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20. ABSTRACT (Continued)

data processing procedures that include preprocessing, preliminary data quality evaluation, and orbit determination. Also included is an overview of the hardware operation required for the correct interpretation of the SST Doppler, refraction, and timing data.

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INTRODUCTION

The Navigation Package (NAVPAC) is a satellite-borne data collection system developed for the Defense Mapping Agency (DMA). It consists of two sensor subsystems:

1. A receiver/data subsystem capable of tracking up to three Navy Navigation Satellites (NAVSATs) simultaneously and recording integrated Doppler, first-order ionospheric refraction, and timing information.
2. A miniature electrostatic accelerometer (MESA) capable of measuring accelerations due to atmospheric drag, winds, and orbit adjust thrusts.

The receiver/data subsystem, which also provides the power, command, telemetry, and MESA data storage functions was designed and built by the Applied Physics Laboratory of Johns Hopkins University (APL/JHU) according to the specifications provided by the Naval Surface Weapons Center (NSWC). The MESA was designed and built by Bell Aerospace TEXTRON according to the specifications provided by the Air Force Geophysics Laboratory (AFGL) and NSWC.

The NAVPAC satellite-to-satellite tracking (SST) Doppler data and the accelerometer data are used by NSWC for orbit determination for the host satellite. In the future, these data may also be used for gravity field model improvement. The MESA data are also used by AFGL for atmospheric density and wind studies.

NAVPAC stores the collected data in a core storage unit until it becomes full (approximately one rev). At that time, the data are automatically transferred to an onboard tape recorder. Daily, the data accumulated on this tape recorder are telemetered to a ground station, recorded on analog tape, and provided to NSWC. Here, the analog tape is digitized, preprocessed in the "Quick Look" mode using preliminary trajectories, and a preliminary data quality evaluation is done. Later, the data are grouped for a given time span and preprocessed in the "Normal" mode using the precise NAVSAT trajectories. After preprocessing, the data are combined with the station tracking data for the host satellite in NSWC's orbit computation program, CELEST, to produce the required ephemerides.

The purpose of this report is to provide a detailed mathematical description of the NAVPAC satellite-to-satellite tracking data processing procedures from preprocessing to orbit determination including an overview of the hardware operation required for the correct interpretation of the Doppler, refraction, and timing data. A similar description for the NAVPAC accelerometer data is given in Reference 1.

RECEIVER/DATA SUBSYSTEM OVERVIEW

The NAVPAC receiver/data subsystem includes three identical, independent receivers for phase-locked loop tracking of the coherent 400/150-MHz signal pair broadcast by the Navy Navigation Satellites. Each receiver is similar in design to the ground-based Geociever tracking equipment. Both are designed in the "master/slave" configuration; i.e., the scaled signal dynamics from the

400-MHz channel is subtracted from the 150-MHz channel leaving only signal variations due to the difference in ionospheric refraction effects on the two signals. The low channel can track a satellite only when the high channel is phase-locked.

The signal dynamics that the NAVPAC receivers must track are over twice what a ground-based Geociever would see. The maximum Doppler shift is ± 20 kHz, the maximum Doppler rate is 400 Hz/sec, and the maximum Doppler acceleration is ± 6 Hz/sec². Figure 1 shows worst-case plots for these three parameters for a hypothetical satellite-to-satellite pass. Since the actual NAVSAT broadcast frequencies are offset by ~ 80 ppm (~ 32 kHz) at 400 MHz, but the local NAVPAC oscillator is not offset, the resulting Doppler frequencies range from 12 to 52 kHz. Acquisition always occurs in the 12- to 32-kHz range at elevation angles no greater than $\sim 10^\circ$ as viewed from the host satellite. Each receiver sweeps in this frequency range asynchronously with the others with each sweep taking 40 sec (20 kHz divided by 500 Hz/sec). If a signal is detected by the high channel, the sweep stops and NAVSAT-type modulation is searched for. If it is not found, the sweep is restarted and a bogie counter is incremented. If the correct modulation is present, an indicator is sent to the data subsystem to initiate the Doppler count and to the low channel to allow it to begin tracking.

The low channel actually tracks the frequency given by

$$f_L = 4(e_L - \frac{3}{8} e_H)$$

where

e_L = error in frequency at 150 MHz due to refraction
 e_H = error in frequency at 400 MHz due to refraction

To first order, the error due to refraction is inversely proportional to the frequency. Therefore $e_L = 8/3 e_H$. Substituting this into the above equation gives

$$f_L = 4(e_L - \frac{3}{8} e_H) = 4(\frac{8}{3} e_H - \frac{3}{8} e_H) = 9\frac{1}{6} e_H$$

Thus, the low-channel tracked frequency is approximately 9-1/6 times the error in frequency at 400 MHz due to refraction. The low channel can track frequencies less than ± 600 Hz with a maximum rate of 50 Hz/sec. The refraction count takes place in the data subsystem. If a loop open and sweeping indicator is received by this subsystem at any time during the count, the count is stopped and the refraction count for this measurement is set to zero. (In the original NAVPAC design the loop not phase-locked indicator was used instead.)

Extensive priority logic is built into the receiver subsystem controller. If one receiver is locked onto and tracking a NAVSAT signal and another receiver encounters the same signal during its normal sweep mode, the sweep stops and the Doppler frequencies are compared. If they are the same (within 25 Hz), the original receiver has priority and continues to track while the other receiver resumes its sweep. Whenever phase-lock is lost for 6 sec, the receiver reverts to the sweep mode to try to reacquire the signal. It now sweeps over a frequency range starting at $f_{LL} - 1.4$ kHz (where f_{LL} is the frequency at which loss of lock occurred) and extends 20 kHz or to 52 kHz whichever is less. This sweep range is chosen because in certain cases positive Doppler rates are possible.

ORBITAL PLANE SEP 180 deg

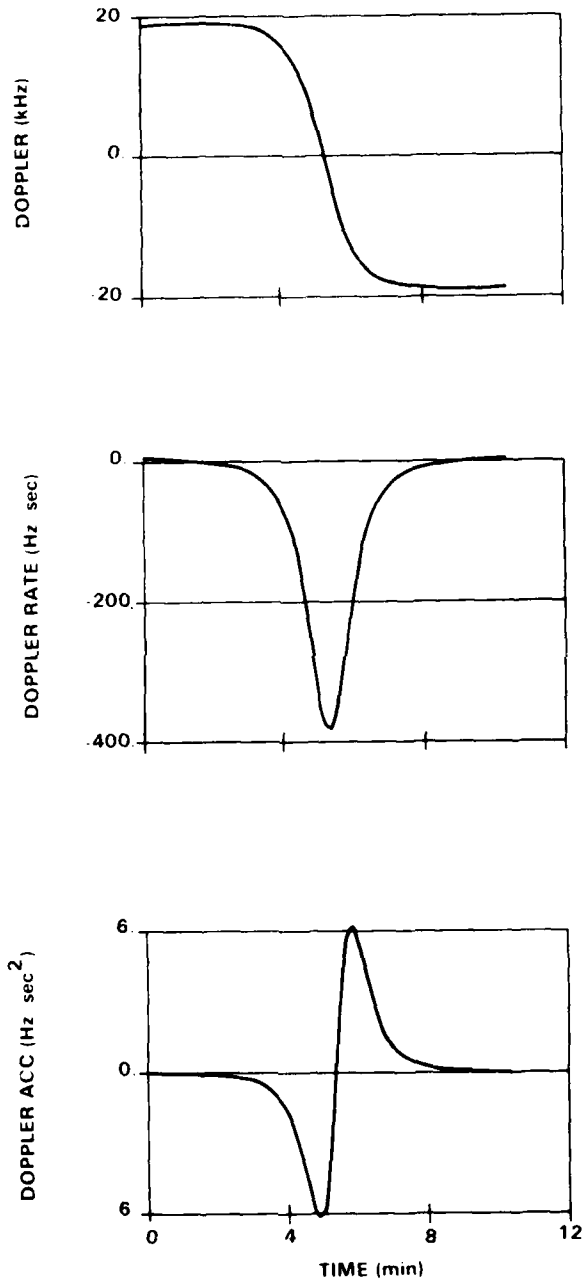


Figure 1. Worst-case Signal Dynamics

Figure 2 shows the dynamics of a hypothetical satellite-to-satellite pass with large positive Doppler rates. This sweep continues until the signal is reacquired or four times at which time the receiver resets its sweep range back to 12 to 32 kHz. It is possible that one of the other two receivers could lock on to this signal before the original receiver reacquires it. In this case the new receiver has priority. Another important case is when two receivers are tracking separate signals and their Doppler curves cross. Several results are possible. The receivers may continue to track their original signals. One receiver may switch to the other's signal in which case the receiver with priority (determined by the order in which they started tracking) continues to track the common signal and the other reverts to the reacquisition sweep mode. It may reacquire the remaining signal present or the third receiver may acquire this signal. Another possible outcome is that the receivers will just switch signals without any loss of lock. In all cases the priority logic circuitry ensures that the same signal is never tracked by more than one receiver.

This logic can be disabled by command allowing all three receivers to track the same satellite. This is called the priority defeat mode. In this mode the Doppler count intervals are not simultaneous. This is because the sweeps are asynchronous and acquisition is independent for each receiver. Commands also exist for turning off individual receivers and disabling the modulation search circuitry.

Timing for the Doppler data is based on a master clock frequency of 833.333 kHz derived from an ultrastable 5-MHz quartz crystal oscillator. This clock frequency corresponds to a time resolution of 1.2 μ sec. The time code generator (TCG) is then just a 42-bit counter that increments every 1.2 μ sec and starts when the data subsystem is commanded on. This counter recycles to 0 approximately every 61 days or when reset by command. Whenever the data subsystem receives a high-channel, phase-locked indicator from a receiver, the Doppler counter starts at the first negative zero crossing. Cycles are counted for exactly 30 NAVPAC sec (25×10^{12} TCG counts). At this time a time over counter (TOC) starts counting at 1.2 μ sec steps until the next negative zero crossing occurs. Also at this time the TCG is read out and put in the data stream along with a receiver identification (ID) and is identified as a Doppler time mark (DTM). (The TCG is not read out at the beginning of the count just in case the count is not completed.) The Doppler count register now contains one less than the total number of Doppler counts collected in an interval of $30 + \text{TOC} \times 1.2 \times 10^{-6}$ NAVPAC seconds. A Doppler word (DW) is then placed in the data stream containing the Doppler count, time over count, and receiver ID. At the end of the time over count, the Doppler counter restarts at zero and this sequence is continued as long as phase-lock is maintained. Doppler time marks are placed in the data stream every 8 Doppler words as long as the receiver has not lost lock.

In parallel with the Doppler count is a refraction count that terminates at the end of the 30-sec portion of the Doppler count. (An error of 1 refraction cycle corresponds to an error in the range difference measurement of approximately 8 cm.) The Doppler word mentioned above also contains the refraction count.

All of the Doppler timing is an internal NAVPAC function based on the output of the local oscillator driving the TCG. The exact start time of the master clock and the exact frequency at any instant are not known. To allow for conversion from NAVPAC time to absolute time (UTC) in software, NAVPAC contains a navigation message recovery (NMR) unit. This unit demodulates the signal to get the navigation message on the 400-MHz channel and decodes the two-minute time mark (TMTM) and satellite ID. These NAVSAT TMTMs are kept synchronized with UTC very accurately. The NMR unit cycles among the receivers until a Doppler lock indication is present.

