, which has smaller deflections, to slab 5, which has greater deflections. In doing so, the solutions will either be divergent or erroneous.


Figure 5. Illustration of slab numbering system

## Relaxation factors

27. In the iterations with respect to load transfer, the vertical shear forces at a specified node on a specified joint are checked for convergence. If the shear forces at the joints are changed too much between two iterations, the solution may diverge and the shear forces become unreasonably large. To ensure convergence, a self-adjusting relaxation factor is incorporated in the program. More information in this respect can be found in paragraphs 13 and 40 of this report.

Efficiencies of shear and moment transfer
28. Detailed explanations of the definitions of efficiencies of shear and moment transfer are given in Report 1 of this series. It should be reiterated that the efficiency of shear transfer is defined as a ratio between the deflection of the unloaded, or less loaded, slab and the defletion of the loaded slab. Also, the efficiency of moment transfer is the ratio between the actual moment and the moment in the case of 100 percent moment transfer. One hundred percent efficiency of
moment transfer occurs when the rotations at both sides of the joints are the same, and consequently the moments at both sides of the joint are the same. Zero percent efficiency of moment transfer means that the crack opening is so large that moment does not exist along the joint. For an efficiency of 50 percent moment transfer, the moments are 50 percent of those computed from the 100 percent efficiency of moment transfer, and the moments on both sides of the joint are still equal. This is why when the efficiency of moment transfer for a certain joint is some value other than zero or $l$, it is necessary to first run the problem with 100 percent efficiency.
29. When a joint has 100 percent efficiency for both shear and moment transfer, the cracks along the joint actually do not exist. A joint with 100 percent efficiency for shear transfer but zero percent efficiency for moment transfer physically means that the dowel bars placed along the joints are so strong that the deflections (and also the shear forces or stresses) on both sides of the joint are the same, but because the crack opening is so large, moments cannot be carried along the joint at all. The joint reacts as a hinge in which the shear force is 100 percent transferred through the joint, but the moment is zero. Half bandwidth
30. The definition of a half bandwidth of a matrix can be found in any structures book. The size of the half bandwidth directly influences the size of the storage space. A proper nodal numbering system may reduce the size of the half bandwidth. This is illustrated in the two different numbering systems shown in Figure 6. Both slabs in Figures 6 a and 6 b have 20 nodes and 12 elements, but the half bandwidth for the arrangement shown in Figure 6 a is $(4+2) \times 3=18$ and that of Figure $6 b$ is $(5+2) \times 3=2$ ?. The rule of thumb is to arrange the finite element grid with the side having fewer nodes in the vertical direction. Note the rule used in the programs in the nodal point-numbering system is to 80 from left to right and to increase from bottom to top. Weight of concrete slab
31. In the classical Westergaard solution, the weight of the slab is not considered in the computation. Consideration of the weight

a. HALF BANDWIDTH $=(4+2) \times 3=18$

b. HALF BANDWIDTH
$=(5+2) \times 3=21$

Figure 6. Influence of finite element arrangement on the size of half bandwidth
of the slab is an option in this computer program. When temperature and loads are not considered and the subgrade is uniform and in full contact with the slab, the weight of the slab only causes the slab to settle uniformly and induces no bending in the slab. Consequently, stresses are not induced in the siab. In some cases, the consideration of the weight of the slab is mandatory, as discussed below.
32. The major difference in procedure between full and partial contact between the slab and the subgrade is that it is not necessary to consider the weight of the slab in the case of full contact, but the weight of the slab must be considered in the case of partial contact; otherwise, the solution may diverge.
33. When problems involve temperature warping, the weight of the slab must be considered to avoid the possible divergence of the solution. This is particularly true when gaps exist under some nodes. For the case of partial contact, the weight of the slab must be considered even when temperature is not considered.
Selected points of stress computations
34. While the displacements are computed automatically for every
nodal point, the stresses are computed only on request. The stress matrix is used each time stresses at a nodal point are computed. Some computer time can be saved if the stresses at only a few selected nodes are computed.
Analysis of two-layer slabs
35. The program can be applied to two-layer slabs, either bonded or unbonded. The derivation of the two-layer system is presented in Appendix A.

## Temperature considerations

36. When the temperature is considered, the dimensions of each slab have to be identical; otherwise, the execution of the program will be terminated. Also, the thickness of each slab has to be uniform. The deformed surfaces of the slabs are assumed in the program to be spherical. This assumption is not valid when the thicknesses of the slabs are not uni form.
37. The computed initial curlings are independent of the arrangement of the finite element grid pattern and concrete slab unit weight. The amount of initial curling at each node is computed by means of Equation $10 b$ in Report 1 of this series. The only variable in Equation $10 b$ is the distance $R$ between the center of each slab and the node where the curling is computed.
Correctness and divergence
of the obtained solution
38. Users of the computer program should always be scrupulous with the computed results. Stresses and deflections may be computed and tabulated, but the values still may not be meaningful. Certain features in the program deserve special attention and are explained below.
39. Number of iterations. When the number of iterations with respect to shear transfer $I C$ has reached the maximum allowable number of iterations-ICL and ICLF (Item 6 of Table 2), the solution has not converged (or the specified criterion was too difficult to meet). The problem should be recomputed with larger values of ICLF (and also ICL in certain cases). However, it may be wise at this stage to see whether
the solution obtained is good enough for eng: sering purposes. In some cases, a solution may not be obtainable if the convergence criterion is too strict. The same reasoning can be used when checking the number of iterations with respect to subgrade contact NIC against the maximum allowable number of iterations (NCYCLE). The value of ICL is not as critical as the value of ICLF ; however, a large difference betweon the actual value of IC (printed in the output) and specified ICL is not recommended.
40. Reduction of relaxation factor RFI. If convergent results cannot be obtained, the program reduces the factor automatically. Too small a value of the relaxation factor results in too small of a shear transfer across the joint during each iteration; consequently, the computed results could be erroneous because the convergerce of the solution is artificially enforced. It was found that when the number of slab NSLAB is large, such as 7 , and when the solution is difficult to converge, the option stated at the bottom of paragraph 13 of this report should be waived. To reduce the relaxation factor too rapidly could cause the solution to diverge.
41. Large number of concrete slabs. When a large number of slabs are involved in the computation, it is reasonable to have a lesser number of elements in the slabs that are far away from the load. However, users should be cautioned that the size of the elements next to the joints in these slabs should not be too large. Otherwise, the subgrade reactive forces along the joint in those slabs tend to become too large and affect the overall computed results without causing any divergence. The reason is that the joint where convergence criteria are checked is not at slabs far away from the load; convergent solutions may be obtained but the results may not be correct.
42. Slab numbering systems. The rule used for the numbering of the slab system was explained earlier in this section. Incorrect use results in either solution divergence or erroneous results. In the former case, the user has the chance to locate the mistake since the solution has not been obtained yet. In the latter case, however, the stresses and deflections are computed and tabulated but the accuracies
of the results are doubtful, depending on how incorrectly the slabs are numbered. Unfortunately, a warning system cannot be established in the program when the slabs are numbered incorrectly; users are thus urged to be cautious in numbering the slabs.
43. Symmetries. When symmetry in a given direction is used and deflections and stresses across a certain joint are supposed to be equal, the efficiency of load transfer across the joint should be input only as 100 percent. Otherwise, erroneous results will be computed.

Input Guide
44. The input guide for the program is given in Table 2, with detailed explanations of each entry presented as follows:
a. Item 1: Number of Runs Card (I5).

$\frac{\text { Notes }}{\text { (1) }} \frac{\text { Columns }}{1-5} \quad \frac{\text { Variables }}{\text { NRUN }} \frac{\text { Entry }}{$|  Number of runs to be  |
| :--- |
|  computed  |}

NOTES:
(1) The number of runs is first specified at the onset of computations. The nature of the problems in each individual run is generally different. However, results of one run can be used in the next run immediately followed by the input NREAD or NSTORE . They are explained in Item 6.
b. Item 2: Identification Card (ABO).

| Notes | Columns |
| :--- | :--- |
| $1-80$ | $\frac{\text { Variables }}{\text { TITLE(12) }} \quad$Enter the heading in- <br> formation to be printed <br> with the output |

NOTES:
(1) Begin each new run with a new heading card.
c. Item 3: Dimension of Matrices Card (16I5).

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (1) | 1-5 | NSLAB | Number of slabs in the model |
| (2) | 6-10 | NJOINT | Number of joints in the model |

Input Guide for WESLIQID--Pavements on Winkler Foundation
(Sheet 1 of 9)
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Table 2 (Continued)

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Table 2 (Continued)

Table 2 (Concluded)

(Sheet 9 of 9 )

| Notes | Columns | Variables |
| :---: | :---: | :---: |
| (3) | 11-15 | LNOBD |
| (4) | 16-20 | ICUD |
| (5) | 21-25 | LCLD |
| (6) | 26-30 | NNPD |
| (6) | 31-36 | NELD |


| Entry |
| :--- |
| Declared dimension of |
| stiffness matrices $C$ |
| and $G$ |
| Declared dimension of |
| matrix CU |
| Declared dimension of |
| matrix CL |
| Declared number of nodal |
| points, equaling 200 |
| Declared element number, |
| equaling l30 |

NOTES:
(1) The program is dimensioned for 9 slabs. If NSLAB is greater than nine, all subscripts with a dimension of 9 must be increased. When NSLAB is large, say greater than 5, element size at slabs away from the slab should be selected with care. Discussion in this report is given in Part IV in the section of the correctness of the obtained solution. For $N S L A B=1$, the iterations between each slab are not performed.
(2) The program is dimensioned for 12 joints. If NJOINT is greater than l2, all subscripts with a dimension of 12 must be increased.
(3) The value of LNOBD is the declared dimension of stiffness matrix of $C$ and $G$ and must be identical to the ones specified in the main program. The required dimension of $C$ and $G$ will be computed and printed in the main program. If the computed dimension exceeds the input declared dimension (LNOBD), an error message will be printed, and the execution of the problem will be terminated. When this happens, the dimensions of $C$ and $G$ in the main program must be increased to the computed value. If LNOBD is mistakenly input less than the dimension of $C$ and $G$ specified in the main program but is more than that computed, the program will be executed with no error. The dimension of LNOBD can be computed by means of the equation
$\begin{gathered}\sum_{1}^{\text {NSLAB }} \\ 1\end{gathered} \quad[\operatorname{NX}(I) \times N Y(I)] \times 3 \times H B$
where $N X(I) \times N Y(I)$ equals the total nodal points in slab $I$, 3 is the number of equations at each node, and $H B$ is the half bandwidth of slab $I$ and is equal to $[N Y(I)+2] \times 3$.
(4) LCUD is the declared dimension of matrix CU, which is the upper band matrix to be stored at the joints and must be input identically with the $C U$ in the main program. The computed dimension will be printed in the main program. If the computed dimension is greater than the declared dimension, an error message will be printed, and the execution of the program will be terminate. The dimension of $C U$ is difficult to determine since it depends on the joint conditions. A value of 1000 may be used and can be modified later.
(5) LCLD is the declared dimension of matrix CL, which is the lower band matrix to be stored at the joints and must be input identically with $C L$ in the main program. Similarly to LCUD, the dimension of $C L$ is difficult to determine. A value of 500 may be used.
(6) The present dimensions declared in the progiam for NNPD and NELD are for 200 nodes and 130 elements, respectively. If the computed numbers of nodes and elements exceed declared, the program will be stopped.
d. Item 4: Element Coordinates Cards (16I5). (1)

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
|  | 1-5 | NX(I) | Number of nodal point in X -direction in slab I |
|  | 6-10 | NY(I) | Number of nodal point in Y-direction in slab I |
| (2) | 11-15 | JONO( $\mathrm{I}, \mathrm{l}$ ) | Joint number on left side of slab I |
| (2) | 16-20 | JONO ( 1,2 ) | Joint number on right side of slab I |
| (2) | 21-25 | JONO ( 1,3 ) | Joint number on lower side of slab I |
| (2) | 26-30 | JONO ( 1,4 ) | Joint number on upper side of slab I |

NOTES:
(1) If the number of slab is greater than 1 , continue to next data card using same format until the number of slab (NSLAB) is satisfied.
(2) The slabs are numbered according to the magnitude of load; i.e., the slab subjected to the largest load is numbered first. Detailed explanation of the numbering system is given earlier in this Part. The joints can be numbered in any arbitrary order. The joint number is zero for free edge. For the case of a single slab, the joint numbers should all be zeros. Figures shown in the example problems


NOTES:
(1) If the number of joint is greater than 1 , continue to the next data card using same format until the number of joint (NJOINT) is satisfied.
(2) $\operatorname{EFF}(N J O I N T, j)$ is the efficiency of load transfer for each joint, with subscript $j$ equal to $l$ for shear transfer and 2 for moment transfer. The program will change the subscript from 2 to 3 if the moment is with respect to the Yaxis, instead of with respect to the X-axis. The value of efficiency across a joint varies from 0 to 1 . If the efficiency of moment transfer for a certain joint is other than 0 or $l$, it is necessary to run the problem twice. The first run uses an efficiency of $l$ and determines the moments at the joint for 100 percent moment transfer. Depending on the efficiency of moment transfer, the second run will assign the appropriate moment at each of the nodes along the joint. These two runs can be performed at the same time with the second run immediately following the first. They can also be run separately by reading in the 100 percent moments at those joints whose efficiency is not zero or 100 percent. In this case, NREAD (Item 6) should be set to one. If LTR (input in Item 11) is equal to 1 or 2, $\mathrm{EFF}(1,1)$ must be input as 1. However, it does not mean that 100 percent shear transfer is used in the program.
f. Item 6: Miscellaneous Data Cards.

