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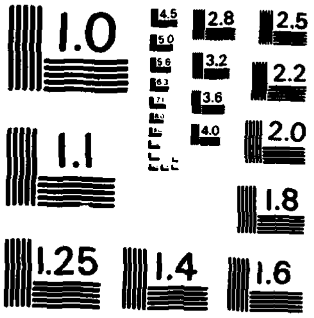
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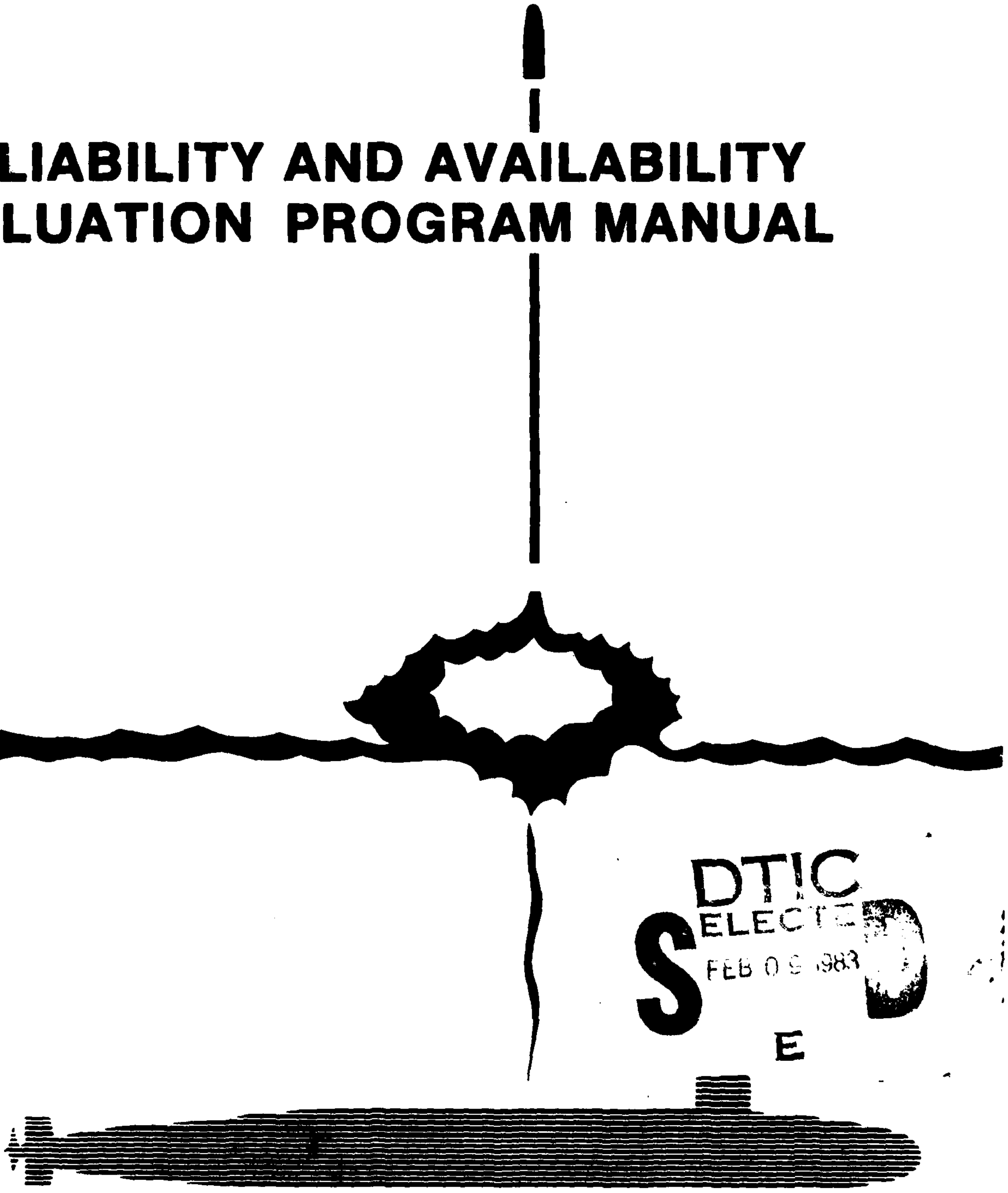
NAVSEA OD 29304B
1 NOVEMBER 1982

RELIABILITY AND AVAILABILITY EVALUATION PROGRAM MANUAL



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NAVSEA OD 29304B

RELIABILITY AND AVAILABILITY EVALUATION PROGRAM MANUAL

1. NAVSEA OD 29304B is a comprehensive practical guide for assessing the reliability and availability of the Strategic Weapon System subsystems and lower level assemblies. Originally published in 1965, NAVSEA OD 29304B has been revised to reflect the assessment requirements of newly revised NAVSEA OD 21549A "Technical Program Management Requirements for Navy Strategic Systems Project Office Acquisitions", to update the material presented in NAVORD OD 29304A (1973) "Reliability Evaluation Program Manual" and to incorporate and update the material presented in NAVORD OD 43251 "Availability Evaluation Program Manual."

2. It is intended that this manual be reviewed periodically to insure its accuracy and currency. Users of the manual are encouraged to report any errors discovered and any recommendations for improvement to Department of the Navy, Strategic Systems Project Office (Attn: Code 2014), Washington, D.C. 20376.

3. Copies of NAVSEA OD 29304B may be obtained from Defense Documentation Center, Cameron Station, Alexandria, Virginia 22314.

Andrew DePrete

ANDREW DEPRETE
By direction

NAVSEA OD 29304B
1 NOVEMBER 1982

RELIABILITY AND AVAILABILITY EVALUATION PROGRAM MANUAL

prepared for

Department of the Navy
Strategic Systems Project Office

prepared by

Evaluation Associates, Inc.

Contracts

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FOREWORD

The basic purpose of this document is to provide guidelines for SSPO personnel and contractors in the evaluation and documentation of SWS program life cycle reliability and availability. Each section of the document achieves this purpose by providing a detailed discussion of the various reliability and availability topics and techniques necessary for an evaluation program. Section 5 (Assessment of Component Reliability), 6 (Software Evaluation) and 7 (Assessing System Reliability and Availability) while providing detailed discussions and numerous examples, deal with preliminary, incomplete and rapidly growing topics. These sections may also be controversial due to the discussions of Bayesian Methods and Duane or software models which have not been sufficiently tested on all classes of data to have gained general acceptance. However, these sections provide guidelines for quantitative data analysis and an orientation for reliability and availability analysts and other interested program personnel. If the analyst is reasonably sure that the distribution being evaluated is not exponential, then sections 5 and 7 contain references to other material which would be needed to handle state-of-the-art problems.

Originally published in 1965, NAVWEPS OD 29304[1] prescribed uniform procedures for measuring the reliability of subsystems, beginning in research and development. The manual's purpose was to provide a practical method for making reliability measurements, measurements directly related to mission requirements and useful in both development and operational program phases. Following its introduction the manual gained acceptance, particularly after independent studies confirmed the accuracy and validity of its techniques, even in conditions where the quality of input data was severely limited. The manual's techniques represented the "state of the art" in reliability analysis at that time. The methodology was based on "classical" methods, as any subjective or pre-existing knowledge of hardware characteristics was excluded from consideration and analysis was based solely on test results.

The manual was first revised in 1973[3]. Bayesian statistical methods, which permitted formal inclusion of prior information directly with test data for economical evaluation programs, were included in this revision. In many instances reliability is more efficiently defined in terms of performance variables, rather than in terms of absolute success or failure. Here the methods of variables statistics permit more efficient usage of test results, with consequent cost reduction to the program budget. Variables methods compatible with the Basic and the Bayesian Method of the OD were also incorporated in 1973. In addition to including these analytical methods, new material was added in 1973 covering reliability prediction and apportionment.

The treatment of data system requirements was expanded in 1973 to accommodate evaluation of modified and operational systems in development. Tests specifically performed for reliability demonstration were discussed and the conditions under which they may be desirable were considered. An effort was made to amplify the presentation throughout, to incorporate aspects of operational readiness evaluation contained in NAVORD OD 43251[4], and to maintain compatibility with NAVORD OD 42282[5], Integrated Test Program Manual.

This latest revision has been prepared to update and expand both the methods and scope of NAVORD OD 29304A[3], and to address the reliability and availability *evaluation* requirements of NAVSEA OD 21549[6]. This section on methods (Section 5) for assessing reliability of system elements has been expanded to cover a wider variety of statistical models and to incorporate reliability growth models. Section 6 has been included to cover reliability evaluation of software. The section on system assessment (Section 7) has been expanded to provide procedures for combining the expanded hardware models and the software models to yield system mission-phase reliability and availability values. Section 8 has been added to cover reliability demonstration. The material in all other sections has been updated in order to be compatible with the additions described above. Fault tree analysis has been included in the section on reliability analysis as a procedure for evaluating designs considering undesirable events in an operational environment.

This document supersedes NAVORD OD 29304A[3] and NAVORD OD 43251[4]. The Statistical Addendum [2], NAVORD OD 29304/Addendum, has not been superseded.

REFERENCES

1. NAVWEPS OD 29304: Guide Manual for Reliability Measurement Program, 1965.
2. NAVORD OD 29304/Addendum: Statistical Exposition of Guide Manual for Reliability Measurement Program, 1967.
3. NAVORD OD 29304A: Reliability Evaluation Program Manual, 1973.
4. NAVORD OD 43251: Availability Evaluation Program Manual, 1970.
5. NAVORD OD 42282: Integrated Test Program Manual, 1973.
6. NAVSEA OD 21549A: Technical Program Management Requirements for Navy SSPO Acquisitions, 1981.

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GLOSSARY

ACCURACY	A term denoting the closeness of measured values to the true value of a quantity, taking into consideration both systematic error (bias) and random error (variance).
ALERT TIME	Time during which a system is on station ready for operation.
ALGORITHM	A fixed step-by-step procedure to carry out a given computational or logical formulation.
ALLOCATION	(See APPORTIONMENT)
APPORTIONMENT	A process of assigning goals or requirements to items in a system in accordance with a logical scheme.
ASSESSMENT	The use of test data and/or operational service data to form estimates of population parameters and to evaluate the precision of those estimates (synonym – Estimation).
ATTRIBUTE	A characteristic or property that is appraised in terms of whether it does (or does not) exist with respect to a given requirement.
AVAILABILITY, A	*Availability is equivalent to Operational Readiness Reliability in this manual.*
AVAILABILITY, APPARENT	For an item checked out at intervals, the quotient of apparent up time divided by apparent up time plus apparent down time. Apparent availability is greater than operational availability when failure detection is not immediate.
AVAILABILITY, INHERENT, A_i	Availability with respect to failure only, under ideal support conditions; an intrinsic hardware characteristic. It is estimated by the ratio of total operating time plus total alert time to the sum of total operating time plus total alert time plus total corrective maintenance time.
AVAILABILITY, INTERVAL, $\bar{A}_T$	The time average of pointwise availability over intervals of stated length, T.
AVAILABILITY, OPERATIONAL, A_o	Availability in the actual operating environment; a function of facility characteristics as well as hardware. It is estimated as the ratio of operating time plus alert time to total calendar time. Equivalent to operational readiness.

AVAILABILITY PHASE	Any phase of a mission when an availability figure-of-merit applies, i.e., failures are permissible if system is up when needed.
AVAILABILITY, POINTWISE, A(t)	The probability that an item will be operable at a stated instant in time.
AVAILABILITY, STEADY STATE, A	The limit of interval availability as time increases without limit ($T \rightarrow \infty$). It is estimated by the up time ratio or the expression MTBF divided by MTBF plus MTTR.
BAYES' EQUATION	$P(A\mid B) = P(A)\dfrac{P(B\mid A)}{P(B)}$ where the various terms may be defined and illustrated as follows: A = A hypothesis or statement of belief; for example the failure rate is .007 failures per million hours. B = Evidence such as a test result, bearing upon the truth or credibility of the hypothesis; for example the component experienced no failures in 1,000 hours of simulated mission testing. P(A) = The prior probability or degree of belief in the truth of hypothesis A before test information B becomes available. P(A|B) = The posterior or updated measure of belief in hypothesis A given the impact of evidence B. P(B|A) = The likelihood or probability of the evidence given the truth of the hypothesis. P(B) = The probability of the evidence B evaluated over the ensemble of possible hypotheses A.
BAYESIAN METHODS	Statistical procedures that allow information available prior to testing to be combined with test data by means of Bayes' equation.
BIAS	A measure of error that occurs systematically in a series of measurements. The difference between a true value and the limiting means of repeated measurements is termed bias.
BINOMIAL DISTRIBUTION	Consider a series of n independent events, each event having only two possible states with probabilities of occurrence P and 1-P. If n is fixed, a random variable X is said to have a Binomial Distribution with parameter P when $P[X=x] = \binom{n}{x} P^{x} (1-P)^{n-x},\ x = 0, 1, 2, \ldots, n$

COMPONENT	The first indenture level below an equipment. A combination of parts, devices and structure, usually self contained, which performs a distinct function (acts on one or more inputs to produce appropriate outputs) in the operation of an equipment; for example, a converter, a gas generator, an amplifier.
CONFIDENCE COEFFICIENT	A measure of assurance that a statement based upon statistical (frequency) data is correct. The probability that an unknown parameter lies within a stated interval or is greater than or less than some stated value.
CONFIDENCE INTERVAL	A region within which an unknown parameter is said to lie with stated probability. The region is two sided when both upper and lower limits are specified. It is one sided when only the upper or the lower limit is specified.
CONFIDENCE LIMIT	A bound of a confidence interval.
CONSTANT FAILURE RATE (CFR)	Characterizes an item with constant (Instantaneous) Hazard Rate $h(t)$.
CUMULATIVE DISTRIBUTION FUNCTION (CDF)	The probability, $F(x)$ that a random variable x takes a value less than or equal to x.
CUT	A set of items in a higher level assembly which, if all are failed, indicate the higher level assembly has also failed.
DECREASING FAILURE RATE (DFR)	Characterizes an item with decreasing hazard rate. This may occur, for instance, during the early part of the life of an item.
DEGREE OF BELIEF	A Bayesian term associated with the probability of a hypothesis.
DEMONSTRATION	Formal measurement of system characteristics with statistical confidence by testing or operation.
DENSITY FUNCTION	See Probability Density Function
DESIGN OBJECTIVE	A desired value or goal relative to a stated device parameter or characteristic, established as guidance for designers.
DESIGN REQUIREMENT	A required value relative to a stated device parameter or characteristic. In the case of RMA design requirements, normal practice is to specify a design requirement along with an appropriate producer's risk and a minimum acceptable value along with an appropriate consumer's risk.

EQUIPMENT	The first indenture level below a subsystem.
ESTIMATION	The use of test and/or operational data to form estimates of population parameters and to evaluate the precision of those estimates (synonyms – Assessment and Measurement).
EVALUATION	A broad term used to encompass prediction, measurement, and demonstration.
EXPECTED VALUE	The first moment about the origin of a probability distribution. Also called mean. May be estimated by the arithmetic average.
EXPONENTIAL DISTRIBUTION	A probability distribution having the density function $f(t) = \lambda e^{-\lambda t}$ where λ, the failure rate, is a constant. Under very general conditions it is the distribution of time between successive failures of complex systems.
EXPONENTIAL MODEL	In reliability engineering, a model based on the assumption that times t between successive failures are described by the exponential distribution.
FAILURE	Performance below a specified minimum level or outside a specified tolerance interval.
FAILURE, NON-RELEVANT	Failure not applicable to the computation of reliability.
FAILURE, RELEVANT	Failure attributable to a deficiency of design, manufacture or materials of the failed device; applicable to the computation of reliability.
FAILURE RATE	When not further qualified, denotes hazard rate.
FBM WEAPON SYSTEM	The SSBN submarine, together with its supporting tactical subsystems – missile, fire control, guidance, MTRE, navigation, launcher, and ship support.
FIGURE OF MERIT	An index or quantitative measure of merit used to characterize an item for analysis or comparison.
FIRMWARE	Denotes a logical element performing like software but built as hardware which is part of a system or computer.
HARDWARE	A general term denoting physical elements of a system.

HAZARD RATE, h(t)	Also called the conditional failure density, represents the probability that an item still functioning at time t will fail in the interval $(t, t + \Delta t)$, where Δt is an infinitesimal time increment. Hazard rate is synonymous to conditional failure rate or instantaneous failure rate.
HYPOTHESIS	In Statistics, an assertion about the value or distribution of one or more random variables.
HYPOTHESIS TESTING	The use of test data to assess the weight of evidence for or against a hypothesis and to accept or reject the hypothesis based on statistical decision rules.
INCREASING FAILURE RATE (IFR)	Increasing hazard rate generally characteristic of limited life items.
INDENTURE LEVELS	A hierarchical structure of hardware complexity: System Subsystem Equipment Component Module Part or Component Part
INTERVAL ESTIMATE	An interval asserted to enclose a defined set with stated probability. Examples: a confidence interval to include a parameter, a prediction interval to include a set of observations, a tolerance interval to include a population fraction.
ITEM	A general term denoting physical elements of a system.
LOG NORMAL DISTRIBUTION	Statistical distribution which characterizes times to failure of items displaying normally distributed logarithms of times to failure.
MAINTAINABILITY	A measure of the ability of an item to be maintained. Mean preventive maintenance time, mean repair time, and mean down time are commonly used indices of maintainability. (The often-encountered definition of maintainability as the probability of repair within a stated time is not used because that probability is not used in the availability expressions and computations in this manual.)
MAINTENANCE	All actions necessary for retaining an item in, or restoring it to, a specified condition.

MAINTENANCE CONSTRAINTS	Limitations on the quantity and/or quality of maintenance available to a system in use.
MAINTENANCE, CORRECTIVE	Unscheduled actions performed, as a result of an item failure, to repair an item and restore it to a specified condition.
MAINTENANCE, PREVENTIVE	Actions performed on a scheduled or routine basis in an attempt to retain an item in a specified condition by providing systematic inspection, detection and prevention of incipient failure.
MAXIMUM LIKELIHOOD METHOD	A method which consists in expressing the likelihood function of a set of failure times, or other data, and to determine, from the maximum value of the likelihood, a "best" estimate of one or several parameters of the statistical distribution which characterizes the data.
MEAN, μ	The first moment of a probability distribution about its origin; the expected value of a random variable. The mean is the most commonly used measure of central tendency. Estimated by an arithmetic average.
MEAN DOWN TIME	The expected or average down time.
MEAN-TIME-BETWEEN-FAILURES (MTBF)	The same as Mean-Time-To-Failure for non-repairable items. It is often employed specifically for repairable items when it denotes the mean time (or cycles, miles, events) between successive failures. MTBF is often estimated by dividing the total operating time for like items by the total number of failures encountered.
MEAN-TIME-TO-FAILURE (MTTF)	In a component or a system, the mean time to first failure. If f(t) is the Probability Density Function of the component, or system, then MTTF = $\int_0^\infty t[f(t)]\,dt = \int_0^\infty R(t)dt$. Similar formulas hold if measure of life units are cycles, miles, events, etc. rather than time. For repairable exponential elements, MTTF denotes also the mean time to successive failures.
MEAN-TIME-TO-REPAIR (MTTR)	The mathematical expectation of time to repair an item. Often estimated as the total repair time divided by the total number of repair actions during a given time period.
MEASUREMENT	Evaluation of the characteristics of an item by observation of its performance in test or operational service.

MINIMAL CUT	A cut from which no item can be removed while maintaining a cut.
MODULE	The first indenture level below a component. An onboard-replaceable item; for example, a Type 3 Module.
NORMAL DISTRIBUTION	The most prominent continuous distribution in statistics, frequently referred to as the Gaussian or bell-shaped distribution. Its density function is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad -\infty < x < \infty$ with mean μ and variance σ^2. The theoretical justification for the normal distribution lies in the central-limit theorem, which shows that under very broad conditions the distribution of the average of n independent observations from any distribution approaches a normal distribution as n becomes large.
NORMAL VARIABLE	A random variable that is normally distributed. In situations where a random variable represents the total effect of many "small" independent causes, each with mutually independent errors, the central limit theorem leads to the prospect that the variable will be normally distributed.
OPERATING MODE	A specific pattern of system operation in which a designated subset of the system's functions are realizable (e.g., standby mode, tracking mode, search mode).
OPERATIONAL READINESS	Operational availability of a weapon system in fleet service.
OPERATIONAL READINESS RELIABILITY (ORR)	The probability that at any point in time the system is either operating satisfactorily or ready to be placed in operation on demand when used under stated conditions.
PARAMETER	A performance characteristic (e.g., voltage, pressure, time to peak pressure, velocity, etc.) that may be measured on a continuum (variable) or go – no go (attribute) basis.
PERCENTILE	If p percent of the values taken by a variable X are less than or equal to x, then x is defined as the pth percentile of X.
POINT ESTIMATE	A single-valued estimate of a population parameter.

POISSON DISTRIBUTION	A probability distribution applicable to situations where a large number of observations is involved and the probability of an event occurring in any specific observation is very small. It has the density equation $f(x) = [e^{-\lambda} \lambda^{x}]/x!$
POISSON EXPECTATION	The mean μ of the Poisson distribution. In reliability problems the product λt is often a Poisson expectation.
"POSTERIOR" DISTRIBUTION	In Bayesian terminology, a Probability Density Function $g(\Theta\|\mu)$ which modified a prior distribution $g(\Theta)$ through the use of Bayes' theorem when test data, such as times to failure or to repair, became available.
PRECISION	A measure of the scatter of repeated measurements about their mean.
PREDICTION	Judgement of the characteristics of an item by means of engineering analysis, using generic data and/or historical data obtained from antecedent items.
"PRIOR" DISTRIBUTION	In Bayesian terminology, a Probability Density Function $g(\Theta)$ used as some prior probabilistic degree of belief on one (Θ) or several parameters of a function or of any failure or repair model.
PROBABILITY DENSITY FUNCTION (PDF)	In the univariate case, it is a continuous function $f(x)$ of a random variable x such that its integral $\int_a^b f(x)\,dx$ represents the probability of x assuming a value between a and b. The integral over all x is 1. In the multivariate case $f(x)$ becomes $f(x^{(1)}, x^{(2)}, \ldots, x^{(n)})$. The n-fold integral over all $x^{(i)}$'s is 1.
PROBABILITY MASS FUNCTION	It is the discrete analogue $f(x_i)$ of a statistical probability density function $f(x)$.
PROGRAMS	Programs, specifically digital programs, are self-contained sets of instructions capable of performing a specified function in the absence of other programs.
QUALITY ASSURANCE	A planned and systematic pattern of all actions necessary to provide adequate confidence that material conforms to established technical requirements and achieves satisfactory performance in service.

RANDOM VARIABLE	An output of an experiment which may take any of the values of a specified set with a specified relative frequency or probability.
REDUNDANCY	The existence of more than one means for accomplishing a given function. All means of accomplishing the function need not necessarily be identical.
REDUNDANCY, ACTIVE	The redundancy wherein all redundant items are operating simultaneously.
REDUNDANCY, STANDBY	That redundancy wherein the alternative means of performing the function is inoperative until needed and is switched on upon failure of the primary means of performing the function.
REJECT RATE	The percent of units found defective by testing and then rejected.
RELIABILITY, R	The probability that an item will perform its intended function without failure for a specified interval under stated conditions, given that it is up (operable) at the beginning of the interval.
RELIABILITY PHASE	Any phase of a mission during which a reliability figure of merit applies (i.e., no failure is permissible). e.g., launch, flight.
REPLACEMENT RATE	The rate at which spares are consumed. This is usually greater than the failure rate due to replacement of non-failed hardware. The reciprocal is mean-time-between-replacements.
RISK, CONSUMER'S, β	The probability that a test will accept by chance a device or lot having a characteristic equal to a specified unacceptable level.
RISK, PRODUCER'S, α	The probability that a test will reject by chance a device or lot having a characteristic equal to a specified desired level.
SOFTWARE	Idealized set of instructions which constitute the essence of computer programs, subprograms, and routines. Also contents of operating and maintenance manuals which show how to run or modify programs or equipment. (Software is to be distinguished from the hardware which supports the set of instructions (e.g., punched cards, magnetic tape, core memory, paper of manuals).
SOFTWARE ERROR	An incorrect statement or logical fault residing in the coded instructions of the software.

SOFTWARE FAILURE	Software error revealed during execution of software.
SOFTWARE RELIABILITY	The probability that a given software program will operate without error for a specified time in a specified mission.
SPECIFICATION	With respect to RMA requirements, a complete specification provides a design requirement and the associated producer's risk and a test requirement (minimum acceptable requirement) and the associated consumer's risk.
STATE A	A hardware/software state in which the item is non-operating but must be operational in a later mission phase.
STATE B	A hardware/software state in which the item is non-operating and must not operate prematurely. It must survive and be operational in a later mission phase.
STATE C	A hardware/software state in which the item is operating. The duration of operation is measured in cycles or discrete events.
STATE D	A hardware/software state in which the item is operating. The duration of operation is measured in units of time (e.g., hours and minutes).
SUBSYSTEM	The first indenture level below the system, SWS. Examples; fire control, missile and navigation, etc.
SUCCESS CRITERIA	The minimum functional performance required of an item for mission success.
SYSTEM EFFECTIVENESS	A measure of the degree to which an item can be expected to achieve a set of specific mission requirements and which may be expressed in terms of availability, reliability and performance capability.
SYSTEM STATE	A designation of system status at a particular time with respect to operable and inoperable equipments. An n-equipment system can exist in 2^n states ranging from all equipments up to all equipments down.
TACTICAL	Pertaining to or necessary for the primary mission of the weapon system.
TIME	When used herein without a modifier the word time is interpreted to mean calendar time.

TIME (Continued)

Active Time – That time during which an item is in the operational inventory. For the FBMWS/SWS, the patrol period. It is the time base for availability calculations in this book.

Alert Time – Time during which a system is ready for operation. For FBMWS/SWS on station awaiting designated targets.

Checkout Time – That element of Repair Time during which repair is being confirmed and verified to be satisfactory.

Corrective Maintenance Time (Repair Time) – That element of Downtime in which work is done to repair trouble or failure. Includes time to obtain tools, documents and spares from local stock rooms, set them up for repair, troubleshoot, test spares if necessary, effect repair, make necessary adjustments and calibrations, confirm the repair by test if necessary and close up the repaired item. It specifically excludes time devoted to off-line repair of any item that was replaced. It also excludes elements of Delay Time such as meals, sleep, administrative delays including the postponement of repair by managerial decision, awaiting spares from off-site or remote locations, etc.

Downtime – Total time during which an item is not in condition to perform its intended function.

Fault Correction Time – That element of Repair Time during which a failure is corrected by (a) repairing in place, (b) removing, repairing, and replacing, or (c) removing and replacing with a like serviceable item.

Fault Location Time – That element of Repair Time during which testing and analysis is performed on an item to isolate a failure.

Inactive Time – That time during which an item is not in active inventory, therefore, not expected to be operable, for the FBMWS/SWS, time not spent on patrol. Not included in availability calculations in this manual.

Item Obtainment Time – That element of Repair Time during which the needed item or items are obtained from stockrooms within the facility. For the FBMWS/SWS, time to obtain items from the ship's stores.

TIME (Continued)	*Maintenance Time* – That part of Downtime when maintenance work is actually being done. *Mission Time* – That element of Uptime when an item is performing its designated mission. *Modification Time* – That element of Downtime during which specific modifications or retrofits are made to an item to add to or improve its characteristics. *Operating Time* – Cumulative Operating Time in testing or use. *Preparation Time* – That element of Repair Time needed to obtain the necessary test equipment and maintenance manuals and set up the necessary equipment. *Reaction Time* – That element of Uptime needed to initiate mission functions, measured from the time a command is received. *Uptime* – That element of Active Time when an item is up, i.e., alert, reacting or performing mission functions. *Uptime, Apparent* – That element of Active Time when an item is thought to be up. Apparent Uptime may be greater than Uptime when failure detection is not immediate.
UPTIME RATIO	The quotient of Uptime divided by Uptime plus Downtime. The Uptime ratio is a statistical estimate of steady-state availability.
VARIABLE	A characteristic or property that is appraised in terms of scalar values.
VARIABLES METHOD	A method whereby the value of certain measurable parameters are equated to "failures" or "errors" if they lie beyond the range of specified critical values. The values of the parameters are often assumed to be normally distributed.
VARIANCE, σ^2	The second moment about the mean of a probability distribution. A measure of the dispersion of random variable about its mean value. In testing, variance is a measure of random errors in a series of measurements.

VARIANCE OF ESTIMATE	The variance associated with a parameter estimate obtained by sampling. For example, $\sigma_{\bar{x}}^2 = \sigma^2/n$ is the variance of estimate of the mean $\bar{x}$ of samples of size n from the distribution of the random variable x having variance σ^2. The square root of the variance of estimate is called the Standard Error.
WEIBULL DISTRIBUTION	The distribution of the smallest value of life, strength or similar property among a sample of components is modeled by the pdf $f(t) = \begin{cases} \left[\beta t^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}}\right] / \alpha^{\beta} & t \geqslant 0, \alpha, \beta > 0 \\ 0 & t < 0 \end{cases}$ The function, called the Weibull model, applies when failure is determined by the strength of a weakest link. The function was introduced by Weibull on empirical grounds based on studies of material strength. It was later derived by Freudenthal and Gumbel from extreme value theory, as the type III asymptote of the minimum extreme value among measurements modeled by an initial distribution bounded below. The location parameter α is minimum expected life; the shape parameter β is a slope. There is also a three parameter Weibull Distribution which involves a scale parameter representing absolute minimum life.

LIST OF SYMBOLS, ACRONYMS AND ABBREVIATIONS

Symbols

a	Coefficient used to compute confidence interval.
A	Steady-state availability.
A_a	Achieved availability. Availability with respect to failure (corrective maintenance) and preventive maintenance jointly.
A_i	Inherent availability. Availability with respect to failure (corrective maintenance).
A_o	Operational availability. Availability attained in actual use.
A_p	Availability with respect to preventive maintenance.
$A(t)$	Pointwise availability.
$\overline{A}_T$	Interval availability.
c, C	A cut or set of components which if all fail will fail the subsystem.
C	Confidence level; in basic method the ratio σ^2/λ.
d	Length of component reliability confidence interval. Also observed downtime in an interval.
D	Largest absolute deviation; also downtime.
e	Base of natural logarithms.
f	Degrees of freedom.
$f(\ )$	Probability density function, or probability mass function.
F	Fractile of the cumulative F–distribution.
g	Event that at least one cut in G occurs.
G	An independent group of cuts.
$h(t)$	Hazard rate function.
i, j, k	Subscripts denoting respectively component, environment, test state.
k	Factor used with accelerated testing.

K	Standard normal deviate.
K_{RCnf}	Tolerance factor for reliability R, confidence C, sample size n, degrees of freedom f.
L	Subscript denoting lower confidence limit.
m	Number of successful units in m of n configuration; also number of observed repairs.
M	Mean of a normally distributed random variable; also equivalent number of missions.
M_c	Corrective maintenance time; repair time.
$\bar{M}_c$	Mean corrective maintenance time; MTTR.
$\bar{M}_{c_G}$	Geometric mean corrective maintenance time.
n	Sample size; also number of variables in model.
n( )	Normal PDF.
N	Number of tests. Also number of items on test.
N( )	Normal CDF.
p	Subscript denoting parallel-related components.
P	Probability.
Q	Unreliability, (1-R).
R	Reliability.
s	Subscript denoting serial-related components; also sample standard deviation: also number of successes in a test.
S	Safety.
t	Actual test time; also the student's -t statistic.
t_i^*	Time between the (i-1)th and the ith failure when n items are placed on test without replacement.
$\bar{t}_i$	Time to failure of the ith item since the beginning of testing.
t_x	Time to failure of the last failing item.
T	Planned test time, also a period of fixed or nominal length.
u	Observed uptime in an interval; also subscript denoting upper confidence limit.
U	Unavailability; also uptime.

U_i	Unavailability with respect to failure; inherent unavailability.
U_p	Unavailability with respect to preventive maintenance.
W	Statistic used in W-test of normality; also ratio of two random variables.
x	Number of failures.
$\overline{X}$	Mean of a sample.
α	Producer's risk; also coefficient used to convert test or operating time (or cycles) to equivalent missions.
β	Consumer's risk; also a bias correction factor in the basic method.
γ	Significance level.
$\gamma(\ )$	Gamma density function.
Γ	Gamma function.
$\Lambda(\ )$	Gamma CDF.
δ	Delta function.
Δ	An incremental change in the value of a variable.
ϵ	Limiting acceptable risk; also error.
θ	Mean time to failure, also mean time between failures.
λ	Failure rate.
μ	Mean of a normally distributed random variable; also repair rate
η	Warrantee period.
ν	Reciprocal of $\overline{M}_{c_G}$.
π	3.14159
Π	Symbol denoting product.
ρ_{ij}	Simple coefficient of correlation between i th and j th variables.
$\rho_{i \cdot jk}$	Multiple correlation coefficient for i th variable.
σ	Standard deviation of a normally distributed random variable.
σ^2	Variance of a normally distributed random variable.

Σ	Symbol denoting summation.
τ	In the Bayesian method, prior or pseudo test time.
ϕ	In the Bayesian method, prior or pseudo failures.
χ^2	Chi-square statistic.
$\hat{\ }$	Indicates statistical point estimate.
$'$	Prime symbol denotes predicted value.
$*$	Star symbol denotes required value, or objective.
$\bar{\ }$	Bar denotes negation (e.g. $\bar{A}$ denotes not A); also denotes average value.
$\cup$	Union symbol, interpreted as the logical "or".
$\cap$	Intersection symbol, interpreted as the logical "and".
$\sim$	Is distributed as.
$\rightarrow$	Becomes.
§	Paragraph.

Acronyms and Abbreviations

AO	Approximately Optimum.
BAN	Best Asymptotically Normal.
BICS	Boolean Indicated Cut Sets.
BLU	Best Linear Unbiased.
CA	Corrective Action.
CAR	Corrective Action Report.
CDF	Cumulative Distribution Function.
CDR	Critical Design Review.
CDRL	Contract Data Requirement List.
CFR	Constant (Instantaneous) Failure Rate.
CLIFS	Coordination, Life, Interchangeability, Function and Safety.
DFR	Decreasing (Instantaneous) Failure Rate.
DSARC	Defense Systems Acquisition Review Council.
EDM	Engineering Development Model.
FBMWS	Fleet Ballistic Missile Weapon System.
FLTAC	Fleet Analysis Center.
FMECA	Failure Mode, Effects, and Criticality Analysis.
FOT	Follow-on Operational Test.
FSR	Failure Summary Report.
FTA	Fault Tree Analysis.
IFR	Increasing (Instantaneous) Failure Rate.
ITP	Integrated Test Program.
ITPP	Integrated Test Program Plan.
LCLS	Lower Critical Limit Specification.
MEC	Mission Essentiality Code.

MIL HDBK	Military Handbook.
MIL STD	Military Standard.
ML	Maximum Likelihood.
MLE	Maximum Likelihood Estimate.
MTBF	Mean Time Between Failures.
MTTF	Mean Time to Failure.
MTTR	Mean Time to Repair.
MVU	Minimum Variance Unbiased.
NAVMAT	Naval Material Command.
NAVORD	Naval Ordnance Systems Command.
NAVSEA	Naval Sea Systems Command.
NAVWEPS	Bureau of Naval Weapons.
OD	Ordnance Document.
OPEVAL	Operational Evaluation.
OT	Operational Test.
PAT	Production Assessment Test.
PDF	Probability Density Function.
PDR	Preliminary Design Review.
PEM	Performance Evaluation Missile.
PERT	Program Evaluation Review Technique.
PLS	Production Lot Sampling.
PMF	Probability Mass Function.
POMP	POSEIDON Modification Program.
PPM	Program Plan Matrix.
PRST	Probability Ratio Sequential Test.
PUAD	Parts Usage and Application Data.

RDT&E	Research, Development, Test and Evaluation.
RMA	Reliability, Maintainability and Availability.
R&R	Repair and Refurbishment, Repair and Replacement, etc.
RV	Random Variable.
SITP	Shipyard Installation Test Program.
SOR	Specific Operational Requirement.
SOW	Statement of Work.
SPALT	SSPO Alteration.
SRA	System Requirements Analysis.
SSPO	Strategic Systems Project Office.
SWS	Strategic Weapon System.
TAAF	Test Analyze and Fix.
TDP	Technical Development Plan.
TECHEVAL	Technical Evaluation.
TOG	Technical Objectives and Guidelines Document.
TRD	Technical Requirements Document.
UCLS	Upper Critical Limit Specification.

Section 1
INTRODUCTION

1.1 PURPOSE

The Strategic Systems Project Office (SSPO) requires each prime contractor to satisfy specified reliability, maintainability and availability (RMA) operational objectives and RMA evaluation requirements. These requirements are primarily specified through NAVSEA OD 21549[1], and by means of Technical Objectives and Guidelines Documents (TOGs), weapon specifications, and SSPO policy statements cited in applicable contracts.

The primary purpose of this document is to provide SSPO contractors with a set of RMA evaluation methodology and techniques to facilitate compliance with contractual RMA evaluation requirements and to foster the achievement of specified RMA operational objectives.

1.2 SCOPE

This document is concerned primarily with evaluating the reliability and availability of strategic weapon systems and subsystems throughout the life-cycle of the program, including concept development, advanced development, full-scale development, production and deployment. However, not all ~~SSPO~~ contractors build subsystems; some supply equipments or other lower level assemblies. The methods presented in this manual are applicable to devices of any assembly level. Thus, the term system, as used herein, should be interpreted as the highest level assembly provided by a contractor. Evaluation as treated herein is normally quantitative, however, in some cases (e.g., the logistic phase of a mission), qualitative evaluation is performed. Evaluation, as discussed herein, is intended to encompass those technical program management elements described in the paragraphs of NAVSEA OD 21549[1] listed in Figure 1-1.

The major objectives of the evaluation program are:

- to establish reliability design criteria
- to provide periodic assessments of achieved reliability
- to identify RMA problem areas
- to provide evidence of compliance with contractual requirements.

It is recognized that many functions other than evaluation are also necessary to an effective reliability and availability program. Figure 1-2 lists some of them.

While it is fully recognized that many of the functions listed in figure 1-2 are major contributors to the design of reliable and maintainable systems and to the detection and correction of reliability and maintainability problems, detailed discussion of those functions is beyond the scope of this manual. They are discussed herein only to the extent that they impact the evaluation functions.

1.3 APPLICABILITY

The evaluation methods described in this document are intended for use by SSPO contractors and subcontractors to the extent specified in their contracts, throughout the entire contractual period. It should be noted that the methods used in this manual may be used on equipment procured for training as well as on tactical equipment. NAVSEA OD 21549[1] is structured for use in a variety of contract phases (e.g., concept development, advanced development, full-scale development, production and deployment). The OD also recognizes that hardware and software are procured at various indenture levels (e.g., subsystem, equipment, component). Not all requirements are uniformly applicable in all contract phases. NAVSEA OD 21549[1] provides a vehicle (Appendix B) to permit tailoring RMA program requirements to contract needs.

While standardization is desirable, it is recognized that new methods may be developed in the future, which offer significant advantages and that special situations may arise in a program for which the methods discussed herein may not be optimal. Although SSPO contractors are encouraged to use the methods described herein, they are also encouraged to develop their own methods for special situations. It is intended that this manual be reviewed periodically to insure its accuracy and currency. Users are encouraged to report errors discovered and recommendations for improvement to:

Department of the Navy
Strategic Systems Project Office
Washington, D.C. 20376
Attention: Code SP2014

While primarily intended for use in SSPO programs, the methods presented in this manual are equally applicable for evaluating the reliability and availability of many complex military or industrial products.

1.4 REFERENCES

1. NAVSEA OD 21549A: Technical Program Management Requirements for Navy SSPO Acquisitions.

PARAGRAPH	TITLE
3.3.1	Reliability Evaluation
3.3.1.1	Reliability Modeling
3.3.1.2	Reliability Apportionment
3.3.1.3	Reliability Predictions
3.3.1.4	Failure Mode, Effects and Criticality Analysis
3.3.1.5	Reliability Data
3.3.1.6	Reliability Assessment
3.3.1.7	Reliability Demonstration
3.3.1.8	Reliability Status Reports
3.3.2.1	Maintainability Apportionment
3.3.2.2	Maintainability Predictions
3.3.2.4	Maintainability Data
3.3.2.5	Maintainability Assessment

Figure 1-1. Evaluation Functions Included in NAVSEA OD 21549[1]

PARAGRAPH	TITLE
3.1.8	Corrective Action System
3.2	Design Control
3.3.2.3	Maintainability Analysis
3.3.2.6	Maintainability Demonstration
3.3.2.7	Maintainability Status Reports
3.4	Test Programs
3.5	Configuration Management Program
3.6	Procurement Control
3.7	Manufacturing Control
3.8	Quality Control

Figure 1-2. Reliability and Maintainability Program Functions Other Than Evaluation Included in NAVSEA OD 21549[1]

Section 2
RELIABILITY AND AVAILABILITY

2.1 IMPORTANCE OF RELIABILITY AND AVAILABILITY

A major task facing a system manager early in a development program is the establishment of system requirements. Among the important quantitative system requirements are those specified for reliability and availability.

Reliability is important because of the *threat of catastrophic loss implicit in functional failure of any part of a weapon system in fleet service.*

Availability is of concern when repair of failures can be achieved during a mission phase. The randomness of demands on systems in service often permits repair of failure without degradation of mission performance. Degradation in performance occurs only when a demand occurs at a time when the system is down for repair or undergoing preventive maintenance (including calibration).

SSPO requires timely and accurate evaluation of reliability and availability, beginning in the development phase of weapon system procurement, as an input to decision making and program control. The evaluation process as defined by this document involves apportionments and predictions followed by measurement and is supported by a variety of analyses (e.g., mission analysis; system analysis; analysis of the integrated test program; failure mode, effects and criticality analysis). The principal purpose of apportionments is to develop reliability and availability design criteria for system elements. The principal purpose of predictions is to provide periodic forecasts of the reliability and availability of the projected (final) design. Apportionments and predictions are followed by objective measurement of reliability and availability beginning when the earliest fabricated units become available for testing. The total life history of development hardware is observed. A data system is used to record development history and to facilitate computation of point and interval estimates of reliability and availability. Often, early test data are derived from tests of equipments or other subordinate assembly levels. System estimates are synthesized by means of a mathematical model. Later, these are supplemented by tests of larger assemblies and the system. Use of a system mathematical model takes advantage of the variety of tests performed at various assembly levels during development. Alternatively, test data can be taken only at system level. In either case the data system provides for largely automated preparation and updating of numerical reports in standardized formats suited to the needs of program management. At all stages of the evaluation (prediction and measurement), results are compared to specified requirements to determine if corrective action is required.

2.2 NATURE OF RELIABILITY AND AVAILABILITY

2.2.1 Nature of Availability

Availability is a dimensionless number defined on the interval $[0, 1]$. Availability is defined as the probability that a system will be up or operable when called upon during a mission. This definition implies the need for analysis of both the system and its mission in quantifying availability. The FBMWS/SWS TOGs have an operational readiness reliability objective. *Availability is equivalent to Operational Readiness Reliability in this manual.* It is an appropriate index during the operational readiness portion of the FBMWS/SWS mission.

A system is down when it is not usable because it is undergoing maintenance [corrective or preventive (including calibration)]; it is available only when it is up with respect to both corrective and preventive maintenance. Thus, system availability is equal to or greater than the product of its availability with respect to failure (corrective maintenance, A_c), and its availability with respect to preventive maintenance, A_p.

$$A \geqslant A_c \cdot A_p \qquad (2\text{-}1)$$

Availability may be greater than the product because of dependencies between preventive and corrective maintenance and because preventive maintenance may often be shortened or postponed when demand occurs. Because of these aspects, this manual will treat availability only from a corrective maintenance viewpoint. The subscript c in A_c will be dropped. Availability must be represented by an equation embodying appropriate assumptions as to the failure and maintenance processes applicable to the system under mission conditions.

Many different availability indices can be derived, because the functions and requirements of systems differ widely; thus no single index or figure-of-merit can meaningfully represent availability for all systems[1]. Moreover, availability may depend on the care and skill with which a system is transported, operated and maintained, as well as on the system design. However, in this manual, use of the term is limited to measures which are primarily properties of system design. This manual assumes an adequate number of spares, etc. for system availability and does not address measures of availability which are primarily reflections of logistic support, spares provisioning and similar factors.

The availability, A, of a system is estimated by a function of mean time between failure (MTBF) and mean time to repair (MTTR),

$$A = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \qquad (2\text{-}2)$$

When the system has exponential failure and repair rates, $\lambda = 1/\text{MTBF}$ and $\mu = 1/\text{MTTR}$. This substitution leads directly to equation 2-4.

The availability of a system over a period of time, such as a submarine patrol of duration T, is known as interval availability and is given by equation 2-3.

$$\overline{A}_T = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{(\lambda+\mu)^2 T}\left[1 - e^{-(\lambda+\mu)T}\right] \qquad (2\text{-}3)$$

where

μ is the system repair rate.

λ is the system failure rate.

Equation 2-3 includes a steady-state term and a transient term.

As T, the mission duration, is increased the interval availability approaches a constant value, designated steady-state availability, which is written:

$$A = \lim_{T\to\infty} \overline{A}_T = \frac{\mu}{\lambda+\mu} \qquad (2\text{-}4)$$

For missions of sufficient length, the transient term in equation 2-3 is ignored and steady-state availability is taken as the applicable measure. Throughout this manual it is assumed that the relevant mission is long enough to justify the use of steady-state availability. Any error stemming from this assumption will tend to render the analysis conservative; that is, interval availability is always greater than steady-state availability.

For many systems, immediate failure detection and repair are not achievable for all failure modes. Some modes of failure will be automatically detected and alarmed, others will be periodically detected during the patrol, and still others will not be detectable during a patrol. This fact gives rise to the concepts of actual and apparent availability. Apparent availability is a measure of the observed status. Actual availability is an index which accounts for the undetected failures by predicting the number of failures that exist but will not be detected. This index is of primary use to system planners, permitting them to perform trade-off studies leading to extra failure detection capability. Before using any availability equation, the analyst must determine that failure detection and repair is a viable option. If it is not, reliability is the appropriate measure.

2.2.2 Dependence of System Availability on the Reliability and Maintainability of System Elements

The availability exhibited by a system during periods of stated length is a statistically distributed variable. Its distribution is jointly determined by the system's reliability and maintainability, which in turn depend on the corresponding properties of the equipments that constitute the system.

Reliability determines the frequency of unscheduled downtimes; maintainability determines their durations and the duration of downtimes for other purposes (e.g., scheduled preventative maintenance and calibration). In its initial configuration the system will possess certain inherent levels of reliability and maintainability which together establish an upper limit on the availability attainable by the system. Beyond this limit, additional effort can be applied to improve item reliability or maintainability or both, in order to increase availability. Figure 2-1 illustrates the dependence of availability on reliability and maintainability. The relative impacts of reliability and maintainability upon availability are often studied in specific situations and the results used in trade-offs and the setting of RMA specifications.

2.2.3 Nature of Reliability

Reliability is defined as the probability of functioning without failure during a specified mission or a portion of a mission. It is an

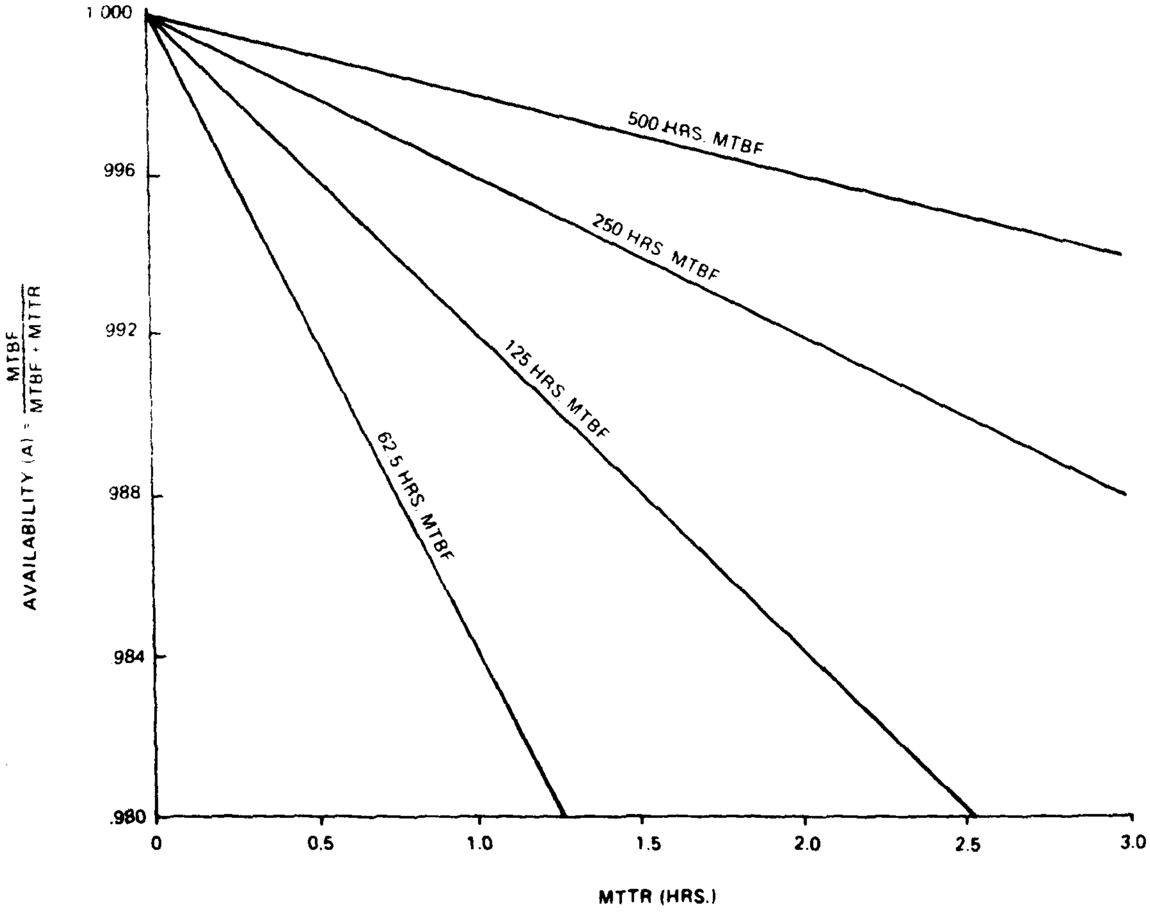

Figure 2-1. Dependence of Availability (A) Upon Reliability (MTBF) and Maintainability (MTTR)

index appropriate to the launch and flight phases of the FBMWS/SWS mission. Reliability also applies to any phase where continuous or standby operation is required and repair is not possible. The definition presupposes explicit definition of a nominal design mission or range of possible missions, and also assumes explicit success criteria, a particular subset of hardware and software functions necessary to the success of the primary mission.

Reliability is expressed as a dimensionless real number in the interval [0, 1], where zero represents certainty of failure and unity represents certainty of success. A general mathematical expression for the reliability of time dependent devices is:

$$R(T) = e^{-\int_0^T h(t)\,dt} \qquad (2\text{-}5)$$

where T is mission duration, $R(0) = 1$ and $h(t)$ is the so-called hazard rate or instantaneous rate of failure. While $h(t)$ is formally time-dependent, both theory and experience support the supposition that for complex systems the rate can often be approximated by a constant after an initial early mortality period and prior to wear out [2-Chapters 4 and 5]. This characteristic shape of the hazard function has given rise to the familiar "bathtub curve", figure 2-2.

Figure 2-2 is a general portrayal of time-varying reliability characteristics of equipment through their life cycles. Good reliability practice is to operate equipment only during its useful life period, between the so-called green-time line, T_G, and red-time line, T_R. In this flat portion of the curve $h(t)$ is called failure rate and is designated by the symbol λ; its reciprocal, $1/\lambda$, is the mean time between failure, MTBF. It can be shown that the probability density of time to failure in this region, $f(t)$ - the unconditional instantaneous rate of failure - is described by the exponential distribution, a one-parameter function fully defined by the single parameter λ. An equivalent expression for reliability in terms of $f(t)$ is:

$$R(T) = \int_T^{\infty} f(t)\,dt \qquad (2\text{-}6)$$

which, for the exponential case, is

$$R(t) = e^{-\lambda t} \qquad (2\text{-}7)$$

Clearly evaluation of λ or MTBF is equivalent to evaluation of the complete reliability function over the entire range of t, wherever the exponential model applies.

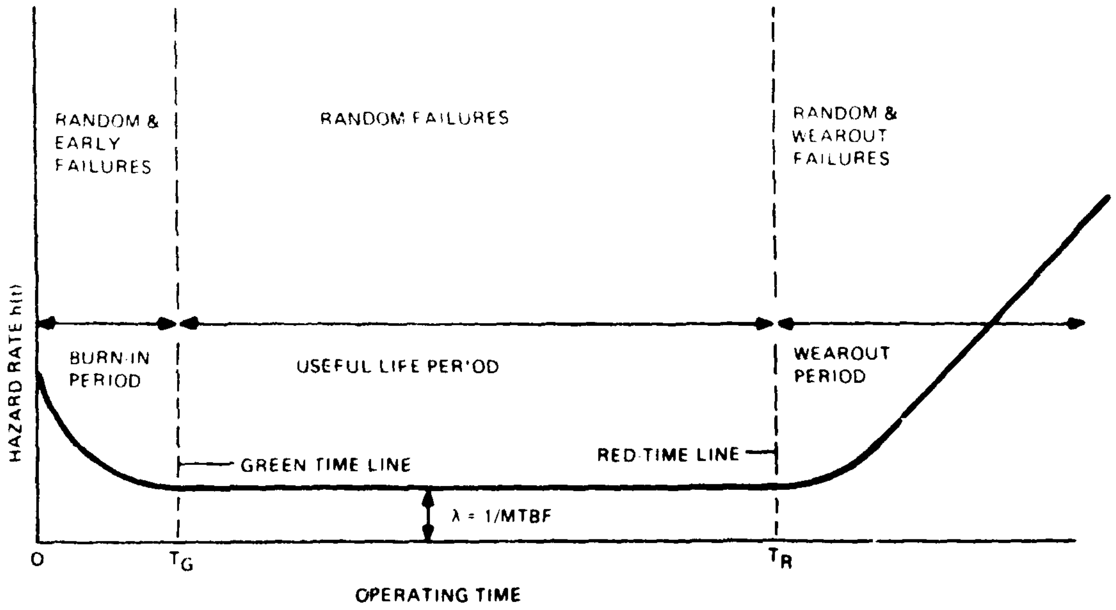

Figure 2-2. Hazard Rate as a Function of Operating Time

If a system remains in service beyond T_R, its failure rate increases rapidly until the system can no longer be supported economically by available spares and maintenance facilities.

When hazard rates are time-varying, as is often true of simple devices, the resulting failure distributions and reliability functions are generally much more complex than in the exponential case [3-Chapter 2]. For such devices, definition of a complete reliability function may require that two or three parameters be given, although a discrete value of R(t) may always be defined directly for a mission of given length.

A number of researchers in the reliability field [4-pages 402-407 for example] have suggested that the advent of semiconductor devices has reduced equipment failure rates and lengthened their useful life beyond the normal use period for a system. In effect, these researchers claim that the failure characteristics of electronic equipment have changed and that present day electronic equipment has a decreasing failure rate (DFR). It has been shown [5-pages 375-383] that mixtures of exponentials have a DFR.

Consider for example two components with constant failure rates λ_1 and λ_2 where $\lambda_1 > \lambda_2$. One thousand of these components are put in service (500 of each type). Initially, the failure rate is $(\lambda_1 + \lambda_2)/2$ but after 500 failures (without replacement) the failure rate of the surviving population would be lower since the expected number surviving from λ_2 population is higher than the expected number surviving from the λ_1 population. The mixture has a DFR.

An important consideration with DFR devices is when to put them into service. When the failure rate is decreasing rapidly (e.g., as in the burn-in period of figure 2-2) a minimum burn-in period should be required and service use should begin after T_G. Even with DFR, service use may be permitted when the hazard rate reaches an acceptably low level. Of course if screening or burn-in is economical it may be cost effective to perform more screening, etc. prior to use. Figure 2-2 may still illustrate this process with minor changes. The useful life period would have a decreasing slope and T_R is not reached during the useful life of the system.

A curve similar to figure 2-2 applies for one-shot devices. In this case the reliability is

$$R = 1 - \frac{x}{n} \qquad (2\text{-}8)$$

where

x is the number of failures
n is the sample size

The primary reason for the decrease prior to T_G in figures 2-2 and 2-3 is that manufacturing defects are being eliminated by test and inspection. The increase after T_R is due to phenomena such as mechanical wear and chemical deterioration. Maintenance should be performed when an item reaches T_R.

The hazard rate for equipment which operates continuously or periodically, such as electronic equipment or jet engines is most often given as a failure rate, λ, and is expressed in failures per million hours or failures per mission. When λ is constant, MTBF = 1/λ. The hazard rate for one-shot devices is a decimal value between 0 and 1 and is usually estimated by the ratio of total failures to total trials.

2.2.4 Nature of Maintainability

Maintainability is a characteristic of system design and installation. Mean preventive maintenance time, mean repair time, and mean down time are commonly used indices of maintainability.

In this manual, assessment of maintenance actions will be limited to actions involved in corrective maintenance (reference corrective maintenance time figure 2-4) that is, modification time, delay time and preventive maintenance down time will be excluded from estimates of mean time to repair.

2.3 THE SPECIFICATION OF RMA REQUIREMENTS

MIL-STD-490[6] provides general guidelines for the preparation of specifications The specification addressed in this manual is the specification of meaningful numerical RMA requirements to guide design and procurement activities and to facilitiate evaluation of the system as development progresses.

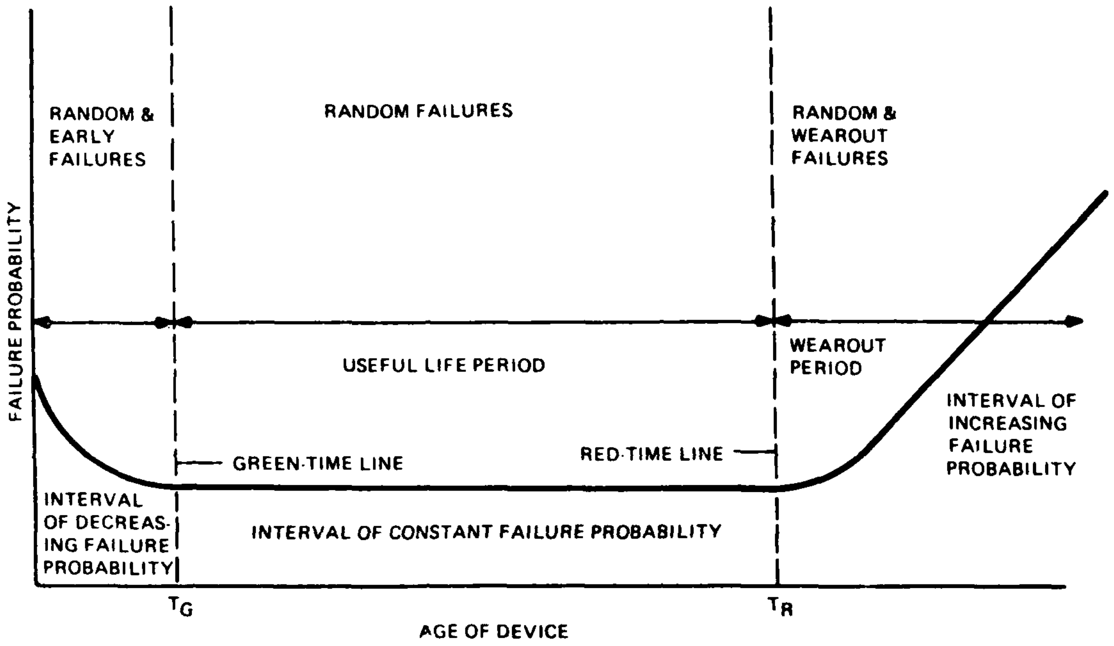

Figure 2-3. Failure Probability Curve as a Function of Age of One-Shot Devices

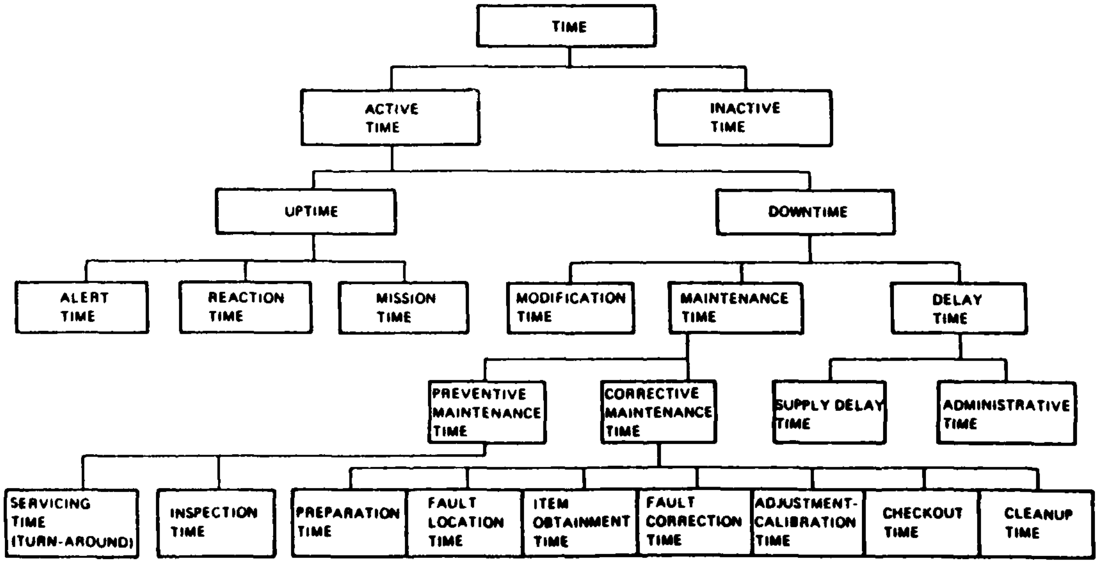

Figure 2-4. Time Relationships

An explicit definition in engineering terms of the intended mission or range of missions and the environments associated with them is essential to proper specification of reliability and availability at every assembly level. The assumption that reliability and availability characteristics are system parameters independent of mission variables is a common error in preparing specifications. Reliability and availability depend critically on the mission. Both the numerical values and the functional forms of the applicable indices depend on the mission. Equally as necessary as mission definition is a numerical requirement applicable to the weapon system or other top-level procurement item; it is the requirement from which system reliability and availability requirements must be derived, and to which subsequent evaluation results must be related.

It is normal practice to establish quantitative requirements for each phase of a tactical mission. Consider, for example, a submarine that goes to sea on periodic patrols and will fire missiles at assigned targets, only if such action is ordered. If no missiles are fired on a given patrol, the operational readiness phase represents the entire mission. If, however, one or more missiles are fired during the patrol, the mission consists of three phases, namely operational readiness, launch and flight. In this example quantitative requirements would be established for the three tactical phases, as indicated below:

Operational Readiness Reliability	Launch Reliability	Flight Reliability
R^*_{OR}	R^*_L	R^*_F

When mission phase reliabilities are statistically independent, the system reliability requirement, R^*_S is:

$$R^*_S = R^*_{OR} \, R^*_L \, R^*_F = A^* \, R^*_L \, R^*_F \tag{2-9}$$

since by definition in this manual $A^* = R^*_{OR}$.

When they are not independent, methods such as those covered in § 4.2.2.1.2.4 are required.

The superscript symbol (*), to be read star, affixed to each symbol identifies a requirement or objective accordingly, the system development program must be tailored to meet or exceed each specified numerical requirement.

The logistic phase of the mission consists of the transportation, handling and storage, installation and check out that takes place prior to the mission and any necessary refit. Numerical requirements are not normally provided for the logistic phase. The system development program must consider the logistic phase and insure that the system is not degraded by the logistic environment.

Availability objectives may be assigned to certain support equipment used in the processing, test and inspection portion of the logistics phase to assure an adequate flow of ready for issue equipment to the fleet.

2.3.1 Specification for Availability

It is not only possible, but frequently desirable, to establish a single quantitative requirement for availability, designated herein as A^*. Its value must necessarily lie in the interval 0 to 1. For tactical reasons it should be as close to 1 as cost, life cycle support requirements and state-of-the-art permit. The value selected for A^* will depend upon a variety of factors such as tactical mission needs, anticipated complexity of the system, expected reliabilities of system elements, maintenance policy (e.g., what equipment will be removed and replaced during the mission if failure occurs, spares stocking points, etc.), capabilities of both diagnostic equipment and operators in fault isolation, and consequences of non-spareable equipment failures. The selection of the value for A^* should, therefore, be done only after all relevant factors, such as those cited above, are analyzed and trade-off decisions are made.

Specifications of a single value of A^* provides a wide trade-off region for reliability (MTBF) and maintainability (MTTR). In many cases it is necessary to constrain this choice by specifying a minimum value of MTBF or a maximum value of MTTR. The advantage of including constraints on reliability or maintainability is that they preclude undesirable system trade-offs (e.g., a computer with an MTBF of 2 hours and an MTTR of .1 hours would have an availability of 0.95 but the probability of successfully running a program requiring 2 hours would be less than 37 percent). As shown in figure 2-1 specifying any two of the RMA parameters fixes the third (e.g., if MTBF = 62.5 hours and MTTR = 1 hour, A must be .984).

2.3.2 Specification for Reliability

Reliability may be specified as a constraint on availability as indicated in § 2.3.1. However, some mission phases do not permit repair. In these cases reliability must be specified as it is the only RMA parameter of interest.

As in the case of availability, a single quantitative requirement for reliability designated herein as R^* can be established. The specification of design requirements or objectives is intended to guide the design effort. Reliability prediction such as prescribed by MIL-STD-756[7] is often used as a measure of compliance.

2.3.3 Specification for Maintainability

Frequently quantitative values are established for system maintainability (M) via equation (2-2) given previously for availability (A). When this equation is solved for mean-time-to-repair (MTTR), we have

$$\text{MTTR} = \frac{(1\text{-}A)}{A} \times \text{MTBF} \qquad (2\text{-}10)$$

Consequently, if both availability and reliability are specified, maintainability (MTTR) is also specified. As previously indicated, there are numerous combinations of R and M that can be chosen to meet availability requirements. The one that is chosen should be one that is an optimum when all principal relevant factors are considered such as R and M program costs, state-of-the-art, and maintenance policy.

In addition to specifying MTTR, it is often desirable to also specify a maximum time to repair. This maximum repair time is that value below which a specified percent of all corrective maintenance tasks should be completed. It is customary for this value to be synonymous with the 95th percentile point in the distribution of corrective maintenance down times. It is often desirable to specify diagnostic requirements (e.g., fault detection capability, fault isolation time) to restrict the R&M trade off region.

2.3.4 Demonstration Requirements

The discussion of demonstration is limited to reliability demonstration in this manual. A maintainability demonstration would be designed to prove, by test, that the actual MTTR is less than the specified value (MTTR*) and that 95% of the repair times are less than the specified maximum repair time [8]. An availability demonstration could directly demonstrate that the A^* requirement is met or it could measure the reliability and maintainability parameters and determine the achieved availability.

Demonstration tests are formal tests designed to accept or reject the hypothesis that the design meets the requirement. The actual reliability achieved by a system cannot be measured as a point on the probability scale, with positive statistical confidence during the system's life. Enforcement of the specification is, therefore, facilitated by providing suitable tests to demonstrate compliance with its requirements.

In this manual we identify and illustrate the application of several principal types of reliability demonstrations in Section 8.

A complete reliability specification is shown in figure 2-5.

Quantities Specified			
Design Objective	Producer's Risk	Minimum Reliability	Consumer's Risk
R^*	α	R_L^*	β

Figure 2-5. Complete Reliability Specification

R^* is the design requirement, that is the reliability which must be exhibited in operational use. The contractor should conduct his design program to meet this requirement.

α is the producer's risk, the probability that a design with reliability of R^* will fail the demonstration test.

R_L^* is a demonstration test parameter, commonly referred to as the minimum acceptable reliability.

β is the consumer's risk. It is the probability that a design with reliability of R_L^* will pass the demonstration test.

The decimal values assigned to the R^* and R_L^* in figure 2-5 are converted to MTBF values θ_0 and θ_1, respectively when a demonstration test plan from MIL-STD-781[9] is to be used.

In those cases where the mission of a weapon system includes more than one phase, it may not always be necessary to establish R demonstration requirements for the product for each mission phase. In these instances, a possible approach is to select the mission phase which has the most stringent requirements for the product and demonstrate that the product meets these most stringent requirements.

2.4 REFERENCES

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2. Bazovsky, I., Reliability Theory and Practice, Prentice Hall, Inc. 1961.

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4. Wong, K. L.: Unified Field (Failure) Theory Demise of the Bathtub Curve. Proceedings Annual Reliability and Maintainability Symposium 1981.

5. Proschan, F.: Theoretical Explanation of Observed Decreasing Failure Rate. Technometrics, V. 5, No. 3, 1963.

6. MIL-STD-490: Specification Practices.

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9. MIL-STD-781: Reliability Tests Exponential Distribution.

Section 3

ESTABLISHMENT AND MANAGEMENT OF A RELIABILITY AND AVAILABILITY EVALUATION PROGRAM

3.1 BASIC REQUIREMENTS

One of the major tasks facing the Strategic Systems Project Office (SSPO) is the establishment of system requirements. Among the important quantitative system requirements are those established for reliability, maintainability, and availability (RMA).