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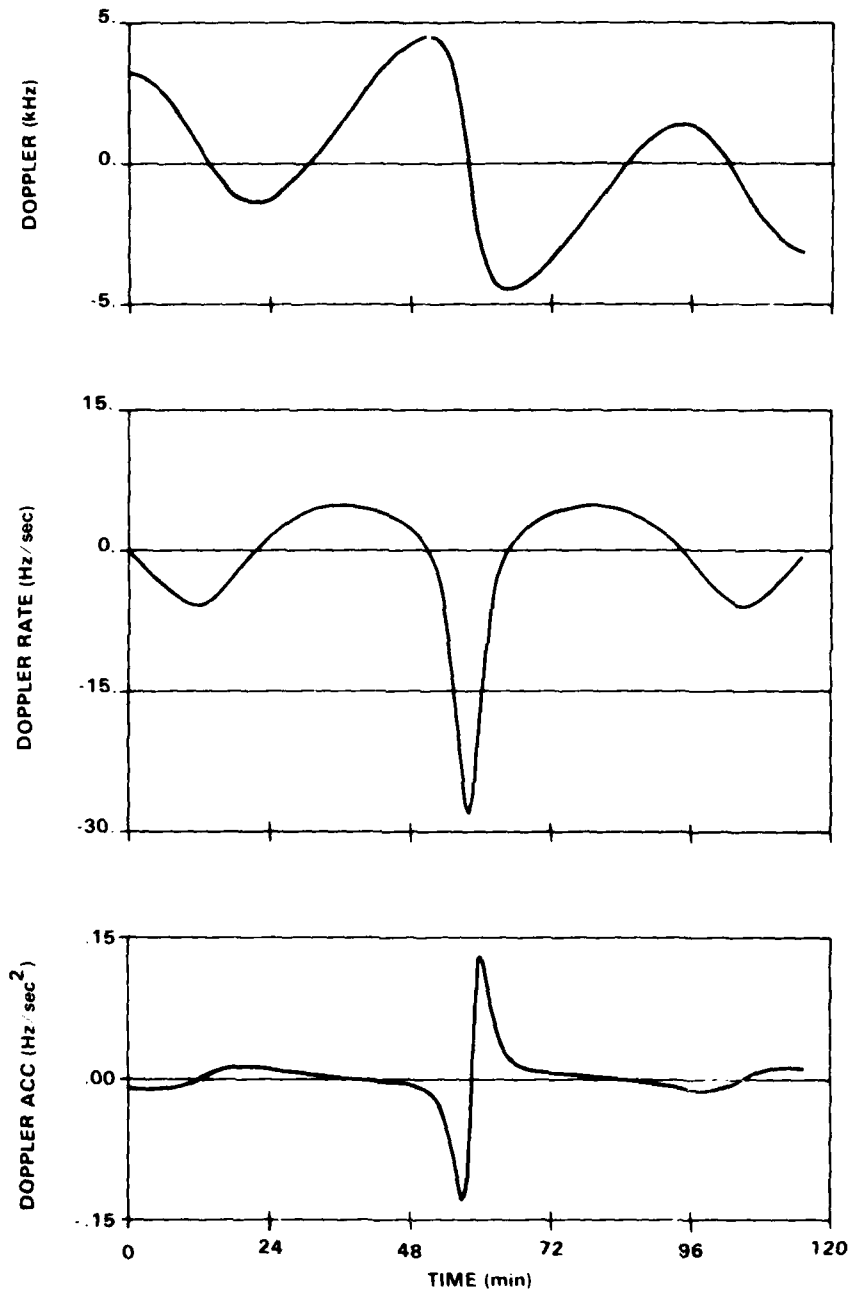


Figure 2. Signal Dynamics for Pass with Large Positive Doppler Rates

It then decodes the signal in order to recover the TMTM and satellite ID. If 4 min have elapsed without a successful recovery, the NMR cycles to the next locked receiver. When a TMTM is decoded, the least significant 36 bits of the TCG register along with the satellite and receiver IDs are placed in the data stream identified as a TMTM. This portion of the TCG register recycles to 0 approximately every 23 hr.

TMTMs are the only link between a receiver and the satellite it is tracking. It is possible for a TMTM to occur in the data stream before a DTM signifying acquisition if the TMTM occurs during the first 30-sec Doppler count. A TMTM may also occur during a Doppler count that is not completed due to loss of lock. It is also possible for a TMTM from a particular satellite to be obtained by one receiver and actual acquisition of this satellite (defined by the first complete Doppler count) may be on another receiver. If three satellites are being tracked simultaneously, as much as 6 min can elapse between indications that each receiver is still tracking the same satellite.

DTMs, DWs, and TMTMs are placed in the data stream as they are collected regardless of which receiver they come from or which satellites are being tracked. NAVPAC also has the capability to time tag three external events. All of the corresponding data words are 48 bits as described in Figure 3.

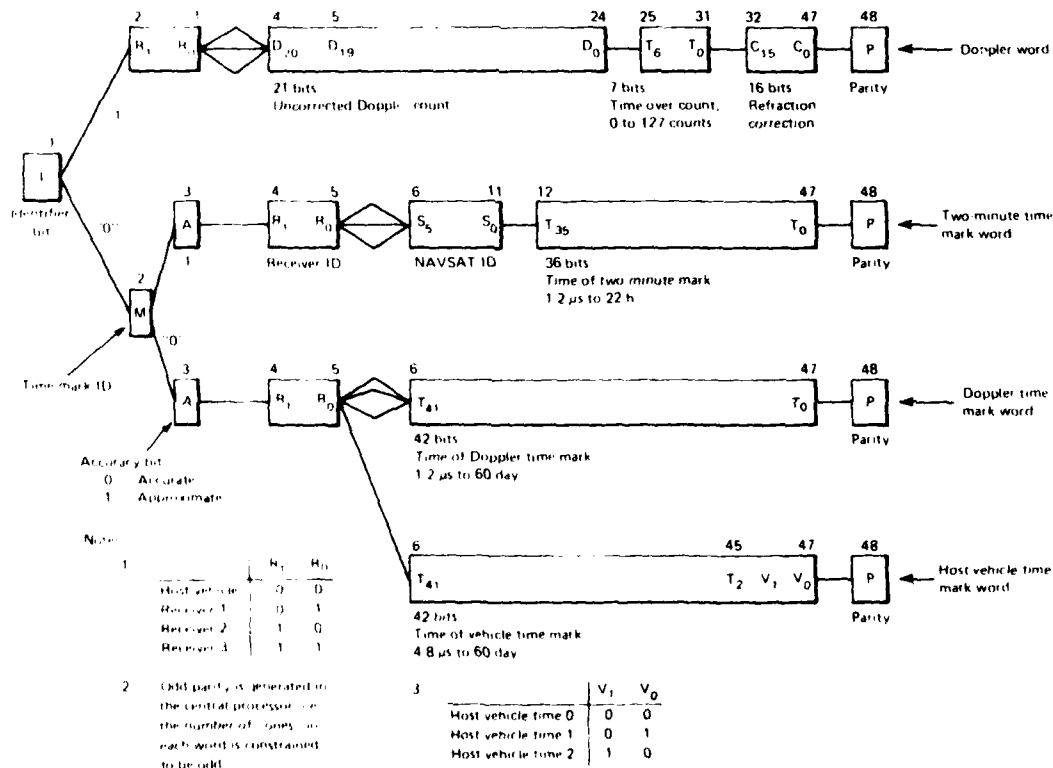


Figure 3. Format of the 48-Bit Data Word²

The NAVPAC data subsystem also collects both analog and discrete telemetry from the receivers. The analog data, which are digitized into 8-bit words, consist of AGC readings for all receivers (high and low channels) and are sampled twice each telemetry frame span of 114.5 sec. The discrete data consist of such quantities as Doppler lock and sweep status indicators for all receivers (high and low channels) and bogie counters for each high channel. These are sampled once every 114.5 sec. NAVPAC also has a troubleshooting telemetry collection mode in which data are sampled at eight times the normal rate.

The above discussion was only intended to give the reader a sufficient understanding of the receiver/data subsystem operation for correctly interpreting the NAVPAC Doppler, refraction, and timing data. Reference 2 contains a complete detailed description of the NAVPAC hardware built by APL/JHU.

DATA PREPROCESSING (NAVPAC)

The primary purpose of preprocessing the raw NAVPAC Doppler data (using the NAVPAC computer program) is to convert it into a form usable by the QSOLVE data quality evaluation program and by the CELFST orbit computation program. In addition, information about the NAVPAC clock is generated for use in correctly time-tagging the telemetry and accelerometer data. Preprocessing involves four passes through the data to accomplish the following:

1. Calibrating the clock
2. Sorting data into passes
3. Time-tagging and converting counts to range differences
4. Reordering passes by NAVSAT

The raw data are input as a stream of 48-bit words as defined in the previous section. Two modes of preprocessing are possible: "Quick Look" and "Normal." These are identical except that the data for only a subset of the possible NAVSATs are processed in the "Quick Look" mode.

CALIBRATING THE CLOCK

As mentioned in the receiver/data subsystem overview, the TMTMs relate NAVPAC time (contents of a counter incrementing every 1.2 NAVPAC μ sec) to Coordinated Universal Time (UTC). The input NAVPAC clock reset time is only an approximate UTC time at which the data subsystem on command was executed. This uncertainty along with the fact that the time increment is not exactly 1.2 UTC μ sec and varies continuously imply that some type of adjustment needs to be made to the NAVPAC times (DTMs, host vehicle time marks (HVTMs), and telemetry and accelerometer time marks) based on the received TMTMs.

To ensure that the displayed times for all the printed raw Doppler data can later be compared to calibrated times for analysis purposes, the reset time is recomputed based on the first TMTM for

which trajectory information is available. The following definitions are based on the convention that τ represents a relative time and t an absolute time:

$$\begin{aligned} \tau_{\text{TMTM}} &= \text{NAV PAC time of TMTM in counts} \times 1.2 \times 10^{-6} \text{ sec} \\ \Delta t_{\text{TT}} &= \text{transmission time} = \frac{\rho(t_{\text{TMTM}})}{c} = \frac{|r_{\text{NS}} - r_{\text{HS}}|}{c} \\ \Delta t_{\text{DD}} &= \text{decode delay} \\ \Delta t_{\text{RD}_i} &= i^{\text{th}} \text{ receiver delay} \\ t_{\text{R}} &= \text{input reset time} \end{aligned}$$

$t_{\text{R}} + \tau_{\text{TMTM}}$ is first rounded to the nearest even minute mark, t_{TMTM} . The approximate NAVPAC time at the time of emission of the TMTM by the satellite, τ'_{TMTM} , is then given by

$$\tau'_{\text{TMTM}} = \tau_{\text{TMTM}} - \Delta t_{\text{TT}} - \Delta t_{\text{DD}} - \Delta t_{\text{RD}_i}$$

The recomputed reset time is then given by

$$t_{\text{R}_{\text{recomputed}}} = t_{\text{TMTM}} - \tau'_{\text{TMTM}}$$

In addition, the next two TMTMs are compared against the first to verify that the first was *not* grossly in error. This recomputed reset time is used in computing all times displayed in the raw data printout.

In the first pass through the data, quantities for each TMTM needed for the clock calibration are computed as given later in this section. Also if the NAVPAC clock recycles or has recycled, the DTMs and HVTMs are adjusted accordingly where the increment is integer multiples of $2^{42} = 4398046511104$ (approximately 61 days 2 hr). Since only 36 bits of the 42-bit time word are present in the TMTMs, each TMTM count is compared against the previous DTM and adjusted in increments of $2^{36} = 68719476736$ (approximately 23 hr) until larger. All TMTMs, DTMs, and HVTMs are given an associated day, hour, and minute label and every data word (including Doppler words) is given a time tag in seconds from the beginning of the day in the raw data printout. The DTM times printed have 30 sec subtracted from them to indicate the start time of the following Doppler count. For this reason TMTMs and DTMs may appear to be out of order in the printout. The time tag associated with each DW is the end time of that particular count to the nearest second. The begin time of a particular count is either the end time of the last count (on the same receiver) or the time of the preceding DTM. The refraction count present in each DW has had $2^{15} = 32768$ subtracted from its raw value to get the true refraction count (which may be positive or negative).

To relate the satellite ID in the TMTM data word to the NSWC satellite numbers a table look-up procedure is used. Table 1 gives the correspondence between TMTM IDs, NSWC satellite numbers, and API satellite numbers for all current NAVSATS. The printed IDs are the NSWC satellite numbers unless no match is found in which case the original recovered ID is printed and

the TMTM is deleted from any further processing. The total number of bad IDs is printed after the pass summary table to be described later.

Table 1. Satellite Numbers

TMTM ID	NSWC	APL
36	58	30120
40	59	30130
56	60	30140
28	68	30190
16	77	30200
52	64	30180
62	86	30460 (TIP II)
63	90	30470 (TIP III)
32	93	30110 (TRANSAT)

After identifying the satellite from which the TMTM was received, $t_{R_{recomputed}} + \tau_{TMTM}$ is rounded to the nearest even minute mark, t_{TMTM} . Next the time of transmission, Δt_{TT} , is computed as

$$\Delta t_{TT} = \frac{\rho(t_{TMTM})}{c} = \frac{|r_{NS} - r_{HS}|}{c}$$

where r_{NS} = NAVSAT position at t_{TMTM} } t_{TMTM} is assumed to be a trajectory
 r_{HS} = host satellite position at t_{TMTM} } timeline for each satellite

Then the approximate UTC time, t'_{TMTM} , at which the TMTM was received by NAVPAC is given by:

$$t'_{TMTM} = t_{TMTM} + \Delta t_{TT} + \Delta t_{DD} + \Delta t_{RD_i}$$

where Δt_{DD} = decode delay = .007373 sec (the same for each receiver)

and Δt_{RD_i} = i^{th} receiver delay (different for each receiver) $i = 1, 2, \text{ or } 3$

This is approximate because none of the three terms added to t_{TMTM} is known exactly, e.g., Δt_{RD_i} is actually a function of signal strength. The above computation is repeated for each TMTM for which Δt_{TT} can be computed resulting in two values for each time mark $-\tau_{TMTM}$ and t'_{TMTM} . This correspondence between NAVPAC and UTC times is also in error because the NAVSAT clocks are not synchronized exactly with UTC or each other.