Card 1 (915)

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (1) | 1-5 | NLAYER | Number of layer in the concrete slab, either 1 or 2 |
| (2) | 6-10 | NBOND | Bond between two layers in the concrete slab: <br> EQ. 1 only one layer exists or when two layers are bonded |


| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
|  |  |  | EQ. 2 if two layers are not bonded |
| (3) | 11-15 | NOTCON | Total number of nodes at which reactive pressure is initially set at zero |
| (4) | 16-20 | NGAP | Total number of nodes at which a gap exists between slab and subgrade; assign zero if no gap exists |
| (5) | 21-25 | NCYCLE | Maximum number of cycles for checking subgrade contact; generally use 10 or more |
| Also, when LTR is equal to 1 or 2 , the efficiency of moment transfer should always be zero. In most cases, solution convergence is much more difficult when the efficiency of moment transfer is not zero. The following tabulation shows the proper use of joint efficiency: |  |  |  |
| LTR |  | Efficiency of Shear Transfer | Efficiency of Moment Transfer |
| 0 |  | Open | Open |
| 1 |  | 1 | 0 |
| 2 |  | 1 | 0 |
| (6) | 26-30 | NSTORE | Options for thermal stress and thermal deflections: |
|  |  |  | EQ. 0 need not be read in from data cards punched |
|  |  |  | EQ. 1 needs to be read in from data cards punched |
|  |  |  | EQ. 2 the values deter- <br> mined from the previous problems are used |
| (7) | 31-35 | NREAD | A parameter indicating whether any moments at joint are to be read from data cards: |
|  |  |  | EQ. 0 no |
|  |  |  | EQ. 1 yes |


| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (8) | 36-40 | INDP | EQ. 0 yes, i.e., dependent |
|  |  |  | EQ.1 no, i.e., independent |
| (9) | 41-45 | NPRINT | Number of nodes at which stresses and deflections are to be printed |

NOTES:
(1) Description of the bond between the two layers of concrete slab can be found in Appendix A.
(2) Derivation of composite modulus and Poisson's ratio for bonded layers can be found in Appendix $A$.
(3) If the subgrade soil at certain nodal points is known to be not in contact with the pavement due to pumping or plastic deformation, the subgrade reactive pressure at these nodes can be initially set at zero to obtain speeding convergence. If NCYCLE $=1$ (NCYCLE is listed in the following card), these nodes will never be in contact. If NCYCLE $>1$, these nodes may or may not be in contact, depending on calculated results.
(4) Description of gaps can be found in Part II of Report 1 of this series. Note that gaps to not include those induced by the temperature warping but those due to pumping or plastic deformation. However, it is difficult to separate the gaps caused by temperature warping and other sources in the fields. If it is believed that the measured gaps include the temperature warps, the computation should be carried out by setting NTEMP $=0$.
(5) If a Westergaard solution is desired, NCYCLE should be set to l. In so doing, the subgrade is always in full contact with the slab, even though the slab should be curled up and leaving gaps between the slab and the subgrade soil due to either load or temperature differential.
(6) In the area of pavement design, engineers are interested in stresses induced by the applied load and the temperature warping. In the area of pavement research, however, engineers tend to measure only stresses due to the applied load because thermal stresses are difficult to measure. To compute stresses and deflections by the load alone, two separate but consecutive runs have to be conducted. The first run computes the thermal stresses alone. This is done by setting NSTORE $=0, ~ N W T=1, N T E M P=1, N G A P>0$ (if it is the case), NOTCON > 0 (if it is the case), INDP $=1$, and NLOAD $=0$ in the first run. In the second run, the stresses induced by the applied load and the temperature warping are
computed by setting $\operatorname{NSTORE}=2, ~ N W T=1, ~ N T E M P=1$, NGAP > 0 (if it is the case), NOTCON > 0 (if it is the case), $\operatorname{INDP}=0$, and NLOAD equal to the actual number of loads. The differences between those computed in the first and second runs are the stresses and deflections due to the applied load alone and are computed and printed as output data by the computer. Note that when temperature is considered, the slab and the subgrade may be in partial contact; the principal of superposition may no longer be held true (see paragraph 47 of Report 1 of this series). It should also be pointed out in the case of the first and second runs discussed above, the input measured gaps should not include the gaps due to the temperature warping because they are to be computed. More discussions on this can be found in the explanation of NGAP in note 4 of this item.
(7) If the problem involves the efficiency of moment transfer for a certain joint that is other than 0 or 1 and the moments at this joint with 100 percent moment transfer are known, they can be read in at this point by setting NREAD $=1$. Users should refer to the notes in Item 5, the joint efficiency cards, and Item 21, the efficiency of moment card.
(8) When the stresses due to temperature (see NSTORE) or moments computed at 100 percent moment transfer (see NREAD) computed in the previous run are used in this run, this run is not considered to be independent and INDP should be 0 , otherwise INDP is 1. Since the results from the previous runs are used in this run, the relaxation factor RFI used in the last iteration cycle in the previous run should be used in this run to obtain faster convergence.
(9) The deflections at each node are computed in the program, but the stresses at any node are computed only on request.
Card 2 (9I5)

Notes
(1)
Columns
$1-5$

$$
\begin{equation*}
6-10 \quad \text { ICX } \tag{2}
\end{equation*}
$$

Variables
NTEMP

Entry
Condition of temperature warping:
EQ. 0 temperature gradient is zero
EQ. 1 temperature gradient is not zero
A parameter indicating whether temperature curling exists in the X-direction:

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
|  |  |  | EQ. 0 no |
|  |  |  | EQ. 1 yes |
| (2) | 11-15 | ICY | A similar parameter in the $Y$-direction |
| (3) | 16-20 | NLOAD | Number of elements on which load is applied; use 0 if there is no load |
| (4) | 21-25 | NMCF | Number of concentrated nodal forces and moments which are to be read in; assign 0 if no moments or forces are applied |
| (5) | 26-30 | NWT | Weight of slab consideration: |
|  |  |  | EQ. 0 weight is not considered |
|  |  |  | EQ. 1 weight is considered |
| (6) | 31-35 | NMT | Number of cases to be solved for moment transfer |
| (7) | 36-40 | NSX | Number of nodal points on X -axis, which is an axis of symmetry; assign 0 if X -axis is not an axis of symmetry |
| (7) | 41-45 | NSY | Number of nodal points on Y-axis, which is an axis of symmetry; assign 0 if Y-axis is not an axis of symmetry |

NOTES:
(1) When temperature is considered in the problem, the program works only when the slabs have identical dimensions. Also, erroneous results will be obtained if NAT(I) in card 4 of Item 6 is not 0 .
(2) Most pavement slabs have temperature warping in both $X$ - and Y-directions. However, in the case of a continuously reinforced concrete pavement, temperature warping should not be considered in the longitudinal direction if cracks in the pavement do not exist. Otherwise, the amount of curling will be too large.
(3) Because the uniformly applied surface load at each element is lumped into concentrated loads at the four nodal
points, the accuracy of the solution can be improved if the size of the elements at which the loads are applied is reduced.
(4) The concentrated force is considered to be positive if it is acting downward and is negative if it is acting upward. Positive moment follows right-hand screw system (see Figure 1 of Report 1). The program is dimensioned for 200 concentrated forces and moments. If NMCF is greater than 200, dimensions of NFF, NFI, and NF must be increased.
(5) In the original Westergaard solution, the pavement slab was considered to be weightless, but temperature could be considered. Note that if the subgrade is assumed to be in full contact with the slab, the consideration of slab weight affects only deflections but not stresses. However, when the slab is in partial contact with the subgrade, slab weight has a significant effect on slab stresses.
(6) If the efficiency of moment transfer of a joint is 0.5 , and it is desired to obtain solutions not only for an efficiency of 0.5 but also for efficiencies of 0.75 and 0.25 , assign NMT to 3 and $C M(N M T)$ in Item 21 to $1.0,1.5$, and 0.5 , respectively. Set $N M T=1$ if the efficiencies of moment transfer are zeros.
(7) The explanations on symmetry can be found earlier in this Part. When subgrade stresses and deflections are computed, symmetry should be used with caution. When either NSX or NSY is not zero, the total number of nodal reactive forces is reduced one half, and when both NSX and NSY are not zero, the total number of nodal reactive forces is reduced to one quarter. Symmetry should not be used at nodes along a joint.
Card 3 ( $815, ~ I 10)$

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (1) | 1-5 | JNCK | Joint number used to check convergence; one joint only |
| (2) | 6-10 | NBCK | Beginning node at the specified joint (JNCK) used for checking convergence. If NSLAB $=1$, use any integer number |
| (2) | 11-15 | NECK | Ending node at the joint used for checking convergence. If $N S L A B=1$, use any integer number |

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| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (2) | 16-20 | NNCK | A specified node between and including NBCK and NECK, which is used for determining whether the relaxation factor RFI should be reduced. If NSLAB $=1$, use any integer number |
| (3) | 21-25 | ICL | Maximum number of iterations allowed for coarse control; generally use 150 |
| (3) | 26-30 | ICLF | Maximum number of iterations allowed for fine control; generally use 300 |
| (4) | 31-35 | IGNOR | A parameter indicating whether the reduction of relaxation factor RFI should be ignored: |
|  |  |  | EQ. 0 if RFI is reduced <br> EQ. 1 if RFI is not reduced whenever the results diverge |
| (5) | $36-40$ | MNRFR | Maximum number of times for which relaxation factor is allowed to reduce in half; generally use 10 or more |
| (6) | 41-50 | MAXFAJ | Maximum shear force at one node that may exist along the joint |