The Department of Defense and the Navy Department have recognized the need for properly specifying both quantitative and qualitative RMA requirements for many years. Recent high-level documents support this need, e.g.:

DoD Directive 5000.3 [1]
DoD Directive 5000.40 [2]
SECNAVINST 3900.36 [3]
NAVMATINST 3000.1 [4]
NAVMATINST 5430.53 [5]
CNM Procurement Policy Memorandum #15 [6]
NAVMAT 09H Guideline Policy #1 [7]
NAVMAT 09H Guideline Policy #2 [8]
NAVMAT 09H Guideline Policy #3 [9]

SSPO has and will continue to emphasize the importance of properly specifying quantitative and evaluation program requirements for RMA in its weapon system procurements. RMA evaluation program requirements are tailored to each specific procurement based on the Technical Objectives and Guidelines Document (TOG) and the checklist (Appendix B) of NAVSEA OD 21549[10].

3.1.1 Review of Basic Requirements

One of the first steps that the contractor should take in planning the RMA evaluation program is to review contractual requirements invoked via the completed checklist (Appendix B) of NAVSEA OD 21549[10]. This checklist will not only indicate which RMA evaluation program elements are invoked on the contractor, but also what kind of modifications, if any, have been made to the basic requirements. The purpose of this review is to establish the scope of the evaluation program. The review must also include the examination of contractual quantitative RMA requirements derived from the TOG and invoked by contractual specifications to establish the degree of difficulty anticipated in meeting these requirements.

3.1.2 Review of Company Policies

Contractor management must review company policies, procedures, guidelines and operating instructions in light of the review conducted in § 3.1.1. Necessary modifications, additions, and updates should be undertaken for this program.

3.2 ELEMENTS OF RMA PROGRAM PLANNING AND MANAGEMENT

The following RMA program elements are required by NAVSEA OD 21549[10].

3.2.1 Management Policy

The contractor must establish and maintain a documented policy for fulfilling contractual RMA requirements. Statements of policy shall form the basic guidelines and the internal company authority for developing and implementing the RMA program. Specific responsibilities must be assigned and action authorities clearly delineated. Personnel performing RMA evaluation functions must have sufficient, well defined responsibility, authority and organizational independence to fulfill specified requirements, to identify and evaluate RMA problems and to accomplish and verify corrective action.

3.2.2 Program Planning

Planning of the RMA evaluation program must be initiated at the earliest feasible moment to assure that methods and controls necessary for fulfilling specified requirements are developed in advance of the necessity for their implementation and maintained as necessary throughout applicable life-cycle phases. The program should use established procedures and instructions, augmented as necessary, to meet RMA requirements. These procedures and instructions must be complete and concise, of a type appropriate for the functions to be controlled, define the responsibilities, and provide the methods and criteria for performance.

Planning should:

- demonstrate an awareness, recognition, and organized approach to the achievement of RMA requirements.
- assure that adequate controls are maintained throughout all phases of contract performance.
- provide for smooth transition of the RMA program throughout all phases of contract performance.
- provide objective evidence of the effective implementation and operation of the RMA program.

3.2.3 Program Plan Matrix

The contractor shall prepare a Program Plan Matrix (PPM) to indicate the means of complying with specified RMA evaluation requirements. The PPM requirements for evaluation are the following. Identify, by paragraph number of NAVSEA OD 21549[10], the documents that satisfy each RMA requirement and the organization that has primary responsibility for implementation. When documents are not available or are inadequate for satisfying specific requirements, additional documents required should be identified and an estimated completion date specified. The PPM should be maintained to reflect current documentation and organizational responsibilities. In addition, the PPM should include milestones and schedules for accomplishing each RMA requirement.

3.2.4 Integrated Data System

The contractor shall establish and maintain a system for the effective collection, control, processing and use of data generated to support: design engineering, the quality, reliability and maintainability programs; the subsystem safety program and the corrective action system. The collection, processing, storage, maintenance, retrieval, control and distribution procedures should be designed to meet these detailed data needs. The system should be designed to assure that records of similar or related data elements from various contractor internal functional areas, subcontractors and other external sources are compatible for the purpose of retrieval and analysis. The supporting documentation should include:

- A list of logs, forms, and other media used to record data, along with the description and storage location of these documents.
- A list and description of output reports, indicating the preparing organization and periodicity.
- A tabulation of applicable procedures that describe the preparation and flow of input data and output reports.

Additional details are provided later in this manual on data needs for RMA.

3.2.5 Corrective Action System

The contractor shall establish and maintain a system for corrective action of problems/failures. The system shall include: reporting of problems/failures, investigation, analysis, and performance of actions to correct problems/failures and preclude recurrence. The system should use problem/failure data from tests and inspections throughout the contract effort. Reporting of problem/failure data and corrective actions should be in a form to assure smooth transition and integration through the various phases of program performance.

A key element of a development program is effective corrective action (CA). Repair and maintenance actions are considered disposition CA since they only affect the failed item. Effective CA will eliminate or reduce future item rejections and failures. CA may apply to both relevant/non-relevant as well as verified/not verified (false alarm) failures. (Note: non-relevant and not verified failures also force equipment out of service.)

Figure 3-1 shows the elements of a simplified corrective action system, and the manner in which it provides feedback to manufacturing and R&R activities to stimulate reliability growth.

CA usually cannot be determined at the time the failure is observed. It may not even be determinable after verification testing and teardown. Detailed failure analysis is usually required to develop effective CA. This includes a review of all data (observations, test findings, lab analysis, etc.) to make a determination as to the cause of failure; then effective CA can be developed and approved through the corrective action board (CAB). The results of the failure analysis should be documented in a comprehensive failure analysis report which is conclusionary in nature (i.e., specifies the results of the analysis, the cause of failure, CA implemented, implementation data, etc.).

The results of the failure reports are then used to measure the effectiveness of corrective actions. Failure concurrence with CA implemented, can be plotted as a function of time to determine CA effectiveness.

3.2.6 Documentation

The contractor shall develop and maintain those documents necessary to fulfill specified RMA requirements. A system for scheduling and monitoring the preparation of these documents should be maintained to assure that preparation is timely in relation to program milestones.

3.2.7 Audits

The contractor shall audit his RMA program periodically to determine compliance with specified RMA requirements. Audits should be planned to begin at the start of the contract and should be conducted throughout the life of the contract. Planning should consider program milestones and delineate criteria for audit scope and frequency. Audits should be performed by an independent audit group or by personnel not having responsibilities in the area being audited. Results of each audit should be documented in a report to appropriate managers and supervisors. Action should be taken to assure timely correction of deficiencies and follow-up performed to verify effectiveness of corrective action.

3.2.8 Configuration Management

The contractor shall establish and implement a program for configuration management of deliverable hardware and software (including training and special test and inspection equipment with concomitant software). The program shall assure implementation of requirements for configuration identification, control, status accounting and verification in accordance with SSPINST 4130.4[11].

3.2.9 SPALT Management

The contractor shall establish and implement a program for monitoring and providing data regarding the proposal, development, production and implementation of each essential SPALT in accordance with SSPINST P4720.1[12].

3.3 ELEMENTS OF A RELIABILITY AND AVAILABILITY EVALUATION PROGRAM

The basic elements of a reliability and availability evaluation program and the flow of associated activities, promulgated in NAVSEA OD 21549[12], are shown in figure 3-2. The various elements are discussed in detail elsewhere in this manual. The paragraphs are identified in the figure.

3.3.1 Reliability and Availability Analysis

A major element of an evaluation program is reliability and availability analysis. This analysis consists of three major functions; mission analysis, system analysis, and analysis of the integrated test program, and is discussed in Section 4. It is supported by:

- A data system function responsible for collecting, controlling, processing, and using data from tests, operations and maintenance activities.

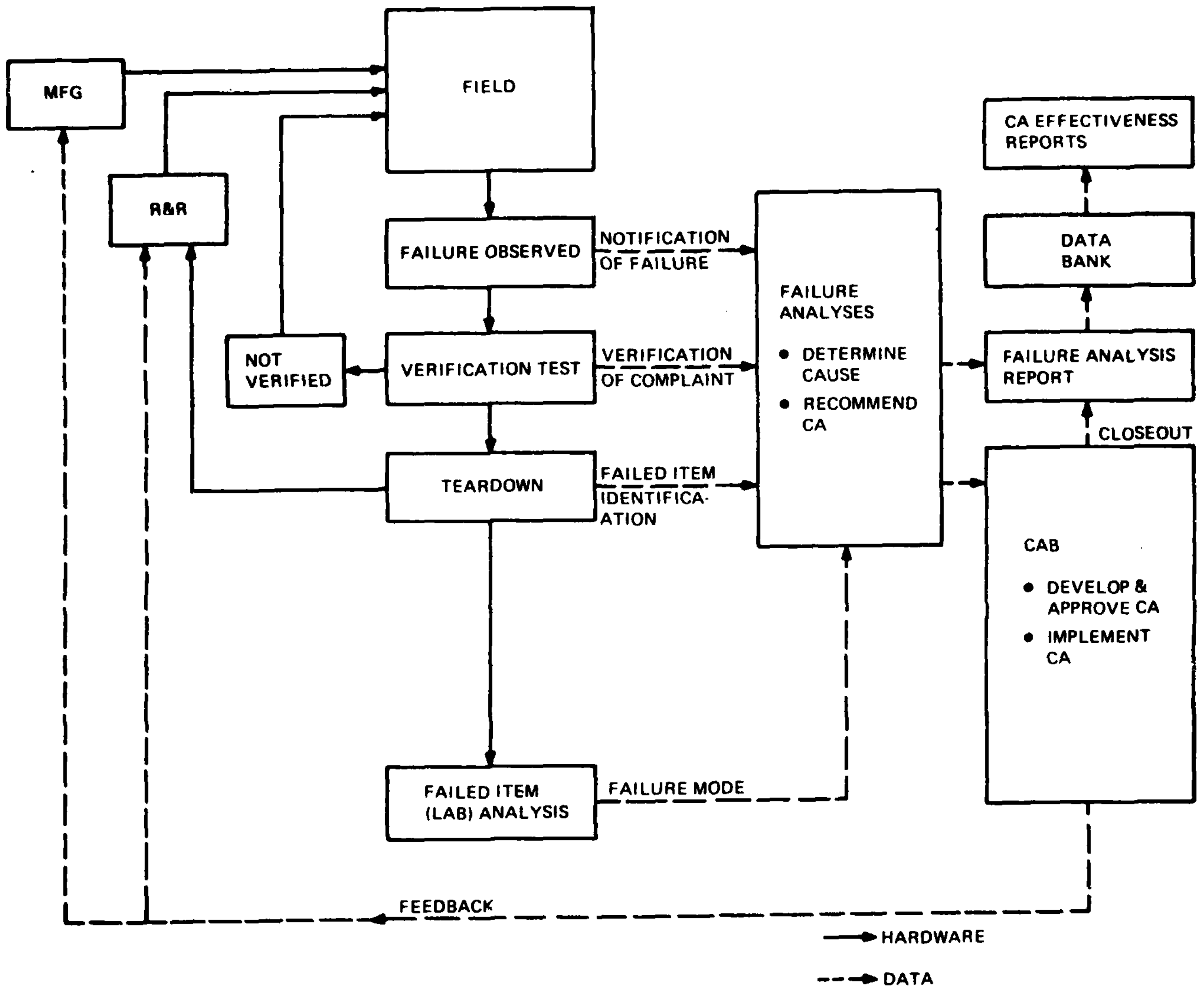

Figure 3-1. Simplified Corrective Action System

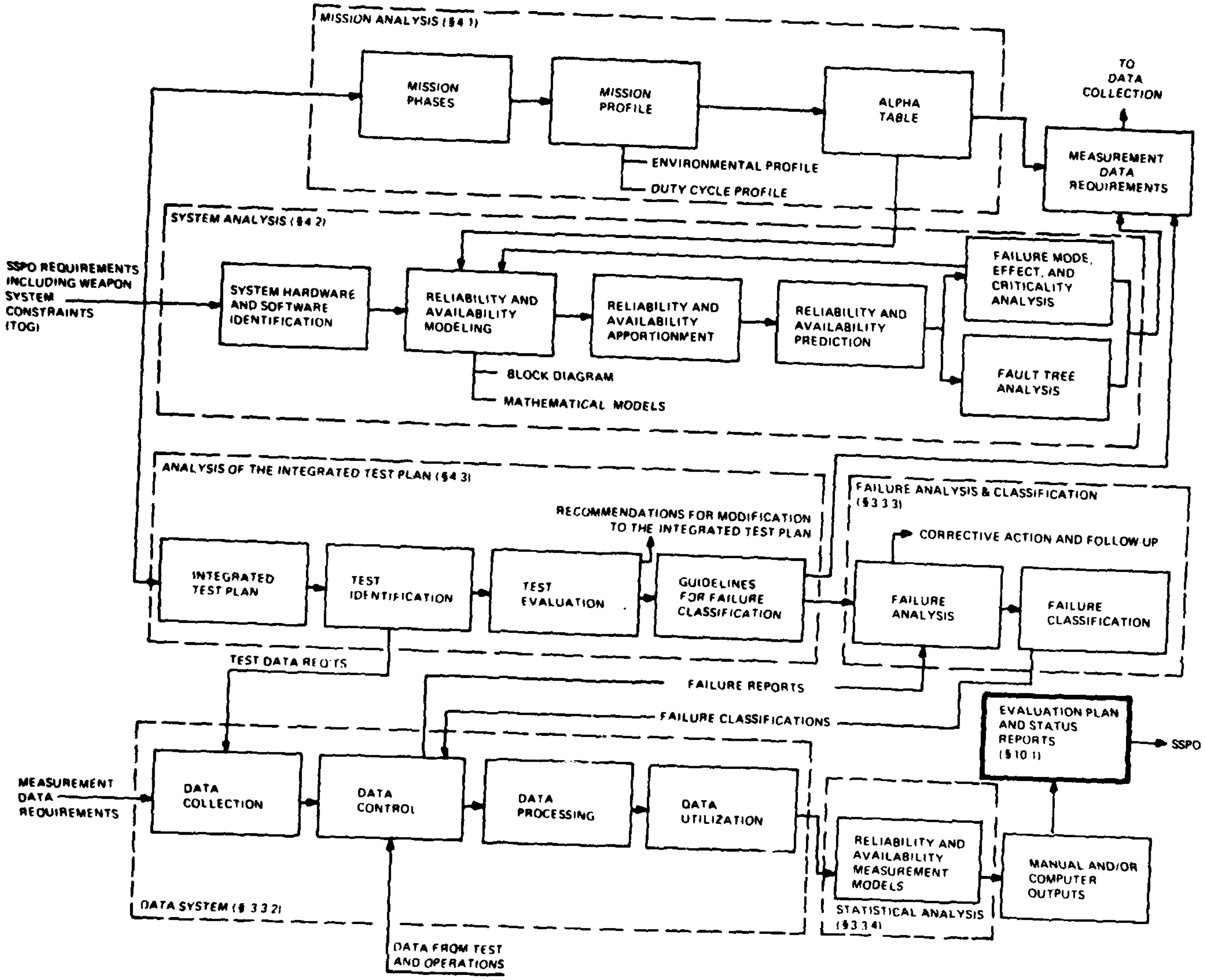

Figure 3-2. Flow of Activities in a Reliability and Availability Evaluation Program

- A failure analysis and classification function responsible for analyzing and classifying each failure, providing corrective action and closing out each failure.
- A statistical analysis function for necessary statistical support, including determining confidence bounds on estimated reliability and availability.

3.3.2 Data System

A data system is needed to support reliability and availability evaluation. Each of the three functions of reliability and availability analysis (mission analysis, system analysis, and analysis of the integrated test plan) help to generate evaluation data requirements. System analysis defines the hardware and the software for which data are to be collected: mission analysis defines the applicable environments and duty cycles; analysis of the integrated test program identifies tests relevant to reliability and availability evaluation.

Four functions comprise a data system. They are collection, control, processing and utilization. An effective data system embodying each of these must be established during the development phase. Its basic structure should be sufficiently flexible so that with minimum modifications, principally to the collection and control functions, subsequent operational data from subsystems in fleet service can be employed to extend evaluation into the fleet use phase.

Data systems are discussed in detail in Section 9.

3.3.3 Failure Analysis and Classification

The primary goal of failure analysis is reliability and availability improvement. This improvement is obtained through the engineering analysis of each failure in order to identify the mechanism and cause of failure and to recommend appropriate corrective action

Failure analysis supports the reliability and availability evaluation process by enabling proper classifications to be made of each failure. Guidelines for failure classification should be established before testing begins. Various classifications are useful for reliability and availability evaluation. Most important is that each failure must be classified as relevant or non-relevant; only relevant failures are counted in computing reliability and availability. Rules for making this classification should be established. Other classification criteria such as severity (catastrophic, critical, major, minor) may also impact reliability and availability measurement and should be defined specifically for each system.

The failure analysis system is a closed-loop process, in which all failures are evaluated to determine the need for corrective action and each failure is closed out by an appropriate corrective action.

3.3.4 Statistical Analysis

Reliability and availability parameter estimates are obtained by using appropriate models. This manual describes several methods approved for selected use with proper application in SSPO programs.

3.4 IMPLEMENTING A RELIABILITY AND AVAILABILITY EVALUATION PROGRAM

In support of its decision-making functions, SSPO requires reliability and availability evaluations beginning in the development phase, before the first fabrication of hardware and software for testing and continuing through the system's operational or fleet use phase. Implementation of a program to meet these needs is a joint activity of the weapon system manager and the system contractors. To this end, SSPO weapon system management specifies top-level system reliability and availability requirements at the onset of the development program. SSPO management also stipulates the need for reliability and availability evaluation as part of the Weapon System Requirements Specification and in companion documents such as NAVSEA OD 21549[10]. In support of the contractor's analytical tasks, as described in Section 4 of this manual, SSPO also provides information on the intended mission, system interfaces, schedules, and the operational use, and logistic environments. Later, SSPO management will also function to review and integrate contractor's outputs for the evaluation of the reliability and availability of the complete weapon system.

The contractor's responsibilities include establishing reliability and availability evaluation as program activities by means of policy directives and designation of organizational responsibilities and authority for the analysis, data, reporting, and corrective action functions which comprise the program. Figure 3-3 illustrates SSPO/contractor responsibilities and relationships in a weapon system evaluation program.

To implement the evaluation program for a particular system, the contractor must perform the mission and system analysis described herein, develop the necessary mathematical models for reliability and availability, and determine the type and quantity of data necessary to solve the models. The contractor must analyze the integrated test program to determine the quantity and quality of data applicable to reliability and availability evaluation. Procedures, instructions and forms must be developed to collect, monitor and process data from the test and operational sites. A manual system, computer programs or a combination thereof for processing the data and generating tabular portions of reports must also be developed. Most important, if the evaluation program is to be of benefit to the project, is the need for a suitable internal information loop to assure that contractor management and other cognizant project personnel are made aware of the current system reliability and availability, particularly system elements that may require corrective action to achieve satisfactory reliability or availability. Timeliness is of the essence.

Most contractors have company policies, procedures, and operating instructions which cover reliability and availability in detail. These should be reviewed against the requirements for the particular system, and the current contractor organization structure, to assure that they are complete and up-to-date.

With the beginning of testing, the data system is implemented to collect, control, process and utilize data to measure reliability and availability. Status reports are issued at intervals as required by contract. The information loop is closed by feedback of evaluation results, through the contractor's management structure, to the engineering, production and assurance activities.

3.5 REFERENCES

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2. DoD Directive 5000.40: Reliability and Maintainability, 1980.
3. SECNAVINST 3900.36: Reliability and Maintainability of Naval Material, Policy for, 1960.
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5. NAVMATINST 5430.53: CNM Staff Directorate for Reliability and Maintainability, 1974.
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7. NAVMAT 09H Guideline Policy #1: Specification of Reliability and Maintainability Requirements in Navy Contracts, 1973.
8. NAVMAT 09H Guideline Policy #2: Specification of Reliability and Maintainability Requirements in Navy Contracts, 1974.
9. NAVMAT 09H Guideline Policy #3: Generation of Mission Profiles, 1974.
10. NAVSEA OD 21549A: Technical Program Management Requirements for Navy SSPO Acquisitions, 1981.
11. SSPINST 4130.4: Configuration Management Plans; Requirements for, 1971.
12. SSPINST P4720.1: SPALT Policies and Procedures for Alteration of FBM Weapon System Equipment.

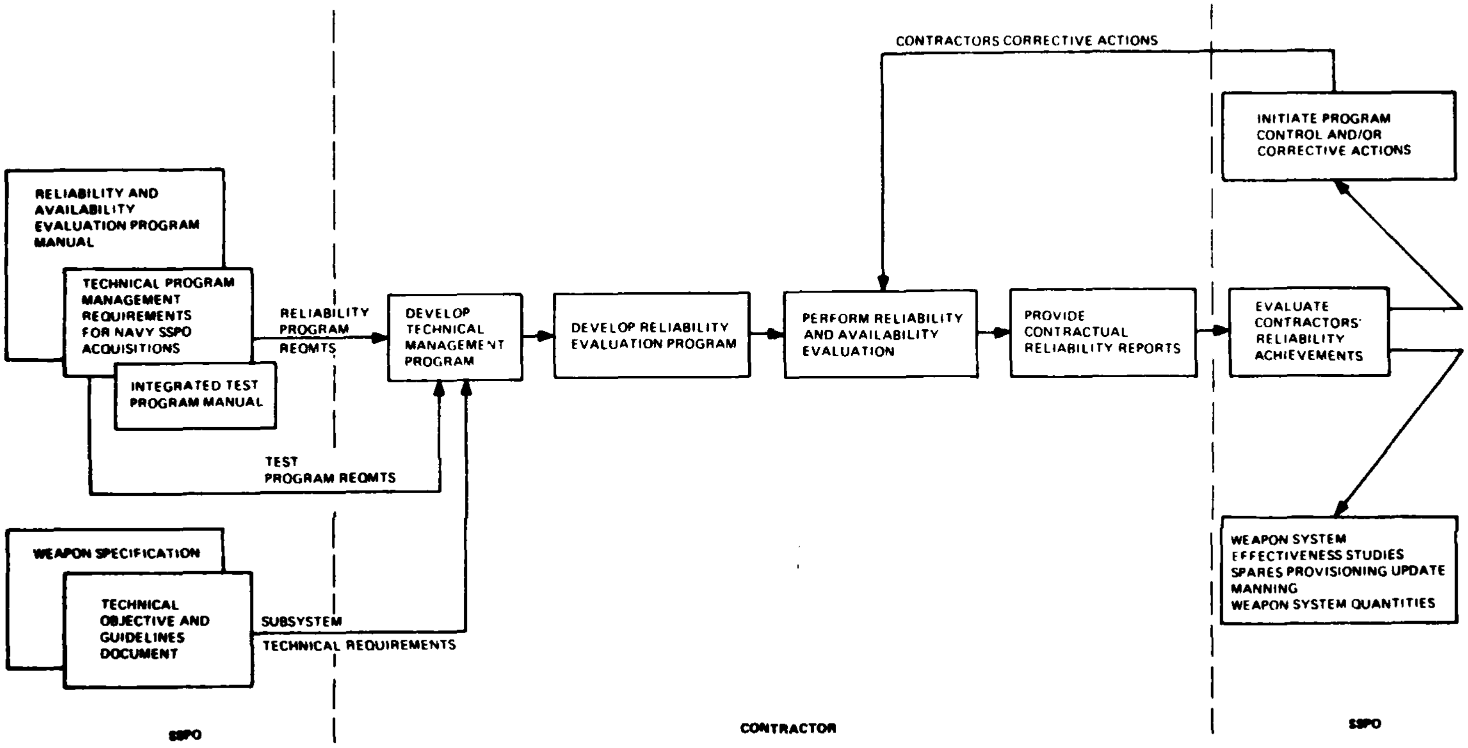


Figure 3-3. Responsibilities and Relationships in a Weapon System Reliability and Availability Evaluation Program

Section 4
RELIABILITY AND AVAILABILITY ANALYSIS

Three types of analysis are discussed in this section, namely, mission analysis, system analysis, and analysis of the integrated test program.

4.1 MISSION ANALYSIS

Mission analysis is a process for defining with precision the mission for which availability and reliability are to be evaluated. Mission analysis identifies hardware and software performance functions, success criteria, duty cycles, environmental stress levels, and time of exposure to each environment. The TOG is used as the baseline for the mission analysis. The results of the mission analysis are used in the development of design specifications for the subsystem and lower level elements. Mission analysis for reliability and availability evaluation includes the definition of mission phases and the development of mission (environmental stress and duty cycle) profiles.

4.1.1 Definition of Mission Phases

The mission is represented as a sequence of events. When a range of alternative missions is possible, each is examined as a distinct event sequence. Each total mission is then separated into phases. For systems such as FBMWS/SWS four phases are normally defined:

a. Logistic phase (including transportation, handling, storage, processing, refit, and test).
b. Operational readiness phase.
c. Launch phase.
d. Flight phase.

A reliability figure of merit is usually appropriate to any mission phase that begins with a demand on the system or device under analysis, such that the mission cannot be successful if failure occurs at any time during the phase. Normally, the launch and flight phases are in this category. An availability figure of merit is appropriate to any mission phase in which success can follow failure and repair. The operational readiness phase is in this category.

The general sequence of mission phases for a FBMWS/SWS patrol is shown in figures 4-1 and 4-2. (It should be noted that the logistic phase occurs before and after a patrol).

The weapon system's tactical mission is defined for a patrol period, t_p, which in the absence of a demand for launching of missiles is of nominal duration, T. For this mission the entire patrol is the operational readiness phase, (Figure 4-1A). An availability figure of merit is appropriate in this case.

Should a demand be made on the weapon system, that demand terminates the operational readiness phase and initiates the ensuing launch phase which may include a hold period at time t_1, and flight phase at time t_2 (Figure 4-1B).

A successful mission precludes failure of any on-line, non-redundant system element during the launch phase, regardless of whether the element is repairable. This is a valid constraint for purposes of hardware evaluation, even though it may be possible in a tactical situation for the submarine's crew to effect one or more missile launches despite certain equipment failures. Thus, the figure of merit during the launch phase (and of flight hardware and software in the flight phase) is reliability or probability of failure-free operation.

In the development of the mission profiles for each phase, the Analysis for Design paragraph of NAVSEA OD 21549[1] requires that the most severe design constraints be identified.

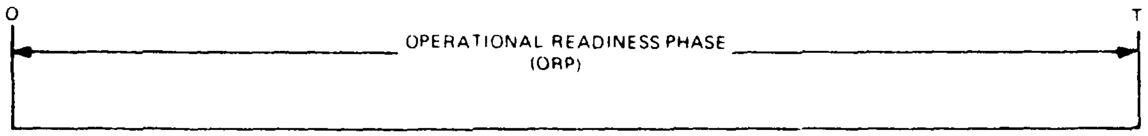


Figure 4-1A. Patrol Without Launch

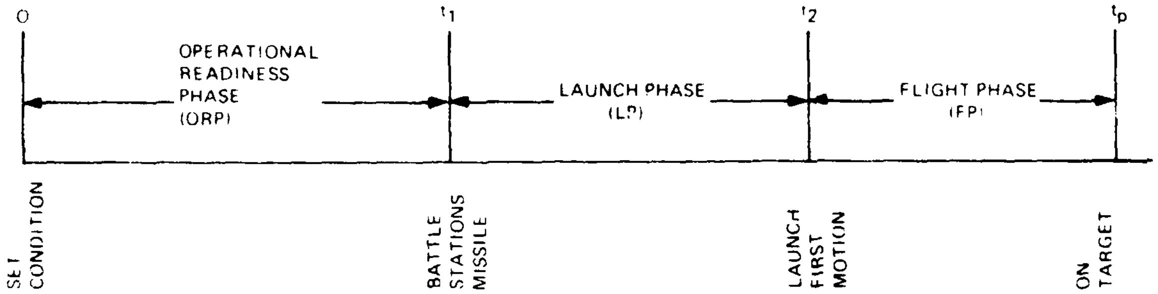


Figure 4-1B. Patrol With Launch

Figure 4-1. Definition of Tactical Mission Phases

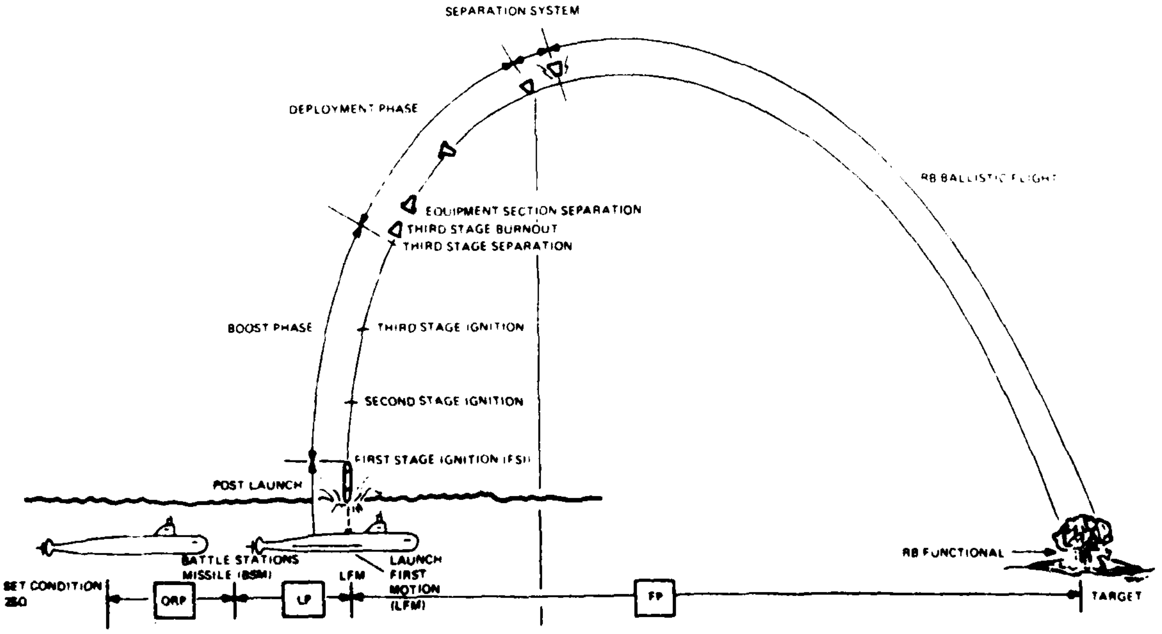


Figure 4-2. Tactical Mission Phases

4.1.2 Development of Mission Profiles

A mission profile is developed for each phase defined. Two kinds of information are developed in the preparation of a mission profile: an environmental profile containing the level and duration of exposure to each applicable environment stress and the related duty cycle profile (i.e., whether the device is operating, non-operating, or cycling) for the hardware or software.

To prepare a mission profile the analyst lists the operational modes of the system in each mission phase. Performance functions required by each mode in each phase are then listed and associated with the hardware and software necessary to accomplish them. A form having the general information content illustrated in figure 4-3 is helpful in organizing this portion of the analysis. In general, not all of a system's functions will be equally important to the mission. Thus it is necessary to define the minimum limits of successful performance for purposes of reliability and availability analysis. This is accomplished by listing that subset of the performance functions that is essential to the primary mission.

Performance times required for each of the essential functions are then listed. Where performance times are random variables, their maximum values should be used to ensure a conservative approach. Environmental levels that depart significantly from room ambient are then listed, together with maximum times of system exposure to those environments for each operational hardware state (A, B, C, D) as defined below.

Reliability models can sometimes be simplified considerably by adopting the mission as a common unit of time. When this is done the term T representing mission length becomes unity and drops from the reliability equation, failure rates are expressed in failures per mission, the reliability under evaluation is $R(T) = e^{-\int_0^T h(t)\,dt}$ or $R(1) = e^{-\int_0^1 h(t)dt}$ and for the exponential case this becomes $R(T) = e^{-\lambda T}$ and $R(1) = e^{-\lambda}$ and need not be treated as a time function. All elements of the system are analyzed in terms of their defined mission. This normalization simplifies the task of combining element reliabilities in system models.

The coefficient α is used to convert test time from units of minutes or cycles to units of equivalent missions. For a given hardware item, alpha is a reciprocal of mission exposure time, either environmental or operational. Its dimensions are missions per minute or cycle, whichever is appropriate to the device. Figure 4-4 is an example of a table of α coefficients for hardware elements of a system. The following information is also tabulated for each component:

Environmental - Mission environmental stresses experienced by the component. Operating life is considered as a separate environment.

Hardware State - Four hardware states are defined as listed below.

Mission Phase:				Phase Duration: Space or Distance:		
System Mode	Function	Related Hardware and Software	Function and time Duration	Success Criteria	Duty Cycle	Environment and Time Duration

Figure 4-3. Information Required for the Development of Mission Profiles

HARDWARE	HARDWARE DRAWING NUMBER	PHASE #1						PHASE #2										PHASE #3					
		LIFE		HIGH TEMPERATURE				LIFE		HIGH TEMPERATURE				VIBRATION				LIFE		HIGH TEMPERATURE			
		C	D	A	B	C	D	C	D	A	B	C	D	A	B	C	D	C	D	A	B	C	D
RECEIVER	194C796P1		1/5				1		1/20				1/20				1/10		1/5				1/2
TRANSOLATOR	194C543G3		1/5				1		1/20				1/20				1/10		1/5				1/2
PROGRAMMER	194C696G1		1/5				1		1/20				1/20				1/10		1/5				1/2
BAROSWITCH	863C799G1	1				1		1/3				1/3				1/3							
PULSE GENERATOR	215E175G1		1				1		1/5	1/15			1/5	1/5			1/5		1/2				1/2
FLASHING LIGHT	681C643G3		1				1		1/5	1/15			1/5	1/5			1/5		1/2				1/2
BEACON	563E617G1		1				1		1/5	1/15			1/5	1/5									
VEHICLE CONTROLLER	179D869G3		1/5				1		1/20				1/20				1/10		1/5				1/2
PROPULSION	215E168G2		1/5				1		1/20				1/20				1/10		1/5				1/2
BATTERIES	179B970P2		1/5				1		1/20				1/20				1/10		1/5				1/2
TV CAMERA	215E133G1			1					1/20				1/20				1/10						
SENSORS	926B975G2		1/5				1		1/20				1/20				1/10						
MULTICODER	926B972G1		1/5				1		1/20				1/20				1/10						
AMPLIFIER	604D147G1		1/5				1		1/20				1/20				1/10						
TRANSMITTER	926B979P1		1/3				1		1/12	1/8			1/12				1/10						

State A – **Non-operating but must survive and be operational in a later mission phase.**

State B – **Non-operating but must not operate prematurely and must be operational in a later mission phase.**

State C – **Operating.** *Duration countable in cycles or discrete events.*

State D – **Operating.** Duration measurable in units of time.

Each Alpha Value represents, for the specific hardware indicated, the reciprocal of either

(a) The amount of time, in minutes, that a piece of hardware is expected to be exposed to an environmental state;

(b) The number of cycles that a piece of hardware is expected to be exposed to an environment and state during one mission phase.

Note: The environment alpha value may be corrected, in some instances, by an accelerating factor.

The software alpha values would follow those for hardware, only the life column in each phase is applicable for software.

Figure 4-4. Alpha Table

State A – Non-operating but must survive and be operational in a later mission phase.
State B – Non-operating but must not operate prematurely and must be operational in a later mission phase.
State C – Operating. Duration countable in cycles or discrete events.
State D – Operating. Duration measurable in units of time.

4.1.2.1 The Environmental Profile

The environmental profile is an important element of the mission profile. It lists anticipated exposure of each hardware item to environmental stresses (e.g., temperature, vibration, shock, acceleration, radiation) during each mission phase.

An environmental profile includes as a minimum the time duration of each mission phase and the level of each environment imposed on the item during each mission phase.

Environmental Profile information is required for the development of hardware specifications and the planning of test activities to ensure that hardware is designed to survive the environments that will be encountered in all mission phases including transportation, storage, handling, mating and checkout, as well as in use, and that the ability to withstand all environments is verified in the integrated test program by test, analysis, or both.

Environmental Profiles are not required for software. Software consists of instructions, independent of the storage medium, and is therefore not directly affected by the physical environment. Software may be stored on magnetic tapes or disks, paper tapes or cards, or in a core memory. The hardware in which software is stored is often affected by its physical environment (e.g., magnetic or electrostatic fields). A hostile environment may erase all or part of a correct software program if the storage medium is not properly protected, but this is considered a hardware problem. The physical environment may indirectly affect software performance in other ways. The response of input sensors to environmental variables may affect the logical path taken through a computer program, thereby revealing a software error not present in other paths. In this case also, the environment has not actually affected the software's reliability; the defect was always present, though unrevealed.

4.1.2.2 The Duty Cycle Profile

The duty cycle profile defines the state (operating, non-operating or cycling) of each item in a system during each mission phase. It includes as a minimum; 1) the time duration, distance, number of cycles, etc. of each mission phase, 2) a description of what each item must do during the mission phase including its success or failure criteria, 3) anticipated total time, cycles, etc. in each state (operating, non-operating, cycling) during each mission phase. Exposure time is normally measured in minutes for operating and non-operating states. The total number of cycles occurring during the mission phase, is used for the cycling state. A duty cycle profile is of less importance during the logistic phase, when most equipment is not operating, however it must be considered when applicable.

4.1.2.3 Alpha Values

The alpha value for each environment is the reciprocal of the time (in minutes, cycles or other appropriate units) during which the system will be exposed to the environment during the mission. Its purpose is to convert failures per unit time or cycle to failures per mission. The alpha values normalize data to failures per mission permitting them to be combined conveniently.

Example: An equipment consists of three serially related components; it operates in a benign environment and has a one-hour mission.

The duty cycles of the components are:

Component A	operates one hour
Component B	operates 40 minutes
Component C	operates 20 minutes

Alpha values for the components are 1/60, 1/40, 1/20 respectively.

Now if 1,000 hours of equipment level testing and 1,000 hours of component testing are accumulated on each component, total accumulated test time on each component in equivalent missions is,

Component	From Equipment Test (Equiv. Missions)	From Component Test			Total Equiv. Missions
		Minutes	Alpha Values	Equiv. Missions	
A	1,000	60.000	1/60	1,000	2,000
B	1,000	60.000	1/40	1,500	2,500
C	1,000	60,000	1/20	3,000	4,000

The need to normalize to equivalent missions as a time base stems from the desire to use component-level test data in conjunction with equipment-level test data. A contractor using only equipment level tests would not need to normalize to equivalent missions.

The failures charged to each component can be divided by the component time in missions; the sum of these failure rates is the equipment failure rate in failures per mission.

Non-operating failure rates can be added if they are significant. In benign environments non-operating failure rates are usually negligible. The *non-operating* alpha values for components B and C would be 1/20 and 1/40 respectively, since the non-operating times during each sixty minute mission are twenty minutes for component B and forty minutes for component C. Since component A operates continuously, its non-operating failure rate would not enter the calculations.

Alpha values are also needed when combined environments are experienced in a mission. In these cases, test data are normalized to equivalent missions for each environment separately. For environments such as temperature and vibration, both non-operating and operating failure rates and alpha values are appropriate, since test data can provide appropriate estimates of the failure rates. *Caution:* It is always necessary to relate the test environment to the mission environment (e.g., § 4.1.2.4).

4.1.2.4 Acceleration Factors

Normal practice in the analysis of an integrated test program is to accept environmental test data as useful for reliability evaluation if and only if the test environment is essentially the same as the mission environment. This must usually be evaluated on an environment by environment basis, because the ability to accurately duplicate the combined environmental effects that occur in most missions does not exist in many test facilities.

In some cases contractors may plan to run accelerated tests. Accelerated test data can be used for reliability evaluation when the contractor has data to relate the failure rate in test to the failure rate in the mission. Such a relationship is often expressed by a k-factor or acceleration factor. For example, if one hour of test in some accelerated environment is known to be equivalent to two hours of mission exposure, the acceleration factor, k, would be 2. The test time in equivalent missions is then $k\alpha t$. This procedure is somewhat similar to the use of environmental factors (Π_E) in MIL-HDBK-217[2] predictions.

Conceptually, k-factors can take on values less than one, but the purpose of accelerated testing is to reduce test time, thus k-factors are normally greater than one. MIL-HDBK-217[2] cautions that extrapolation of environmental modifiers is completely invalid. A similar caution applies to the use of k-factors. The contractor must have data to justify the use of a k-factor used in reliability evaluation or demonstration.

4.2 SYSTEM ANALYSIS

System analysis is a study of the means by which the hardware and software that comprise a system are able to respond to the demands of the mission. As used in this manual, the term encompasses seven activities: 1) listing the system configuration and performance functions, 2) reliability and availability modeling, 3) reliability and availability apportionment, 4) reliability and availability prediction, 5) failure mode, effects and criticality analysis, 6) fault tree analysis, and 7) development of measurement data requirements.

4.2.1 Listing System Configuration and Performance Functions

Configuration of the system's hardware can be defined by a hardware list indentured by assembly level. Various assembly level categories are commonly used. A typical set of categories is:

- System
  - Subsystem
    - Equipment
      - Component
        - Module
          - Part

While hardware listing is the simpler approach, configuration can also be defined by tabulating the hardware required to realize system functions. A functional listing is of particular value when a system is complex, multifunctional or has many interfaces with other systems. Both approaches are illustrated in figure 4-5.

The configuration of a software system can also be defined as an indentured list. When software development is done under structured programming, self-contained programs, subprograms and routines, each separately compilable and independently testable, are programmed to perform one or more well-defined functional tasks. But it is usually simpler to list software elements by function. Usually a list of modules can be directly extracted from a functional diagram of a software system. Figure 4-6 is a simplified functional diagram of a land-based radar software system that filters raw radar data to predict the trajectory of a tracked missile. This information can be used to keep the radar "on track". The components of the software system are the executive program, the Kalman filter, coordinate transformation, predicted trajectory subprograms, and integration and matrix inversion routines which support the Kalman filter. A software system is composed of programming instructions. Each indentured element must be given an identifying number and subsequent changes must be closely controlled by a configuration management system. The media on which the instructions are stored are part of the computer hardware.

Function		Component	
A Vehicle Control			
A1	Command Link	11	Receiver
		4	Transolator
		8	Programmer
		9	Baroswitch
A2	Tracking	13	Pulse Generator
		14	Flashing Light
		15	Beacon
A3	Vehicle Operation	1	Vehicle Controller
		2	Propulsion
A4	Power Supply	3	Batteries
B	Payload Operation	10	TV Camera
C	Monitoring	5	Sensors
		6	Multicoder
		7	Amplifier
		12	Transmitter

COMPONENT LISTED BY MISSION FUNCTION

Equipment	Component	
Vehicle	1	Vehicle Controller
	2	Propulsion
	3	Batteries
Telemetry and Command	4	Translator
	5	Sensors
	6	Multicoder
	7	Amplifier
	8	Programmer
	9	Baroswitch
Payload	10	TV Camera
Communications	11	Receiver
	12	Transmitter
	13	Pulse Generator
	14	Flashing Light
	15	Beacon

COMPONENTS LISTED BY EQUIPMENT BLOCKS

Figure 4-5. Component Listings

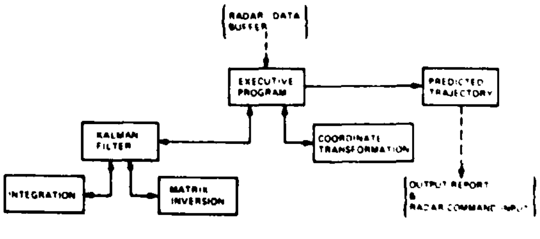


Figure 4-6. Trajectory Prediction Software System

4.2.2 Reliability and Availability Modeling

4.2.2.1 Reliability Modeling

A reliability model represents the manner in which the reliability of a system depends on the reliability of the system's constituent elements. A reliability model consists of a reliability block diagram and one or more mathematical equations.

4.2.2.1.1 Reliability Block Diagram

A reliability block diagram is a logic diagram, a graphic analog of logical events that result in success or failure of the system. In this sense it differs from a functional block diagram, which is a schematic representation of a system. (Note: The block diagrams used in a FMECA are functional diagrams.)

Figure 4-7 is a reliability block diagram of a simple subsystem containing three equipments, all of which must operate for subsystem success.

Figure 4-7. Serial Subsystem

The logical expression for subsystem success conveyed by the block diagram is

$$S = A \cap B \cap C, \text{ where}$$

S represents the event subsystem success, and the intersection symbol, $\cap$, is interpreted as "and". Also the reliability or probability of subsystem success, P(S), is given by

$$P(S) = P(A \cap B \cap C)$$

If failures of A, B, and C are statistically independent, P(S) can be written

$$P(S) = P(A) \cdot P(B) \cdot P(C)$$

where P(A), P(B), and P(C) are the probabilities of the events A, B, and C respectively.

A reliability block diagram models redundancy in terms of parallel paths. Figure 4-8 shows a subsystem in which equipments A and B and either C or D must operate for the subsystem to be successful.

The logical expression for subsystem success is

$$S = A \cap B \cap (C \cup D), \text{ where}$$

the union symbol, $\cup$, is interpreted as "or" and subsystem reliability is

$$P(S) = P\,[A \cap B \cap (C \cup D)]$$

Again, given equipment independence,

$$P(S) = P(A) \cdot P(B) \cdot [P(C) + P(D) - P(C) \cdot P(D)]$$

Reliability block diagrams can be prepared for many systems using only simple series-parallel combinations. But more complex logical structures are often encountered. Many systems contain switchable standby elements that are not activated until one or more primary elements fail. Redundant configurations of the m-out-of-n type [Appendix E] are also common, as are majority voting schemes and similar arrangements. A single component may be used to back up two or more parallel redundant elements, or multiple standby elements can be used to back-up a single primary element. The possible configurations are limitless.

It is possible that a fan or a battery which is not part of the subsystem could be required for cooling or graceful degradation, respectively, in either figure 4-7 or 4-8. It is important that the model reflect this dependency when it exists. The failure mode, effects, and criticality analysis (FMECA) described in § 4.2.5 provides a useful input to modeling for dependencies of this nature.

Usage rules also affect the structure of a reliability block diagram. For example, a system may contain three identical equipments, subjected during the mission to varying levels

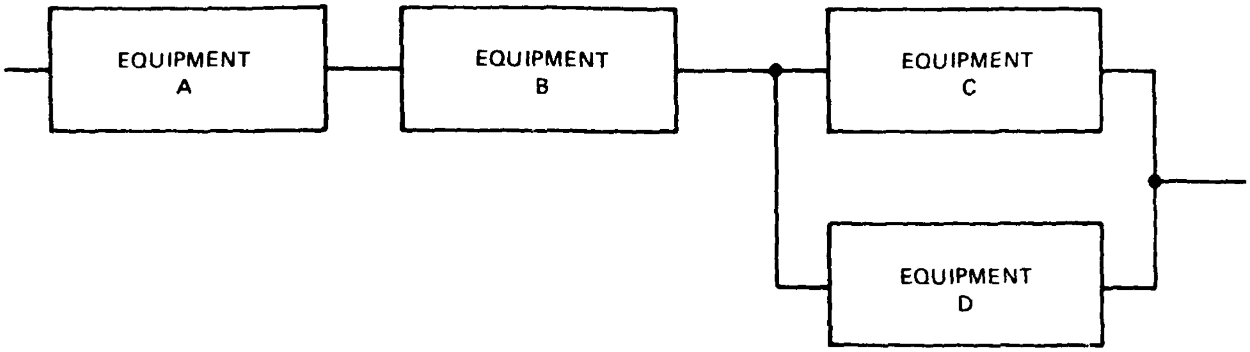


Figure 4-8. Series-Parallel Subsystem

of functional loading. Suppose that the mission consists of three phases and that the load profile demands that at least one equipment must operate during the first phase, at least two of the three must operate during the second phase, and all three equipments must operate during the final phase. Then the reliability block diagram will be radically different for each phase, although the equipment configuration will not change (Figure 4-9). Further, it is possible for such a system to complete one phase reliably, yet be unavailable to begin the next.

In choosing among various block diagram formats it should be remembered that a block diagram should provide quick and easy insight into the logical relationships that determine the success or failure of the system being modeled. Figures 4-10 and 4-11 show two block diagrams of a typical trip function. Most would agree that the diagram in figure 4-11 is more quickly and easily understood than its equivalent shown in figure 4-10.

4.2.2.1.2 Mathematical Models – Reliability

A mathematical model is an algebraic analog of the block diagram. It is prepared by review of the block diagram. In developing reliability models of the attribute type, three assumptions are usually applied to elements of the system: 1) only two element states are recognized – operable or failed, 2) repair is not considered; a failed element is considered to remain failed for the duration of the mission, 3) elements are statistically independent; failure of a given element does not affect the probability of failure of any other element.

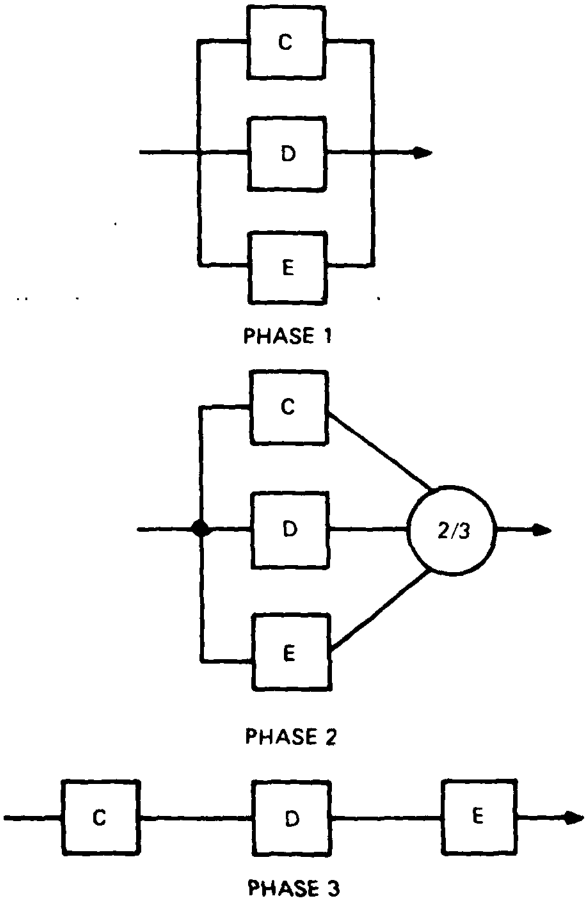


Figure 4-9. Reliability Model by Mission Phases

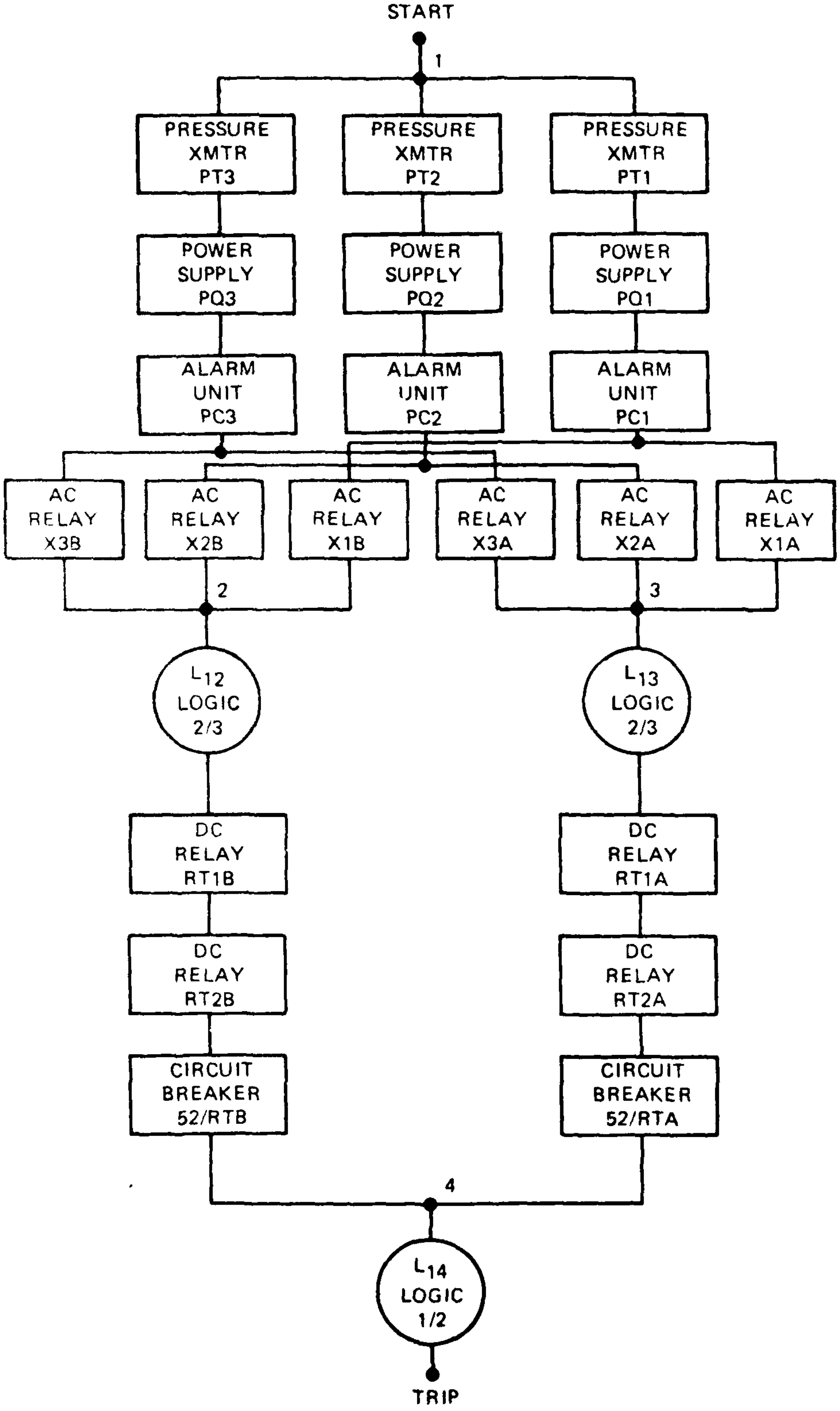


Figure 4-10. Reliability Block Diagram
Circuit Breaker Trip Function

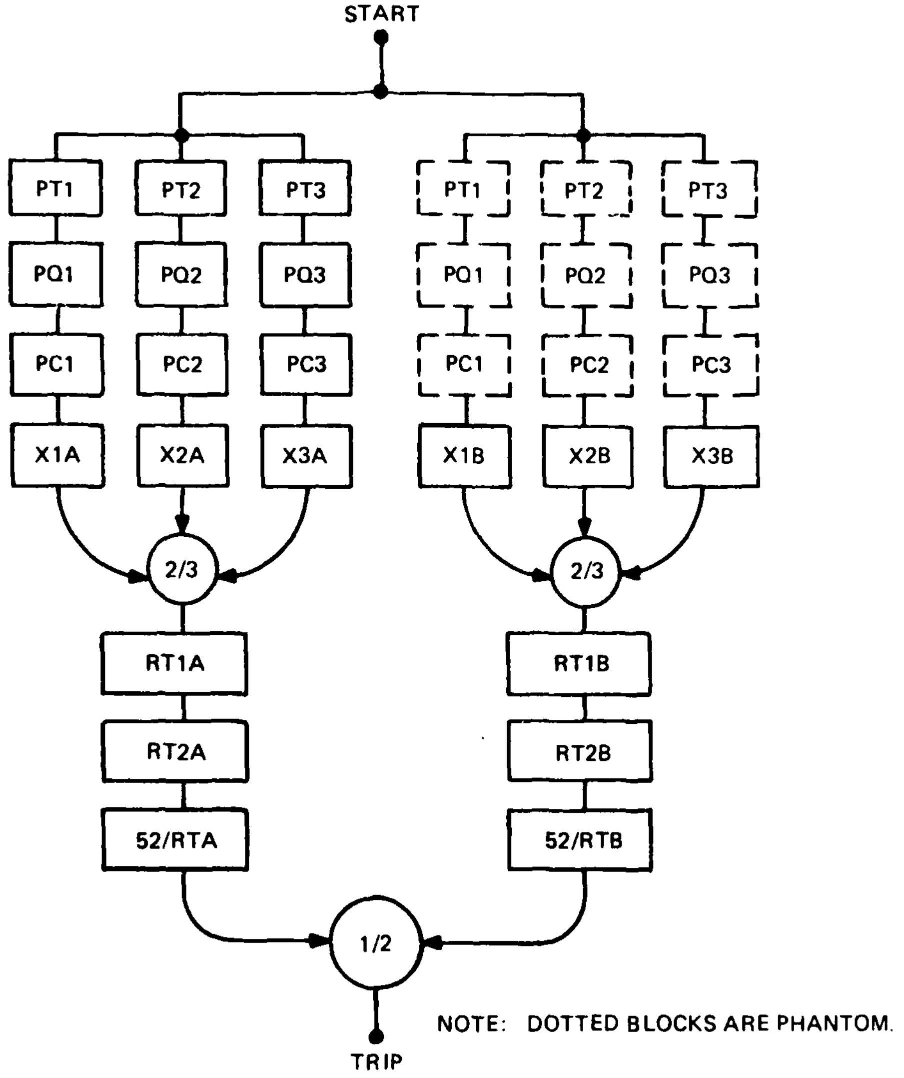


Figure 4-11. Alternative Reliability Block Diagram – Circuit Breaker Trip Function

Under these assumptions $R_A + Q_A = 1$, where R_A is the reliability of element A, the event probability, P(A), and Q_A is the unreliability of element A, the event probability, $P(\bar{A})$. If an element is used N times, $(R + Q)^N = 1$ and if N different items are used, $\prod_{i=1}^{N} (R_i + Q_i) = 1$.

4.2.2.1.2.1 Directly Written Models

Where the block diagram reflects simple series-parallel logic, a reliability model can be written using well known combinatorial equations.

For series related elements:

$$R_S = R_A \cdot R_B \cdot R_C \cdot \cdot \cdot = \prod_{\text{all } i} R_i \quad (4\text{-}1)$$

and for parallel redundant elements where one must operate,

$$R_S = 1 - [(1-R_A)(1-R_B)(1-R_C) \cdot \cdot \cdot] \quad (4\text{-}2)$$
$$= 1 - \prod_{\text{all } i} (1-R_i)$$

If the elements are identical and there are n of them, equations (4-1) and (4-2) reduce respectively to

$$R_S = R^n \qquad (4\text{-}3)$$

and

$$R_S = 1 - (1 - R)^n \qquad (4\text{-}4)$$

For n identical parallel redundant elements, any m of which are required to operate,

$$R_S = \sum_{x=m}^{n} \binom{n}{x} R^x (1 - R)^{n-x} \qquad (4\text{-}5)$$

where

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} \qquad 0! = 1 \qquad n! = (n)(n-1)\cdots(1).$$

The phase models corresponding to figure 4-9 can easily be written for phases 1 and 3 using equations (4-2) and (4-1) respectively.

$$R_{S_{(\text{Phase } 1)}} = 1-(1-R_C)(1-R_D)(1-R_E)$$

$$R_{S_{(\text{Phase } 3)}} = R_C\ R_D\ R_E$$

But none of the equations above are adequate for phase 2, although equations (4-5) would solve phase 2 if all the elements were identical. The equation for phase 2 is developed using Binomial Modeling concepts in § 4.2.2.1.2.2 and is presented in figure 4-12.

4.2.2.1.2.2 Binomial Modeling

Models involving two-state components can always be written using the binomial relationship $R + Q = 1$, but the process becomes tedious as the number of components increases. A system comprised of n two-state elements can assume any of 2^n unique states. Some of them correspond to system success, others to system failure.

It is assumed that every component begins its mission in an operable condition and can complete the mission in either of two states—operable or failed. The probability that it will complete the mission in an operable state is its reliability R, the probability that it will complete the mission in a failed state is $1-R = Q$. If a system consists of two components, the system can complete its mission in any one of four states. If the components are statistically independent, the state probabilities can be enumerated by a simple expansion.

$$(R_1+Q_1)(R_2+Q_2) = R_1R_2 + R_1Q_2 + R_2Q_1 + Q_1Q_2 = 1$$

The four terms of the expansion represent respectively the joint probabilities that both components will be successful, that component 1 will be successful and component 2 will fail, that component 2 will be successful and component 1 will fail, and that both components will fail. Because the four states collectively exhaust all possibilities, the probabilities add to unity.

If the components are serially related in the system, only the full success state represents system success and the first term of the expansion is the system reliability model. However, if the components are related in active parallel redundancy, three of the four states represent system success and only the total failure state corresponds to system failure. In this case the first three terms of the expression form the system's reliability model.

It can readily be seen that a system of three components can complete a mission in any one of eight possible states. Dropping subscripts for the sake of notational brevity gives

$$(R+Q)^3 = R^3 + 3R^2Q + 3RQ^2 + Q^3 = 1$$

The respective terms represent the probabilities of the one state in which all three components complete the mission successfully, three states in which two components are successful while one component fails, three states in which one component is successful while two components fail, and one state in which all three components fail. Again, system reliability depends on how many of these eight possible states correspond to a successful mission.

For example, in the three component system described in figure 4-9 Phase 2, the possible system states are shown in figure 4-12, four of them represent system success. In this example two of the three components are required for system success. Therefore, all states with two or more S's represent system

State		Component			System
No.	Description	C	D	E	
1	1 way - 3 succeed	S	S	S	S
2	3 ways - 2 succeed and 1 fail	F	S	S	S
3		S	F	S	S
4		S	S	F	S
5	3 ways - 1 succeeds and 2 fail	F	F	S	F
6		F	S	F	F
7		S	F	F	F
8	1 way - 3 fail	F	F	F	F

Figure 4-12. List of Possible States for the Three Component System Depicted in Figure 4-9, Phase 2

success; those with two or more F's represent system failure. The reliability model for figure 4-9 Phase 2 can now be written by summing the four system success states or by subtracting the sum of the system's four failed states from unity. Using the four success states we have:

$$R_S = R_C R_D R_E + (1-R_C)R_D R_E + R_C(1-R_D)R_E + R_C R_D(1-R_E)$$

4.2.2.1.2.3 Conditional Probability Modeling

It is often convenient to simplify models that lack the simple series-parallel structure by using the relationship:

$$R_S = P(S|A) \cdot P(A) + P(S|\overline{A}) \cdot P(\overline{A}) \quad (4\text{-}6)$$

This statement indicates that the reliability of the system is the probability that the system works given that A works, times the probability that A works, plus the probability that the system works given that A fails, times the probability that A fails. A can be an element or group of elements in the system. For example, in figure 4-13, given that A does not fail, the system succeeds if either D or E does not fail.

$$P(S|A)P(A) = [1-(1-R_D)(1-R_E)]R_A$$

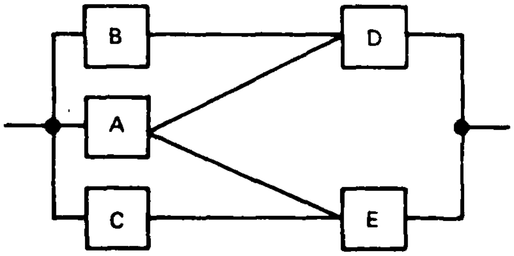


Figure 4-13. System Model

Given that A fails, the system can only succeed if either B and D or C and E do not fail.

$$P(S|\overline{A})P(\overline{A}) = [1-(1-R_B R_D)(1-R_C R_E)](1-R_A)$$

Thus the system reliability model is the sum of both expressions.

$$R_S = [1-(1-R_D)(1-R_E)]R_A + [1-(1-R_B R_D)(1-R_C R_E)](1-R_A)$$

MIL-HDBK-217[2] discusses system reliability modeling.

4.2.2.1.2.4 Models by Minimum Cuts Method

Generally it is easy to write directly the reliability model for a system of simple configuration intended for a single-phase mission. But when the system lacks the series-parallel structure or the mission consists of multiple phases, the model may be much more difficult to write and to evaluate. It may not be sufficient to solve such a model for each phase independently of the other phases, since the conditional reliability thus found is the probability of success in a particular phase, given that all components of the system are operable at the beginning of the phase. For redundant systems, this condition may not be met as it is possible to enter any but the first phase with some elements failed but the system operable. Under these conditions a system may complete its current phase reliably, but be unavailable to begin the next phase. What is sought is the probability of system success in any phase, conditioned on success in the preceding phase, rather than conditioned on all elements beginning the current phase unfailed. The method of minimum cuts [3, 4 pages 136-139, 5 pages 329-338] is a powerful analytical tool for treating this class of problems. Dr. C. Persels' approach [3] is followed in this manual.

A cut is defined as a group of components which, if all fail, will fail the system. A minimum cut is a cut having the property that if any failed component is analytically deleted from the cut the remaining components no longer comprise a cut.

The steps in the minimum cut method are:

1. Find the minimal cuts for each phase
2. Combine phase minimal cuts into mission minimal cuts
3. Group dependent mission minimal cuts
4. Find the probability that at least one minimal cut in a group will fail
5. Combine group probabilities

An example of a two phase system (Figure 4-14) is carried along to illustrate the method. Subscripts denote phases. It can be seen that, for example, failure of component E in phase 1 would not fail the system in phase 1, but would render it incapable of performing phase 2.

Finding the Minimal Cuts for Each Phase

The minimal cuts for each phase shown in figure 4-14 are easily obtained by inspection. The formal procedure is:

1. Obtain Boolean expression for system success
2. Complement the expression
3. Place in disjunctive form
4. Expressions between the ORs are the minimal cuts.

Subscripts in figure 4-14 denote the mission phase. Thus $\overline{A}_1$ is the event that component A fails in phase 1. We will let c_i^j stand

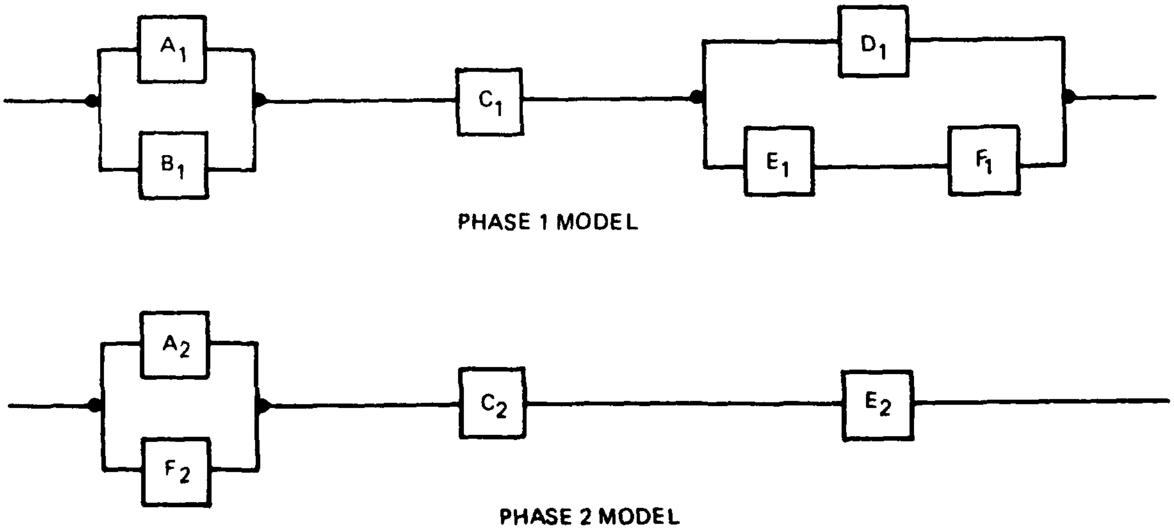


Figure 4-14. Two Phase System Model

for cut j in phase i.

The Boolean expressions are:

Phase 1

$$S_1 = [A_1 \cup B_1] \cap [C_1] \cap [D_1 \cup (E_1 \cap F_1)]$$

Phase 2

$$S_2 = [A_2 \cup F_2] \cap C_2 \cap E_2$$

Their complements are:

$$\overline{S}_1 = [\overline{A}_1 \cap \overline{B}_1] \cup [\overline{C}_1] \cup [\overline{D}_1 \cap (\overline{E}_1 \cup \overline{F}_1)]$$

$$\overline{S}_2 = [\overline{A}_2 \cap \overline{F}_2] \cup \overline{C}_2 \cup \overline{E}_2$$

In the disjunctive form:

$$\overline{S}_1 = [\overline{A}_1 \cap \overline{B}_1] \cup [\overline{C}_1] \cup [\overline{D}_1 \cap \overline{E}_1] \cup [\overline{D}_1 \cap \overline{F}_1]$$

$$\overline{S}_2 = [\overline{A}_2 \cap \overline{F}_2] \cup \overline{C}_2 \cup \overline{E}_2$$

Therefore the minimal cuts for Phase 1 are:

$$c_1^1 = [\overline{A}_1 \cap \overline{B}_1]$$

$$c_1^2 = [\overline{C}_1]$$

$$c_1^3 = [\overline{D}_1 \cap \overline{E}_1]$$

$$c_1^4 = [\overline{D}_1 \cap \overline{F}_1]$$

and the minimal cuts for Phase 2 are:

$$c_2^1 = [\overline{A}_2 \cap \overline{F}_2]$$

$$c_2^2 = [\overline{C}_2] \qquad (4\text{-}7)$$

$$c_2^3 = [\overline{E}_2]$$

Probability of system failure is equal to the probability that all of the elements in at least one of the minimal cuts will fail. Probability of system success in a phase is equal to 1-P (all components in at least one cut fail). Thus,

$$P(S_1) = 1 - P[c_1^1 \cup c_1^2 \cup c_1^3 \cup c_1^4]$$

$$P(S_2) = 1 - P[c_2^1 \cup c_2^2 \cup c_2^3]$$

Combining Phase Minimal Cuts into Mission Minimal Cuts

For a multiphase mission, minimal cuts must be found that represent the system from initiation of the mission through the phase being evalutated. These are called mission minimum cuts, and are designated by capital C. They are found by combining previous mission minimum cuts with current phase minimum cuts obtained for each phase without regard to any other phase.