A polynomial equation is used to convert from a NAVPAC time, τ , to a UTC time, t , as follows:

$$t = t_{R_{recomputed}} + A + (1 + B)\tau + C\tau^2 \quad (1)$$

This can be rewritten as:

$$t - t_{R_{recomputed}} - \tau = A + B\tau + C\tau^2$$

For each TMTM, an approximate value for t is computed giving an equation of the form:

$$t'_{TMTM_j} - t_{R_{recomputed}} - \tau_{TMTM_j} = A + B\tau_{TMTM_j} + C\tau_{TMTM_j}^2 \quad j = 1, 2, \dots, N_{TMTM}$$

This is an overdetermined system of linear equations in the unknowns A , B , and C and can be solved using least-squares techniques without a priori information. The normal equations are given by:

$$\begin{pmatrix} N_{TMTM} & \sum_j \tau_{TMTM_j} & \sum_j \tau_{TMTM_j}^2 \\ \sum_j \tau_{TMTM_j} & \sum_j \tau_{TMTM_j}^2 & \sum_j \tau_{TMTM_j}^3 \\ \sum_j \tau_{TMTM_j}^2 & \sum_j \tau_{TMTM_j}^3 & \sum_j \tau_{TMTM_j}^4 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} \sum_j t''_{TMTM_j} \\ \sum_j \tau_{TMTM_j} t''_{TMTM_j} \\ \sum_j \tau_{TMTM_j}^2 t''_{TMTM_j} \end{pmatrix}$$

where $t''_{TMTM_j} = t'_{TMTM_j} - t_{R_{recomputed}} - \tau_{TMTM_j}$

and all sums are from $j = 1$ to N_{TMTM}

After these equations are solved for A , B , and C , the individual residuals of fit, ϵ_j , are computed where

$$\epsilon_j = (t'_{TMTM_j} - t_{R_{recomputed}} - \tau_{TMTM_j}) - (A + B\tau_{TMTM_j} + C\tau_{TMTM_j}^2)$$

The root mean square (RMS) of these residuals is then computed

$$\epsilon_{RMS} = \left(\frac{\sum_j \epsilon_j^2}{N_{TMTM}} \right)^{1/2}$$

and all TMTM for which $|\epsilon_j| > \sigma_{tol} \epsilon_{RMS}$ are deleted and the least-squares solution repeated. The above iterative procedure is done until no more TMTMs are deleted or until the maximum number of iterations is reached. For each iteration, the NAVPAC time count and the corresponding residual for all deleted points are printed. For the last iteration, the calibrated TMTMs, a correction term, the t''_{TMTM_j} terms, and the final residuals are printed along with the original and final number of TMTMs, the number of iterations, the RMS of the residuals (in μsec), the re-computed reset time, and the A , B , and C values. The correction term mentioned above is given by $B\tau_{TMTM_j} + C\tau_{TMTM_j}^2$. This is the clock correction due only to the oscillator frequency offset (related to B) and frequency drift (related to C).

Equation (1) with $t_{R_{recomputed}}$, A , B , and C as determined above is then used to convert any NAVPAC time, τ , to its equivalent UTC time, t . After the clock calibration but still in this first

pass through the data, the HVTMs, which have already been separated by type, are calibrated and the appropriate delays (different for each type) subtracted out.

SORTING DATA INTO PASSES

Because of simultaneous tracking of up to three NAVSATS, the possible receiver switching that can occur, and the loss of lock and reacquisition possibilities, sorting of the Doppler data into passes is a complicated procedure. The key data words in the sorting are the TMTMs, since they provide the only connection between receiver number and satellite ID. All data for legitimate satellite IDs are sorted. The following description gives a set of rules for assigning data to passes based on DTMs and TMTMs.

If a DTM occurs more than 10 min after the last DTM from any receiver, it is assumed that all previously active passes have ended and a possible new pass has begun. Four DWs are required before a new pass is declared active. If a DTM occurs on a receiver that has been inactive, the assumption is made that this indicates the start of a new pass. If the DTM is the second DTM from a receiver without an intervening TMTM, a pass has started but the satellite has not been identified. If a DTM and a TMTM have already been found on this receiver, there are two cases to consider: 1) If the time since the last DTM is exactly 4 NAVPAC minutes + $\frac{8}{15}$ TOC, all DWs between these DTMs are assigned to the current pass. 2) If this time difference is not exact, future TMTMs are checked to see if a) the receiver lost the signal but reacquired it later in which case the pass is continued or b) a different satellite ID is found in which case the DTM defines the beginning of a new pass. Also in case b) the previous pass is not terminated because the possibility still exists that this satellite may be reacquired by another receiver.

When a TMTM is being processed, its receiver number and satellite ID are compared against previous TMTMs until at least a partial match is found or 10 min have elapsed. Four outcomes are possible:

1. Receiver and satellite match — This indicates that between the two TMTMs in question all the data from this receiver are from the same satellite. This is normally the case.
2. Receiver and satellite do not match — This gives no information on the present receiver so the next TMTM back is checked. If no match is found, it is assumed that the current TMTM indicates the beginning of a new pass.
3. Receiver matches, satellites do not — This indicates that the receiver was previously tracking another satellite. The present TMTM then may indicate the beginning of a new pass or a receiver switch. The satellite that was previously tracked is searched for later on another receiver.
4. Satellite matches, receivers do not — This also indicates a switch. The pass from the previous receiver must be kept active in case its satellite ID reappears later.

When a satellite switch occurs without the involved receiver losing lock, an ambiguity exists as to during exactly which Doppler count the switch took place. It is assumed that all DWs since the last TMTM come from the most recent satellite tracked by this receiver.

For data collected in the priority defeat mode, the sorting is simplified. All tests for switching are deleted, each receiver is treated independently, and the only requirement is to determine when a particular pass ends and the next one begins.

The main result of this pass through the raw SSI Doppler data is the assignment of pass numbers and the corresponding satellite ID to every DTM and DW.

TIME TAGGING AND CONVERTING COUNTS TO RANGE DIFFERENCES

The data are now reprocessed in order to assign calibrated times to each observation and to convert from uncorrected Doppler counts at 400 MHz and refraction counts to first-order refraction corrected range difference data with a nominal frequency bias removed. In addition, refraction counts are examined for zero or large values in which case the data are tagged.

Each pass begins with a DTM. Since the time tag associated with a range difference observation is the end time of the corresponding Doppler count, a dummy observation must be included at the beginning of each pass which specifies the start time of the first count. This dummy observation consists of setting the observed value to 0, and the observation sigma to 1000. The UTC time, t , corresponding to a DTM is given by

$$t_{DTM} = (\tau_{DTM} - 30.)_{cal} \quad \text{This notation } ()_{cal} \text{ means substitution of the NAVPAC time within } () \text{ into the time calibration Equation (1).}$$

The UTC time associated with the first observation immediately following the DTM is given by

$$t_{obs_1} = (\tau_{DTM} + TOC_1)_{cal}$$

Therefore, the UTC time associated with the i^{th} observation from this same receiver given the NAVPAC time associated with the $i - 1^{st}$ observation is given by

$$t_{obs_i} = (\tau_{obs_{i-1}} + 30. + TOC_i)_{cal} \quad (2)$$

where $\tau_{obs_i} = \tau_{DTM} + (i - 1) \times 30. + TOC_1 + \dots + TOC_{i-1}$

In all these expressions, the TOC is actually the TOC from the DW multiplied by 1.2×10^{-6} sec. This procedure continues until the next DTM is encountered for the current pass. If it is from the same receiver, 8 DWs from this receiver have been processed since the last DTM, and

$$\tau_{DTM_k} = \tau_{DTM_{k-1}} + 240. + \sum_{i=1}^8 TOC_i$$

then no loss of lock has occurred and therefore timing is continued based on Equation (2). In all other cases, timing is restarted based on this new DTM with a dummy observation. Time tagging for up to three passes can be taking place concurrently. The only timing ambiguity exists when a pass switches from one receiver to another without a loss of lock (no DTM). In this case, the observation times must be based on the next available DTM from this receiver by subtracting $30. + TOC_1$ from each uncalibrated observation time starting with the first observation after the DTM and then

calibrating the result. All UTC observation times are converted to a day number and seconds from the beginning of this day after calibration.

For each Doppler word, the Doppler count (DC_i) and refraction count (RC_i) are combined as follows to get the range difference observation

$$RD_i = \frac{c}{f + f_{\text{offset}}} \left[(DC_i - \frac{6}{55} RC_i) + f_{\text{offset}} (30 + TOC_i) \right]$$

where DC_i = Doppler count at 400 MHz (raw Doppler count + 1 if $TOC_i > 0$.)

RC_i = refraction count (raw refraction count - 32768)

$f_{\text{offset}} = \begin{cases} 32000 \text{ Hz (} 80 \text{ ppm at 400 MHz) for satellites 58, 59, 60, 64, 68, and 77} \\ 33792 \text{ Hz (} 84.48 \text{ ppm at 400 MHz) for all other satellites} \end{cases}$

$30 + TOC_i$ = Doppler count interval in NAVPAC seconds

c = speed of light (in km sec)

f = nominal NAVPAC frequency = 400×10^6 Hz

Each observation is assigned an observation sigma of $(30 + TOC_i)^{-1/2}$. However, if $RC_i = 0$ or $RC_i > 2000$, the sigma is set to 1000. Also, if $|RD_i| > 500$ km, the observation value is set to 0.

After all the DIMs and DWs for a given pass are processed, the pass is assigned a type number based on the angular separation of the host satellite and NAVSAT orbit planes. Table 2 defines the type numbers as a function of the approximate right ascension of the ascending node difference. Pass types 2 and 3 are usually short (less than 1/3 of a rev) and have a unique time of closest approach (TCA) with range differences negative before and positive after this time. An approximate TCA is defined as the first observation time for which the observation changes sign. If only a portion of the actual satellite pass is present with all observations of the same sign, then the approximate TCA is defined as the last observation time, if all observations are negative and the first observation time, if all observations are positive.

Table 2.

Type Number	Magnitude of Right Ascension Difference ($^{\circ}$)
1	0 to 45
2	45 to 135
3	135 to 180

Type 1 passes are much longer than the other two types (up to 2 hr) and may have more than one real TCA. These passes are usually segmented and each segment treated as a separate pass in further processing. An example of the Doppler curve for a hypothetical pass of this type was given in Figure 2. A plot of the range difference values, which are actually used in the segmenting

procedure, would have a similar shape but with the signs reversed. Note that there are long periods when the Doppler shift is increasing (range differences are decreasing). The segmenting starts by searching the range difference values for a change in sign or 35 min whichever comes first. A change in sign from - to + is a normal TCA. A change from + to - is also called a TCA but is actually a time of furthest approach (zero Doppler shift but a local maximum instead a local minimum range). If no TCA is found, the pass is segmented at this time and a type number 6 assigned to the pass along with a TCA defined by $-(t_{\text{last obs.}} - t_{\text{first obs.}})^2$. The negative sign attached to this value indicates that it is not a true TCA. If a TCA is found, the next local minimum or maximum or a time span of 35 min defines the end of this segment. If the TCA corresponds to a - to + sign change, type number 4 is assigned to the pass. If it corresponds to a + to - sign change, type number 5 is assigned. If a pass segment as defined above is less than 10 min long, the segment is extended to the next local extremum or another 25 min. If the last observation in the pass is reached and the current segment is less than 10 min long, this segment is combined with the previous segment. Finally, if the last observation is reached without any segmenting being done, the pass type is left at 1 and again $-(t_{\text{last obs.}} - t_{\text{first obs.}})^2$ is assigned as the TCA. All of the above results in pass segments less than 45 min in duration. If the segmenting takes place during an observation span where no loss of lock occurs, the first observation of the new segment must be a dummy observation with the observation time set equal to that of the last observation in the previous segment.