NOTES:
(1) The most efficient joint for checking convergence is the joint ciosest to the heaviest loads. If NSLAB = 1 , JNCK can be any integer number.
(2) For a joint along the X -direction, the node is numbered from left to right; for a joint along the Y-direction, it is numbered from bottom to top. For instance, if the joint is along the Y-direction and there are seven nodes in the joint, and if the middle three nodes are used for checking convergence, thus $N B C K=3$ and $N E C K=5$. If $N S L A B=1$, NBCK, NECK, and NNCK can be any integer number.
(3) Coarse control is used before the subgrade contact condition is determined and fine control is used afterwards. For a given contact ec dition, coarse control is used to check the load transfer. Once the subgrade contact condition is finally determined, fine control is used to obtain accurate solutions. If NCYCLE $=1$, coarse control is still used prior to the use of fine control. In some problems, ICLF may be exhausted before the criterion DELF is satisfied. Before rejecting the solution, it may be wise to check to see how far the solution is from satisfying the criterion. For instance, if DELF $=0.001$ and computed divergence is 0.002 or 0.0025 and the computed results seem to be reasonable, the solution may be considered acceptable. In some problems, it may be very hard to satisfy the specified convergence criterion. Note: ICLF should always be greater than ICL.
(4) IGNOR is used to increase the flexibility of the program. In some cases, it may be desirable to check the convergence condition when the relaxation factor is fixed at a certain value. The numerical technique used in this program involves an iterative procedure in which a solution may not always be feasible. If a solution is not obtained and if it is noticed from the printed output that the solution was convergent at a reasonable rate during a particular cycle (or relaxation factor), the problem should be run again using the particular relaxation factor and setting IGNOR to 1 . In this case, the maximum number of iterations may need to be increased.
(5) If NSLAB $=1$, MNRFR can be any integer number.
(6) If the computed shear force at any node along the joint exceeds MAXFAJ, the relaxation factor will be reduced by one half and iterations restarted. Proper selection of MAXFAJ will expedite the convergence of the solution; however, the value of MAXFAJ varies with the problem. MAXFAJ can be estimated as the shear force acting on the particular node at which the convergence criterion is checked. If input MAXFAJ is less than the computed shear force, the solution will be difficult to converge. If this is the case, change the value according to the printed output or simply use a large number such as 5 or 10 times greater than the total load applied on that slab. The use of a larger value of MAXFAJ would ensure that the relaxation factor RFI is not reduced faster than necessary, and also it would not seriously affect the convergence, because when the solution is divergent and the relaxation factor RFI needs to be reduced, the computed shear forces at the joint tend to become extremely large and exceed the value of specified MAXFAJ, resulting in a reduction of RFI value. Consequently, too large a MAXFAJ tends to increase the computer
time but too small a MAXFAJ would reduce the RFI too rapidly and cause slow convergence or divergence. If temperature alone is considered, the shear force at a dowel bar at the joint should be equal to one quarter of the dead weight of the grid element at which the dowel is connected, which should be a very small force. For simplicity, a larger MAXFAJ can be used, such as from 500 to $10,0001 \mathrm{lb}$. If $N S L A B=1$, MAXFAJ can be any integer number.
Card 4 (8I5)

Notes Columns Variables

6-10 NAT(1)

11-15 NAT(2)
(1)
(2)

21-25 ILPR
(2)

26-30 NPUNCH

Entry
Number of additional moduli of subgrade reaction to be read in; assign 0 if the subgrade modulus is uniform throughout
Number of additional thicknesses to be read in for the top layer; assign 0 if thickness is uniform throughout
Number of additional thicknesses to be read in for the bottom layer; assign 0 if thickness is uniform throughout or NLAYER $=1$
First cycle at which displacements are to be printed; if IFPR $=0$, no displacements will be printed until the end
Last cycle at which displacements are to be printed; ILPR should be equal to or greater than IFPR

Option for punching values of thermal stresses and deflections on cards:
EQ. 0 no
EQ. 1 yes

## NOTES :

(1) Computed displacements during iteration may be printed for inspection. If it is desired to print out the displacements computed at second and third cycles, set IFPR to 2 and ILPR to 3.
(2) Cards can be punched if NPUNCH is equal to 1. NPUNCH is used when either NSTORE $=1$ or NREAD $=1$.
Card 5 (3F10.5, 3E10.3, F10.5)

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (1) | 1-10 | $\operatorname{SUBMOD}(1)$ | Modulus of subgrade reaction, $k$, in pci |
| (2) | 11-20 | TEMP | Difference in temperature, in degrees Fahrenheit, between top and bottom of slab: |
|  |  |  | EQ.positive slab curled upward |
|  |  |  | EQ.negative slab curled downward |
| (3) | 21-30 | RFI | ```Initial relaxation factor at the joint; generally use 0.5``` |
| (4) | $31-40$ | DEL | Tolerance of convergence for coarse control; usually use 0.01 |
| (4) | $41-50$ | DELF | Tolerance of convergence for fine control; usually use 0.001 |
| (5) | 51-60 | YMSB | Young's modulus of dowel bars |
| (5) | 61-70 | PRSB | Poisson's ratio of dowel bars |
| (6) | 71-75 | NCOMP | Number of subgrade elastic moduli |

NOTES:
(1) If subgrade modulus is not uniform, $\operatorname{SUBMOD}(1)$ is the modulus of the uniform part; while the modulus SUBMOD(I) at node $I$, which is different from $\operatorname{SUBMOD}(1)$, will be read in later.
(2) If two layers are considered in the computation, the coefficient of thermal expansion is assumed to be equal for both layers.
(3) If convergent results cannot be obtained, the program will adjust the factor automatically. If LTR $=1$ or 2 and a small spring constant or amount of dowels is used, a smaller RFI is recommended to reduce the number of iterations.
(4) DEL and DELF correspond to ICL and ICLD, respectively, in card 2 of this table. In the program when the ratio of the difference of shear force between two consecutive iterations to be shear force is greater than the specified DEL or DELF, the iteration cycle starts again.
(5) Any number can be used if LTR is not equal to 2.
(6) If stresses and deflections in the subgrade need not be computed, NCOMP must be input as zero. The value of the elastic modulus of the subgrade $E$ (in psi) should correspond to the modulus of subgrade reaction $k$ (in pci). Since direct relation between $E$ and $k$ does not exist, a trial-and-error method may have to be employed to determine an appropriate $E$ value. Therefore, a number of subgrade $E$ values may have to be used in computations.
g. Item 7: Subgrade Moduli Card (5(I5, Fl0.5)).

Note: Use a blank card if the subgrade has a uniform subgrade modulus.

| Notes | Columns | $\frac{\text { Variables }}{\text { (1) }}$NS(I)  Node number at which sub- <br> grade modulus is to be <br> specified <br> (1) $6-15$ $\operatorname{SUBMOD}(\operatorname{NS}(I))$Subgrade modulus at node <br> NS(I) |
| :--- | :---: | :---: | :--- |

NOTES:
(1) Report NS(I) and $\operatorname{SUBMOD}(N S(I))$ for each node at which the modulus is different from SUBMOD(1).
h. Item 8: Nodal Points Coordinate Cards (8(F10.5)). (1)

X-coordinate card

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (2) | 1-10 | $X(1,1)$ | X-coordinate of the first node in slab I |
| (2) | 11-20 | $X(1,2)$ | X-coordinate of the second node in slab I |
|  |  | : | : |
|  |  | : | : |
| (2) |  | $X(I, N X(I))$ | X-coordinate of the last node in slab I |

Y-coordinate card

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (2) | 1-10 | $Y(I, I)$ | Y-coordinate of the first node in slab I |
| (2) | 11-20 | $Y(1,2)$ | Y-coordinate of the second node in slab I |
|  |  | : | : |
|  |  | : | : |
| (2) |  | $Y(I, N Y(I))$ | Y-coordinate of the last node in slab I |

NOTES:
(1) If the number of slab is greater than 1 , continue to the next data card after the nodal points on Y-axis are input, using the same format until the number of slab (NSLAB) is satisfied.
(2) Both $X$ - and Y-coordinates starting from 0 and increasing from left to right for the X-coordinate and increasing from bottom to top for the Y-coordinate. If the value NX or NY in a slab exceeds 8, continue the input to the second data card.
i. Item 9: Layer Properties Cards (2(2F10.5, El0.3)).(1)

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (2) | 1-10 | $T(1,1)$ | Thickness of layer 1 |
|  | 11-20 | PR(1) | Poisson's ratio of layer 1 |
|  | 21-30 | YM(1) | Young's modulus of layer l |
|  | 31-40 | $T(1,2)$ | 'Thickness of layer 2 |
|  | 41-50 | PR(2) | Poisson's ratio of layer 2 |
|  | 51-60 | YM(2) | Young's modulus of layer 2 |

NOTES:
(1) If the number of leyer is 1 , stop the input at column 30.
(2) If thickness is not uniform, thicknesses different from $T(I, I)$ will be read in later.
j. Item 10: Slab Thickness Card (5(I5, F10.5)).

Note: If the thickness is uniform in the layer (i.e., $\mathrm{NAT}(1)=0$ ), place a blank card for that layer. Two blank cards are required if thicknesses in both layers are uniform (i.e., if $\operatorname{NAT}(2)$ also is zero).

| $\frac{\text { Notes }}{(2)}$ | $\frac{\text { Columns }}{1-5}$ | $\frac{\text { Variables }}{\operatorname{NT}(I)}$ | Nodal number at which <br> thickness is to be <br> specified |
| :---: | :---: | :---: | :---: |
| (2) | $6-10$ | $\mathrm{~T}(\mathrm{NT}(\mathrm{I}))$ | Thickness at node $\mathrm{NT}(\mathrm{I})$ |

NOTES:
(1) Continue the input for other thicknesses that are different from $T(l, N L A Y E R)$, until $N A T(j)$ is satisfied, where $j$ varies from 1 to 2 . If the number of additional thicknesses is greater than 5, continue the input to next data card. If the number of additional thicknesses to be specified exceeds 25 , the dimension of variable NT should be increased accordingly.
(2) If the number of layer is 2, continue to next data card using same format.
k. Item 11: Joint Information Cards (I5, F10.3, 3F10.5, 2F10.3, F10.5). (1)
Note: Use a blank card if the number of slab is l, i.e., $\overline{\mathrm{NSLAB}}=1$. If the number of slab is greater than 1 and if $\operatorname{LTR}(I)$ for joint $I$ is 0 , use a blank card for joint $I$. For instance, if a pavement system has four joints and joints l-3 have $\operatorname{LTR}(I)=0$ and joint 4 has $\operatorname{LTR}(I)=2$, use three blank cards and specify the joint detail in the fourth card.
$\frac{\text { otes }}{(2)} \frac{\text { Columns }}{1-5} \quad \frac{\text { Variables }}{\text { LTR }(I)}$
Entry

| Method for specifying |
| :--- |
| shear transfer at joint |
| I: |

EQ. $0 \quad$| efficiency of shear |
| :--- |
| transfer is |
| specified |

EQ. $1 \quad$| a spring constant |
| :--- |
|  |
| is specified |

| EQ. 2 data on dowel bars |
| :--- |
| $\quad$ are provided |
| Spring constant for |
| aggregate interlock or |
| key joint at joint $I$ |


| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (4) | 16-25 | $B D(I)$ | Bar diameter at joint I |
| (4) | 26-35 | BS(I) | Bar spacing at joint I |
| (4) | 36-45 | WJ(I) | Width of joint I |
| (4) | 46-55 | $\operatorname{SCKV}(1,1)$ | Initial value for modulus of dowel support (or steel concrete $k$ value) at joint I |
| (4) | 56-65 | $\operatorname{SCKV}(1,2)$ | Final value for modulus of dowel support at joint I |
| (5) | 66-75 | DCGF (I) | Deformation of concrete when good fit is obtained |

NOTES:
(1) If the number of joint is greater than 1 , continue to next data card using same format.
(2) The efficiency of shear transfer is defined as a ratio between the deflection of the unloaded, or less loaded, slab and the deflection of the loaded slab.
(3) If LTR is not specified to 1, SPCON may be assigned 0 , blank, or any value. However, 0 or blank is preferred.
(4) If LTK is not equal to $2, \mathrm{BD}, \mathrm{BS}, \mathrm{WJ}$, $\operatorname{SCKV}(N J O I N T, 1), \operatorname{SCKV}(N J O I N T, 2)$, or $\operatorname{DCGF}(I)$ may be left 0 , blank, or any value. Zero or blank is preferred.
(5) When the deformation of concrete under dowel is smaller than DCGF , SCKV(NJOINT, I) is needed; when greater, $\operatorname{SCKV}(N J O I N T, 1)$ and $\operatorname{SCKV}(N J O I N T, 2)$ are input the same. Leave blank if LTR is not equal to 2. Detailed explanation on DCGF and SCKV can be found in Part II of Report 1 of this series. The normal range for SCKV is between 300,000 and $1,500,000$ pci.

1. Item 12: Total Uniformly Applied Load Card (Fl2.2).

Note: Skip this card if there is no load uniformly applied on the slabs, i.e., NLOAD $=0$. A blank card is not needed.
Notes
(1) Columns

$1-12$$\frac{\text { Variables }}{\text { RLOAD }} \quad$| Total uniformly applied |
| :--- |
| load on the slab |

NOTES:
(1) The total load refers to the uniformly applied load only. The total load should be divided by 2 or 4 if it is
symmetric with respect to one axis (X- or Y-axis) or both $X$ - and $Y$-axis, respectively. Additional point loads applied at nodal points are excluded.
m. Item 13: Loading Cards (I5, 5F10.5).

Note: Use a blank card if there is no load uniformly applied on the slabs, i.e., NLOAD $=0$.