Since phase 1 is the first phase, mission cuts are identical to the phase cuts.

$$c_1^1 = C_1^1 \quad c_1^2 = C_1^2 \quad c_1^3 = C_1^3 \quad c_1^4 = C_1^4$$

Superscripts are ordinal indices used to distinguish among cuts.

Mission cuts through phase 2 are obtained by combining phase 2 cuts with previous mission cuts (i.e., with phase 1 cuts). A cut must appear once and only once on the list. Therefore, three rules are followed:

1. Current phase cuts become current mission cuts. (In the notation used below, the symbol → is read as "becomes").
2. Any previous mission cut which includes a current phase cut is dropped. (This prevents the cut from being listed twice).
3. Any previous mission cut which does not contain a current phase cut becomes a current mission cut. (This assures that the cut will continue to be listed exactly once).

In the previous example,

$C_1^1 \rightarrow C_2^1$ (rule 3)

C_1^2 is dropped and $c_2^2 \rightarrow C_2^2$ (rule 2)

C_1^3 is dropped and $c_2^3 \rightarrow C_2^3$ (rule 2)

$C_1^4 \rightarrow C_2^4$ (rule 3)

$c_2^1 \rightarrow C_2^5$ (rule 1)

In summary:

$$C_2^1 = \overline{A}_1 \cap \overline{B}_1$$

$$C_2^2 = \overline{C}_2$$

$$C_2^3 = \overline{E}_2$$

$$C_2^4 = \overline{D}_1 \cap \overline{F}_1$$

$$C_2^5 = \overline{A}_2 \cap \overline{F}_2$$

represent the mission minimal cuts for the two phase mission illustrated in figure 4-14. Additional phases are processed in like manner.

The evaluation of mission cuts through a given phase i will give the unconditional probability of subsystem success through that phase.

Grouping Dependent Mission Minimal Cuts

It is an infrequent circumstance that all mission cuts are independent. But they can be collected into independent or disjoint groups, each group containing only mission cuts having components in common. Each cut in a group "chains" to at least one other mission cut in the same group by having one or more components in common with it. In the example under discussion,

$$\text{Group 1} = [C_2^1, C_2^5, C_2^4] = [(\overline{A}_1 \cap \overline{B}_1), (\overline{A}_2 \cap \overline{F}_2), (\overline{D}_1 \cap \overline{F}_1)]$$

$$\text{Group 2} = [C_2^2] = [\overline{C}_2]$$

$$\text{Group 3} = [C_2^3] = [\overline{E}_2]$$

The reliability of the system for the two phase mission is:

$$P(S) = \prod_{i=1}^{N} [1 - P(\overline{G}_i)] \qquad (4\text{-}8)$$

where N is three in this case since there are three groups.

Finding The Probability That At Least One Cut In A Group Will Fail

The number of cuts in a group is in general N. To determine the probability that at least one cut in a group will fail, we sum the N first order terms and subtract the $\binom{N}{2}$ second order terms (all combinations of cuts taken two at a time), we add the $\binom{N}{3}$ third order terms, proceeding with alternating algebraic signs until we come to the one, $\binom{N}{N}$, Nth order term.

For the first group we would have:

$$\begin{aligned} P(\overline{G}_1) &= P[C_2^1 \cup C_2^5 \cup C_2^4] \\ &= + [+P(C_2^1) + P(C_2^5) + P(C_2^4)] \\ &\quad - [P(C_2^1 \cap C_2^5) + P(C_2^1 \cap C_2^4) \\ &\quad + P(C_2^5 \cap C_2^4)] \\ &\quad + [P(C_2^1 \cap C_2^5 \cap C_2^4)] \end{aligned}$$

This can also be written as:

$$\begin{aligned} P(\overline{G}_1) = &+ [+P(\overline{A}_1 \cap \overline{B}_1) + P(\overline{A}_2 \cap \overline{F}_2) \qquad (4\text{-}9) \\ &+ P(\overline{D}_1 \cap \overline{F}_1)] \\ &- \Big\{ +P[(\overline{A}_1 \cap \overline{B}_1) \cap (\overline{A}_2 \cap \overline{F}_2)] \\ &+ P[(\overline{A}_1 \cap \overline{B}_1) \cap (\overline{D}_1 \cap \overline{F}_1)] \\ &+ P[(\overline{A}_2 \cap \overline{F}_2) \cap (\overline{D}_1 \cap \overline{F}_1)] \Big\} \\ &+ P[(\overline{A}_1 \cap \overline{B}_1) \cap (\overline{A}_2 \cap \overline{F}_2) \cap (\overline{D}_1 \cap \overline{F}_1)] \end{aligned}$$

Note for group 1, N=3. Therefore, there are $\binom{N}{1}$ or 3x2x1/((1) x (2x1)) or 3 first order terms, $\binom{N}{2}$ or (3x2x1/((2x1) x (1x1)) or 3 second order terms, and $\binom{N}{N}$ or 1 third order term. The algebraic signs alternate plus for the first order terms, minus for the second order terms and plus for the third order terms. In this example, both group 2 and group 3 have only one term, N=1:

$$P(\overline{G}_2) = P(C_2^2) = P[\overline{C}_2] \qquad (4\text{-}10)$$

$$P(\overline{G}_3) = P(C_2^3) = P[\overline{E}_2] \qquad (4\text{-}11)$$

Combining Group Probabilities

$$P(\overline{S}) = P[\overline{G}_1 \cup \overline{G}_2 \cup \overline{G}_3 \cup \cdots \overline{G}_N]$$

The number of groups is in general N. To determine the probability that at least one group will fail, sum the N first order terms and subtract the $\binom{N}{2}$ second order terms (all combinations of groups taken two at a time), add the $\binom{N}{3}$ third order terms, proceeding with alternating algebraic signs until the one, $\binom{N}{N}$, N^{th} order term is reached.

Since there are three independent groups in the system:

$$P(\overline{S}) = +[+P(\overline{G}_1) + P(\overline{G}_2) + P(\overline{G}_3)] \quad (4\text{-}12)$$
$$-\Big\{+P[(\overline{G}_1) \cap (\overline{G}_2)]$$
$$+P[(\overline{G}_1) \cap (\overline{G}_3)] + P[(\overline{G}_2) \cap (\overline{G}_3)]\Big\}$$
$$+ \quad P[(\overline{G}_1) \cap (\overline{G}_2) \cap (\overline{G}_3)]$$

It should be noted that this expression has seven terms. If the five mission cuts had been used directly, without grouping dependent cuts, (a permissible approach), there would have been thirty-one terms in the expression for P(S): $\binom{N}{i}$ or 5 first order, 10 second order, 10 third order, 5 fourth order and 1 fifth order terms.

If a solution for the first phase is desired, it is efficient to group the cuts in the first phase. The groups for phase 1 would be:

$$\text{Group 1} = [C_1^1] = [\overline{A}_1 \cap \overline{B}_1]$$
$$\text{Group 2} = [C_1^2] = [\overline{C}_1] \quad (4\text{-}13)$$
$$\text{Group 3} = [C_1^3, C_1^4] = [(\overline{D}_1 \cap \overline{E}_1), (\overline{D}_1 \cap \overline{F}_1)]$$

The probability of system success is:

$$P(S) = 1 - P(\overline{S})$$

Approximations

A conservative estimate of system reliability for a multiphase mission is available by truncating the expression for $P(\overline{S})$ at any negative sign. The expression was:

$$P(\overline{S}) = +[+P(\overline{G}_1) + P(\overline{G}_2) + P(\overline{G}_3)]$$
$$-\Big\{+P[(\overline{G}_1) \cap (\overline{G}_2)]$$
$$+P[(\overline{G}_1) \cap (\overline{G}_3)] + P[(\overline{G}_2) \cap (\overline{G}_3)]\Big\}$$
$$+ \quad +P[(\overline{G}_1) \cap (\overline{G}_2) \cap (\overline{G}_3)]$$

The approximation is obtained by truncating:

$$P(\overline{S}) = +[+P(\overline{G}_1) + P(\overline{G}_2) + P(\overline{G}_3)]$$
$$P(S) = 1 - P(\overline{S})$$

The error will be less than the first term eliminated in this case:

$$-\Big\{P[(\overline{G}_1) \cap (\overline{G}_2)] + P[(\overline{G}_1) \cap (\overline{G}_3)]$$
$$+ P[(\overline{G}_2) \cap (\overline{G}_3)]\Big\}$$

Numerical Example

In the model illustrated in figure 4-14, assume that the probabilities of success for each element in phase 1 are:

$P(A_1) = 0.9500$
$P(B_1) = 0.9000$
$P(C_1) = 0.9600$
$P(D_1) = 0.9300$
$P(E_1) = 0.9700$
$P(F_1) = 0.9400$

The probabilities of success of each element in phase 2, given that the element has survived phase 1, are:

$P(A_2 | A_1) = 0.9700$
$P(E_2 | E_1) = 0.9100$
$P(F_2 | F_1) = 0.9000$
$P(C_2 | C_1) = 0.9200$

Then the following complementary probabilities are true:

$P(\overline{A}_1) = 0.0500$
$P(\overline{B}_1) = 0.1000$
$P(\overline{A}_2 | A_1) = 0.0300$
$P(\overline{E}_2 | E_1) = 0.0900$
$P(\overline{C}_1) = 0.0400$
$P(\overline{D}_1) = 0.0700$
$P(\overline{F}_2 | F_1) = 0.1000$
$P(\overline{E}_1) = 0.0300$
$P(\overline{F}_1) = 0.0600$
$P(\overline{C}_2 | C_1) = 0.0800$

Because unconditional probabilities are used in the minimum cuts method, the unconditional probabilities of phase 2 are needed and are as follows:

$$P(A_2) = P(A_2|A_1)P(A_1) + P(A_2|\overline{A}_1)P(\overline{A}_1)$$

$$= (0.9700)(0.9500) + (0)(0.0500)$$

$$= 0.9215$$

$$P(\overline{A}_2) = 1 - P(A_2) = .0785$$

$$P(F_2) = P(F_2|F_1)P(F_1) + P(F_2|\overline{F}_1)P(\overline{F}_1)$$

$$= (0.9000)(0.9400) + (0)(0.0600)$$

$$= 0.8460$$

$$P(\overline{F}_2) = 1 - P(F_2) = 0.1540$$

$$P(C_2) = P(C_2|C_1)P(C_1) + P(C_2|\overline{C}_1)P(\overline{C}_1)$$

$$= (0.9200)(0.9600) + (0)(0.0400)$$

$$= 0.8832$$

$$P(\overline{C}_2) = 1 - P(C_2) = 0.1168$$

$$P(E_2) = P(E_2|E_1)P(E_1) + P(E_2|\overline{E}_1)P(\overline{E}_1)$$

$$= (0.9100)(0.9700 + (0)(0.0300)$$

$$= 0.8827$$

$$P(\overline{E}_2) = 1 - P(E_2) = 0.1173$$

Exact reliability of two-phase mission using equation (4-9) for Group 1,

$$P(\overline{G}_1) = P(\overline{A}_1)P(\overline{B}_1) + P(\overline{A}_2)P(\overline{F}_2) + P(\overline{D}_1)P(\overline{F}_1)$$

$$- P(\overline{B}_1)P(\overline{A}_1)P(\overline{F}_2)$$

$$- P(\overline{A}_1)P(\overline{B}_1)P(\overline{D}_1)P(\overline{F}_1)$$

$$- P(\overline{A}_2)P(\overline{F}_1)P(\overline{D}_1)$$

$$+ P(\overline{B}_1)P(\overline{A}_1)P(\overline{F}_1)P(\overline{D}_1)$$

$$P(\overline{G}_1) = 0.0202$$

Similarly, from equation (4-10), F [illegible] = $P(\overline{C}_2) = 0.1168$. From equation [illegible] 1), $P(\overline{G}_3) = P(\overline{E}_2) = 0.1173$.

Finally, using equation (4-8), which reflects the fact that the groups are independent, the two-phase mission reliability R_S is

$$R_S = [1-P(\overline{G}_1)]\,[1-P(\overline{G}_2)]\,[1-P(\overline{G}_3)]$$

$$= (.9798)(0.8832)(0.8827) \tag{4-14}$$

$$= .7639$$

Multiplying Phase Reliabilities

Since the phases of the two phase mission are not independent, it is not correct to multiply the reliability of phase 1 by the reliability of phase 2 to obtain mission reliability.

Phase 1

Groups for phase 1 are given in equation 4-13.

$$G_1 = \overline{A}_1 \cap \overline{B}_1$$

$$G_2 = \overline{C}_1$$

$$G_3 = \{\overline{D}_1 \cap \overline{E}_1, \overline{D}_1 \cap \overline{F}_1\}$$

Phase 2

Groups for phase 2 are the minimal cuts of equation 4-7.

$$G_1 = \overline{A}_2 \cap \overline{F}_2$$

$$G_2 = \overline{C}_2$$

$$G_3 = \overline{F}_2$$

Then, for phase 1

$$P(\overline{G}_1) = P(\overline{A}_1)P(\overline{B}_1) \qquad = 0.0050$$

$$P(\overline{G}_2) = P(\overline{C}_1) \qquad = 0.0400$$

$$P(\overline{G}_3) = P[(\overline{D}_1 \cap \overline{E}_1) \cup (\overline{D}_1 \cap \overline{F}_1) \qquad = 0.0062$$

$$= P(\overline{D}_1)P(\overline{E}_1) + P(\overline{D}_1)P(\overline{F}_1)$$

$$- P(\overline{D}_1)P(\overline{E}_1)P(\overline{F}_1)$$

and for phase 2

$$P(\bar{G}_1) = P(\bar{A}_2)P(\bar{F}_2) = 0.0121$$
$$P(\bar{G}_2) = P(\bar{C}_2) = 0.1168$$
$$P(\bar{G}_3) = P(\bar{E}_2) = 0.1173$$

and,

$$R_{(\text{Phase 1})} = (.9950)(.9600)(.9938) = .9493$$
$$R_{(\text{Phase 2})} = (.9879)(.8832)(.8827) = .7702$$

The product (.9493) (.7702) is .7312 which underestimates the true mission reliability of 0.7639 (Equation 4-14), a significant underestimate.

4.2.2.2 Availability Modeling

An availability model indicates the manner in which the availability of a system depends on the reliability of the system's constituent elements and on their maintenance characteristics. (Note: as was indicated in § 2.2.1, this manual treats availability from a corrective maintenance viewpoint.) This availability model consists of a block diagram and one or more mathematical equations.

The functional configuration of the system in each of its operating modes is defined by means of block diagrams, normally one diagram for each mode. In the diagram each block represents an equipment or group of equipments. The directions of functional flows are labeled and inputs and outputs are identified. Thus a functional block diagram is a graphical representation of the dependence of system performance on the operability of its hardware elements. In addition, an equipment "tree" diagram, based on packaging rather than functional relationships should be supplied, detailing hardware down to and including the component level. Figure A-1 is a typical system block diagram. The system is completely analyzed in Appendix A.

4.2.2.2.1 Availability Block Diagrams

The block diagrams used in availability analysis are essentially the same as those described in § 4.2.2.1.1.

4.2.2.2.2 Mathematical Models - Availability

The models of § 4.2.2.1.2 are applicable to availability with the appropriate changes in symbols if the block can be repaired during the mission phase. [i.e., A (availability) for R (reliability) and U (unavailability) for Q (Unreliability).]

A subsystem consisting of n equipments in series is available when all n equipments are available. Thus, if

$$A_i = \frac{\text{MTBF}_i}{\text{MTBF}_i + \text{MTTR}_i} \qquad (4\text{-}15)$$

is the availability of the ith equipment, and if all equipments fail and are repaired independently, the availability of the subsystem is

$$A = \prod_{i=1}^{n} A_i \qquad (4\text{-}16)$$

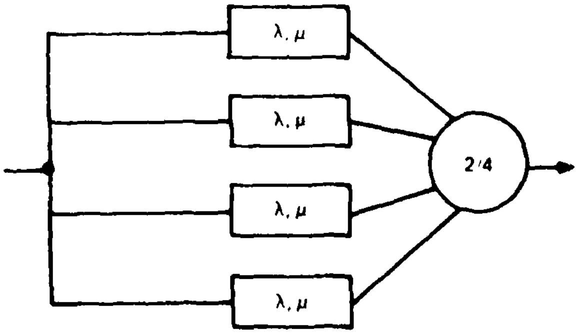
The availability of a subsystem of n identical equipments in parallel, any m of which are required to be up for the system to be up, is given by

$$A = \sum_{x=0}^{n-m} \binom{n}{x} A_i^{n-x} (1-A_i)^x \qquad (4\text{-}17)$$

where

$$\binom{n}{x} = [n!]/[(n-x)!(x)!]$$

For a system such as



where repair is not permitted, the MTBF (Figure E-11) is

$$\text{MTBF} = 1.083333/\lambda$$

The subsystem degrades as failures occur to a 2 of 3 and then a 2 of 2 subsystem before the subsystem goes down.

If repair is permitted, figure E-11 shows the MTBF to be

$$\text{MTBF} = 0.083333\,\mu^2/\lambda^3$$

When a failure occurs repair is initiated immediately. The subsystem goes to 2 of 3 when the first failure occurs. It only goes to 2 of 2 if a second failure takes place before the first repair. For a repair rate of 1 repair/hour and a failure rate of .001 failures/hour, the MTBF of the subsystem without repair is 1,083.333 hours and the MTBF will repair is over 83 million hours.

4.2.3 Reliability and Availability Apportionment

Apportionment is the process by which requirements are allocated from the system level to lower assembly levels. System requirements will have been established by the procuring activity prior to full scale development.

System requirements stem from or relate to mission needs. Usually more than one approach is available to fulfill a need or to satisfy a mission requirement. Therefore, trade-offs may be made between competing system designs, such as redundancy or derating approaches.

It should be noted that requirements are being apportioned Requirements should not be set arbitrarily since changes in requirements normally incur a cost penalty.

4.2.3.1 Reliability Apportionment

Reliability requirements must be apportioned prior to design. Because apportionment is properly completed before the design, apportionment techniques are of necessity somewhat subjective. As an example, consider a hypothetical re-entry system with design requirements as shown in figure 4-15. When the apportionment establishes the design requirement, it influences design decisions such as redundancy, part quality and electrical, mechanical and thermal derating philosophy.

Essential functions of the system include separation, environmental control, decoy, maneuvering, arming and fuzing, and attitude control. Without necessarily defining the hardware concept (e.g., the attitude control system could use hot gas, cold gas, rocket, spring, or other means to accomplish its function) to be used, relative weights are assigned to each function in the categories of criticality, complexity, design maturity, and severity of mission profile. Other categories (e.g., MEC codes could be used as a category) may be appropriate for certain systems. Relative weights (scores) are assigned on a 1 to 10

Reliability Specification				
System	Reliability Objective	Producer's Risk	Minimum Acceptable Reliability	Consumer's Risk
Re-entry Vehicle	.9512	20%	.8607	20%

Figure 4-15. System Reliability Requirements

scale. In all categories except criticality, high scores represent more design difficulty i.e., 10 is more complex on complexity scale; 10 is the least advanced on the design mature scale; 10 has most difficult mission to survive on mission profile scale. Criticality is a measure of the importance of a function to a successful mission. If a loss of the function aborts the mission a score of 1 is appropriate; 10 implies little effect on the mission if the function is lost. While this scoring is reversed from the other categories, it is correct because it requires high reliability for elements that are most essential to the mission.

There are many methods for assigning scores. The system engineer can assign them. A group of cognizant people can assign them independently, then resolve significant differences by discussion or by taking an average score. Various paired comparison schemes can also be used.

Complexity

A score of ten could be assigned to the most complex device (e.g., estimate the number of active and passive parts to develop a score). Relative scores would rank other devices 1-10.

Maturity Score

A score of ten could be assigned to new devices (i.e., advances in state-of-the-art or questionable characteristics). Mature devices should be rated 1-9 depending on the degree of maturity.

Mission Profile

A score of ten could be assigned to the devices that must survive the most severe mission profile (e.g., the re-entry system must survive launch and re-entry environments and would be assigned a 10; other missile body segments complete their mission before re-entry and would receive correspondingly lower scores). On board the submarine where the environment is essentially the same for many items, the duty cycle is used to establish the relative scores.

Criticality

A score of one is assigned to all items whose failure would result in the loss of the mission. Higher scores are assigned to items that are backed up or may not impact the main mission (e.g., a decoy system failure would not prevent the re-entry system from reaching its target, particularly an undefended target).

Figure 4-16 illustrates the mechanics of the apportionment of a system reliability requirement of 0.9512 (failure rate 0.0500 failures/mission) after scores have been assigned on a 1-10 scale in each of the four categories. The scores are summed for each function and presented in column six, titled function score. The result of summing column six is 103, the total re-entry vehicle score. The failure rate allocated to each function is obtained by multiplying the system failure rate (0.0500) by the ratio of the function score for the appropriate function over the total system score of 103, as shown in column seven. The last columns show the apportioned failure rates and reliability requirements.

It should be noted that when a function or hardware component whose reliability is known is to be used in a new design, the proper approach is to recognize this fact initially and factor this function or component out of the apportionment. Since the reliability of the function or component is known, and no resources are to be applied to modify it, it would be inconsistent with the objectives of the apportionment process to require other than its known reliability. Of course, later tradeoff studies may suggest the desirability of expending effort to improve the reliability of a component of established design.

Assume the Command Link Function of figure 4-16 is known to have a reliability of 0.9910 based upon extensive testing in an earlier program, and that this function is to be used without change. Apportionment can proceed as follows:

System failure rate requirement	0.0500
Known failure rates (In this case the failure rate of the command link function.)	-0.0090
Apportion to remaining functions	0.0410

Figure 4-17 shows that using the command link function, which is less reliable than its initial apportionment, increases the design requirements for the other functions. Figure 4-18 illustrates apportionment of the monitoring function failure rate to the component level.

As the design progresses, feasibility studies are normally conducted to determine if candidate designs will meet their requirements. Reliability prediction is a feasibility study which permits comparison of the predicted reliabilities with required reliabilities. When a prediction is lower than the requirement, it is desirable to assign resources in the manner which achieves the necessary reliability improvement at minimum cost. While general strategies, such as increased derating or use of higher grade parts, produce an overall gain in reliability, concentration on specific problem areas by adding redundancy or redesigning a component or equipment may be more cost-effective.

For a few items substantive improvement may not be feasible. This might be true for example, for an off-the-shelf component of established design, which had already benefited from one or more product improvement programs. In general, an analyst can not formulate component reliability as a deterministic function of resources expended for reliability improvement, but it is often possible to estimate these functional relationships subjectively using past experience as a guide. Reliability improvements realizable by various actions can be listed along with expected, optimistic, and pessimistic predictions of the cost of each action. With the above information available, simple cost-benefit analysis techniques can be applied to allocate resources effectively.

When it is determined that redundancy is required to meet a reliability requirement, the analyst can make use of techniques such as Kettelle's algorithm (cost & reliability trade-off) [6, 7] for optimal allocation of redundancy.

Figures 4-16 through 4-18 allocate the allowable failure rate; it is also possible using similar methods to allocate the failure probability, Q, or the reliability, R, directly.

Function	Complexity Score	Maturity Score	Severity of Mission Profile score	Criticality Score	Function Score	Failure Rate Allocation	Apportioned	
							Failure Rate per Mission	Reliability
Command Link	7	6	2	1	16	16/103x.0500	.0078	.9922
Tracking	6	5	2	8	21	21/103x.0500	.0102	.9898
Vehicle Operation	10	7	2	6	25	25/103x.0500	.0121	.9880
Power Supply	1	2	2	1	6	6/103x.0500	.0029	.9971
Payload Operation	1	5	2	7	15	15/103x.0500	.0073	.9927
Monitoring	8	3	2	7	20	20/103x.0500	.0097	.9904
Total Score 103								
Re-entry Vehicle Reliability Design Objective							.0500	.9512

Figure 4-16. Functional Reliability Apportionment

Function	Complexity Score	Maturity Score	Severity of Mission Profile score	Criticality Score	Function Score	Failure Rate Allocation	Apportioned	
							Failure Rate per Mission	Reliability
Command Link	. . .	. . .	. . .	. . .	. . .	.0090	.0090	.9910
Tracking	6	5	2	8	21	21/87x.0410	.0099	.9902
Vehicle Operation	10	7	2	6	25	25/87x.0410	.0118	.9883
Power Supply	1	2	2	1	6	6/87x.0410	.0028	.9972
Payload Operation	1	5	2	7	15	15/87x.0410	.0071	.9929
Monitoring	8	3	2	7	20	20/87x.0410	.0094	.9906
Total Score 87								
Re-entry Vehicle Reliability Design Objective							.0500	.9512

Figure 4-17. Functional Reliability Apportionment with Command Link Reliability Known

Component	Complexity Score	Maturity Score	Severity of Mission Profile score	Criticality Score	Component Score	Failure Rate Allocation	Apportioned	
							Failure Rate per Mission	Reliability
Sensor	2	7	8	5	22	22/81x.0094	.0026	.9974
Multicoder	7	6	5	1	19	19/81x.0094	.0022	.9978
Amplifier	3	8	6	8	25	25/81x.0094	.0029	.9971
Transmitter	5	2	6	2	15	15/81x.0094	.0017	.9983
Total Score 81								
Monitoring Function Reliability Design Objective							.0094	.9906

Figure 4-18. Component Reliability Apportionment of Monitoring Function

4.2.3.2 Availability Apportionment

System availability is a joint function of the availability of the system's equipments. Each of these, in turn, is a joint function of the reliability and maintainability of the equipment, which can combine in various proportions to yield a given level of availability. In its initial configuration a system will possess certain inherent levels of reliability and maintainability, which together establish an upper limit on the availability attainable by the system. Beyond this limit, additional effort can be applied to develop either increased reliability or improved maintainability or both, in order to increase availability. Figure 2-1 illustrates the dependence of availability on reliability and maintainability. Specifications of an availability requirement defines the ordinate on the curve of figure 2-1. Move horizontally to an MTBF curve and vertically downward to the corresponding MTTR. A trade-off region exists along the ordinate since an infinite number of MTBF-MTTR pairs can satisfy the requirement. A variety of criteria can form a basis for such a trade-off. One method considers the marginal costs of improving reliability and maintainability. Appendix A includes an example of such a trade-off.

When a trade-off has been made, as the final step of system analysis, availability requirements may be apportioned to equipments in a consistent and logical manner. The objective of apportionment is to provide goals against which the availability growth of the system's elements can be measured, and to provide designers with goals for reliability and maintainability. A procedure for reliability apportionment, considering the factors of complexity, state-of-the-art, duty cycle, and criticality was described in the previous paragraph. Apportionment of maintainability can be done in similar manner, based on one or more of the same factors or based on considerations of location, packaging or physical configuration of the system. But unlike reliability, maintainability requirements cannot logically be apportioned to assembly levels below the lowest levels specified as repairable on-site under the users' maintenance policies.

The MTTR of a system is the average of its subsystem or equipment MTTRs, each weighted by the failure rate λ_i of the subsystem or equipment and written:

$$\mathrm{MTTR}_s = \frac{\Sigma(\lambda_i \mathrm{MTTR}_i)}{\Sigma\lambda_i} \cdot$$

If the system is series related with exponential failure and repair rates, its failure rate is $\lambda_s = \Sigma\lambda_i$ as previously defined. Thus the system repair rate $\mu_s = 1/\mathrm{MTTR}_s$ can be written as:

$$\mu_s = \frac{\lambda_s}{\Sigma(\lambda_i/\mu_i)} \cdot$$

A commonly used criterion for maintainability apportionment is the condition

$$\frac{\lambda_1}{\mu_1} > \frac{\lambda_2}{\mu_2} > \cdots \frac{\lambda_i}{\mu_i}$$

which results in subsystem MTTR apportionments inversely proportioned to subsystem failure rates; that is lower MTTR requirements are assigned to subsystems having higher failure rates. Applying that condition to a system of n subsystems gives:

$$\mu_i = n\lambda_i\,(\mu_s/\lambda_s) \cdot$$

For example, assume a shipboard catapult consisting of launching, braking and retraction subsystems has a system requirement of 1850 mean launch cycles between failure. The system availability requirement is A = .98 and the reliability apportionment process has resulted in the failure rate apportionments shown in figure 4-19. Average launch rate is

SUBSYSTEM	COMPLEXITY SCORE	STATE-OF-THE-ART SCORE	CRITICALITY SCORE	TOTAL SCORE	WEIGHTING FACTOR	APPORTIONED[1] FAILURE RATE (F CYCLE)	APPORTIONED MTBF (CYCLES)
Launching	9	10	1	20	20/53	20.40×10^{-5}	4900
Brake	7	10	1	18	18/53	18.36×10^{-5}	5450
Retraction	4	3	8	15	15/53	15.30×10^{-5}	6535
Total				$\Sigma = 53$		54.06×10^{-5}	1850

(1) Failure Rate = Weighting Factor x 1/1850

Figure 4-19. Catapult Model and Apportioned Subsystem Failure Rates

10.67 launches per hour, so the apportioned system MTBF is 1850/10.67 = 173.38 hours. Thus, the .98 availability requirement dictates MTTR not greater than

$$\text{MTTR}_s = \text{MTBF}_s \left(\frac{1\text{-A}}{\text{A}}\right) = 173.38 \left(\frac{.02}{.98}\right)$$

$$= 3.53 \text{ hours.}$$

Subsystem MTTR requirements are to be apportioned so that MTTR_s = 3.53 hours. Therefore the expression

$$\mu_i = 3\left(\frac{1850}{3.53}\right)\lambda_i = 1572.2\lambda_i$$

is applied to each subsystem failure rate yielding the MTTR requirements listed in figure 4-20.

SUBSYSTEM	APPORTIONED λ (FAILURES/LAUNCH)	APPORTIONED MTTR ($1/\mu_i$)
Launching	20.40×10^{-5}	3.11 hours
Brake	18.36×10^{-5}	3.46
Retraction	15.30×10^{-5}	4.15
System	54.06×10^{-5}	3.53

Figure 4-20. Apportioned Subsystem MTTRs

Maintainability apportionment can be simplified in cases where it can be shown that peripheral conditions unique to the operational environment, such as access limitations or the availability of diagnostic equipment, are the principal factors acting to determine maintenance time. In many such instances, the variable element of maintenance time is tightly distributed about a central value fixed by factors such as those noted above which are outside the designer's control. Where the dispersion is a negligible fraction of the central value, maintenance time may be treated as a constant for analytical purposes. If the quantity of preventive maintenance and the downtime necessitated by the need for preventive maintenance are significant functions of system design, these factors contribute additional degrees of freedom to the apportionment task.

By the time trade-off studies are completed, specifications containing tentative reliability and maintainability requirements, determined subjectively, may already have been written for many equipments. Specifications prepared in that manner are not optimal because, by implementing the results of the trade-offs, the same availability may be achieved with reduced expenditure of effort or more availability may be gained for the same effort. Thus, when a contractor apportions reliability and maintainability goals, in effect he apportions resources and effort as well. One of the purposes of apportionment is to permit equipment requirements to be defined objectively, so that the system requirements can be realized in a timely and economical manner. If the prediction indicates that the system will not meet its apportioned availability or reliability requirements, then additional design effort is required. Even when the prediction indicates that a system can be expected to meet or exceed its requirements, trade-off studies may be useful to optimize the design [8].

4.2.4 Reliability and Availability Prediction

4.2.4.1 Reliability Prediction

Prediction is accomplished by solving the reliability model using appropriate failure rates at part or component level. Failure rates for use in prediction can come from sources such as MIL-HDBK-217[2]; NPRD-1, Nonelectronic Parts Reliability Data [9]; the Government-Industry Data Exchange Program (GIDEP), or may be derived by the contractor by observation of his own products in tests or in service, if a sufficiently large body of such data can be obtained for study. Failure rates must be corrected for applied and induced stress levels and duty cycles as determined by the mission analysis.

4.2.4.1.1 Purpose of Prediction

Reliability prediction shall be used in formulating design decisions. The reliability prediction should begin in the design phase and continue during the design effort. Early predictions may be based primarily on part counts or known reliability of similar components. As design information becomes available, predictions can be updated using stress data on specific parts and reflecting the actual components utilized in the design. Reliability prediction has several purposes,

a. as a basis of selection among competing designs (predictions should use same data sources and assumptions),

b. to disclose critical or reliability limiting items in the design,

c. sensitivity of design to electrical stress, thermal stress and part quality,

d as a basis for reliability trade-offs among system components,

e. to describe numerically the inherent reliability of the design,

f. to provide inputs to: Design Review; Failure Mode, Effects, and Criticality Analysis (FMECA); Maintainability Analysis; Safety Analysis; Logistic Support; and Thermal Design.

4.2.4.1.2 Policy

SSPO policy as reflected in NAVSEA OD 21549[1] is to require initial, intermediate and final reliability predictions. An initial prediction forecasts the reliability of the projected final product. This forecast is based on the characteristics of the early design and improvements expected during the development phase. An intermediate prediction updates the initial forecast. The update is based on increased design information, including environmental data and internal stress information. A final prediction is based on the design submitted for final design review. It predicts the operational reliability of the item based on all relevant information available at that point in the program.

Predictions should be performed using the most realistic failure rates available. Data from almost identical hardware used in almost identical applications should provide a more realistic data base for predictions than average failure rates from MIL-HDBK-217[2] and RADC publications. The depth of the prediction analysis should be consistent with the level of design definition available.

4.2.4.1.3 Prediction Methods

Reliability prediction requires knowledge of the quality of parts that will be employed (commercial, JAN, JANTX, JANTXV), the level to which the parts will be screened, the temperature at which the parts will be used, the degree to which the parts will be electrically and environmentally derated, and any redundancies employed. Figures 4-21 and 4-22 illustrate the prediction process. MIL-HDBK-217[2] describes the most widely used prediction methods for electrical, electromechanical and electronic parts.

The reliability prediction of nonelectronic parts, such as gaskets, seals, valves, clutches, etc., is accomplished using various sources of failure rate data. These sources include (a) NPRD-1, "Nonelectronic Parts Reliability Data" [9], published by the Reliability Analysis Center at Rome Air Development Center, (b) GIDEP (Government and Industry Data Exchange Program, (c) Vendor data and (d) In-house data.

The NPRD-1 document [9] provides failure rates, including a mean and upper and lower limits on a 60% confidence interval, for a limited number of devices. GIDEP provides failure rates, including a mean and upper and lower limits on a 90% confidence interval, for a wide variety of devices reflecting various environments such as ground, ground mobile, jet aircraft, missiles, etc. Vendor and in-house data serves as a failure data source for peculiar equipment supplied by the manufacturer and/or equipment designer. It may reflect qualification or environmental test results or actual field use. In general, the failure data from these various sources is to be considered as generic and representative of the device of interest. Care must be exercised in selecting a failure rate for a particular device from these sources to assure optimum correspondence between the device of interest and the data source relative to design similarity and use environment.

Early prediction assumptions generally include:

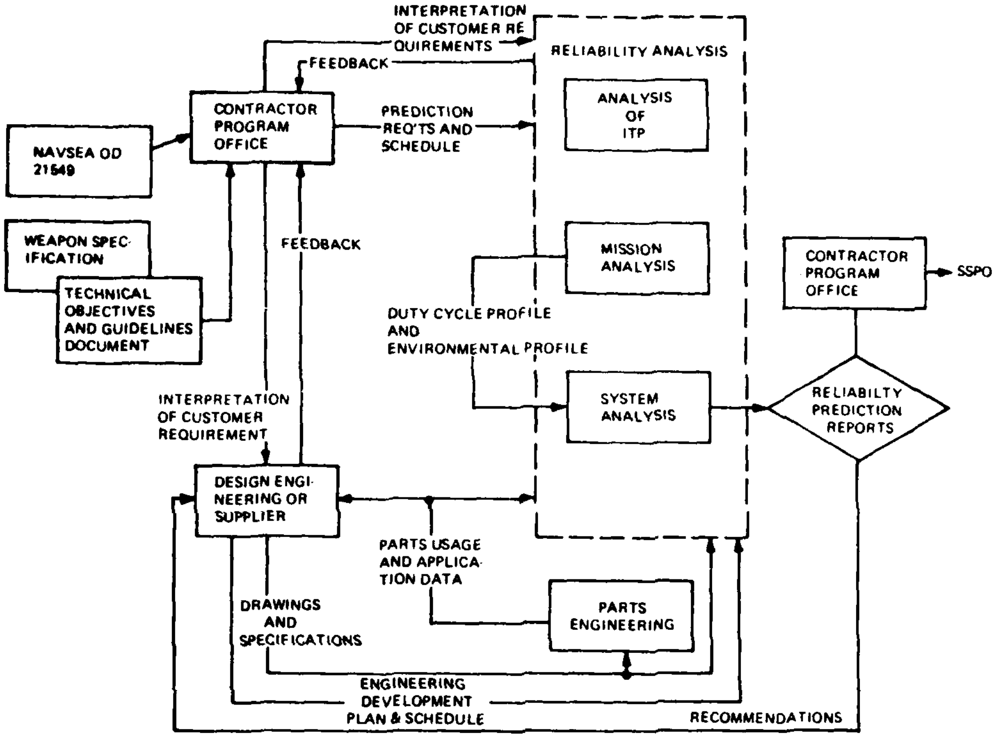
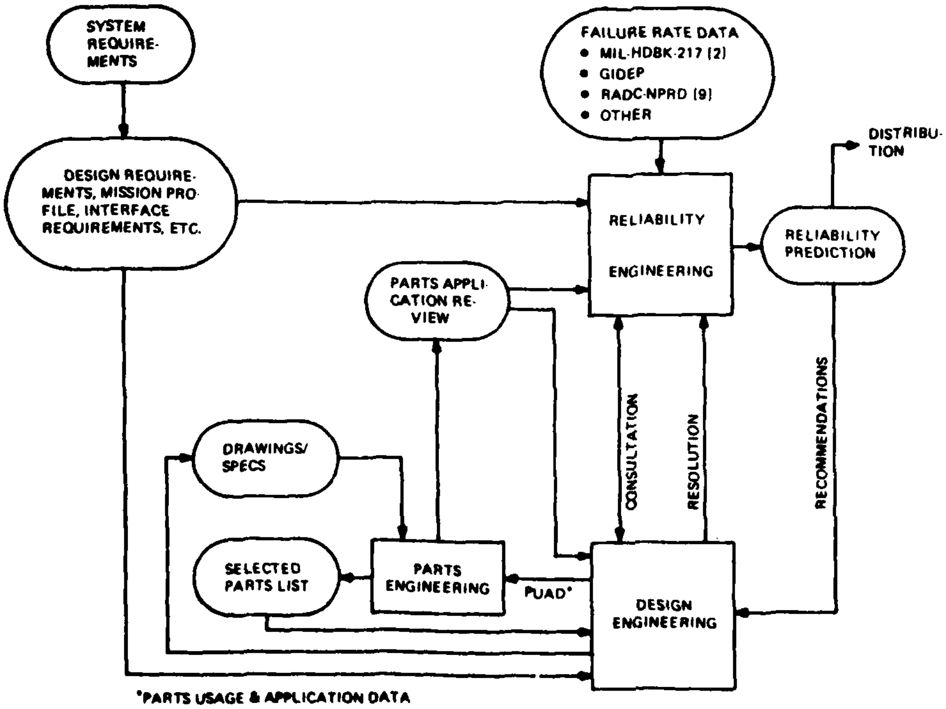


Figure 4-21. Flow of Reliability Prediction Activities and Information



*PARTS USAGE & APPLICATION DATA

Figure 4-22. Reliability Prediction Activity

- Part Quality - Early predictions are usually based on assumptions as to the quality of parts and screening. The predictions are refined when the quality of parts is known. MIL-HDBK-217[2] uses a factor, Π_Q, to account for part quality.

- Use Environment - In early predictions it may be necessary to make assumptions regarding the environment in which parts will be used. Later, analyses will provide better knowledge of the use environment. MIL-HDBK-217[2] uses a factor, Π_E, to account for the use environment.

- Application Review - Early predictions are usually based on assumed derating rules (e.g., all parts will be used at 50% of rated load). During the program, a part usage and application review should establish more accurate application and derating factors for each part. Knowledge of the actual derating, duty cycle, temperature limits and similar application factors permits much more accurate estimates of failure rates. The analyst should look for overstressed parts and call them to the attention of cognizant line and management personnel. It should be noted that SSPO requires that worst case conditions [e.g., environmental, duty cycle and derating] be used in prediction work.

4.2.4.1.4 Prediction Report

A prediction report should contain the best estimate of the reliability of the final design, based on information available when the prediction is made. The report must clearly identify the sources of all data used. See section 10 for examples of prediction work sheets.

When the predicted reliability is below the required reliability, the report should provide recommendations for improvement. The cognizant design engineering group should indicate actions planned or taken to improve reliability. Reliability demonstration testing should not be started while predicted reliability is below the requirement. Instead, redesign should be undertaken and verified by performing evaluation tests of the redesign effort.

4.2.4.2 Availability Prediction

Availability prediction is accomplished by predicting MTBF and MTTR. Development of reliability predictions has been discussed above. MTBF is the arithmetic mean or statistical expectation of time between successive failures. Prediction of maintainability indices is discussed below.

4.2.4.2.1 Analysis of Corrective Maintenance Tasks and Prediction of Availability with Respect to Failure

After the availability model has been written, a listing based on the maintenance concept, is made of corrective maintenance tasks that can arise because of failures of each of the equipment blocks, together with estimates of their failure rates and repair times (from maintainability predictions—see section 10 for worksheet examples). Repair time should include fault detection and isolation capabilities. A form such as figure 4-23 can be used to expedite the analysis. For series equipment, the sum of the failure rates of the components is a prediction of the equipment failure rate. The sum of the $\lambda \overline{M}_c$ column divided by the equipment failure rate is a prediction of the expected repair time. The final column is an approximation of the inherent availability or fractional up-time of the equipment block with respect to failure.

The prediction may be based on any of the procedures of MIL-HDBK-217[2], MIL-STD-756[10] and/or MIL-HDBK-472[11]. Source data may be based on historical experience, subjective evaluation, expert judgment or direct measurement of reliability and maintainability characteristics of elements of the system. However, the contractor may elect to use a non-standard method specifically applicable to the type of hardware comprising the system, subject to approval by SSPO.

Rules for developing system parameters from those of lower assemblies depend on the usual assumptions of statistical independence and exponential behavior. Experience has shown that these assumptions are valid for many systems.

Subsystem:								
(1) Equipment	(2) Component	(3) Predicted Failure Rate λ failures/hour	(4) Recovery Action		(5) Predicted MTTR $\overline{M}_c$ (hours)	(6) $\lambda\overline{M}_c$ failures	(7) $U = \dfrac{\overline{M}_c}{\lambda^{-1} + \overline{M}_c}$	(8) A = 1-U
			Repair	Replace				
A	1	$\lambda_1 = .018723$	X		$\overline{M}_{c_1} = 0.5$	.0093615	.009275	.9907
	2	$\lambda_2 = .016457$	X		$\overline{M}_{c_2} = 1.0$	.0164570	.016191	.9838
	3	$\lambda_3 = .020034$		X	$\overline{M}_{c_3} = 1.5$	.0300510	.029174	.9708
Equipment A	–	$\lambda_A = .055214$	–	–	*1.0119	.0558695	.052915	.9471
B	1	$\lambda_1 = .022345$		X	$\overline{M}_{c_1} = 1.5$	.0335175	.032431	.9676
	2	$\lambda_2 = .021691$		X	$\overline{M}_{c_2} = 1.2$	.0260292	.025369	.9746
Equipment B	–	$\lambda_B = .044036$	–	–	*1.3522	.0595467	.056199	.9438
Subsystem	–	$\lambda_S = .099250$	–	–	*1.1629	.1154162	.103475	.8965

*Calculated using column (6) ($\lambda\overline{M}_c$) over column (3) (λ)

Figure 4-23. Analysis of Corrective Maintenance Tasks

MTTR^{-} ($\overline{M}_c$) is found as the average of mean repair times of all components, repairable on line, weighted for the relative failure rate of each component.

$$\overline{M}_c = \frac{\sum_{i=1}^{n} (\lambda_i \overline{M}_{c_i})}{\sum_{i=1}^{n} \lambda_i} \qquad (4\text{-}18)$$

4.2.5 Failure Mode, Effects, and Criticality Analysis

Failure Mode, Effects, and Criticality Analysis (FMECA) is a systematic, organized design evaluation procedure which:

a. Identifies potential failure modes, their causes and method of detection [visual, manual, automatic (PM/FL)] at the level of hardware of interest (e.g., system, subsystem, equipment or component). This includes impact of dummy loads, fans, and air conditioners which can fail and pull the system down.

b. Determines, by analysis and evaluation, the effect of each failure mode on the hardware element in which it occurs, on the next higher assembly and ultimately on system operation, mission objectives and crew safety.

c. Establishes criticality level of each failure mode, permitting ranking of failure modes relative to effect on the mission.

d. Predicts probability of occurrence of each failure mode, permitting ranking of failure modes relative to likelihood.

e. Provides a suitable basis for assigning priorities to failure mode corrective actions through the joint consideration of criticality and probability of occurrence.

f. Documents results in an orderly fashion to highlight deficiencies (reliability, detection capability) and safety problems, recommend corrective action, identify changes needed in test programs, and aid in the development of operating and maintenance manuals.

g. Provides timely inputs to design reviews.

h. Provides feedback of information to cognizant contractor organizations (e.g., test, design, reliability, maintainability, systems).

4.2.5.1 Purpose of the FMECA

The purpose of FMECA is to evaluate the design by analysis in the early design stages. Specifically, the FMECA is to identify potential failure modes and to define their criticality so that informed decisions can be made about the worthiness of the design and the necessity for corrective actions.

FMECA is performed in order to prevent problems, to eliminate failure modes during early design stages and before they actually occur in operational use.

The results of the FMECA are valuable in test program planning and in determining the need for automatic monitoring, fault detection or alarm design features.

A completed FMECA consists of:

a. An orderly list of failure modes and their causes.

b. A classification or ranking of the failure modes with regard to their impact on performance and safety.

c. The probability of occurrence of the failure mode, a ranking with regard to expected frequency of occurrence.

d. An identification of existing design features (e.g., isolation or fault tolerant techniques), screening procedures (e.g., improved part quality), etc. that will minimize or obviate the effects of potential failure modes or reduce their probability.

e. Recommendations for precluding or circumventing significant failure modes or for reducing their probability of occurrence.

f. A description of alarms or other means of detecting the failure mode and the frequency with which the mode can be detected; (e.g., instantaneously, during daily checkout, etc.).

g. Criteria for test planning and the design of test and checkout systems which are responsive to identified failure modes and safety hazards.

h. Criteria for logistics planning and maintainability analysis by inclusion of information for selection of preventive maintenance points and development of trouble shooting guides.

i. Identification of single failure points in circuits for worst case analysis; failure modes involving parameter drifts may require worst case analysis to determine criticality (see NAVSEA OD 21549[1] for most severe design analysis requirements).

j. Input data for trade-off studies and for establishing corrective action priorities.

k. Historical documentation for future reference to aid in analysis of test and/or field failures for consideration of design changes, and as an aid in future development efforts.

4.2.5.2 FMECA Method

FMECA is an interdisciplinary study requiring skills in system and equipment (hardware and software) design, reliability analysis and data utilization, maintainability, safety, probability concepts, testing, modeling, and associated mathematics. While an individual may be assigned responsibility for a FMECA, he will require team support in order to produce results of significant substance, because so many skills are involved in the analysis.

FMECA is a detailed analysis of an equipment, subsystem or system. It is necessary to understand how the device operates and how it interfaces with other devices to perform a mission. The analyst must explore the effects of various part faults or functional failures on the equipment and ultimately the system [12, 13, 14, 15].

a. Gathering Information

A variety of information is required to produce a meaningful FMECA. Figure 4-24 identifies the types of information which are usually accumulated prior to performing a FMECA. Figure 4-25 shows how individual pieces of information and tasks are organized to produce a FMECA. Figure 4-26 illustrates a functional block diagram useful for performing a FMECA.

	INFORMATION	USUAL SOURCE
1.	System-Mission Description	System Specification Operational Profile Mission Analysis (Analysis for Design)
2	System/Subsystem/ Component Functions	Design Sketches, Drawings, Functional Block Diagrams
3.	Environmental Conditions	System Specification, Thermal, Dynamic, Structural & Mechanical Analyses
4.	System/Subsystem/ Component/Part failure modes individual and critical faults	System/Hardware design activity and reliability organization
5.	Failure rates for each mode identified	Past Experience, Handbooks, Related Products, Vendor Data, Published Papers, GIDEP
6.	System/Subsystem/ Component Reliability Block Diagram and mathematical model	System Specification, Mission Analyses, Design Drawings, Functional Block Diagrams
7.	Maintenance Concepts	System Requirements Analysis

Figure 4-24. General Steps, Information, Sources and Interfaces for FMECA

Analysts performing a FMECA must first acquire full understanding of the design and how it works, then focus on how the design can fail.

FMECA should consider the lowest hardware level for which adequate design definition exists. Every credible potential failure mode should be identified and classified relative to probability of occurrence and effect on the system, mission, or crew safety.

Compensating features and existing detection capability are analyzed and additional compensating features or detection capability are recommended for every failure mode for which there is a significant probability of aborting the mission or creating an unacceptable safety hazard.

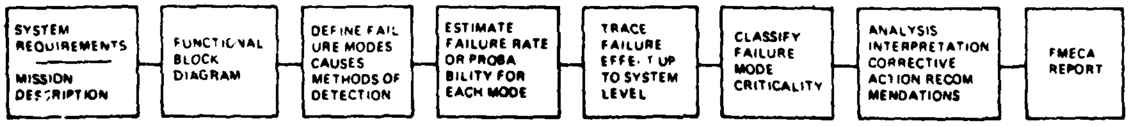


Figure 4-25. FMECA Process

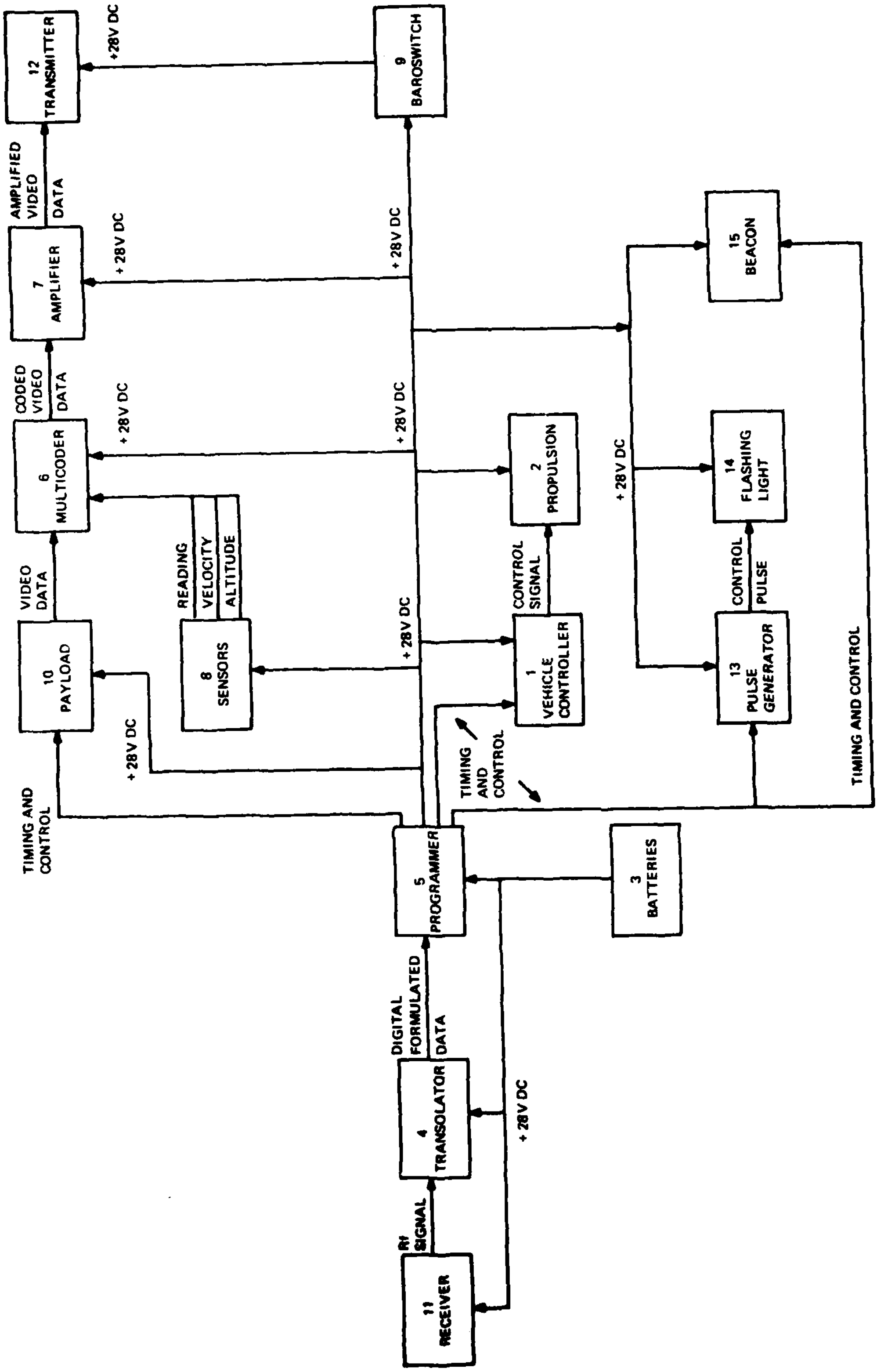


Figure 4-26. Functional Block Diagram of Reconnaissance Vehicle Subsystem

FMECA is a single-point failure analysis technique; i.e., each failure mode is considered individually. For example, the analysis could be performed on a top-down breakdown item that is critical. The analysis should be performed to the level where the problem can be identified and corrective action can be taken. Modes are ranked to reflect both the probability of occurrence and the severity of the failure effect relative to the hardware's mission or on crew safety. Corrective action should consider use of higher reliability parts, redundancy, alternate modes of operation, use of protective devices, improved maintenance accessibility or as candidates for developmental testing and reliability improvement programs. All of this information is documented by completing FMECA forms.

In addition to the expected environments of the operating mission, a FMECA should also take into account failure modes and effects associated with transportation and storage environments. The effects of these environments are of particular concern for hardware that cannot be tested effectively (i.e., arming and fuzing systems, squibs). However, even for hardware that can be tested, these environments affect availability.

b. FMECA Scheduling

To influence design decisions, FMECA must be completed and results available when design reviews are held. Design reviews represent key milestones in a system development program. An initial FMECA should be available to support preliminary design review (PDR) and an updated FMECA should be available for critical design review (CDR).

c. FMECA Worksheet

A number of generally similar forms are in use for documenting FMECA. While no one form is applicable to all programs, it is usually easy to adapt a form to a specific program. Figure 4-27 presents typical FMECA worksheets.

d. FMECA Procedure

A FMECA is completed as follows:

Step 1. Block Diagram – A functional block diagram may be prepared to describe relationships among elements of the hardware at a particular assembly level. For example, in a diagram of a subsystem, each block represents an equipment; in an equipment diagram each block represents a component. The diagram should make clear the functional relationships of each block to the others; the nature and magnitude of inputs and outputs should also be labeled. Each block may be designated by an item number for use in completing the FMECA form. Figure 4-26 is an example of a functional block diagram at subsystem level. It partitions hardware for analysis at the equipment level. Since not all hardware fits the typical part-component-equipment-subsystem-system pattern, cases may be encountered where the system must be partitioned arbitrarily. In those cases the hardware should be grouped for analysis in the way that seems simplest and most logical. It may be better to base such a grouping on functional rather than placement or packaging considerations.

Step 2. Failure Modes – Each block of the block diagram is considered in succession. All credible failure modes are listed, both degradative and catastrophic. It is important to list not only what is expected to happen but rather every failure mode than can happen. This usually requires some consolidation of simple failure events, particularly when considering higher levels of assembly such as equipments or subsystems. For example, all of the numerous failures that can affect an amplifier in continuous use can be summarized in three failure modes–no output, gain out of tolerance, noise or distortion of the amplified signal. Similarly, for many switches, all failure possibilities can be summarized by considering that the switches may fail open when they should remain closed, may close (short) when they should be open, or may have excessively high circuit resistance. In reviewing the ways hardware can fail, it is important to assess possible effects of environmental stresses as well as operating stresses. For this reason it is important that the best available environmental envelope be prepared during mission analysis before beginning FMECA.

Step 3. Causes of Failure – Beside each failure mode are listed all the causes believed capable of giving rise to the failure. This step may also call for summarizing some of the

FAILURE MODE, EFFECTS, AND CRITICALITY ANALYSIS

Sheet 1 of 10

Indenture Level No. 1 Reconnaissance Vehicle System
Indenture Level No. 2 Recovery Aids
Indenture Level No. 3 Flashing Light
Indenture Level No. 4
Drawing 123C456P03

Contract or Project Order N00030-
Organization Reliability
Prepared By A. Engineer
Approved By A. Boss
Date December 1981

OUTPUT SPECIFICATION FUNCTIONAL DESCRIPTION	FAILURE MODE		POSSIBLE CAUSES	SYMPTOMS DETECTABILITY	EFFECTS OF FAILURE		EXISTING COMPENSATING PROVISION	FC	FP	REMARKS AND RECOMMENDATIONS
	SN	DESCRIPTION			LOCAL EFFECT	END EFFECT				
<u>Pulse Generator</u> (PG) Electronic circuit that interrupts the input voltage to the light.	PG1	Constant output	Short circuit	Visual observation	Light does not flash; constant on	Difficult to visually locate system for recovery	Beacon, item 15, sends a signal which is used for locating purposes	2	2	Incorporate redundant pulse generator
	PG2	No output	Open circuit	Visual observation	Light does not turn on			2	2	
<u>Flashing Light</u> (FL) High intensity filament lamp operated off of the input from the pulse generator.	FL1	Fails to light	Open filament	Visual observation	Light is not on	Difficult to visually locate system for recovery	Beacon, item 15, sends a signal which is used for locating purposes	2	1	Use two lights in parallel

FC - Failure Classification (e.g., 1-Catastrophic, 2-Critical, 3-Major, 4-Minor)

FP - Failure Probability (e.g., 1 - $Q_i \leq 0.01\ Q_T$, 2 - $0.010Q_T < Q_i \leq 0.1Q_T$, 3 - $0.10Q_T < Q_i \leq 0.20Q_T$, 4 - $0.20Q_T < Q_i$)

Q_T = Total System Failure Probability

Figure 4-27A. Typical FMECA Worksheet

FAILURE MODE, EFFECTS, AND CRITICALITY ANALYSIS

PROGRAM: MULTIPLEX SYSTEM

INITIAL INDENTURE LEVEL: Local Multiplexer (LM)

INDENTURE LEVEL 2: RF Transceiver 4C

Date December 1981

Prepared by A. Engineer

OUTPUT SPECIFICATION FUNCTIONAL DESCRIPTION	FAILURE MODE		POSSIBLE CAUSES	SYMPTOMS DETECTABILITY	EFFECTS OF FAILURE		EXISTING COMPENSATING PROVISION	FS *	FP **	REMARKS AND RECOMMENDATIONS
	SN	DESCRIPTION			LOCAL EFFECT	END EFFECT				
4C.09 Provide clock for on-module frequency reference	4C.09 -01	Fail to provide clock	Clock is generated internally	Maintenance Unit detects loss of communication via this cable and LM reports no channel offers are being received. Fault code printed by MU.	Clock is used as frequency reference for transmitter and both receivers, so all functions are lost.	Loss of communication via this cable	Alternate RF Transceiver	1	1	

FS = Failure Severity

Level 1 - Minor - Neglible Effect

Level 2 - Major. Some System Degradation

Level 3 - Critical. Severe System Degradation

Level 4 - Catastrophic. Loss of System

FP = Failure Probability

Level 1 = $\leq 0.01\ Q_T$

Level 2 = $0.01\ Q_T < Q_i \leq 0.10\ Q_T$

Level 3 = $0.10\ Q_T < Q_i \leq 0.20\ Q_T$

Level 4 = $0.20\ Q_T < Q_i$

Q_T = Total System Failure Probability

Q_i = Failure Probability of ith Failure Mode

Figure 4-27B. Typical FMECA Worksheet

FAILURE MODE, EFFECTS, AND CRITICALITY ANALYSIS

PROGRAM: MULTIPLEX SYSTEM

INITIAL INDENTURE LEVEL: Local Multiplexer (LM)

INDENTURE LEVEL 2: Power Supply 4A/47

Date December 1981

Prepared by A. Engineer

OUTPUT SPECIFICATION FUNCTIONAL DESCRIPTION	FAILURE MODE		POSSIBLE CAUSES	SYMPTOMS DETECTABILITY	EFFECTS OF FAILURE		EXISTING COMPENSATING PROVISION	FS *	FP **	REMARKS AND RECOMMENDATIONS
	SN	DESCRIPTION			LOCAL EFFECT	END EFFECT				
4A.01 Provide ±5 vdc and +15 vdc to LM modules	4A.01 -01	Fail to provide power to LM modules	Power Supply converts 115 vac, 60 Hz or back-up power to the required dc outputs	Redundant LM reports loss of power for this LM, fault code printed by maintenance unit	Loss of power to LM modules (Type B)	Loss of LM	Back-up supply	1	3	LM modules may load-down power supply
47.01 Provide routing of 115 vac, 60 Hz ships power	47.01 -01	Fail to provide 115 vac, 60 Hz ships power	115 vac from ship	Loss of power is reported to Maintenance Unit by redundant LM, fault code printed	Loss of 115 vac to EMI Filter (Type A)	Loss of LM if Back-up power is not available	Back-up supply	1	2	AC power indicator on LM will be extinguished

FS = Failure Severity

Level 1 - Minor - Neglible Effect

Level 2 - Major Some System Degradation

Level 3 - Critical. Severe System Degradation

Level 4 - Catastrophic. Loss of System

FP = Failure Probability

Level 1 = $\leq 0.01\ Q_T$

Level 2 = $0.01\ Q_T < Q_i \leq 0.10\ Q_T$

Level 3 = $0.10\ Q_T < Q_i \leq 0.20\ Q_T$

Level 4 = $0.20\ Q_T < Q_i$

Q_T = Total System Failure Probability

Q_i = Failure Probability of ith Failure Mode.

Figure 4-27C. Typical FMECA Worksheet

with the failure mode and determines its effect, whereas FTA starts with a specific undesired event and determines the potential causes using combinatorial logic. FTA is a valuable tool for summarizing the results of a FMECA and is also useful for preparing trouble shooting manuals.

4.2.6.1 FTA Method

FTA involves the following procedure:

a. *System Definition*

Define (1) performance and safety of the system, (2) relationships between system performance of lower level assemblies, (3) human, hardware, and software interfaces, and (4) operation of the system.

b. *Mission Definition*

Define (1) mission phases, (2) environmental profile, (3) duty cycle profile, and (4) success criteria.

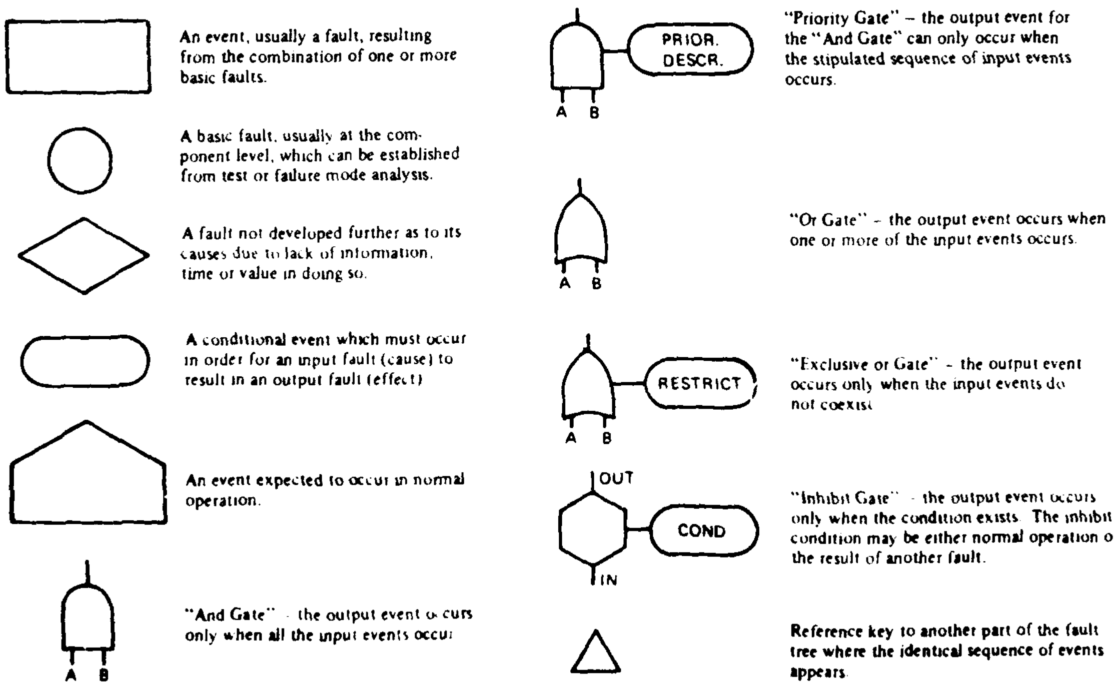
c. *Block Diagrams*

Block diagrams depict system functional flow at the circuit/component level. Operator interactions and external interfaces are also included in the diagrams.

d. *Fault Tree Construction*

Fault tree construction begins with the identification of the undesired event that will form the top level of the tree. This is normally a high level event involving nonperformance of a required function at the system level, although top events are sometimes stated in non-hardware terms such as occurrence of personnel injury. For example, in the case of a nuclear power station, the top event in a reactor control system tree might be core melt while the top event in the safety system tree could be the release of a specified quantity of radiation.

The fault tree is developed using logic symbols (Figure 4-28) to trace downward from the top event, through all levels of sub-events, to all of the elementary events which contribute to causing the top event. The level of



An event, usually a fault, resulting from the combination of one or more basic faults.

A basic fault, usually at the component level, which can be established from test or failure mode analysis.

A fault not developed further as to its causes due to lack of information, time or value in doing so.

A conditional event which must occur in order for an input fault (cause) to result in an output fault (effect)

An event expected to occur in normal operation.

"And Gate" - the output event occurs only when all the input events occur

"Priority Gate" – the output event for the "And Gate" can only occur when the stipulated sequence of input events occurs.

"Or Gate" - the output event occurs when one or more of the input events occurs.

"Exclusive or Gate" - the output event occurs only when the input events do not coexist

"Inhibit Gate" - the output event occurs only when the condition exists. The inhibit condition may be either normal operation or the result of another fault.

Reference key to another part of the fault tree where the identical sequence of events appears.

Figure 4-28. Logic Symbols, Used in Fault Tree Diagrams possible causes in categories. For example, it might not be possible to list every circuit condition that could lead to loss of output from an amplifier, but they might be summarized as "circuit failures" when analyzing the subsystem. Later, each would be reviewed as analysis proceeded down to the amplifier itself and the circuits within it.

Step 4. Method of Detection Provisions for detecting the failure mode when it occurs are listed. Some failure modes may cause automatic shut down to prevent additional damage; others may result in audible or visual alarms, and still others may go undetected.

Step 5. Effect on function or Item Performance – The immediate result of each assumed failure at the next higher level of assembly is described. It is important to list the local effects of the failure rather than to make a judgment of the overall significance of the failure to the system's performance.

Step 6. Effect on System Performance – The result of each assumed failure mode on the system or highest assembly level item being developed is described.

Step 7. Failure Classification – Failure modes may be classified by the approximate degree of degradation resulting from each mode. A code such as that shown below is usually adequate.

Catastrophic – Failure that will create a safety hazard (death or injury), or significant system loss.

Critical – Failure that will degrade the system beyond acceptable limits.

Major – Failure that will degrade the system beyond acceptable limits but can be adequately controlled or countered by alternate means.

Minor – Failure that does not degrade overall system performance beyond acceptable limits.

Each contractor must define these terms specifically for his subsystem.

Step 8. Failure Probability – The frequency of each failure mode or the probability of the mode, can be estimated using methods such as those given in MIL-HDBK-217[2]. Often a simple scoring scheme is used to group modes by relative probability.

Step 9. Compensating Provisions/Conditions – Compensating provisions embodied in the design, such as redundant channels, higher reliability parts, should be listed for each significant failure mode. It should be stated whether the compensation is total or partial, and whether resorting to the compensating provisions will limit or reduce efficiency.

Step 10. Comments and/or Recommendations for Design Improvement Additional provisions that might feasibly be included in the design should be listed. Additional redundancy and alternate modes of operation are examples of compensatory provisions that might be indicated where a failure would be critical to the mission. Where a single part failure mode is identified as catastrophic or critical to the mission, one or more recommendations for improving the reliability of the design should always be made.

Step 11. Closing the Loop For a FMECA to be effective, it is important that responsible engineering management be an integral part of the FMECA process. Recommendations growing out of the analysis should be evaluated by management for feasibility and cost of implementation. Corrective action decisions resulting from this evaluation must then be followed up and closed out by management, closing the loop on the FMECA process.