Various quantities designed to provide pertinent information about each NAVPAC pass are computed and entered in a pass summary table. A subset of these quantities are also totalled over all passes. Elevation and azimuth angles relative to the host satellite at rise, TCA, and set are computed for each pass after a more exact TCA has been determined. TCA is refined by iterating the following formula

$$TCA_{j+1} = TCA_j - (\bar{\rho}^T \dot{\bar{\rho}})(TCA_j) / (\bar{\rho}^T \dot{\bar{\rho}} + \bar{\rho}^T \ddot{\bar{\rho}})(TCA_j)$$

until $|TCA_{j+1} - TCA_j| < .05$ sec

where TCA_0 = assigned TCA value (as defined above)

$$\rho = r_{NS} - r_{HS} = \text{range vector at } TCA_j$$

$$\dot{\rho} = \dot{r}_{NS} - \dot{r}_{HS} \text{ and } \ddot{\rho} = \ddot{r}_{NS} - \ddot{r}_{HS}$$

$$r_{NS} = \text{NAVSAT inertial position at } TCA_j$$

$$r_{HS} = \text{host satellite inertial position at } TCA_j$$

This iteration also works for type 5 passes. If the assigned TCA is negative (types 1 or 6), no iteration takes place because no real TCA exists for these pass segments. The elevation angle to the NAVSAT is defined as the angle between the range vector, ρ , and a plane perpendicular to the vector from the center of the earth to the host satellite, r_{HS} . A positive elevation angle means the NAVSAT is above this plane.

$$\text{Elevation angle} = 90^\circ - \text{Arccos} \left(\frac{\rho \cdot r_{HS}}{|\rho| |r_{HS}|} \right)$$

The azimuth angle is defined as the angle in the plane perpendicular to r_{HS} , P_{HS} , between the instantaneous orbit plane's intersection with P_{HS} and the projection of the range vector, ρ , on P_{HS} measured clockwise.

$$\text{If } \theta = \text{Arccos} \left(\frac{(r_{HS} \times r_{NS}) \cdot (r_{HS} \times \dot{r}_{HS})}{|r_{HS} \times r_{NS}| |r_{HS} \times \dot{r}_{HS}|} \right) \quad 0. \leq \theta \leq 180^\circ$$

$$\text{then the azimuth angle} = \begin{cases} 360^\circ - \theta & \text{if } \frac{r_{HS} \times \dot{r}_{HS}}{|r_{HS} \times \dot{r}_{HS}|} \cdot r_{NS} > 0. \\ \theta & \text{otherwise} \end{cases}$$

In addition, at TCA an approximate longitude and latitude of the subsatellite point is computed as follows:

$$\text{Define } \Delta T = \text{TCA} - t_{V.E.}$$

$$\psi = \arctan(y_{HS}/x_{HS}) - \text{mod} \left(\frac{\pi}{180} \omega \Delta T, 360^\circ \right)$$

$$x = (x_{HS}^2 + y_{HS}^2)^{1/2} \cos \psi$$

$$y = (x_{HS}^2 + y_{HS}^2)^{1/2} \sin \psi$$

Then the longitude and latitude are given approximately by:

$$\lambda = \arctan(y/x) \quad 0. \leq \lambda < 360^\circ$$

$$\phi = \text{Arctan} \left(\frac{z_{HS}}{(x_{HS}^2 + y_{HS}^2)^{1/2}} \right) \quad -90. \leq \phi \leq 90^\circ$$

where $t_{V.E.}$ = recent time of vernal equinox

$$\omega = \text{Earth's sidereal rotation rate} = 7.292115855 \times 10^{-5} \text{ rad/sec}$$

r_{HS} = host satellite inertial position at TCA

The number of losses of lock and reacquisition for each pass is computed by counting the number of 0. observation values and subtracting 1 for the first dummy observation in the pass and subtracting the number of observations set to 0. because of failure of the range difference absolute tolerance test. These losses of lock are also summed over all passes. The number of receiver switches for each pass and over all passes are also computed. Also for each pass, the number of zero refraction count observations is computed. In addition, the percent of zero refraction count observations for all passes is determined.

This pass through the NAVPAC Doppler data results in a CELEST-formatted time-corrected observation file with the pass ordering based on the sorting procedure. The header record for each pass contains the pass numbers as they appear in the pass summary table (for use in QSOLVE) and the pass type number (for use in QSOLVE and CELEST).

REORDERING PASSES BY NAVSAT

The last pass through the NAVPAC Doppler data sorts the passes by NAVSAT, which automatically time orders the data for each satellite by TCA except for any segmented passes of priority defeat data. In this case, all the pass segments from one receiver appear in sequence followed by the same sequence for each of the other two receivers. The data are sorted by NAVSAT because both QSOLVE and CELEST process all the data from one satellite before going to the next satellite.

PRELIMINARY DATA QUALITY EVALUATION (QSOLVE)

The NAVPAC SST data quality is evaluated within a few days after data collection using the QSOLVE computer program. This program is similar to the CELEST FILTER section but contains features pertinent to more timely data analysis. It requires a CELEST-formatted perturbed trajectory for the host satellite and inertial trajectories for the NAVSATs (up to 6). In addition, the program uses the SST observation file created by the NAVPAC program. Only the data corresponding to the overlapping host and NAVSAT trajectory spans are processed. The data for each NAVSAT are processed a pass at a time sequentially. The results for each pass are summarized at the end of the run and labelled with a pass number used to relate the results back to the NAVPAC program output.

QSOLVE employs a least-squares differential correction technique to remove any signal left in the residuals for a pass (based on preliminary trajectories) and estimates the variance of the data based on linearly adjusted residuals after fit. The assumption is made that the residuals can be expressed as a linear combination of the parameters to be solved for. Points inconsistent with the rest of the points are deleted and another fit done. This iterative procedure continues until either the maximum number of iterations is reached or until two consecutive iterations with no points deleted occur. The parameters that can be adjusted to remove any signal present are frequency bias, frequency drift, and radial, along-track, and cross-track position and velocity of the host satellite at TCA. No parameters related to the NAVSAT trajectory are present in this adjustment.

For each pass (or segment of a pass for long passes that have been split), the evaluation proceeds in the following steps:

1. Compute the TCA for the pass using the iterative procedure given in the Data Preprocessing section. If the initial TCA is negative, no TCA iteration is possible. Compute the elevation angle at TCA given by the formulas in the Data Preprocessing section. Compute the rotation matrix to be

used to transform the equations from the inertial Cartesian reference frame to the RAC (radial, along-track, and cross-track) reference frame at TCA as follows:

$$\text{Let } U = \frac{\bar{r}_{HS}}{|\bar{r}_{HS}|}$$

$$W = \frac{\bar{r}_{HS} \times \dot{\bar{r}}_{HS}}{|\bar{r}_{HS} \times \dot{\bar{r}}_{HS}|}$$

$$V = W \times U$$

where \bar{r}_{HS} and $\dot{\bar{r}}_{HS}$ are the host satellite's inertial position and velocity vectors at TCA

$$\text{then } R_{TCA} = (U \ V \ W)$$

$$3 \times 3$$

$$\text{Also save } \psi(TCA) = \frac{\partial \bar{r}_{HS} \ \dot{\bar{r}}_{HS}(TCA)}{\partial e_{HS}} = 6 \times 6 \text{ state transition matrix}$$

2. For each observation in the pass do the following:

a. Interpolate off of the host satellite trajectory to get $\bar{r}_{HS}(t_{obs_i})$ and

$$\psi_r(t_{obs_i}) = \frac{\partial \bar{r}_{HS}(t_{obs_i})}{\partial e_{HS}} = \text{upper } 3 \times 6 \text{ portion of the state transition matrix.}$$

Interpolate off of the NAVSAT trajectory to get $r_{NS}(t_{obs_i})$. To get r_{NS} at the time the signal left the NAVSAT an iterative procedure is required.

$$\text{Let } \Delta t_{TT}^{(0)} = |r_{NS}(t_0) - \bar{r}_{HS}(t_0)| / c \quad t_0 = t_{obs_i}$$

$$\text{Define } t_1 = t_0 - \Delta t_{TT}^{(0)}$$

Compute a new transmission time estimate

$$\Delta t_{TT}^{(1)} = |r_{NS}(t_1) - \bar{r}_{HS}(t_0)| / c$$

If $|\Delta t_{TT}^{(1)} - \Delta t_{TT}^{(0)}| > 10^{-6}$ sec, continue this procedure by letting $t_{j+1} = t_0 - \Delta t_{TT}^{(j)}$ and

iterating until $|\Delta t_{TT}^{(j+1)} - \Delta t_{TT}^{(j)}| < 10^{-6}$ sec

Then compute and save:

$$\rho(t_{obs_i}) = r_{NS}(t_{obs_i} - \Delta t_{TT}^{(j)}) - \bar{r}_{HS}(t_{obs_i})$$

$$\text{and } \rho = |\rho|$$

b. Test to see if the geometric line of sight between the host satellite and NAVSAT passes within h_{min} kilometer of a spherical earth's surface (SST observability test). This is done to eliminate all observations containing a significant tropospheric refraction effect.

Let $R_{max} = R_{Earth} + h_{min}$

Define $\theta_1 = \text{Arccos} \left(\frac{R_{max}}{|r_{HS}|} \right)$
 $\theta_2 = \text{Arccos} \left(\frac{R_{max}}{|r_{NS}|} \right)$

} Quantities in parentheses are set = 1, if > 1, to handle the case when $h_{min} >$ the altitude of the satellite

$\theta = \theta_1 + \theta_2 =$ maximum possible central angle for viewing (see Figure 4)

and $\phi = \text{Arccos} \left(\frac{r_{HS} \cdot r_{NS}}{|r_{HS}| |r_{NS}|} \right)$

If $\phi > \theta$, the observation is deleted from further processing.

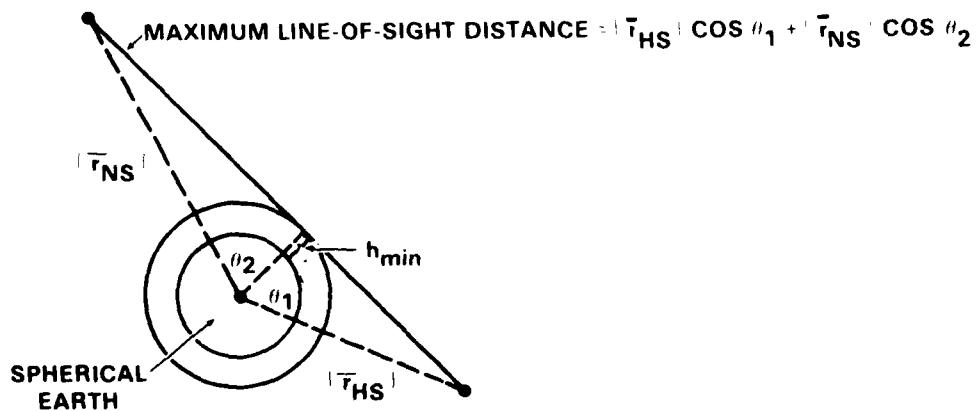


Figure 4. SST Observability Geometry

c. Assuming that the relevant quantities for the observation at t_{obs_i} have been saved:

1) Obtain the computed range difference value:

$$C_{RD_i} = \rho(t_{obs_i}) - \rho(t_{obs_{i-1}})$$

2) Compute the required partial derivatives of ρ at t_{obs_i} :

$$\frac{\partial \rho(t_{obs_i})}{\partial f_b} = - \frac{c}{f_o} (t_{obs_i} - TCA)$$

c = speed of light in km/sec

$$f_o = 10^6$$

$$\frac{\partial \rho(t_{obs_i})}{\partial f_d} = - \frac{c}{2f_o} (t_{obs_i} - TCA)^2 \times 86400$$

f_b in ppm

f_d in ppm/day

$$\frac{\partial \rho(t_{obs_i})}{\partial r \dot{r}(TCA)} = \frac{\partial \rho(t_{obs_i})}{\partial r_{HS}(t_{obs_i})} \psi_r(t_{obs_i}) \psi^{-1}(TCA)$$

where

$$\frac{\partial \rho(t_{obs_i})}{\partial r_{HS}(t_{obs_i})} = - \frac{\rho(t_{obs_i})}{|\rho(t_{obs_i})|}$$

$$\psi_r(t_{obs_i}) = \frac{\partial r_{HS}(t_{obs_i})}{\partial e_{HS}}$$

3) Form the A_i matrix and $O - C_i$ for this observation, as follows:

$$A_i = \begin{pmatrix} \frac{\partial D_i}{\partial f_b} & \frac{\partial D_i}{\partial f_d} & \frac{\partial D_i}{\partial r \dot{r}(TCA)} \\ 1 \times 8 \end{pmatrix} \quad \text{and} \quad O - C_i = \text{Observed value}_i - C_{RD_i}$$

where

$$\frac{\partial D_i}{\partial f_b} = \frac{\partial \rho(t_{obs_i})}{\partial f_b} - \frac{\partial \rho(t_{obs_{i-1}})}{\partial f_b}$$

$$\frac{\partial D_i}{\partial f_d} = \frac{\partial \rho(t_{obs_i})}{\partial f_d} - \frac{\partial \rho(t_{obs_{i-1}})}{\partial f_d}$$

$$\frac{\partial D_i}{\partial r \dot{r}(TCA)} = \frac{\partial \rho(t_{obs_i})}{\partial r \dot{r}(TCA)} - \frac{\partial \rho(t_{obs_{i-1}})}{\partial r \dot{r}(TCA)}$$

4) Compute the weighted sum of squares of the original O - Cs:

$$\text{weighted SOS} = \sum_i \frac{(O - C)_{org_i}^2}{\sigma_i^2}$$

3. If the number of observations remaining is < 8 , processing for this pass is terminated. Return to step 1. for the next pass.