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (2) | 1-5 | $\mathrm{NL}(\mathrm{I})$ | Element number over which load is applied |
| (3) | 6-15 | $\mathrm{XDA}(1,1)$ | Lower limit of loaded area in element $I$ in X-direction |
| (3) | 16-25 | $\mathrm{XDA}(\mathrm{I}, 2)$ | Upper limit of loaded area in element I in X-direction |
| (4) | 26-35 | $\operatorname{YDA}(1,1)$ | Lower limit of loaded area in element $I$ in Y-direction |
| (4) | 36-45 | $\operatorname{YDA}(1,2)$ | Upper limit of loaded area in element $I$ in Y-direction |
|  | 46-55 | $Q(I)$ | Uniformly applied pressure in element I |

NOTES:
(1) If the number of loaded elements is greater than 1 , continue to next data card using same format.
(2) Beginning from the first slab and ending at the last slab, the nodes and elements are numbered consecutively from bottom to top and then from left to right, as shown in Figure 2.
(3) Use -1 to +1 if the load covers the whole length of element.
(4) Use -1 to +1 if the load covers the whole width of element.
n. Item 14: Subgrade Contact Card (16I5).

Note: Use a blank card if the slabs are initially in full contact with the subgrade, i.e., NOTCON $=0$.

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (1) | 1-5 | $\operatorname{NODNC}(\mathrm{I})$ | Nodal number at which reactive pressure is initially assumed $0, I=$ 1,NOTCON |

NOTES:
(1) Continue the input until the number of NOTCON is satisfied. Continue to next data card if NOTCON is greater than 16.
ㅇ. Item 15: Stresses Print Card (16I5).
Note: Use a blank card if stresses at all nodal points are printed, i.e., NPRINT $=\Sigma(N X(I) \times N Y(I)), I=1, N S L A L$

Notes
Columns
Variables $\mathrm{NP}(\mathrm{I})$

| Entry |
| :--- |
| Nodal number whose |
| stresses are to be |
| printed, $I=1, \quad$ NPRINT |

NOTES:
(1) Deflections are printed for all nodal points.
(2) Continue the input until the number of NPRINT is satisfied. Continue to next data card if NPRINT is greater than 16.
p. Item 16: Symmetry Cards.

Card 1: symmetry on $X$-axis (16I5)
Note: Use a blank card if X -axis is not an axis of symmetry, i.e., NSX $=0$.

| Notes | Columns <br> $1-5$ | Variables <br> NODSX(1) |
| :---: | :---: | :---: | | First nodal number on |
| :--- |
| 6-10 |

Card 2: symmetry on Y-axis (16I5)
Note: Use a blank card if Y-axis is not an axis of symmetry, i.e., NSY $=0$.

| Notes Columns | $\frac{\text { Variables }}{1-5}$ |  |
| :---: | :---: | :--- |
|  | NODSY(1) Entry  <br> $6-10$ $\operatorname{NODSY}(2)$ | First nodal number on <br> Y-axis |
|  |  | Second nodal number on <br> Y-axis |

Notes Columns $\frac{\text { Variables }}{\text { NODSY(NSY) }} \frac{\text { Entry }}{$|  Last nodal number on  |
| :--- |
|  Y-axis  |}

g. Item 17: Thermal Stresses and Thermal Deflections Read In Card.

Note: Use a blank card if NSTORE is not equal to 1 . One blank card takes care of both STRSTO and FSTORE .
Card 1: Stresses (6F10.5)

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (1) | 1-10 | STRSTO( $\mathrm{I}, \mathrm{l}, \mathrm{L}$ ) | Stress $\sigma_{x}$ in node $I$, layer $L=1$ |
| (1) | 11-20 | $\operatorname{STRSTO}(\mathrm{I}, 2, \mathrm{~L})$ | Stress $\sigma_{y}$ in node $I$, layer $L=1$ |
| (1) | 21-30 | $\operatorname{STRSTO}(\mathrm{I}, 3, \mathrm{~L})$ | Shear stress $\tau$ in node I, layer $L=1$ |
| (1) | $31-40$ | $\operatorname{STRSTO}(\mathrm{I}, 4, \mathrm{~L})$ | Major principal stress in node $I$, layer $L=1$ |
| (1) | 41-50 | $\operatorname{STRSTO}(\mathrm{I}, 5, \mathrm{~L})$ | Minor principal stress in node $I$, layer $L=1$ |
| (1) | 51-60 | $\operatorname{STRSTO}(\mathrm{I}, 6, \mathrm{~L})$ | Maximum shear stress in node I, layer L = 1 |

NOTES:
(1) Each data card includes the six stress components for a nodal point. Repeat the data card at the same format for other nodal points, starting from node 1 to the last node (LNP). If the slab has a second layer, repeat the data cards with the same format for $\mathrm{L}=2$.

Card 2: vertical deflections (8F10.5)

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (1) | 1-10 | FSTORE(1) | Vertical deflection at node 1 |
| (1) | 11-20 | FSTORE(2) | Vertical deflection at node 2 |
|  |  | : | : |
|  |  | : | : |
| (1) |  | FSTORE(LNP) | Vertical deflection at node LNP |
| NOTES: |  |  |  |
| $\text { (1) } \mathrm{L}$ slabs | is the idered. | 1 number of | dal points for all the |

r. Item 18: Gaps Read In Card (5(I5, F10.5)).

Note: Use a blank card if NGAP $=0$.

| (1) | Columns | Variables |  |
| :--- | :--- | :--- | :--- |
| $1-5$ NG(I) Nodal number at which gap <br> between slab and subgrade <br> is specified <br> (2) $6-15$ $\operatorname{CURL}(N G(I))$Amount of gap at node |  |  | $N G(I)$ |

NOTES:
(1) Continue the input for other nodes at which the gap between slab and subgrade is specified until the number of NGAP is satisfied. If NGAP is greater than 5, continue the input to next data card.
(2) Gap is positive and precompression is negative.
s. Item 19: Concentrated Forces or Moments Card (4 (I5, I5, F10.2)).
Note: Use a blank card if there are no concentrated forces or moments, i.e., NMCF $: 0$.

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
|  | 1-5 | NFF ( I ) | Nodal number at which concentrated forces or moments are specified |
| (1) | 6-10 | NFI( I ) | Nature of specified force at node I |
| (2) | 11-20 | $\begin{aligned} & \mathrm{FO}(\mathrm{NFF}(\mathrm{I})-1) \times 3 \\ & +\mathrm{NFI}(\mathrm{I}) \end{aligned}$ | Concentrated force or moment at node I |

NOTES :
(1) $\mathrm{NFI}(\mathrm{I})=1$ for vertical force, 2 for moment about X axis, and 3 for moment about Y-axis.
(2) The magnitude of concentrated force or moment is input in. The equation number is related to nodal number by (NFF(I)-1)) $\times 3+\mathrm{NFI}(I)$. For instance, if a moment about Y-axis is applied at node 13 , the equation number will be (13-1) $\times 3+3=39$. Note that the nodes are numbered consecutively from bottom to top and then from left to right beginning from the first slab and ending at the last slab.
t. Item 20: Moments at Joints Read In Cards (6F12.3). (1) Note: Use a blank card if NREAD is equal to 0 .

Notes
(2)
(2)

Columns
1-12
13-24 $\operatorname{FAJ}(I, 2)$

Entry
Moment at first node at joint I computed in the previous run with 100 percent moment transfer

Moment at second node at joint I computed in the previous run with 100 percent moment transfer

Moment at the last node at joint I computed in the previous run with 100 percent moment transfer

NOTES:
(1) If the number of joints is greater than 1 , continue to next data card using same format.
(2) This card is needed only when the efficiency of moment transfer is not equal to 0 or 1 ; otherwise, this card should be skipped. For instance, if the efficiencies of moment transfer at joints $1,2,3$, and 4 are $0.3,0,1$, and 0.7 , respectively, only joint 1 and joint 4 data cards are needed.
u. Item 21: Efficiency of Moment Card (8(F10.5)).

Note: Use a blank card if efficiencies of moment transfer are zeros, i.e., $\operatorname{EFF}(I, 2)=0$.

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (1) | 1-10 | CM(1) | Multiplying factor for efficiency of moment transfer for case 1 |
| (1) |  | CN ( NMT ) | Multiplying factor for efficiency of moment transfer for case NMT |

## NOTES:

(1) The use of $C M$ in the program is to facilitate the input format when several efficiencies of moment transfer are involved. For instance, if efficiencies of $0.5,0.25$, and 0.1 are to be computed, an efficiency of 1 should first be computed in the first run with $N R E A D=0$. In the second run, NREAD is still 0 and the values of EFF in Item 5 should be set to 0.5 , and $C M^{\prime}$ s in this table should be set as $1,0.5$, and 0.2 because the products of $0.5 \times 1,0.5 \times$ 0.5 , and $0.5 \times 0.2$ are $0.5,0.25$, and 0.1 , respectively, which are the efficiencies to be computed. The program is
developed in such a way that erroneous results will be computed if EFF in the second run is set as 1.0 , and the CM's are set as $0.5,0.25$, and 0.1 . If the results of a particular run are not used in the following run, $C M$ must be set to 0 .
v. Item 22: Subgrade Stresses Card

Note: Use a blank card if computation of subgrade stresses and deflections is not needed, i.e., YMSS = 0 . One blank card takes care of $N Z, N R, Z Z(I), X R(I)$, and $Y R(I)$. Card 1: values of modulus and Poisson's ratio (4 (Flo.2, F5.1)).

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
|  | 1-10 | $E(1)$ | Elastic modulus of the subgrade for the first computation |
|  | 11-15 | $\nu(1)$ | Poisson's ratio of the subgrade for the first computation |
|  |  | : | : |
|  |  | : | : |
|  |  | E(NCOMP) | Elastic modulus of the subgrade for the NCOMP ${ }^{\text {th }}$ computation |
|  |  | $v($ NCOMP $)$ | Poisson's ratio of the subgrade for the NCOMP computation |

Card 2: number of computations (2110)


| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
|  |  | ZZ(NZ) | Depth of the last computation |
| Card 4: offset card (8F10.5) |  |  |  |
| Notes | Columns | Variables | Entry |
| (1) | 1-10 | XR(1) | X-coordinate of first computation |
|  | 11-20 | YR(1) | Y-coordinate of first computation |
|  | 21-30 | $\mathrm{XR}(2)$ | X-coordinate of second computation |
|  | $31-40$ | YR(2) | Y-coordinate of second computation |
|  | : | : |  |
|  | : | : |  |
|  |  | XR ( NR) | X-coordinate of last computation |
|  |  | YR(NR) | Y-coordinate of last computation |

## NOTES:

(1) Computations at each offset point are made at all the NZ depths. The origin of the coordinates is at nodal point l, i.e., node 1 of slab l. Referring to the nodal numbers shown in Figure 2, if stresses and deflections in the subgrade soil at various depths at three locations are to be computed, the first location is directly under node 1 , the second location is the midpoint between nodes 5 and 8 , and the third location is at the center of element 13. The input values of $X R(1), Y R(1), X R(2), Y R(2), X R(3)$, and $\operatorname{YR}(3)$ should thus be $0 ., 0 ., 135 ., 90 .,-135$. , and -135 . Note that nodes $1,16,21$, and 36 shear the same location.
w. Item 23: Subgrade Stress Directly under a Node and Joint Card.