When a FMECA is reported, the results of the analysis should be summarized in an executive summary. The summary should include a listing of the important failure modes disclosed in the analysis and recommendations for eliminating them or reducing their impact. Management can then consider and act on the recommendations. The FMECA worksheets should be included in the FMECA report for review by interested parties and as documentation of the analysis

4.2.6 Fault Tree Analysis

Fault Tree Analysis (FTA) [16] is a design evaluation procedure which:

a. Identifies undesired events.

b. Graphically traces, from each undesired event selected, through hardware failures, software and human errors which could cause the event.

c. Estimates the probability of undesired events.

FTA differs from FMECA. FMECA begins

AD-A124 294 RELIABILITY AND AVAILABILITY EVALUATION PROGRAM MANUAL
(U) EVALUATION ASSOCIATES INC BALA CYNWYD PA 01 NOV 82
NAVSEA-OD-29304B N00030-79-C-0163

UNCLASSIFIED F/G 14/4 NL

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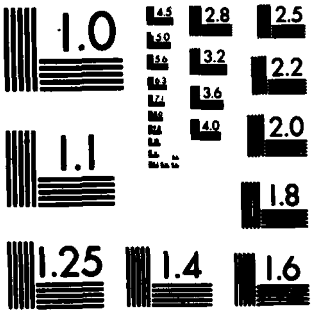

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

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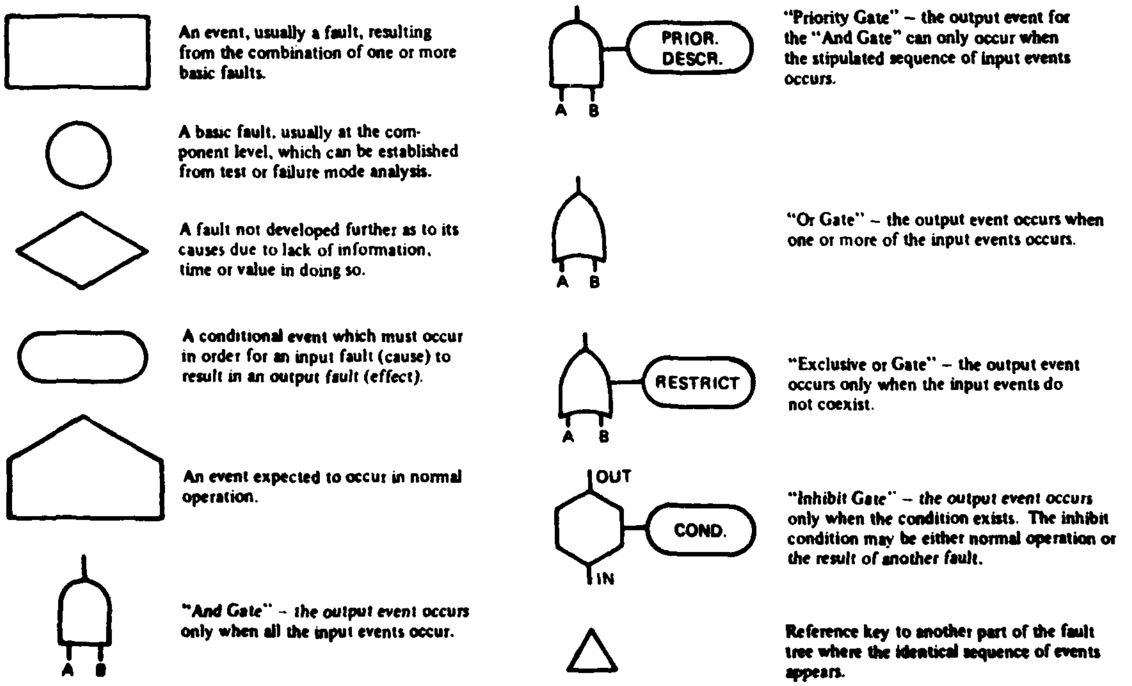
d. *Fault Tree Construction*

Fault tree construction begins with the identification of the undesired event that will form the top level of the tree. This is normally a high level event involving non-performance of a required function at the system level, although top events are sometimes stated in non-hardware terms such as occurrence of personnel injury. For example, in the case of a nuclear power station, the top event in a reactor control system tree might be core melt while the top event in the safety system tree could be the release of a specified quantity of radiation.

The fault tree is developed using logic symbols (Figure 4-28) to trace downward from the top event, through all levels of sub-events, to all of the elementary events which contribute to causing the top event. The level of



An event, usually a fault, resulting from the combination of one or more basic faults.

A basic fault, usually at the component level, which can be established from test or failure mode analysis.

A fault not developed further as to its causes due to lack of information, time or value in doing so.

A conditional event which must occur in order for an input fault (cause) to result in an output fault (effect).

An event expected to occur in normal operation.

"And Gate" – the output event occurs only when all the input events occur.

"Priority Gate" – the output event for the "And Gate" can only occur when the stipulated sequence of input events occurs.

"Or Gate" – the output event occurs when one or more of the input events occurs.

"Exclusive or Gate" – the output event occurs only when the input events do not coexist.

"Inhibit Gate" – the output event occurs only when the condition exists. The inhibit condition may be either normal operation or the result of another fault.

Reference key to another part of the fault tree where the identical sequence of events appears.

Figure 4-28. Logic Symbols, Used in Fault Tree Diagrams elementary events can be arbitrarily defined for any fault tree; however, it usually refers to (1) hardware failures of basic components such as those found in MIL-HDBK-217[2] and other failure rate handbooks, (2) human actions, (3) software errors and (4) occurrences of nature such as fire, water, wind and earthquakes.

The tree is constructed by linking the top event to its immediate causes or sub-events using the appropriate gate symbol from figure 4-28, then linking these sub-events to their causes and continuing in turn until the desired elementary cause level is reached. In the nuclear power example, the core melt tree might look like figure 4-29.

The top event is linked to its immediate causes by an AND gate since core melt can occur only if the reactor control systems and safety systems fail simultaneously. Developing the safety system branch of the tree, the next gate would also be an AND gate for the typical redundant safety system; however, the first gate under a particular safety channel would be an OR since any of the events shown could disable that safety channel.

Fault trees are subjected to both qualitative analysis and quantitative analysis. Qualitative analysis consists of determining the various combinations of elementary events that will cause the top event to occur and is used to: locate single point failures, assess criticality of components, identify common mode failures, evaluate redundancy and determine the relative importance of general fault categories; i.e., hardware, software, human error and nature. Quantitative analysis consists of determing the probability of occurrence of the top event from the probabilities of occurrence of the elementary input events. Quantitative analysis is always preceded by qualitative analysis since the quantification methods given here are valid only under certain conditions and those conditions are insured by performance of the indicated qualitative analysis.

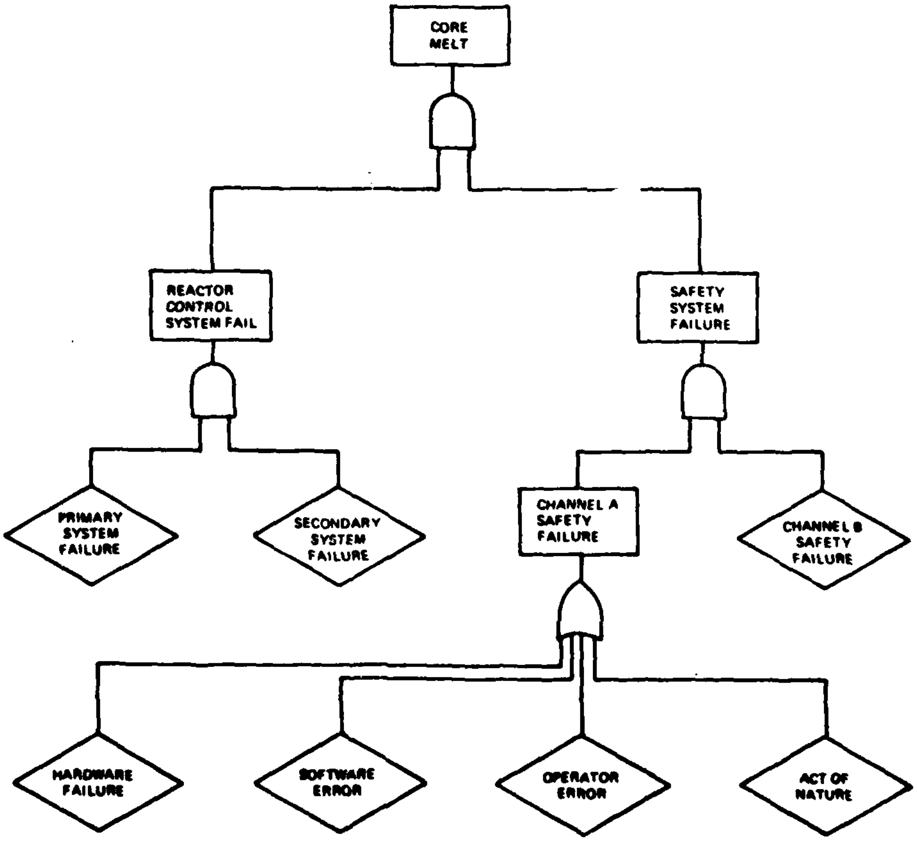


Figure 4-29. Fault Tree – Core Melt

e. *Qualitative Analysis*

For a simple fault tree the relationship between top event occurrence and elementary event occurrence can be determined by inspection. In the tree of figure 4-30, it can be seen that the top event A will occur in either of two cases; (1) event D occurs or (2) events E and F occur together. The tree can be redrawn as in figure 4-31 to more clearly reflect the true relation of event A to input events D, E and F.

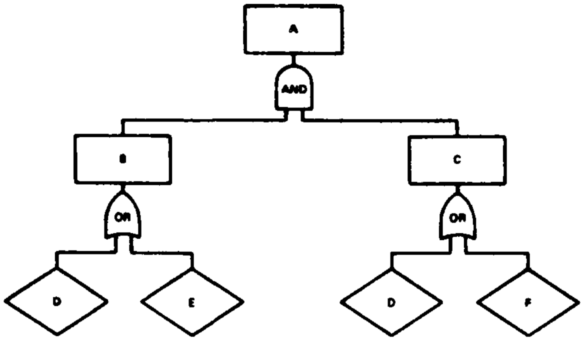


Figure 4-30. Fault Tree Example

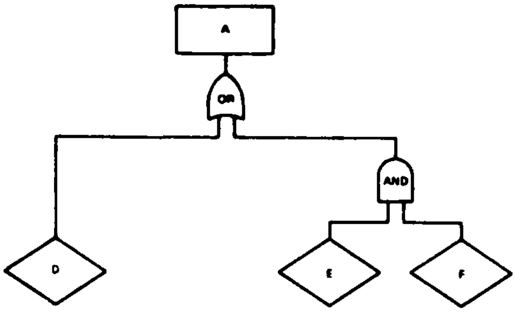


Figure 4-31. Fault Tree Example – Redrawn

In a more complex tree it is quite difficult and sometimes impossible to reduce the tree to its simplest form by inspection. The method of minimal cut sets, described in § 4.2.2.1.2.4, may be applied to provide a systematic way of reducing any fault tree to a form which is free of repeated inputs and therefore amenable to quantitative analysis. The idea of the cut set algorithm, first stated by Fussell and Vesely [17], is to replace each gate by its inputs of gates and basic events until a list matrix is constructed that contains only basic events. The algorithm is based on the fact that an AND gate increases the size of a cut set while an OR gate increases the number of cut sets, hence AND gate inputs are listed as column entries in a single row while OR gate inputs are listed as row entries in a single column.

To illustrate the application of the Fussell Vesely algorithm, consider the example fault tree in figure 4-32. The top event gate, G-O is an OR gate therefore the list matrix is begun by listing its inputs on separate rows:

1
G-1
2

Since any one of these events can cause the top event to occur, each will be a member of a separate cut set. Since G-1 is an OR gate, it is replaced by its inputs listed in separate rows:

1
G-2
G-3
2

G-2 is an AND gate, hence when it is replaced its inputs are entered in one row as:

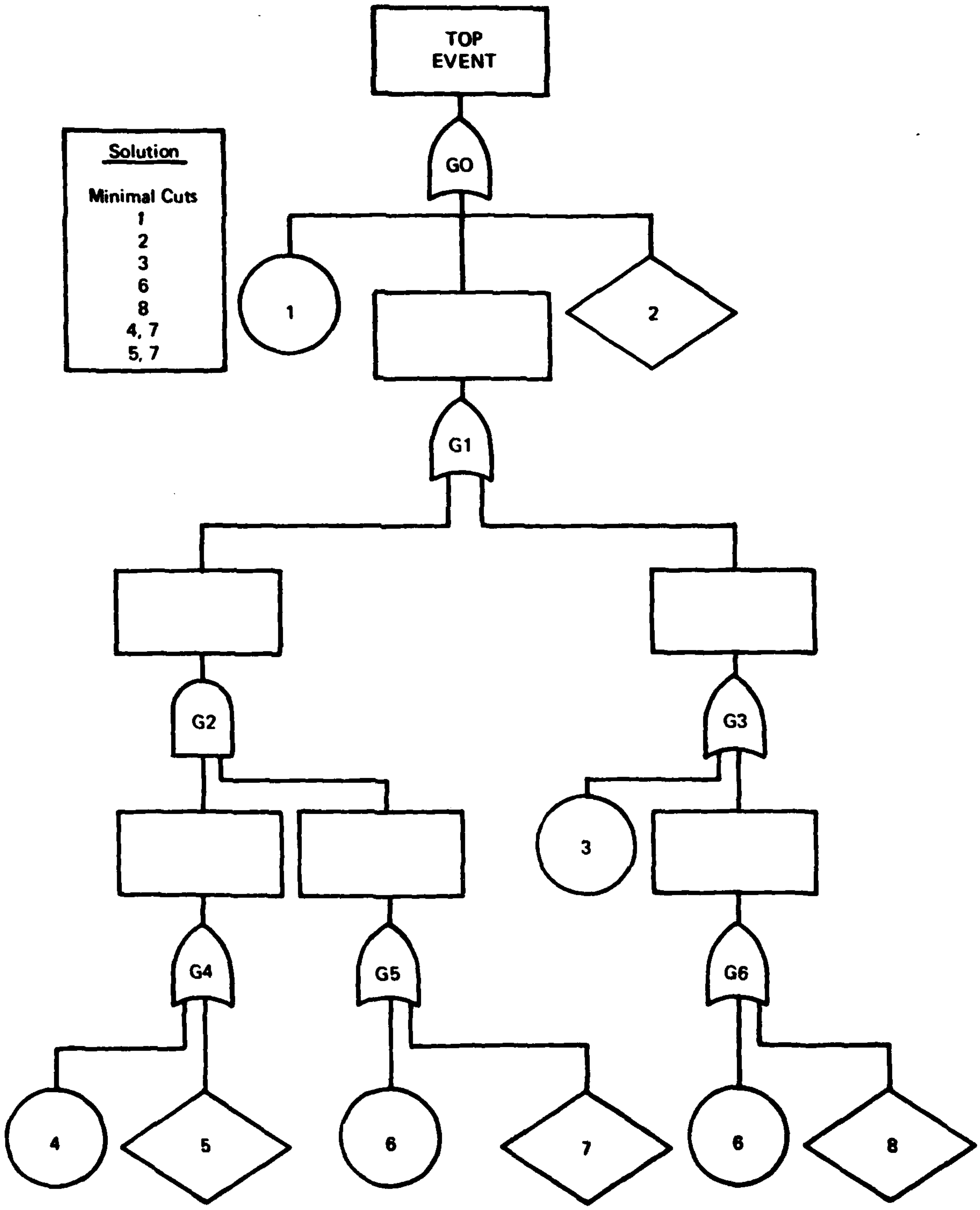
1
G-4, G-5
G-3
2

Now replacing OR gate G-4 we get:

1
4, G-5
5, G-5
G-3
2

Replacing G-5 produces

1
4,6
4,7
5,6
5,7
G-3
2



Solution

Minimal Cuts
1
2
3
6
8
4, 7
5, 7

Figure 4-32. Fault Tree and Solution

Elimination of G-3 creates the list

1
4,6
4,7
5,6
5,7
3
G-6
2

The final list is obtained by replacing G-6.

1
4,6
4,7
5,6
5,7
3
6
8
2

The cut sets obtained by this algorithm are called Boolean Indicated Cut Sets (BICS) since they will not be minimal unless there are no replications of basic events. When basic events are repeated, as in the example, then the BICS list generated by the algorithm must be reduced by inspection. In the list of BICS for the example 6 is a cut set of size one, hence all larger cut sets that contain 6, [4,6] and [5,6] are deleted, making the list of minimal cut sets

1
2
3
6
8
4,7
5,7

Although the Fussell Vesely algorithm can, in theory, be applied to the reduction of any fault tree, in practice, its use is best restricted to fault trees of modest size. Hand application of the algorithm to large trees is quite tedious and likely to result in errors. Computer programs to implement the algorithm are available, [18, 19] however, other computerized methods [20, 21, 22, 23] may prove more efficient for use with very large trees.

Another qualitative procedure that may be used in fault tree analysis is the determination of minimal path sets. A path set is a set of basic events whose non-occurrence insures the non-occurrence of the top event. A path set is minimal if it cannot be further reduced and still remain a path set. Path set determination may be used to identify areas in which redundancy would be beneficial.

The first step in finding the minimal path sets for a fault tree is to construct the dual (complement) of the tree by replacing OR gates with AND gates and AND gates with OR gates in the original tree and by replacing the occurrence of basic events with the non-occurrence of those events. The minimal cut sets of the dual tree are then obtained, these are the minimal path sets of the original tree.

f. *Quantitative Analysis*

After a fault tree has been constructed and reduced to minimal cut sets, the probability of occurrence of the top event can be determined by propagating the probabilities of occurrence of the basic events upward through the tree.

The probability of occurrence of the output event of an AND gate is found by multiplying the probabilities of occurrence of the input events. The probability of occurrence of the output event of an OR gate is approximately equal to the sum of the probabilities of occurrence of the input events. The exact probability for the output of an OR gate may be found from

$$\sum_{i=1}^{n} P_i - \sum_{i=1}^{n} \prod_{i<j} P_i P_j + \sum_{i=1}^{n} \prod_{i<j<k} P_i P_j P_k - \cdots$$

which for two inputs is

$$P(O) = P(A) + P(B) - P(A)\,P(B)$$

for three inputs is

$$\begin{aligned} P(O) = {} & [P(A) + P(B) + P(C)] - [P(A)\,P(B) \\ & + P(A)\,P(C) + P(B)\,P(C)] \\ & + [P(A)\,P(B)\,P(C)] \end{aligned}$$

and for four inputs is

$$\begin{aligned}
P(O) = {} & [P(A) + P(B) + P(C) + P(D)] \\
& - [P(A)\,P(B) + P(A)\,P(C) \\
& + P(A)\,P(D) + P(B)\,P(C) \\
& + P(B)\,P(D) + P(C)\,P(D)] \\
& + [P(A)\,P(B)\,P(C) \\
& + P(A)\,P(B)\,P(D) \\
& + P(A)\,P(C)\,P(D) \\
& + P(B)\,P(C)\,P(D)] \\
& - [P(A)\,P(B)\,P(C)\,P(D)]
\end{aligned}$$

The exact method is difficult to employ, either by hand or by computer, for large number of inputs and should be used only when the approximation is not sufficiently accurate. The accuracy of the approximation is a function of both the number of inputs and the magnitude of the input probabilities; it decreases with increasing number of inputs and with increasing input probabilities. The approximate value will always be larger than the true value, with an error of about 5% for 10 inputs with probabilities 1×10^{-2}. The error percentage will change by one order of magnitude for each order of magnitude change in either the number of inputs or the input probabilities.

It must be remembered that the AND gate computational procedure and the exact method for OR gates both require that all gate inputs be independent. This condition is insured throughout the tree if either there are no repeated basic inputs or if the replications are eliminated by reduction of the tree to minimal cut sets. To illustrate the importance of meeting this condition, consider the fault tree in figure 4-30 and let $P(D) = P(E) = P(F) = 1 \times 10^{-6}$. Then $P(B) = P(C) = 2 \times 10^{-6}$ and $P(A) = [P(B)]\,[P(C)] = 4 \times 10^{-12}$ with the calculation of P(A) violating the condition of independence, since D is an ultimate cause of both events B and C. P(A) correctly computed from the minimal cut sets of figure 4-31, is 1.000001×10^{-6}. The error encountered in using the AND gate multiplicative role incorrectly will depend on the number of replications involved as well as the value of the input probabilities involved. Over the range of input probabilities typically found in fault trees, the error can always be expected to be greater than 50%.

The computation of top event probability is made easier in the case of large fault trees by use of one of several available computer programs [19, 21, 22, 23].

4.2.7 Comparison of FMECA and FTA Methods

The FMECA analyst determines the assembly level at which the FMECA design analysis is to begin. Failure modes are postulated for each element at that level and the effects of the failure modes are traced upward. The analysis is inductive; reasoning is from the particular to the general. There is little chance for omission because each failure mode of each hardware element is examined in turn.

A principal advantage of the FMECA is its procedural simplicity. The analytical process is straightforward and permits complete and orderly evaluation of a design.

Disadvantages of FMECA are that the method considers only single failures and cannot readily examine the effects of human errors or other factors external to the system. Nor does it lend itself to assessing secondary effects of a failure. For example, a power supply may become overloaded due to a short circuited crystal; the overload, in turn, may result in a reduction of output from the power supply, causing other system effects which may not be readily discerned from the FMECA process. Multiple hardware failures or human errors can also result in consequences difficult to identify by FMECA.

Reference is sometimes made to "top down" FMECA, and this is often confused with fault tree analysis. In a top down FMECA, modes of failure of an assembly are postulated and their effects on personnel and equipment mission are determined. The failure modes are then traced back through the assembly to determine their causes.

Fault tree analysis begins with selection of an undesired event. The analyst then works downward to identify possible hardware malfunctions and human or software errors that lead to the undesired event. The analysis is carried down to a hardware or operating level where failure rate data are available or can be developed. The method is deductive; reasoning is from the general to the particular. Advantages of the FTA are its ability to encompass external factors such as human error, sabotage or natural disasters within the formalism of the analysis. Fault trees also tend to be easy to read and interpret. A major disadvantage of the FTA method is that there is no way to be sure every fault path has been included in the analysis. Examples of fault tree development are illustrated in figures 4-33 and 4-34.

It can readily be seen that FTA conveniently handles the machine-operator interfaces. However, it is not easy to be sure that all possible causes of the top event have been considered. And it is necessary to draw a fault tree for each top event of interest. It is not usually convenient to model a combination of undesired events in a single fault tree because the same item may appear at several points in the tree.

A fault tree has direct visual impact and for that reason is often useful for summarizing a FMECA. Thus, a FTA is often provided in the management summary of a FMECA. A fault tree can aid in the development of repair and test manuals by providing a graphic means for tracing from a system fault to the associated hardware failure(s).

Figure 4-35 compares the FMECA and FTA methods.

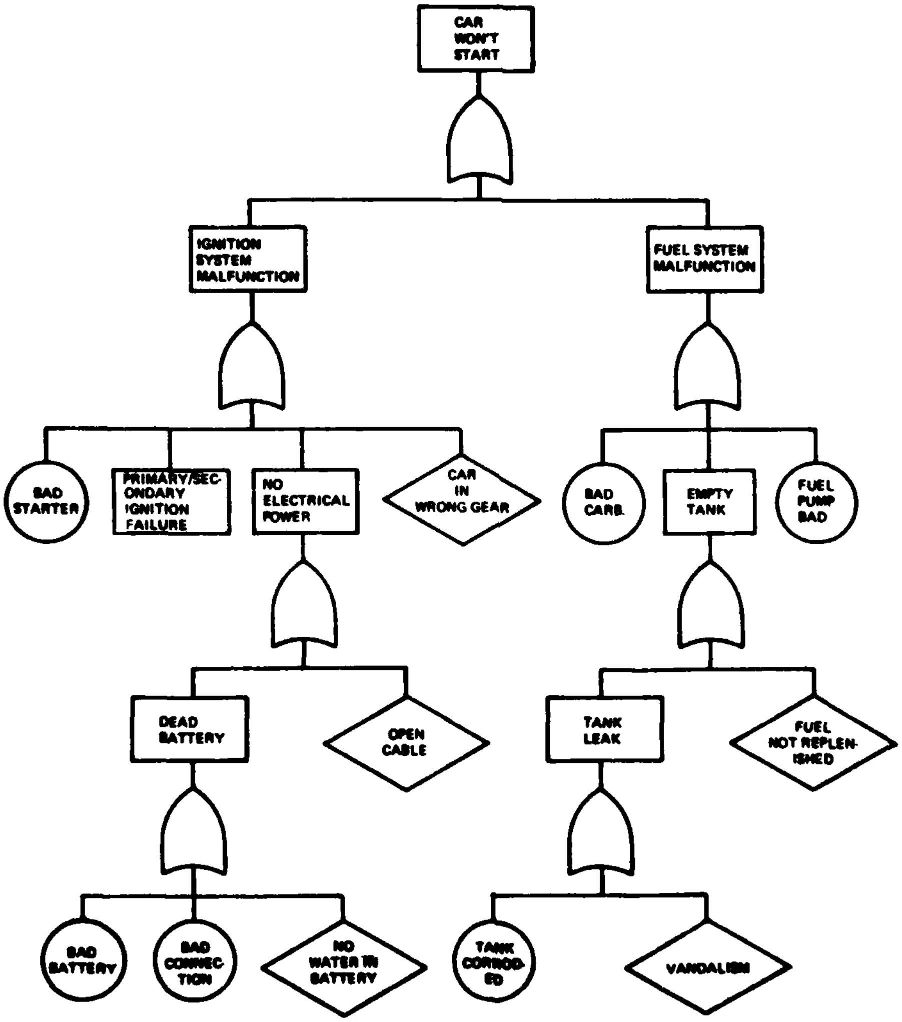


Figure 4-33. Example of Fault Tree Development

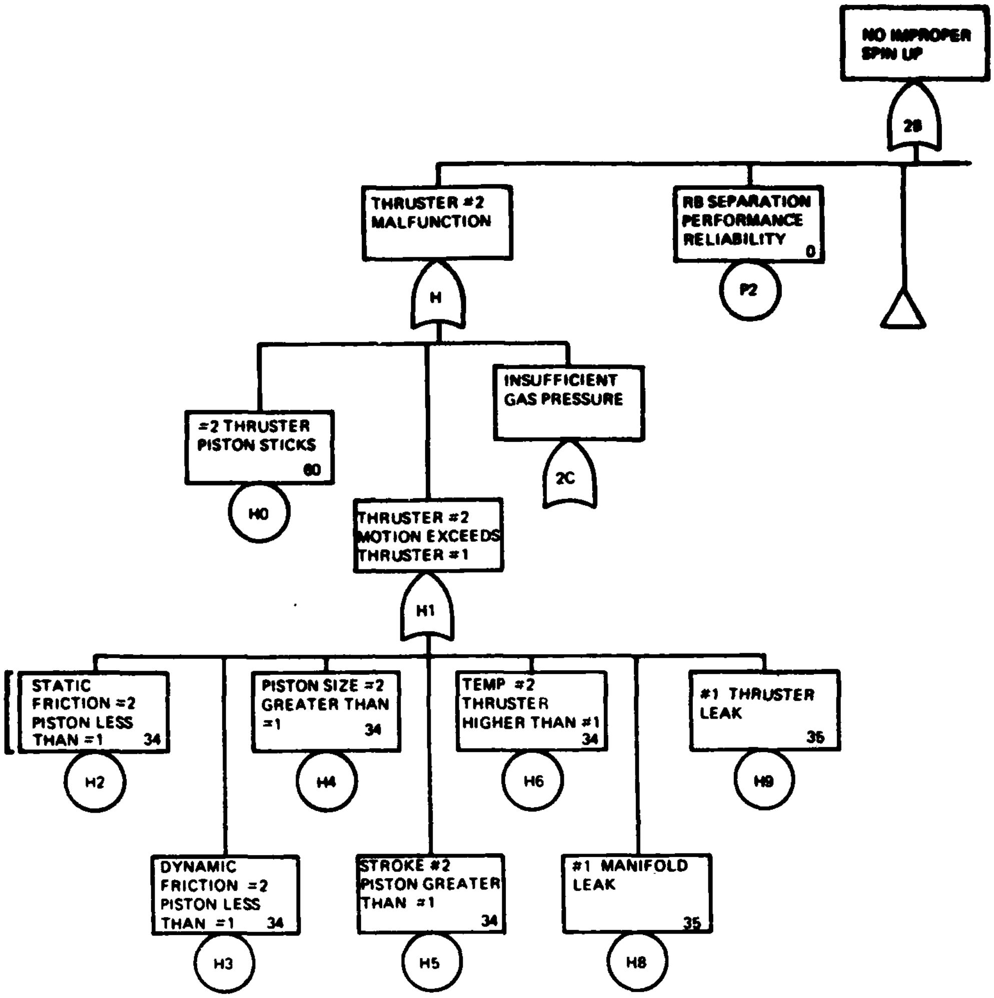


Figure 4-34. Portion of RB Deployment Fault Tree

	Cost		Modeling		Completeness	Human Factors	Corrective Action
	System	Events	System	Events			
FMECA	Low	N/A	Superior	N/A	Superior - All hardware failures considered	Difficult to take into consideration	Superior
FTA	High	Low	Difficult All system failures (events) must be known	Superior - multiple failures and human factors considered	Difficult to assure that nothing was overlooked	Superior	Adequate

Figure 4-35. FMECA/FTA Comparison

4.3 ANALYSIS OF INTEGRATED TEST PROGRAM FOR RELIABILITY AND AVAILABILITY EVALUATION

The general practice on FBMWS/SWS programs has been to minimize the planning of tests specifically for reliability or availability demonstration, while making maximum use of data evolved from tests performed for other reasons. The integrated test program approach reflects this philosophy. NAVORD OD 42282[24], Integrated Test Program Manual, describes the approach in detail. When this approach, an efficient method for major weapon systems, is used, the evaluation task is to analyze the integrated test program. There are two major sub-tasks–test identification and test evaluation.

4.3.1 Test Identification

Throughout the development and pilot production phases of a program, the contractor's test program must provide the data for reliability and availability evaluation. An Integrated Test Program Plan (ITPP) reflecting planning and control of all testing activities is required by SSPO. The ITPP sets forth the purposes and extent of the test program and is an essential input to planning for reliability and availability evaluation. The value of planned tests for reliability and availability measurement can be estimated by analysis of the ITPP, which can then be expanded or amended as program requirements may dictate. Specific tests that will contribute data for reliability and availability measurement are first identified. This is accomplished by completing a form such as the Test Identification Form shown in figure 4-36.

To complete the analysis, it is necessary to resolve any questions that may arise as to whether tests of individual equipments or other portions of the system will be employed for reliability measurement, as well as tests of the full system configuration. In general, tests of hardware at lower assembly levels can be

Test Ident. No.	Type of Test	Level of Test	Purpose of Test	Hardware Involved	Test Duration	Cycles/ Operating Time	Pass/Fail Criteria	Instrumentation Requirements	Data to be used for Reliability, Availability Evaluation

Figure 4-36. Reliability/Availability Test Identification Form

accepted as contributing to the evaluation data base, whenever the criteria of substantial mission equivalence can be satisfied with respect to operating environment, functional use and maintenance conditions. With these criteria as guidance, analysis of planned tests can readily be made.

The first step in the analysis is to evaluate each planned test to determine its purpose, the hardware involved, and the estimated duration of the test. This is done by filling out the test identification form. A decision whether to use the data for reliability and availability measurement is based on this evaluation. If the data are to be used, then specific pass/fail criteria and cycle/operating time designation for each portion of the test should be stated and any special instrumentation requirements indicated. The test identification form contains the following fields:

Test Identification Number – a number assigned to fully identify the test.

Type of Test – List the type of test such as Development, Engineering Evaluation, or Qualification.

Level of Test – indicate whether the test is at system, subsystem, equipment, or component level.

Purpose of Test – summarize the purpose of the test and include a reference to the particular paragraph of the test plan that describes the test in detail.

Hardware Involved – list the component breakdown for equipment, subsystem and system tests. Where a standard configuration is involved, this can be a reference to a standard list.

Test Duration – estimate the amount of operating or environmental exposure time that will be accumulated during the test. Provide separate estimates for each component if this is necessary for system, subsystem or equipment tests.

Cycle/Operating Time – define whether the test results are to be reported as cycles and/or operating time. This decision is made from review of the mission profile.

Pass/Fail Criteria – define a specific criteria for determining whether the test should be considered a success or failure. This might be a time or environmental level, threshold or specific readings for particular performance parameters.

Instrumentation Requirements – indicate the basis for measuring operating time or cycles, such as the time the power supply is activated or the number of times a particular switch is actuated.

Data to be used for Reliability/Availability Evaluation – indicate, by inserting a "yes" or "no" in this column; whether the test data will be used for reliability and availability measurement purpose.

4.3.2 Test Evaluation

The expected contribution of the tests selected for reliability and availability evaluation is estimated by completing a form such as the Test Evaluation Form shown in figure 4-37.

A review of the tests selected for use in reliability and availability evaluation should assure that adequate sample sizes are provided; that is, the data used for evaluation is not limited to repeated testing of 1 or 2 units.

From the test evaluation form, a summation is made of the estimated test times for a component in each level of test–component, equipment, subsystem, and system. This result is converted into equivalent missions through multiplication by the component alpha value obtained from the mission profile. A comparison of the estimated number of equivalent missions with the number necessary to demonstrate the apportioned reliability/availability at the desired confidence level will indicate whether the test program for that component is adequate. This evaluation should be done for each significant mission environment to which the component will be exposed. Each component and equipment group should be evaluated in turn.

In the exponential case, an estimate of the reliability lower bound which the planned testing would produce is obtained by multiplying the predicted failure rate (failures per mission) by the estimated equivalent missions to be produced by the planned testing. This product is the expected number of failures. Using this value of the expected number of failures and the estimated equivalent missions enter tables such as NAVWEPS OD 30668 [25] to determine the lower bound the

Test Ident. No.	Hardware Name	Test Environment	Estimated Test Time				Alpha Valve	Estimated Equivalent Missions	Estimated R_L
			Subsystem	Equipment	Component	Total			

Figure 4-37. Reliability/Availability Test Evaluation Form

planned data is expected to produce. If this is too far below the predicted value consideration can be given to planning additional tests.

Example

Predicted Failure Rate = .006 failures/ mission

Estimated Equivalent Missions = 1,000 mission

The product (.006) (1,000) = 6 failures

Selecting the eighty percent confidence level from tables [25] or Appendix E figure E-2.

$$R_L = 0.9910$$

which can be compared with $R' = 0.9940$, the predicted value.

Similar procedures can be used for other distributional forms.

The test evaluation form, figure 4-37 contains the following fields:

Test Identification Number – a number assigned to fully identify the test.

Hardware Name – state the equipment being evaluated. Evaluate the equipment first, then the components within it, then the next equipment and its components, etc.

Test Environment – list the mission environments to be evaluated for each hardware element.

Estimated Test Time – list the estimated test time totals for each level of test–system, subsystem, equipment, or component and also the sum.

Alpha Value – transfer the appropriate alpha value for the environment from the mission profile.

Estimated Equivalent Missions – this is the product of the alpha value and the sum of the estimated test time/environment.

The number of equivalent missions should be approximately equal for a balanced test program. Since the ITP approach makes use of all applicable test data including tests not planned specifically for reliability/availability evaluation, it is possible for imbalance to occur. The analyst should understand the reasons for an unbalanced program when it occurs.

Estimated R_L – this column provides the estimated lower bound on reliability at the desired confidence level which the test program is expected to produce.

Evaluation of the integrated test program may indicate an unbalanced plan (i.e., widely varying values of estimated equivalent mission at the same hardware level) or insufficient testing (i.e., a value of R_L far below the predicted value R′).

Analysis of the reasons for an unbalanced program or insufficient testing of some items often leads to recommendations for improving the ITPP.

4.3.3 Data Classification

Test data (time and failure information) from tests considered non-relevant for reliability purposes are excluded for purposes of evaluation. The failures should be analyzed and reported in the failure summary report and should be classified non-relevant since they come from a non-relevant test.

Test data from tests considered relevant for reliability purposes can be used for evaluation. Rules for establishing the relevance of failures in these tests should be carefully established. It is often desirable but not always accurate to eliminate (consider non-relevant) failures due to human error, test equipment error, and similar causes even when the failure occurs in relevant test programs. The contractor should establish, in his reliability evaluation plan the rules to be used for this purpose.

Elimination of relevant failures may be desirable after corrective action has been incorporated to eliminate a failure mode. (Note: this should not be permitted when reliability growth models are being employed, as the growth model requires these data.) The contractor should establish criteria for the amount of failure-free tests data required on the new design before the relevant failures can be eliminated. Provisions for re-inserting all failures removed must be available if the failure mode recurs. The contractor should also establish the policy for using the test time when failures are removed.

All failures should be reported in the failure summary report. The failure classification, non-relevant, relevant, or non-relevant previously classified relevant, defines failures to be used in reliability/availability calculations.

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Section 5
ASSESSMENT OF COMPONENT RELIABILITY

This section deals with techniques for assessing the reliability of components, based on test data generated in component level tests. It identifies those failure models and methods most effective in reliability assessments and provides guidelines for their selection and application. The section provides a road map (Figure 5-4) approach for selection of the model most appropriate for the data being assessed. Assessment consists of obtaining point, and interval estimates of reliability, failure rate, and MTBF from component test data, and when reliability growth is present, of obtaining trend lines and reliability bounds reflecting the uncertainty in trend lines as a function of calendar time.

Software reliability assessment is considered in section 6 which contains all information on software, and component availability is treated in section 7, the system assessment section, since availability is most meaningfully stated at the system or equipment levels.

A number of statistical terms and symbols used in this section are defined in more detail in figure 5-1 than was presented in the glossary.

5.1 OVERVIEW OF THE ASSESSMENT PROBLEM

Reliability information consists generally in a set of times to failure, cycles to failure, stress/strength parameter performance and pass/fail data for one or several items on test. Using these data, point, interval, or trend estimates of reliability parameters or measures such as λ, θ, or R are made.

There are many types of components, types of tests and ways of testing that provide data for reliability assessment. There are many probabilistic models of failure used in reliability assessment, such as, the exponential, binomial, normal, lognormal, Weibull, gamma, beta, extreme-value, cited or referenced in this section. The most applicable model for a given situation is sometimes known or assumed in advance of testing from an engineering analysis of the failure/repair process or from failure data/model analysis for similar components previously tested. It is always preferable to verify any assumed failure model by a goodness-of-fit test of the new failure data to the model when sufficient data have been accumulated.

In development programs there is, additionally, the possibility of reliability growth with its measures characterized by trends and testing schedules dependent on calendar time.

Given these hardware and testing considerations, statistical methods are also diverse, featuring: the classical approach with decisions and numerical results dependent respectively on tests of hypothesis and on point and interval estimates inferred strictly from the test data at hand; the variables approach which replaces the classical pass/fail concept with the concept of critical continuous parameters, which are often normally distributed and which are either "within specs" or "out of specs"; and the Bayesian approach which allows the inclusion of prior information in specified dosage of strength by means of "prior" distributions and parameters, and the update of this information with current test data in the form of "posterior" distributions and parameters.

The "real world" of hardware and testing is related in figure 5-2 to the probabilistic models of failure/repair and to the assessment methods described in this section. The first two columns of figure 5-2 show, respectively, the categories of components found in complex systems, and examples of such

Term or Symbol	Name	Definition & Explanation
CDF, $Q(t)$, $G(x)$, $Q(x)$	Cumulative Distribution Function	$Q(t)$ is the probability that the variate T takes a value less than or equal to t. $Q(t) = P_r(T < t)$. In reliability, when t is time, $Q(t)$ is often the unreliability or probability of failure in the time interval $(0, t)$
PDF, $f(t)$, $g(x)$, $f(x)$	Probability Density Function	$f(t)$ is a function whose integral over the range $t_1 < t < t_2$ is equal to the probability that the variate takes a value in that range. $f(t) = dQ(t)/dt$ In reliability, when t is time, $f(t)$ is often a failure rate PDF.
PMF	Probability Mass Function	The collection of numbers (p_i) satisfying $P(X=x_i)=p_i > 0$ for all i and $\sum_{i=1}^{\infty} p_i = 1$
$R(t)$, R	Reliability Function	Probability of no failure up to time t. $R(t) = 1 - Q(t)$
$A(t)$, A	Availability Function	$A(t)$: The probability that a system is operational at the instant of time, t A : $\lim_{t \to \infty} A(t) = MTBF/(MTBF + MTTR)$
$h(t)$	Hazard Rate or Failure Rate	The ratio of the failure rate PDF to the reliability at time t. $h(t) = f(t)/R(t)$
$\bar{X}$	Sample Mean	$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$, ML estimator for a normal distribution
MTTF, θ	Mean Time to Failure (or simply Mean if the PDF is not a failure model)	$MTTF = \int_0^{\infty} t\, f(t)\, dt = \int_0^{\infty} R(t)\, dt$ for a continuous $f(t)$ or $R(t)$ $MTTF = \sum_{x=0}^{\infty} x f(x)$ for a discrete $f(x)$
MTTR	Mean Time to Repair	Mean Corrective Maintenance Time following System entry into a Down State (Ref. figure 2-4)
MTBF, θ	Mean Time Between Failure	Same as MTTF but often used when repairs are considered.
s^2	Sample Variance	$\frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^2$, unbiased ML estimator for a normal distribution
σ^2, μ_2, v	Population Variance	Second moment about the Population Mean $\sigma^2 = \int_0^{\infty} (t - \mu)^2 f(t)dt$ or $\int_0^{\infty} (x - \mu)^2 f(x)dx$
σ	Population Standard Deviation	$\sqrt{V}$ where V = Population Variance
Prior	Prior Probability Density Function	In Bayesian terminology, a PDF $g(\theta)$ used as some prior probabilistic degree of belief on one (θ) or several parameters of a function or of any failure model.
Posterior	Posterior Probability Distribution Function	In Bayesian terminology, modification of the Prior through the use of Bayes' theorem when test data becomes available: $g(\theta \mid Data) = f(Data \mid \theta) g(\theta) / f(Data)$
Variables, VAEP	Variables Method	A method, whereby the value of certain measurable parameters are equated to "failures" if they lie beyond the range of specified critical values. The values of the parameters are often assumed to be normally distributed.

Figure 5-1. Definition of Statistical Terms or Symbols Used in Section 5

Term or Symbol	Name	Definition & Explanation
t_i	ith ordered - time to Failure	Time to Failure ordered by magnitude
M_C	Time to Repair	Same as Corrective Maintenance Time (Ref. figure 2-4). Used in computing MTTR on previous page
HPP	Homogeneous Poisson Process	λ is constant in calendar time in Poisson Expression
NHPP	Non-Homogeneous Poisson Process	$\lambda(t)$ changes in calendar time in Poisson Expression
IID	Identically Independently Distributed	Characterizes Failure/Repair Process which is purely random and calendar time independent. There are no trends.
$N(\mu, \sigma)$	Normal PDF	The normal PDF with mean μ and standard deviation σ
$\chi^2_{1-\alpha, \nu}$	Chi-Square CDF	The 1-α percentile of the χ^2 distribution with ν degrees of freedom
$F_{\gamma;\nu 1,\nu 2}$	F CDF	The γ percentile of the F distribution with $\nu 1$ and $\nu 2$ degrees of freedom
$t_{\alpha, \nu}$	Student-t CDF	The α percentile of the Student-t distribution with ν degrees of freedom
R_L	Lower Bound on Reliability	R_L is a number such that $P(R_L > R) < \alpha$, where $1 - \alpha$ is the confidence level
λ_u	Upper Bound on Failure Rate	λ_u is a number such that $P(\lambda_u < \lambda) < \alpha$, where $1 - \alpha$ is the confidence level
$MTTF_L, \theta_L$	Lower Bound on Mean Time To Failure	$MTTF_L$ is a number such that $P(MTTF_L > MTTF) < \alpha$, where $1 - \alpha$ is the confidence level
Piggyback	"Piggyback" Test	Test not primarily designed for the purpose of obtaining reliability data
SITP		Shipyard Installation Test Program
DASO		Demonstration and Shakedown Operation
OT/FOT		Operational Test Follow On Operational Test
IFR	Increasing Failure Rate	Increasing Hazard Rate
CFR	Constant Failure Rate	Constant Hazard Rate (Characterizes the exponential)
DFR	Decreasing Failure Rate	Decreasing Hazard Rate
SUP	Supremum	Least Upper Bound
λ	Greek Lambda	Constant failure rate parameter
μ	Greek Mu	Constant repair rate parameter
7(3) 22		Refers to the 1st value (7), the increment (3), and the final value (22) of the argument of a tabulated function. These values are here 7, 10, 13, 16, 19, and 22.
η	Greek Eta	Warrantee Period

Figure 5-1. Definition of Statistical Terms or Symbols Used in Section 5 (Continued)

components found in the Navy's inventory. The third and fourth columns show, respectively, the types of tests and success criteria used in assessment. Notice that some of the tests such as integrity or performance tests may be used as "piggyback" tests for failure/repair data, but that they may be insufficient for a thorough assessment of reliability and availability or for a true reliability/availability demonstration (see Section 8) if they do not have the sample sizes or the homogeneity of conditions necessary to demonstrate specified reliability/availability values at specified confidence levels. The data, however, may still be usable for a preliminary assessment. Column five identifies the primary measures of equipment behavior used in reliability assessments. As to be expected, these measures are different for each equipment category. Column six shows the most applicable distributions as models of failure by equipment category.

In this section only the exponential, binomial, normal, lognormal, and Weibull distributions are the object of a full assessment description. Their statistical descriptions are given in § 5.2. The binomial distribution is used *mostly* for data from pass/fail test, and for time truncated cycles to failure tests, the exponential models failure/repair data with constant failure/repair rate (CFR), the normal models data *from* "aging" components with increasing failure rate (IFR) and the variables approach to reliability assessment, the lognormal models skewed failure/repair data with increasing failure/repair rates (IFR), and the Weibull models failure data with DFR, CFR, or IFR characteristics.

Other probability distributions used in assessment, such as the Poisson, the gamma, the beta, the hypergeometric, the inverted gamma and the ubiquitous t, F, and χ^2 are either mentioned or used with appropriate references.

Finally, column seven of figure 5-2 lists the candidate methods for reliability assessment. The Rubinstein method mentioned in this column is applicable to exponential components with data originating from testing one component at a time in mixed censoring Life Tests, tests which are quite general since they are inclusive of both Type I and Type II tests described below. The method allows assessment of reliability for components tested for different failure modes and for different operating conditions.

5.1.1 Types of Tests by Method of Implementation or Stoppage

It is important to consider the various ways tests can be implemented, and test data obtained, since the assessment formulas used in the remainder of this section are critically dependent on the manner in which test data have been obtained.

(a) *In Uncensored Life Tests*, which are sometimes costly or diffucult to schedule, n identical items are placed on test and are monitored for times to failure (e.g. in minutes) or for cycles to failure until all items have failed.

(b) *In Type I Life Censoring Tests, or Time Truncated Tests*, n identical items are placed on test for a predetermined amount of test time T or number of cycles, and the times to failure (e.g. in minutes) or the cycles to failure of the x items which fail are recorded, unless there are no failures. These tests can be implemented *with* or *without replacement* of a failed item.

(c) *In Type II Life Censoring Tests* or *Tests to Failure*, n identical items are placed on test until a predetermined number of failures (x) occur. The times or cycles to failure of the x items are recorded. These tests can be implemented *with* or *without replacement* of a failed item.

(d) *In One Component at a Time Mixed Censoring Life Tests*, which are used for bulky or expensive components, or when only a single testing device is available, testing may take place one component at a time. The test may terminate either by failure or accumulation of planned test times. Notice that data from both (b) and (c) above are consistent with this manner of testing, but that the reverse is not true.

(1) Equipment Category	(2) Examples	(3) Tests Used for R Measurement	(4) Success Criteria	(5) R Measures of Primary Interest	(6) Most Applicable Probability Distributions	(7) Candidate Assessment Methods* Name	 Ref.Paras.Herein
One-shot Mechanisms	Igniters Exploding Bridge Wires Explosive bolts Solid Rocket Motors Energy Transfer System Harness	Performance under mission environments (W/O aging) (W/aging)	All performance parameter values must be within design limits. Structural integrity must be maintained long enough to complete assigned function. Must be in an "up" state upon operational demand.	Ratio of number of successful tests to total number of tests.	Binomial Negative Binomial Beta** Uniform**	Classical Rubinstein R Growth Bayesian***	5.4.1 5.4.5 5.4.4 5.4.6
Continuous or periodic operating devices	Electrical: Computers Gyros Generators Sonars Alarms Batteries Mechanical: Pumps Hydraulic Controls	Performance under mission environments and mission duration (W/O aging) (W/aging) Life tests (test to failure) Stress tests	All performance parameter values, including integrity, must remain within design limits during periods of operational demands and must be in an "up" state upon operational demand.	Time to first failure (TTFF) Mean time to first failure (MTTFF) Mean time between failures (MTBF) Distribution of values of each key performance parameter in relation to its design limits.	Exponential (W/CFR) Two parameter Weibull (W/DFR or W/IFR) Gamma** Normal Lognormal (when data distribution is skewed)	Classical Rubinstein R Growth Bayesian*** Variables	5.4.1 5.4.5 5.4.4 5.4.6 5.4.2
	Structures	Stress to failure Integrity tests under normal usage	Strength distribution must not overlap stress distribution	Proportion of strength distribution that lies in the failure region Failure stress level (e.g. burst strength of pressure vessels)	Normal Lognormal (when data distribution is skewed)	Classical Variables Stress/ Strength	5.4.1 5.4.2 5.4.3
	Cyclical: Relays Switches Circuit Breakers	Life tests (test to failure) Integrity tests under normal usage	All cyclical devices must perform assigned function when called upon for the required number of cycles without loss of integrity.	Cycles to first failure (c) Mean cycles to failure (c)	Exponential (W/CFR) Two parameter Weibull (W/IFR)	Classical Rubinstein R Growth	5.4.1 5.4.5 5.4.4

*See Figure 5-11 for relevant equations to compute point estimates and confidence limits for population parameters.
**If Bayesian Method is used.
***When prior data is to be combined with new test data.

Figure 5-2. Basic Tests and Supporting Information Used in Assessment of System Element R and A

(e) *In Stress/Strength Tests*, the test is generally independent of time. The data consists of applied stresses [e.g., in pounds per inch (PSI)] versus the stress percentile, and of the strength (e.g. in PSI) versus the strength percentile.

(f) *In Life Tests for Repairable Items*, where items may be bulky and expensive, n items are often monitored through long periods of operating life where their times/cycles to failure and times to repair are recorded, as well as the calendar time of failure, repair, overhaul and other important events.

(g) *In Pass/Fail Tests*, N times out of a grand total of M items (which may be taken to be infinite if M >> N) are tested. The result of the test is time independent. The items are classified as x "defectives", and s = N-x "non-defective".

Many other types of tests, and of test data exist, but only the situations described above will be considered in this section.

5.1.2 Influence of Equipment Maturity and Category of Life Cycle Testing Upon Applicable Reliability Assessment Methodology and Failure Models

As previously discussed in § 5.1 and documented in figure 5-2, the most applicable failure models and reliability assessment methods are functions of the type of equipment subjected to test. While this is true, the most applicable models and methods are also influenced significantly by the maturity of the product design at the time of its test and the kind of life cycle test involved. This dependence is described in figure 5-3.

5.1.3 Quality of Point and Interval Estimators

When speaking of either Classical, Bayesian, or Variables methods an important topic is the quality of the point and interval estimators used for R/A assessment. There are, for instance, Maximum Likelihood (ML), Minimum Variance Unbiased (MVU), Best Linear Unbiased (BLU), Maximum Information (MI), Best Asymptotic Normal (BAN) and many other methods [4, 9] used to formulate estimators. These lead to estimators endowed with lesser or greater amounts of desirable qualities; such as, unbiasedness, consistency, asymptotic efficiency, efficiency, minimum variance, sufficiency, and invariance [3, 4, 11].

In many cases, a method is selected only because it is the only tractable one, that is, it gives an estimator where the other methods could not because of inherent mathematical difficulties.

The quality of estimators most sought out is *unbiasedness*, which is the quality that the expected value of the estimate of a parameter or the reliability and availability measure is equal to the parameter or measure being estimated.

Another valuable quality is *minimum variance unbiasedness*, that is, the property of an estimator to cluster as closely as possible about the true value of the quantity which is being estimated.

Also, it should be noted that while some point and interval estimation methods are computationally straightforward, others, particularly interval estimations with certain types of test data, are state-of-the-art or cannot be performed exactly without excessive computational labor. A number of situations can be tackled only by making simplifying assumptions, by settling for asymptotic results rather than exact ones, by using numerical approximations, or by Monte-Carlo simulation.

In many cases, however, the lack of quality or quantity of test data (such as may arise from "piggyback" tests) does not warrant seeking out the most exacting methods. In these cases assumptions of exponentiality or normality, or neglect of test conditions can be entertained, provided that the results obtained with such assumptions are presented with an estimate of all errors, *including* the assumptional errors.

Graphical methods should not be neglected in this connection and must not be underrated. Whenever possible graphical methods should be backed by analytical techniques (e.g., Goodness-of-Fit Tests), but as indicated in [22] even the analytical techniques cannot distinguish significantly between similarly shaped Weibull, Lognormal or normal distributions with fewer than 40 samples.

		Categories of Life Cycle Tests								
						Production				
		DTs (Dev. Tests)	EETs (Engrg. Eval Tests) (Less Qual.& Demo.)	Qual.	R Demonstration	PAT	Aging Tests	SITP	DASO	OT/POT
Are test data likely to be homogeneous?		No	No - for new product designs or existing designs undergoing major redesign Yes - for mature designs	Yes	Yes	Yes - unless production goes out of control after an initial learning period.	Yes - to the point where quality degradation begins.	Not until debugging of installed equipment is completed.	Yes - after an initial learning period.	Yes - except where major SPALTing is required.
Can test data be used effectively for R assessment?		No	Yes	Yes - when used to support EET and R Demo. data.	Yes - but primarily to provide additional evidence of meeting R design objectives.	Limited application due to small quantities of data.	(Same as for PAT)	Yes - for devices that can be operated continuously or periodically in a shipboard installed environment after C/O.	Yes	Yes
One-shot devices	To calculate: R, R_L	N.A.	● Use attributes R growth model (e.g. Duane) for components undergoing periodic design change ● Use Binomial (or Negative Binomial) for mature or stabilized designs.	● Use Binomial model ● If variables data are available on performance parameters use: ● Variables R model ● Normal Dist. model, and/or ● Analysis of variance (ANOVA)	● Use attributes Sequential Ratio Test (SRT) analogous to MIL-STD-781 type tests.	● Use trend analysis ● Use statistical tests of hypotheses	● Use trend analysis to detect if and when degradation begins	N.A. - One-shot devices are not tested during SITP	● Use Binomial R model	● Use Binomial R model
Continuously or Periodically Operating devices	To calculate: $\hat{R}$, R_L, $\hat{\lambda}$, λ_u, $\hat{\theta}$, θ_L, etc.	N.A.	● Use experimental R growth model (e.g. Duane) if components are undergoing periodic design change and failure data are times to failure, otherwise ● Select most applicable failure model IAW figure 5-4.	● Use most applicable failure model by analysis IAW Figure 5-4	● Use MIL-STD-781 type plans.	● Use trend analyses. ● Use statistical tests of hypotheses	● Use trend analyses to detect if and when degradation begins.	● Use exponential R growth model if R growth is expected. Otherwise: ● Select most applicable failure model IAW figure 5-4.	● Select most applicable failure model IAW figure 5-4.	● Select most applicable failure model IAW figure 5-4.

Figure 5-3. Influence of Component Maturity and Category of Life Cycle Testing Upon Applicable R Assessment Methodology and Failure Models

5.2 STATISTICAL DISTRIBUTIONS USED AS MODELS OF FAILURE (OR REPAIR)

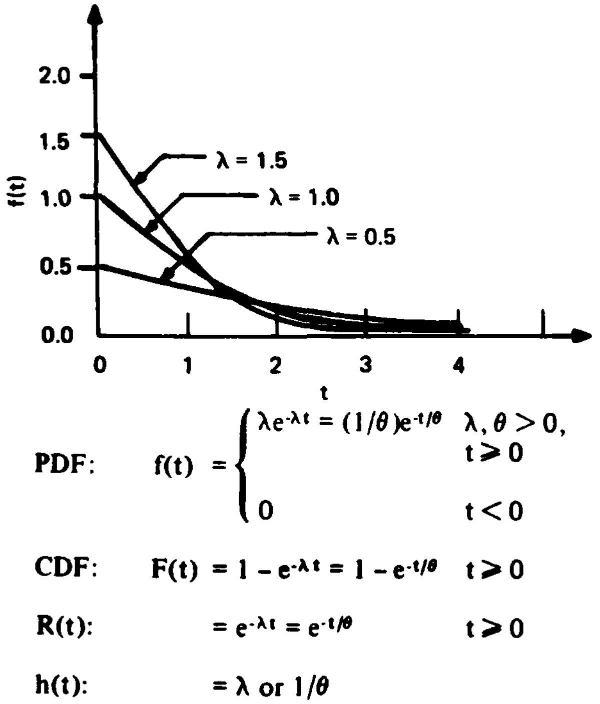
This subsection presents in pictorial and narrative form the statistical properties and applicability of some common distributions used as models of failure (or repair). Distributions covered and their order of coverage are:

§ 5.2.1 Exponential Distribution
§ 5.2.2 Binomial Distribution
§ 5.2.3 Normal Distribution
§ 5.2.4 Lognormal Distribution
§ 5.2.5 Weibull Distribution

In addition § 5.2.6 identifies many other distributions that have found application to reliability and availability modeling in special situations.

5.2.1 The Exponential Distribution

5.2.1.1 Without a Warrantee Period



PDF: $f(t) = \begin{cases} \lambda e^{-\lambda t} = (1/\theta) e^{-t/\theta} & \lambda, \theta > 0, \; t \geq 0 \\ 0 & t < 0 \end{cases}$

CDF: $F(t) = 1 - e^{-\lambda t} = 1 - e^{-t/\theta} \quad t \geq 0$

R(t): $= e^{-\lambda t} = e^{-t/\theta} \quad t \geq 0$

h(t): $= \lambda$ or $1/\theta$

MTTF: $= 1/\lambda$ or θ

Variance: $= 1/\lambda^2$ or θ^2

Process:

The basic failure process underlying the exponential is the Poisson failure process $g(x,t) = (e^{-\lambda t} (\lambda t)^x)/x!$, where $g(x,t)$ represents the probability that exactly x failures will occur in the time interval from 0 to t. Because reliability is defined as the probability *of no failure* from 0 to t, $g(0,t) = R(t) = e^{-\lambda t}$.

The assumptions under which the expression for $g(x,t)$ is derived are:

1) Component failure occurs when a random external disturbance or shock induces a component failure.

2) The number of shocks during any interval of time is independent of the number of shocks occurring during other intervals of time.

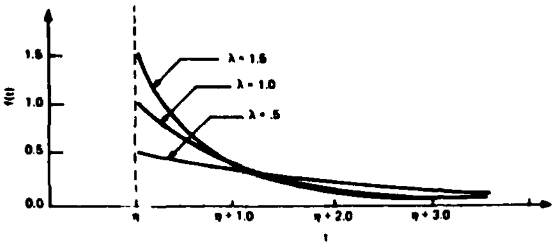
3) The probability of exactly one shock in a given interval of time is proportional to the length of time, with a constant of proportionality λ.

An important feature of the exponential failure model is that the constancy of the hazard rate λ implies that reliability is a function of time but not of the age of a component. The component does not wear out, but fails only because of random shocks, i.e., momentary concentrations of stress in excess of strength. There is, however, another aspect of the exponential which makes it applicable as a failure model even when the Poisson assumptions are not fulfilled. It has been shown that if a component is made of many elements, each having a different failure distribution, even a failure distribution exhibiting wearout, then the component will tend to exhibit asymptotically a constant hazard rate as time goes on [49].

Applicability of exponential distribution (without a warrantee period) as a hardware failure model:

Applies to most electronic components and complex systems (i.e., Central Navigation Computer, Ship Inertial Navigation System, Guidance Systems, Electrical Interconnects, Power Distribution, Servo-Mechanisms, etc. . . .). Also applicable as a hardware repair model. Has been proposed for several software failure models.

5.2.1.2 With a Warrantee Period, η.



PDF: $f(t) = \lambda e^{-\lambda(t-\eta)}$ $\quad \lambda, \eta > 0, t \geqslant \eta$

CDF: $F(t) = 1 - e^{-\lambda(t-\eta)}$ $\quad t \geqslant \eta$

R(t): $= e^{-\lambda(t-\eta)}$

h(t): $= \lambda$ $\quad$ for $t \geqslant \eta$

MTTF: usually not of interest

Variance: $= 1/\lambda^2$ $\quad$ for $t \geqslant \eta$

Applicability of the exponential distribution (with a warrantee period) as a hardware failure model:

Applies to the same components and systems shown in § 5.2.1.1, but which have a warrantee period η during which no failure can occur.

5.2.2 The Binomial Distribution

PMF:

$$g(x) = \begin{cases} \binom{N}{x} R^{N-x}(1-R)^x & 0 < R < 1, \\ & x = 0, 1, 2, \\ & \ldots N \\ 0, \text{otherwise} & \end{cases}$$

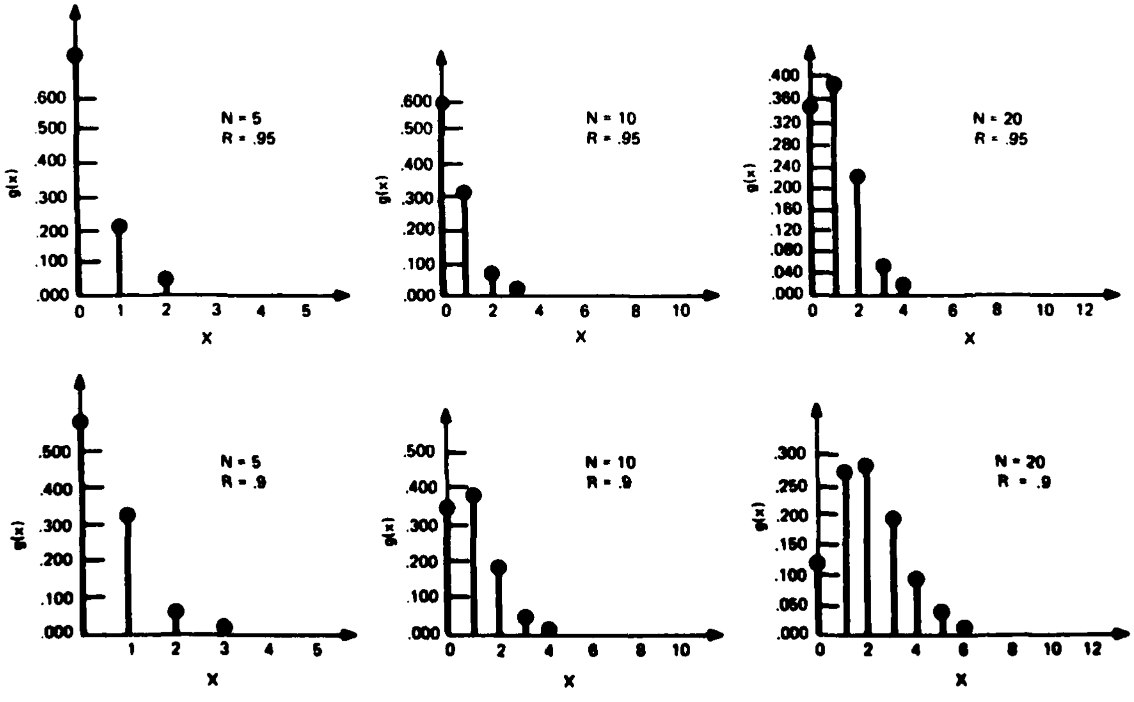
CDF: $G(x) = \sum_{i=0}^{x} \binom{N}{i} R^{N-i}(1-R)^i$

Mean: NR

Variance: NR(1-R) or NRQ (Q = 1-R)

Process:

The binomial density function arises from a Bernoulli process, a process in which an event, such as a success, can occur with constant probability R, or a complementary event, such as a failure, can occur with constant probability Q, (Q = 1-R). In each trial under these conditions, the form of the binomial PMF shown above, with R representing the probability of success per trial,



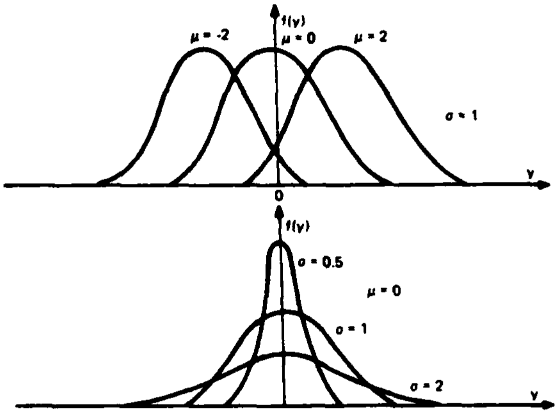
BINOMIAL DISTRIBUTIONS

yields the probability, g(N-x), that exactly N-x successes are obtained in N trials. Examples of reliability calculations using binomial tables are given in Appendix E, § E.1.3. Figure E-3 provides Binomial Tables (80% confidence).

Applicability of the binomial as a failure model:

Applies to one-shot devices, failed or successful items in sampled lots. One-shot devices commonly found in FBMWS/SWS include: igniters, stage separation devices, energy transfer system harness, propulsion devices such as rocket motors, launcher devices such as igniters or grain (propellant), warhead fuzing devices.

5.2.3 The Normal Distribution



PDF: $$f(y) = \frac{1}{\sigma\sqrt{(2\pi)}}\, e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$$

$-\infty < y < \infty, \sigma > 0,$

$-\infty < \mu < \infty$

CDF: $F(y) = \Phi(y)$

$$= \frac{1}{\sigma\sqrt{(2\pi)}} \int_{-\infty}^{y} e^{-\frac{1}{2}\left(\frac{\xi-\mu}{\sigma}\right)^2} d\xi$$

Mean: μ

Variance: σ^2

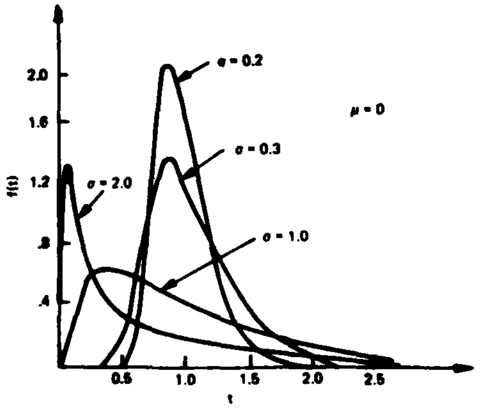
Process:

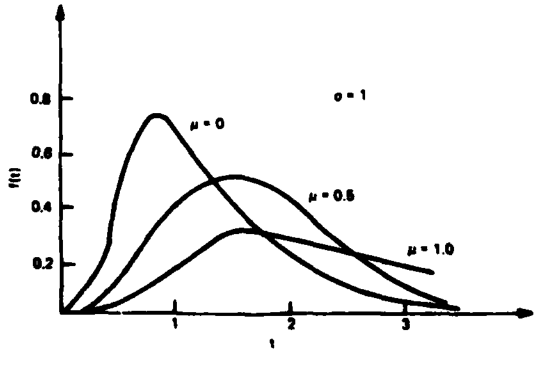
The failure process underlying the normal has been described in [9]. It arises naturally when a component's performance depends on a critical parameter which, because of manufacturing variability, has an initial value α_0 that is normally distributed. Assume also that this parameter varies during operation so that its value $\alpha = u(\alpha_0, t)$ is a function of time, the unit fails if it exceeds a value α_1. Let τ be the instant of failure such that $u(\tau, \alpha_0) = \alpha_1$ or $\tau = \phi(\alpha_0, \alpha)$. It can be shown that under these conditions τ is also distributed normally. If the mean life of the item is denoted by $E\{\tau\} = \mu$ and σ is the standard deviation of τ, with $\alpha << \mu$, then the normal failure PDF results.

Applicability of the normal distribution:

Critical parameters of hardware exhibiting symmetrical variability, for example when times to failure are normally distributed about some mean value, μ. This is often expected when hardware enters its wearout phase. Variables measurements are required.

5.2.4 The Lognormal Distribution





Note: μ and σ are parameters of the lognormal distribution. They do not have the usual meaning of mean and standard deviation as for the normal distribution.

PDF: $f(t) = \frac{1}{\sigma t\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln(t)-\mu}{\sigma}\right)^2} \quad \sigma, t > 0$

CDF: $F(t) = \int f(\xi)\, d\xi$

R(t): $1 - F(t)$

MTTF: $\exp[(\sigma^2/2)+\mu)]$

Variance of t: $[\exp(\sigma^2 + 2\mu)]\ [\exp(\sigma^2)-1]$

Variance of ln(t): σ^2

Mean of ln(t): μ

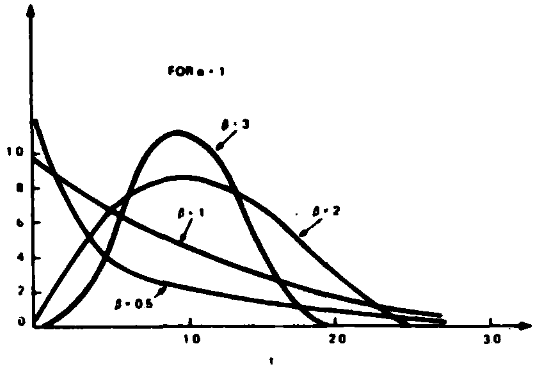
Process:

The lognormal failure process is explained [9] for a progressive fracture failure mechanism. Let $X_1 < X_2 \ldots < X_n$ be a sequence of random variables that denote the sizes of a fatigue crack at successive stages of its growth. A proportional effect model is assumed for the growth of these cracks such that the crack growth at stage i, $\Delta X_i = X_i - X_{i-1}$, is randomly proportional to the size of the crack, X_{i-1}, and that the item fails when the crack size reaches X_n. When $\Delta X_i \rightarrow 0$, as n becomes large, this model leads to the lognormal failure distribution.