4. Initialize the following quantities:

$$\text{Sigma multiplier} = \sigma_{mult.} = 1. \quad \text{Sigma tolerance} = \sigma_{tol.} = 10000. \quad N_S = 0$$

5. Do absolute and sigma tolerance testing on each point. (On the first pass through this computation, the original O - Cs are tested. Also those observations with $\sigma_i = 1000$. are deleted from further processing)

If $|O - C|_{adj_i} > \text{absolute tolerance}$, delete the point from further processing

If $|O - C|_{adj_i} > \sigma_{tol.} \sigma_{mult.} \sigma_i$, delete the point

where σ_i is the original observation sigma on the input SST observation file

6. Again if the number of observations remaining is < 8 , processing for this pass is terminated. Return to step 1. for the next pass.

7. Compute the estimated variance for the remaining pass data:

$$\text{Estimated variance} = \frac{\sum_{i=1}^{N_R} (O - C)_{adj_i}^2}{N_R + N_A - N_S}$$

where N_R = no. of points remaining
 N_A = no. of parameters with a priori sigmas
 N_S = no. of parameters solved for

8. If zero points were deleted on this iteration and also on the previous iteration, go to step 16. Also increment the iteration number, and, if greater than the maximum, go to step 16. Otherwise continue with the next step.

9. Recompute the sigma tolerance

$$\sigma_{tol.} = \left(2. + \frac{N_o}{N_o + 50.} \right) \frac{N_o}{N_R}$$

where N_o = no. of observations present after the observability test was completed.

10. Compute the pass normal equations for the remaining observations using their most recently estimated variances as follows:

$$B\Delta p = F$$

where $B = A^T W A$ $A = \begin{pmatrix} A_1 \\ \vdots \\ A_{N_R} \end{pmatrix}$ $O - C = \begin{pmatrix} O - C_1 \\ \vdots \\ O - C_{N_R} \end{pmatrix}$

and $F = A^T W (O - C)_{org.}$

$$W = \text{diag} \left(\frac{1}{(\sigma_i \sigma_{mult})^2} \right) \quad i = 1, 2, \dots, N_R$$

11. Transform these normal equations so that the position and velocity adjustments are in the RAC coordinate system instead of the ECI coordinate system at TCA:

Define $R = \begin{pmatrix} 1 & 0 & & \\ 0 & 1 & & \\ & & O & \\ & & R_{TCA} & \\ & & O & R_{TCA} \end{pmatrix}$

$$B_R \Delta p_R = F_R$$

where $B_R = R^T B R$

and $F_R = R^T F$

12. Add the a priori parameter information into B_R : i.e., add $1/\sigma_p^2$ to each diagonal for which $\sigma_p \neq 0$. Do a singular solution of the resulting normal equations using the PASMAT routine from CHEST (see Reference 3) to get Δp_R and B_R^{-1} . Δp_R will have zero rows and B_R^{-1} will have zero rows and columns corresponding to parameters not solved for.

Compute $\Delta p = R \Delta p_R$

and $B^{-1} = R B_R^{-1} R^T$

13. Compute the weighted sum of squares for the points used in the above solution

$$\text{weighted SOS} = \sum_{i=1}^{N_R} \frac{(O - C)_{\text{org}_i}^2}{(\sigma_i \sigma_{\text{mult}})^2}$$

and a linear adjustment to this quantity $F^T \Delta p$. Then, define the predicted signal to noise

$$S/N_{\text{Pred.}} = \left(\frac{\text{weighted SOS} - 1^T \Delta p}{N_R + N_A - N_S} \right)^{1/2}$$

where N_S = no. of parameters solved for in PASMAT

Redefine σ_{mult} , as

$$\sigma_{\text{mult. (new)}} = S/N_{\text{pred.}} \times \sigma_{\text{mult. (old)}}$$

14. Linearly adjust the $O - C$ s

$$(O - C)_{\text{adj.}} = (O - C)_{\text{org.}} - \Lambda \Delta p$$

and compute an estimate of the variance before tolerance testing

$$\text{Estimated variance} = \frac{\sum_{i=1}^{N_R} (O - C)_{\text{adj}_i}^2}{N_R + N_A - N_S}$$

15. Return to step 5. to continue the processing.

16. Processing for the current pass has now been completed. Information for a summary table is saved and step 1. is initiated for the next pass.

Various levels of printout are available during each iteration above. The most useful is a printer plot of the adjusted $O - C$ s in which the horizontal spacing between points is uniform even if the observations are not uniformly spaced. Any signal left in the adjusted $O - C$ s is usually apparent in these plots.

A summary table is given after all passes have been processed. It contains the following information grouped by NAVSAT and ordered by TCA for each NAVSAT:

1. Two pass numbers — the first is the pass number assigned in QSOLVE and the second was assigned in the NAVPAC program.

2. No. of iterations

3. No. of points deleted by the SST observability test
4. No. of points deleted by tolerance testing
5. No. of points remaining in final iteration
6. % of points deleted by tolerance testing
7. Receiver number(s) (contained on the SST observation file)
8. TCA (day, seconds) and elevation angle at TCA
9. Maximum $O - C_{adj}$ and square root of the estimated variance based on the final iteration adjusted $O - C$'s (labelled RMS).
10. Position navigations in the RAC reference frame and their sigmas (from $\overline{\Delta p_R}$ and B_R^{-1}) (both are 0.0, if the parameter was not solved for)
11. Frequency bias solution, f_b , in ppm
12. Frequency drift solution, f_d , in ppm/day
13. NSWC satellite no

ORBIT DETERMINATION (CELEST)

GENERAL

After the NAVPAC SST data have been preprocessed, it is in the proper form for use in CELEST - NSWC's orbit computation program. CELEST employs a classical weighted least-squares differential correction technique to fit satellite initial conditions and force and measurement model parameters to various types of observations. The program consists of four major sections:

1. ORBGEN generates the reference trajectories and dynamic partial derivatives
2. FILTER edits and determines weights for the data and forms pass matrices
3. BSOLVR expands and combines pass matrices to obtain solutions and computes diagnostics
4. COVAR propagates the solutions and their covariances to produce the fitted trajectory

CELEST is unlike most orbit computation programs in that it employs the "pass matrix" concept. Pass matrices are essentially normal equations based on data from each pass separately. The FILTER forms the pass matrices as an integral part of its data editing and weight determination procedure.

In the BSOLVR section, the solution for a given span (either a long or short arc) is based on pass matrices with TCAs in this span only. These so-called canonical pass matrices are updated to the epoch of the fit span and then certain expansions have to be performed to adjust the equations to reflect the actual drag and thrust profiles. Reference 3 contains a complete mathematical description of the CELEST program before it was modified to handle the NAVPAC-related data processing procedures.

The purpose of the rest of this report is to describe the modifications and additions to CELEST that were specifically designed to refine the processing procedures for station tracking data for the host satellite and/or the NAVSATs (single-satellite mode) and to make possible processing of the NAVPAC SST data (multisatellite mode). Modifications that affect the NAVSAT processing were required because the precise NAVSAT trajectories (in the form of hybrid trajectories) are required for the NAVPAC SST data processing procedures.

INTEGRATION (ORBGEN)

All sections of CELEST were modified to accommodate up to eight satellites simultaneously the host satellite and up to seven NAVSATs. ORBGEN has been expanded to integrate up to eight trajectories of the same type (inertial, perturbed, or earth-fixed) in one run done sequentially. If the initial conditions for the NAVSATs are not given at the same time, each set can be updated or backdated to a general epoch defined as the host satellite trajectory epoch. The only restrictions on these integrations are that all NAVSAT integrations must use the same gravity field model and are limited to two drag segments and one thrust. The primary use for this capability is for simulation studies. In practice, all NAVSAT trajectories required for NAVPAC data preprocessing and CELEST processing are determined independently by running CELEST in the single-satellite mode using station tracking data.

Perturbed trajectories (time histories of position and partials of position with respect to initial conditions and force model parameters) are required in CELEST for all satellites for which data are to be processed. Hybrid trajectories (modified perturbed trajectories defined under the COVAR section below) are actually required for the NAVSATs when processing NAVPAC SST data. The epochs of the NAVSAT trajectories must be equal to or earlier than the host satellite trajectory epoch. In the multisatellite mode, if the solution has not converged and reintegration using improved initial conditions from the fit is required, only the host satellite trajectory is reintegrated. Any solved-for corrections to the NAVSAT initial conditions are ignored.

ORBGEN is also used to generate the host satellite perturbed trajectory for use in the NAVPAC program "Quick Look" route and QSOLVE. For this the ARDC 1959 density model was incorporated into CELEST. This model is defined as follows:

For altitudes $76. \leq h < 108$, nmi:

$$\rho = \rho_{76} \left(\frac{76.}{h} \right)^{7.18} \left[\left(\frac{108. - h}{32.} \right) + .85 F_{10.7} \left(\frac{h - 76.}{32.} \right)^{4/3} \right] \\ \times \left[1. + \left(\frac{h - 76.}{1224.} \right) (1. + \cos \psi)^3 \right]$$

where

$$\begin{aligned} \rho &= \text{density in slugs/ft}^3 \\ \rho_{76} &= \text{density at 76 nmi in slugs/ft}^3 \\ h &= \text{satellite's geocentric altitude above its subpoint in nmi} \\ F_{10.7} &= \text{solar flux at 10.7-cm wavelength} \\ \cos \psi &= \frac{1}{|r|} [(x\ell_s + ym_s) \cos \theta + (y\ell_s - xm_s) \sin \theta + zn_s] \end{aligned}$$

where

$$\begin{aligned} r &= \text{satellite position vector} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (|r|^2 = x^2 + y^2 + z^2) \\ \ell_s &= \cos \lambda_s \\ m_s &= \sin \lambda_s \cos I \\ n_s &= \sin \lambda_s \sin I \\ \theta &= 0.55 \text{ rad} \\ \lambda_s &= \left(\frac{\pi d}{182.625} \right) - 1.41 + .0335 \sin \left(\frac{\pi d}{182.625} \right) \\ d &= \text{no. of days elapsed since 31 December 1957 to epoch day (1 yr = 365.25 days)} \\ I &= \text{orbital inclination (in rad)} \end{aligned}$$

For altitudes $108. \leq h < 378$. nmi

$$\rho = \rho_0(h) .85 F_{10.7} \left[1. + .02375 (e^{.0102h} - 1.9) (1.0 + \cos \psi)^3 \right]$$

where $\rho_0(h) = e^{2.302585093(-15.738 - .00368h + 6.363e^{-.0048h})}$

These computations result in densities in slugs/ft³ and need to be converted to kg/km³ by multiplication by 5.15375×10^{11} . This model can be used for determining the drag accelerations to be used in the integration of the equations of motion only. The variational equations were not modified to accommodate this model; i.e., $\partial \rho / \partial r$ is still derived from the standard CELEST density model.

DATA EDITING, WEIGHT DETERMINATION, AND PASS MATRIX FORMATION (FILTER)

General

The purpose of the CELEST FILTER section is to edit and assign weights to the data based on its noise variance and to form pass matrices for use in BSOLVR. For the NAVPAC SST data (Class 4, Type 2), basically the same procedures as defined in Reference 3 for ground station tracking (GT) data are used with modifications made to account for the fact that the data are collected by an orbiting satellite instead of a ground station. These modifications are given below. The FILTER section can process GT data only, SST data only, or both (done sequentially - GT then SST) and generate the appropriate pass matrices. Modifications that affect the GT data processing procedures (for both the NAVSATs and the host satellite) were included as part of the NAVPAC changes. These are described next.

GT-Related Modifications

The primary GT-related modification was adjustment of the processing procedures to account for the fact that the transmitting antenna is displaced from the center of gravity for each satellite. It is assumed that the satellite is oriented with its negative vertical axis through its center of gravity (c.g.) pointed at the center of the earth and no rotation about this axis (yaw motion) is possible. This is an approximation, since the NAVSAT and host satellite stabilization systems keep this axis oriented normal to the ellipsoidal earth using gravity gradient and active control techniques, respectively. The antenna offset from the c.g. must be taken into account when determining the computed value for each observation and its partial derivatives. The reference trajectory used is assumed to be the trajectory of the center of gravity of the satellite.

For range difference data types (Class 9, Types 8, 9, and 5), the inertial position of the transmitting satellite at the time of signal emission is adjusted as follows:

Compute a rotation matrix R_{RAC} by

$$R_{RAC} = \begin{pmatrix} \frac{\mathbf{r}}{|\mathbf{r}|} & \frac{(\mathbf{r} \times \dot{\mathbf{r}}) \times \mathbf{r}}{|\mathbf{r} \times \dot{\mathbf{r}}| |\mathbf{r}|} & \frac{\mathbf{r} \times \dot{\mathbf{r}}}{|\mathbf{r} \times \dot{\mathbf{r}}|} \end{pmatrix} = (U \ V \ W)$$

where $\mathbf{r}, \dot{\mathbf{r}} = \mathbf{r}_{c.g.}, \dot{\mathbf{r}}_{c.g.}$ = satellite's position and velocity vectors at time of signal emission

This defines the assumed body-fixed axes orientation in inertial space.