Card 1: number of locations and information (8I5)

| Notes | Columns | $\frac{\text { Variables }}{\text { (1) }}$ |  |
| :--- | :---: | :--- | :--- | | NAJ |
| :--- | | Number of locations di- |
| :--- |
| (2) |


| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (2) | 11-15 | NJP(2) | Number of nodal points share the same location, second location |
|  | : | : | : |
|  | : | - | : |
| (2) |  | NJP(NAJ) | Number of nodal points share the same location, NAJ ${ }^{\text {th }}$ location |

Card 2: nodal points sharing the same locations (4I5)
Note: Skip this card if computation is not made at locations along the joint, i.e., NAJ $=0$.

| Notes | Columns | Variables | Entry |
| :---: | :---: | :---: | :---: |
| (3) | 1-5 | $\operatorname{JON}(1,1)$ | Nodal number at one side of the joint that has the smallest nodal number, first location |
| (3) | 6-10 | $\operatorname{JON}(1,2)$ | Nodal number at the other side of a joint of node JON(1,1) that shares the same location with node $\operatorname{JON}(1,1)$, second location |
| (4) | 11-15 | JON $(1,3)$ | Nodal number at the other side of a joint of either node JON(1,1) or node JON $(1,2)$ that shares the same location with nodes $\operatorname{JON}(1,1)$ and $\operatorname{JON}(1,2)$, first location |
| (4) | 16-20 | $\operatorname{JON}(1,4)$ | Nodal number at the other side of a joint of either node JON(1,1) or node $\operatorname{JON}(1,2)$ or node $\operatorname{JON}(1,3)$ that shares the same location with these three nodes, first location |

Note: If there are more computations at locations along the joint, i.e., NAJ is greater than l, repeat the data cards with the same format for the second location: $\operatorname{JON}(2,1), \operatorname{JON}(2,2), \operatorname{JON}(2,3)$, and $\operatorname{JON}(2,4)$; the third location: $\operatorname{JON}(3,1), \operatorname{JON}(3,2), \operatorname{JON}(3,3)$, and JON(3,4) ; ... the last (NAJ) location: JON(NAJ,1), JON(NAJ, 2) , JON(NAJ, 3), and JON(NAJ, 4).

## NOTES:

(1) For a single slab, i.e., $N S L A B=1$, NAJ should be input as zero. If NSLAB is greater than 1 and if the computation is made at locations directly under a node and also along a joint, NAJ is the total number of such locations.
(2) For NSLAB $=1$, $\mathrm{NJP}(\mathrm{I})$ should be skipped. If NSLAB is greater than 1 and NAJ is greater than zero, $N J P(I)$ is the number of nodal points sharing the same location. The sequential order of inputting nodal information for nodes located along the joints is of vital importance. A slight mistake in the order will result in erroneous results. The basic rule is to input nodal information of nodes of smaller numbers prior to larger numbers. This can best be illustrated by an example. For the four-slab pavement system shown in Figure 2, if the stresses and deflections under nodes 1,2, $3,4,6,10,19,20,23$, and 25 at various depths are to be computed, NAJ should be input as 7 , as only 7 nodal points are located along the joint, i.e., $1,2,3,4,10,19$, and 20. The sequential order for inputting NJP(I), (I21,NAJ) should be $1,2,3,4,10,19$, and 20 . At the location of nodal point 1 , since nodes 16,21 , and 36 share the same location with node l, $\operatorname{NJP}(1)$ should thus be input as 4. At nodal point 2 , node 17 shares the same location with node 2 and thus $\operatorname{NJP}(2)$ is equal to 2. Similarly, the values of $\mathrm{NJP}(3), \mathrm{NJP}(4), \mathrm{NJP}(5), \mathrm{NJP}(6)$, and $\mathrm{NJP}(7)$ should all be 2. It should be pointed out that if computations at node 24 are desired, node 4 should be used in the input to replace node 24 , as 4 is smaller than 24 and also as nodes 4 and 24 share the same location.
(3) Since NAJ $=7$, seven separate data cards are needed to indicate the nodal numbers of these 7 special computation locations. The first card corresponding to NJP(1) should be input as $1,21,36$, and 16 . It is of vital importance to input first nodal number 1 ; the order of the other three nodal numbers is of no importance. In other words, this card can be input as either 116236 or 1362116 , or 121636 , or 1163621 , or 1361621 . The important rule is to input first the smallest nodal number of the form nodal numbers.
The second card (of card 2) corresponding to $\operatorname{NJP}(2)$ should be input as 217 . Nodal number 2 is input prior to nodal number 17 as 2 is smaller than 17 . Similarly, the third card (of card 2) corresponding to $\mathrm{NJP}(3)$ is 318 ; the fourth card (of card 2) corresponding to $N J P(4)$ is 424 ; the fifth card (of card 2) corresponding to $\operatorname{NJP}(5)$ is 1030 ; the sixth card (of card 2) corresponding to $\operatorname{NJP}(6)$ is 1934 ; and the seventh card (of card 2) corresponding to $\operatorname{NJP}(7)$ is 2035 .
(4) $\operatorname{JON}(1,3)$ and $\operatorname{JON}(1,4)$ are not needed if $N J P$ equals

2; $\operatorname{JON}(1,4)$ is not needed if NJP equals 3. Both $\operatorname{JON}(1,3)$ and $\operatorname{JON}(1,4)$ are needed if NJP equals 4.
45. In this Part of the report, the input data of five example problems are presented. Printouts of the computer output for three example problems are presented and explained.

Example Problem 1: A Single Slab with Many Input Options
46. Figure 7 shows the finite element grid of a single slab. The nodes and elements are numbered consecutively from bottom to top and then from left to right. The input data consist of the following information:
a. The soncrete slab is 10 in. thick with a Young's modulus of $6,000,000$ psi and a Poisson's ratio of 0.2 . The slab is underlain with a 4-in. stabilized layer,


Figure 7. Finite element layout for Example Problem 1
which has a modulus of $500,000 \mathrm{psi}$ and a Poisson's ratio of 0.2. The condition of the interface between the layers is bonded. The subgrade has a $k$ value of 100 pci.
b. The thickness of the layers and the modulus of subgrade reaction $k$ are not uniform throughout the slab. Table 3 gives the additional $k$ values and thicknesses at particular nodal points.
c. Gaps exist under the pavement at 22 nodal points. The amounts of gaps are 0.5 in. at nodes 1 to 9 and 0.25 in. at nodes 10 to 18 , and also at nodes $82,83,91$, and 92.
d. The slab is subjected to four concentrated loads, 72,000 Ib each, at nodes 62, 71, 80, and 89. There is no uniformly applied load.
e. Stresses at all nodes (99) are printed.

Table 3
Additional Subgrade $k$ Values and Thicknesses

| Node | Layer Thicknesses, in. |  | Subgrade | k | Value, pci |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Top Layer | Bottom Layer |  |  |  |
| 1 | 12 | 2 |  | 65 |  |
| 2 | 3 | 3 |  | 65 |  |
| 3 | 13 | 1 |  | 65 |  |
| 4 | 9 | 5 |  | 65 |  |
| 5 | 8 | 4 |  | 65 |  |
| 6 | - | 10 |  | -- |  |

47. The input data for Example Problem 1 are given in Table 4. The readers should refer to the input guide, as necessary.

Example Problem 2: A Single Slab With Separate Runs
for Computing the Stresses and Deflections Due
to the Applied Load Alone
48. The purpose of Example Problem 2 is to illustrate the inpu procedure to compute stresses and deflections induced by the applied load alone. The reason for the need of this computation is explained in the input variable NSTORE (Item 6 of Table 2).
49. The finite element grid shown in Figure 7 is also used in this example problem. Input data used in Example Problem 7 are used except for the following differences:
Table 4. Example Problem 1--Input Data for Single Slab with Many Options

(Continued)
Table 4 (Concluded)

a. The concrete slab is 15 in. thick and the slab is not underlain with a stabilized layer. Also, there is no gap under the pavement. The thickness of the slab and the subgrade $k$ values are uniform throughout the pavement.
b. A positive temperature differential of $+3^{\circ} \mathrm{F}$ per in. of pavement is assumed. For a l5-in. concrete slab, the total difference between the top and the bottom of the slab is $+45^{\circ} \mathrm{F}$.
c. The slab is subjected to a uniformly applied load at the corner of the slab, i.e., element 50.
d. There are only 20 nodal points where the stresses are computed and printed.
50. The input data for this problem are given in Table 5. In the first run, the stresses and deflections due to temperature, sla' weight, and gaps are computed. This is done by defining the following variables in the input data as follows:
a. $\operatorname{NLOAD}=0, \quad \mathrm{NMCF}=0, \quad \mathrm{NWT}=1, \quad \mathrm{NTEMP}=1, \mathrm{TEMP}=$ 45 , NCYCLE $\geq 10$, and NGAP equals the exact number of nodes where gaps exist.
b. NSTORE $=0$ because thermal stresses and deflections are not read in from data cards.
c. $\operatorname{INDP}=1$ because the computation does not depend on the results of the previous run.
51. In the second run, the stresses and deflections due to the applied load, temperature, and gaps are computed. This is done by defining the following variables in the input data as follows:
a. NMCF $=0$, NWT $=1, \quad$ NTEMP $=1$, TEMP $=45$, NCYCLE $\geq 10$, and NLOAD and NGAP equal the exact number of nodes where gaps exist.
b. NSTORE $=2$ because the stresses and deflections due to the thermal effect computed in the first run should be used in the second run.
c. INDP $=0$ because this run is not independent of the previous run.
52. Once the stresses and deflections due to the applied load and temperature are computed, the differences between the results computed in the first and second runs are those due to the applied load alone. Such a computation was made and the computer output is presented later in this section in Computer Output 3 with detailed explanation.
Table 5.

Table 5 (Continued)


(Continued)
(Sheet 2 of 3 )
(pəpntouoj) $\varsigma$ әтqв山


## Example Problem 3: A Two-Slab Pavement System 2 Symmetrical Along the $X$-Axis

53. The slabs involved are 15 by 12.5 ft . A uniformly applied load with a 7.5 - by l0-in. rectangular imprint is applied near the center of the joint. Because of symmetry, it is only necessary to use half of the slabs for the computations. Figure 4 shows the finite element grid for the problem. According to the principle that the slab carrying the heaviest load is numbered first, the left slab is designated as slab 1 and the right slab as slab 2. Beginning from the first slab and ending at the last slab, the nodes and elements are numbered consecutively from bottom to top and then from left to right. The input data for the problem are presented in Table 6, with the special features listed below:
a. The efficiency of shear transfer across the joint is assumed to be 100 percent and the efficiency of moment transfer is zero percent.
b. Assuming good fit between the steel and the concrete, the initial and final values for the modulus of dowel support $K$ are equal. The value of DCGF (item 11) is arbitrarily assumed to be 1 in . Since the deformation of concrete does not exceed 1 in., initial $K$ value is always used in the computation.
c. Because of symmetry, half of the total load is used in the problem, which is applied at element 43.
d. Because the problem is symmetrical about the X-axis, the nodal numbers lying on the $X$-axis of symmetry are $1,8,15,22,29,36,43,50,57,64,71,78,85,92$, 99, 106, and 113.
e. Nodal points 57,58 , and 59 are used for checking the convergence. Nodal point 57 is used for determining whether the relaxation factor RFI should be reduced. Accordingly, the values of NBCK, NECK, and NNCK should be 1,3 , and 1 , respectively.

## Example Problem 4: A Nine-Slab Pavement System

54. Figure 8 shows the finite element grid for a nine-slab pavement system. The system has a total of 196 nodal points, 122 elements, and 12 joints. In the real case, the number of elements may
Table 6. Example Problem 3--Input Data for a Two-Slab System ${ }_{2}$ Symmetrical Along the X-Axis

|  | FORTRAN STATEMENT |  |  |  |  |  |  |  | ioentricatiom |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1, 1 |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| 1-2 2 |  | 1. 9639 , 1.300 | 200.1200 | 130 |  |  | -6icher |  |  |
| 1 1, 9 |  | - | $\ldots 0.1$ |  |  |  | ,1-1.1. |  |  |
| -1/ ${ }^{8}$ |  | 131 | L. 2 , |  |  |  |  |  |  |
| 1,4.40909_1-1 0,09090 |  |  |  |  |  |  |  |  |  |
| 1-1 |  | 0.1 | $1+1$ |  | $26,1+1$ | 1 |  |  |  |
| -1, 0 | 1 | 9.1 | $1+1 p_{1}+10$ |  | 9 $\mathrm{Q}_{1}$ | 1 |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| 1 | $\cdots \times 1$ |  | 1.- 1.el-1.10 | 1. 1 , -1-1-1-1 | -1.1.-1.1.el | -1, +1-1 | , |  |  |
| 1.360. | د | -1-10,09090.1. | -1. 9.35090 | -1-1 1.popgras. |  | 1. 3,008-09 | -0,20000 |  |  |
|  |  |  |  |  |  |  |  |  |  |
| L, | .9. | $\pm 35$ | 1.50 | $12+1$ |  | - | $\ldots$, |  | 143.5 |
|  | , 359. |  | 1.1.1., 1. |  | -1.1 1 0... |  |  |  |  |
| Les. | -19. |  |  | 1 1.1.und 135 . | 1-1 1-1.c. 150 | -1.c. 79. | -1, 1.190 |  |  |
|  |  | -1-1-1-10. |  | -1-1-1-1 | -1, 6 ¢ 6 . | . ب. | , . 125 |  | 50. |
| $1 \ldots \downarrow$ - |  |  |  |  |  |  |  |  |  |
| L12. |  | 1.1.1.1. 0.15 | 0.5009007 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| 1.... | 7509.90 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
| 포소, cam: |  |  | 4Ixs gupcrupe. |  |  |  |  |  |  |
| -1, |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