Applicability of the lognormal distribution as a failure model:

Applies to component measurements when the distribution of these measurements are skewed and the distribution can be normalized by using the logarithms of each measurement. Used for assessing the reliability of structural components. Also used as a repair model.

5.2.5 The Weibull Distribution (two parameter)



PDF: $f(t) = \begin{cases} \beta t^{\beta-1} e^{-(t/\alpha)^\beta}/\alpha^\beta & t \geq 0, \alpha, \beta > 0 \\ 0 & t < 0 \end{cases}$

CDF: $F(t) = 1 - e^{-(t/\alpha)^\beta}$

R(t): $e^{-(t/\alpha)^\beta}$

h(t): $\beta t^{\beta-1}/\alpha^\beta$

MTTF: $\alpha\Gamma((\beta+1)/\beta)$

Variance: $\alpha^2\ [\Gamma((\beta+2)/\beta) - \Gamma^2((\beta+1)/\beta)]$

Process:

The failure process underlying the Weibull failure model has been conceptualized as a chain of links; the links are not all equally strong but are chosen from a population having a single distribution of breaking strengths. Stress is applied to the chain as a whole and is assumed to be applied equally to each link. The chain breaks (component fails) when its weakest link fails. Then the probability distribution of the time to failure of such a component is a Weibull.

Applicability of the Weibull Distribution as a failure model:

Normally applied to strength of structures, electrical connections subjected to physicochemical degradation. Simple devices which display IFR ($\alpha > 1$) or DFR ($\alpha < 1$) characteristics.

5.2.6 Other Distributions Used in Reliability Modeling

Many more distributions have been used in modeling reliability and availability. The following list is far from exhaustive.

a. The Birnbaum-Saunders Fatigue Life Distribution [9]. Applicable to Structures.

b. The Competing-Risk Model [9]. Applicable to more complex elements, components where there is more than one mechanism for failure.

c. The Mixed Distribution Models [9]. Applicable to physical dimensions of mass produced items.

d. The General Distribution to time to failure [9]. Applicable to truly complex elements, components or equipments which display a typical "bathtub" hazard rate.

e. The Piecewise Linear Models [24]. Applicable to complex items for which an empirical "bathtub" hazard rate is modeled by means of linear segments of the form $h(t) = a_i - b_i(t_i - t_{i-1})$.

f. The Polynomial Reliability Model [24]. Applicable to complex items susceptible to being modeled empirically by a polynomial of high degree.

g. The mixed distribution method of Calvin [25]. This method makes use of a single continuous equation to describe a "bathtub" reliability model. Known failure mechanisms of the component are incorporated in the model through an "additive" procedure. Up to two unknown failure processes can be considered to build up the bathtub curve.

h. The Bayesian Beta prior and conjugate Beta posterior of the binomial failure model [9].

i. The Bayesian Beta prior and conjugate Beta posterior of the negative binomial failure model [9].

j. The Bayesian gamma prior and conjugate gamma posterior of the exponential failure model [9].

k. The Bayesian Inverted gamma prior and conjugate inverted gamma posterior of the Weibull failure model [9].

l. The General Failure Rate Function Model [26] which can be made to fit empirically many types of data.

m. The Mixed Weibull-Gamma Distribution Model [26] which has a bathtub shape and can be made to fit empirically equipment exhibiting infant mortality and wearout.

n. The uniform and truncated uniform prior on the Binomial Failure Model.

o. The Bivariate exponential model of reliability [26] which is applicable to components subjected to three different types of Poisson disturbances.

p. The three major types of Extreme-Value Distributions [5, 9] used in corrosion problems and in stress-strength interference models.

In addition, many other statistical distributions are used in software reliability and as Reliability Growth models.

5.3 SELECTION OF A FAILURE MODEL

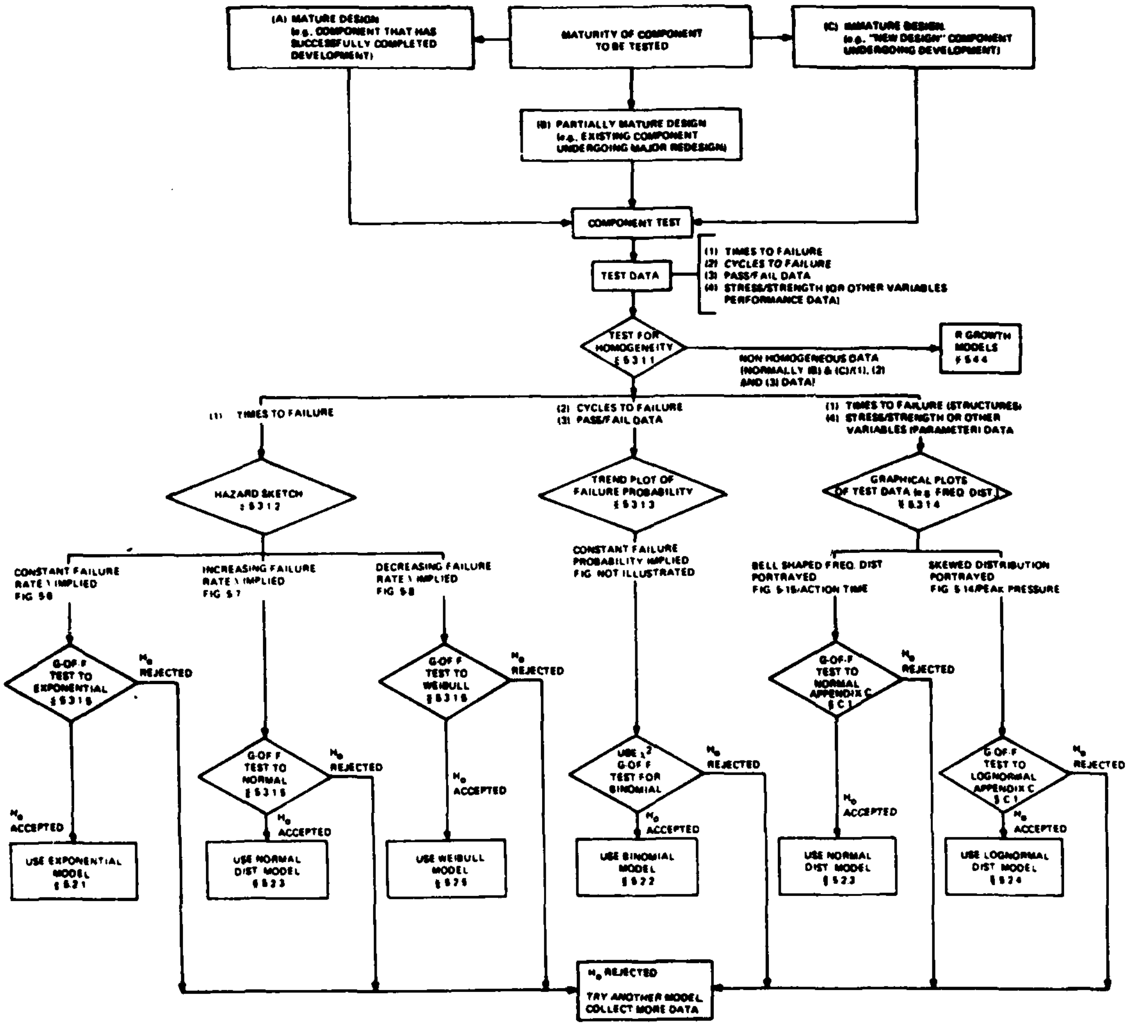
This paragraph treats the problem of what to do when failure data are available and one is confronted with a preliminary selection of models for the data at hand. A suggested procedure is presented next in the form of a roadmap.

5.3.1 Roadmap for the Selection of a Failure Model

Figure 5-4 shows the various factors and options involved in the selection of a failure model when test data are available. The roadmap recognizes that components placed on test may fall into any one of three categories of design maturity, namely: (a) a fully mature design, (b) a partially mature design and (c) a new, immature design. The roadmap also recognizes that test data generated and used for point, interval and trend estimates of component reliability may be any one of the following: (1) times to failure, (2) cycles to failure, or (3) pass/fail data; it may include also (or only) stress/strength or other variables (parameter) performance data. The type of equipment or component tested will normally dictate the type of data generated and collected (also see section 9).

5.3.1.1 Test of Data for Homogeneity

After data, e.g. failure, stress/strength parameters, etc. are at hand, one of the first questions to be answered is: Are the test data homogeneous? That is, did the sample data all come from the same parent population? In



LEGEND: G-OF-F – GOODNESS OF FIT; H_0 – NULL HYPOTHESIS, IF REJECTED CONSIDER OTHER FAILURE MODELS

Figure 5-4. Roadmap for the Selection of a Failure Model real life, a positive answer generally requires that all hardware items used in the test sample have been built from the same drawing, and under manufacturing operations that were in a state of statistical control. Items built to a mature design and under fully developed manufacturing procedures and controls are expected to yield homogeneous failure data or stress/strength variables performance data under similar test conditions. Conversely, items built to a changing design or under changing manufacturing processes are expected to yield non-homogeneous data because of inherent differences in the makeup of the test items. In many instances the analyst may have a high confidence, based on a review of the above factors and their relevance to a specific application, that the test data are either homogeneous or non-homogeneous. If any doubt exists, the analyst should find one or more of the following statistical techniques helpful in conducting homogeneity analyses: trend analysis based on hypothesis testing (see an example below), preparation of sample frequency distributions (see an example in § 5.4.4), or quality control chart analysis [50].

Homogeneity Test by Trend Analysis

As stated in [1], the simplest means to detect a trend is visually, by plotting cumulative number of failures versus cumulative operating time, using data in its *original chronological order*. If there is reliability growth, times between failures become larger, and the plot is concave down. If reliability decays, times between failure become smaller, and the plot is concave up (see figure 5-5).

For a quantitative trend test for time to failure data, Laplace's test can be employed as indicated in [1]. Assume that x failures have been experienced during a test. Laplace's test uses the u statistic of Equation (5-1) which is almost distributed as N(0, 1) for $x \geqslant 3$.

$$u = \left(\frac{\sum_{i=1}^{x} t_i}{xT} - \frac{1}{2} \right) (12x)^{1/2} \qquad (5\text{-}1)$$

T, which is generally unknown, is approximated by the largest time to failure observed.

The null hypothesis of the test (see Appendix C) is that the data originates from a Homogeneous Poisson Process (HPP). If u is small, reliability growth is probably happening. If u is large, reliability degradation is the likely situation.

Example:

Six failures occur at times 0.5 day, 1 day, 2 days, 2 weeks, 6 weeks, and 16 weeks for an element which is replaced on test after repair and modification. Assume times to repair and modify are negligible. Testing terminates at

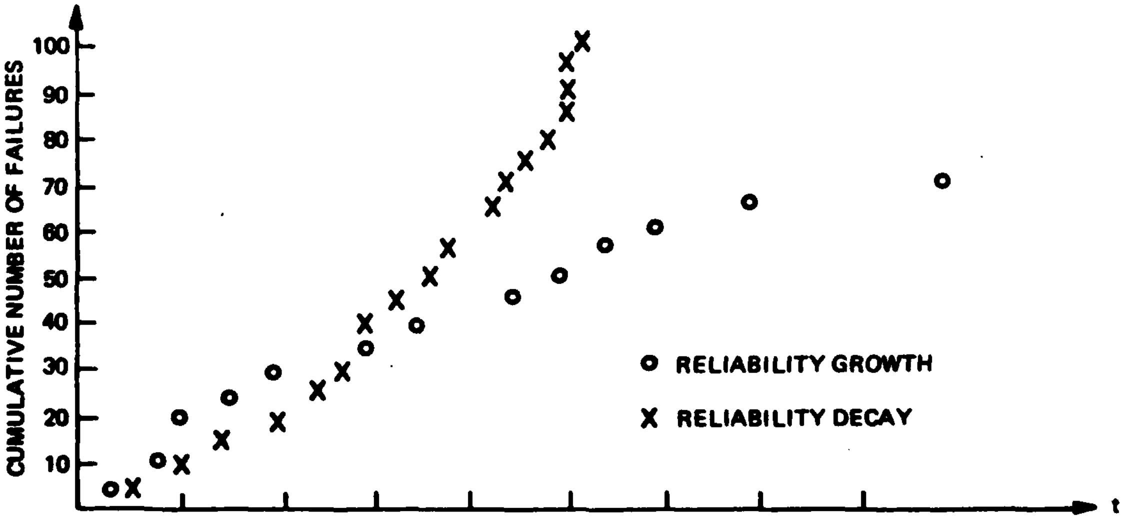


Figure 5-5. Visual Reliability Trend Analysis

20 weeks. Clearly this element has undergone reliability growth. But how sure are we of this? The t_i's (interval between failures), in weeks, are 0.07, 0.07, 0.14, 1.71, 4, and 10; $x = 6; T = 10; u = -1.97$.

A cumulative normal distribution table [13] indicates that the critical region to the left of -1.97 is 0.0244, so that the data presented fails the HPP (null) hypothesis at the 2.5% level of significance (since 2.5% is still >0.0244), i.e. data is non-homogeneous.

Several additional methods of trend detection are presented in [2] and [3].

Returning to figure 5-4, it is observed that if the results of the homogeneity test indicate the failure data to be non-homogeneous, the use of a R growth model is warranted (§ 5.4.4). If the data are judged to be homogeneous, the data should next be analyzed to determine the appropriate failure model.

5.3.1.2 Selection of a Failure Model Based on Hazard Rate Sketching or Hazard Rate Plotting

The next step to failure model selection is sketching or plotting the data. Hazard rate sketching which works for time to failure data reordered by magnitude, consists in sketching an approximation to the hazard rate of the failure data at hand and comparing visually the sketch obtained with the theoretical hazard rates of the main probability models, as illustrated in figures 5-6 through 5-8.

Notice that, for the three distributions considered, the following results are always true.

Exponential:	The hazard rate is a constant (λ or $1/\theta$)
Normal:	The hazard rate increases
Weibull:	The hazard rate increases ($\beta > 1$), remains constant ($\beta = 1$), or decreases ($\beta < 1$)

Example of Hazard Rate Sketching Based on Times to Failure

Times of failure of the example of § 5.3.1.2 were originally labeled:

$t_1 = 12, t_2 = 6, t_3 = 19, t_4 = 17, t_5 = 2, t_6 = 8, t_7 = 16, t_8 = 7, t_9 = 14, t_{10} = 19, t_{11} = 11$ and $t_{12} = 5$.

For the purpose of further analysis and to indicate that time reordering by magnitude is taking place, we relabel, such that:

$t_{(1)} = 2, t_{(2)} = 5, t_{(3)} = 6, t_{(4)} = 7, t_{(5)} = 8, t_{(6)} = 11, t_{(7)} = 12, t_{(8)} = 14, t_{(9)} = 16, t_{(10)} = 17, t_{(11)} = 19, t_{(12)} = 19$.

Figure 5-9 shows a hazard sketch (ordinary graph paper) of these ordered times to failure data. The usual approximation, based on the mean rank, for the hazard rate at the ith ordered failure time $t_{(i)}$ is given by:

$$\hat{h}\left[t_{(i)}\right] = 1/\left\{\left[t_{(i+1)} - t_{(i)}\right] \left[x - i + 1\right]\right\} \quad (5\text{-}2)$$

For small samples, however, say for $x \leq 8$, a better approximation based on the median rank [5, page 31] or [7] is given by:

$$\hat{h}\left[t_{(i)}\right] = 1/\left\{\left[t_{(i+1)} - t_{(i)}\right]\left[x - i + 0.7\right]\right\} \quad (5\text{-}3)$$

Using equation 5-2 on the data above one encounters a problem. The last two $t_{(i)}$'s, $t_{(11)}$ and $t_{(12)}$ are equal and the approximation to the hazard rate goes to infinity in this case at $t_{(11)}$. Before deciding what to do about it, one may sketch the hazard rate using (5-2) up to $t_{(10)} = 17$ (see figure 5-9). The hazard rate thus drawn shows a rapid increase. One cannot include $\hat{h}(t_{(11)})$ in the sketch but $\hat{h}(t_{(11)}) \rightarrow \infty$ does not any longer seem odd when one attempts to draw a smooth dashed line to represent a probable hazard rate through the points. Perhaps that line should be boldly inflected upward near $t = 19$ to indicate that $\hat{h}(t_{(11)})$ is not so much an "outlier" as an indication of rapidly increasing trend. However, $\hat{h}(t_{(11)})$ *could* have been an outlier. Hazard rate sketching has the drawback of being "noisy" in the sense that it magnifies the effect of a bad point. Should such an "outlier" be suspected, it is better to include it in a preliminary sketch than to dismiss it as unrepresentative before viewing its effect.

Since the data sketched in figure 5-9 were times to failure, reference to figures 5-7 and 5-8 indicate that both the Normal and the Weibull are candidate distributions for times to failure. At this point a Goodness-of-Fit test of the data could be made to check conformance of the data to each of the two candidate distributions. If neither distribution

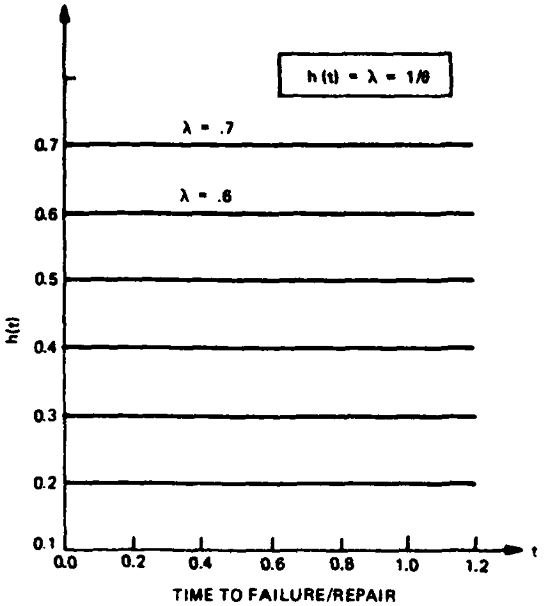


Figure 5-6. Exponential Hazard Rate.

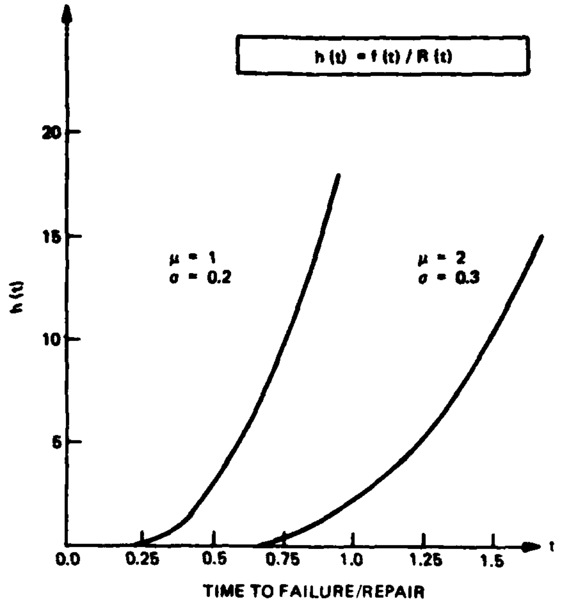


Figure 5-7. Normal Hazard Rate

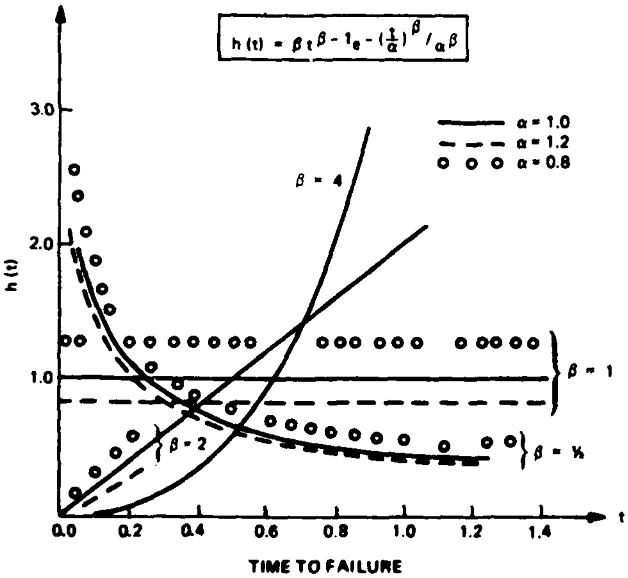
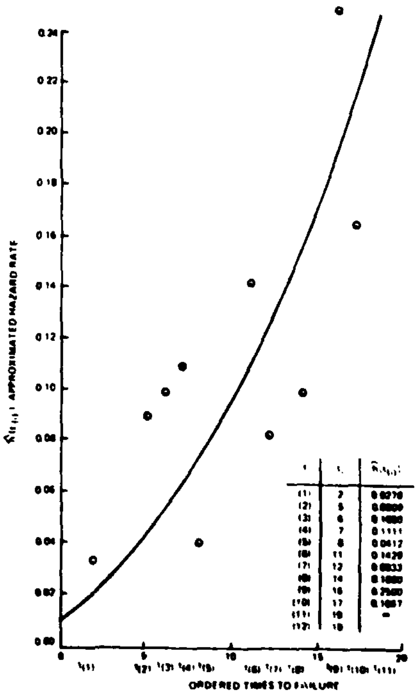


Figure 5-8. Weibull Hazard Rate



i	t_i	$\hat{h}(t_{(i)})$
(1)	2	0.0278
(2)	5	0.0909
(3)	6	0.1000
(4)	7	0.1111
(5)	8	0.0417
(6)	11	0.1429
(7)	12	0.0833
(8)	14	0.1000
(9)	16	0.2500
(10)	17	0.1667
(11)	19	∞
(12)	19	

Figure 5-9. Hazard Rate Sketching

were rejected by the Goodness of Fit test, the analyst should select that distribution more easily applied.

Example of Weibull Hazard Rate Plotting

Special graph paper is commercially available for Weibull Hazard Rate Plotting [20], [21]. On this graph paper, the hazard rate from Weibull failure times appears as a straight line. The graph paper also allows the estimation of α and β (see § 5.2.5) with simple geometric constructions.

The data on this page have been gathered or calculated:

The Weibull hazard plot corresponding to these data points is shown in figure 5-10. The plot is of ordered times to failure vs cumulative hazard rate, and is a straight line for the Weibull on log-log graph paper.

The advantage of hazard rate plotting, as expounded in [17], [18], and [19] is that it accommodates arbitrarily censored data. Hazard rate paper also exists for the Log-normal, and is simple to construct for the exponential [18].

Ordered Failure No. (i)	Ordered Time to Failure $t_{(i)}$	Empirical Hazard $\hat{h}[t_{(i)}] = \left\{[t_{(i+1)}-t_{(i)}]\ [n-i+1]\right\}^{-1}$	Cumulative Hazard
(1)	0.1	.00000	.00000
(2)	0.1	.08621	.08621
(3)	0.5	.08928	.17549
(4)	0.9	.12346	.24895
(5)	1.2	.01923	.31818
(6)	3.2	.02857	.34675
(7)	4.6	.02451	.37126
(8)	6.3	.03344	.40471
(9)	7.6	.00797	.41268
(10)	13.3	.00134	.41403
(11)	48.7	.00500	.46187
(12)	48.7	.04785	.50972
(13)	49.8	.00245	.51218
(14)	82.4	.03267	.54486
(15)	84.2	.00086	.54572
(16)	156.4	.01418	.55991
(17)	161.1	.01347	.57338
(18)	166.4	.00059	.57398
(19)	296.0	.00303	.57701
(20)	323.5	.00411	.58112
(21)	345.6	.00265	.58377
(22)	383.3	.00703	.59081
(23)	399.1	.00625	.59706
(24)	419.1	.00068	.59774
(25)	628.0	.01666	.61441
(26)	638.0	.00061	.61502
(27)	963.6	.00119	.61622
(28)	1173.0	.00043	.61737
(29)	1529.9	.00021	
(30)	3833.9		

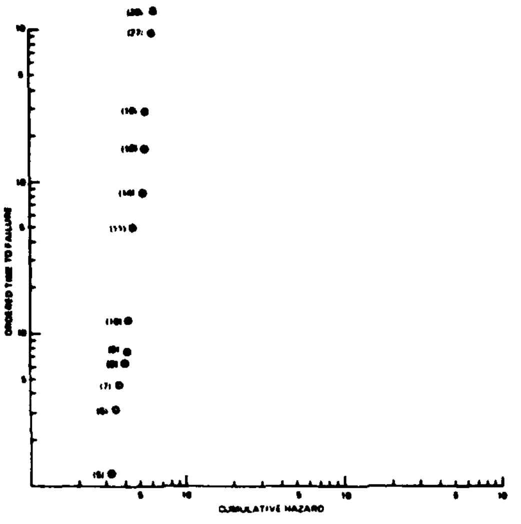


Figure 5-10. Weibull Hazard Plot

5.3.1.3 Selection of a Failure Model Based on Trend Plots of Failure Probability (Cycles to Failure and Pass/Fail Data)

The objectives of this step will generally have been accomplished in the homogeneity tests of cycles to failure and pass/fail data cited in the early steps of figure 5-4. If not done previously, moving averages of the test data can be plotted to display presence or absence of significant trends. Absence of a significant trend, plus random scatter of plotted points within postulated two-sided binomial distribution control limits, would support a conclusion that the test data came from a population with constant failure probability; hence the use of binomial failure model would be warranted.

5.3.1.4 Plot of Test Data in Frequency Distribution Form

A graphic portrayal of the test data in frequency distribution form (Figure 5-12) is very helpful in displaying symmetrical or asymmetrical properties of the distribution of stress/strength/parameter performance data. A symmetrical, bell-shaped frequency distribution (as for Action Time in figure 5-14, § 5.4.2.3.2) would be considered as evidence that the parent distribution is Normal. Also, when sample data from a Normal population are plotted on Normal probability paper, the cumulative percentage points should fall on or close to a straight line, as in figure 5-15, § 5.4.2.3.2. If the frequency distribution is skewed (as for Peak Pressure in figure 5-14, § 5.4.2.3.2), one should plot the data on another candidate probability density paper, e.g., lognormal probability paper. If a linear trend is observed (as in figure 5-16, § 5.4.2.3.2), one may assume the sample data came from a Lognormal population.

5.3.1.5 Verification (or Refutation) of the Applicability of the Candidates Failure Model by a Goodness-of-Fit Test*

The last step in the selection of a failure model (figure 5-4) is to perform a goodness-of-fit test of the sample data to each candidate failure model. The two dominant goodness-of-fit tests described in statistical textbooks [3], [9] are the Kolmogorov-Smirnov Test and the Chi-Square Test. These tests are also discussed in Appendix C.

A Goodness-of-Fit Test for the Weibull

A Goodness-of-Fit test applicable to uncensored or censored samples of the two-parameter Weibull is described in [46]. A more accessible presentation of the same method is given in [5].

This method consists in calculating the statistic

$$S = \frac{\displaystyle\sum_{i=[x/2]+1}^{x-1} \left\{ \frac{\ln t_{(i+1)} - \ln t_{(i)}}{M_i} \right\}}{\displaystyle\sum_{i=1}^{x-1} \left\{ \frac{\ln t_{(i+1)} - \ln t_{(i)}}{M_i} \right\}} \qquad (5\text{-}4)$$

***Note of Caution**: Confidence in the results of a goodness-of-fit test is influenced greatly by sample size. Sample sizes of the order of 100 or more are generally required to be reasonably confident of rejecting a bad failure model. When sample sizes used are appreciably less than 100, the analyst may find that no candidate failure model will be rejected, irrespective of the number of models analyzed.

(i)	$t_{(i)}$	Ln $t_{(i)}$	$u_{(i)}$=Ln $t_{(i+1)}$-Ln $t_{(i)}$	M_i	$u_{(i)}/M_i$	$\Sigma u_{(i)}/M_i$
1	0.1	-2.303	0	1.020551	0	0
2	0.1	-2.303	1.609	0.521285	3.087	3.087
3	0.5	-0.693	0.588	0.355415	1.654	4.741
4	0.9	-0.105	0.288	0.272945	1.054	5.795
5	1.2	0.182	0.981	0.223885	4.381	10.176
6	3.2	1.163	0.363	0.191578	1.894	12.070
7	4.6	1.526	0.314	0.168899	1.862	13.933
8	6.3	1.841	0.188	0.152286	1.232	15.164
9	7.6	2.028	0.560	0.139783	4.003	19.168
10	13.3	2.588	1.298	0.130219	9.967	29.135
11	48.7	3.866	0.000	0.122871	0.000	29.135
12	48.7	3.866	0.022	0.117274	0.190	29.325
13	49.8	3.908	0.504	0.113132	4.451	33.777
14	82.4	4.412	0.022	0.110268	0.196	33.973
15	84.2	4.433	0.619	0.108598	5.702	39.675
16	156.4	5.052	0.030	0.108124	0.274	39.948
17	161.1	5.082	0.032	0.108944	0.297	40.246
18	166.4	5.114	0.576	0.111289	5.175	45.421
19	296.0	5.690	0.089	0.115596	0.769	46.189
20	323.5	5.779	0.066	0.122683	0.539	46.728
21	345.6	5.845	0.104	0.134165	0.772	47.500
22	383.3	5.949	0.040	0.153650	0.263	47.763
23	399.1	5.989	0.049	0.191137	0.256	48.019
24	419.1	6.038	0.404	0.289773	1.396	49.414
25	628.0	6.443				

where [x/2] denotes the greatest integer $<$ x/2, i.e. for x=25, [x/2] = 12. The M_i and the critical values of S have been tabulated [5] for i=3 (1)25.

The test cannot be directly applied to the data of § 5.3.1.3 because it exceeds 25 data points, but since the method is applicable to censored data, it is at least possible to take the first 25 ordered times and perform a Goodness-of-Fit test on these points.

From the entries in the last column of the data on this page:

$$S = \frac{49.414 - 29.325}{49.414} = 0.407.$$

The critical value of S at α = 5% level of significance is 0.65 in the referenced tables [5-Appendix 13]. Even for α = 25% level of significance, the critical value of S is still 0.56. The hypothesis that the 1st 25 ordered times of the given sample are Weibull is therefore accepted. If one wished to include the 5 points which have not been considered, one would also perform the test on the last 25 points.

Goodness-of-Fit Tests for the Normal

Lilliefors modification of the Kolmogorov-Smirnov Test [16, 5] applies to distributions which are assumed to be, under the null hypothesis, normal with unknown μ and σ. Since this test applies to lognormal repair times as well, if the logarithms of the repair times are considered instead of the repair times themselves, an example of application of this test to data transformed to normal is shown in appendix C § C.1. Another goodness-of-fit test for the normal is the Shapiro and Wilks W- Test illustrated in figure 5-17.

Goodness-of-Fit Test for the Exponential

Lilliefors modification of the Kolmogorov-Smirnov test for the exponential case is useful both in reliability and availability assessment and is illustrated in appendix C § C.1.

5.4 RELIABILITY ASSESSMENT METHODS AND ILLUSTRATED APPLICATIONS

Once the model (distribution) has been determined, the data must be analyzed to provide reliability assessment. This subsection contains basic descriptions of the principal and current methods used for assessing hardware (and software) reliability. They include the following in their order of presentation:

§ 5.4.1 Classical Methods
§ 5.4.2 Variables Reliability Methods Based on the Normal Distribution
§ 5.4.3 R Assessment Method Based on Stress-Strength Interference Model
§ 5.4.4 Methods for Assessing R Growth
§ 5.4.5 Rubinstein Method
§ 5.4.6 Bayesian Methods
§ 5.4.7 Methods for adjusting R Estimation, Derating and Uprating.

Illustrated applications of these methods are provided.

5.4.1 Classical Methods for Estimating Hardware (and Software) Reliability

Classical methods used for estimating hardware (and software) reliability are loosely defined as those reliability point, interval and trend estimation techniques based upon fundamental statistical distributions including the Binomial, Exponential, Normal and Weibull. These methods are documented in summary form in figure 5-11. Their applications are illustrated on a sample basis below.

5.4.1.1 Reliability Point and Interval Estimation Using the Exponential Model

Point and interval estimation formulas and results depend on the testing procedure (§ 5.1.1) from which failure data are obtained and the mathematical technique used. This point will be made clear in the following subparagraphs. As stated in § 5.1.3 the most desirable R and A estimator are the *unbiased* and *minimum variance* ones.

5.4.1.1.1 Maximum Likelihood (ML) Point Estimations of λ, θ, R, and Calculations of λ_u (upper confidence limit of λ).

Another method for obtaining reliability estimates is the Maximum Likelihood (ML) Method. ML estimates for type II censoring without and with replacement and type I censoring without and with replacement situations are presented.

5.4.1.1.1.1 Type II Censoring without Replacement

It is shown by making use of order-statistics in [9] that the ML estimator of θ for failure truncated tests is:

$$\widehat{\theta} = \left(\sum_{i=1}^{x} \tilde{t}_i + (n-x)\tilde{t}_x \right) / x \qquad (5\text{-}5)$$

where n is the number of items on test, $\tilde{t}_i$ represents the time to failure of each item on test since the beginning of testing, and $\tilde{t}_x$ the time to failure of the last item to fail.

Example Type II. Censoring Without Replacement:

Assume that 15 exponential items are put on test and the decision is made to stop testing after the 8th failure. The recorded times to failure in *mission equivalents* are:

$\tilde{t}_1 = 2.504$, $\tilde{t}_2 = 4.877$, $\tilde{t}_3 = 7.657$, $\tilde{t}_4 = 11.170$,
$\tilde{t}_5 = 14.675$, $\tilde{t}_6 = 25.423$, $\tilde{t}_7 = 28.075$, and
$\tilde{t}_8 = 57.588$.

Then, from equation 5-5:

$$\widehat{\theta} = [151.969 + (15\text{-}8)(57.588)]/8 = 69.385 \text{ missions,}$$

$$\widehat{\lambda} = \frac{1}{\widehat{\theta}} = 0.0144123 \text{ failure/mission}$$

and

$$\widehat{R} \text{ (For 1 mission)} = e^{-\widehat{\lambda}(1)} = 0.9857$$

Exhibit #	Item Classification	Statistics		Raw Data					Type of Test						
	Classification by Performance	Failure Process Principal Reliability Measure	IFR, CFR, DFR	n - # units on test	t - test time	c - # test cycles	S - # successes	X - # failures	Time Independent or uncensored	Type I censoring w/o replacement	Type I censoring w/ replacement	Type II censoring w/o replacement	Type II censoring w/ replacement	Point Estimates of Reliability (R), MTTF (θ), Hazard Rate (λ) or other Specified Parameters	One-Sided Confidence Limits at 100 $(1-\alpha)$% Confidence for R, θ, λ or Other Specified Parameters
1	All hardware elements/ components, but more particularly one-shot devices. Also, all test outcomes classified in terms of "successes & failures"	Bernoulli process (R) Binomial sampling $\binom{n}{x}(1-R)^x(R)^{n-x}$ R=R		X n fixed, x variable					X					$\hat{R}=s/n$ or $(n-x)/n$ $\widehat{\theta,\lambda}$, N/A	R_L=value of R such that: $\sum_{r=0}^{x}\binom{n}{r}(1-R)^r(R)^{n-r}=\alpha$ or $R_L=s/[s+(n-s+1)F(\alpha,df_1,df_2)]$ where $df_1=2(n-s+1)$, $df_2=2s$. or R_L obtained by method of Randomized intervals. or R_L from Normal Approximation valid for $n(1-R)>5$, $0.1<R<0.9$ or $n(1-R)<25$, all R. or R_L obtained from Poisson Approximation for $R>0.9$
2	All hardware elements/ components, but more particularly one-shot devices. Also, all test outcomes classified in terms of "successes and failures"	Bernoulli process: Negative Binomial, Pascal sampling $\binom{n-1}{x-1}(1-R)^x(R)^{n-x}$ R=R		X n variable, x fixed					X					$\hat{R}=\frac{s}{n-1}$ or $\frac{n-x}{n-1}$. θ, λ, N/A	R_L = value of R such that $\sum_{r=0}^{x-1}\binom{n-1}{r}(1-R)^rR^{n-1-r}=\alpha$
3	All hardware elements/ components, but more particularly continuously operating, electro-mechanical devices	Poisson Process & Exponential PDF $g(x)=e^{-\lambda t}[(\lambda t)^x/x!]$ $R(t)=e^{-\lambda t}$	CFR	X	X	X	X			X				$\hat{\theta}=(\sum_{i=1}^{x}t_i+(n-x)t_0)/x$, $x\neq0$, $\hat{\lambda}=1/\hat{\theta}$ $\hat{R}=e^{-\hat{\lambda}t}$ or $e^{-t/\hat{\theta}}$ $\hat{R}(MVU)=(1-t_m/\sum_{i=1}^{n}t_i)^{n-1}$ uncensored t_m=mission, & t_i=test time	$R_L=\beta_\alpha(n-x,x+1)$ $\theta_L=t_o/\ln(1/R_L)$ $\lambda_u=1/\theta_L$

Figure 5-11. Classical Methodology Summary for Assessing Component R, MTTF, and Failure Rate

Exhibit #	Item Classification	Statistics		Raw Data					Type of Test						
	Classification by Performance	Failure Process Principal Reliability Measure	IFR, CFR, DFR	n - # units on test	t - test time	c - # test cycles	S - # successes	X - # failures	Time Independent or uncensored	Type I censoring w/o replacement	Type I censoring w/ replacement	Type II censoring w/o replacement	Type II censoring w/ replacement	Point Estimates of Reliability (R), MTTF (θ), Hazard Rate (λ) or other Specified Parameters	One-Sided Confidence Limits at 100 (1-α)% Confidence for R, θ, λ or Other Specified Parameters
4	**All hardware elements/ components, but more particularly continuously operating, electro & electro-mechanical devices.**	Poisson Process & Exponential PDF $g(x)=e^{-\lambda t}[(\lambda t)^x/x!]$ $R(t)=e^{-\lambda t}$	CFR	X	X		X					X		$\hat{\theta} = (\sum_{i=1}^{x} t_i+(n-x)t_x)/x,\ x \neq 0$ $\tilde{\theta}$ (unbiased) $= x\hat{\theta}/(x+1)$ $\hat{\lambda} = 1/\hat{\theta},\ \tilde{\lambda} = 1/\tilde{\theta}$ $\hat{R} = e^{-\hat{\lambda}t}$ or $e^{-t/\hat{\theta}}$ $\tilde{R} = e^{-\tilde{\lambda}t}$ or $e^{-t/\tilde{\theta}}$	$\theta_L = 2x\hat{\theta}/\chi^2 (2x)\, 1-\alpha$ $\lambda_u = 1/\theta_L$ $R_L = e^{-\lambda_u t}$ or e^{-t/θ_L}
5	**All hardware elements/ components, but more particularly continuously operating electro & electro-mechanical devices.**	Poisson Process & Exponential R $g(x)=e^{-\lambda t}[(\lambda t)^x/x!]$ $R(t)=e^{-\lambda t}$	CFR	X	X		X				X			$\hat{\theta}=nt_o/x,\ r \neq 0$ $\hat{\lambda} = 1/\hat{\theta}$ $\hat{R} = e^{-\hat{\lambda}t}$ or $e^{-t/\hat{\theta}}$	$\theta_L = 2nt_o/\chi^2(2x+2)\, 1-\alpha$ $\lambda_u = 1/\theta_L$ $R_L = e^{-\lambda_u t}$ or e^{-t/θ_L}
6	**All hardware elements/ components but more particularly continuously operating electro & electro-mechanical devices.**	Poisson Process & Exponential R $g(x)=e^{-\lambda t}[(\lambda t)^x/x!]$ $R(t)=e^{-\lambda t}$	CFR	X	X	X	X						X	$\hat{\theta} = nt_x/x,\ x \neq 0$ $\hat{\lambda} = 1/\hat{\theta}$ $\hat{R} = e^{-\hat{\lambda}t}$ or $e^{-t/\hat{\theta}}$	$\theta_L = 2nt_x/\chi^2(2x)\, 1-\alpha$ $\lambda_u = 1/\theta_L$ $R_L = e^{-\lambda_u t}$ or e^{-t/θ_L}
7	**All hardware elements/ components but more particularly continuously or periodically operating electro & electro-mechanical devices.**	Poisson Process Exponential (Incomplete) $f(x)=e^{-\lambda t}(\lambda t)^x/x!$ $R(t)=\begin{cases} e^{-\lambda t}, & \lambda > 0 \\ 0, & t < 0 \end{cases}$ (Only time t_x of last failure is observed)	CFR	X	X	X	X					X (Incomplete)		$\hat{\theta} = \beta_{x,n} t_x,\ x < 2n/3$ where $\beta_{x,n}$ is tabulated $\hat{\lambda} = 1/\hat{\theta}$ $\hat{R} = e^{-\hat{\lambda}t}$ or $e^{-t/\hat{\theta}}$	Not available.
8	**All hardware elements/ components, but more particularly continuously or periodically operating electro & electro-mechanical devices which exhibit a threshold or guarantee period u during which they do not fail.**	Poisson Process Two Parameter Exponential Distribution. $f(x) = e^{-\lambda(t-\mu)}(\lambda(t-\mu))^x/x!$ $R(t) = \begin{cases} e^{-\lambda(t-\mu)}, & t \geq \mu, \\ 0 & t<\mu, \lambda>0, \mu>0 \end{cases}$	0 failure rate for $t < \mu$, then CFR	X	X	X	X					X		$\hat{\theta} = \sum_{i=1}^{x} (t_i+(n-x)t_x-nt_1)/x,\ x \neq 0$ $\hat{\mu} = t_1$ $\tilde{\theta}$ (MV, unbiased) $= (x/x-1)\hat{\theta}$ $\tilde{\mu} = t_1 - \tilde{\theta}/n$ $\hat{\lambda} = 1/\hat{\theta},\ \tilde{\lambda} = 1/\tilde{\theta}$	$\theta_L = 2(x-1)\tilde{\theta}/\chi^2_{(2x-2)\, 1-\alpha}$ $\mu_L = t_1 - \tilde{\theta}F(1-\alpha, 2, 2(x-1))/n$ $\mu_u = t_1,\ \lambda_u = 1/\theta_L$ $R_L = e^{-\lambda_u(t-\mu_L)}$

Figure 5-11. Classical Methodology Summary for Assessing Component R, MTTF, and Failure Rate (Continued)

Exhibit #	Item Classification	Statistics		Raw Data					Type of Test						
	Classification by Performance	Failure Process Principal Reliability Measure	IFR, CFR DFR	n - # units on test	t - test time	c - # test cycles	S - # successes	X - # failures	Time independent or uncensored	Type I censoring w/o replacement	Type I censoring w/ replacement	Type II censoring w/o replacement	Type II censoring w/ replacement	Point Estimates of Reliability (R), MTTF (θ), Hazard Rate (λ) or Other Specified Parameters	One-Sided Confidence Limits at 100 $(1-\alpha)$% Confidence for R, θ, λ or Other Specified Parameters
9	All hardware components/ elements, particularly those which display increasing or decreasing hazard rates, such as structural elements and mechanical devices subject to wear or tear and exhibiting a weakest spot.	Two parameter Weibull $R(t) = \begin{cases} e^{-(t/\alpha)^{\beta}} & t, \alpha > 0 \\ 0 & t < 0 \end{cases}$ where β is a shape parameter and α is a scale parameter. $R(t) = \frac{e^{-kt^{m+1}}}{m+1}$	IFR if $\beta > 1$, CFR if $\beta = 1$, DFR if $\beta < 1$	X	X	X			X					$\widehat{\alpha}$ and $\widehat{\beta}$ obtained from iterative solution of $\widehat{\alpha} = \frac{1}{n}\left\{\sum_{i=1}^{n} t_i^{\widehat{\beta}}\right\}^{\frac{1}{\widehat{\beta}}}$ $\widehat{\beta} = n\left[\frac{1}{\widehat{\alpha}}\right]^{\widehat{\beta}}\left(\sum_{i=1}^{n} t_i^{\widehat{\beta}} \ln t_i - \sum_{i=1}^{n} \ln t_i\right)$ (ML Biased) $\widehat{R} = \begin{cases} e^{-(t/\alpha)^{\beta}}, & t, \widehat{\alpha} > 0 \\ 0, & t < 0 \end{cases}$	A variety of competing methods proposed for the interval estimation of α and β.
10	Same as 9	Three Parameter Weibull. $R(t) = \begin{cases} e^{-\left(\frac{t-\gamma}{\alpha}\right)^{\beta}}, & t > \gamma,\ \alpha > 0 \\ 0, & t < \gamma \end{cases}$ where γ is a threshold parameter	IFR if $\beta > 1$, CFR if $\beta = 1$, DFR if $\beta < 1$	X	X	X			X					A variety of competing methods proposed for the point estimation of γ, α and β.	A variety of competing methods Proposed for the interval estimation of γ, α, and β.
11	Structural or other component/elements which are characterized by a normally distributed key parameter. Failure will occur if parameter is out of tolerance.	Normal Failure Model $R(t) = (1/2\pi)^{1/2} \times \int_{\frac{t-t_0}{\sigma}} \exp(-x^2/2)\,dx$ $= 1-\phi[(t-T_0)/\sigma]$	IFR	X	X				X					$\widehat{T}_0 = \frac{1}{n}\sum_{i=1}^{n} t_i$ $\widehat{\sigma} = s = \frac{1}{n}\sum_{i=1}^{n}(t_i - \widehat{T}_0)^{1/2}$ $\widehat{\sigma}$ (Unbiased) $= \frac{1}{n-1}\sum_{i=1}^{n}(t_i - \widehat{T}_0)^{1/2}$	Approximate limits for R, T_0, and σ can be obtained from normal theory. Exact limits require tables of tolerance limits.
12	Structures subject to fatigue cracks. Also, semi-conductor components. $f(t) = \begin{cases} \frac{e^{-\frac{1}{2}\left(\frac{\ln(t)-\mu}{\sigma}\right)^2}}{\sqrt{2\Pi}\,\sigma t}, & t > 0 \\ 0, & t < 0 \end{cases}$	Log Normal Failure Model	IFR, then DFR	X	X									$\mu = \frac{1}{\sigma}\sum_{i=1}^{n} \ln(t_i)$ $\widehat{\sigma}^2 = \frac{\sum_{i=1}^{n}(\ln(t_i))^2 - n\left[\sum_{i=1}^{n}\ln(t_i)/n\right]^2}{n}$ $\sigma_u = [n\sigma^2/\chi^2\alpha, (n-1)]^{1/2}$ $\widehat{\sigma}^2$ (Unbiased) $= \frac{n}{n-1}\widehat{\sigma}^2$	Limits on μ and σ from normal theory μ has a t-distribution Limits on reliability obtainable from non-central t-distribution tables

Figure 5-11. Classical Methodology Summary for Assessing Component R, MTTF, and Failure Rate (Continued)

It must be noted that, in the ML method it is permissible to replace a parameter by its estimate in a function to obtain a proper estimate of the function. Other methods do not generally have this property.

The ML method does not guarantee unbiasedness of the estimate, however, and while $\hat{\theta}$ is unbiased for this type of test with exponential components, $\hat{\lambda}$ is not. An unbiased estimate of $\hat{\lambda}$ is given by:

$$\hat{\lambda}_{\text{Unbiased}} = \left(\frac{x-1}{x}\right)\hat{\lambda} \qquad (5\text{-}6)$$

For the data above: $\hat{\lambda}_{\text{Unbiased}} = 0.0126108$ failures/mission.

The upper confidence limit, λ_u, for λ is given in [9] (also see Appendix D § D.1) by:

$$\lambda_u = \hat{\lambda}\chi^2_{1-\alpha;2x}/2x \qquad (5\text{-}7)$$

For a confidence $\gamma = 0.8 = 1-\alpha$, from Appendix E, figure E-1.

$$\chi^2_{0.8;16} = 20.465$$

therefore

$$\lambda_u = 0.0184341 \text{ failures/mission}$$

If only N = 8 items had been placed on test and the same times to failure had been obtained, then the use of the same formulae would have given:

$$\hat{\theta} = \frac{(151.969 + 0)}{8} = 18.996 \text{ missions}$$

$$\hat{\lambda} = \frac{1}{\hat{\theta}} = 0.0526427 \text{ failures/mission}$$

$$\hat{\lambda}_{\text{Unbiased}} = \frac{x-1}{x}$$

$$\hat{\lambda} = 0.046062 \text{ failures/missions}$$

$$\lambda_u = \frac{(0.0526427)(20.465)}{16}$$

$$= 0.0673333 \text{ failures/mission}$$

The uncensored test is of special interest since the original data were obtained from a Monte-Carlo simulation with λ selected to be 0.07. The fact that λ_u did not bracket the actual λ expresses the occurrence of one in five chances of failing to do so from a random sample (80% confidence).

5.4.1.1.1.2 Type II Censoring with Replacement

It is shown in [9], that:

$$\hat{\lambda} = x/N\tilde{t}_x \qquad (5\text{-}8)$$

$$\lambda_u = \chi^2_{1-\alpha;2x}/(2N\tilde{t}_x) \qquad (5\text{-}9)$$

Example Type II. Censoring with Replacement:

Assume N = 15 and the same data as in § 5.4.1.1.1.1, then:

$$\hat{\lambda} = 8/(15)(57.588)$$
$$= 0.00926119 \text{ failures/mission}$$

and, at the 80% confidence level:

$$\lambda_u = (20.465)/(30)(57.588)$$
$$= 0.0118456 \text{ failures/mission}$$

5.4.1.1.1.3 Type I Censoring without Replacement

Again from [9], we have:

$$\hat{\lambda} = x/\left(\sum_{i=1}^{x}\tilde{t}_i + (N-x)\,T\right) \qquad (5\text{-}10)$$

where T is the preassigned test duration.

$\hat{\lambda}$ is unbiased for this type of test with exponential components, but $\hat{\theta}$ is biased.

The problem of obtaining a theoretically satisfying lower confidence limit on θ is shown to be extremely complex in [9, p. 173-174], where only a *conservative* solution is provided.

5.4.1.1.1.4 Type I Censoring with Replacement

From [9],

$$\hat{\lambda} = x/NT \qquad (5\text{-}11)$$

Again $\hat{\lambda}$ is unbiased but $\hat{\theta} = 1/\hat{\lambda}$ is biased. An unbiased estimator of $\hat{\theta}$ is

$$\hat{\theta}_{\text{Unbiased}} = \frac{x}{x+1}\hat{\theta} \qquad (5\text{-}12)$$

A *conservative* formulation for λ_u from [9], also derived from a different viewpoint in Appendix D § D.1 is:

$$\lambda_u = \chi^2_{1-\alpha;2x+2}/2T \qquad (5\text{-}13)$$

Example Type I. Censoring with Replacement

A test with a single replaceable item on test is to run for 100 missions. The *ordered* times to failure, in missions, are:

$\bar{t}_1 = 0.045$, $\bar{t}_2 = 0.538$, $\bar{t}_3 = 4.000$, $\bar{t}_4 = 8.303$,
$\bar{t}_5 = 12.518$, $\bar{t}_6 = 23.962$, and $\bar{t}_7 = 32.350$.

Then:

$$\begin{aligned}\hat{\lambda} &= 7/(1)(100)\\ &= 0.0700000 \text{ failures/mission}\end{aligned}$$

$$\hat{\theta} = \frac{1}{\hat{\lambda}} = 14.2857 \text{ missions}$$

$$\hat{\theta}_{\text{Unbiased}} = 12.50000 \text{ missions}$$

And at the 80% confidence level,

$$\begin{aligned}\lambda_u &= \chi^2_{0.8;16}/(2)(100)\\ &= .102325 \text{ failures/mission}\end{aligned}$$

The data in this example were obtained with a Monte-Carlo simulation of the exponential where the "true" value of λ was set to be 0.05 failures/mission.

5.4.1.2 Reliability Assessment Point and Interval Estimation Using the Binomial Model

The binomial is applicable in life-testing situations when times to failure are unknown or irrelevant. Also, the binomial is applicable in Bernoulli from large lots, or sampling with replacement, so that reliability may be assumed to remain constant in successive trials.

If N is large with respect to the population M of components, i.e., N/M > 0.1, the hypergeometric distribution should be used, rather than the binomial [11, p. 40-43].

If testing is effected on a *variable* number of units N until x failures occur, then the appropriate PMF for N considered as a random variable is not the binomial but the negative binomial. This method of sampling for defectives (which is sometimes preferred because it is less costly than Bernoulli sampling when expensive units are destroyed in testing, for instance) is called Pascal sampling [12].

Reliability assessments which make use of the binomial PMF and CDF are illustrated in this subparagraph by means of the following examples.

Example, Pass/Fail or Bernoulli Data:

Twenty-five one-shot items are test fired. Test results show one item out of 25 failed performance requirements.

Denoting the number of items on test by N, the number of successful items by s and the number of failed items by x, then:

$$\hat{R} = s/N \qquad (5\text{-}14)$$

or

$$\hat{R} = (N-x)/N \qquad (5\text{-}15)$$

and the lower reliability limit satisfies the relation:

$$L = \sum_{i=x+1}^{n} \binom{N}{i} R_L^{\,N-i}(1-R_L)^i \qquad (5\text{-}16)$$

For the data presented:

$$\hat{R} = \frac{24}{25} = 0.96$$

and, solving iteratively for R_L, L = .80,

$$0.8 = \sum_{i=2}^{25} \left(\frac{25}{i}\right) R_L^{\,25-i}(1-R_L)^i.$$

$$R_L = 0.8849$$

This value for R_L can also be read directly from figure E-3 in Appendix E.

Note: The data for this case was actually Monte-Carlo simulated with a true R = 0.91.

Example, Cyclic Data:

A replaceable item is put on test for 100 cycles. Each cycle is a success if the item is operating after the cycle. The test results produce 1 failure in the 100 cycle test.

Equations 5-15 and 5-16 still apply to this situation, but N becomes n_c, the number of test cycles, s is now the number of successful cycles, and x, the number of failed cycles. Then:

$$\hat{R} = 99/100 = 0.99, \text{ and}$$

$$R_L = 0.9703 \text{ (from figure E-3)}$$

where $L = 0.80$

Assume that a mission requires 10 cycles. The probability of completing a mission is $(P_{cycle})^{10}$ where P_{cycle} is the probability of completing a cycle. Therefore,

$$P_{cycle} = \hat{R} \text{ and } P_{cycle_L} = R_L$$

then $\hat{R}_{Mission} = (0.99)^{10} = 0.9044$

and $R_{L_{Mission}} \neq (0.9703)^{10} \doteq 0.7397$

[**Caution**: The component lower bounds on reliability *cannot* be used (combined) in a system model to obtain a system lower bound.]

5.4.1.3 Reliability Assessment for Normal Times to Failure

A simple example will be given here to illustrate one possible application of the Normal model (see § 5.2.3).

Example, When Times to Failure are Normally Distributed:

A single replaceable item is put on test until five failures are observed. The times to failure in mission are:

$t_1 = 2.6003$, $t_2 = 3.1467$, $t_3 = 3.0685$, $t_4 = 3.2501$, and $t_5 = 1.8684$.

Assume that, even though it is difficult to discriminate with only 5 failures between a normal distribution of failure times and other distributions, the data is indeed normally distributed.

Then a point estimate of MTTF is:

$$\widehat{MTTF} = \sum_{i=1}^{x} t_i/x \qquad (5\text{-}17)$$

also, as usual for the Normal:

$$\hat{\sigma} = s = \sqrt{\frac{1}{x-1} \sum_{i=1}^{x} (t_i - \bar{t})^2} \qquad (5\text{-}18)$$

A lower limit at L confidence for MTTF is obtained by:

$$MTTF_L = \widehat{MTTF} - t_{L;x-1}\, \hat{\sigma}/\sqrt{x} \qquad (5\text{-}19)$$

where $t_{L;x-1}$ is the L-percentile of the Student-t distribution with x-1 degrees of freedom.

For the given data:

$$\widehat{MTTF} = 13.934/5 = 2.7868 \text{ missions,}$$

$$\hat{\sigma} = 0.57048 \text{ and}$$

$$MTTF_L = 2.7868 - \frac{(1.533)(0.57048)}{2.2361}$$

$$= 2.3957 \text{ missions}$$

where $L = 0.80$.

A simplistic approach to find an 80% lower bound on reliability is to assume

$$\sigma = \hat{\sigma} \text{ and } MTTF = MTTF_L.$$

This leads to

$$R_L = \int_1^{\infty} \text{Normal}(\mu = 2.3457, \sigma = 0.57048) = 0.993$$

where $L = 0.80$.

Note: The data were obtained from a Monte-Carlo simulation of normally distributed times to failure with actual MTTF = 2.5 and actual $\sigma = 0.5$

5.4.2 Variables Reliability Methods Based on the Normal Distribution

An important use of the normal distribution in the assessment of product reliability is as a basic PDF for the "variables" method. The variables method applies when reliability is best defined in terms of performance variables, such as a re-entry angle of attack for a missile, or critical parameters, such as a current or an applied difference of potential being within critical limits, rather than in terms of success of failure attributes. In most cases, the variables method assumes that the performance parameter is normally distributed or can be normalized by transformation, with a true unknown mean, μ, and standard deviation, σ. At the end of testing, the initial parameter sample mean, $\overline{X}$, and sample deviation, s, are available to estimate reliability at the desired confidence level. With these sample statistics, reliability calculations are performed using tolerance factors available in many statistical texts [e.g., (11, p. 311-318)].

5.4.2.1 Two-Sided Case

The two-sided case deals with the proportion (reliability) of a parameter X that lies between $\overline{X} - Ks = LCLS$ and $\overline{X} + Ks = UCLS$, at confidence γ, where LCLS is the lower critical limit specification, UCLS is the upper critical limit specification, and $\overline{X}$ and s are measured from test data. It is assumed that LCLS and UCLS are symmetric about $\overline{X}$. Mathematically,

$$P[P(\overline{X}-Ks \leq X \leq \overline{X}+Ks) \geq R] = \gamma \qquad (5\text{-}20)$$

where R is reliability.

Tables of two-sided tolerance factors [13], tabulated against R and γ allow combinations of R and γ to be found which satisfy (5-20). Appendix E, figures E-4 and E-5, provide tables of two-sided tolerance factors at the 50 and 80 percent confidence levels. To illustrate the method, tables are not employed here but an approximate equation [11] is used. This equation is:

$$K_R = K \Big/ \left[\left(1+\frac{1}{2N}\right) \sqrt{(N-1)/\chi^2_\alpha} \right] \qquad (5\text{-}21)$$

In equation 5-21, K_R is the normal deviate for the reliability sought, K is the tolerance factor obtained as $K = (\overline{X} - LCLS)/s$ or $(UCLS - \overline{X})/s$, N is the test sample size and χ^2_α is the chi-square CDF with $\alpha = 1 - \gamma$.

Example, Two-Sided Case

Assume that n = 5, $\overline{X}$ is measured to be 8, s = 0.484, LCLS = 6, UCLS = 10, then K = 4.09. Assume also that the desired confidence γ is 0.80, then $\chi^2_\alpha = \chi^2_{1-\gamma} = \chi^2_{0.20} = 1.648$ (for 4 degrees of freedom). Equation 5-21 gives $K_R = 2.39$ which is the normal deviate corresponding to $R_L = 0.992$ at 80% confidence. If another confidence level were selected, the R_L would not be 0.992. K_R, hence the reliability could also be specified, and the confidence of reliability computed if desired.

If LCLS and UCLS are not symmetric about $\overline{X}$, the calculation of reliability and confidence is much more involved [51].

5.4.2.2 One-Sided Case

Often, a parameter will lead to a failure only if it exceeds or is below a critical value. In this case, a one-sided equation is formed, either $P[P(X \leq \overline{X} + Ks) \leq R] = \gamma$ or $P[P(X \leq \overline{X} - Ks) \leq R] = \gamma$. Tables of one-sided tolerance factors [13] tabulated against R and γ permit combinations of R and γ to be found which satisfy these equations. Appendix E presents a table of one-sided tolerance factors in figure E-5.

Alternately, an approximation formula [14] can be used, as follows:

$$K_R = K - K_\gamma \frac{1}{N} + k^2/2f \qquad (5\text{-}22)$$

where $K = (SL-\overline{X})/s$, SL is the one-sided specification limit, K_γ is the normal deviate at confidence γ, K_R is the normal deviate for the reliability sought.

For example, if K equals 2.31 for N = 10 items on test, and γ is selected as 0.95, then

$K_\gamma = 1.645$, and $K_R = 1.274$. From a table of normal deviates: $R = 0.899$ (at 95 confidence).

5.4.2.3 Demskey's Extension of the Variables Method

The variables technique [VAEP] proposed by Demskey [14] is applicable when reliability is defined entirely in terms of the probability that one or more performance variables will jointly take values within specified limits. In addition, the distribution of each variable must be continuous and definable by its low-order moments. Test data consist of repeated measurements of the variables taken under conditions pertinent to the mission. The data for each variable are tested for goodness-of-fit to a normal distribution. If the fit is unacceptable, various transformations of the data are tried until an acceptable normal fit is achieved. Then tolerance limit theory is applied to obtain point and confidence limit estimates of reliability for each variable. The calculated limits within which a predicted proportion of a normal population will fall are defined by $\overline{X} \pm K_{RCnf}s$, where the subscripts denote reliability, confidence, sample size and degrees of freedom (usually n-1), respectively*, and where s is the sample standard deviation.

Quantitative data are obtained for each variable for each unit of a test sample. The data for each variable are examined to estimate the distribution of the parent population. Examination may include the preparation of a frequency distribution of the sample data, a plot of the data on probability paper, a test for goodness-of-fit to an assumed distribution, or a test for normality such as the W-test[2],[3]. If the parent population is non-normal, a function is determined and applied to transform the sampling distribution into a normal distribution. The mean μ and the standard deviation σ of the normal population are then estimated from the normalized sample data. Tolerance limit theory for normal distributions is applied to obtain a point estimate and lower confidence limit of reliability for each variable, where reliability of the variable is defined as the proportion of the parent population within the specified tolerance interval. Finally, the reliability estimates for all variables are combined by means of a mathematical model to obtain point and lower confidence limit estimates of device reliability.

In many instances normally distributed data are not to be expected and transformation is necessary or desirable. Efforts to establish or attain reasonable normality of variables data are justified by the mathematical efficiency and ease of application of that distribution, by the availability of published tolerance limit factors for that distribution, and because many statistical tests of hypotheses presume normality of the parent populations.

Nevertheless, instances arise when error associated with the use of normal theory are unacceptable and it is necessary to work directly with untransformed data. Moreover, tolerance intervals can be computed directly for many distributions of interest, including the exponential, Weibull, extreme value, Cauchy and logistic [7]. Thus, if there is an a priori reason to believe that data are distributed according to one of these forms, transformation may not be necessary. Non-parametric (distribution free) tolerance intervals may also be computed using tables provided by Somerville [8]. However, reliability estimates based on non-parametric tolerance intervals will be less precise than those developed from data derived from known distributions and should be used only if all else fails.

5.4.2.3.1 Model for Independent Variables

Let the reliability of an equipment be determined by the numerical values taken by each of m continuously distributed independent variables $X_1, X_2, \ldots X_m$ with lower and upper specification limits L_i and U_i respectively, where $i=1, 2, \ldots, m$. The equipment fails if any of the m variables fall outside its specification limits. The general reliability series model then is:

$$R = [P(L_1 < X_1 < U_1)] \cdot [P(L_2 < X_2 < U_2)] \cdot\cdot\cdot\cdot\cdot [P(L_m < X_m < U_m)] \qquad (5\text{-}23)$$

---

*Tables of K_{RCnf} are provided in Figures E-5 through E-7.

or

$$R = \prod_{i=1}^{m} R_i \qquad (5\text{-}24)$$

Omission of one or more relevant parameters from the model will usually result in optimistically high reliability estimates; conversely, inclusion of non-relevant parameters in the model will often result in underestimating the reliability.

If an equipment is in a success state when any one of its m variables takes a value within specified limits, the reliability model is that of a parallel system.

$$R = 1 - \prod_{i=1}^{m} (1-R_i) \qquad (5\text{-}25)$$

5.4.2.3.2 Model for Dependent Variables

When reliability is high and t = 1 mission, $\lambda_i \approx 1 - R_i$. For this case equipment reliability, when the variables are correlated (dependent), is estimated by the model

$$\widehat{R} = 1 - \sum_{i=1}^{m} \widehat{\lambda}_i + \left(\frac{\sum_{i=1}^{m} \rho_{ij}}{m^2}\right) \sum_{i=1}^{m} \rho_{i \cdot jk} \cdot \widehat{\lambda}_i \qquad (5\text{-}26)$$

where:

m = number of performance variables in the model.

ρ_{ij} = significant simple coefficient of correlation between ith and jth variable.

$\rho_{i \cdot jk}$ = significant coefficient of multiple correlation for the ith variable, holding constant the effects of the jth, kth, etc. variables. It may be interpreted as the simple coefficient of correlation between the actual values of the ith variable and those predicted by regressing that variable against all the others.

In the case where the model variables are mutually independent the correlation coefficients are all identically zero and the model reduces to (see equation 5-24):

$$\widehat{R} = \prod_{i=1}^{m} \widehat{R}_i \simeq 1 - \sum_{i=1}^{m} \widehat{\lambda}_i \qquad (5\text{-}27)$$

5.4.2.3.3 Computation of Tolerance Factors

The limits within which a predicted proportion of a normal population will fall are defined by $\overline{X} \pm K_{RCnf}s$, where $\overline{X}$ is the sample mean and where the subscripts denote respectively, reliability, confidence, sample size and degrees of freedom (usually n-1), and s is the sample standard deviation:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}} \qquad (5\text{-}28)$$

These limits converge to the familiar $\overline{X} \pm Ks$ for large samples, where the unsubscripted K is the standard normal deviate. A tolerance factor may be estimated for a given specification limit (SL) by:

$$K_{RCnf} = \frac{SL - \overline{X}}{s} \qquad (5\text{-}29)$$

One-sided factors can be computed using the relationships:

$$K_{RCnf} = \frac{K_R + K_C \left[\frac{1}{n}\left(1 - \frac{K_c{}^2}{2f}\right) + \left(\frac{K_R^2}{2f}\right)\right]^{1/2}}{1 - \frac{K_c^2}{2f}} \qquad (5\text{-}30)$$

where K_R and K_c are, respectively, the standard normal deviate for reliability R and confidence C, n is sample size and f is degrees of freedom. Two-sided factors are obtained using:

$$K_{RCnf} = \left(1 + \frac{1}{2n}\right) K_R \sqrt{f/\chi_c^2} \qquad (5\text{-}31)$$

where χ_c^2 is the chi-square statistic for confidence C with f degrees of freedom. In the absence of tolerance limit tables*[20], these equations can be used to estimate reliability or confidence by solving for K_R or χ_c^2 and referring these computed statistics to published tables of the standard normal distribution or chi-square distribution.

---

*Limited one-sided and two-sided tolerance factors are available in Appendix E.

Having computed K_{RCnf} from test data by means of equation 5-29, point and interval reliability estimates for the ith model variable are computed by reference to tabular values of K_{RCnf}. $\widehat{R}_i$ is the reliability tabulated for K_{RCnf} at confidence C = .50. R_{L_i} is the reliability tabulated at confidence C = .80.

Then a series subsystem point estimate $\widehat{R}$ is obtained from equation 5-26 or 5-27 depending on whether the dependent or independent model is used. A confidence interval estimate for a dependent serial system is:

$$R_L = \widehat{R} - \widehat{R} \left[\sum_{i=1}^{m} \frac{d_i^2}{\widehat{R}_i^2} \right. + 2 \left. \left[\sum_{i=1}^{m} \rho_{i(i+1)} \frac{d_{i+1} d_i}{\widehat{R}_{i+1} \widehat{R}_i} \right] \right\}^{1/2} \tag{5-32}$$

For an independent serial system:

$$R_L = \widehat{R} - \widehat{R} \left[\sum_{i=1}^{m} \frac{d_i^2}{\widehat{R}_i^2} \right]^{1/2} \tag{5-33}$$

In both of the above:

$$d_i = \widehat{R}_i - R_{L_i} \tag{5-34}$$

For a parallel system of identical independent elements, the point estimate is:

$$\widehat{R} \cong 1 - \prod_{i=1}^{n} \widehat{\lambda}_i \tag{5-35}$$

and the confidence limit estimate is

$$R_L = \widehat{R} - n \widehat{\lambda}_i^{n-1} d_i \tag{5-36}$$

When zero failures are observed, an adjustment can be made to obtain a finite estimate of variance and hence of the confidence interval. The adjustment is to redefine the component point estimate as $\widehat{\lambda}_i = n/4$ in the confidence interval equation 5-36.

Series-parallel configurations can be treated by stepwise reduction to an equivalent series model.

5.4.2.3.4 Outliers

Data classified as outliers by the Dixon test [9] or similar tests, and not explainable as test errors, can be treated as observations from subpopulations. When there are two or more outlier data points, the mean and variance of the subpopulation are estimated in the usual manner. A single datum is taken as an estimate of the subpopulation mean; the subpopulation's variance is assumed to be identical to that of the main population. Statistics resulting from the main sample and outlier sample are frequency-weighted.