Then $\mathbf{r}_{ant.} = \mathbf{r}_{c.g.} + R_{RAC} \Delta \mathbf{r}_{ant.}$

where $\Delta \mathbf{r}_{ant.}$ = antenna offsets with respect to the c.g. in the body-fixed radial, along-track, and cross-track (RAC) directions

In addition, the inertial velocity of the transmitting satellite for Doppler frequency data (Class 7, Type 7, NAVSATs only) is adjusted as follows:

$$\dot{\mathbf{r}}_{ant.} = \dot{\mathbf{r}}_{c.g.} + \boldsymbol{\omega} \times R_{RAC} \Delta \mathbf{r}_{ant.}$$

where $\boldsymbol{\omega} = \frac{1}{|\mathbf{r}|} (\mathbf{V} \cdot \dot{\mathbf{r}}) \mathbf{W} + \frac{|\mathbf{r}|}{|\mathbf{r} \times \dot{\mathbf{r}}|} (\mathbf{W} \cdot \ddot{\mathbf{r}}) \mathbf{U} \approx$ RAC frame angular velocity vector

$\ddot{\mathbf{r}}$ = satellite acceleration vector

Another modification involves the use of temperature and pressure values as functions of station height as input to the modified Hopfield tropospheric refraction correction model. If the pressure and relative humidity on the observation file header for a given pass are both zero, the following formulas are used to define the weather parameters:

$$\begin{aligned} \text{Temperature (}^\circ\text{C)} &= 15. - 6.5 h \\ \text{Pressure (millibars)} &= 1013.25e^{-.119913 h} \\ \text{Relative humidity} &= 80. \end{aligned}$$

where h = station height above the reference ellipsoid in km

SST-Related Modifications

The general flow of the NAVPAC SST data processing procedures in the FILTER section is the same as for station tracking data. Data editing and weight determination are an integral part of the pass matrix generation. Since the NAVPAC SST observation file has the data time ordered by NAVSAT, all data for one NAVSAT is processed consecutively. Each SST pass consists of a header with identifying information and observations in the form {time, observation, sigma}.

The TCA iteration for each pass is identical to that described in the Data Preprocessing section of this report as are the computations of the zenith angle (90° -elevation angle) and azimuth angle at TCA. Another computation to be done for TCA is the formation of a rotation matrix, R_{TCA} , required to convert from the inertial reference frame to the navigation reference frame.

$$R_{TCA} = \begin{pmatrix} \hat{r} & (\hat{r} \times \hat{i}) \times \hat{r} & \hat{r} \times \hat{i} \\ |\hat{r}| & |(\hat{r} \times \hat{i}) \times \hat{r}| & |\hat{r} \times \hat{i}| \end{pmatrix}_{3 \times 3}$$

where $\hat{r}, \hat{i} = \hat{r}_{HS}(TCA), \hat{i}_{HS}(TCA)$

Also the two 6×6 state transition matrices

$$\begin{pmatrix} \dot{\psi}_{HS}(TCA) \\ \dot{\psi}_{HS}(TCA) \end{pmatrix} \text{ and } \begin{pmatrix} \dot{\psi}_{NS}(TCA) \\ \dot{\psi}_{NS}(TCA) \end{pmatrix}$$

need to be saved for use in transforming the pass normal equations.

For each observation in the pass, the following items are peculiar to the NAVPAC SST data:

1) No station coordinates or tropospheric refraction correction computations are applicable to the SST data. This also implies that no ABCD (earth-fixed to inertial transformation) matrices are required since all computations involve trajectories that are given in the inertial reference frame.

2) Observations with $\sigma = 1000$, are deleted and counted as deweighted points even though a non-zero observation value may be present. These correspond to zero refraction count observations.

3) The SST observability test as described in the Preliminary Data Quality Evaluation section of this report replaces the zenith angle tolerance test.

4) The time transmission correction, Δt_{TT} , is applied at the transmitter (NAVSAT) as described in the same section mentioned in 3) above.

5) If the host satellite data reduction is being done using the WGS-72 station coordinates, then an adjustment to the NAVSAT's inertial position is required after the time transmission correction is applied. This is necessary because the NAVSAT trajectories are computed using NWL-9Z station

coordinates that differ from the WGS-72 coordinates by 0.26'' of arc in longitude. The adjustment is done as follows:

$$r_{NS, \text{adj.}} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} r_{NS} = R_{NW1-9Z \rightarrow WGS-72} r_{NS}$$

where $\theta \approx 7.222 \times 10^{-5} \text{ deg} = 0.26''$

These adjusted NAVSAT coordinates are then used in all further observation processing.

6) The antenna offsets for each satellite are accounted for in obtaining the computed observation value and partial derivatives of the data with respect to satellite coordinates. For the host satellite, define the R_{HS} rotation matrix by

$$R_{HS} = \begin{pmatrix} r_{HS} & (r_{HS} \times \dot{r}_{HS}) \times r_{HS} & r_{HS} \times \dot{r}_{HS} \\ |r_{HS}| & |(r_{HS} \times \dot{r}_{HS}) \times r_{HS}| & |r_{HS} \times \dot{r}_{HS}| \end{pmatrix}$$

where r_{HS} , \dot{r}_{HS} are host satellite position and velocity from the trajectory (i.e., for the c.g.) at time of signal reception (t_{obs} .)

then $r_{HS, \text{ant.}} = r_{HS} + R_{HS} \Delta r_{HS, \text{ant.}}$

where $r_{HS, \text{ant.}}$ = inertial position of the HS receiving antenna at t_{obs} .

$\Delta r_{HS, \text{ant.}}$ = antenna offsets wrt the c.g. in body-fixed radial, along-track, and cross-track directions

For the NAVSAT, define the R_{NS} rotation matrix by the same formula with r_{NS} and \dot{r}_{NS} replacing r_{HS} and \dot{r}_{HS} where these are defined at the time of signal emission ($t_{\text{obs}} - \Delta t_{TT}$)

$$r_{NS, \text{ant.}} = r_{NS, \text{adj.}} + R_{NS} \Delta r_{NS, \text{ant.}}$$

where $r_{NS, \text{ant.}}$ = inertial position of the NS transmitting antenna at $t_{\text{obs}} - \Delta t_{TT}$

$\Delta r_{NS, \text{ant.}}$ = antenna offsets wrt the c.g. in body-fixed radial, along-track, and cross-track directions.

The radial NAVSAT antenna offset can also be used to adjust the NAVSAT's position to account for the possible difference between the central force term μ_{NS} used in the NAVSAT orbit fit and μ_{HS} used in the host satellite orbit fit.

This offset is given by

$$\Delta r_{NS, \text{radial}} = \frac{a_{NS}}{3} \frac{\Delta \mu}{\mu_{NS}}$$

where a_{NS} = semimajor axis of NAVSAT orbit in km

$$\Delta \mu = \mu_{HS} - \mu_{NS} \quad \text{in km}^3/\text{sec}^2$$

7) The observation equation for the NAVPAC SST range difference (RD) data is given as:

$$RD = \left[\rho + \frac{c}{f_o} f_b (t - TCA) \right]_{t_{\text{obs}} - \Delta t}^{t_{\text{obs}}}$$

where $\rho = |r_{NS, \text{ant.}}(t - \Delta t_{TT}) - r_{HS, \text{ant.}}(t)|$ in km

$$f_o = 10^6 \quad (\text{gives } f_b \text{ units of ppm})$$

$$f_b = \text{frequency bias (nominally zero)}$$

Δt = range difference time interval in seconds ($t_{\text{obs}} - \Delta t$ is the observation time of the previous observation)

The required partial derivatives for ρ are given as:

$$\frac{\partial \rho}{\partial r_{HS}} = -\hat{\rho}^T \quad \frac{\partial \rho}{\partial r_{NS}} = \hat{\rho}^T$$

$$\frac{\partial \rho}{\partial p_k} = -\hat{\rho}^T \frac{\partial r_{HS}}{\partial p_k} \quad p_k = \text{HS orbit or force model parameter}$$

$$\frac{\partial \rho}{\partial q_k} = \hat{\rho}^T \frac{\partial r_{NS}}{\partial q_k} \quad q_k = \text{NS orbit or force model parameter}$$

where

$$\frac{\partial r_{HS}}{\partial p_k} \quad \text{and} \quad \frac{\partial r_{NS}}{\partial q_k}$$

are obtained from the appropriate perturbed trajectory at the specified times. The HS and NS epochs may be different however.

Then the partial derivatives for RD are given as:

$$\frac{\partial RD}{\partial p_k} = \frac{\partial \rho(t_{obs})}{\partial p_k} - \frac{\partial \rho(t_{obs} - \Delta t)}{\partial p_k}$$

$$\frac{\partial RD}{\partial q_k} = \frac{\partial \rho(t_{obs})}{\partial q_k} - \frac{\partial \rho(t_{obs} - \Delta t)}{\partial q_k}$$

$$\frac{\partial RD}{\partial f_b} = \frac{c}{f_o} \Delta t$$

A row of the A-matrix (partials of data with respect to parameters) has the parameters (columns) ordered as follows:

$$(p^T \ q^T \ f_b) \quad \text{maximum of 23 parameters}$$

where $p^T = (e_{HS}^T \ C_{D_{HS}}^T \ T_{HS}^T \ K_{R_{HS}})$

$$q^T = (e_{NS}^T \ C_{D_{NS}}^T \ T_{NS}^T \ K_{R_{NS}})$$

and drag, thrust, and radiation pressure parameters are optional.

After all the observations for a pass have been processed as described above, the A and $O-C$ matrices for all observations are converted to the normal equation matrices B and \bar{E} as follows:

$$B = A^T W A \quad \bar{E} = A^T W (O - C)$$

where

$$W = \text{diag} \left(\frac{1}{\sigma_{obs_i}^2} \right)$$

$$\sigma_{obs_i}^2 = \text{noise variance on the } i^{\text{th}} \text{ observation}$$

For the adjustment procedure (required for data editing and weight determination) a subset of the above normal equations is used and is represented by

$$B_o = \begin{pmatrix} B_{b,b} & B_{b,1} & B_{b,2} \\ B_{b,1}^T & B_{1,1} & B_{1,2} \\ B_{b,2}^T & B_{1,2}^T & B_{2,2} \end{pmatrix} \quad \text{and} \quad \bar{E}_o = \begin{pmatrix} f_b \\ f_1 \\ f_2 \end{pmatrix}$$

13 x 13 13 x 1

where subscript b refers to frequency bias (1)
 subscript 1 refers to host satellite orbital elements (6)
 subscript 2 refers to NAVSAT orbital elements (6)

For the NAVPAC SST data, the predicted signal to noise for a pass is given as

$$S/N_{\text{pred.}} = \left(\frac{\text{Variance} - F_o^2 \Delta p_o}{N_R + N_A - N_S} \right)^{1/2}$$

where N_R = no. of points remaining
 N_A = no. of parameters with a priori sigmas = 7
 N_S = no. of parameters solved for in PASMAT

As was mentioned earlier, QSOLVE and the CELEST FILTER section are similar in the way each determines the noise variance on the data for each pass. The main differences are listed below:

- 1) FILTER can also adjust NAVSAT position and velocity components.
- 2) QSOLVE can solve for an additional bias parameter - frequency drift.
- 3) Each iteration in FILTER retests points that were previously tagged. In QSOLVE, once a point is deleted it is never reconsidered in later iterations.
- 4) The sigma tolerance can be fixed by input in FILTER but not in QSOLVE.
- 5) The estimate of data quality in CELEST is called the "Filtered Noise" and is defined as

$$\text{Filtered noise} = \left(\frac{\sum_{i=1}^{N_R} (\sigma_i \sigma_{\text{mult.}})^2}{N_R} \right)^{1/2}$$

QSOLVE uses the adjusted O - Cs to estimate the standard deviation of the noise defined as

$$\text{Noise s.d.} = \left(\frac{\sum_{i=1}^{N_R} (O - C)_{\text{adj.}}^2}{N_R + N_A - N_S} \right)^{1/2}$$

SOLUTIONS AND DIAGNOSTICS (BSOLVR)

General

BSOLVR, the solution and diagnostic section of CELEST, has been modified to process the NAVPAC SST data separately or simultaneously with station tracking data for the host satellite. The program can accommodate parameters for up to seven NAVSATs along with the host satellite parameters. The total number of arc parameters is limited to 87 with a maximum of 39 host satellite arc parameters allowed.

If station data pass matrices are present, they are time updated, expanded, bias eliminated, and summed first. If NAVPAC SST data pass matrices are also present, they are processed similarly for

each NAVSAT, and then combined with the GT pass matrices to obtain the solution for all arc parameters simultaneously. The epoch for all arc parameters is defined as the host satellite trajectory epoch for long-arc fits and the program determined epoch for short-arc fits. For this reason, the NAVSAT trajectory epochs must be the same or earlier than the host satellite trajectory epoch.