Table 6 （Concluded）

|  |  |  |  | FORTRAN STA | tement | $3{ }^{5}$ | ss | －icatins． |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 15．．．22 | 29.1 .136 | 13.1 .2 .150 | C． 57. | 60 | ， 63 | 64．．． 65 |
| 1.16 | 67 | 68.1 .68 | 70．．．72 | 78. | s2．＿1． 29 |  |  |  |
|  |  | 15， 122 | 2a．．．36 | $13.1+150$ | $57.1+6$ | 71． 21.38 | 2 | 92．． 106 |
| 1．．．123 |  |  |  |  |  |  |  |  |
|  | \％ 3 85 |  |  |  |  |  | 1 |  |
|  | man |  |  |  | ＋11．1．1． |  | ＋1．1． |  |
| Mrs， | 10，cupr |  | ，1－11 | ．．．1． | ， |  |  |  |
| muricen | Ho，coue |  | ${ }^{8}$ |  |  | い」しやら2し |  | 11－1．4．．． |
| nenr，cura： | maras |  |  | － |  | 1．11．1．1．1．1 | 14t1．t14 | 1．1．．．1．．．．－ |
| 1，c．0 | 000. |  |  |  | L1．141－1－14 | 11．11．161 | 1．1．1．．．．． | 1．1．1． |
| 1. |  |  |  | －1－1－4． |  |  | 1．1．1．1． | 以＋＋，－－－ |
|  |  |  |  | 山上1， | い | 1．1．1．t．t．t | 1．1．1．1．1． | －－ |
| 1. |  |  | －1．1 |  | ＋ | －12．1＋1＋1， | ，－ | 1．1．1．1． |
| 1. |  | 以1－1 | 1．14114 | －111 | い小 | ，．1．1．．． | ．．．．．．．．．． |  |
|  |  |  |  | 11．1．1．1．1 |  | ＋1－1．．．1．1． | 1．14151．1． |  |
|  |  |  |  | 土 |  |  | ＋1－1．1． |  |
| 1－1． |  | －1． |  | －1， |  | －1－1－1上」－1 |  |  |
| 1， |  |  |  |  |  |  | い1－1－L |  |
| 1. |  |  |  |  |  |  |  | ．．．．．．．．．． |
| 1. |  | － |  |  |  |  |  |  |
|  |  | －1．1－1 |  |  |  | －1 |  |  |
| 1， |  |  |  |  |  |  |  |  |
| 1， |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| mocosamers |  |  |  |  |  |  | page | or |




Figure 8. Finite element layout for Example Problem 4
need to be more to obtain more accurate results, but the dimensions of several variables have to be increased in the program.
55. In Figure 8, the slab is numbered according to the magnitude of load and the expected magnitude of shear forces transmitted across the joint. Since the load is applied at the center slab, it is numbered slab 1. Because the load is applied at the upper right corner of the
slab, greatest shear forces will be transferred to the right middle and upper middle slabs; therefore, they are numbered slabs 2 and 3, respectively. The choice between slabs 2 and 3 is arbitrary because the shear forces transferred from slab 1 to these two slabs are expected to be the same in magnitude. For the same reason, the right upper slab is numbered slab 4 and the left middle and lower middle slabs are numbered slabs 5 and 6, respectively. Similar to slabs 2 and 3, the choice between slabs 5 and 6 is arbitrary. Similarly, slabs 7, 8, and 9 are numbered.
56. Beginning with slab 1 and ending at slab 9, the nodes and elements are numbered consecutively from bottom to top and then from left to right as shown in Figure 8.
57. The joints can be numbered arbitrarily. However, the joints shown in Figure 8 are numbered according to the magnitude of shear transfer across the joint. For illustrative purposes, the use of the element coordinates card for slab 1 is explained. Slab 1 has five nodal points in the $X$-direction and five nodal points in the Y-direction, and the slab is surrounded by joint 5 on the left, joint 1 on the right, joint 6 at the bottom, and joint 2 at the top. The element coordinates cart in Item 4 of Table 2 should then be input as 5162 . The same logic is used in the input for the other slabs.
58. The input data for the problem are presented in Table 7. Special features of the input are listed below:
a. At joints 1 to 10 , the efficiency of shear transfer across the joint is assumed to be 100 percent and the efficiency of moment transfer is zero percent. At joint ll, a spring constant of 1000 psi is specified for shear transfer and zero percent for moment transfer. At joint 12 dowel bars are used for a shear transfer device and, similar to all other joints, moment transfer is assumed to be zero percent. The bars have a diameter of 1 in. and are spaced 18 in. apart. The final value for modulus of dowel support $\operatorname{SCKV}(12,2)$ is assumed to be $1,500,000$ psi but the initial value $\operatorname{SCKV}(12,1)$ is assumed to be $600,000 \mathrm{psi}$ when the deformation of concrete DCGF(12) is less than 0.01 in . It should be pointed out that the unusual and varied combination of load transfer options across the joints used in this problem is merely for illustrative purposes.
Table 7. Example Problem 4--Input Data for a Nine-Slab Pavement System

Table 7 (Continued)

|  |  |  |  | fortran sta | stament |  |  |  | MEntifthtion |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | -0,009090. |  |  |  |  |  |  |  |
|  |  | 0, | $1+1.0$ | 0 | . ... 35 |  |  |  |  |
|  |  | $0_{1}$ | , Lo+1+10 | -1...1 | 0 |  |  |  |  |
|  |  | [-1.4.1.es | -1.49 . . 289 | . . . $0_{2}$, . . 25 | . 60000 |  |  |  |  |
| [.. ${ }^{\circ}$ |  | $1 .+0_{1+1} 0^{0}$ | $\ldots 0_{1} 0_{1} \ldots \rho$ |  |  |  |  |  |  |
| 1, 209. |  | 上上29 9 + + | 11.19.149 | [ : 1 1.peorex | 2.0002-03. | - $3,00008+07$ | 1.9.2 |  |  |
|  | 20,400 | rtoun svpause spay | $\mathrm{Br}_{2}$ |  |  |  |  |  |  |
|  | -19. | 1. $1.1+1+1+60$ | , 220. | 256. | 180. |  |  |  |  |
|  | $\square 9$. | $12.1 .1+1.150$ | , 120. | , 256. | ', , 1800 |  |  |  |  |
|  | -1.9. |  | . . 60. |  | 1. 1880 |  |  |  |  |
|  | - | -1-1-L1, 60. |  |  | $\therefore 280$. |  |  |  |  |
|  | P. | - $1 .+\ldots+\ldots+150$ | , 120. | , 256. | . 280. |  |  |  |  |
|  | - 9. | - | -1,1-5 60. | . . . . . . . 220. |  |  |  |  |  |
|  | -1, 9. | -1, +1-2+2.34 | L-1 : . . , , , 690. | . . . . . . . 220. | 280. |  |  |  |  |
|  |  | -1.c. | -1-1.1. $\mathrm{S}_{1} 60$. | . 220. | 180. |  |  | - |  |
|  | - 1 ¢ | , 60 | 1. . 220. | , 280. |  |  |  |  |  |
|  | [-1 ${ }^{\text {P }}$ |  | , 220. | , 256. | 180. |  |  |  | 1-1.-4-2 |
|  | - 2. | -2-1. 60 | , 1220. | 256. | 180. |  |  |  |  |
|  | $\ldots$ | -n 60. | 120. | . 280. |  |  |  |  |  |
|  | - 0 | , , 60 | 220. | , 280. |  |  |  |  |  |
|  | - 2 | , 2 l | , 60 | , 220. | 180. |  |  |  |  |
| - | 0. | 24. | .-.1-1.-1, 60. | 220. | . 288. |  |  |  |  |
|  | Q. |  |  |  |  |  |  |  |  |
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Table 7 (Concluded)

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(Sheet 3 of 3)
b. A 60,000-1b single-wheel load with a contact area of 500 sq in. is applied at element 16. The loading card in Item 13 of Table 12 should thus be from -0.875 to 1 in both X - and $Y$-directions. With such a division of contact area in the element, the actual tire imprint has a contact area of 506.25 sq in. and a contact pressure of 118.51851 psi.
c. Nodal points 23, 24, and 25 at joint 1 are used for checking convergence and nodal point 25 is used for determining whether the relaxation factor RFI should be reduced. Therefore, JNCK, NBCK, NECK, and NNCK in Item 4 of Table 2 are input as 1, 3, 5, and 5, respectively.
$\frac{\text { Example Problem 5: A Four-Slab Pavement System with } 50}{\text { and Zero Percent Moment Transfer Along the Joints }}$
59. As was explained in the input guide (Table 2), when moment transfer is other than 100 or zero percent, a separate computer run with 100 percent moment transfer must first be made and the computed moments along the joints are then used in the following run. Example Problem 5 presents the input data for such a case.
60. Figure 2 shows the finite element layout for the problem. Two uniformly applied loads are placed at the two upper slabs near the corner joints. Each load has a magnitude nf $51,840 \mathrm{lb}$ and a dimension of 36 by 36 in . Because the loads are equal in magnitude, the designa$t+j n$ of slab 1 and slab 2 is arbitrary. Similarly, the designation of slab 3 and slab 4 is also arbitrary. Ince the slab numbers are deter$\because i r \cdot i$, berinning from slab 1 and ending at slab 4 , the nodes and ele-ner:- are numbered consecutively from bottom to top and then from left - : irft as shown in Figure 2. It should be pointed out that, for the -. rne: ience of presenting and explaining the output results of the com$\therefore \because \cdot i n$, minimum number of elements is used in this problem. The ...ntit: and discussion of the computer output is at the end of this $\because \cdot$.
-.. The input data for the problem are presented in Table 8, . : evial features of the problem described below.
Table 8. Example Problem 5--Input Data for a Four-Slab Pavement System

Table 8 (Continued)

(Sheet 2 of 4)
(Continued)
Table 8 (Continued)

(Continued)

a. Slab 1 has three nodal points in both $X$ - and $Y$ directions, and has joint 3 on the left and joint 2 at the bottom. The slab coordinates card in Item 4 of Table 2 for slabl should thus be read as 33302 . The same reasoning is used for the other three slabs.
$\underline{b}$. The difference in the input data between the first and second run lies in the following variables: (1) $\operatorname{EFF}(I, 2)$ of the joint efficiency card in Item 5 of Table 2, (2) NMT , number of cases to be solved for moment transfer in card 2 of Item 4 of Table 2, (3) INDP in card 1 of Item 4 of Table 2, and (4) CM(j) of the efficiency of moment card in Item 21 of Table 2. They are discussed separately as follows:
(1) In the first run $\operatorname{EFF}(I, 2)$ is input as 1.0, i.e., 100 percent efficiency, and the number of cases to be solved NMT and multiplying factor $C M(1)$ are both l. INDP is also equal to 1.
(2) In the second run, $\operatorname{EFF}(I, 2)$ should be input as 0.5 and $N M T=2$ ( 50 percent and zero percent moment transfers). INDP $=0$ because moments computed at 100 percent moment transfer computed in the first run are used in this run. $C M(1)$ and $C M(2)$ are input as 2.0 and 0 , respectively, because $1.0(\operatorname{CM}(1)) \times 0.5(\operatorname{EFF}(I, 2))=0.5$, i.e., 50 percent moment transfer, and $0(C M(2)) \times$ $0.5(\operatorname{EFF}(I, 2))=0$, i.e., zero percent and moment transfer. Provisions are made in the program that other combinations of $E F F$ and $C M$ can be used. For instance, $\operatorname{EFF}(1,2)$ can be 0.25 and $C M(1)$ and $\mathrm{CM}(2)$ can be 2.0 and 0 , respectively. However, erroneous results will be computed if EFF is set as 1.0 .
The moments along the joints computed in the first run are stored automatically in the memory. They are multiplied by the coefficient 0.5 or 0 and are used at the joints in the second run.
c. Nodes 1 and 4 at joint 2 are used to check convergence, and node 1 is used for determining whether the relaxation factor RFI should be reduced. Therefore, JNCK, NBCK, NECK, and NNCK should be input as $2,1,2$, and 1 , respectively, in card 3 of Item 4 of Table 2.
d. Dowel bars are used in all four joints to transfer shear forces. Bars 0.5 in. thick spaced 36 in. center-to-center are used in joints 1 and 2 , and 1.0 -in. bars spaced 12 in. center-to-center are used in joints 3 and 4. A good fit is assumed for all four joints and thus $\operatorname{DCBF}(j)$ is arbitrarily selected as 1 in. in Item 11 of Table 2, i.e., a very large value.
62. If it is desired to determine the stresses and deflections due to load alone and the coefficients of moment transfer across the joints are between 0 and 100 percent, the procedure of using NSTORE $=2$ in card 1 of Item 5 of Table 2 becomes rather complicated. It is suggested that the stresses due to temperature alone and the stresses due to temperature and load be computed separately. The stress from the temperature alone may be subtracted from the stress from the temperature and load to give the stresses due to load alone.
63. The use of four slabs in this example is for illustration only. Because of symmetry with respect to the $Y$-axis, only the right half, or slabs 1 and 3, need actually be considered.