For a single outlier, the failure rate model is:

$$\widehat{\lambda} = \frac{1}{n} \widehat{\lambda}_0 + \frac{n-1}{n} \widehat{\lambda}_p \tag{5-37}$$

where $\widehat{\lambda}_0$ and $\widehat{\lambda}_p$ are the point failure rate estimates for the outlier and main group respectively. The weighted estimate of failure rate is then used in the point and interval reliability equations exactly as an unweighted estimate would be used.

Example, variables model

Fifty solid rocket motors are test fired; thrust-time profile and chamber pressure (psia) measurements are tabulated below as peak pressure and action time (Figures 5-12 and 5-13). The logarithms of peak pressure values are added to the tables for subsequent use in developing reliability estimates.

Frequency distributions using ordered test data are shown in figure 5-14, parts 1 and 2. Action time is plotted on normal probability paper, figure 5-15. Peak pressure is plotted on log normal probability paper, figure 5-16. Several statistical quantities required for analysis are computed.

$$n_T = 50$$

$$\Sigma T = 6005.7$$

$$\Sigma T^2 = 723{,}024.39$$

$$(\Sigma T)^2 / n_T = \frac{(6005.7)^2}{50} = 721{,}368.6498$$

Test Unit Number	Action Time T (seconds)	Peak Pressure P (psia)	Log Peak Pressure Z (log psia)
1	114.2	1136	3.05538
2	123.1	1386	3.14176
3	108.8	1947	3.28937
4	118.4	1247	3.09587
5	119.8	1256	3.09899
6	122.2	1672	3.22324
7	115.8	1171	3.06856
8	117.4	1286	3.10924
9	117.2	1214	3.08422
10	120.7	1370	3.13672
11	118.6	1443	3.15927
12	114.8	1580	3.19866
13	113.9	1352	3.13098
14	122.5	1554	3.19145
15	120.4	2666	3.42586
16	123.4	2116	3.32552
17	105.8	1532	3.18526
18	131.8	1578	3.19811
19	118.3	1408	3.14860
20	125.6	1761	3.24576
21	113.8	1525	3.18327
22	122.9	2222	3.34674
23	136.3	2059	3.31366
24	119.6	1233	3.09096
25	118.2	1387	3.14208
26	116.5	2050	3.31175
27	113.7	1129	3.05269
28	111.9	1401	3.14644
29	126.0	1662	3.22063
30	113.9	1080	3.03342
31	123.3	1730	3.23805
32	122.5	1456	3.15316
33	120.4	1345	3.12872
34	123.1	1467	3.16643
35	122.9	1351	3.13066
36	122.0	1364	3.13481
37	126.7	1239	3.09307
38	126.4	2408	3.38166
39	124.6	1775	3.24920
40	121.4	1676	3.22427
41	118.2	1408	3.14860
42	126.4	1685	3.22660
43	121.0	1541	3.18780
44	119.2	1623	3.21032
45	112.8	1908	3.28058
46	126.6	2742	3.43807
47	114.5	1458	3.16376
48	127.3	1862	3.26998
49	113.1	1308	3.11661
50	127.8	1834	3.26340

Figure 5-12. Sample Test Data (Hypothetical)

Ordinal Index i	Action Time, T (seconds)	Peak Pressure, P (psia)	Log Peak Pressure Z (log psia)
1	105.8	1080	3.03342
2	108.8	1129	3.05269
3	111.9	1136	3.05538
4	112.8	1171	3.06856
5	113.1	1214	3.08422
6	113.7	1233	3.09096
7	113.8	1239	3.09307
8	113.9	1247	3.09587
9	113.9	1256	3.09899
10	114.2	1286	3.10924
11	114.5	1308	3.11661
12	114.8	1345	3.12872
13	115.8	1351	3.13066
14	116.5	1352	3.13098
15	117.2	1364	3.13481
16	117.4	1370	3.13672
17	118.2	1386	3.14176
18	118.2	1387	3.14208
19	118.3	1401	3.14644
20	118.4	1408	3.14860
21	118.6	1408	3.14860
22	119.2	1443	3.15927
23	119.6	1456	3.16316
24	119.8	1458	3.16376
25	120.4	1467	3.16643
26	120.4	1525	3.18327
27	120.7	1532	3.18526
28	121.0	1541	3.18780
29	121.4	1554	3.19145
30	122.0	1578	3.19811
31	122.2	1580	3.19866
32	122.5	1623	3.21032
33	122.5	1662	3.22063
34	122.9	1672	3.22324
35	122.9	1676	3.22427
36	123.1	1685	3.22660
37	123.1	1730	3.23805
38	123.3	1761	3.24576
39	123.4	1775	3.24920
40	124.6	1834	3.26340
41	125.6	1862	3.26998
42	126.0	1908	3.28058
43	126.4	1947	3.28937
44	126.4	2050	3.31175
45	126.6	2059	3.31366
46	126.7	2166	3.32552
47	127.3	2222	3.34674
48	127.8	2408	3.38166
49	131.8	2666	3.42586
50	136.3	2742	3.43807

Figure 5-13. Sample Data and Transformed Data Presented in Order of Magnitude, Low to High

S_T^2 (Sum of Squares) = $\Sigma T^2 - [(\Sigma T)^2/n_T] = 1{,}655.7402$

$$S_T = \sqrt{\frac{1{,}655.7402}{49}} = 5.81$$

$$\overline{T} = \frac{6005.7}{50} = 120.11$$

$n_p = 50$

$\Sigma P = 79603$

$\Sigma P^2 = 133{,}657{,}959$

$$(\Sigma P)^2/n_P = \frac{(79603)^2}{50} = 126{,}732.752$$

S_P^2 (Sum of Squares) = $\Sigma P^2 \; [(\Sigma P)^2/n_P]$ = 6,925,207.

$$S_P = \sqrt{\frac{6{,}925{,}207}{49}} = 375.9$$

$$\overline{P} = \frac{79603}{50} = 1592.1$$

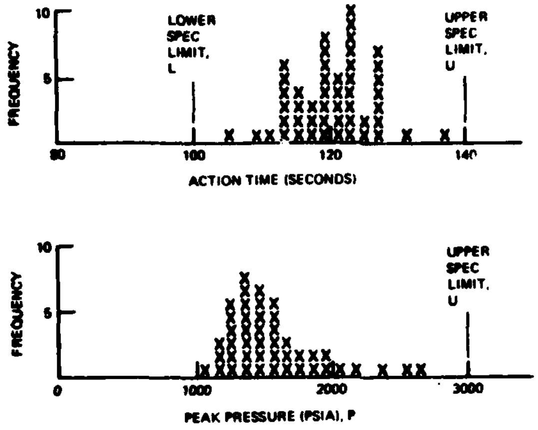
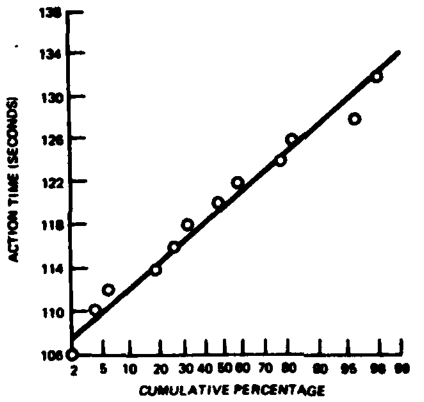


Figure 5-14. Frequency Distributions for Sample Test Data



NOTE. STRAIGHT LINE REPRESENTS THE PARENT DISTRIBUTION N(120,6) FROM WHICH SAMPLE WAS DRAWN.

Figure 5-15. Normal Probability Plot for Action Time

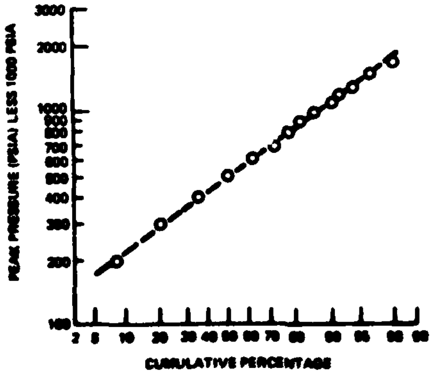


Figure 5-16. Log Normal Probability Plot of Peak Pressure

$n_z = 50$

$\Sigma Z = 159.57021$

$\Sigma Z^2 = 509.689050550$

$$(\Sigma Z)^2/n_z = \frac{(159.57021)^2}{50} = 509.253038389$$

$$S_z^2 = \Sigma Z^2 - (\Sigma Z)^2/n_z = .43601216$$

$$s_z = s_{\log P} = \sqrt{\frac{.43601216}{49}} = .09433$$

$$\overline{Z} = \log P = \frac{159.57021}{50} = 3.19140$$

$\Sigma ZT = 19,176.764081$

where P is peak pressure, T is action time, and Z is log peak pressure.

The degree of correlation between variable Z and T is expressed by the simple correlation coefficient ρ_{ZT} where,

$$\rho_{ZT} = \frac{[\Sigma ZT - (\Sigma Z)(\Sigma T)/n]}{\sqrt{[\Sigma Z^2 - (\Sigma Z)^2/n] \cdot [\Sigma T^2 - (\Sigma T)^2/n]}} \qquad (5\text{-}38)$$

Inserting appropriate quantities from Figure 5-18, we have $\rho_{ZT} = .4014$.

All data are tested for normality by the Shapiro and Wilks W-test, figure 5-17. It is concluded that the variables action time, T and log of peak pressure, Z are normally distributed.

Since the estimated correlation of action time, T and log peak pressure Z is statistically significant at the .01 significance level, the correlated model is applied to estimate rocket performance reliability based on T and Z data.

Using equation 5-29, compute two one-sided tolerance intervals for action time T.

$$(K_{RCnf})_U = \frac{U_T - T}{S_T} = \frac{140\text{-}120.11}{5.81} = 3.423$$

$$(K_{RCnf})_L = \frac{L_T - T}{S_T} = \frac{100\text{-}120.11}{5.81} = -3.461$$

Interpolating linearly, using a Table of K_{RCnf}:

K_{RCnf}	R
3.0902	.999
3.423	$\boxed{.9995}$
3.7190	.9999
3.0902	.999
3.461	$\boxed{.9996}$
3.7190	.9999

Excluding .0005 on one side of the distribution and .0004 on the other side, a reliability of $R_T = .9991$ is estimated for the action time variable. This process is repeated for the 80 percent confidence limit.

K_{RCnf}	R
3.4031	.999
3.423	$\boxed{.9991}$
4.0862	.9999
3.4031	.999
3.461	$\boxed{.9992}$
4.0862	.9999

Excluding .0009 on one side of the distribution and .0008 on the other, gives $R_{T_L} = .9983$ at 80 percent confidence for the action time variable.

As an alternative, equation 5-30 could be solved for K_R with respect to each specification limit separately. Rearranging equation 5-30 and referring to a table of the standard normal distribution:

$$K_R = K_{RCnf} - K_C\sqrt{\frac{K_{RCnf}}{2f} - \frac{1}{n}} \qquad (5\text{-}39)$$

$$= 3.423 - .842\sqrt{\frac{3.423}{98} - \frac{1}{50}}$$

$$= 3.320$$

The corresponding R for the upper limit is .9995, about as interpolated previously. For the lower limit of action time:

$$K_R = K_{RCnf} - K_C\sqrt{\frac{K_{RCnf}}{2f} - \frac{1}{n}}$$

Part 1: Action Time, T (Seconds)					
k	i	Descending Values of T*	i	Ascending Values of T*	A_{n-k+1}**
1	50	136.3	1	105.8	.3751
2	49	131.8	2	108.8	.2574
3	48	127.8	3	111.9	.2260
4	47	127.3	4	112.8	.2032
5	46	126.7	5	113.1	.1847
6	45	126.6	6	113.7	.1691
7	44	126.4	7	113.8	.1554
8	43	126.4	8	113.9	.1430
9	42	126.0	9	113.9	.1317
10	41	125.6	10	114.2	.1212
11	40	124.6	11	114.5	.1113
12	39	123.4	12	114.8	.1020
13	38	123.3	13	115.8	.0932
14	37	123.1	14	116.5	.0846
15	36	123.1	15	117.2	.0764
16	35	122.9	16	117.4	.0685
17	34	122.9	17	118.2	.0608
18	33	122.5	18	118.2	.0532
19	32	122.5	19	118.3	.0459
20	31	122.2	20	118.4	.0386
21	30	122.0	21	118.6	.0314
22	29	121.4	22	119.2	.0244
23	28	121.0	23	119.6	.0174
24	27	120.7	24	119.8	.0104
25	26	120.4	25	120.4	.0035

Part 2: Log Peak Pressure (log psia), Z					
k	i	Descending Values of Z - 3	i	Ascending Values of Z - 3	A_{n-k+1}
1	50	.43807	1	.03342	.3751
2	49	.42586	2	.05269	.2574
3	48	.38166	3	.05538	.2260
4	47	.34674	4	.06856	.2032
5	46	.32552	5	.08422	.1847
6	45	.31366	6	.09096	.1691
7	44	.31175	7	.09307	.1554
8	43	.28937	8	.09587	.1430
9	42	.28058	9	.09899	.1317
10	41	.26998	10	.10924	.1212
11	40	.26340	11	.11661	.1113
12	39	.24920	12	.12872	.1020
13	38	.24576	13	.13066	.0932
14	37	.23805	14	.13098	.0846
15	36	.22660	15	.13481	.0764
16	35	.22427	16	.13672	.0685
17	34	.22324	17	.14176	.0608
18	33	.22063	18	.14208	.0532
19	32	.21032	19	.14644	.0459
20	31	.19866	20	.14860	.0386
21	30	.19811	21	.14860	.0314
22	29	.19145	22	.15927	.0244
23	28	.18780	23	.16316	.0174
24	27	.18526	24	.16376	.0104
25	26	.18327	25	.16643	.0035

$i = 1.2. \ldots .n$

$k = n/2 = \frac{50}{2} = 25$

$b_I = .3751\,(136.3 - 105.8) + .2574(131.8 - 108.8) + \cdots + .0035(120.4 - 120.4) = 40.44899$

$S_T{}^2 = \Sigma T^2 - (\Sigma T)^2/n = 723,024.39 - 721,368.6498 = 1655.7402$

$$W_T = \frac{b_T{}^2}{S_T{}^2} = \frac{40.44899^2}{1655.7402} = .988$$

$P(W = .955) = .10$

$P(W = .974) = .50$

and $P(W_T = .988) > .50$.

$b_z = .64385118$

$S_z^2 = .43601200$

$$W_z = \frac{b_z^2}{S_z^2} = .9508$$

$P(W_z = .9508) \cong .075$

* *from Figure 5-13*

**from Hahn, G. and Shapiro, C.S. – Statistical Methods in Engineering–Wiley and Sons, 1967

Figure 5-17. W-Tests for Normality

$$= 3.461 - .842\sqrt{\frac{3.461}{98} - \frac{1}{50}}$$

$$= 3.346$$

$\widehat{R}$ for the lower limit is .9996 as previously found. Therefore, $\widehat{R}_T = .9991$, which agrees with the results obtained using tables.

Proceeding to the log pressure variable Z,

$$U_Z = \log 3000 \text{ psia} - 3.47712$$

$$(K_{RCnf})_U = \frac{U_Z - \overline{Z}}{s_Z}$$

$$= \frac{3.47712 - 3.19140}{.09433} = 3.028$$

Interpolation gives:

$$\widehat{R}_Z = .998$$

$$R_{Z_L} = .996 \text{ at 80 percent confidence}$$

To estimate motor reliability equation 5-26 is solved. The approximations $\widehat{\lambda}_T \cong 1 - \widehat{R}_T = .0009$ and $\widehat{\lambda}_Z \cong 1 - \widehat{R}_Z = .002$ are used.

m = 2	i	$\widehat{R}_i$	$\widehat{\lambda}_i$	ρ_{ij}	$\rho_{ij}\widehat{\lambda}_i$	$\Sigma\rho_{ij}/m^2$
	1	.9991	.0009	.4014	.00036	.2007
	2	.998	.002	.4014	.0008	
			.0029		.00116	

$$\widehat{R} = 1 - \sum_{i=1}^{m} \widehat{\lambda}_i + \left(\frac{\sum_{i=1}^{m} \rho_{ij}}{m^2} \right) \sum_{i=1}^{m} \rho_{ij} \widehat{\lambda}_i$$

$$= 1 - .0029 + (.2007)(.00116)$$

$$= .9973$$

The confidence limit is found using equation 5-32.

m = 2	i	$\widehat{R}_i$	R_{i_L}	d_i	$d_i/\widehat{R}_i$
	1	.9991	.9983	.0008	.0008
	2	.998	.996	.002	.002

$$R_L = \widehat{R} - \widehat{R}\left\{\left[\sum_{i=1}^{m} \frac{d_i^2}{\widehat{R}_i^2}\right] + 2\left[\sum_{i=1}^{m} \rho_{ij} \frac{d_{i+1} \cdot d_i}{\widehat{R}_{i+1} \cdot \widehat{R}_i}\right]\right\}^{1/2}$$

$$= .9973 - .9973\,(.0024)$$

$$= .9949 \text{ at 80\% confidence.}$$

5.4.3 Reliability Assessment Method Based on Stress-Strength Interference Models

Reliability models for structures are usually complex models of the Weibull or Lognormal type, or stress-strength models.

5.4.3.1 Stress-Strength Interference, Normal Distribution

In [12], a model of normally distributed stress and strength is presented with an example (see Figure 5-18). In the figure, s is a normally distributed stress with mean μ_s and standard deviation σ_s, and v is a normally distributed strength with mean μ_v and standard deviation σ_v. The interference area is a high-stress low strength area where failure is likely to occur. In the example, burst pressures of rocket chambers may be known to be normally distributed with mean $\mu_v = 800$ psia and standard deviation $\sigma_v = 100$ psia. Twenty (20) test firings of a solid-propellant rocket engine are made and the maximum value of chamber pressure is evaluated for each firing. The sample mean measured maximum pressure is found to be $\overline{s} = 400$ psia and

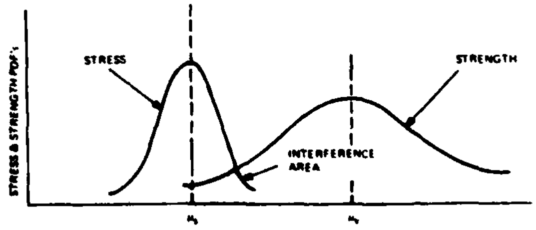


Figure 5-18. Stress-Strength Interference Probability Density Functions

the sample standard deviation $\hat{\sigma}_s = 25$ psia. It is shown in the example that the reliability estimate for the engine is:

$$\hat{R} = \phi \left[\frac{\mu_v - \bar{s}}{\sqrt{\sigma_v^2 + \sigma_s^2}} \right] = 0.9996 \qquad (5\text{-}40)$$

where ϕ is the normal CDF. An approximate lower confidence bound R_L, is given by:

$$R_L = \hat{R} - K_{0.90} V \qquad (5\text{-}41)$$

where K is the normal deviate for 90% confidence, and V, the variance of $\hat{R}$, is approximated through a complicated expression (see [12]), valid when $\hat{R}$ is high. The result is $R_L = 0.9993$ where $L = 0.90$. Additional point and interval reliability estimation examples for structures (normal PDF case) are given in [23].

5.4.3.2 Stress-Strength Interference, Weibull and Other Distributions

Reference [5] presents examples of reliability point and interval estimation for Weibull, lognormal, and exponential stresses and strengths, and for such combinations as exponential stress with normal strengths, normal stress with exponential strengths, and normal stress with Weibull strengths. Point and interval estimates of reliability for these cases are given only for situations where the parameters of the distributions are known, rather than estimated from test data.

5.4.3.3 Time Dependent Stress-Strength Models

Models have been developed to account for repeated application of stresses, as well as change in the distribution of strength caused by aging or cumulative damage. Details on these models are given in [5].

5.4.4 Methods for Assessing R Growth

When hardware (and software) elements of a system are modified throughout the test program, as failure modes are uncovered by testing, product reliability should show steady growth which can be measured and forecast by a mathematical model.

In [44], seven criteria are proposed for reliability growth models. They are:

a. The model should provide for reliability estimation and future prediction.

b. The model should have minimum bias and minimum variance.

c. A method for interval estimates should be inherent in the model.

d. The model should be relatively insensitive to external factors such as data grouping, the range of reliability or rapidity of growth.

e. Estimation techniques should be compatible with digital computing methods.

f. The model should reflect what is really happening.

To these desirable features of reliability growth model critiera, can be added the quantitative criteria of goodness-of-fit of actual failure data.

In [44], thirty-nine models of reliability growth proposed in the literature were assessed against the qualitative requirements described in the previous paragraph. The principal finding of the report was that the Duane reliability growth model was equal or preferable to all others considered with respect to the given critieria. Reference [45] indicates that while Duane is the preferred model in many situations and is the most generally applicable model, other models may be better in specific applications. The Duane model is discussed here and is defined as:

$$\lambda(t) = K(1-\alpha)t^{-\alpha} \qquad (5\text{-}42)$$

where $\lambda(t)$ is the failure rate at time t, while K and α are parameters to be estimated. Other forms of the Dunae model are:

1. $\lambda_\Sigma = Kt^{-\alpha}$, where λ_Σ = cumulative failure rate, K and α are parameters to be estimated from data, and t = total operating hours, cycles or missions.

2. $q_\Sigma = KN^{-\alpha}$ (applicable to attribute data), where q_Σ = cumulative probability of failure, N = number of trials.

The Duane growth model plots as a straight line on log-log paper (see § 6.5). In order to test its validity for a particular set of failure data, it is recommended that a Cramer-Von Mises goodness-of-fit test be performed first. The description of the goodness-of-fit test is delayed until point and interval estimates are presented in the next paragraph, because the test requires a knowledge of these estimates before it can be applied.

5.4.4.1 Point and Interval Estimates in the Duane Reliability Growth Model

The point and interval estimates presented here are those of the Army Materiel Systems Analysis Agency (AMSAA) model which is a close relative of the Duane model and is described in [47]. It is shown in [47] that point estimates of 1-α and K may be obtained from the simultaneous solution of the following pairs of equations:

For failure truncated tests:

$$\hat{\alpha} = 1-(x-2)/\left[(x-1)\ln t_x - \sum_{i=1}^{x-1} \ln t_i\right] \quad (5\text{-}43)$$

$$\hat{K} = x/t_x^{\widehat{(1-\alpha)}} \quad (5\text{-}44)$$

For time-truncated tests:

$$\hat{\alpha} = 1-(x-1)/\left[(x \ln T - \sum_{i=1}^{x} \ln t_i)\right] \quad (5\text{-}45)$$

$$\hat{K} = x/T^{\widehat{(1-\alpha)}} \quad (5\text{-}46)$$

In the expressions above, x is the total number of failures, t_i, the time to failure for the ith failure, and T the test duration.

The expressions above are valid even for small samples. They yield a point estimate of failure rate in the Duane model given by:

$$\hat{\lambda}(t) = \hat{K}(1-\hat{\alpha})t^{-\hat{\alpha}} \quad (5\text{-}47)$$

According to [47], an estimate of the MTBF, $\hat{\theta}(t)$, is

$$\hat{\theta}(t) = 1/\hat{\lambda}(t) \quad (5\text{-}48)$$

A lower bound on MTBF, θ_L, can be calculated [48] from the two-sided tables of [47], which are reproduced in figure 5-19, and 5-20, as follows:

In order to find a γ lower bound on θ, θ_γ, for instance, one should take the appropriate L factor from the 2γ-1 column to obtain:

$$\theta(t)_\gamma = L_{(2\gamma-1)}\hat{\theta}(t) \quad (5\text{-}49)$$

If, for instance, 10 failures have occurred in failure terminated tests, then:

$$\theta(t)_{0.90} = L_{0.80}\hat{\theta}(t) = 0.6852\,\hat{\theta}(t)$$

Example, Duane Growth Model for Time Truncated Tests.

Figure 5-21 shows synthetic failure data with corresponding calculations of $\hat{\alpha}(t)$ and $\theta_L(t)$. It was assumed that testing was test time truncated with T after each failure halfway before the next failure. The estimation was obtained with equations 5-45 and 5-48. Notice that while the estimated $\hat{\alpha}$ and $\hat{K}$ show increasing stability with x, indicating the possibility of an adequate fit to the Duane model (see § 5.4.4.2 for a confirmation), $\hat{\theta}$ and θ_L show the increased values with time expected from the achievement of reliability growth.

5.4.4.2 Goodness-of-Fit to Duane Reliability Growth Model

A Cramer-Von Mises Goodness-of-Fit test for the Duane R growth model is presented in [47]. The test statistic is, for failure-truncated tests:

$$C^2_{x-1} = \frac{1}{12(x-1)} + \sum_{i=1}^{x-1}\left[\left(\frac{t_i}{t_x}\right)^{1-\hat{\alpha}} - \frac{2i-1}{2(x-1)}\right]^2 \quad (5\text{-}50)$$

and for time-truncated tests:

$$C^2_{x-1} = \frac{1}{12(x-1)} + \sum_{i=1}^{x-1}\left[\left(\frac{t_i}{T}\right)^{1-\hat{\alpha}} - \frac{2i-1}{2(x-1)}\right]^2 \quad (5\text{-}51)$$

The critical values of C^2_{x-1} are shown in figure 5-22.

Example, Goodness of Fit to Duane Model For Time Truncated Test Data.

Equation 5-51 was used on the data displayed in figure 5-21 to generate the test

γ	.80		.90		.95		.98	
x	L	U	L	U	L	U	L	U
2	.8065	33.76	.5552	72.67	.4099	151.5	.2944	389.9
3	.6840	8.927	.5137	14.24	.4054	21.96	.3119	37.60
4	.6601	5.328	.5174	7.651	.4225	10.65	.3368	15.96
5	.6568	4.000	.5290	5.424	.4415	7.147	.3603	9.995
6	.6600	3.321	.5421	4.339	.4595	5.521	.3815	7.388
7	.6656	2.910	.5548	3.702	.4760	4.595	.4003	5.963
8	.6720	2.634	.5668	3.284	.4910	4.002	.4173	5.074
9	.6787	2.436	.5780	2.989	.5046	3.589	.4327	4.469
10	.6852	2.287	.5883	2.770	.5171	3.286	.4467	4.032
11	.6915	2.170	.5979	2.600	.5285	3.054	.4595	3.702
12	.6975	2.076	.6067	2.464	.5391	2.870	.4712	3.443
13	.7033	1.998	.6150	2.353	.5488	2.721	.4821	3.235
14	.7087	1.933	.6227	2.260	.5579	2.597	.4923	3.064
15	.7139	1.877	.6299	2.182	.5664	2.493	.5017	2.921
16	.7188	1.829	.6367	2.144	.5743	2.404	.5106	2.800
17	.7234	1.788	.6431	2.056	.5818	2.327	.5189	2.695
18	.7278	1.751	.6491	2.004	.5888	2.259	.5267	2.604
19	.7320	1.718	.6547	1.959	.5954	2.200	.5341	2.524
20	.7360	1.688	.6601	1.918	.6016	2.147	.5411	2.453
21	.7398	1.662	.6652	1.881	.6076	2.099	.5478	2.390
22	.7434	1.638	.6701	1.848	.6132	2.056	.5541	2.333
23	.7469	1.616	.6747	1.818	.6186	2.017	.5601	2.281
24	.7502	1.596	.6791	1.790	.6237	1.982	.5659	2.235
25	.7534	1.578	.6833	1.765	.6286	1.949	.5714	2.192
26	.7565	1.561	.6873	1.742	.6333	1.919	.5766	2.153
27	.7594	1.545	.6912	1.720	.6378	1.892	.5817	2.116
28	.7622	1.530	.6949	1.700	.6421	1.866	.5865	2.083
29	.7649	1.516	.6985	1.682	.6462	1.842	.5912	2.052
30	.7676	1.504	.7019	1.664	.6502	1.820	.5957	2.023
35	.7794	1.450	.7173	1.592	.6681	1.729	.6158	1.905
40	.7894	1.410	.7303	1.538	.6832	1.660	.6328	1.816
45	.7981	1.378	.7415	1.495	.6962	1.606	.6476	1.747
50	.8057	1.352	.7513	1.460	.7076	1.562	.6605	1.692
60	.8184	1.312	.7678	1.407	.7267	1.496	.6823	1.607
70	.8288	1.282	.7811	1.367	.7423	1.447	.7000	1.546
80	.8375	1.259	.7922	1.337	.7553	1.409	.7148	1.499
100	.8514	1.225	.8100	1.293	.7759	1.355	.7384	1.431

Figure 5-19. Confidence Intervals for MTBF in the Duane Growth Model from Failure Terminated Test

γ / x	.80		.90		.95		.98	
	L	U	L	U	L	U	L	U
2	.261	18.66	.200	38.66	.159	78.66	.124	198.7
3	.333	6.326	.263	9.736	.217	14.55	.174	24.10
4	.385	4.243	.312	5.947	.262	8.093	.215	11.81
5	.426	3.386	.352	4.517	.300	5.862	.250	8.043
6	.459	2.915	.385	3.764	.331	4.738	.280	6.254
7	.487	2.616	.412	3.298	.358	4.061	.305	5.216
8	.511	2.407	.436	2.981	.382	3.609	.328	4.539
9	.531	2.254	.457	2.750	.403	3.285	.349	4.064
10	.549	2.136	.476	2.575	.421	3.042	.367	3.712
11	.565	2.041	.492	2.436	.438	2.852	.384	3.441
12	.579	1.965	.507	2.324	.453	2.699	.399	3.226
13	.592	1.901	.521	2.232	.467	2.574	.413	3.050
14	.604	1.846	.533	2.153	.480	2.469	.426	2.904
15	.614	1.800	.545	2.087	.492	2.379	.438	2.781
16	.624	1.759	.556	2.029	.503	2.302	.449	2.675
17	.633	1.723	.565	1.978	.513	2.235	.460	2.584
18	.642	1.692	.575	1.933	.523	2.176	.470	2.503
19	.650	1.663	.583	1.893	.532	2.123	.479	2.432
20	.657	1.638	.591	1.858	.540	2.076	.488	2.369
21	.664	1.615	.599	1.825	.548	2.034	.496	2.313
22	.670	1.594	.606	1.796	.556	1.996	.504	2.261
23	.676	1.574	.613	1.769	.563	1.961	.511	2.215
24	.682	1.557	.619	1.745	.570	1.929	.518	2.173
25	.687	1.540	.625	1.722	.576	1.900	.525	2.134
26	.692	1.525	.631	1.701	.582	1.873	.531	2.098
27	.697	1.511	.636	1.682	.588	1.848	.537	2.068
28	.702	1.498	.641	1.664	.594	1.825	.543	2.035
29	.706	1.486	.646	1.647	.599	1.803	.549	2.006
30	.711	1.475	.651	1.631	.604	1.783	.554	1.980
35	.729	1.427	.672	1.565	.627	1.699	.579	1.870
40	.745	1.390	.690	1.515	.646	1.635	.599	1.788
45	.758	1.361	.705	1.476	.662	1.585	.617	1.723
50	.769	1.337	.718	1.443	.676	1.544	.632	1.671
60	.787	1.300	.739	1.393	.700	1.481	.657	1.591
70	.801	1.272	.756	1.356	.718	1.435	.678	1.533
80	.813	1.251	.769	1.328	.734	1.399	.695	1.488
100	.831	1.219	.791	1.286	.758	1.347	.722	1.423

Figure 5-20. Confidence Intervals for MTBF in the Duane Growth Model from Time Terminated Test

x	T	t_i	$\hat{a}(t)$	$\hat{K}(t)$	$\hat{\theta}(t)$	$\theta(t)_{0.90}$
1	125	50				
2	300	200	0.545	0.0746	529	106
3	550	400	0.464	0.0677	442	116
4	850	700	0.426	0.0625	455	142
5	1250	1000	0.428	0.0677	497	175
6	1600	1500	0.397	0.0585	517	199
7	1850	1700	0.351	0.0456	461	190
8	2100	2000	0.313	0.0364	430	187
9	2350	2200	0.283	0.0305	402	184
10	2700	2500	0.279	0.0301	407	194
11	3200	2900	0.299	0.0350	444	218
12	3750	3500	0.316	0.0396	488	247
13	4500	4000	0.348	0.0496	552	287
14	5500	5000	0.384	0.0644	662	353
15	6500	6000	0.405	0.0752	755	411
16	8500	7000	0.459	0.1124	958	533
17	13500	10000	0.548	0.2184	1585	896
18	18750	17000	0.587	0.2910	2519	1448
19	21000	20500	0.583	0.2842	2760	1609
20	22000	21500	0.569	0.2555	2652	1567
21	23000	22500	0.556	0.2304	2556	1531
22	24000	23500	0.543	0.2084	2470	1497
23	25000	24500	0.530	0.1890	2393	1467
24	26250	25500	0.521	0.1752	2345	1452
25	27500	27000	0.511	0.1626	2323	1452
26	28500	28000	0.500	0.1486	2261	1427
27	29800	29000	0.492	0.1392	2228	1417
28	31300	30600	0.486	0.1325	2232	1431
29	32750	32000	0.480	0.1258	2225	1437
30	33750	33500	0.470	0.1157	2189	1425

Figure 5-21. Reliability Growth Failure Data and Estimated Parameters

statistic C^2_{x-1} shown in figure 5-23. As can be seen, from the fact that the calculated values of C^2_{x-1} in figure 5-23 are all smaller than the 0.01 critical values of C^2_{x-1} in figure 5-22, the goodness-of-fit hypothesis is not rejected at the 0.99 significance level for large values of x.

5.4.5 The Rubinstein Method

This method termed the Basic Method in revision A of this manual, was developed by David Rubinstein in 1958. It treats the mixed-censoring one-component-at-a-time test case, and adjusts for multiple environments and for testing at various assembly levels. The equations given in the following subparagraphs are derived in [10].

5.4.5.1 Bias Arising from Mixed Truncation of Tests

Tests may terminate either by failure or accumulation of planned test times. Generally, both truncation policies are employed, so that test data for components in a system are neither completely Poisson, nor completely binomial. Under these conditions, the component failure rate estimate, $\widehat{\lambda} = \Sigma X / \Sigma t$, where ΣX is total observed failures and Σt is total actual test duration in equivalent missions, is biased. The bias is most significant early in a test program when data are few. The estimate given below, equation 5-52, contains a provision for correcting this bias.

5.4.5.2 Unbiased Estimation of Failure Rates and Their Variances

An estimate, which contains a bias correction factor, is:

$$\widehat{\lambda} = \frac{\sum_{i=1}^{N} X_i}{\sum_{i=1}^{N} t_i} \left[1 + \frac{\sum_{i=1}^{N} T_i^2}{2 \left(\sum_{i=1}^{N} T_i \right)^2} \right]^{-1} \qquad (5\text{-}52)$$

Equation 5-52 is valid when the failure rate is small. The summation

$$\sum_{i=1}^{N} T_i$$

is the sum of the planned test times for the N tests. As an example, assume three units of a component are tested. The stress applied during the test will occur for 6 minutes of the design mission. Thus, 1 hour of test time is equivalent to 10 missions (α = 10). Two of

	Level of Significance, α				
x-1	.20	.15	.10	.05	.01
2	.138	.149	.162	.175	.186
3	.121	.135	.154	.184	.231
4	.121	.136	.155	.191	.279
5	.121	.137	.160	.199	.295
6	.123	.139	.162	.204	.307
7	.124	.140	.165	.208	.316
8	.124	.141	.165	.210	.319
9	.125	.142	.167	.212	.323
10	.125	.142	.167	.212	.324
15	.126	.144	.169	.215	.327
20	.128	.146	.172	.217	.333
30	.128	.146	.172	.218	.333
60	.128	.147	.173	.221	.333
100	.129	.147	.173	.221	.336

For x-1 > 100, use values for x-1 = 100

Figure 5-22. Critical Values of C^2_{x-1} – Parametric Form of the Cramer-Von Mises Statistic for the AMSAA Model

the three units complete the test without failure; the third fails after 1 hour. Figure 5-24 tabulates the test data for this example. The bias corrected estimate of failure rate is

$$\widehat{\lambda} = \left[1 + \frac{1}{2}\frac{\Sigma T_i^2}{(\Sigma T_i)^2}\right]^{-1} \frac{\sum_{i=1}^{N} X_i}{\sum_{i=1}^{N} t_i}$$

$$= \left[1 + \frac{1}{2}\frac{(5400)}{(120)^2}\right]^{-1} \left(\frac{1}{100}\right)$$

$$= (.842)\,(.010)$$

$$= .00842 \text{ failures/mission}$$

x	T	t_i	C^2_{x-1}
1	125	50	
2	300	200	0.0844
3	550	400	0.0514
4	850	700	0.0424
5	1250	1000	0.0291
6	1600	1500	0.0383
7	1850	1700	0.0170
8	2100	2000	0.0158
9	2350	2200	0.0164
10	2700	2500	0.0189
11	3200	2900	0.0207
12	3750	3500	0.0246
13	4500	4000	0.0252
14	5500	5000	0.0443
15	6500	6000	0.0571
16	8500	7000	0.0549
17	13500	10000	0.1154
18	18750	17000	0.2736
19	21000	20500	0.2827
20	22000	21500	0.2305
21	23000	22500	0.1932
22	24000	23500	0.1678
23	25000	24500	0.1520
24	26250	25500	0.1408
25	27500	27000	0.1370
26	28500	28000	0.1375
27	29800	29000	0.1395
28	31300	30600	0.1385
29	32750	32000	0.1400
30	33750	33500	0.1433

Figure 5-23. Goodness-of-Fit Statistic for the Duane Model

Unit Serial No.	α Missions HR.	Planned Test Time T		T^2	Actual Test Time T		Failures X
		HR.	Missions		HR.	Missions	
1	10	3	30	900	3	30	0
2	10	6	60	3600	6	60	0
3	10	3	30	900	1	10	1
Sum N = 3			120	5400		100	1
Sum Squared			14,400				

Figure 5-24. Hypothetical Set of Test Data

A simpler approximation to the bias correction factor, which gives a slightly conservative result (exact when all T_i are equal), does not require storage of the planned test time. This approximation of equation 5-52 is:

$$\hat{\lambda} = \frac{\sum_{i=1}^{N} X_i}{\sum_{i=1}^{N} t_i} \left(1 + \frac{1}{2N}\right)^{-1} \qquad (5\text{-}53)$$

where N is the number of tests.

Since

$$\left[1 + \frac{1}{2N}\right]^{-1} = \frac{2N}{2N+1} = \frac{6}{7} = 0.857.$$

Then:

$\hat{\lambda} = (0.01)(0.857) = 0.00857$ failures/mission.

The variance of the unbiased estimate is estimated by:

$$\sigma^2_{\hat{\lambda}_i} = \frac{\hat{\lambda}}{\sum_{i=1}^{N} t_i} = \frac{.00857}{100} = .0000857$$

5.4.5.3 Combining Estimates from Different Environments and Test States

Failure rates and variance of estimate are computed individually for each component (i), and combinations of environments (j) and test states (k) (see § 4.1.2). If the j-environments and the k-test states used to test a given component are representative of operational environments and demands, the operational failure rate, λ, of a given component can be estimated by use of equation 5-54, where i is a constant and identifies the component of interest.

$$\hat{\lambda}_i = \sum_j \sum_k \hat{\lambda}_{ijk} \qquad (5\text{-}54)$$

Equation 5-55 can be used in a similar manner to calculate the variance of the estimated λ for component i.

$$\sigma^2_{\hat{\lambda}_i} = \sum_j \sum_k \sigma^2_{ijk} = \sum_j \sum_k \frac{\hat{\lambda}_{ijk}}{t_{ijk}} \qquad (5\text{-}55)$$

Finally, component reliability estimates are computed using the summed failure rate estimates:

$$\hat{R}_i = e^{-\hat{\lambda}_i} \qquad (5\text{-}56)$$

5.4.5.4 Confidence Limits on Failure Rates and Reliability

All statistical estimates based on sampling are subject to uncertainty, therefore, it is necessary to calculate confidence limits. Such calculations are not readily amenable to shifts from one assembly level to another, so confidence limits for components are difficult to translate to higher levels. The equation given here for confidence limits addresses this problem. It is based on normal distirubtion theory and has been corrected to compensate for the fact that failure rate, λ, is usually a small value.

In general, only upper confidence limits are of interest for failure rates. The upper limit for a component is computed as:

$$\lambda_{u_i} = \frac{2\hat{\lambda}_i + (\beta K)^2 \hat{C}_i + \sqrt{4\hat{\lambda}_i (\beta K)^2 \hat{C}_i + (\beta K)^4 \hat{C}_i^{\,2}}}{2} \qquad (5\text{-}57)$$

where

$$\hat{C}_i = \frac{\sigma^2_{\hat{\lambda}_i}}{\hat{\lambda}_i} \qquad (5\text{-}58)$$

and K is the standard normal deviate for specified confidence level (e.g., 0.842 for 80% confidence, 1.282 for 90% confidence).

While normal distirubtion theory is not completely appropriate for small values of λ the procedures given above compensate for that difficulty. However, as the confidence level is reduced toward 50%, an additional modification becomes appropriate. This bias *correction*, called β, is particularly important when few failures have been observed. Thus, the product βK is used rather than K in the equations. For 100α% confidence and X failures, β is computed from the relationship:

$$\beta = \frac{X_u - X}{K\sqrt{X_u}} \qquad (5\text{-}59)$$

where

$$X_u = \chi^2_{\gamma:2x+2}/2$$

Then,

$$\beta = \frac{(\chi^2/2) - X}{K\sqrt{\chi^2/2}}$$

For example, given zero failures and a specified confidence level of 80 percent, it is quickly found, using tables (e.g., Appendix E figure E-1):

$$x_u = 0.5\chi^2_{80:2} = (0.5)(3.219) = 1.6095$$

and

$$\beta = \frac{1.6095 - 0}{0.842\sqrt{1.6095}} = 1.507$$

In figure 5-25, values of β for 80% confidence are listed.

When *no failures have occurred*, the upper limit of failure rate is defined as:

$$\lambda_{u_i} = \frac{(\beta K)^2}{\text{smallest } t_{ijk}} \qquad (5\text{-}60)$$

Number of Failures X	Beta Value	Number of Failures X	Beta Value	Number of Failures X	Beta Value
0	1.5074	10	1.1740	20	1.1297
1	1.3694	11	1.1674	21	1.1270
2	1.3091	12	1.1614	22	1.1244
3	1.2721	13	1.1561	23	1.1220
4	1.2471	14	1.1513	24	1.1197
5	1.2280	15	1.1469	25	1.1175
6	1.2130	16	1.1429	26	1.1155
7	1.2007	17	1.1392	27	1.1136
8	1.1904	18	1.1358	28	1.1117
9	1.1817	19	1.1326	29	1.1100

Figure 5-25. Beta Correction Factors for 80 Percent *Confidence*

λ_u is the larger of equations 5-57 and 5-60 using the smallest t_{ijk} with zero failures.

5.4.5.5 Unequal Numbers of Tests in Multiple Environments

The Rubinstein method can be used to assess reliability when data is from different test states and multiple environments, as shown in the following example:

Two stress environments are defined: high temperature *(h) and vibration (v). In each of* these environments a component is tested non-operating (a) and operating (d). Figure 5-26 summarizes the mission exposure times for the component where α provides the equivalent missions.

Environment	Test States	Mission Exposure Time (Min)	Mission/ Min. α
High Temperature (h)	Non-operating (a) Operating (d)	10.00 0.50	0.10 2.00
Vibration (v)	Non-operating (a) Operating (d)	0.25 20.00	4.00 0.05

Figure 5-26. Mission Exposure times for a Component by Environment and Test States

For the component in high temperature and non-operating (ha) four tests are performed with the following durations (in equivalent missions) and the following results.

Test 1: Component failed after 142 minutes of a scheduled 300 minute test (14.2 equivalent missions).

Test 2: Component did not fail in 300 minutes of testing (30 equivalent missions).

Test 3: Component failed after 147 minutes of a scheduled 300 minute test (14.7 equivalent missions).

Test 4: Component did not fail in 300 minutes of testing (30 equivalent missions).

These tests are summarized on the first line of figure 5-27. Three other sets of tests, in test states hd, va, and vd, were performed on the component. The test data are summarized in figure 5-27. λ_{ijk}'s are calculated using equation 5-53.

ijk	Test Time (Minutes)	α	t_{ijk} (eq. missions)	No. of Tests n_{ijk}	No. of Fail. x_{ijk}	Failure Rate λ_{ijk}
1ha	889.0	0.10	88.9	4-	2	0.0200
1hd	45.45	2.00	90.9	5	1	0.0100
1va	21.425	4.00	85.7	3	1	0.0100
1vd	2000.0	0.05	100.0	3	0	0.0000

Figure 5-27. Test Data Summary.

Estimates of the combined failure rate and reliability are:

$$\widehat{\lambda}_1 = \widehat{\lambda}_{1ha} + \widehat{\lambda}_{1hd} + \widehat{\lambda}_{1va} + \widehat{\lambda}_{1vd}$$

$$= .02 + .01 + .01 + .00 = .04$$

$$R(\text{Mission}) = e^{-.04} = 0.9608$$

Underlying these estimates are assumptions that the component is described by the exponential failure law, and that it is permissible to add the failure rates, (i.e., the failure rates in each environment and test state are independent). The assumptions are set forth and fully discussed in [10].

The method also derives lower reliability confidence bounds (LRCB) for components (and series-parallel groups of components) using an approximate method consistent with the mixed censoring, one-component-at-a-time method of testing (see Section 7).

Point and interval reliability estimates for binomial test data (defective vs non-defective classification of test results) can also be derived using the Rubinstein Method.

5.4.6 Bayesian Methods and Their Statistical Formulation

In principle, all of the component failure distribution models discussed herein can be used for Bayesian reliability assessment. Traditional (frequentist) statisticians consider the parameters of these distributions as fixed, though unknown, and estimable if sufficient test data are available. In Bayesian estimation a degree of belief viewpoint about the possible values the distribution parameters can assume replaces the frequentist's assumption of fixed parameters. This degree of belief can be based on engineering judgment or on previous data in accordance with the rules shown in § 5.4.6.9. It is embodied in the values given "prior" parameters of a "prior" distribution, values which reflect the "weakness" or "strength" of the degree of belief.

As test data become available, the prior distribution is modified by the data to form a posterior distribution by the use of Bayes' Theorem. Bayes' Theorem can be written generally as:

$$P_r(\theta|u) = P_r(u|\theta) \cdot P_r(\theta)/P_r(u) \qquad (5\text{-}61)$$

In reliability applications, θ is an unknown parameter of a failure model or a function of the unknown parameter, such as reliability itself. $P_r(\theta)$ is some prior probabilistic degree of belief about a value or a set of values of the parameter θ, u represents a statistical summary of failure data from a test, such as a combination of ordered test failure times $t_1, t_2, t_3, \ldots t_n$, or the number of failures x among n units tested. $P_r(u|\theta)$ is the conditional probability that u is observed, given a particular value of θ. $P_r(u)$ is the unconditional probability of u, and $P_r(\theta|u)$ is the posterior probability of θ given that u has occurred.

Although Bayes' Theorem can be used as stated with prior probabilities ascribed to discrete values of θ, in many cases of interest prior probabilities are described as a continuous PDF $g(\theta)$ over all possible θ.

The form taken by Bayes' formula when $u = (t_1, t_2, \ldots t_n)$ is a set of times to failure, for instance as in [1]:

$$g(\theta|u) = f(u|\theta)\, g(\theta)/f(u) \qquad (5\text{-}62)$$

where: $f(u|\theta)$ is the joint (PDF) of a sample of size N from $f(t|\theta)$, the sampling PDF of the random variable t (time to failure). $g(\theta)$ is the prior PDF on θ, also called the mixing or the compounding PDF. f(u) is the joint density of the sample observations and is equal to $\int_{-\infty}^{\infty} f(u,\theta)d\theta$. $g(\theta|u)$ is the posterior PDF of θ and is equal to $f(u,\theta)/f(u)$.

5.4.6.1 Priors and Posteriors for Reliability Model Parameters

Selection and justification of a prior is often difficult. In some cases selection has been based on convenience, as in the case of so-called "conjugate priors", distributions such that prior and posterior have the same functional form. Ideally, a prior should be selected on the basis of test data which, if sufficient, permits inferences about the parameter(s). When such data are scarce, more or less arbitrary judgments must be made about the prior if a Bayesian method is to be used. This sometimes leads to the adoption of a "flat or uniform prior", which embodies the concept of minimum information about values the parameter can assume.

5.4.6.2 The Bayesian Binomial Failure Model, Bernoulli Sampling

The Bayesian binomial model corresponds to the frequentist binomial model where n units are tested and x defectives are found in the lot (Bernoulli Sampling). Each unit tested in the Bayesian binomial model has a sampling distribution;

$$f(y|R) = R^y (1\text{-}R)^{1-y}, 0 < R < 1, y = (0,1) \quad (5\text{-}63)$$

where y = 0 indicates a defective unit, and y = 1 a non-defective unit. In the Bayesian binomial model R is not considered fixed, but a random variable, assigned a prior distribution g(R). Several choices of a prior are available. Additional information is given in [9].

5.4.6.3 The Uniform Prior

Selection of a uniform prior on R, expressed as:

$$g(R) = \begin{cases} 1 & 0 \leq R \leq 1 \\ 0 & \text{elsewhere} \end{cases} \qquad (5\text{-}64)$$

presumes complete ignorance, that is, that nothing is known a priori about R, so R can assume any value between 0 and 1 with equal probability.

In this formulation, the sampling distribution of the variable y is:

$$f(y_1, \cdots y_n|R) = R^s(1\text{-}R)^{n-s}$$

where s is the number of successes out of N units on test and is equal to

$$\sum_{i=1}^{n} y_i$$

The posterior pdf of R is $g(R|y_1, \cdots y_n)$ which is a beta distribution with parameters p = s+1 and Q = n-s+1 and is equal to:

$$g(R|y_1, \cdots y_n) = \frac{\Gamma(n+2)}{\Gamma(s+1)\,\Gamma(n\text{-}s\text{-}1)} R^s (1\text{-}R)^{n-s} \quad (5\text{-}65)$$

The posterior mean $E[R|y_1, y_2, \cdots y_n]$ is the mean of the beta distribution, which yields a posterior estimate for R, $\hat{R} = (s+1)/(n+2$ or $\hat{R} = (n\text{-}x+1)/(n+2)$. Notice that the prior mean for R was E[R] = 1/2, and that the posterior estimate of R reevaluates the mean on the basis of the observations.

5.4.6.4 Truncated Uniform Prior

For a truncated uniform prior it is assumed that there is a basis for believing that the component reliability R is at least as large as R_0, but that it can assume any value from R_0 to 1 with equal likelihood. Mathematically, the Prior on R is:

$$g(R) = \begin{cases} \frac{1}{1\text{-}R_0}, & R_0 > 0, R_0 \leq R \leq 1 \\ 0, & \text{elsewhere} \end{cases} \qquad (5\text{-}66)$$

The mean of the posterior PDF of R yields:

$$\hat{R} \equiv E\left\{R|y_1, \cdots y_n\right\} = \frac{s+1}{n+2}\left\{\frac{1-I_{R_0}(s+2, n-s+1)}{1-I_{R_0}(s+1, n-s+1)}\right\} \tag{5-67}$$

where s is the number of successes among n units on test, and I_{R_0} is the cumulative beta distribution evaluated at R_0.

5.4.6.5 The Beta Prior

The uniform prior and truncated uniform prior on R are less often applied than the beta prior, which has two advantages. The beta distribution has two extra fixed parameters, ℓ and m, which allow a greater flexibility in fitting test data, and the beta PDF is the conjugate for binomial test data, i.e., the posterior PDF on R is also a beta distribution. The prior is:

$$g(R,\ell,m) = \frac{\Gamma(\ell+m)}{\Gamma(\ell)\Gamma(m)} R^{\ell-1}(1-R)^{m-1} \tag{5-68}$$

$$\ell, m > 0, 0 \leqslant R \leqslant 1$$

The posterior is:

$$g(\theta|X_1, \cdots X_n) = \frac{\Gamma(\ell+m+n)}{\Gamma(\ell+s)\Gamma(m+n-s)} R^{\ell+s-1}(1-R)^{m+n-s-1} \tag{5-69}$$

$$\ell, m > 0, 0 < R < 1$$

where s is the number of successes out of n units tested. The posterior mean yields an estimate of R:

$$\hat{R} = \frac{\ell+s}{\ell+m+n}$$

Example, Bayesian Method

In the following example, $\hat{R}$ is rewritten in the form

$$\hat{R} = \frac{\ell+s}{\ell+s+m+n-s} = \frac{\ell+s}{\ell+m+s+x}$$

where x is the number of failures. In this expression for $\hat{R}$, ℓ represents the number of prior successes, and m the number of prior failures.

A. *Prior and Posterior Data*

The data, obtained for an ejector gas generator, were:

Prior number of successes: $\ell = 3500$
Prior number of failures: $m = 1.30$
Number of test successes: $s = 90$
Number of test failures: $x = 0.00$

B. *Calculations*

$s' = \ell + s = 3500 + 90 = 3590$
$x' = m + x = 1.30 + 0.00 = 1.30$

(1) *Reliability Point Estimates:*

(a) Prior:

$$\hat{R} = \frac{\ell}{\ell+m} = \frac{3500}{3500+1.30} = .99963$$

(b) Posterior:

$$\hat{R}' = \frac{s'}{s'+x'} = \frac{3590}{3590+1.30} = .99964$$

(2) Reliability one-sided confidence estimates

(a) Prior PDF at $\alpha = .99$:

$$\frac{\Gamma(3501.3)}{\Gamma(3500)\,\Gamma(1.3)} \int_0^{R_L} P^{(3500-1)}(1-P)^{(1.30-1)}dP = 1-.99$$

solving for R_L, $R_L = .99852$

At $\alpha = .95$:

$$\frac{\Gamma(3501.3)}{\Gamma(3500)\,\Gamma(1.3)} \int_0^{R_L} P^{(3500-1)}(1-P)^{(1.30-1)}dP = 1-.95$$

solving for R_L, $R_L = .99900$

At $\alpha = .80$:

$$\frac{\Gamma(3501.3)}{\Gamma(3500)\,\Gamma(1.3)} \int_0^{R_L} P^{(3500-1)}(1-P)^{(1.30-1)}dP = 1-.80$$

solving for R_L, $R_L = .99942$

(b) Posterior PDF:

At $\alpha = .99$:

$$\frac{\Gamma(3591.30)}{\Gamma(3590)\,\Gamma(1.30)} \int_0^{R_L} P^{(3590-1)}(1-P)^{(1.30-1)}\,dP = 1-.99$$

solving for R_L, $R_L = .99855$

At $\alpha = .95$:

$$\frac{\Gamma(3591.30)}{\Gamma(3590)\,\Gamma(1.30)} \int_0^{R_L} P^{(3590-1)}(1-P)^{(1.30-1)}\,dP = 1-.95$$

solving for R_L, $R_L = .99902$

At $\alpha = .80$:

$$\frac{\Gamma(3591.30)}{\Gamma(3590)\,\Gamma(1.30)} \int_0^{R_L} P^{(3590-1)}(1-P)^{(1.30-1)}\,dP = 1-.80$$

solving for R_L, $R_L = .99944$

C. *Calculation Summary*

	Prior	*Posterior*
1. Reliability Point Estimate	.99963	.99964
2. Reliability One-Sided Lower Confidence Estimates:		
$\alpha = .99$	.99852	.99855
$\alpha = .95$	.99900	.99902
$\alpha = .80$	.99942	.99944

5.4.6.6 Empirical Priors

Certain objections to beta priors on the reliability of binomial components have led to the use of empirical priors. Strong beta priors used in conjunction with binomial component data can lead to unrealistic posterior estimates of reliability, when a string of failures occurs. For instance, [35] shows that if 99 prior successes with 100 prior trials are assumed for the prior with a corresponding estimate R = 0.99 for prior reliability, a string of 10 successive failures would yield a posterior estimate $\widehat{R} = 0.90$. This high value of component reliability after such a string of failures is hardly credible. It is also shown that use of very weak priors gives unrealistic LRCB's in the no failure case.

In [42,35] a case is made for the employment of discrete empirical priors, where prior probability assignments are made for discrete ranges of reliability. The posterior reliability in this case is also discrete and empirical, that is, both prior and posterior estimates appear as tabulated functions rather than as analytic expressions. In many cases posterior reliability estimates are more realistic with empirical priors than when beta priors are used with binomial component test data.

5.4.6.7 The Pascal Process

This case corresponds to the negative binomial model. The test sampling process is Pascal and consists in testing units one after the other until x = n - s defectives have been found. The number of units tested n, is a random variable, while x is a constant selected before testing. With a beta prior on R the posterior becomes:

$$g(R \mid n_1 \cdot \cdot\, n_s) = \frac{\Gamma(\ell+m+\Sigma n_i)}{\Gamma(\ell+s)\,\Gamma(m+\Sigma n_i-s)} R^{\ell+s-1}(1-R)^{m+\Sigma n_i-s-1} \tag{5-70}$$

which is also a beta PDF. An estimate of R is the posterior mean:

$$\widehat{R} = \frac{\ell + s}{\ell + m + \Sigma n_i} \tag{5-71}$$

In the above equations, s is the number of required successes or survivals, n_i represents the number of units actually tested to obtain one survival for each i, and Σn_i is

$$\sum_{i=1}^{n} n_i = n.$$

As an example, assume that a total of 15 survivals is required and that a sequence of defectives d, and nondefectives $\bar{d}$, is as follows:

$$\underbrace{\bar{d}\quad d}_{n_1}\ \underbrace{d}_{n_2}\ \underbrace{\bar{d}\quad d}_{n_3}\ \underbrace{d}_{n_4}\ \underbrace{d}_{n_5}\ \underbrace{d}_{n_6}\ \underbrace{\bar{d}\quad \bar{d}\,d}_{n_7}\ \underbrace{d}_{n_8}\ \underbrace{d}_{n_9}$$

$$\underbrace{d}_{n_{10}}\ \underbrace{d}_{n_{11}}\ \underbrace{d}_{n_{12}}\ \underbrace{d}_{n_{13}}\ \underbrace{d}_{n_{14}}\ \underbrace{d}_{n_{15}}$$

then:

$$n_2 = n_4 = n_5 = n_6 = n_8 = n_9 = n_{10} = n_{11} =$$

$$n_{12} = n_{13} = n_{14} = n_{15} = 1$$

$$n_1 = n_3 = 2$$

$$n_7 = 3$$

If $\ell = m = 0$, then

$$\widehat{R} = \frac{s}{\Sigma n_j} = \frac{15}{19} = 0.789.$$

This point estimate of R is the same as for the corresponding frequentist case. If, on the other hand, $\ell = 1$, $m = 2$, where ℓ and m embody some prior knowledge of R, using equation 5-71, we have:

$$\widehat{R} = \frac{\ell + s}{\ell + m + \Sigma n_i} = \frac{1 + 15}{3 + 19} = 0.727$$

5.4.6.8 Exponential Reliability Model, Poisson Process

A Poisson process is characterized by constant failure rate λ. If the prior on λ is selected to be Gamma with parameters τ and φ, then

$$g(\lambda,\tau,\phi) = \begin{cases} \frac{\tau^{\phi}}{\Gamma(\phi)} \lambda^{\phi-1} e^{-\tau\lambda} & \tau,\phi,\lambda > 0 \\ 0 & \text{elsewhere} \end{cases} \qquad (5\text{-}72)$$

Then, the posterior is also Gamma, regardless of whether the test is Type I Censored (fixed test time T) or Type II Censored (fixed number of failures x).

Letting T represent the fixed testing time in Type I censoring, and

$$T = \sum_{i=1}^{n} t_i$$

in Type II censoring where the t_i's represent the time of failure of the ith unit, and x is the number of failures, the posterior on λ is given by:

$$g(\lambda|x) = g(\lambda|T) = \frac{(\tau+T)^{\phi+x}}{\Gamma(\phi+x)} \lambda^{\phi+x-1} e^{-\lambda(\tau+T)} \qquad (5\text{-}73)$$

In either censoring case, an estimate of λ is given by the posterior mean:

$$\widehat{\lambda} = \frac{\phi + x}{\tau + T} \qquad (5\text{-}74)$$

A problem with prior data, such as φ and τ, and actually observed data, such as x and T, is that they may be incompatible. A test of hypothesis at a significance level α (see Appendix C) has been devised for the acceptance or rejection of the prior $\widehat{\lambda}_0 = \phi/\tau$ after x and T have been observed. This test is:

$$\frac{\chi^2\ (2x, \alpha/2)}{2T} < \lambda < \frac{\chi^2\ (2x+2, 1\text{-}\alpha/2)}{2T} \qquad (5\text{-}75)$$

If $\widehat{\lambda}_0$ lies between the indicated $\chi^2{}_{(\;)}/2T$ limits, it is accepted, otherwise it is rejected. Only after this test has been performed and $\widehat{\lambda}_0$ has been accepted is it permissible to compute an upper limit on failure rate, using prior data, as follows:

$$\lambda_u = \frac{\chi^2\ (2[\phi+x]+2, 1\text{-}\alpha)}{2(\tau+T)} \qquad (5\text{-}76)$$

Also:

$$R_L = e^{-\lambda_u} \qquad (5\text{-}77)$$

Example, Bayesian Process, Poisson:

Assume that the prior failure rate has been estimated to be $\widehat{\lambda}_0 = \phi/\tau = 2/600 = .00333$. Assume also that the failure data observed later was the same as given in the example of § 5.4.1.1.1 with x = 8 failures and T = 555 hours. Selecting α = 0.2, we have:

$$\frac{\chi^2\ (16, 0.1)}{1110} < \lambda < \frac{\chi^2\ (18, .9)}{1110} \qquad (5\text{-}78)$$

or:

$$0.00839 < \lambda < 0.02341$$

Since $\widehat{\lambda}_0$ is outside the interval, the prior data and the test data are deemed to come from separate populations and should not be combined. Both the prior and the test data should be examined for error. Unless an error is found in the test, normal practice is to discard the prior and accept the test data.

5.4.6.9 Prior Strength

The fixed parameters of a prior distribution embody its strength. For instance, the posterior estimate of λ, $\hat{\lambda}$, has been given as $(\phi+x)/(\tau+T)$. The corresponding prior $\hat{\lambda}_0$ is readily seen to be ϕ/τ from the form of the conjugate gamma.

ϕ can be construed as pseudo-prior test failures, and τ as pseudo-prior test time. Then the prior is weak when small values of ϕ and τ are selected, and strong when large values of these fixed parameters are used. Strong priors are harder to "wash out" as test data accrue.

Predicted reliability can be used as a basis for establishing a prior best estimate. It fixes the ratio ϕ/τ and the selection of τ establishes prior strength and defines the entire prior distribution. If τ is large, the prior is strong and relatively difficult for test data to modify. Small τ yields a weak prior easily discounted by test data.

Figure 5-28 presents informal rules for choosing τ. They have been found by experience to yield acceptable results for many types of subsystems. The factor M is total expected program test time expressed in equivalent missions. The factor t is total historical test time.

To use categories 1-4 of figure 5-28, it is necessary to analyze the proposed test program for the component under study to determine the expected number of equivalent missions, M, that will be derived from testing. After this number has been determined, select the category that best characterizes the information available for the component. The number of expected missions is then multiplied by the accompanying fraction to obtain the number of prior missions τ. The failure rate that has been predicted becomes the estimated prior failure rate $\hat{\lambda}_0$. For categories 1-4 of the figure, the product of the multiplier and M provides the number of prior missions τ. In categories 5-8 of the table, prior data consist of a total number of equivalent missions τ. This completes the selection of the prior parameters ϕ and τ; $\hat{\lambda}_0 = \phi/\tau$, so that ϕ, which is defined as the number of prior failures, is uniquely determined when τ is selected. It should be noted that ϕ need not be a whole number.

Two key points should be noted when using these techniques to set prior parameters. When the prior information is somewhat subjective (Categories 1-4) the weight it will bear in the final joint estimate will vary from 1/32 to at most 1/4 the weight of objective test data. This is a control against overemphasis of a prior judgement that may be somewhat in error. When frequency data are used fully, (Category 8), the Bayesian framework, although still formally employed, is equivalent to treating prior information as though it were early objective test information on the component in question.

Category	Description	τ
1	Informed Qualitative Judgement	1/32M
2	Based Primarily upon Generic Part Handbook Data	1/16M
3	Partly based upon Generic Part Handbook Data; partly upon Similar Part Handbook Data	1/8M
4	Based Primarily upon Similar Part Handbook Data	1/4M
5	Frequency Data Solely: Similar Parts, Similar Applications	0.1t
6	Frequency Data Solely: Identical Parts, Similar Applications	0.2t
7	Frequency Data Solely: Similar Parts, Identical Applications	0.3t
8	Frequency Data Solely: Identical Parts, Identical Applications	1.0t

Figure 5-28. Decision Rules for Estimating Prior Strength

5.4.6.10 Point and Interval Estimation of Bayesian Parameter

In Bayesian analysis, it is assumed that [10]:

a) A prior PDF on θ: $g(\theta)$, has been selected. It is desired to estimate the value of θ from the observations $(t_1, t_2, \cdots t_n)$ each drawn from $f(u|\theta)$. Many different estimates exist, but a commonly used method to obtain a "best" estimate is to require that the loss function $(\hat{\theta}-\theta)^2$ be minimized. This leads to a point estimate of θ, $\hat{\theta}$, by the means of the posterior PDF. Therefore:

$$\hat{\theta} = E(\theta, u) = \int_{-\infty}^{\infty} \theta g(\theta|u)\, d\theta \qquad (5\text{-}70)$$

To obtain an upper bound θ_u or a lower bound θ_L for θ at confidence $1 - \alpha$, it is only necessary to calculate:

$$\int_{\theta_u}^{\infty} g(\theta|u)\, d\theta = \alpha \text{ or } \int_{-\infty}^{\theta_L} g(\theta|u)\, d\theta = \alpha \quad (5\text{-}71)$$

Other interval estimates for θ are also easy to obtain by similar integrals. Once the posterior is known in Bayesian methods that use discrete empirical priors. point and interval estimates of the parameter of a failure model can be obtained directly from the discrete posterior parameter histogram.

b) One random observation is obtained from the prior distribution on θ.

c) A random sample is drawn from $f(u|\theta)$.

Figure 5-29 provides a summary chart of Bayesian methods for assessing R, MTBF, failure rate, and other distributional parameters.

5.4.6.11 Assessing Validity of Bayesian Methods in Presence of Erroneous Priors

In [43] a study was performed to show the influence of erroneous prior point estimate and strength on Bayesian reliability estimates. The errors studied in the model were: a) incorrect distributional form of the prior, b) incorrect mean for the prior, and c) incorrect number of failures, set to be zero or three times the expected number.

The errors were investigated by Monte Carlo simulation, using weak, moderate and strong priors. It was concluded that when the test data distribution is correctly identified, but the strength of the prior is incorrectly chosen (*usually too strong*), a Bayesian prediction may seriously overstate reliability early in the program. The degree of overstatement is both a function of the size of the error and the assigned strength or degree of belief in that prior. Error in the estimated mean is particularly important. Without reasonable assurance that a prior, even if weak, is positioned close to the subsequent performance of the equipment under test, Bayesian methods are likely to give misleading results. Great caution should therefore be exercised in utilizing Bayesian methods. It is important, in particular, not to use Bayesian methods as an excuse for not collecting data. The penalty is, of course, that the prior estimate will be given too much final credence, when in fact, it should be used only as an initial educated guess to be confirmed or denied in due time by newly collected data.

5.4.6.12 Summary Chart for Bayesian Methods

Figure 5-29 provides a summary chart of Bayesian methods for assessing R, MTBF, failure rate, and other distributional parameters.