Two modifications to BSOLVR were implemented to improve the long-arc fit diagnostic capabilities of the program. The first was an option to compute navigations before fit without doing cross-pass filtering. The second was an option to cross-pass filter based on ratios of navigations to their estimated uncertainties instead of navigations. This ratio is called a "signal to noise" and is given by

$$S/N(\Delta r_i) = \frac{\Delta r_i}{(\sigma_{\Delta r_i}^2 + \sigma_{\text{orbit}_i}^2)^{1/2}} \quad i \text{ index signifies a particular navigation component}$$

where Δr_i = navigation value
 $\sigma_{\Delta r_i}$ = formal navigation standard deviation from the covariance matrix of the solution
 σ_{orbit_i} = input a priori component uncertainty

In addition, all RWS navigations are then computed as follows:

$$RWS \Delta r_j = \left(\frac{\sum_j \left[\frac{\Delta r_j^2}{(\sigma_{\Delta r_j}^2 + \sigma_{\text{orbit}_j}^2)} \right]_j}{\sum_j \left[\frac{1}{(\sigma_{\Delta r_j}^2 + \sigma_{\text{orbit}_j}^2)} \right]_j} \right)^{1/2} \quad j \text{ index signifies a particular pass}$$

The same a priori orbit sigmas are used for station and SST cross-pass filtering even though their navigation reference frames have only the host satellite along-track direction in common.

The SST-related modifications are given next followed by an alternate pass-matrix expansion technique for the host satellite parameters called the multiple-velocity expansion method.

SST-Related Modifications

Adjustment of the SST pass matrices to reference the fit epoch, t_s , and expansion to accommodate the drag and thrust segmentation are done as follows:

Let $B(t_0)$ and $F(t_0)$ represent the canonical pass matrix and its right-hand side for a particular pass. Updating these normal equations to be for orbital elements at t_s results in the following matrices:

$$B(t_s) = [\psi^{-1}(t_s)\Gamma(t_s)]^T B(t_0) [\psi^{-1}(t_s)\Gamma(t_s)] \quad F(t_s) = [\psi^{-1}(t_s)\Gamma(t_s)]^T F(t_0)$$

where $\psi(t_s) =$

$$\begin{pmatrix} \psi_{HS}(t_s) & & & \\ & I_1 & & \\ & & \psi_{NS}(t_s) & \\ & & & I_2 \end{pmatrix}$$

$$\psi_{HS}(t_s) = \frac{\partial \bar{r}_{HS} \dot{r}_{HS}(t_s)}{\partial \bar{e}_{HS}(t_0)} = 6 \times 6 \text{ state transition matrix from the HS perturbed trajectory}$$

$$\psi_{NS}(t_s) = \frac{\partial \bar{r}_{NS} \dot{r}_{NS}(t_s)}{\partial \bar{e}_{NS}(t'_0)} = 6 \times 6 \text{ state transition matrix from the NS hybrid trajectory}$$

t_0 and t'_0 are the epochs of the corresponding trajectories that may not be equal.

and $T(t_s) =$

$$\begin{pmatrix} T_{HS}(t_s) & & & \\ & I_1 & & \\ & & T_{NS}(t_s) & \\ & & & I_2 \end{pmatrix}$$

$$\left. \begin{aligned} F_{HS}(t_s) &= \frac{\partial r_{HS} \dot{r}_{HS}(t_s)}{\partial e_{HS}(t_s)} \\ F_{NS}(t_s) &= \frac{\partial r_{NS} \dot{r}_{NS}(t_s)}{\partial e_{NS}(t_s)} \end{aligned} \right\} \begin{array}{l} 6 \times 6 \text{ matrices obtained from the PARDEL subroutine} \\ \text{of CELEST} \end{array}$$

I_1 and I_2 are identity matrices of different dimensions

The expansion to accommodate the drag and thrust profiles and the radiation pressure update for the host satellite is done by treating all the NAVSAT parameters and the frequency bias parameter as the bias parameters are treated presently in the station pass-matrix expansion procedure (see Reference 3). After this is completed, the same procedure is used to expand the NAVSAT dynamic parameter sections only the expanded host satellite parameters and frequency bias are now treated as biases. The formulas for each of the expansions above are a function of the SST pass TCA time relative to the drag and thrust segment times. For the NAVSATs, a maximum of two drag segments and one thrust is allowed.

The expanded pass matrix after the a priori frequency bias information is included has the form:

$$B_e = \begin{pmatrix} B_{o,o} & B_{o,b} \\ B_{o,b}^T & B_{b,b} \end{pmatrix} \quad \text{and} \quad E_e = \begin{pmatrix} E_o \\ E_b \end{pmatrix}$$

where $B_{b,b} = B(t_s)_{b,b} + \frac{1}{\sigma_{f_b}^2}$

The next step is to formally eliminate the frequency bias parameter from the corresponding equations. This results in the following matrices:

$$B_{elim.} = B_{o,o} - B_{o,b} B_{b,b}^{-1} B_{o,b}^T \quad \text{and} \quad E_{elim.} = E_o - B_{o,b} B_{b,b}^{-1} E_b$$

These eliminated matrices (containing arc parameters only) are then summed for all passes for a given NAVSAT within the fit span except for those that have been tagged. This results in up to N (no. of NAVSATs) sets of normal equations given by:

$$\begin{pmatrix} B_{HS,HS}^{(i)} & B_{HS,NS_i} \\ B_{HS,NS_i}^T & B_{NS_i,NS_i} \end{pmatrix} \begin{pmatrix} \Delta p_{HS} \\ \Delta p_{NS_i} \end{pmatrix} = \begin{pmatrix} F_{HS}^{(i)} \\ F_{NS_i} \end{pmatrix} \quad i = 1, 2, \dots, N$$

If station tracking data are present, it would already be reduced to a system of arc normal equations for the host satellite parameters given by

$$B_{HS}^{Sta.} \Delta p_{HS} = F_{HS}^{Sta.}$$

These matrices are then combined as follows to form the arc normal equations:

$$\begin{pmatrix}
 B_{HIS}^{Sta} + \sum_1 B_{HIS,HIS}^{(i)} & B_{HIS,NS_1} & B_{HIS,NS_2} & \dots & B_{HIS,NS_N} \\
 & B_{HIS,NS_1}^I & & & \\
 & & B_{NS_1,NS_1} & & \\
 & & & B_{NS_2,NS_2} & 0 \\
 & & & & \ddots \\
 & & & & & B_{NS_N,NS_N}
 \end{pmatrix}
 \begin{pmatrix}
 \Delta p_{HIS} \\
 \Delta p_{NS_1} \\
 \Delta p_{NS_2} \\
 \vdots \\
 \Delta p_{NS_N}
 \end{pmatrix}
 =
 \begin{pmatrix}
 I_{HIS}^{Sta} + \sum_1 I_{HIS}^{(i)} \\
 I_{NS_1} \\
 I_{NS_2} \\
 \vdots \\
 I_{NS_N}
 \end{pmatrix}$$

$$B \Delta p = E$$

Next, the a priori information on each host satellite and NAVSAT arc parameter is introduced into the normal equations by adding terms of the form $1/\sigma_p^2$ to each diagonal element. Different values of σ_p for the same parameter but for different NAVSATs can be used. B is then inverted and the equations solved to get the arc parameter solution $\Delta p = B^{-1} E$ and the covariance matrix B^{-1} .

Signal-to-noise values (old and bias-reduced) are computed separately for GT and SST data and also combined. The formulas for the SST signal-to-noise quantities are identical to the those used for the GT data. The combined signal-to-noise values are given as follows:

$$SN(\text{old}) = \left(\frac{\left(\sum_{i=1}^{N_{GT}} N_i \right) S N_{GT}^2(\text{old}) + \left(\sum_{i=1}^{N_{SST}} M_i \right) S N_{SST}^2(\text{old})}{\sum_{i=1}^{N_{GT}} N_i + \sum_{i=1}^{N_{SST}} M_i} \right)^{1/2}$$

$$SN(\text{bias-reduced}) = \left(\frac{\left(\sum_{i=1}^{N_{GT}} N_i \right) S N_{GT}^2(\text{adj.}) + \left(\sum_{i=1}^{N_{SST}} M_i \right) S N_{SST}^2(\text{adj.})}{\sum_{i=1}^{N_{GT}} N_i + \sum_{i=1}^{N_{SST}} M_i} \right)^{1/2}$$

where $S/N_{GT}(\text{old}) = \text{signal to noise before fit for GT data}$
 $S/N_{SST}(\text{old}) = \text{signal to noise before fit for SST data}$
 $N_i = \text{no. of observations in } i^{\text{th}} \text{ GT pass}$
 $M_i = \text{no. of observations in } i^{\text{th}} \text{ SST pass}$
 $N_{GT} = \text{total no. of GT passes}$
 $N_{SST} = \text{total no. of SST passes}$
 $S/N_{GT}(\text{adj.}) = \text{bias-reduced signal to noise for GT data}$
 $S/N_{SST}(\text{adj.}) = \text{bias-reduced signal to noise for SST data}$

The predicted signal to noise for the arc is then given by

$$S/N(\text{predicted}) = \left(S/N^2(\text{bias-reduced}) - \frac{E^T \bar{\Delta p}}{\begin{pmatrix} N_{GT} & N_{SST} \\ \sum_{i=1} N_i & \sum_{i=1} M_i \end{pmatrix}} \right)^{1/2}$$

and the percent change in variance is given by

$$\text{Percent change in variance} = \frac{100 E^T \bar{\Delta p}}{\begin{pmatrix} N_{GT} & N_{SST} \\ \sum_{i=1} N_i & \sum_{i=1} M_i \end{pmatrix} S/N^2(\text{bias-reduced})}$$

Navigations are computed for each pass based on the expanded pass matrices. For each station pass matrix, the $\bar{\Delta p}_{H_0}$ solution augmented with zeroes for the pass bias parameters is used to linearly adjust the right-hand side and then the present form 3 navigation procedure is followed. For each NAVPAC SST pass matrix, the navigation procedure is as follows:

1) The sections of the expanded pass matrix corresponding to the NAVSAT orbital elements and the frequency bias before the bias elimination was performed are required for the navigation procedure.

$$B'_e = \begin{pmatrix} B'_{o,o} & B'_{o,b} \\ B'^T_{o,b} & B_{b,b} \end{pmatrix} \quad E'_e = \begin{pmatrix} E'_o \\ E'_b \end{pmatrix}$$

7×7 7×1

where ' indicates only those rows and columns corresponding to orbital elements for the NAVSAT are present.

2) The right-hand side is then linearly adjusted to account for the solution for the host satellite parameters only

$$F_e'' = \begin{pmatrix} F_o' & B_{HS,NS}^T & \Delta P_{HS} \\ F_b' & B_{HS,b}^T & \Delta P_{HS} \end{pmatrix} \begin{cases} 0 \\ 1 \end{cases} \quad \text{where} \quad B_{o,o} = \begin{pmatrix} B_{HS,HS} & B_{HS,NS} \\ B_{HS,NS}^T & B_{NS,NS} \end{pmatrix}$$

$$B_{o,b} = \begin{pmatrix} B_{HS,b} \\ B_{NS,b} \end{pmatrix}$$

3) B_e' and F_e'' are then transformed to reference NAVSAT position and velocity in the HS RAC reference frame at TCA

$$B_e'(TCA) = R^T \psi^{-1}(TCA) B_e' \psi^{-1}(TCA) R$$

$$F_e''(TCA) = R^T \psi^{-1}(TCA) F_e''$$

where

$$\psi(TCA) = \frac{\partial r_{NS} \dot{r}_{NS}(TCA)}{\partial e_{NS}(t_s)}$$

$$R = \begin{pmatrix} R_{TCA} & 0 & 0 \\ 0 & R_{TCA} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

R_{TCA} = RAC transformation at TCA for the pass defined in the FILTER section

4) The resulting system of navigation equations $B_e'(TCA) \Delta \bar{N} = F_e''(TCA)$ is then solved using the PASMAT subroutine that determines which components (may include velocity components) can be solved for. PASMAT returns zero values for a component and its sigma if it is not solved for.

After the navigations are performed for all SST passes, the RMS and RWS navigations for each NAVSAT and combined are computed. The RMS navigation computations account for the fact that not all navigation components are present for each SST pass.