## Computer Output 1

64. Table 9 shows the Computer Output 1 printout for Example Problem 5. For clarification, the input data for each run are first printed. Therefore, any mistakes in the input data can be easily checked. For convenience of explanation, entry numbers are used in places where explanations are needed. In many places the output printout is self-explanatory.

Entry 1
65. $\operatorname{IFF}(I, 1)$ is input as 1.0 because $\operatorname{LRT}=2$ (Item II of Table 2). $\operatorname{EFF}(I, 2)$ is also input as 1.0 because the efficiency of moment transfer in the first run is 100 percent.
Entry?
66. Referring to joint 1 of Figure 2, the initial starting nodal numbers at the left side of the joint are nodes 10 and 30 , and the last nodes at the right are nodes 16 and 36 . For joint 3 in the up-and-down direction, the starting nodal numbers at the bottom are nodes 1 and 16 , and the last nodes a's the top are nodes 3 and 18. The information printed out can be useful to verify whether the finite element grid coordinate system is input correctly.

Entry 3
67. Unless all joints are 100 percent efficient, a problem


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Table 9 (Continued)




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Table 9 (Continued)


exists at the junction of four slabs. In Figure 9, both nodes 16 and 21 impose a deflection to node 36. A question then immediately arises as to which node, 16 or 21 , should be used. To facilitate the analysis, it is assumed that the node having a smaller nodal number should be used. In this case, only node 16 can impose a deflection on node 36 , while node 21 is not connected to node 36 . This is shown in Figure 9 where slabs 3 and 4 are not connected at the junction. The omission of shear transfer between nodes 21 and 36 yields greater stresses and dispiacements in the pavement and is therefore on the safe side.

63. Another problem exists at node 16 because a deflection is imposed from node 1 and at the same time a reactive force from node 36 (due to the deflection of slab 4). Because the deflection is fixed at node 16 , the imposed reactive force actually has no effect on the solution. Since node 16 and node 1 are connected by a dowel bar, the reactive force at node 36 is not imposed to node 16 but is transferred to node $I$, or the firct node on joint 3 .
64. The output information in Entry 3 explains how the shear forces are transferred across the joints. Before going into detail, the definitions of IST, NJT, and NKT are explained as follows:


Figure 9. Shear transfer at the junction of four slabs
a. IST is an identification for shear transfer at corners of the slabs. It is two-dimensional in the program as $\operatorname{IST}(N J O I N T, i)$, where $i=1$ and 2. The left or bottom node on the given joint is indicated by setting $i=1$ and the right or top node by setting $i=2$. An IST of 0 indicates that there is no shear transfer at the node across the joint; 1 indicates that there is a regular shear transfer, and 2 indicates that the shear force at node NKT of joint NJT must be transferred here.
b. NJT is the joint number from which shear is transferred. NJT is also two-dimensional in the program as NJT(NJOINT,i), where $i=1,2$. The meaning of the indexes is the same as in IST. The program wili print 0 if $\operatorname{IST}(N J O I N T, i)=0$ or 1 .
c. NKT is the nodal number of joint NJT from which shear is transferred. It should be noted that the nodal number here is defined differently from those shown in Figure 1. Node l. is the node either at far left or at the very bottom, then counting from left to right or from bottom to top. NKT is also two-dimensional in the program similar to $\mathrm{NJT}(\mathrm{NJOINT}, i)$. Also, the program will print 0 if $\operatorname{IST}(J O I N T, i)=0$ or 1 .
70. In Entry 3, two values of IST, NJT, and NKT are printed for each joint. The first number refers to the node either at the left or at the bottom of the joint; the second number refers to the node either at the ighi or at the top of the joint.
71. Referring to Figure 2, the shear transfer at two end nodes of joint 1 is regular, so the values of IST are both printed as 1 and consequently the values of NJT and NKT are all zeros. The shear transfer at the bottom node of joint 3 is more complicated. At node 16 , it is not necessary to impose a reactive force from node 36 because a deflection is imposed from node 1 so that the force at node 36 of joint l is directly transferred to node l. Note that node 36 is the third node (counting from the left) at joint 1 ; the values of IST, NJT, and NKT at the lower end of joint 3 are thus 2 , 1 , and 3, respectively. This means that the shear force at node 36 of joint 1 is transferred to the node at the lower end of joint 3.
72. The information printed out in Entry 3 is quite involved and is difficult to understand. Fortunately, complete appreciation of

Entry 3 by the user is not required because such an understanding is not a prerequisite to the use of other output data. Entry 4
73. Entry 4 prints out the computed dimensions of matrices and other information. Note that the computed values are less than those declared.

## Entry 5

74. Initial curlings and gaps are deformations due to temperature and gaps. Initial curlings are computed solely based on the temperature differential, and the concrete weight and subgrade reactive forces are not considered. Since temperature and gaps are not considered in the example problem, the values printed out in Entry 5 are all zeros. Note that when temperature is considered, the initial curling of the slabs should be symmetrical, provided that the thicknesses of the slabs are uniform and the finite element grid patterns are not far off from being symmetrical. When the user is skeptical about the computed stresses and deflections, the output shown in Entry 5 should first be checked.

Entry 6
75. Because of the method used in specifying the uniformly applied load, a small difference may exist between the actual load and the input (or calculated) load. The printout in Entry 6 is presented for visual inspection. In the program, the operation will be terminated for the particular run when the difference between the actual and the calculated load exceeds 3 percent.
Entry 7
76. The variable ICC in the program refers to the number of iteration cycle for checking the subgrade contact. In this example computer output NCYCLE $=1$, so ICC is limited to 1 . Entry 8
77. The differences between the two iterations are generally decreasing, indicating the solution is converging. The iteration continues until the ratio of the difference in values becomes smaller than the specified DEL or DELF .

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80. 'the displacments and stresses are printed when the convoremer requirements are met. Positive stress indicates that the slab hat: compression at the top and temsion at the bottom and nerative stress indicates the opposita. 'lhe symbols of stress XY, mador, minor, shear, and remetion stand for shear stress, mafor primeipal stress, minor prineipal stress, miximum shear stress, and subrowh reactive stress, respectively. The suberade reactive stress is computed as the product ot modulus ot subrrade reation $k$ (pei) and slab deflection (in.) so it has a unit of pil. To obtain the total reactive force acting at the node, the subriade reartive sitress should be multiplied by the affeeted area.

Entry 1:
11. The stressos and displacements are computed for one more iteration for inspertion of converenence by the user. When the solution correctly converpes, the difierences in the computed results between t.wo iterations should be insipmiricant. Otherwise, the solution is not converrent.

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## Table 10 (Continued)



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## Table 10 (Continued)

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Table 10 (Concluded)

problem, which is to compute stresses and deflections for a symmetrically loaded square slab subjected to both temperature warping and applied load. Gaps with a maximum magnitude of 1 in. exist in the subgrade near the load. Because of symmetry, only one quarter of the slab is computed. The $80,000-1 b$ load ( $\mathrm{p}=200 \mathrm{psi}$ ) is applied at the center of the slab and the temperature differential is $3.75^{\circ} \mathrm{F}$ per inch of the slab, causing the slab to curl upward. Figure 10 shows the finite element grid pattern of the slab. The purpose ol this example printout is to show the differences among the initial gap, deflection, and final gap. Similar to the previous example output, entry numbers are used in places where explanations are needed. In places where similar explanations are given in the previous example output, they are not repeated.

## Entry 1

83. Because the uniformly applied load is applied at the center of the slab, the case is symmetrical with respect to both the $X$ - and Y-axis. It is thus necessary to consider only one quarter of the slab in the computation. According to Figure 10, the nodal numbers that are symmetrical along the $X$-axis are $1,8,15,22,29,36$, and 43 , and the nodal numbers that are symmetrical along the Y-axis are 1, 2, 3, 4, 5, 6 , and 7. The computation is made on the quarter slab with $\sigma_{x}=0$ and $\sigma_{y}=0$ at each nodal point along the $X$ - and $Y$-axis, respectively. Entry ?
84. The initial gaps at each nodal point are printed as was
input.

## Entry 3

85. The iritial curlings are computed based on the input temperature differential. The slabs are assumed to be weightless, and the subgrade reactive forces are not considered. The magnitude of the curling is measured from the initial subgrade surface to the warped bottom surface of the slab. Positive curling (or gap) indicates that the warped slab at the particular node is above the initial subgrade surface; i.e., the slab is warped upward. Negative curling indicates that


Figure 10. Finite element layout for Computer Output 2 the warped slab is below the initial subgrade surface; i.e., the slab at the particular node is sinking into the ground.
86. It should be noted that when gaps exist under the slab, the initial curlings are combined with the gaps. For instance, at node 1 the initial curling is zero because node 1 is located at the center of the slab. But because a l-in. gap exists beneath node $I$ and because curling is defined to be the distance from the initial subgrade surface to the warped slab, the initial curling at node 1 becomes 1 in., as shown in Entry 3. Similarly, the actual curling at node 2 is 0.00094 in .
curling upward and since the initial gap at node 2 is 0.8 in., the initial curling at node 2 becomes 0.80094 , as shown in Entry 3. Entry 4
87. The total load applied on the 300 - by 300 -in. slab is 8,000 lb. Because only one quarter of the slab is used in the computation, both the input load and calculated load are $20,000 \mathrm{Ib}$ in magnitude. Entry 5
88. Displacements are induced by the load and the subgrade reactive forces and are measured from the initial warped surface to the new surface. Note that the applied load generally makes the slab move downward and the subgrade reactive forces push the slab upward. Positive deflection indicates downward movement, and negative deflection indicates upward movement. Entry 5 shows that all the deflections are positive, indicating the deflections are a downward movement from the warped up.
Entry 6
89. The gap or precompression is computed as the difference between the initial curling and the deflection. Sign convention used in the initial curling (Entry 3) is used in Entry 6. At node 1, the gap is 0.77679 in., which is computed as the difference between the upward initial curling of 1 in . (Entry 3) and the downard deflection of 0.2232 in. (Entry 5); a positive gap indicates that node 1 is 0.77679 in. above the initial subgrade surface. At node 5 , the initial curling is 0.07594 in. (above the initial subgrade surface) and the deflection is 0.1848 in. (downward movement under the load and the subgrade reactive forces), so the precompression becomes -0.10885 in. sinking into the ground.

## Entry 7

90. During the first cycle of iteration, a full subgrade contact condition is assumed, except at nodes where gaps are specified. The gaps shown in Entry 6 indicate that many nodal points have lost the subgrade support, i.e., the subgrade contact condition has changed. Therefore, computations start again based on the new subgrade contact
condition shown in Entry 6 and the deflected surface shown in Entry 3 (initial curling).

## Entry 8

91. The deflections are measured from the initial curling (or precompression) shown in Entry 3.

## Entry 9

92. The gaps or precompressions are the differences between the initial curlings (Entry 3) and the deflections (Entry 8). The sign at each node is compared with those shown in Entry 6, and since the signs at some nodes have changed, the computation starts again with the new subgrade contact condition shown in Entry 9. The iteration repeats until the sign of either the gap or the precompression at each node no longer changes.