An explanation of the headings of figure 5-29 is:

Heading	*Meaning*	*Reference/ Paragraph*
Exhibit #	Refers to corresponding classical models	Figure 5-11
Item Classification	Refers to corresponding classical models	Figure 5-11 and § 5.4.1
Failure/Reliability Model	Refers to corresponding classical models	Figure 5-11 and § 5.4.1
Raw Data & Type of Test	Refers to corresponding classical models	Figure 5-11 and § 5.4.1
Assumed Prior	Prior Distribution	§ 5.4.6
Posterior	Posterior Distribution	§ 5.4.6
Point Estimates (Posterior Mean)	Point Estimates	§ 5.4.6
One-Sided Confidence Limits	Interval Estimates	§ 5.4.6

5.4.7 Adjustments to Reliability Estimation, Derating and Uprating

Systems that operate in space, undersea or in other extreme environments may be tested in ground test environments which may not be able to simulate the mission environment realistically. For example, it may not be possible to simulate the heating and stresses experienced by a re-entry vehicle at maximum

Exhibit #		Item Classification	Failure/Reliability Model	Raw Data & Type of Test	Assumed Prior	Posterior
#	Corresponding Classical From Figure 5-11	Classification By Performance			G_i	G_f
1	1	All hardware elements/components, but more particularly one-shot devices and items which can be classified as "defective" or "non-defective."	Bernoulli Process Binomial Sampling $g(x)=\binom{n}{x}(1-R)^x(R)^{n-x}$ R=R	n items on test, x defectives observed	$g_i(R)$, uniform on R. $\begin{cases}1 & 0<R<1\\0 & R>1, R<0\end{cases}$	$g_f=\frac{\Gamma(n+2)R^s(1-R)^{n-s}}{\Gamma(s+1)\Gamma(n-s+1)}$ (<u>NOT</u> conjugate)
.1	1	All hardware elements/components, but more particularly one-shot devices and items which can be classified as "defective" or "non-defective."	Bernoulli Process Binomial Sampling $g(x)=\binom{n}{x}(1-R)^x(R)^{n-x}$ R=R	n items on test, x defectives observed.	$g_i(R)$ uniform on 1-R $\begin{cases}\frac{1}{1-R_o}, & R_o<R<1, R_o>0\\0 & \text{elsewhere}\end{cases}$	$g_{f_o}=g_f\Big/\int_{R_o}^{1}g_f dR$ with g_f defined as in Exhibit #1 (<u>NOT</u> conjugate)
.2	1	All hardware elements/components, but more particularly one-shot devices and items which can be classified as "defective" or "non-defective."	Bernoulli Process Binomial Sampling $g(x)=\binom{n}{x}(1-R)^x(R)^{n-x}$	n items on test, x defectives observed	$g_i(R, \ell, m)$, β on R $g_i(R,\ell,m)=\frac{\Gamma(\ell+m)R^{\ell-1}(1-R)^{m-1}}{\Gamma(\ell)\Gamma(m)}$ $\ell, m>0, 0<R<1$	$g_f=\frac{\Gamma(\ell+m+n)R^{\ell+s-1}(1-R)^{m+n-s+1}}{\Gamma(\ell+s)\Gamma(m+n-s)}$ $\ell, m>0, 0<R<1$ (conjugate)

Part 1 of 2

Figure 5-29. Bayesian Methodology, Summary for Assessing Component R, MTTF, Failure Rate & Distributional Parameters

Point Estimates (Posterior Mean)			One-Sided Confidence Limits (From Posterior PDF) at 100(1 - α)% Confidence			
Reliability	MTTF	Failure Rate	Reliability	Mean Time To Failure	Failure Rate	Posterior Variance
$\hat{R}$	$\hat{\theta}$ (Or Other Specified Parameters)	$\hat{\lambda}$	R_L	θ_L (Or Other Specified Parameters)	λ_U	(Also Posterior Risk) V
1 1 $\frac{s+1}{n+2}$ or $\frac{n-x+1}{n+2}$	N/A	N/A	R_L determined from: $\int_{R_L}^{1}$ (Posterior)dR=100α%	N/A	N/A	$\frac{(s+2)(n-s)}{(n+2)^2(n+3)}$
1.1 1 $\frac{(s+1)}{(n+2)} \frac{1-I_{R_o}(s+2, n-s+1)}{1-I_{R_o}(s+1, n-s+1)}$	N/A	N/A	R_L determined from: $\int_{R_L}^{1}$ (Posterior)dR=100α%	N/A	N/A	
1.2 1 $\frac{\ell+s}{\ell+m+n}$	N/A	N/A	$R_L = \left[\left(\frac{m+n-s}{\ell+s}\right)\left(\frac{1}{Fd}\right)+1\right]^{-1}$ F with s(ℓ+s), 2(m+n-s) degrees of freedom.	N/A	N/A	$\frac{(\ell+s)(m+n-s)}{(\ell+m+n)(\ell+m+n+1)}$

Part 2 of 2

Figure 5-29. Bayesian Methodology, Summary for Assessing Component R, MTTF, Failure Rate & Distributional Parameters (Continued)

Exhibit #		Item Classification	Failure/Reliability Model	Raw Data & Type of Test	Assumed Prior	Posterior
#	Corresponding Classical From Figure 5-11	Classification By Performance			G_i	G_f
2	2	**All hardware elements/components, but more particularly one-shot devices and items which can be classified as "defective" or "non-defective."**	Bernoulli Process Pascal Sampling	Number of items on test n is variable. The number of "non-defectives" s is fixed.	β on R $g_i(R,\ell,m)=\frac{\Gamma(\ell+m)\, R^{\ell-1}(1-R)^{m-1}}{\Gamma(\ell)\,\Gamma(m)}$ $\ell,m>0,\ 0<R<1$	$g_f=\frac{\Gamma(\ell+m+\Sigma n_j)\, R^{\ell+s-1}(1-R)^{m+\Sigma n_i-s-1}}{\Gamma(\ell+s)\,\Gamma(m+\Sigma n_i-s)}$
3	3	**All hardware elements/components, but more particularly continuously operating electro & electro-mechanical devices.**	Poisson Process. $g(x)=e^{-\lambda t}(\lambda t)^x/x!$ Exponential Reliability $R(t)=e^{-\lambda t}$	Type I censoring test. Testing time t_o is fixed. x is the number of failures observed for a (replaced) item on test.	Gamma on λ, the failure rate. $g(\lambda,\alpha\beta)=\begin{cases}\frac{\alpha^{\beta}\lambda^{\beta-1}e^{-\alpha\lambda}}{\Gamma(\beta)} & \alpha,\beta,\lambda>0\\ 0 & \text{elsewhere}\end{cases}$	$g_f=\frac{(\alpha+t_o)^{\beta+x}\lambda^{\beta+x-1}e^{-\lambda(\alpha+t_o)}}{\Gamma(\beta+x)}$ (conjugate)
4	4	**All hardware elements/components, but more particularly continuously operating electro & electro-mechanical devices.**	Poisson Process $g(x)=e^{-\lambda t}(\lambda t)^x/x!$ Exponential Reliability $R(t)=e^{-\lambda t}$	Type II censoring test. Testing time is variable. Test terminates when a (replaced) item has failed x times.	Gamma on λ, the failure rate. $g(\lambda,\alpha\beta)=\begin{cases}\frac{\alpha\beta\,\lambda^{\beta-1}e^{-\alpha\lambda}}{\Gamma(\beta)} & \alpha,\beta,\lambda>0\\ 0 & \text{elsewhere}\end{cases}$	$g_f=\frac{(\alpha+\Sigma t_i)^{\beta+x}\lambda^{\beta+x-1}e^{-\lambda(\alpha+\Sigma t_i)}}{\Gamma(\beta+x)}$

Part 1 of 2

Figure 5-29. Bayesian Methodology, Summary for Assessing Component R, MTTF, Failure Rate & Distributional Parameters (Continued)

Point Estimates (Posterior Mean)			One-Sided Confidence Limits (From Posterior PDF) at 100(1-α)% Confidence			
Reliability	MTTF	Failure Rate	Reliability	Mean Time To Failure	Failure Rate	Posterior Variance
$\hat{R}$	$\hat{\theta}$ (Or Other Specified Parameters)	$\hat{\lambda}$	R_L	θ_L (Or Other Specified Parameters)	λ_U	(Also Posterior Risk) V
2 2 $\frac{\ell+s}{\ell+m+\Sigma n_i}$	N/A	N/A	$R_L = \left[\left(\frac{m+n-s}{\ell+s}\right)\left(\frac{1}{F_\alpha}\right)+1\right]^{-1}$ F with $2(\ell+s)$, $2(m+n-s)$ degrees of freedom.	N/A	N/A	
3 3 $e^{-\hat{\lambda}t}$	$\frac{\xi+t_o}{\beta+x}$	$\frac{\beta+x}{\xi+t_o}$	$e^{-\lambda_u t}$	$\theta_L = 1/\lambda_u$	Obtained from the solution to $\int_o^{\lambda_u} (Posterior) d\lambda = 1-\alpha$ or $\lambda_u = \chi^2_{2(\beta+x),\, 1-\alpha}$	$\frac{\beta+x}{(\xi+t_o)^2}$
4 4 $e^{-\hat{\lambda}t}$	$\frac{\xi+\Sigma t_i}{\beta+x}$	$\frac{\beta+x}{\xi+\Sigma t_i}$	$e^{-\lambda_u t}$	$\theta_L = \frac{1}{\lambda_u}$	Obtained from the solution of: $\int_o^{\lambda_u} (Posterior) d\lambda = 1-\alpha$ or: $\lambda_u = \chi^2_{2(\beta+x),\, 1-\alpha}$	$\frac{\beta+x}{(\xi+t_o)^2}$

Part 2 of 2

Figure 5-29. Bayesian Methodology, Summary for Assessing Component R, MTTF, Failure Rate & Distributional Parameters (Continued)

Exhibit #		Item Classification	Failure/Reliability Model	Raw Data & Type of Test	Assumed Prior	Posterior
#	Corresponding Classical From Figure 5-11	Classification by Performance			G_i	G_f
5	9	All hardware components/elements, particularly those which display increasing or decreasing Hazard Rates, such as Structural Elements and Mechanical Devices subject to wear or tear and exhibiting a weakest spot.	Two Parameter Weibull. $R(t) = \begin{cases} e^{-\left(\frac{t}{\alpha}\right)^{\beta}} & : t, e > 0 \\ 0 & t < 0 \end{cases}$ where β is a shape parameter and α is a scale parameter. $R(t) = e^{-kt^{m+1}/m+1}$	Type II Censoring.	β is assumed known. The prior is on $\xi = \alpha^{\beta}$, and is an inverted gamma. $g_i = (\Gamma(\alpha)\beta(\alpha))^{-1}\ \xi^{-\alpha-1} \exp(-1/\beta\xi)$ with $(\alpha, \beta, \xi > 0)$	The posterior is an inverted gamma with parameters. $\alpha^1 = \alpha + x$ and $\beta^1 = \beta/\left[(\sum_{i=1}^{n} t_i^{\beta} + (n-x)t_x^{\beta})\beta + 1\right]$ (conjugate)

Part 1 of 2

Figure 5-29. Bayesian Methodology, Summary for Assessing Component R, MTTF, Failure Rate & Distributional Parameters (Continued)

Point Estimates (Posterior Mean)			One-Sided Confidence Limits (From Posterior PDF) at 100 (1-α)% Confidence			
Reliability	MTTF	Failure Rate	Reliability	Mean Time To Failure	Failure Rate	Posterior Variance
$\hat{R}$	$\hat{\theta}$ (Or Other Specified Parameters)	$\hat{\lambda}$	R_l	θ_l (Or Other Specified Parameters)	λ_{ll}	(Also Posterior Risk) V
5 9 With β known, Reliability is estimated as follows: Let: $W=\left[\sum_{i=1}^{n} t_i^{\beta}+(n-x)t_x^{\beta}\right]$ and $R^*(t) = e^{-t^{\beta}}$ then: $\hat{R}^*(t)=\left[\frac{w+(1/\beta)}{w+(1/\beta)(t)}\right]^{\alpha+x}$	With β known, parameter $\xi = \alpha^{\beta}$ is estimated as follows: Let: $W=\left[\sum_{i=1}^{n} t_i^{\beta}+(n-x)t_x^{\beta}\right]$. then: $\hat{\xi}=(w+1/\beta)/(\alpha+x-1)$			For parameter $\xi=\eta^{\beta}$, obtained from solution to: $\int_0^{\xi_u}(\text{Posterior})d\xi=1-\alpha$		With: $W=\left(\sum_{i-1}^{n} t_i^{\beta}+(n-x)t_x^{\beta}\right)$. Then: $V=\left(\frac{W+1/\beta}{\alpha+x-1}\right)^2 \cdot \frac{1}{\alpha+x-2}$

Part 2 of 2

Figure 5-29. Bayesian Methodology, Summary for Assessing Component R, MTTF, Failure Rate & Distributional Parameters (Continued)

deceleration. Derating has been used to address such problems. As stated in [27] "Data for the generation of derating curves come from several sources, such as life tests of component parts, system life tests, or field operation." A family of curves of failure rate versus thermal stress is derived, with electrical stress as a parameter of each curve. Given mission thermal and electrical stresses, failure rates can be derived from the curves.

For state-of-the-art systems, derating curves for particularly stressed environments, such as re-entry, may not exist, or may be based on engineering studies or a few readings of thermal and stress sensors in flight tests. Under these conditions, the curves may be highly conjectural and subject to appreciable errors. Moreover, the failure process for stresses outside the range of observation may be known only from a set of possibly inaccurate curves.

In [28] k-factors are introduced for derating components. They are defined as weighting factors used to convert time in ground test into pseudo-flight test time. To make the k-factors as realistic as possible, it is necessary to have test data from both ground tests and flight tests. The accuracy of the derating method is dependent on the amount of flight test failure data available. See [30], [31], [32] for applications of k-factors to reliability estimation.

Uprating may apply when systems are purposely stressed in testing beyond the nominal stresses for which reliability indices are sought but not beyond system design limits. Accelerated testing is discussed in [9] with examples of models available in the literature to relate stress factors to certain reliability measures. The models presented are:

a. Power Rule Model

The Power Rule Model [9] can be derived via considerations of kinetic theory and activation energy; where the item MTBF is inversely proportional to the Pth power of applied stress:

$$\theta = \frac{K}{S^P} \qquad (5\text{-}72)$$

where K is a constant to be estimated, P is the power of applied stress, also to be estimated and S is stress. Point and interval estimates of K and P are obtained and used to make inferences about θ at mission stress.

b. Arrhenius Reaction Rate Model

The Arrhenius Reaction Rate Model [9] has applicability to semiconductor materials. It is given by:

$$\lambda = e^{(A-B/T)} \qquad (5\text{-}73)$$

where λ is the hazard rate at absolute temperature T, and A and B are empirical parameters to be estimated from multiple sets of test data.

c. Eyring Model for a Single Stress [9].

In this model, the hazard rate, λ, from the Arrhenius Reaction Rate Model [9] is related to operating temperature T by:

$$\lambda = Te^{(A-B/T)} \qquad (5\text{-}74)$$

d. Generalized Eyring Model [9]

This model is applicable to items subjected to two types of stresses, a thermal stress, T, and a non-thermal one, such as an electric voltage, V. The hazard rate, λ, is then given by:

$$\lambda = AT\,e^{[CV + (DV-B)/KT]} \qquad (5\text{-}75)$$

A, B, C, and D are empirical constants to be estimated.

Use of these models to develop derating and uprating data has often proven difficult [33].

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Section 6
SOFTWARE EVALUATION

This section examines the nature of software errors and describes methods of assessing software reliability which have been tested in service. Software is playing an increasingly critical role in military and industrial systems and software errors can be the cause of system failures.

6.1 DEFINITION OF SOFTWARE

Software has been defined[1] as: written or printed data, such as programs, routines, and symbolic languages, essential to the operation of computers.

Another more general definition[2] has been quoted in [3] as, "Software is information that is: a) structured with logical and functional properties, b) treated and maintained in various forms and representations during its life cycle, c) tailored for machine processing in its fully developed state."

The tangible elements on which software is stored, printed or displayed, such as magnetic or paper tapes, disks, or punched cards, are not considered software, but part of the system hardware.

The distinction between intangible software and the tangible media on which it is recorded is useful for purposes of reliability analysis because the storage media are amenable to treatment by conventional hardware reliability methods, while the intangible software displays characteristics that preclude such methods.

6.2 SOFTWARE FAILURES

Software does not break down as hardware components do when operating or stored for any length of time. Yet software failures in operating systems often occur. Since software failures often occur randomly, and in order to understand how this random occurrence of software failures takes place, it is necessary to review briefly certain characteristics of software and its failure modes, which are not exhibited to the same extent by hardware.

A major difference between hardware and software, which seems at first to preclude a probabilistic definition of software reliability, is the fact that software is unchangeable during repeated operation, in contrast to hardware which may exhibit degradation leading to failures or random operational failures best described in probabilistic terms.

Software, once delivered for operation, does not degrade due to wear or fatigue. But whether stored on tape, disk or cards, software command and control and application programs are identical to themselves until deliberately changed. Very occasionally, a tape or disk may be subjected to a stray magnetic field which may damage the stored data or instructions, but by the intangible definition of software, this is considered a hardware failure.

Duplicate software systems yield identical results, whether correct or not, when operated with a particular set of inputs. Thus, replication does *not* confer the reliability benefits of redundancy as is true for hardware, and there is no component variability to contend with. Moreover, physical environment does not affect the performance of intangible software.

In complex programs conditional branching (IF, GOTO, THEN, ELSE, . . . in FORTRAN, PL/1, or ALGOL) often reflects the whole gamut of possible decisions which a human might make under various contingencies in an operational situation. The loops and feedback paths in even the simplest programs lead rapidly to astronomically large numbers of contingencies, which cannot be enumerated or checked individually by even the fastest computers. As an example,

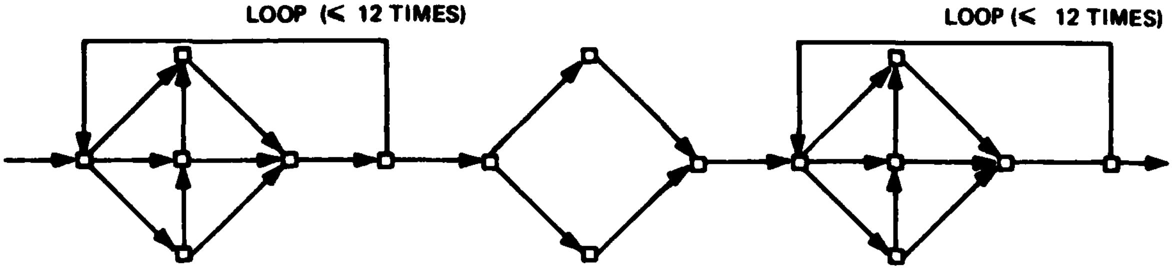

Figure 6-1. "Simple" Logical Flow Chart

the simple program flow-chart[4] of figure 6-1 can be shown to contain about 10^{20} distinct paths. If a computer could check one billion paths per second, well over 3,000 years would be required to check the program exhaustively.

When operations begin, with real, as opposed to constructed, input data, there is no way to know precisely which of the many possible logical paths will be exercised. If the logic in the exercised path is defective at any point, a software failure may occur. Experienced software designers recognize these facts and seek to limit the effects of errors, since the number of errors can be reduced, but generally all errors are never totally eliminated.

Military software depends on numerical algorithms to solve linear or non-linear integro-differential equations of dynamics necessary to steer maneuverable systems, evade enemy anti-missile systems, filter redundant and error-contaminated multiple-sensor data, etc. These algorithms are only approximations of physical laws. In many cases, real-time computational speed constraints compel the use of simplified algorithms, which are in effect approximations of approximations of the physical laws. While these algorithms are supposed to be valid for a specified range of operational conditions, they may not be valid for all conditions within the range, thus a software error may occur. The following example is from an earth orbital simulation program which was "thoroughly" verified and performed flawlessly until, years later, North polar trajectories were specified. The program produced trajectories near the North Pole exhibiting strange positional errors that could not arise from the slowly varying value of gravity over the earth's surface. It was found that the algorithm used for trajectory calculation made use of the tangent of latitude, which either overflowed the computer register or was not calculated accurately enough for values of latitude close to 90°.

Software may interface with other programs or computers, receive data from input sensors, service hundreds of users on an interrupt basis in timesharing systems, drive tapes, disks, plotters or CRT's, and be employed in a multiprocessor environment, with added time phasing and logical complexities, thus encounter many opportunities for failure.

6.2.1 Software Failure Modes and Random Occurrences of Software Failures

Examples of software error modes and some of their possible causes are given below:

a. One of the logical paths is in error for certain operating modes or inputs. This may be due to a coding error, misunderstanding by the programmer of the required logic, or an error in the program performance specification.

b. The computations in application programs are wrong for certain functional arguments or input parameters. The cause may be error in coding, in units defined, in algorithms used or in numerical techniques. Sometimes the analytical expression which the programmer started with is incorrect, and the basic mathematics is in error.

c. A program "hangs up" when directing the activities of multiprocessing equipment. The cause may be found in an error in subroutine calls, programmed interface, or timing and phasing of interrupts.

d. A program displays none of the failure modes cited above, but aborts nevertheless. The cause may be either internally computed quantities become too numerous to be accommodated in core memory, that the phasing of overlaid programs is incorrect, that the supporting compiler is in error, or simply that a very large or very small quantity ($> 10^{99}$ or $< 10^{-99}$, for instance) overflows or exceeds the permissible limits of the hardware or compiler.

e. A program does not hang or fail but it keeps running without doing anything. Under a particular set of input variables or command instructions, the program may be looping endlessly along a faulty logical path.

This brief list of software error modes is by no means exhaustive, but it gives an idea of the kinds of errors often encountered. It also helps to explain the basis of the random distribution of software failures in time. Errors in operational programs are latent; they manifest as failures only when certain combinations of input parameters, commands, options, or data exercise the defective parts of the program. Under a large variety of circumstances, these inputs may be considered to be random sets from all possible sets of inputs. Random sets of inputs, in turn, cause randomly distributed failures in the corresponding outputs. These random output failures which can be analyzed statistically, constitute the statistical basis for the concept of reliability as applied to software.

There is unfortunately no concensus in the literature as to definitions or distinctions among commonly used terms such as software bugs, errors, faults, or failures. In this manual, the term error is used to denote any latent or hidden defect in software.

A software failure is the occurrence or revelation of a software error. A failure can be obvious as when the computer stops operating, or more subtle as when the results of a computation appear suspiciously large or small to an analyst and are verified to be in error by a percentage which may be of little significance in some applications, but may be critical in others (e.g., 2% of the true value).

When an error has been uncovered in a program, it is generally corrected before testing resumes. In the process of correcting the program, other errors may be found. Frequently, a new error is introduced during the correction process. This happens so frequently that some statistical software models have attempted to quantify this source of programming errors.

Some errors are not immediately corrected in the high level language used to write the program, but are corrected for expediency at a lower language level after the high level language has been assembled or compiled. Such a correction is termed a "patch." As the number of patches increases, the likelihood of introducing errors into a program increases greatly. This has become such a problem that MIL-STD-1679 (NAVY)[9] specifies that the total number of patch words in a program shall not exceed 0.005 times the total machine instruction words in the program.

The three terms, Criticality, Severity, and Priority are in common use to describe the impact of software errors on a task or mission. For example, MIL-STD-1679 (NAVY)[9], employes Priority as the technique for error classification. There are five levels of Priority in MIL-STD-1679 (NAVY)[9] which are defined as:

- Priority 1 is assigned to an error that *prevents the accomplishment* of an operational or mission-essential function . . . or which jeopardizes personnel safety.
- Priority 2 is assigned to an error that *adversely affects* the accomplishment of an operational or mission essential function . . . and for which no alternative work around solution exists . . . or which interferes with an operator . . . so as to degrade performance . . . etc.
- Priority 3 is the same as Priority 2, except that there is a reasonable work-around solution.
- Priority 4 is assigned to an error that is an operator inconvenience or annoyance and does not affect a required operational or mission essential function.
- Priority 5 is assigned to all other errors.

6.3 QUANTITATIVE DEFINITION OF SOFTWARE RELIABILITY

Commonly found quantitative definitions[2,5,6,7] for software reliability cover a wide spectrum of concepts. The most useful closely parallel the definition of hardware reliability: software reliability is the probability that a given software program will operate without failure for a specified time in a specified usage environment (i.e., using actual mission data).

6.4 SOFTWARE RELIABILITY PREDICTION

While the literature[5,6,35-39] pertaining to software prediction is extensive, as yet no accurate and generally applicable method has been validated to predict the reliability or availability of software. There are no accepted instruction error rates analogous to the piece part failure rates of MIL-HDBK-217[40], from which software reliability can be predicted. Attempts to derive such elemental rates on the basis of selected attributes of a particular program, such as its "maturity level"[35] or "complexity"[39], have been inconclusive to date. In fact, studies have shown that the most complex modules in a software system frequently contain the least errors. But this finding could be explained by the fact that software managers often assign the most difficult modules to the most experienced programmers.

6.5 SOFTWARE RELIABILITY MEASUREMENT

Software reliability measurement can begin as soon as the software module or program completes initial debugging, but should certainly encompass formal software validation. Reliability is normally measured during acceptance testing before the software is turned over to the user to determine if reliability requirements have been met. This value can also be used to determine the effect on reliability of different development and testing tools and techniques. The measurement also allows a forecast of when testing will be completed and whether a reliability goal can be met. The measurement must take into account differences from the operational environment including test data selection and reliability growth.

Software failure rate is not a constant parameter, but decreases continuously as a result of progressive error detection and removal. The failure rates of interest are the rates observed at various points in the development program, and also the rate forecasted to apply at the beginning of system deployment or service.

The very validity of software reliability measurement methods in existence today is still strongly debated, with certain authors[41] quite opposed to the development of software reliability measures patterned after hardware reliability measures and with methods which attempt to combine hardware and software "failures" in assessing system reliability. Among others who accept a more conventional view of software reliability, there are fundamental differences of opinion about the form of the hazard rate $h(t)$ in software reliability. According to Myers[6], "Proponents of the constant $h(t)$ agree that the inputs appear to be random because the input domain is so large. However, others argue that $h(t)$ increases during the time between errors, using the rationale that the program's inputs gradually close in on the remaining errors. There are others[10] who believe that $h(t)$ decreases with time, arguing that the longer the program runs without encountering an error, the lower the probability of encountering one. Based on the earlier axiom that every time an error is encountered, the probability of encountering one increases, one could postulate that $h(t)$ decreases between errors and it increases whenever an error is detected."

The Duane model which is consistent with a decreasing hazard rate seems to show promise of being applicable to many kinds of software. It has, therefore, been selected for description in this section. In case it proves to give poor forecasts for a particular software project, then other methods such as Shooman's[13,14], Jelinski-Moranda's De-Eutrophication[15,16], Lloyd-Lipow's Modified De-Eutrophication[17, 18], Jelinski-Moranda's Geometric De-Eutrophication[19], Shick and Wolverton's[22,23], Littlewood and Verralls[10] (which reflects programming environmental factors), may be tried.

The Duane Growth Model[26] is a non-homogeneous Poisson process which has been used to model the improvement of many industrial activities. It has recently been applied with success to software reliability[27]. Its functional form is

$$\lambda_\Sigma = \lambda_1 T^{-\alpha} \qquad (6\text{-}1)$$

where

λ_Σ = observed cumulative failure rate (total failures ÷ total running time)

λ_1 = estimated initial failure rate (λ at $T=1$)

T = Σt = observed total operating time (hours, cycles or missions)

α = estimated growth rate parameter

Alternatively, the model can be expressed in terms of MTBF, defined as λ^{-1}.

The model is fitted to the data by computing the value of λ_Σ at each successive software failure and plotting the data points λ_Σ and T on log-log paper (or log λ_Σ and log T on arithmetic paper). Data described by the Duane model will invariably show acceptable linearity as measured by standard correlation indices (e.g. r statistic), and will become progressively more linear because of the smoothing process inherent in plotting successive averages of an increasing sample. This accounts for the excellent visual fit achieved in most Duane plots.

In some programs it has been noted that very early test data (i.e., the first few failures) do not exhibit satisfactory linearity on a Duane plot. This effect, when it occurs, results from the limited ability of early verification testing to simulate operation of the software in a fully developed system. Thus, early verification testing may be viewed as a "benign use environment." Some analysts omit this early data from reliability growth computations; others have successfully fitted the Duane model to such data by applying a constant multiplier, determined empirically, to test time accrued in early tests. When this is done, T/k replaces T in the model. Usually k falls in the range $1 < k < 5$. Contractors may adopt either approach as applicable, but must fully justify the validity of their reasoning in doing so.

The location parameter λ_1 and the slope α of a Duane curve are estimated directly from the graphic plot. Forecasts are made by linear extrapolation of the best-fit line to give a point estimate of λ_Σ for future values to T. This is a valid descriptive statistic under the assumption that growth will continue in the future as in the past. Typical values of α range from .2 to .7 with the average being close to .4.

When software experiences reliability growth, the cumulative failure rate, λ_Σ, is a pessimistic index, biased by early unreliable performance, which is weighed equally with more recent performance. The current failure rate, $\lambda(T)$, is defined as the derivative of the number of failures, X, with respect to operating time, T. Note that this is an unconditional rate, not a hazard rate conditioned on survival to T. It is representative of future performance allowing only for growth that has already taken place. In terms of the growth model,

$$\begin{aligned}\lambda(T) &= \frac{\partial X}{\partial T} = \frac{\partial}{\partial T}(\lambda_\Sigma T)\\ &= \frac{\partial}{\partial T}(\lambda_1 T^{-\alpha} T)\\ &= (1-\alpha)\,\lambda_1 T^{-\alpha}\\ &= (1-\alpha)\,\lambda_\Sigma\end{aligned}$$

It can be seen that the current failure rate improves in parallel with the cumulative failure rate. It is important to remember that this estimate is valid only after several failures have been observed; $\lambda(T)$ should not be computed until a pattern of reliability growth has been established.

Cumulative failure rate, λ_Σ, or MTBF, Θ_Σ, can be computed at any time but is normally computed immediately after every relevant failure, in order to provide proper data points for the growth model. The estimates are $\hat{\lambda}_\Sigma = X/T$ or $\hat{\Theta}_\Sigma = T/X$, where X is total number of relevant failures and T is total operating time. These estimates may be made for subcategories of failures as described above; as a minimum, they should be made for relevant failures of priority 1 and priority 2 taken together as a group.

When a clear growth pattern has been established, the current failure rate or MTBF should be estimated at the same time the cumulative failure rate or MTBF is calculated. The estimates are $\hat{\lambda}(T) = (1-\alpha)\,\hat{\lambda}_\Sigma$ or $\hat{\Theta}(T) = 1/(1-\alpha)\hat{\lambda}_\Sigma$.

The parameter α is the logarithmic growth rate, the slope of the Duane plot. A consequence of a growth process having constant α is that whenever the total test time T doubles, the cumulative failure rate is decreased by a constant factor $m = 2^{-\alpha}$. Growth or learning models are sometimes referred to by m rather than α. Thus, if α = .3, m = .812 and the model could be termed an 81 percent learning curve. Measured values of α in software programs usually fall between .2 and .7, corresponding to learning curves of 87 and 61 percent. It can be seen that the arithmetic rate of improvement declines steadily, corresponding to the diminishing returns property exhibited in most growth processes.

The rate α is computed as

$$\alpha = \frac{\ln\lambda_1 - \ln\lambda_\Sigma}{\ln T} \qquad (6\text{-}2)$$

and should be reported as an indicator of the intensity and effectiveness of project management relative to software reliability improvement. It should be noted that α depends, at least in part, on the level and consistency of management stress on reliability improvement. Therefore, management can improve α by intensifying efforts aimed at detecting and correcting software failure, the primary means by which reliability growth occurs. Routine recalculation of α after each failure will quickly identify changes in this important trend parameter.

Other software models of failure known as the seeding and tagging models have received a great deal of attention in the past few years[31,32]. Inasmuch as they do not, at present, provide time-dependent reliability measures of software, but only an estimate of the number of failures remaining in a computer program, these models are not recommended for reliability assessment.

Example of Duane Modeling of Software Data

The data in Figure 6-2A is derived from weekly summary reports (5 days testing) for priority 1 and 2 software failures during the software development program. Failures were fixed by competent personnel before the next reporting period.

Time	Failures	Cumulative Failures	Failure Rate
Σt (days)	x	Σx	λ_Σ (Failure/Day)
5*	16	16	3.2000
10*	8	24	2.4000
15*	6	30	2.0000
20*	5	35	1.7500
25	10	45	1.8000
30	9	54	1.8000
35	8	62	1.7714
40	10	72	1.8000
45	9	81	1.8000
50	11	92	1.8400
55	6	98	1.7818
60	6	104	1.7333
65	6	110	1.6923
70	5	115	1.6429
75	6	121	1.6133
80	7	128	1.6000
85	6	134	1.5765
90	5	139	1.5444
95	6	145	1.5263
100	5	150	1.5000
105	4	154	1.4667
110	6	160	1.4545
115	7	167	1.4522
120	5	172	1.4333

*Benign Testing Environment

Figure 6-2A. Software Failures from Weekly Summary Reports

During the first 4 weeks (see asterisks in Figure 6-2A), the testing environment was benign. When $\hat{\lambda}_\Sigma = \Sigma x/\Sigma t$ is plotted vs T on log-log paper, figure 6-3 results. Notice the break after t = 20 days, and the two trend lines exhibiting different slopes.

A test environment factor, k, found empirically to be equal to 2 is now applied to the test time up to 20 days, so that the original Σt = 10 days becomes the corrected Σt_c = 5 days, and the original Σt = 20 days becomes the corrected Σt_c = 10 days. For Σt_c = 5, x = 16 + 8 = 24 failures are accrued, and for

$\Sigma t_c = 10$, $x = 6 + 5 = 11$ failures and $\Sigma x = 35$ failures. $\Sigma t = 25$ then becomes $\Sigma t_c = 15$ with $x = 10$ failures, and the last time $\Sigma t = 120$ becomes $\Sigma t_c = 110$ with $x = 5$ failures (Figure 6-2B). Corrected values $\lambda_\Sigma = \Sigma x / \Sigma t_c$ are recomputed and plotted on Figure 6-4. The Duane plot is now approximately straight, with a value of λ_1 estimated from the intersection of a visually fitted straight line and the T = 1 axis. This gives $\hat{\lambda}_1 \cong 5.95$, and from 6-2):

$$\alpha = \frac{\ln(5.95) - \ln(172/110)}{\ln(110)} = 0.284$$

Time	Failures	Cumulative Failures	Failure Rate
Σt (days)	x	Σx	λ_Σ (Failure/Day)
5	24	24	4.8000
10	11	35	3.5000
15	10	45	3.0000
20	9	54	2.7000
25	8	62	2.4800
30	10	72	2.4000
35	9	81	2.3143
40	11	92	2.3000
45	6	98	2.1778
50	6	104	2.0800
55	6	110	2.0000
60	5	115	1.9167
65	6	121	1.8615
70	7	128	1.8286
75	6	134	1.7867
80	5	139	1.7375
85	6	145	1.7059
90	5	150	1.6667
95	4	154	1.6211
100	6	160	1.6000
105	7	167	1.5905
110	5	172	1.5636

Figure 6-2B. Adjusted Software Failure Data

In this example, it is difficult to justify the factor k = 2 used to correct the benign testing environment, except if it is arrived at *before* a Duane plot is evolved. Notice also that the data given is not entirely suitable for a Duane analysis. The 24 failures, for instance, do not occur at time $\Sigma t_c = 5$, but in the interval 0-5 days. One may obtain somewhat more accurate results by using mid interval time markers at 2.5, 7.5, 12.5 days, etc., instead of 5, 10, 15 days, etc., but in any event, one loses information and obtains reduced accuracy when one must use summary results.

For a more thorough treatment, including confidence limits on λ, the expressions of § 5.4.4 can be used.

6.6 COMMENTS ON SOFTWARE-HARDWARE RELIABILITY ESTIMATION

Some care must be exercised if one is to incorporate software reliability in a system. Assume, for instance, that the program described in the example of § 6.5 is to be incorporated in a serial system of components. First, its reliability must be computed. Using $\hat{\lambda} = (1-\hat{\alpha})\hat{\lambda}_1 T^{-\hat{\alpha}}$ at T = 110 days, we obtain:

$$\hat{\lambda} = (1-0.284)(5.95)(110)^{-0.284}$$
$$= 1.12 \text{ failures/day}$$

Assume now that a mission consists in running the program for 3.863 minutes, or 0.002683 days. Then

$$\hat{R} = e^{-\hat{\lambda} t} = e^{-(1.12)(.002683)}$$
$$= 0.997.$$

The system is shown in figure 6-5. R represents a radar with mission reliability 0.996, I represents a mechanical/electrical interface with mission reliability = 0.998, S represents the software with reliability = 0.997, C is the computer hardware with mission reliability = 0.999, and L is a launcher with mission reliability = 0.995. In this simple case, the system reliability, assuming independence of the components, would be simply: (0.996) (0.998) (0.997) (0.999) (0.995) = 0.985, and the unreliability would be 0.015.

Assume now that the computer-software-interface subsystem is inexpensive compared to the radar and launcher, and that it is

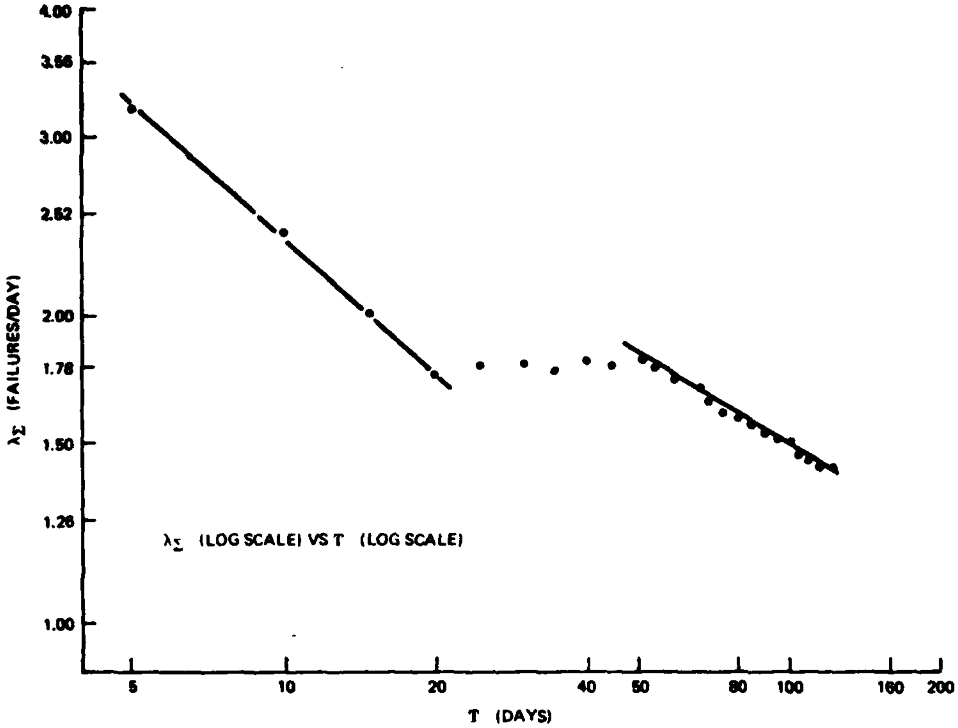

Figure 6-3. Duane Plot: Benign Early Software Testing Environment

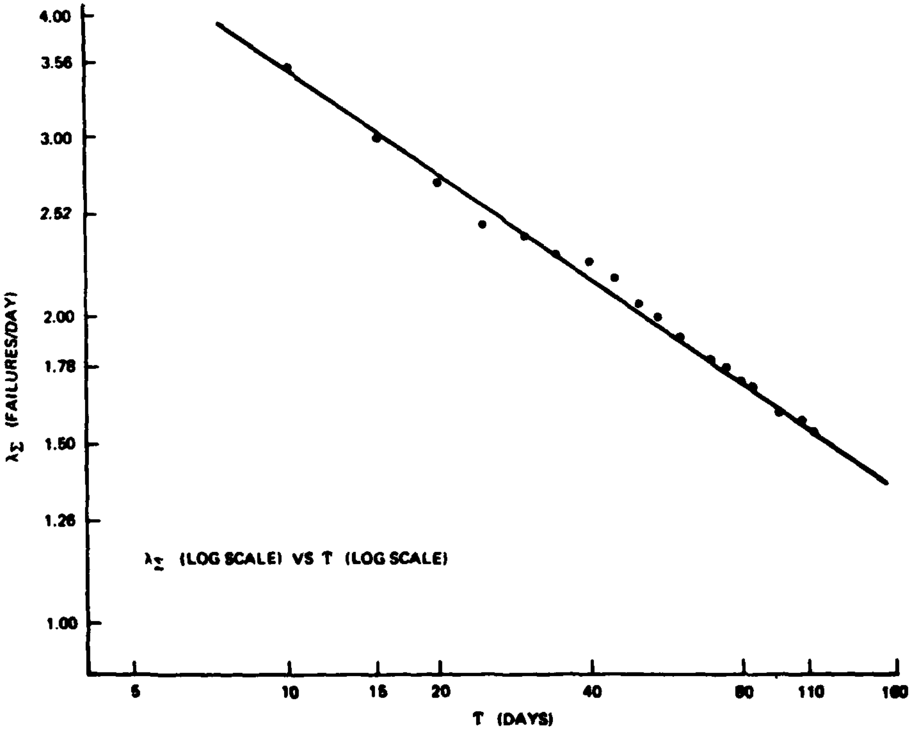

Figure 6-4. Duane Plot: Operational Software Testing Environment

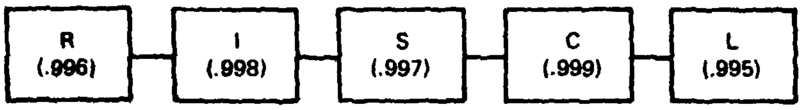

Figure 6-5. *Radar System with Software*

decided to reduce the unreliability by adopting the redundant configuration shown in figure 6-6.

It is not difficult to calculate that the redundant system of independent components, with two interfaces, two programs and two computers has an unreliability of 0.009, which represents an improvement of 40% in unreliability over the simple series configuration. But such a calculation would, in this case, be incorrect for the reasons discussed below.

If S1 fails, then by definition S2 would fail since software is identical and would reach the same point in both programs and would never be able to perform that function (get past the error). Figure 6-6 however, ignores the particular characteristics of "redundant" software which contains identical latent errors. It is then very probable that S1 and S2 will succeed or fail identically if they receive nearly identical inputs I1 and I2. Under these conditions, effectively figure 6-7, S1 and S2 are practically totally dependent, and the system unreliability in this case is 0.012 which represents an improvement of only 20% in unreliability over the simple series configuration (Figure 6-5).

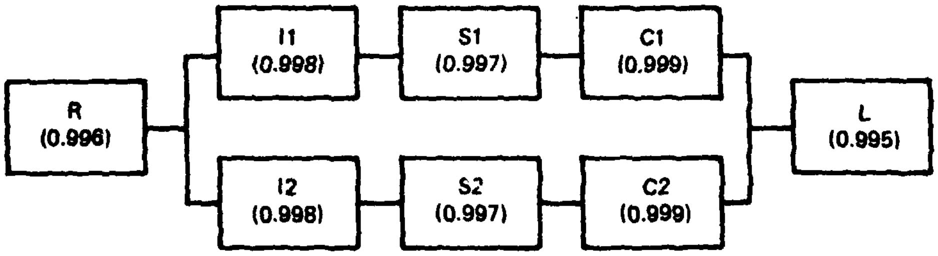

Figure 6-6. Radar System with Computational Redundancy

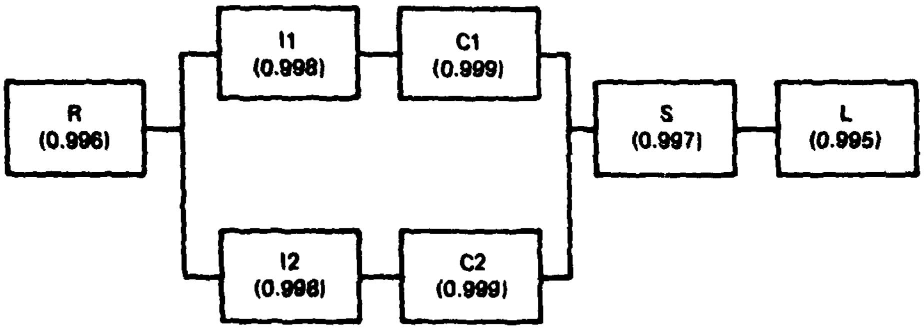

Figure 6-7. Radar System Without Computational Redundancy

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Section 7
ASSESSING SYSTEM RELIABILITY AND AVAILABILITY

Methods for point and interval assessment of the reliability of individual components have been described in Section 5. In that context, the term component was understood to apply to any assembly level, provided that the test data were taken at the assembly level of the component being assessed. In this section, these methods are extended to perform system level reliability and availability assessment. This assessment is also based on the data being derived at the component level, where the components are the constituent elements of the system, but these component assessments are combined to provide estimates of system parameters.

Point estimation of system reliability and availability is by far the simpler and is discussed briefly first. System interval estimation is more complex and is discussed at greater length.

7.1 POINT ESTIMATES FOR SYSTEMS

Point estimates for the reliability and availability of systems can be obtained by inserting the point estimate for each component into the reliability or availability model of the system, and solving the system equations. Some examples are given below.

For a serial system, the reliability of a system composed of n independent components is given by:

$$R_s(t) = \prod_{i=1}^{n} R_i(t) \qquad (7\text{-}1)$$

where $R_i(t)$ is the reliability of the ith element. If estimates $\widehat{R}_i(t)$ of the $R_i(t)$'s have been obtained from test data, then a point estimate of $R_s(t)$ is:

$$\widehat{R}_s(t) = \prod_{i=1}^{n} \widehat{R}_i(t) \qquad (7\text{-}2)$$

For a parallel system of n independent components, the system model yields:

$$R_s(t) = 1 - \prod_{i=1}^{n} [1 - R_i(t)] \qquad (7\text{-}3)$$

If $\widehat{R}_i(t)$ are estimates of $R_i(t)$, then an estimate of system reliability is

$$\widehat{R}_s(t) = 1 - \prod_{i=1}^{n} [1 - \widehat{R}_i(t)] \qquad (7\text{-}4)$$

For mixed series-parallel systems of independent components, all series combinations and all parallel combinations of elements are reduced to single elements through the equations given above, to yield a point estimate of system reliability. The same procedure is applied to the availability model to obtain a point estimate of system availability [§ 4.2.2 discusses modeling].

An example illustrating the above techniques, used in the evaluation of missile flight reliability, is provided in appendix B.

7.1.1 The Rubinstein Method, Serial Systems of Exponential Components

The Rubinstein method fully described in [1] provides an estimate of each component failure rate λ_i and its variance σ_i^2. Because the components are independent and follow an exponential failure law, the point estimate of failure rate for a serial system of n components is:

$$\widehat{\lambda}_s = \sum_{i=1}^{n} \sum_{j} \sum_{k} \widehat{\lambda}_{ijk} \qquad (7\text{-}5)$$

where the subscript i indicates component, j indicates environment and k indicates test state. Then:

$$\hat{R}_s(t) = e^{-\hat{\lambda}_s t} \qquad (7\text{-}6)$$

Because the components are independent, the estimate of the variance of $\hat{\lambda}_s$ is:

$$\hat{\sigma}^2_{\hat{\lambda}_s} = \sum_i \sum_j \sum_k \frac{\hat{\lambda}_{ijk}}{t_{ijk}} \qquad (7\text{-}7)$$

and $\hat{C}_s$, a quantity needed in the sequel is:

$$\hat{C}_s = \hat{\sigma}^2_{\hat{\lambda}_s} / \hat{\lambda}_s \qquad (7\text{-}8)$$

Example, Point Estimate for Two-Component System – Rubinstein Method

A two-component serial system is modeled in figure 7-1. Each component is tested separately. For component 1 in high temperature, non-operating (subscripts 1ha), four tests are performed with the following results:

Test 1. Component failed after 142 minutes of a scheduled 300 minute test (αt = 14.2 equivalent missions).

Test 2. Component did not fail in 300 minutes of testing (αt = 30).

Test 3. Component failed after 147 minutes of a scheduled 300 minutes of testing (αt = 14.7).

Test 4. Component did not fail in 300 minutes of testing (αt = 30).

Then:

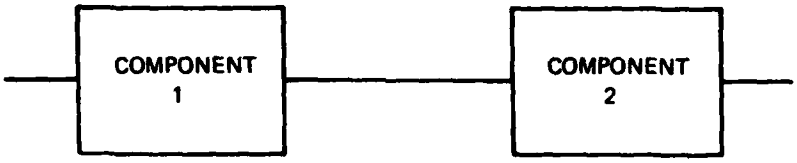
$$\hat{\lambda}_{1ha} = \frac{X_{1ha}}{t_{1ha}} \left(\frac{2N_{1ha}}{2N_{1ha}+1} \right)$$

where:

X_{1ha} is the total number of failures on component 1, environment h, and test condition a.

t_{1ha} is the total test time in equivalent missions on component 1, environment h, and test condition a.

N_{1ha} is the number of units of component 1 tested in environment h and test condition a.



Component	Environment	Test Condition	Mission Exposure Time (Min.)	α $\frac{\text{Missions}}{\text{Minute}}$
1	High Temp (h)	Non-Oper (a)	10.00	0.10
		Operating (d)	0.50	2.00
	Vibration (v)	Non-Oper (a)	0.25	4.00
		Operating (d)	20.00	0.05
2	High Temp (h)	Non-Oper (a)	10.00	0.10
		Operating (d)	1.00	1.00
	Vibration (v)	Non-Oper (a)	NONE	—
		Operating (d)	5.00	0.20

Figure 7-1. Serial Subsystem Block Diagram and Test Data

$$\hat{\lambda}_{1ha} = \frac{(1+0+1+0)}{(14.2+30.0+14.7+30.0)} \cdot \frac{2x4}{(2x4)+1}$$

$$= \left(\frac{2}{88.9}\right)\left(\frac{8}{9}\right)$$

$$= 0.0200 \text{ failures/mission}$$

Figure 7-2 summarizes the test data and failure rate estimates.

ijk	Test Time (minutes)	α	t_{ijk} (M.E.)*	N_{ijk}	X_{ijk}	$\hat{\lambda}_{ijk}$
1ha	889.0	0.10	88.9	4	2	.02
1hd	45.45	2.00	90.9	5	1	.01
1va	21.425	4.00	85.7	3	1	.01
1vd	2000.0	0.05	100.0	3	0	.00
2ha	923.0	0.10	92.3	6	1	.01
2hd	100.0	1.00	100.0	6	0	.00
2vd	500.0	0.20	100.0	4	0	.00

*M.E. = Mission Equivalents

Figure 7-2. Test Data and Failure Rate Estimates

Point estimates are built up by progressive summation:

$$\hat{\lambda}_{1h} = \hat{\lambda}_{1ha} + \hat{\lambda}_{1hd} = .02 + .01$$
$$= .03 \text{ failures/mission}$$

$$\hat{\lambda}_{1V} = \hat{\lambda}_{1Va} + \hat{\lambda}_{1Vd} = .01 + .00$$
$$= .01 \text{ failures/mission}$$

$$\hat{\lambda}_{2h} = \hat{\lambda}_{2ha} + \hat{\lambda}_{2hd} = .01 + .00$$
$$= .01 \text{ failures/mission}$$

$$\lambda_{2V} = \hat{\lambda}_{2Vd} = .00 \text{ failures/mission}$$

and the corresponding reliabilities are (where t = one mission):

$$\hat{R}_{1h} = e^{-.03} = .9705$$
$$\hat{R}_{1V} = e^{-.01} = .9901$$
$$\hat{R}_{2h} = e^{-.01} = .9901$$
$$\hat{R}_{2V} = e^{-.00} = .9999+$$

Perfect reliability, as reflected in the final calculation, is acknowledged to be unattainable. It appears as a consequence of the preceding failure rate estimate λ_{2V} = .00. Neither number can be interpreted as a valid point estimate of a statistical parameter, but rather as an indication that such an estimate cannot be made until at least one failure is observed. For this reason, engineers sometimes omit reporting an estimate of unit reliability or reduce it arbitrarily to .9999+. These precautions are unnecessary if the users understand that such an estimate is not an assertion of certain success. It should be noted that confidence limits computed for the zero failure case are valid.

At component level:

$$\hat{\lambda}_1 = \hat{\lambda}_{1h} + \hat{\lambda}_{1V} = .04 \text{ failures/mission}$$

$$\hat{\lambda}_2 = \hat{\lambda}_{2h} + \hat{\lambda}_{2V} = .01 \text{ failures/mission}$$

At system level:

$$\hat{\lambda}_s = \hat{\lambda}_1 + \hat{\lambda}_2 = .05 \text{ failures/mission}$$

$$\hat{R}_s = e^{-.05} = .9512$$

7.1.2 The Rubinstein Method, Parallel System of Exponential Components

Figure 7-3 models a two-element active parallel system consisting of two of the components examined in the previous example. The failure rate of each component has been estimated as .05 failures/mission.

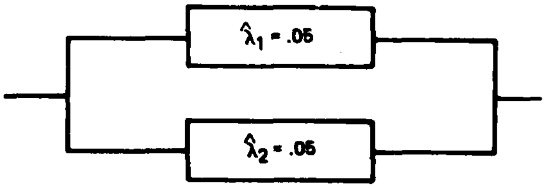


Figure 7-3. Two-Component Parallel System

From equation (7-4),

$$\hat{R}_s = 1 - (1-\hat{R}_1)(1-\hat{R}_2) = e^{-\hat{\lambda}_1} + e^{-\hat{\lambda}_2} - e^{-(\hat{\lambda}_1 + \hat{\lambda}_2)}$$

and

$$\hat{R}_s = .9976$$

7.1.3 Bayesian Approach Used with the Rubinstein Method

The Bayesian formalism makes use of prior knowledge about each component. When the prior failures are ϕ and prior test times are τ, the ratio ϕ/τ determines the mean of the prior failure rate λ_0. with $\sigma^2_{\lambda_0} = \phi/\tau^2$. If the gamma distribution is chosen as a prior, then the posterior distribution will also be gamma with posterior parameters $\phi + X$ and $\tau + t$. Posterior failure rate for component i is:

$$\hat{\lambda}_i = (\phi_i + X_i)/(\tau_i + t_i) \qquad (7\text{-}9)$$

and the variance of $\hat{\lambda}_i$ is given by:

$$\sigma^2_{\hat{\lambda}_i} = (\phi_i + X_i)/(\tau_i + t_i)^2 \qquad (7\text{-}10)$$

Predicted reliability is used as a basis for establishing the prior point estimate. But Bayesian methods require a statement of confidence in the estimate. This is achieved by specifying the variance of the prior estimate. The point estimate fixes the ratio ϕ/τ. Subsequent selection of τ defines the variance ϕ/τ^2. If τ is large, the prior is strong and relatively difficult for test data to modify; if τ is small the prior is weak and easily discounted by test data. Figure 5-28 gives empirical rules for choosing τ.

Bayesian methods require that predictions be available to initiate the computations. Since test data are normally generated for individual component-environment-test state combinations, predictions must be made separately for each such combination. This is a departure from the usual practice of making predictions for components, in which the effects of environment-test state combinations are tacitly aggregated. In most cases, however, the availability of published derating curves and similar application factors allows predictions to be carried down to the necessary detail. When this cannot be done credibly, the user is faced with the possible need to discount some of the test data that will subsequently be obtained, usually at substantial cost to the program. In that instance, the most conservative but least efficient procedure is to employ in the component calculations the minimum test time accrued by that component in any environment-test condition combination. A better procedure is to employ the harmonic mean of the applicable test times. It is given by:

$$t_{i_E} = \left[\frac{1}{mn}\left(\sum_{k=1}^{n} \sum_{j=1}^{n} \frac{1}{t_{ijk}}\right)\right]^{-1} \qquad (7\text{-}11)$$

t_{i_E} = number of effective equivalent missions for ith component

t_{jk} = number of equivalent missions in the jth environment and the kth test condition

The value t_{i_E} is added to τ_i and the number of actual failures x_i is added to ϕ_i for each one of the components of the system. Then the system reliability is recomputed from the system model equations.

Example, Point Estimate of Four-Component Series System Reliability, Bayesian – Rubinstein Method

Assume a four-component series system, with the prediction results set forth below (Figure 7-4).

For Component 1, prior information consists of similar handbook information (Category 4 of figure 5-28), estimated failure rate is 0.0004 failures per mission, and tests planned for the component will total 200 equivalent missions. [Note the symbol λ_o indicates a prediction.]

λ_{o_1} = 0.0004 failures per mission

τ_1 = (1/4) (200) = 50 prior missions

$\phi_1 = \lambda_{o_1} \tau_1$ = (0.0004) (50)

= .02 prior failures

Component 2 has information of a frequency nature that best fits Category 7. Its prior values become:

λ_{o_2} = .0067 failures per mission

τ_2 = (0.3) (600) = 180 prior missions

ϕ_2 = (.0067) (180) = 1.20 prior failures

Component 3 also has frequency data available from a previous program where both the

Component	Prior Failure Rate	Category Selected *	Planned Test Time in Equivalent Missions	Prior Freq. Data	Prior Parameters	
					ϕ	τ
1	0.00040	4	200	N/A	0.02	50
2	0.00670	7	N/A	4/600	1.20	180
3	0.00600	8	N/A	6/1000	6.00	1000
4	0.00011	3	720	N/A	0.01	90
	0.01321					

*See Figure 5-28

Figure 7-4. Bayesian Prediction Results

component and the mission are judged identical to the new program (Category 8).

λ_{o_3} = .0060 failures per mission

τ_3 = (1.0) (1000) = 1000 prior missions

ϕ_3 = (.006) (1000) = 6 prior failures

Information on Component 4 consists of generic data only (Category 3). Analysis of tests planned for component 4 indicates that 720 equivalent missions of testing will be conducted with a predicted failure rate of 0.00011. Then:

λ_{o_4} = .00011 failures per mission

τ_4 = (1/8) (720) = 90 prior missions

ϕ_4 = λ_{04} τ_4 = (.00011) (90)

= .0099 ≅ .01 prior failures

The component failure rates are added to give a prior estimate of system failure rate and system reliability is estimated. These values are:

$$\lambda_{o_S} = 0.0132 \text{ failures/mission}$$

$$R_{o_S} = e^{-.0132} = .9869$$

Assume that three successive reliability reports are prepared during the program, each embodying calculations of reliability. Cumulative test and failure data at each report are tabulated below. Note that the rate of testing is not uniform throughout the program.

To recompute the system estimated λ_s and R_s after the 1st report, for instance, one starts with the prediction model,

$$\lambda_{o_s} = \frac{0.02}{50} + \frac{1.20}{180} + \frac{6.00}{1000} + \frac{0.01}{90}$$

$$= 0.0132 \text{ failures/mission}$$

which is updated with the Report No. 1 data from figure 7-5 as follows:

$$\hat{\lambda}_{S(\text{Report 1})} = \frac{0.02+0}{50+20} + \frac{1.20+1}{180+10} + \frac{6.00+0}{1000+15} + \frac{0.01+0}{90+20}$$

$$= 0.01787 \text{ failures/mission}$$

$$\hat{R}_{S(\text{Report 1})} = e^{-\hat{\lambda}_S(1)} = e^{-0.0179} = 0.9823$$

Report No.	1		2		3	
Component	X	t	X	t	X	t
1	0	20	0	50	0	200
2	1	10	1	20	1	100
3	0	15	1	50	3	500
4	0	20	0	100	0	400

Figure 7-5. Test and Failure Data for Bayesian Analysis

	Prediction		Report 1		Report 2		Report 3	
	λ_0	R_0	$\hat{\lambda}$	$\hat{R}$	$\hat{\lambda}$	$\hat{R}$	$\hat{\lambda}$	$\hat{R}$
Component 1	0.00040	.9996	0.00029	.9997	0.00020	.9998	0.00008	.9999
Component 2	0.00670	.9933	0.01158	.9885	0.01100	.9891	0.00786	.9922
Component 3	0.00600	.9940	0.00591	.9941	0.00667	.9934	0.00600	.9940
Component 4	0.00011	.9999	0.00009	.9999	0.00005	.9999	0.00002	.9999
System	0.01321	.9869	0.01787	.9823	0.01792	.9822	0.01396	.9861

Figure 7-6. Prediction and Posterior Estimates of Failure Rate and Reliability

Proceeding in the same manner for the other reports, the point estimated values of failure rate for these reports are shown in figure 7-6.

7.1.4 Point Estimate of Steady-State Availability for Series and Parallel Systems

A point estimate of steady-state availability for a serial system of n independent exponential components is the product of the estimate of availability for each component, or

$$\hat{A}_s = \prod_{i=1}^{n} \frac{\hat{\mu}_i}{\hat{\lambda}_i + \hat{\mu}_i} \qquad (7\text{-}12)$$

where μ_i and λ_i are respectively the repair rate and failure rate point estimates for the ith component.

For a parallel system of n identical components, m of which must be operable for the system to be available, the point estimate is:

$$\hat{A}_s = \frac{\sum_{i=1}^{n-m} \binom{n}{i} \left(\hat{\mu}\right)^{n-i} \left(\hat{\lambda}\right)^{i}}{(\hat{\mu}+\hat{\lambda})^{n}} \qquad (7\text{-}13)$$

In series-parallel systems of components, this equation can be used to obtain a point estimate of availability for any serial stage. A serial stage is a group of components which may combine in series with other components or stages to form the system, but is itself composed of components or serial strings of components arranged in m-of-n active parallel redundancy. For example, figure 7-7 depicts a system consisting of two serial stages. The serial structure that remains after estimates have been obtained for all stages, permits system availability to be estimated as the product of the stage availabilities, or: $\hat{A}_S = \hat{A}_{S1} \cdot \hat{A}_{S2}$.

7.1.5 Reliability and Availability Assessment of Redundant Non-Repairable and Repairable Systems

The reliability and availability assessment of many systems more complex than series and parallel can be obtained by considering Birth and Death processes and solving the differential equations which arise from a state transition matrix formulation of these processes. A few cases are solved in Appendix D, § D.3, with the results indicated in § 7.1.5.1, § 7.1.5.2 and § 7.1.5.3.

7.1.5.1 MTBF of a 4 of 6 Repairable System with Restricted Repair

Out of 6 identical components in parallel, 4 must be operating for the system to be operational. For such a system with a single repairman available (restricted repair):

$$\text{MTBF} = \frac{1}{\lambda}\left[\frac{2\mu^2 + 12\mu\lambda + 30\lambda^2}{120\lambda^2}\right]$$

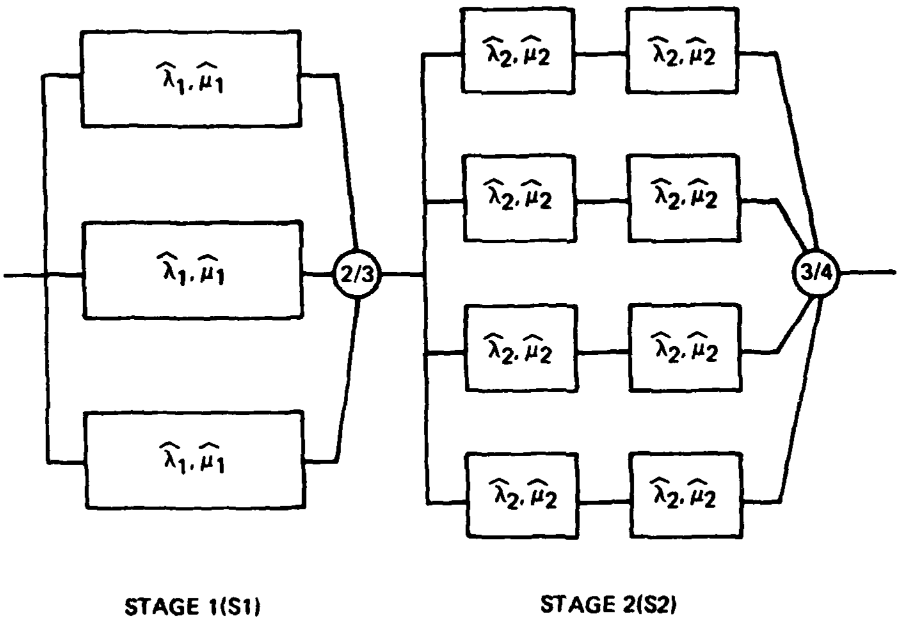
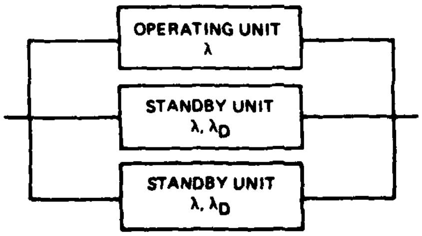


Figure 7-7. System of Two Serial Stages, Each with Identical Components

Thus an estimated MTBF is:

$$\widehat{MTBF} = \frac{1}{\hat{\lambda}}\left[\frac{2(\hat{\mu})^2 + 12\,\hat{\mu}\,\hat{\lambda} + 30(\hat{\lambda})^2}{120(\hat{\lambda})^2}\right]$$

7.1.5.2 Reliability of a 1 of 3 Standby System with Dormant Hazard Rate and No Repair



Assuming perfect switching, λ = operating failure rate, λ_D = dormant failure rate, then:

$$R(t) = e^{-(\lambda+2\lambda_D)t} + \frac{\lambda+2\lambda_D}{\lambda_D}\left(e^{-(\lambda+\lambda_D)t} - e^{-(\lambda+2\lambda_D)t}\right)$$

$$+ \frac{(\lambda+\lambda_D)(\lambda+2\lambda_D)}{\lambda_D}\left(\frac{e^{-(\lambda+2\lambda_D)t} - e^{-\lambda t}}{2\lambda_D}\right.$$

$$\left. - \frac{e^{-(\lambda+\lambda_D)t} - e^{-\lambda t}}{\lambda_D}\right)$$

Again an estimate of R(t), $\hat{R}(t)$, is obtained by replacing λ and λ_D by $\hat{\lambda}$ and $\hat{\lambda}_D$, respectively, in the expression for R(t).

Appendix A provides an example of an availability analysis of a Fire Control Subsystem.

7.1.5.3 MTBF and MTTR of M of N Identical Repairable Components in Parallel with Restricted Repair – The Einhorn Approximations

Only one component is under repair at any one time. If μ is large compared to λ:

$$MTBF = \binom{N}{M-1}\left(\frac{1}{\lambda}\right)^{M-1}\left(\frac{1}{\mu}\right)^{N-M+1}/N \qquad (7\text{-}14)$$

and

$$MTTR = \left(\frac{1}{\mu}\right)/(N-M+1) \qquad (7\text{-}15)$$

Then, point estimates of MTBF and MTTR are:

$$MTBF = \binom{N}{M-1}\left(\frac{1}{\hat{\lambda}}\right)^{M-1}\left(\frac{1}{\hat{\mu}}\right)^{N-M+1}/N$$

and
$$\text{MTTR} = \left(\frac{1}{\hat{\mu}}\right)/(N\text{-}M+1)$$

A vast source of information on Birth and Death processes applied to reliability and availability system assessment is available [2]. Tables for m and n configurations are provided in appendix E, § E.4, figure E-11.

7.1.6 Reliability Point Estimates for Complex Systems

For logically complex systems as illustrated in § 4.2.2.1.2.2 through § 4.2.2.1.2.4, binomial modeling, conditional modeling or minimal cut modeling allows one to express system reliability in terms of the individual component reliabilities. To obtain an estimate of system reliability, it is only necessary to replace known component reliabilities by their estimates in the expression for system reliability.

In § 4.2.2.1.2.2, for instance, the estimated expression for R_S, $\hat{R}_S$, becomes:

$$\hat{R}_S = (1-\hat{R}_C)\hat{R}_D\hat{R}_E + \hat{R}_C(1-\hat{R}_D)\hat{R}_E + \hat{R}_C\hat{R}_D(1-\hat{R}_E) + \hat{R}_C\hat{R}_D\hat{R}_E$$

7.1.7 Reliability Point Estimates for Software-Hardware Systems

Only when different versions of software are operated together is software truly redundant. Replicated software and the frequently used "casualty programs", which are simply subsets of the primary system software, are not redundant from a reliability viewpoint. Non-redundant software is generally represented as a single block in the main sequence of a system reliability block diagram.

When software is incorporated in a system model, the procedure to obtain point estimates of system reliability proceeds strictly as it would if only hardware were involved. That is, the reliability of each block of the diagram is estimated from test data, and the system equation is solved.

See an example of software reliability assessment in § 6.5 and of a "redundant" software-hardware system assessment in § 6.6.

7.2 INTERVAL ESTIMATES FOR SYSTEMS

7.2.1 Series, Parallel and Series-Parallel Systems of Exponential Components – The Rubinstein Method

It has been indicated that for a serial system the Rubinstein method provides a point estimate of system failure rate by solving:

$$\hat{\lambda}_s = \sum_i\sum_j\sum_k \hat{\lambda}_{ijk} \qquad (7\text{-}5)$$

Also, the estimate of $\sigma^2_{\hat{\lambda}_s}$, the variance of $\hat{\lambda}_s$ was shown to be

$$\widehat{\sigma^2_{\hat{\lambda}_s}} = \sum_i\sum_j\sum_k \frac{\hat{\lambda}_{ijk}}{t_{ijk}} \qquad (7\text{-}7)$$

A good approximation for the upper limit at confidence γ for failure is given by:

$$\lambda_u = \max\begin{cases} \dfrac{2\hat{\lambda}+(\beta K)^2\hat{C}+\sqrt{4\hat{\lambda}(\beta K)^2\hat{C}+(\beta K)^4\hat{C}^2}}{2} \\ \dfrac{(\beta K)^2}{\text{smallest } t_{ijk} \text{ (with no failures)}} \end{cases} \qquad (7\text{-}16)$$

where

$$\hat{C}_S = \widehat{\sigma^2_{\hat{\lambda}_S}}/\hat{\lambda}_S \qquad (7\text{-}8)$$

In this expression, K is the standard normal deviate for specified confidence level (e.g., K = 0.842 for 80% confidence), and β is a bias correction factor tabulated in figure 5-25 for a confidence $\gamma = 0.80$. The upper limit on failure rate in the no-failure case is:

$$\lambda_u = \frac{(\beta K)^2}{\text{smallest } t_{ijk}} \qquad (7\text{-}17)$$

A lower confidence limit is of principal interest when reliability is estimated. It is obtained by substituting the corresponding upper limit for failure rate into the reliability equation;

$$R_L = e^{-\lambda_u} \quad \text{Again, } t = \text{one mission.} \qquad (7\text{-}18)$$

With increasing test time the lower reliability confidence limit R_L approaches the best estimate, $\hat{R}$, asymptotically. But $\hat{R}$ may itself increase when corrective actions are effective in eliminating failure modes. If operating experience subsequent to corrective action gives convincing evidence that a failure mode has been eliminated, past failures in that mode may be deleted from the data base used to compute reliability. Under these conditions successive estimates of $\hat{R}$ made during a development program will form a growth curve. Such a curve is of value for visualizing program progress, and with care, it can also be extrapolated to predict future reliability growth.

Example, Interval Estimation for Series System Reliability, Exponential Components – Rubinstein Method

The example of § 7.1.1 used the test data from figure 7-2 to calculate point estimates of failure rate and reliability.