The frequency bias solution for each pass based on the arc solution for the host satellite and NAVSAT parameters is given by:

$$\Delta f_b = B_{bb}^{-1} (F_b' - B_{HS,b}^T \Delta P_{HS} - B_{NS,b}^T \Delta P_{NS})$$

The cross-pass filtering has been extended to include the satellite-to-satellite data. Separate first-order polynomial fits to the SST navigations from all NAVSATS are done, in addition to the fits for the station navigations. The procedure terminates only when the same station passes and the same SST passes are tagged on successive iterations or the maximum number of iterations is reached. For the three SST navigation polynomial fits, only those navigation components that are not zero (i.e., only solved for components) are included in the fits. This means that the number of passes for each polynomial fit will be different for each component and usually less than the total

number of untagged SST passes. The navigation components determined for a particular SST pass do not change from iteration to iteration. Both "total" and "solution" RMS and RWS navigation values are computed for each iteration. "Total" navigation values are based on all passes that are not deleted by input and are the same for all iterations. "Solution" navigation values are based only on the untagged passes used in the solution for each iteration. All SST pass tags are then honored in any subsequent long-arc or short-arc fit.

Multiple-Velocity Pass-Matrix Expansion

The multiple-velocity pass-matrix expansion allows for a simultaneous solution for epoch orbital position and velocity and additional small velocity increments at equally spaced times along the host satellite trajectory. The velocity increments are solved for in the RAC reference frame at each time. This expansion can be used for long arcs as well as short arcs. There is no reintegration possible after a multiple-velocity fit and only propagated trajectories after fit can be computed. This means that a converged orbit using standard techniques needs to be obtained before this solution is performed.

The arc is partitioned with N velocity increment time points $\{t_1, t_2, \dots, t_N\}$ as shown below

- t_0 = epoch of the host satellite trajectory
- t_s = epoch of the fit span
- t_i = end of the fit span



The multiple-velocity pass-matrix expansion is based on the following relationship:

$$\frac{\partial D(t)}{\partial \dot{r}(t_i)} = \left(\frac{\partial D(t)}{\partial r(t_s)} \quad \frac{\partial D(t)}{\partial \dot{r}(t_s)} \right) \left(\psi_{t_s}^{-1}(t_i) \right)_{\dagger}$$

where $t \geq t_i > t_s \geq t_0$

$D(t)$ = observation at time t

$(\)_{\dagger}$ denote the last three columns of a 6×6 matrix

and $\psi_{t_s}^{-1}(t_i) = \psi(t_i) \psi^{-1}(t_s)$

$\psi(\tau) = 6 \times 6$ state transition matrix at $\tau = \frac{\partial r(\tau)}{\partial r(t_0)}$

Define

$$V_i = \left(\psi_{t_s}^{-1}(t_i) \right)_{\dagger} = \frac{\partial r(t_s)}{\partial \dot{r}(t_i)} \quad i = 1, 2, \dots, N$$

6×3

Also define for $i = 1, 2, \dots, N$

$R_i = (U_i \ V_i \ W_i) =$ rotation matrix to go from RAC frame at t_i to the inertial frame

3×3

where

$$U_i = \frac{r(t_i)}{|r(t_i)|}$$

$$W_i = \frac{r(t_i) \times \dot{r}(t_i)}{|r(t_i) \times \dot{r}(t_i)|}$$

$$V_i = W_i \times U_i$$

$r(t_i), \dot{r}(t_i)$ = inertial position and velocity of the host satellite at t_i

Only observations that occur at or after time t_i are used to determine the required velocity increment at t_i . The assumption is made that if for a pass $TCA > t_i$, all the observations in the pass are to be used to solve for $\Delta \dot{v}_i$, the velocity increment at t_i . This allows an expansion of the pass matrix in BSOLVR instead of including the velocity parameters in the formation of the A-matrix for each observation in FHFR.

The starting point for this expansion is the completion of the standard time update and expansion currently done in BSOLVR except for one small change — the updated normal equations involve position and velocity at t_i instead of orbital elements. Each updated set of pass normal equations (for GT or SST data) takes the form:

$$\begin{pmatrix} B_{xx} & B_{xy} \\ B_{xy}^T & B_{yy} \end{pmatrix} \begin{pmatrix} \Delta p_{HS} \\ \Delta p_y \end{pmatrix} = \begin{pmatrix} F_x \\ F_y \end{pmatrix}$$

where x indicates the host satellite position and velocity parameters at t_s
 y indicates all other parameters

and $\Delta p_{HS} = \begin{pmatrix} \Delta r_{HS} \\ \Delta \dot{r}_{HS} \end{pmatrix}$ at t_s

If $t_i \leq TCA < t_{i+1}$, then the velocity parameter expanded normal equations take the following form.

$$\begin{pmatrix} B_{xx} & B_{xx} V_1 R_1 & \dots & B_{xx} V_N R_N & B_{xy} \\ R_1^T V_1^T B_{xx} V_1 R_1 & \dots & R_1^T V_1^T B_{xx} V_N R_N & R_1^T V_1^T B_{xy} \\ \vdots & \ddots & \vdots & \vdots \\ \text{Symmetric} & R_N^T V_N^T B_{xx} V_N R_N & R_N^T V_N^T B_{xy} \\ B_{yy} \end{pmatrix} \begin{pmatrix} \Delta p_{HS} \\ \Delta \dot{v}_1 \\ \vdots \\ \Delta \dot{v}_N \\ \Delta p_y \end{pmatrix} = \begin{pmatrix} F_x \\ R_1^T V_1^T F_x \\ \vdots \\ R_N^T V_N^T F_x \\ F_y \end{pmatrix}$$

where $V_j R_j = (0)$ for $j = i+1, i+2, \dots, N$
 6×3

After this expansion is completed for a particular pass, the biases are formally eliminated. All of the resulting expanded pass matrices for the fit span are then summed and combined (if SST data are present) to form the arc normal equations. A priori sigma information on each velocity increment is included before these equations are solved by adding in terms of the form $1/\sigma_{\Delta v}^2$ to each diagonal element. Separate a priori sigmas for radial, along-track, and cross-track velocity increments are included. All further computations in BSOLVR account for these additional host satellite orbit parameters.

PROPAGATION (COVAR)

This section of CFLEST, which propagates the corrections solved for in the BSOLVR section and applies them to the reference trajectory to get the fitted trajectory, still operates in a single-satellite mode. Therefore only the portion of $\overline{\Delta p}$ and B^{-1} from BSOLVR corresponding to the host satellite are required. Several modifications to this section have been made to accommodate the NAVPAC data processing procedures and are given below.

1) The covariant ephemeris was changed to include the earth-fixed velocity vector and a full 6×6 earth-fixed covariance matrix (position and velocity) at each trajectory timeline.

$$\dot{r}_{EF} = (ABCD) \left[\dot{r}_I - (CD)^T W(CD)r_I \right]$$

$$C_{EF} = H C_I H^T$$

6×6

where r_I, \dot{r}_I, C_I = inertial position, velocity, and covariance matrix

D	=	precession transformation matrix	}	all 3×3
C	=	nutations transformation matrix		
B	=	earth rotation transformation matrix		
A	=	polar motion transformation matrix		

$$W = \begin{pmatrix} 0 & -\tilde{\omega} & 0 \\ \tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{\omega} = .7292115855 \times 10^{-4} \text{ rad/sec} \approx \text{Earth's mean sidereal rotation rate}$$

and $H = \begin{pmatrix} ABCD & 0 \\ -ABWCD & ABCD \end{pmatrix}$

2) An option to apply the rotation matrix, $R_{NWL-9Z-WGS-72}$, defined in the FILTER section to both the propagated inertial positions and velocities at each trajectory timeline has been included in COVAR. This is required if the NAVPAC orbit fits are carried out using NWL-9Y station coordinates. In this case, the rotation would not have been applied to the NAVSAT trajectory when filtering the SST data.

3) The option to output the trajectory of a reference location on the satellite instead of the c.g. has also been included in COVAR. If $\bar{\Delta r}_{ref.}$ represents the coordinates of this reference location in a body-fixed reference system, then the transformation matrix to go from this system to the inertial reference system is defined by

$$R_{ref.} = \begin{pmatrix} \frac{r}{|r|} & \frac{(r \times \dot{r}) \times r}{|(r \times \dot{r}) \times r|} & \frac{r \times \dot{r}}{|r \times \dot{r}|} \end{pmatrix} = (U \ V \ W)$$

where $r, \dot{r} = r_{c.g.}, \dot{r}_{c.g.}$ = propagated inertial position and velocity of the host satellite at the trajectory timeline

(This RAC reference frame is the same as the one used to define antenna offsets in the FILTER section.)

Then $r_{ref.} = r_{c.g.} + R_{ref.} \bar{\Delta r}_{ref.}$

is the inertial position of the reference location.

If, in addition, the inertial velocity is required in COVAR, an adjustment to account for the reference location offset from the c.g. is also required.

$$\dot{r}_{ref.} = \dot{r}_{c.g.} + \omega \times R_{ref.} \bar{\Delta r}_{ref.}$$

where

$$\omega = \frac{1}{|r|} (V \cdot \dot{r})W + \frac{|r|}{|r \times \dot{r}|} (W \cdot \ddot{r})U = \text{RAC frame angular velocity vector}$$

$$\ddot{r} = \text{satellite acceleration vector}$$

The usual reference location would be the electrical phase center of the transmitting antenna (used for the NAVSATS). However, the radial reference location component can also be used to account for differences between the μ value used in the fit and a desired value. For instance if the NAVPAC orbit fits were done in the NWL-9Z reference system (that used for the NAVSATS) and the output trajectory was required in the WGS-72 system (that used for the host satellite) the radial offset would be defined as follows:

$$\Delta r_{HS_{radial}} = \frac{a_{HS}}{3} \frac{\Delta \mu}{\mu_{NS}}$$

where a_{HS} = semimajor axis of the host satellite orbit in km
 $\Delta \mu = \mu_{HS} - \mu_{NS}$ in km^3/sec^2

4) The NAVPAC SST data fits require a special form of NAVSAT trajectory called a hybrid trajectory. A hybrid trajectory is identical to the original perturbed trajectory except the positions have been updated to reflect the results of an orbit fit. The partial derivatives remain unchanged. COVAR was modified to create a hybrid trajectory on option.

5) To get a propagated trajectory and covariance matrix when the multiple-velocity expansion is used in BSOLVR, additional changes to COVAR were required. Figure 5 gives a representation of the actual corrections solved for in the multiple-velocity method. Each $\Delta \bar{v}_i$ is actually an additional correction to the inertial velocity correction at t_i beyond that propagated from all previous $\Delta \bar{v}_j$'s, $j = 1, \dots, i-1$ and Δr_{HS} and $\Delta \dot{r}_{HS}$. This results in a trajectory that is continuous in position but discontinuous in velocity at times t_i , $i = 1, 2, \dots, N$. The state transition matrix required to propagate the correction vector $\Delta \bar{p}_{HS}(t_s)$ and its covariance matrix $B_{HS}^{-1}(t_s)$ to position and velocity corrections and their covariance matrix at time t is given by:

$$\Psi_{t_s}(t) = \begin{pmatrix} \psi_{t_s}(t) & \psi_{t_s}(t)V_1R_1 & \dots & \psi_{t_s}(t)V_NR_N & \psi_d(t) \\ 6 \times (6 + 3N + N_d) & 6 \times 6 & 6 \times 3 & 6 \times 3 & 6 \times N_d \end{pmatrix}$$

$$V_iR_i = \begin{pmatrix} \mathbf{O} & \text{if } t < t_i \\ & 6 \times 3 \end{pmatrix}$$

where the V_i and R_i matrices are defined in the BSOLVR section

$$\psi_{t_s}(t) = \frac{\partial r \dot{r}(t)}{\partial r \dot{r}(t_s)}$$

$$\psi_d(t) = \frac{\partial r \dot{r}(t)}{\partial d(t_s)} \quad \text{where } d \text{ represents all other dynamic parameters (e.g. drag, thrust, or radiation pressure)}$$

N_d = no. of these other dynamic parameters

Then $\begin{pmatrix} \Delta r(t) \\ \Delta \dot{r}(t) \end{pmatrix} = \Psi_{t_s}(t) \Delta \bar{p}_{HS}(t_s)$

and $B^{-1} \Delta \bar{r}, \Delta \dot{\bar{r}}(t) = \Psi_{t_s}(t) B_{HS}^{-1}(t_s) \Psi_{t_s}(t)^T$

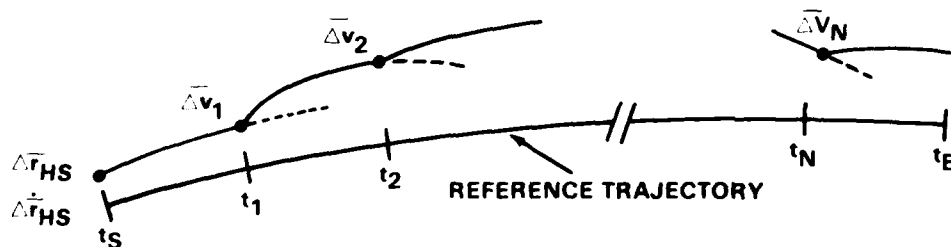


Figure 5. Multiple-Velocity Corrections

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