## Entry 10

93. Entry 10 shows the gaps and precompressions at the end of iteration cycle 3.
Entry 11
94. Entry lib shows the gaps and precompressions at the end of iteration cycle 4. Since the signs at each node shown in Entry 11 do not change from those shown in Entry 10, the criterion for checking subgrade contact is satisfied and the computed deflections shown in Entry lla and gaps and precompressions shown in Entry llb are the final values. Note that the deflections in Entry lla are measured from the initial warped surface in Entry 3.
Entry 12
95. The values shown in Entry 12 are the same as shown in

Entry llb computed during the last iteration cycle, but are different from those shown in Entries 6, 9, and 10 computed during the earlier cycles.
Sign notation used*
96. To clarify the sign notation used in this report, the values of initial curling, deflection, gap, and precompression computed after

[^2]the last iteration at nodes 1, 3, and 5 (Figure 10) are plotted in Figure 11. The initial curlings were computed based on the temperature differential and the assumptions that the slab is weightless and the subgrade reactive forces are inactive. The force-induced deflections computed during each iteration were always measured from the initial curling, not from the initial bottom surface of the slab. It should be noted that stresses in the slab are also computed based on the deflections measured from the initial curled surface, not from the initial surface of the slab. When temperature is not considered and the initial curling does not exist, the deflections are measured from the initial bottom surface of the slab.
97. At node 1 , the sum of the deflection ( 0.1254 in. at Fntry Ila) and the final gap ( 0.87456 in . at Entry Ilb) is equal to the input initial gap ( 1 in.). At node 3 , the sum of the final gap ( 0.38626 in. ) and the deflection ( 0.1222 in.) less the input initial gap ( 0.5 in .) is equal to the difference between the computed initial curling ( 0.50844 in.) and the input initial gap ( 0.5 in.) . Note a $0.00002-i n$. computer round-off error is involved.

## Computer Output 3

98. Table 11 shows the Computer Output 3 printout for Example Problem 2, which is to compute stresses and deflections for a single slab due to the applied load alone. Two runs were conducted consecutively. The first run is made considering only the temperature, slab weight, and gaps, and the second run is made considering the temperature, slab weight, gaps, and the applied load all together. The differences in the computed results of the first and second runs are those due to the applied load alone. This option in the program is activated (in the second run) by setting the variable NSTORE $=2$ (Item 6 of Table 2). The reason for the need to compute stresses due to the applied load alone is explained in the footnote of the variable NSTORE Entry 1
99. NGAP $=0$ because no gap under the slab is assumed. NWT

Computer Output 3 Printout for Example Problem 2
Table 11.

Table 11 (Continued)




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Table 11 (Continued)



















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| $\frac{1}{4}$ | 0.20906 | - | $\mathrm{n} \cdot \mathrm{1sc}^{5150}$ | 3 | 0.08649 | 2 | 0.0779 | ${ }^{3}$ | 0,07910 | 14 | $0.108{ }^{2} 2$ | 7 | 0:14130 |  | 0.17742 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 17 | 0.19880 0.09548 | 10 | c.i 2150 0.1165 0 | 13 | 0.05436 | ${ }^{12}$ | 0.0098 | ${ }_{21}^{13}$ | -0,00502 | 22 | (e. ${ }^{0.10188}$ | ${ }_{2}^{15}$ | -0:0224 | $2{ }^{16}$ | -0.06022 |
| 25 | 0.01933 | 26 | 0.64424 | 47 | 0,06442 | 26 | 0.07151 | 29 | -0,00526 | 30 | -0, ens? 2 | ${ }^{1}$ | toion43 | 32 | -0, 06246 |
| 33 | - 0.04305 | 34 | -r.012\%s | 35 | 9.01010 | s6 | 0.03804 | 37 | 0, 07862 | 38 | 0, 30844 | 39 | -0.0437 | 40 | -0,06135 |
| 41 | -0.06006 | 42 | -f104690 | 4 | -0,01573 | 44 | 0.01436 | 45 | 0,03277 | 46 | 0.10140 | 47 | 0.02208 | 48 | -0,02486 |
| 49 | -0,04516 | 50 | -0.04502 | $\stackrel{1}{4}$ | -0,03333 | 52 | -0:00573 | 53 | 0, 02235 | 54 | 0.04081 | 25 | 0.12574 | 36 | 0,04532 |
| 97 | -0.09337 | 58 | -c.02304 | ¢9 | -0,02757 | 60 | -0.01672 | 61 | 0,00902 | 62 | 0.93547 | 4 | 0.03268 | 64 | 0,14079 |
| 69 | 0.05987 | 80 | n.01034 | 67 | -0.01294 | ${ }^{18}$ | -0,01565 | 6 | -0,00564 | 78 | 0.13814 | 11 | 0.04335 | 12 | 0,06263 |
| 73 | 0.25784 | 74 | 0.07047 | 75 | 0.02655 | 26 | $0 \cdot 00167$ | 77 | -6,00165 | 74 | $0: 00748$ | 19 | 0.07125 | 8 | 0,05663 |
| ${ }^{8!}$ | 0.07252 0.07000 | ${ }_{80}^{82}$ |  | ¢ 6 | 0,09598 | 4 | 0.04414 | 93 | 0,01199 | 9 | 0.08495 | 87 | 0.03215 |  | 0.04940 |
| 97 | 0.06174 | 98 | n,085>s | 89 | 0,10074 |  | , |  | . |  | d, |  | -10 |  | 0. 0402 |


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 （Continued）



and NTEMP are both equal to 1 because both the temperature and weight of the concrete slab are considered. The total temperature differential between the top and the bottom of the slab is $45^{\circ} \mathrm{F}$. NSTORE $=0$ because it is the first run, and $N L O A D=0$ because the load is not considered in the first run. Entry 2
100. The meaning and sign convention of the initial curling and gap can be found in Entry 3 of Computer Output 2. Entry 3
101. The expressions in Entry 3 are the same as those shown in Entries 5, 6, and 7 of Computer Output 2.
Entry 4
102. The expressions in Entry 4 are the same as those in Entries 11 and 12 of Computer Output 2. Entry 5
103. The computed stresses and deflections are stored to be used in the next run.
Entry 6
104. NSTORE $=2$ indicates that the stresses and deflections computed in this run will be subtracted from those computed in the preceding run.

## Entry 7

105. The applied load is considered in the second run.

Entry 8
106. Two sets of stresses are computed and printed. The stresses due to the applied load, temperature, and slab weight are printed in the line where the number of the nodal point is printed. The stresses due to the applied load alone are printed in the line immediately below the printed stresses due to the load, temperature, and slab weight and are printed one space to the right. For instance, at nodal point 71 , stress $\sigma_{x}$ due to the load, temperature, and slab weight is -75.2457 psi and that due to the load alone is only -49.8533 psi .
Entry 9
107. The deflections are those due to the applied load alone,
which are the differences of the computed deflections due to the applied load, slab weight, temperature, and gaps and those due to the slab weight, temperature, and gaps. In this example problem, gaps are not assumed. If they are assumed, the magnitude of the gaps should be those without temperature influence.

## PART VI: CONCLUSIONS AND RECOMMENDATION

108. The computer program WESLIQID has the capacity of obtaining solutions for rigid pavements with discontinuities. The program is versatile because of its various options dealing with problems of different natures. The program is economical to operate and requires only a reasonable amount of core space. It is recommended that the program be used for routine pavement design, analysis, and research purposes.

## APPENDIX A: ANALYSIS OF TWO-LAYER SLABS

1. The program can be applied to two-layer slabs, either bonded or unbonded. Layer 1 has a thickness $t_{1}$, a modulus of elasticity $E_{1}$, and a Poisson's ratio $v_{1}$. Layer 2 has a thickness $t_{2}$, a modulus of elasticity $E_{2}$, and a Poisson's ratio $\nu_{2}$.
2. In the case of unbonded layers, the displacements of both layers are assumed the same, the modulus of rigidity $R$ of the two-layer slab is simply the summation of that of each layer, or

$$
\begin{equation*}
R=\frac{E_{1} t_{1}^{3}}{12\left(1-v_{1}^{2}\right)}+\frac{E_{2} t_{2}^{3}}{12\left(1-v_{2}^{2}\right)} \tag{Al}
\end{equation*}
$$

After the displacements are determined, the stresses in each layer are computed, based on the stress matrix of each.
3. In the case of bonded layers, a composite thickness is used. The composite thickness $t$ can be determined by

$$
\begin{equation*}
t=t_{1}+t_{2} E_{2} / E_{1} \tag{A2}
\end{equation*}
$$

Taking the moment at the surface, the distance of neutral axis from the surface $d_{n}$ can be determined by

$$
\begin{equation*}
d_{n}=\frac{0.5 t_{1}^{2}+t_{2}\left(t_{1}+0.5 t_{2}\right) E_{2} / E_{1}}{t_{1}+t_{2} E_{2} / E_{1}} \tag{A3}
\end{equation*}
$$

The composite moment of inertia $I_{\text {comp }}$ is

$$
\begin{equation*}
I_{\text {comp }}=\frac{1}{12} t_{1}^{3}+t_{1}\left(d_{n}-\frac{t_{1}}{2}\right)^{2}+\frac{1}{12}\left(t_{2}\right)^{3} \cdot \frac{E_{2}}{E_{1}}+t_{2} \cdot \frac{E_{2}}{E_{1}}\left(t_{1}+\frac{t_{2}}{2}-d_{n}\right)^{2} \tag{A4}
\end{equation*}
$$

The composite Poisson's ratio $v_{\text {comp }}$ is

$$
\begin{equation*}
v_{\text {comp }}=\frac{v_{1} t_{1}+v_{2} t_{2} E_{2} / E_{1}}{t_{1}+t_{2} E_{2} / E_{1}} \tag{A5}
\end{equation*}
$$

The modulus of rigidity of the composite slab is

$$
\begin{equation*}
R=\frac{E_{1} I_{\text {comp }}}{I-v_{\text {comp }}^{2}} \tag{A6}
\end{equation*}
$$

After the displacements and moments are determined, the maximum stress in layer $1 \sigma_{1}$ can be obtained by

$$
\begin{equation*}
\sigma_{1}=\frac{M d_{n}}{I_{\operatorname{comp}}} \tag{A7}
\end{equation*}
$$

in which $M$ is the moment in the direction corresponding to the component of stress. The maximum stress in layer $2 \sigma_{2}$ is

$$
\begin{equation*}
\sigma_{2}=\frac{M\left(t_{1}+t_{2}-d_{n}\right) E_{2} / E_{1}}{I_{\operatorname{comp}}} \tag{A8}
\end{equation*}
$$

In accordance with letter from DAEN-RDC, DAEN-ASI dated 22 July 1977, Subject: Facsimile Catalog Cards for Laboratory Technical Publications, a facsimile catalog card in Library of Congress MARC format is reproduced below.

## Chou, Yu T.

Structural analysis computer programs for rigid multicomponent pavement structures with discontinuitiesWESLIQID and WESLAYER : Report 2 : Manual for the WESLIQID Finite Element Program / by Yu T. Chou
(Geotechnical Laboratory, U.S. Arwy Engineer Waterways Experiment Station). -- Vicksburg, Miss. : The Station ; Springfield, Va. : available from NTIS, [1981].

137, 2 p. : ill. ; 27 cm. -- (Technical report / U.S. Army Engineer Waterways Experiment Station ; GL-81-6, Report 2)

Cover title.
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1. Computer programs. 2. Finite element method. 3. Materials--Dynamic testing. 4. Pavements. 5. WESLIQID (Computer programs). I. United States. Army. Corps of Engineers. Office of the Chief of Engineers. II. U.S. Army Engineer Waterways

## Chou, Yu T.

Structural analysis computer programs for rigid : ... 2981. (Card 2)

Experiment Station. Geotechnical Laboratory. III. Title
IV. Series: Technical report (U.S. Army Engineer

Waterways Experiment Station) ; GL-81-6, Report 2. TAT.W34 no.GL-81-6 Report 2


[^0]:    * To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use the following formula: $C=0.555(F-32)$. To obtain Kelvin ( $K$ ) readings, use: $K=0.555(F-32)+273.15$.

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[^2]:    * Readers should consult the sign conventions defined in paragraph 46 of Report 1 of this series.

[^3]:    

[^4]:    