The example is continued to obtain the upper bound on failure rate and the lower bound on reliability at the 80% confidence level for each of the two components and the system.

Equation 7-16 is used to calculate the upper bound on failure rate [Equation 7-17 is the no-failure case].

In order to use equation 7-16, $\hat{\lambda}$, K, β and $\hat{C}$ are required. The $\hat{\lambda}$ values were obtained in § 7.1.1, K, the Normal Deviate, is 0.842 since we selected the 80% confidence level, β may be calculated using equation 5-59 or read directly from figure 5-25, and $\hat{C}$ of $\sigma^2_{\hat{\lambda}}/\hat{\lambda}$ (Equation 7-8).

$$\hat{C}_{1h} = \frac{\underset{k}{\Sigma}(\hat{\lambda}_{1hk}/t_{1hk})}{\hat{\lambda}_{1h}} = \frac{\frac{.02}{88.9}+\frac{.01}{90.9}}{.03} = .011$$

β = 1.2725 for 3 failures and 80% confidence (Figure 5-25). Then, using equations 7-16 and 7-18:

$$\lambda_{u_{1h}} = \frac{2\hat{\lambda}_{1h} + (\beta K)^2 \hat{C}_{1h}}{2} + \frac{\sqrt{4\hat{\lambda}_{1h}(\beta K)^2 \hat{C}_{1h} + (\beta K)^4 \hat{C}_{1h}^{\,2}}}{2}$$

$$= \frac{2(.03) + [(1.272)(0.842)]^2(.011) + \sqrt{.00167338}}{2}$$

= .0568 failures/mission

$$R_{L_{1h}} = e^{-\lambda_{u_{1h}}} = e^{-.0568} = .9448$$

Similarly the calculation for $\lambda_{u_{1v}}$:

$$\hat{C}_{1v} = \frac{\underset{k}{\Sigma}\hat{\lambda}_{1vk}/t_{1vk}}{\hat{\lambda}_{1v}} = \frac{.01/85.7 + 0/100}{.01} = 0.0117$$

Using equation 7-16:

$$\lambda_{u_{1v}} = \text{max.}\begin{cases} \dfrac{2(.01) + [(1.3694)(0.842)]^2(.0117) + \sqrt{.000865}}{2} \\[2ex] \dfrac{[(1.5074)(0.842)]^2}{100} \end{cases}$$

$$\lambda_{u_{1v}} = \text{max.}\begin{cases} 0.0325 \\ 0.0161 \text{ in failures/mission} \end{cases}$$

Therefore:

$\lambda_{u_{1v}}$ = 0.0325 failures/mission

$R_{L_{1v}}$ = 0.9680

By similar methods:

$\lambda_{u_{2h}}$ = 0.0312 failures/mission

$R_{L_{2h}}$ = .9693

And for the zero-failure cases, using equation 7-17:

$$\lambda_{u_{2v}} = \frac{(\beta K)^2}{n} \Sigma \frac{1}{t_{2vK}}$$

$$= \frac{[(1.507)(0.842)]^2}{1} \left(\frac{1}{100}\right)$$

= .0161 failures/mission

$$R_{L_{2v}} = e^{-.0161} = .9840$$

Figure 7-8 summarizes the above results at component-environment level.

Component	Env.	Best Est.		80% Conf.	
		$\hat{\lambda}$	$\hat{R}$	λ_u	R_L
1	High Temp.	.0300	.9705	.0568	.9448
	Vib.	.0100	.9901	.0325	.9680
2	High Temp.	.0100	.9901	.0312	.9693
	Vib.	.0000	.9999	.0161	.9840

Figure 7-8. Component Environment Data

To obtain component estimates, using figures 7-2 and 7-8:

$$\hat{\lambda}_1 = \hat{\lambda}_{1h} + \hat{\lambda}_{1V} = .03 + .01 = .04 \text{ failures/mission}$$

$$\hat{\lambda}_2 = \hat{\lambda}_{2h} + \hat{\lambda}_{2V} = .01 + .00 = .01 \text{ failures/mission}$$

And from equation 7-6, with t = 1 mission:

$$\hat{R}_1 = e^{-.04} = .9608$$

$$\hat{R}_2 = e^{-.01} = .9901$$

Using equation 7-16, with $\hat{C}_1$ and $\hat{C}_2$:

$$\hat{C}_1 = \frac{\frac{.02}{88.9} + \frac{.01}{90.9} + \frac{.01}{85.7} + \frac{.00}{100.0}}{.04} = .011292$$

$$\hat{C}_2 = \frac{\frac{.01}{92.3} + \frac{.00}{100.0} + \frac{.00}{100.0}}{.01} = .010834$$

$$\lambda_{u_1} = \text{max.} \begin{cases} \dfrac{2(.04)+[(1.2471)(0.842)]^2(.011292)+\sqrt{.002147}}{2} \\[2ex] \dfrac{[(1.5074)(.842)]^2}{100} \end{cases}$$

$$\lambda_{u_1} = \text{max.} \begin{cases} 0.0694 \\ 0.0161 \quad \text{in failures/mission} \end{cases}$$

Therefore:

$$\lambda_{u_1} = 0.0694 \text{ failures/mission}$$

$$R_{L_1} = 0.9330$$

Similarly:

$$\lambda_{u_2} = \text{max.} \begin{cases} \dfrac{2(.01)+[(1.3694)(.842)]^2(.010834)+\sqrt{.000784}}{2} \\[2ex] \dfrac{[(1.5074)(.842)]^2}{100} \end{cases}$$

$$\lambda_{u_2} = \text{max.} \begin{cases} 0.0312 \\ 0.0161 \quad \text{in failures/mission} \end{cases}$$

Therefore:

$$\lambda_{u_2} = 0.0312 \text{ failures/mission}$$

$$R_{L_2} = 0.9693$$

Subsystem estimates are calculated:

$$\hat{\lambda}_S = \hat{\lambda}_1 + \hat{\lambda}_2 = .04 + .01 = .05 \text{ failures/mission}$$

$$\hat{R}_S = e^{-.05} = .9512$$

$$\hat{C}_S = \frac{\frac{.02}{88.9} + \frac{.01}{90.9} + \frac{.01}{85.7} + \frac{0}{100} + \frac{.01}{92.3} + \frac{0}{100} + \frac{0}{100}}{.05} = .0112002$$

$$\widehat{\sigma^2_{\hat{\lambda}_S}} = \hat{\lambda}_S \hat{C}_S = (0.05)(.0112002) = 0.00056$$

$$\lambda_{u_S} = .0812 \text{ failures/mission}$$

$$R_{L_S} = e^{-.0812} = .9220$$

In summary,

Component	Mission			
	Best Est.		80% Conf.	
	$\hat{\lambda}$	$\hat{R}$	λ_u	R_L
1	.0400	.9608	.0694	.9330
2	.0100	.9901	.0312	.9693
Subsystem	.0500	.9512	.0812	.9220

completing the example.

Example. Parallel Subsystems with Exponential Components – Rubinstein Method

Returning to the two-subsystem parallel system of §7.1.2 which consists of two of the subsystems described above, the following point estimate was obtained:

$$\hat{R}_p = 1 - (1-R_i)^2 = 1 - (1 - 0.9512)^2$$

$$= .9976$$

A conservative approximation of system reliability in the two-identical-element parallel case, which is reasonably close for low failure rates, is

$$\hat{R}_p = e^{-\hat{\lambda}_p} \simeq e^{-(\hat{\lambda}_s)^2} \qquad (7\text{-}19)$$

where $\hat{\lambda}_p \simeq (\hat{\lambda}_s)^2$. In the above example, for instance,

$$\hat{R}_p \simeq e^{-(.05)^2} = .9975$$

In the examples given above, the implicit use of $R_i{}^2$ or the explicit use of $\hat{\lambda}_p$ as *biased* estimators [1] do not seriously affect the final value of $\hat{R}$. If, however, one were to need many such estimates to estimate the system reliability of a system composed of series-parallel subsystems, then a unidirectional biased estimate for each of the subsystems could accumulate to a large bias for the system. It is therefore preferable, if the illustrated parallel subsystem is only a portion of the total system, to calculate parallel reliability and parallel reliability bounds as follows:

Reliability Lower Bound. Parallel Subsystems with Exponential Components – Rubinstein Method

A relatively bias-free estimate of λ_s^2 is given [1, p. 2-14] by:

$$\widehat{\lambda_s^2} = (\hat{\lambda}_s)^2 - \widehat{\sigma^2_{\hat{\lambda}_s}} \qquad (7\text{-}20)$$

This estimate has approximately the variance:

$$\widehat{\sigma^2_{\widehat{\lambda_s^2}}} \simeq 4(\hat{\lambda}_s)^2\, \widehat{\sigma^2_{\hat{\lambda}_s}} \qquad (7\text{-}21)$$

These approximations permit subsystem reliability to be estimated. In the example, quantities previously found are:

$$\hat{\lambda}_s = 0.05 \text{ failures/mission}$$

$$\widehat{\sigma^2_{\hat{\lambda}_s}} = 0.00056$$

From equation 7-20:

$$\widehat{\lambda_s^2} = (.05)^2 - .00056 = .00194$$

From equation 7-19:

$$\hat{\lambda}_p \cong \widehat{\lambda_s^2} = .00194 \text{ failures/mission}$$

A relatively bias free estimate of R_p (from equation 7-19):

$$\hat{R}_p = e^{-\hat{\lambda}_p} = e^{-.00194} = .9981$$

To compute the lower bound on failure rate an estimate of C_p is required. From equation 7-8:

$$\hat{C}_p = \widehat{\sigma^2_{\hat{\lambda}_p}} \Big/ \hat{\lambda}_p$$

and using equation 7-21:

$$\hat{C}_p \simeq 4(\hat{\lambda}_s)^2\, \widehat{\sigma^2_{\hat{\lambda}_s}} \Big/ \hat{\lambda}_p$$

$$\simeq 4(.05)^2\,(.00056)/.00194$$

$$\simeq .002887$$

Then from equation 7-16:

$$\lambda_{u_p} = \frac{2\hat{\lambda}_p + (\beta K)^2\hat{C}_p + \sqrt{4\hat{\lambda}_p(\beta K)^2\hat{C}_p + (\beta K)^4\,\hat{C}_p{}^2}}{2}$$

where

$$(\beta K)^2\hat{C}_p = [(1.2280)(.842)]^2(.002887)$$

$$= 0.0030865$$

Solving:

$$\lambda_{u_p} = \frac{2(.00194) + 0.0030865 + \sqrt{.00003348}}{2}$$

$$\lambda_{u_p} = 0.0064 \text{ failures/mission}$$

Therefore:

$$R_{L_p} = e^{-0.0064} = .9936$$

The approximation can be extended directly to an n-element parallel configuration p,

$$\hat{\lambda}_p = \prod_{i=1}^{n} \hat{\lambda}_i$$

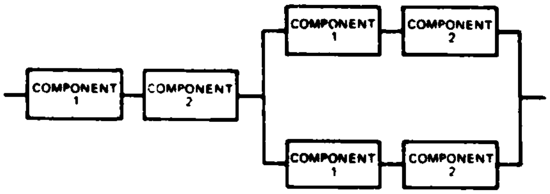
or, for n identical λ_i,

$$\hat{\lambda}_p \cong \hat{\lambda}^n$$

Bias correction equations for parallel configurations of three or more elements are cumbersome and of little overall effect in system calculations where many other elements are in series. Therefore, the estimates $(\hat{\lambda})^3$, $(\hat{\lambda})^4$, etc. may be used directly in most such applications.

Series-Parallel System, Interval Estimation Exponential Components – Rubinstein Method

Consider the following series-parallel system composed of components 1 and 2 in the example of 7.1.1.



This system's structure reduces to

Best estimates are:

$$\hat{\lambda}_{s+p} = \hat{\lambda}_s + \hat{\lambda}_p = .05 + .00194$$

$$= .05194 \text{ failures/mission}$$

$$\hat{R}_{s+p} = e^{-.05194} = .9494$$

where the subscripts indicate the series and parallel groups, respectively.

As previously estimated, the variance of the series portion is:

$$\hat{\sigma}^2_{\hat{\lambda}_s} = .00056$$

To this must be added the variance of the parallel portion, estimated as (Equation 7-21):

$$\hat{\sigma}^2_{\hat{\lambda}_p} = 4(\hat{\lambda}_s)^2\, \hat{\sigma}^2_{\hat{\lambda}_s} = 4(.05)^2\,(.00056)$$

$$= .0000056$$

giving,

$$\hat{\sigma}^2_{\hat{\lambda}_{s+p}} = \hat{\sigma}^2_{\hat{\lambda}_s} + \hat{\sigma}^2_{\hat{\lambda}_p} = .00056 + .0000056$$

$$= .000566$$

Then,

$$\hat{C}_{s+p} = \hat{\sigma}^2_{\hat{\lambda}_{s+p}} / \hat{\lambda}_{s+p} = .000566/.05194$$

$$= .01090$$

and,

$$\lambda_{u_{s+p}} = \frac{2(.05194) + [(1.174)(0.842)]^2\,(.01090) + \sqrt{.00232628}}{2}$$

$$\lambda_{u_{s+p}} = .0814 \text{ failures/mission}$$

$$R_{L_{s+p}} = e^{-.0814} = .9218$$

7.2.2 Interval Estimation for Systems of Exponential Components Using Bayesian Rubinstein Method

For systems, Bayesian estimates combine with test data in a manner precisely analogous to the Rubinstein method; that is, the posterior estimates of $\hat{\lambda}_i = (\phi+x)/(\tau+t)$ and of $\hat{\sigma}^2_{\hat{\lambda}_i} = (\phi+x)/(\tau+t)^2$ are used in the equations:

$$\hat{C}_i = \frac{\hat{\sigma}^2_{\hat{\lambda}_i}}{\hat{\lambda}_i}$$

$$\hat{\lambda}_{u_i} = \frac{2\hat{\lambda}_i + (\beta K)^2\hat{C}_i + \sqrt{4\hat{\lambda}_i(\beta K)^2\hat{C}_i + (\beta K)^4\hat{C}_i^{\,2}}}{2}$$

Since non-integer numbers of failures can appear in the Bayesian formulation, it may be necessary to compute β by linear interpolation between values given in figure 5-25. Note that the zero failure case is not usually applicable since in the Bayesian case ϕ, the pseudo failures, is never zero.

Example, Interval Estimation, Exponential Components, Bayesian Rubinstein Method

The Bayesian upper bound on failure rate is calculated in essentially the same manner as just illustrated for the Rubinstein method. For example, using the information contained in figures 7-4, 7-5, and 7-6 and § 7.1.3 the sample calculation at the system level is illustrated for the prediction and final (report 3) calculation.

Prediction

$$\widehat{C}_{Prior} = \frac{\sum_{i=1}^{n} \frac{\lambda_{o_i}}{T}}{\lambda_{o_{system}}}$$

from figure 7-4 we have:

$$\widehat{C}_{Prior} = \frac{\frac{.00040}{50} + \frac{.00670}{180} + \frac{.00600}{1000} + \frac{.00011}{90}}{0.01321}$$

$$\widehat{C}_{Prior} = 0.00397$$

$$\lambda_{u_o} = \frac{2\lambda_o + (\beta K)^2 \widehat{C}_{Prior} + \sqrt{4\lambda_o (\beta K)^2 \widehat{C}_{Prior} + (\beta K)^4 \widehat{C}^2_{Prior}}}{2}$$

and K is 0.842 for 80% confidence, β (1.1983) is found by interpolation in figure 5-25 at 7.23 failures, $(\Sigma\phi)$.

$$\lambda_{u_{os}} = \frac{2(.01321)+[(1.1983)(.842)]^2(.00397)+\sqrt{.00022989}}{2}$$

The predicted upper bound on system failure rate is:

$$\lambda_{u_{os}} = 0.02281 \text{ failures/mission}$$

The corresponding lower bound reliability prediction is:

$$R_{L_{os}} = e^{-\lambda_{u_{os}}} = 0.9774$$

Final Estimate (after report 3 – figure 7-5):

$$\widehat{C} = \frac{\sum_{i=1}^{n} \frac{\lambda_i}{t_i + \tau_i}}{\lambda_{system}}$$

From figures 7-4, 7-5 and 7-6:

$$\widehat{C}_s = \frac{\frac{.00008}{200+50} + \frac{.00786}{100+180} + \frac{.00600}{500+1000} + \frac{.00002}{400+90}}{.01396}$$

$$\widehat{C}_s = 0.00232$$

$$\lambda_{u_s} = \frac{2\widehat{\lambda}_s + (\beta K)^2 \widehat{C}_s + \sqrt{4\widehat{\lambda}_s (\beta K)^2 \widehat{C}_s + (\beta K)^4 \widehat{C}^2_s}}{2}$$

β = 1.1660 by interpolation in figure 5-25 at 11.23 failures

$$\lambda_{u_s} = \frac{2(.01396)+[(1.166)(.842)]^2(.00232)+\sqrt{.00012987}}{2}$$

$$\lambda_{u_s} = 0.02078 \text{ failures/mission}$$

$$R_{L_s} = e^{-.02078} = .9794$$

	Failure Rate $\widehat{\lambda}_s$	Reliability $\widehat{R}_s$	Failure Rate Upper Bound λ_{u_s}	Reliability Lower Bound R_{L_s}
Prediction	.01321	.9869	.02281	.9774
Report 1	.01787	.9823	.02859	.9718
Report 2	.01792	.9822	.02779	.9726
Report 3	.01396	.9861	.02078	.9794

Figure 7-9. System Results (Estimate and 80% Confidence Bound)

Figure 7-10 shows a comparison of the Rubinstein and Bayesian Rubinstein methods for the example given above. It can be seen that for both the best estimate and upper confidence limit, the earlier estimates of reliability using the Bayesian formulation are higher than by the Rubinstein approach

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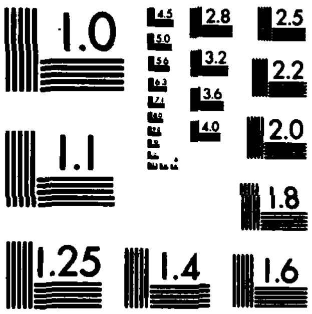

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alone; as test data increase, the measured reliability using the Rubinstein method increases.

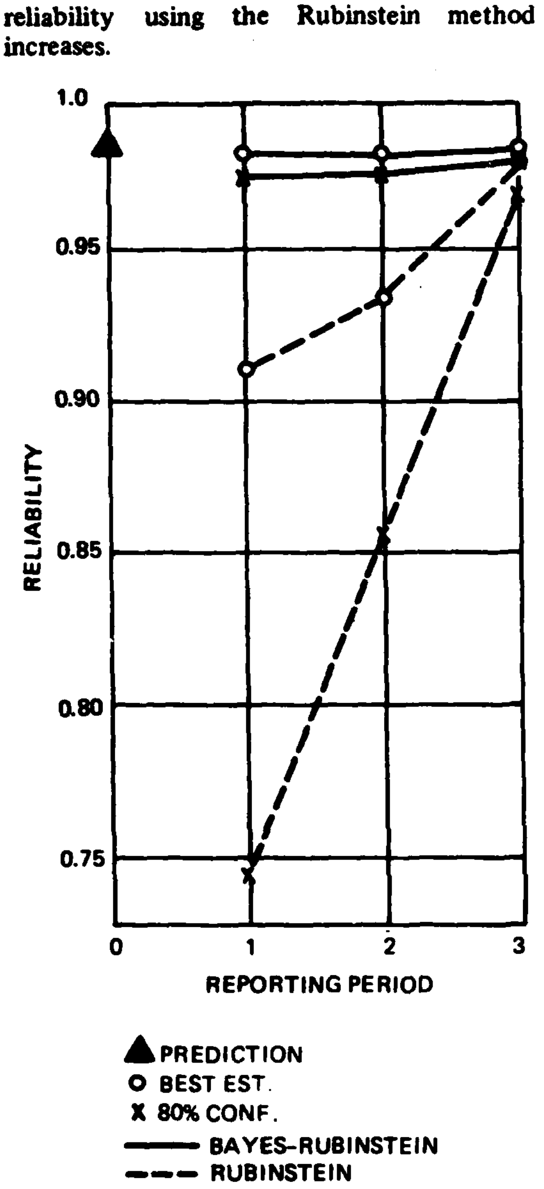

Figure 7-10. Comparison of Rubinstein and Bayesian Rubinstein Techniques of Reliability Measurement

7.2.3 Serial, Parallel, and Complex Systems of Exponential and Binomial Components – Approximation Methods

Ideally, one would like to avail oneself of a method which can handle a) serial systems, b) parallel systems, c) serial-parallel systems, d) complex systems, e) binomial components, f) exponential components, g) Weibull and other distributional components, h) multiple environments, i) multiple test states, j) mixed truncation testing, and which are k) tractable, and l) valid over all ranges of component reliabilities. Unfortunately, such a method does not exist. The best which can be hoped for is that a method will include as many as possible of the attributes a) through l).

The Rubinstein method presented in §7.2.1 and derived in [1] includes all attributes except d) and is untested for g). The Approximately Optimum (AO) method of Mann and Fertig [3, p. 517-524], [4] and Mann and Grubbs [5], makes use of their discovery that $-\ln R_s$, where R_s is a series system reliability, can be well approximated by a non-central chi-square distribution. The AO method does not include attributes d), g), h), i) and j), but is applicable to mixed binomial-exponential components. These two methods seem to be the very best among approximation methods.

If only very reliable systems are considered, and thus if attribute (l) is not a requirement, and if prior engineering knowledge is available, then it can be used in a non-Bayesian sense as a weighing factor in the method presented by Myhre, Rosenfeld and Saunders [6]. The authors have shown that their results are insensitive to fairly significant changes in the weighing factors and claim applicability to attributes a), b), c), d), e), and f) [type I censoring only].

7.2.4 Serial System of Exponential Components – The Fagan-Wilson Simulation Procedure

The procedure [7], which is Monte-Carlo, assumes that each component i follows an exponential failure model $R_i(t) = e^{-\lambda_i t}$, and that n independent components form a system with reliability model,

$$R_S(t) = f[(R_1(t), R_2(t), \ldots R_i(t), \ldots R_n(t)]$$

Tests are assumed to be terminated either at a fixed time t_0 (type I censoring), or after a particular failure occurs (type II censoring). The estimator $\hat{\lambda}_i$ is selected to be x_i/t_o in type I censoring. The distribution of λ_i is known to be $\chi^2_{2x_i+2}/2t_o$.

The procedure consists of generating by computer N chi-square samples for each component, which transform into as many simulated sets of $\hat{R}_i(t) = e^{-\hat{\lambda}_i(t)}$, i = 1, 2, 3, . . . n. The $R_i(t)$ are stored in memory, and N simulated values of $\hat{R}_s(t)$ are generated through the closed form system reliability model [7]. These N values of $\hat{R}_s(t)$ are ordered and 80% system reliability confidence bounds are obtained from the 20 percentile of the resulting histogram.

7.2.5 Interval Estimation for Systems of Components with Non-Exponential or Different PDF's – Monte-Carlo Simulation

Even for simple systems of two serial or two parallel components, closed form or approximation formulas are cumbersome or lacking when the components PDF's are non-exponential or different. Monte-Carlo estimates are then used to obtain a lower reliability bound, an upper bound on failure rate, and even a MTBF, even though it is only a point estimate. (Point estimates of reliability, however, can still be obtained readily by the methods of Section 7.1.)

The Monte-Carlo procedure which may take many different forms requires a great deal of sophistication [3], [7], [8]. For the simple two-component systems illustrated in figure 7-11, the procedure selected is as follows:

Step 1 – Draw a vector of random parameters from the joint PDF of PDF_1 and PDF_2, the PDF's of component 1 and component 2, respectively.

Step 2 – Find numerically, with the vector drawn at step 1 as parameters:

$$\text{MTBF}_{(\text{Series})} = E[\text{PDF}_1(t) \int_t^{\infty} \text{PDF}_2(\tau)d\tau + \text{PDF}_2(t) \int_t^{\infty} \text{PDF}_1(\tau)d\tau]$$

$$\text{MTBF}_{(\text{Parallel})} = E[\text{PDF}_1(t) \int_0^{\infty} \text{PDF}_2(\tau)d\tau + \text{PDF}_2(t) \int_0^{\infty} \text{PDF}_1(\tau)d\tau]$$

where E stands for expected value of

$$R_{(\text{Series})} = \int_t^{\infty} \text{PDF}_1(\tau)d\tau \cdot \int_t^{\infty} \text{PDF}_2(\tau)d\tau$$

$$R_{(\text{Parallel})} = 1 - \int_0^{t} \text{PDF}_1(\tau)d\tau \cdot \int_0^{t} \text{PDF}_2(\tau)d\tau$$

Step 3 – For each Monte-Carlo pass, record the MTBF's and the R's.

Step 4 – Construct a histogram of MTBF's and R's. The means of the resulting PDF's are $\overline{\text{MTBF}_s}$ and $\hat{R}_s$. (As stated before, $\hat{R}_s$ is not really needed since it can be obtained directly from $\hat{R}_1$ and $\hat{R}_2$.) The 20th percentile of the resulting PDF's are the respective 80% bounds on MTBF_s and R_s.

The results of the Monte-Carlo simulation are tabulated in figure 7-12.

7.2.6 System Availability Interval Monte-Carlo Simulation

A Bayesian Monte-Carlo simulation method to estimate the lower availability bound at confidence γ for a serial system of N independent components is presented in this paragraph.

While a point estimate of availability for such a system can be obtained by multiplying the individual estimates of component availability, such a procedure would be incorrect if performed with interval estimates. That is:

$$A_{S_\gamma} \neq \prod_{i=1}^{N} A_{i_\gamma}$$

Where A_{S_γ} represents a system availability bound at confidence γ, and A_{i_γ}, the ith component availability bound at confidence γ.

An interval estimate of system availability can be obtained by Monte-Carlo simulation based on data taken at component level. Input data consist of $\hat{\lambda}$, the point estimate of failure rate, $\hat{\mu}$, the point estimate of repair rate, x, the number of failures, and m, the number of repairs.

The core of the simulation technique is a Bayesian view which considers the true availability as a random variable and synthesizes its distribution $g(A|\hat{\lambda}, \hat{\mu}, x, m)$ conditioned on the estimate or, more correctly,

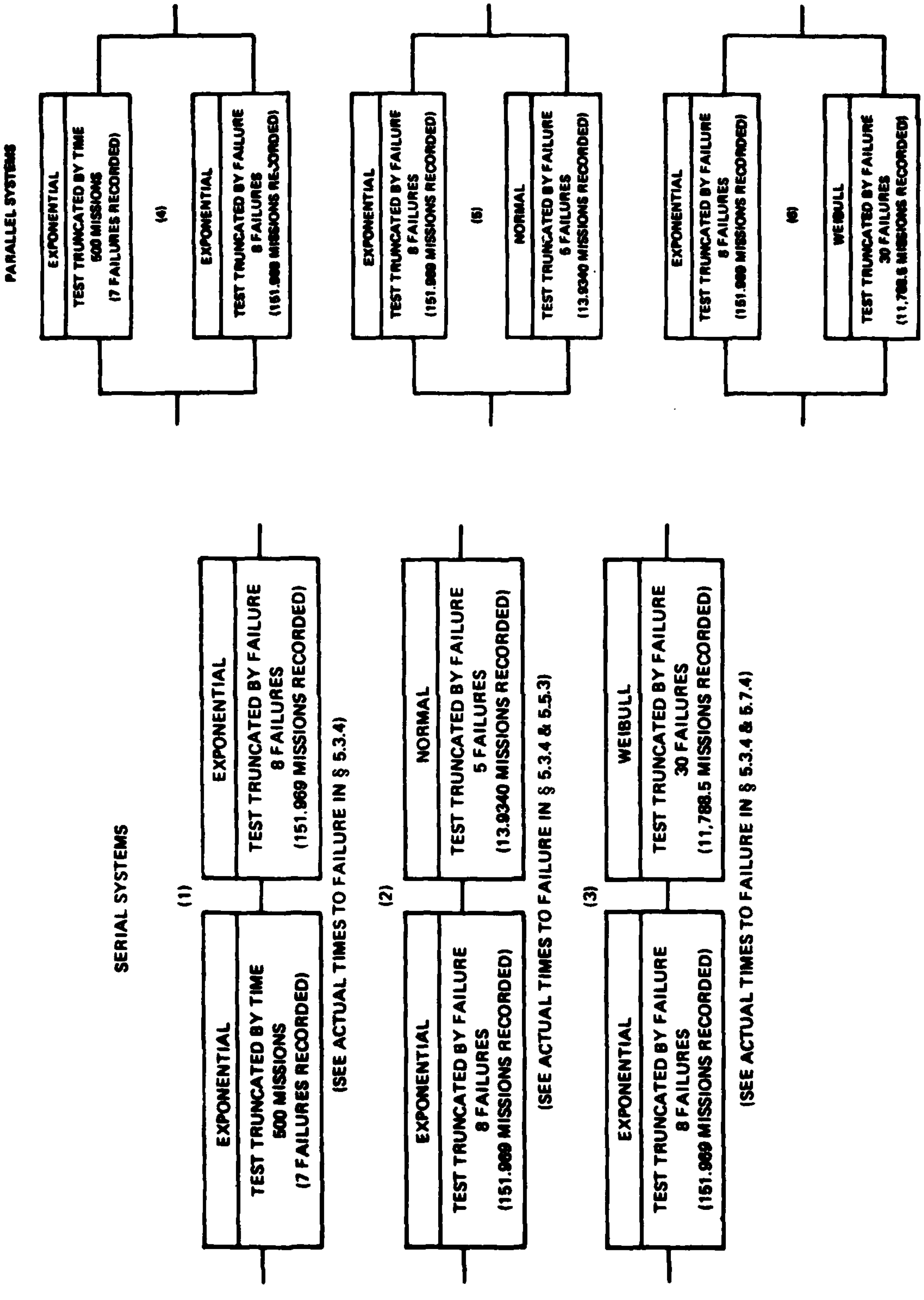

Figure 7-11. Serial and Parallel Systems of Two Components

Case #	Configuration	Components	$\hat{R}_S$	R_L (80% Confidence)	$\widehat{MTBF}_S$	$MTBF_L$ (80% Confidence)
(1)	Serial	Exponential Exponential	0.8755	0.8525	8.030	5.191
(2)	Serial	Exponential Normal	0.9441	0.9291	2.655	2.260
(3)	Serial	Exponential *Weibull*	0.8679	0.8463	15.97	10.55
(4)	Parallel	Exponential Exponential	0.9961	0.9448	27.69	17.65
(5)	Parallel	Exponential Normal	0.9997	0.9998	21.67	10.69
(6)	Parallel	Exponential Weibull	0.9957	0.9945	516.1	277.7

Figure 7-12. Results of Monte-Carlo Simulation

on the data generating the estimate. It has been shown that if u and v are independent random variables having chi-square distributions with 2x and 2m degrees of freedom respectively, then the ratio (u/2x)/(v/2m) has an F distribution with 2x and 2m degrees of freedom. Therefore:

$$\frac{\lambda}{\mu} \sim \frac{\hat{\lambda}}{\hat{\mu}} F_{2x,2m}$$

and an upper confidence bound on λ/μ is

$$\left(\frac{\lambda}{\mu}\right)_{1-\gamma} = \frac{\hat{\lambda}}{\hat{\mu}} F_{2x,2m,1-\gamma}$$

Since a lower confidence bound on availability is given by

$$A_\gamma = \frac{1}{1 + \left(\frac{\lambda}{\mu}\right)_{1-\gamma}} = \frac{1}{1 + \frac{\hat{\lambda}}{\hat{\mu}} F_{2x,2m,1-\gamma}}$$

it is apparent that sampling from $F_{2x,2m}$ is equivalent to sampling from $g(A \mid \hat{\lambda}, \hat{\mu}, x, m)$ by the transformation shown above. It is a simple task to sample from any desired F distribution, beginning with random numbers R[0,1] distributed uniformly on the interval zero to one, or beginning with random numbers N[0,1], distributed normally with zero mean and unit variance. The transformations are

$$N = \left(\sqrt{-2 \ln R_1}\right)(\cos 2\pi R_2)$$

$$v = \sum_{i=1}^{k} N_i^2$$

where v is a chi-square variate with k degrees of freedom. Two chi-square variates are formed by sampling normal variates as follows:

$$v = \sum_{i=1}^{2x} N_i^2$$

and

$$u = \sum_{i=1}^{2m} N_i^2$$

Then an F variate is formed as:

$$y = \frac{v/2x}{u/2m}$$

Note that y is a $F_{2x,2m}$ variate and v and u are independent χ^2_{2x} and χ^2_{2m} variates [8]. Thus 2x+2m independent normal variates must be drawn to construct one F variate, which is then transformed to a sample value A of availability for the component by:

$$A = \frac{1}{1 + \frac{\hat{\lambda}}{\hat{\mu}} y}$$

The computer executes the above algorithm for each component, stores the results, then uses the stored values to solve the system model for system availability. That represents one pass through the simulation procedure. Repetition of the process builds up a histogram of the sample values of system availability which approaches the shape of $g(A \mid \hat{\lambda}, \hat{\mu}, x, m)$ as the number of passes increases. The desired interval estimate is obtained simply by reading the appropriate percentile values.

When the repair time variable is log-normally distributed, the inverse function $p^{-1}(u) = R$ can be solved for the coefficient a in the denominator below by numerical integration on each pass, then substituted into

$$A = \frac{1}{1 + \frac{\hat{\lambda}}{\hat{\mu}} \left(\frac{ae^{\sigma^2/2}}{2x} \right)}$$

Note that a in the denominator is the coefficient of figure E-9 of Appendix E, not availability.

Or, much more simply, a log-normal interval can be formed by sampling the λ and μ variables independently and forming their quotient.

$$\lambda \sim \hat{\lambda} \chi^2 / 2x$$

$$\frac{1}{\nu e^M} \sim \Lambda \left(0, \frac{\sigma^2}{m}\right)$$

Where Λ is the log-normal distribution function, M is the mean of the corresponding normal distribution, ν is the reciprocal of geometric mean corrective maintenance time and z is the standard normal random variate.

$$\ln\left(\frac{1}{\hat{\nu}}\right) - M = z \sim N\left(0, \frac{\sigma^2}{m}\right)$$

$$\mu \sim \exp\left[\ln\left(\frac{1}{\hat{\nu}}\right) - z + \frac{\sigma^2}{2} \right]$$

As in Appendix C, $\sigma^2 = \hat{\sigma}^2$ is assumed. Successive samples of λ and μ are inserted into the expression for A and its histogram is built up. Operating arithmetically with limits computed for each parameter separately will give a much larger "at least" type interval. For example, if A_L is computed using 80% limits on λ and μ, the resulting limit defines a 64% interval for A.

It is easy to read from the histogram a variety of relevant statistics with standard errors which are entirely under control, since they depend only on the number of passes n. Specifically, one can read the mean or expected value, the mode or maximum likelihood value, the median or fifty percent confidence limit, any desired percentiles in order to construct one-sided or two-sided interval estimates, and the range. The standard error of each of these estimates, except the mode, are computable by reference to Kendall [9]. Briefly, the standard errors are, for the mean $\hat{A}$,

$$\sigma_{\hat{A}} = \sigma_A / \sqrt{n}$$

where n is the number of passes, and for the pth and (100-p)th percentiles,

$$\sigma_{A_p} = \phi \sigma_{\hat{A}}$$

Kendall tabulates a few values of the ratio φ, which is symmetrical about the median. When simulating is done often, it is useful to fit a smooth curve (Figure 7-13) and express φ as a function of the desired percentile [7]. The standard error of the mode is available with somewhat greater effort by use of Yasukawa's method [10].

$$\phi = 1.93637 - 2.86403p + 2.86403p^2$$

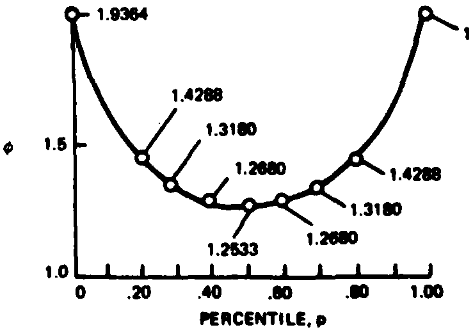

Figure 7-13. Standard Error of a Percentile as a Multiple of Standard Error of Mean

7.3 References

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Section 8
RELIABILITY DEMONSTRATION

8.0 INTRODUCTION

Reliability demonstration is appropriate for newly designed equipment, equipment that has been modified, and equipment that is of unproven reliability or of previously unacceptable reliability. It is best performed at the highest feasible assembly level using equipment as close to the production configuration as possible.

Selection and scheduling of the demonstration test(s) should be an integral part of the Integrated Test Program (ITP), with completion of the demonstration tests and if applicable, retests, to occur prior to starting the production program.

This section provides guidance for approaching reliability demonstration in an orderly and timely way by giving stepwise information on the conduct of a reliability demonstration program from categorization of equipments for demonstration through reporting the final results of the demonstration test.

Minimum contents of the demonstration test plan are provided (§ 8.2.2) along with a "road map" of the reliability demonstration process (Figure 8-1). Additional details on content requirements of the demonstration test plan are provided in § 10.1.1.2 and NAVORD OD 42282.

A brief discussion on the background of reliability demonstration is also provided.

8.1 BACKGROUND

The validity of any demonstration depends on the statistical regularity of the process observed, that is, on the variability of the sample system(s) in repeated tests and operations, similarity of later systems to those comprising the sample, control of test or operating conditions, accuracy of performance measurements, integrity of test results and consistent definition of failure and success in the tests. It also depends to some extent on how well the demonstration model reflects the actual factors that influence system performance.

The measure of reliability most applicable to a particular system and mission should be the specified reliability parameter and the basis for the reliability demonstration. For systems having constant failure rates (exponentially distributed failure times) the customary measures are MTBF or failure rate. Other measures, such as probability of failure or reliability, are customary for ~~one-shot~~ devices, cyclic equipment, and systems with failure times not exponentially distributed.

Traditionally, reliability demonstration has been implemented as hypothesis testing. When this approach is taken, it is possible to estimate the sample size necessary to achieve reliability demonstration with the agreed upon risks. Although interval estimation and hypothesis testing are related, interval estimation cannot address the determination of sample size to satisfy specified decision risks. These risks are commonly referred to as the producer's risk, α (the probability of a test rejecting an item which complies with the design objective), and the consumer's risk, β (the probability of a test accepting an item which has the minimum reliability). These two specified reliability parameters, design objective and minimum reliability, are equivalent to specifying MTBFs (θ_0 and θ_1) for equipments following an exponential distribution. The relationships of θ_0 and θ_1 to α and β are represented in figure 8-2.

When demonstration consists of hypothesis testing, the reliability measure of interest is treated as a constant system parameter not known with precision. The hypothesis testing approach is useful when an adequate test sample is available and when schedules permit the extended testing typically needed. Moreover, hypothesis testing is conceptually valid

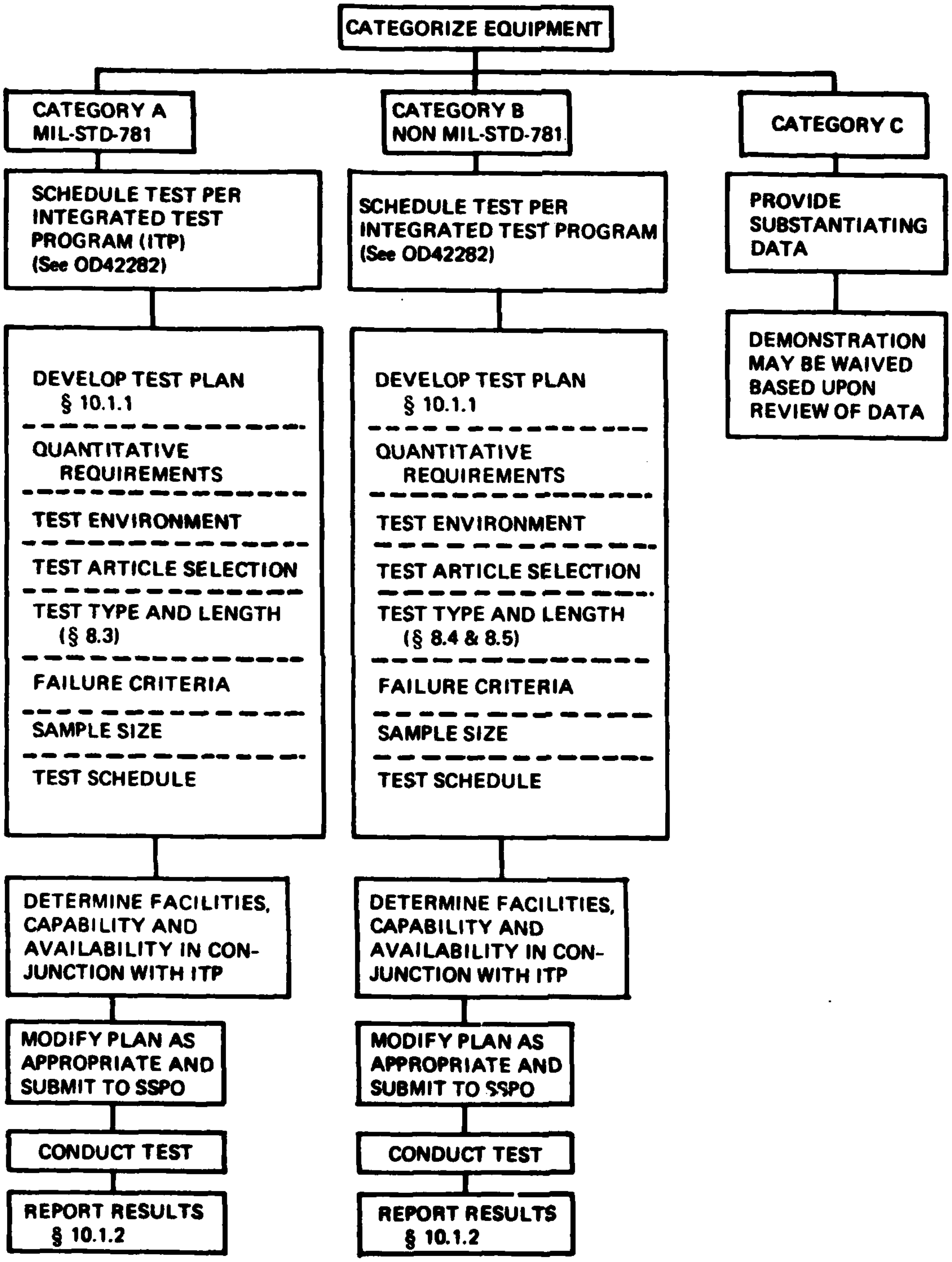


Figure 8-1. Reliability Demonstration Process

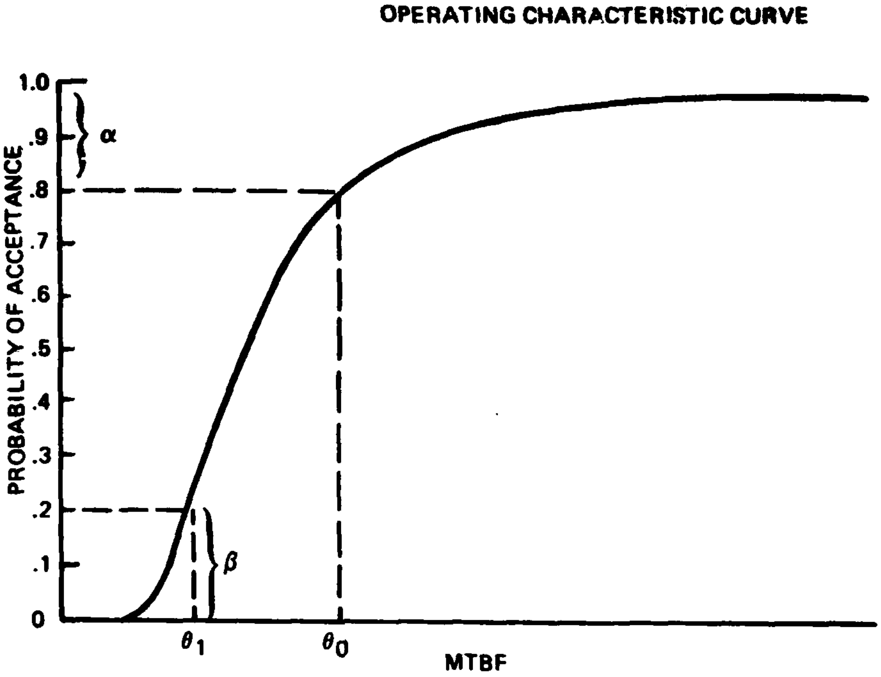


Figure 8-2. $\alpha, \beta, \theta_o, \theta_1$ Relationships

when the system design is fixed and no product improvement program is underway. Military specifications such as MIL-STD-781[1] embody the hypothesis testing approach to demonstration.

MIL-STD-781[1] applies to devices having constant failure rates (exponentially distributed times between failure). It provides fixed length tests based on the chi-square interval method and sequential (variable length) tests based on Wald's sequential testing method. In the C revision of MIL-STD-781[1], fixed length tests vary in length from 1.1 to 45 times the minimum specified MTBF, and have α and β risks ranging between 10 and 30 percent. Sequential tests select between two alternate hypotheses but do *not* provide an estimate of the MTBF expected in service. Thus, sequential tests are not directly applicable when such estimates are needed, however, the data from the tests can be used in other models to obtain the MTBF estimate. The total test time (hence also cost) of a sequential demonstration is a random (though bounded) variable. Usually, however, sequential tests require less time to complete than fixed length tests of equal power. The highest risk sequential test plan of MIL-STD-781C[1] requires from 1.72 to 4.5 times the minimum acceptable (lower test) MTBF (θ_1). Both demonstration approaches of MIL-STD-781[1] are discussed in § 8.3.

When a standard test of MIL-STD-781[1] is invoked by specification, it is general practice to specify the minimum acceptable reliability (R_L*) or MTBF (θ_1), the test selected, and the environmental level at which testing will be performed. This constitutes a complete specification because the test selected contains the discrimination ratio (θ_o/θ_1) and the α and β risks.

Trade-off studies, based on such things as cost, schedule, test samples, need to determine design stability and risk, should be conducted to determine the type of test to be selected.

A more recent concept of demonstration, which is gaining increasing favor, applies to systems undergoing progressive modifications which are taken in response to design improvements and corrective actions for early failures. Reliability and availability measures are treated as variable system properties that improve or grow primarily as a result of progressive weeding out of failure mechanisms by corrective actions as time and failures accumulate. A basic growth model was described empirically in 1964 by J.T. Duane[2], has subsequently gained wide acceptance, and is reflected in standards such as MIL-STD-1635[3] for test-analyze-and-fix (TAAF) programs. Statistical limitations of Duane's model were addressed by Dr. Lawrence Crow[4]. Dr. Crow derived more rigorous estimates of the model's parameters, which is discussed in § 5.4.4. Additional detail is provided in draft MIL-STD-781D[1].

8.2 DEMONSTRATION TESTING

Reliability demonstration testing consists of three major steps: Equipment Categorization, Test Plan Development, and Test Implementation. Figure 8-1 outlines the demonstration process and the following paragraphs provide discussion of equipment categories, test criteria, and test plans.

8.2.1 Procedure

The first step in the demonstration process is the determination of the equipment for which demonstration is required. This is often clearly defined in contractual specifications. Reliability demonstration equipment categories are indicated in figure 8-3. Equipment included in category A should normally be subjected to MIL-STD-781[1] testing. Equipment in category B (e.g. one-shot devices) requires alternate demonstration plans which meet the intent to demonstrate required reliability prior to the production program. Equipment for which there are sufficient patrol data to document and substantiate that the required specified reliability has been achieved may fall in category C. Equipment in category C should not be required to have a formal demonstration of reliability.

The second step in the demonstration process is preparation of the Reliability Demonstration Test Plan (§ 10.1.1.2). In all cases this plan must discuss at least:

Quantitative requirements, § 8.2.2.1,
Test environment, § 8.2.2.2,
Test article selection, § 8.2.2.3,
Test type and length, § 8.2.2.4,
Failure criteria, § 8.2.2.5, and
Sample size, § 8.2.2.6.

Test methods are described in § 8.3 for mature (category A) systems and equipments with constant failure rate. Non-exponential reliability demonstration procedures for mature (category B) systems and equipments are covered in § 8.4. A reliability growth demonstration procedure is discussed in § 8.5.

Equipment Category	Definition
A	Equipment to which MIL-STD-781[1] testing (§ 8.3) applies: 1. Equipment contract requires test 2. Exponential distribution applies 3. New Design 4. Modified Design 5. Existing Design with unproven or previously unacceptable reliability
B	Equipment to which non-MIL-STD-781[1] testing (§ 8.4) applies: 1. Equipment contract requires test 2. Exponential distribution does not apply 3. New Design 4. Modified Design 5. Existing Design with unproven or previously unacceptable reliability
C	Equipment now in use aboard submarines which has exhibited reliability levels equal to or exceeding requirements of the TOG or the subsystem spec.

Figure 8-3. Reliability Demonstration Equipment Categories

The third step in the demonstration process is to conduct the test in accordance with the approved plan and report the results. Reporting is discussed in § 10.1.2.3.

8.2.2 Development of Test Plan/Test Criteria

8.2.2.1 Quantitative Requirements

Design requirements for reliability are derived from the top level requirements for the system which contains the equipment to be tested. Such requirements are based on strategic (mission) objectives and the system reliability objective as apportioned to lower indenture levels. This process establishes meaningful reliability objectives for design to achieve the weapon system objectives.

Minimum acceptable reliability objectives are established by the user (operator) and are based on operational constraints such as availability, reliability and logistics capabilities which affect the user's ability to successfully accomplish the mission. Minimum acceptable reliability is understood as the test requirement to be demonstrated with statistical confidence.

The FBMWS/SWS and constituent subsystems reliability design objectives are specified in the pertinent TOG[11]. Subsystem values are subsequently allocated to lower level system elements, as required, and documented in product specifications. The minimum acceptable reliability for the FBMWS/SWS, established by the CNM[12], is also allocated to lower system elements as required for demonstration test planning. Realistic values for α and β are determined by coordinated consideration of producer/consumer needs, test costs, and other program constraints, e.g. schedules. A complete specification for reliability demonstration is established when quantitative values are assigned to the four parameters shown in figure 8-4.

Quantities Specified			
Design Objective	Producer's Risk	Minimum Reliability	Consumer's Risk
R^* (TOG or Subsystem Spec)	α (Coordinated)	R_L^* (NAVMAT)	β (Coordinated)

Figure 8-4. Complete Reliability Specification

8.2.2.2 Test Level

Results of the mission analysis should be used to determine the environmental conditions and duty cycle to be proposed in the demonstration plan. If a MIL-STD-781[1] test is to be performed, the results of the mission analysis enables the selection of the appropriate test cateogry {e.g., Category 3A, Shipboard Equipment Sheltered of MIL-STD-781C[1]} or justification for a modified test level. If the mission analysis indicates that the MIL-STD-781[1] categories must be modified, then a unique test level shall be established and documented in the test plans and procedures. The Reliability Demonstration Test Plan must address any differences between test environment and mission environments and the effect of these differences on demonstrated reliability.

8.2.2.3 Selection of Test Article

Reliability demonstration tests should be performed using samples of intended production (i.e., manufactured to production drawings using production tooling on production lines, and inspected and tested to approved procedures using production test and measurement equipment).

The demonstration plan should clearly specify the configuration which will enter the test. The test plan should identify any differences between the production configuration and the test sample, and the effect of these differences on demonstrated reliability.

8.2.2.4 *Test Plan Selection*

For equipment in Category A (Figure 8-3) a test plan is selected from MIL-STD-781[1]. It is recognized that when values specified for R^* and R_L^* (Figure 8-4) are converted to θ_u and θ_l, they will not, in general, yield discrimination ratios (θ_u/θ_l) which correspond exactly to the ratios contained in MIL-STD-781[1]. Therefore, some adjustments may be required in selecting the test plan which best satisfies the requirement. Since MIL-STD-781[1] has variable length [probability ratio sequential test (PRST)] and fixed length test plans, trade-off studies should be conducted to determine the type of test which should be performed. Bases for the trade-off studies include cost, schedule, need to determine design stability, and risk. PRST plans have generally been preferred to fixed length test plans. Category B equipment is discussed in § 8.4.

8.2.2.5 Failure Criteria

Failure criteria for each equipment to be tested during the demonstration test must be included in the demonstration plan. The failure categories defined in MIL-STD-781[1] should be used to guide this effort. Particular consideration should be given to the recognition and treatment of procedural errors (e.g., operator induced, inadequate documentation) and software errors. All failures are relevant failures unless proven otherwise.

8.2.2.6 Sample Size

Determine the quantity of each equipment to be used in the reliability demonstration test program. The sample size required should be based upon the expected duration of the test for each equipment, required completion date, possible number of test articles available, and test facilities capability. It should also consider statistical variability (i.e., it is desirable to test a sample large enough to offer some assurance that the tested items are representative of the population) and the need to establish design stability [the effects of time and environments (not considered life tests)]. Equipments characterized by long operating times and comparably high times between failures should be tested in quantity to obtain the maximum amount of test time and information in the shortest calendar time. Note that MIL-STD-781[1] requires that each test sample operate at least one half the average operating time of all equipments on test. A frequent drawback in reliability demonstration testing is that a sample size of one is often used. Better test planning could alleviate this problem in many cases.

8.3 MIL-STD-781[1] DEMONSTRATION TESTS

The test methods described in this paragraph are for equipment that exhibit an exponential distribution of time-to-failure.

8.3.1 Hypothesis Testing (Wald's Probability Ratio Sequential Method)

Sequential tests permit one of three decisions to be made after each observation—accept the test hypothesis, reject the test hypothesis and accept an alternative hypothesis, or continue testing.

Typically the test hypothesis is θ (the true MTBF) = θ_0, (the design value of MTBF). The alternative hypothesis is $\theta = \theta_1$ (the minimum specified value). If $\theta = \theta_0$ the series of observations $x_1, x_2, x_3 \ldots x_n$ is distributed with probability density,

$$p_0(n) = \prod^{n} f(x, \theta_0)$$

where $f(x, \theta_0)$ is the density function applicable to the variable x with parameter θ_0. Likewise, if $\theta = \theta_1$,

$$p_1(n) = \prod^{n} f(x, \theta_1)$$

Thus Wald's method is applicable to any attribute or variable the density function of which is known or assumable a priori.

After each observation the ratio $p_1(n)/p_0(n)$ is tested in the inequality

$$\frac{\beta}{1-\alpha} < \frac{p_1(n)}{p_0(n)} < \frac{1-\beta}{\alpha}$$

where α and β are type I and type II risks respectively. Testing is continued until $p_1(n)/p_0(n)$ fails to satisfy the inequality. The test always converges to a decision and generally requires about half the test time of a fixed length demonstration of equal power.

8.3.1.1 Accept and Reject Criteria

When t is time to failure and $f(t,\theta)$ is an exponential density function, the following test values are computed.

Accept y intercept $$(+h_0) = +\frac{-\ln\frac{\beta}{1-\alpha}}{\frac{1}{\theta_1} - \frac{1}{\theta_0}} \quad (8\text{-}1)$$

Reject y intercept $$(-h_1) = -\frac{\ln\frac{1-\beta}{\alpha}}{\frac{1}{\theta_1} - \frac{1}{\theta_0}} \quad (8\text{-}2)$$

Slope $$(s) = \frac{\ln(\theta_0/\theta_1)}{\frac{1}{\theta_1} - \frac{1}{\theta_0}} \quad (8\text{-}3)$$

The equations for the accept and reject lines are respectively:

$$T = h_0 + sx \qquad (8\text{-}4)$$

$$T = -h_1 + sx \qquad (8\text{-}5)$$

After total unit test time T with x failures observed, decisions are made in accordance with the criteria

Reject if $T < (-h_1 + sx)$

Accept if $T > (h_0 + sx)$

Continue testing if $(-h_1 + sx) < T < (h_0 + sx)$.

The expected total test time to a terminal decision point is:

$\underline{\theta = \theta_0}$

$$E[T] = \theta_0 \left(\frac{(1-\alpha)\ln\frac{\beta}{1-\alpha} + \alpha\ln\frac{1-\beta}{\alpha}}{\ln(\theta_0/\theta_1) + 1 - (\theta_0/\theta_1)} \right) \qquad (8\text{-}6)$$

$\underline{\theta = \theta_1}$

$$E[T] = \theta_1 \left(\frac{\beta\ln\frac{\beta}{1-\alpha} + (1-\beta)\ln\frac{1-\beta}{\alpha}}{\ln(\theta_0/\theta_1) - 1 + (\theta_1/\theta_0)} \right) \qquad (8\text{-}7)$$

Figure 8-5 is an example of a sequential plan with discrimination ratio $(\theta_0/\theta_1) = 2.0{:}1$ and decision risk ($\alpha = \beta = 20$ percent). The plan is obtained as follows:

From equation 8-1:

$$+h_0 = \frac{-\ln\frac{.20}{1-.20}}{\frac{1}{\theta_1} - \frac{1}{\theta_0}} = +1.3863(2\theta_1)$$

since $\theta_0 = 2\theta_1$

$$+h_0 = +2.7726\theta_1$$

Similarly, from equation 8-2:

$$+h_1 = +2.7726\theta_1$$

Using equation 8-3:

$$s = \frac{\ln\left(\frac{2\theta_1}{\theta_1}\right)}{\frac{1}{\theta_1} - \frac{1}{2\theta_1}} = +2\theta_1 \ln 2 \text{ where } \theta_0 = 2\theta_1$$

$$s = +1.3863\theta_1$$

Therefore the accept line is

$$T = h_0 + xs$$

$$= 1.3863(x+2)\theta_1$$

and the reject line is

$$T = -h_1 + xs$$

$$= 1.3863(x-2)\theta_1$$

MIL-STD-781[1] plots failures on the y axis and test time on the x axis. Solving equation 8-4 and 8-5 for x

$$x = \frac{T - h_0}{s} \qquad (8\text{-}8)$$

$$x = \frac{T + h_1}{s} \qquad (8\text{-}9)$$

and inserting the values for h_0, h_1 and s gives

$$x = \frac{T - h_0}{s} = \frac{T}{1.3863\theta_1} - 2$$

$$x = \frac{T + h_1}{s} = \frac{T}{1.3863\theta_1} + 2$$

This information is plotted in figure 8-6.

Figure 8-7 is sequential test plan IVC of MIL-STD-781C[1]. This is similar to figure 8-6 except that the equation for the reject line used in the MIL-STD is

$$x = \frac{T}{1.3863\theta_1} + 1.5$$

(i.e., the reject line dropped 0.5 failures) and truncation lines have been added.

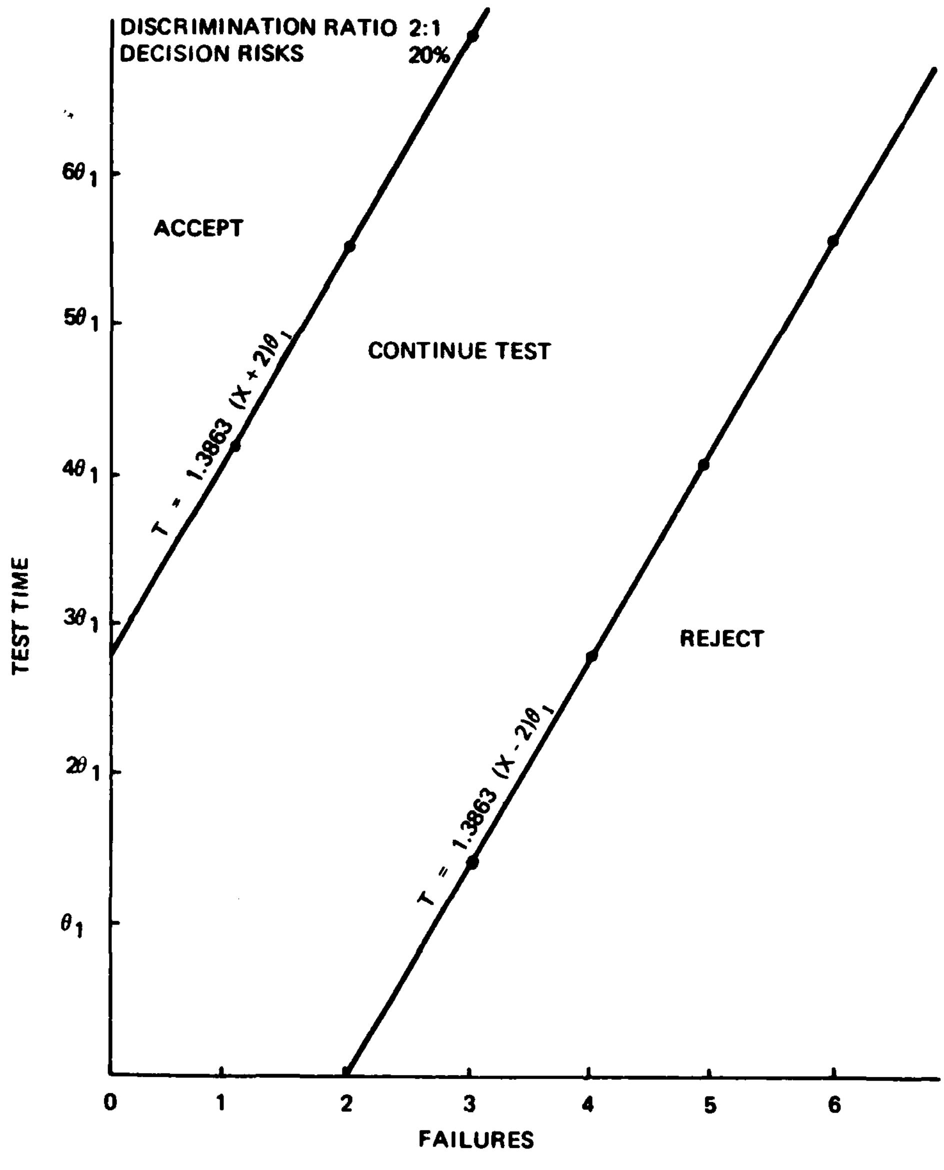


Figure 8-5. Sequential Test Plan Test Time Versus Failures

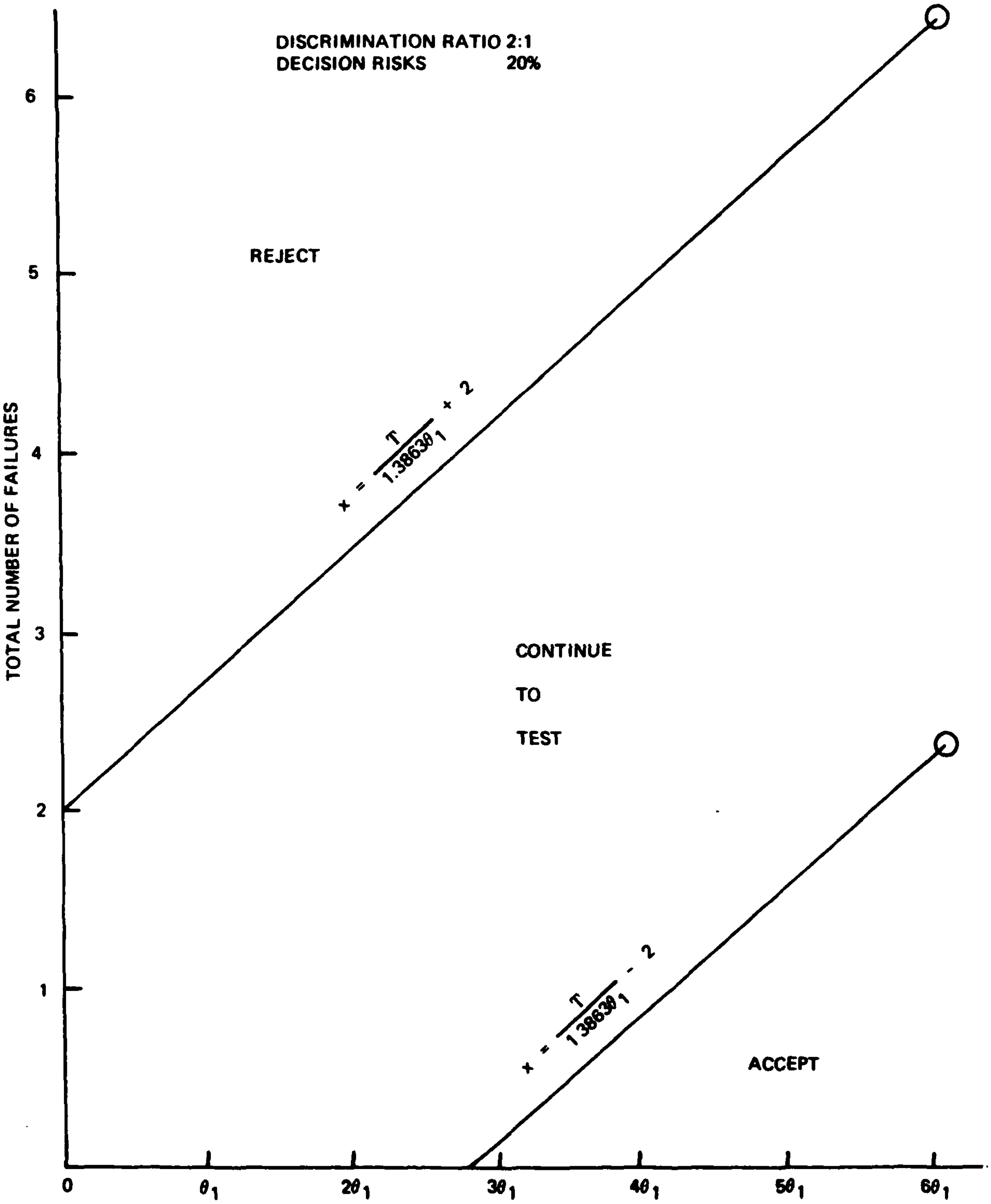
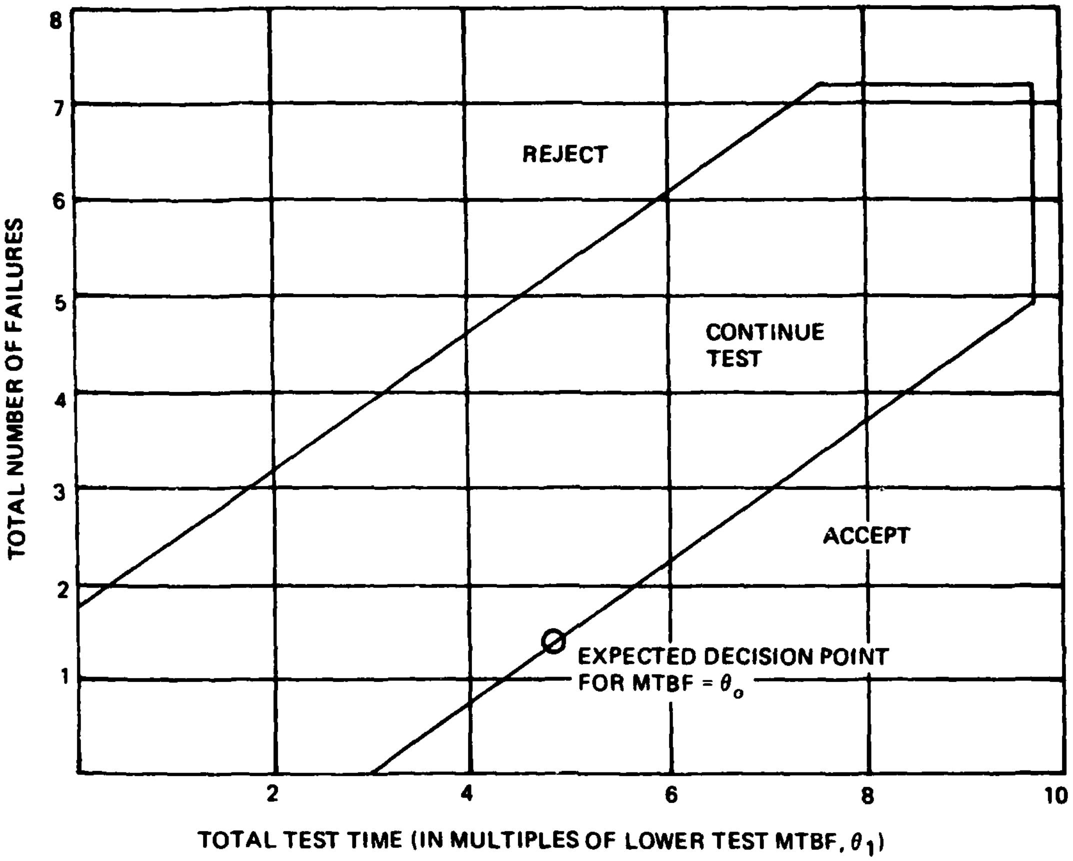


Figure 8-6. Sequential Test Plan, Failures Versus Test Time

DECISION RISKS (NOMINAL) 20 PERCENT
DISCRIMINATION RATIO 2.0 : 1



TOTAL TEST TIME*		
NUMBER OF FAILURES	REJECT (EQUAL OR LESS)	ACCEPT (EQUAL OR MORE)
0	N/A	2.80
1	N/A	4.18
2	.70	5.58
3	2.08	6.96
4	3.46	8.34
5	4.86	9.74
6	6.24	9.74
7	7.62	9.74
8	9.74	N/A

*Total test time is total unit hours of equipment on time and is expressed in multiples of the lower test MTBF. Refer to § 4.5.2.4 of MIL-STD-781C[1] for minimum test time per equipment.

Figure 8-7. Sequential Test Plan IVC of MIL-STD-781C[1] -

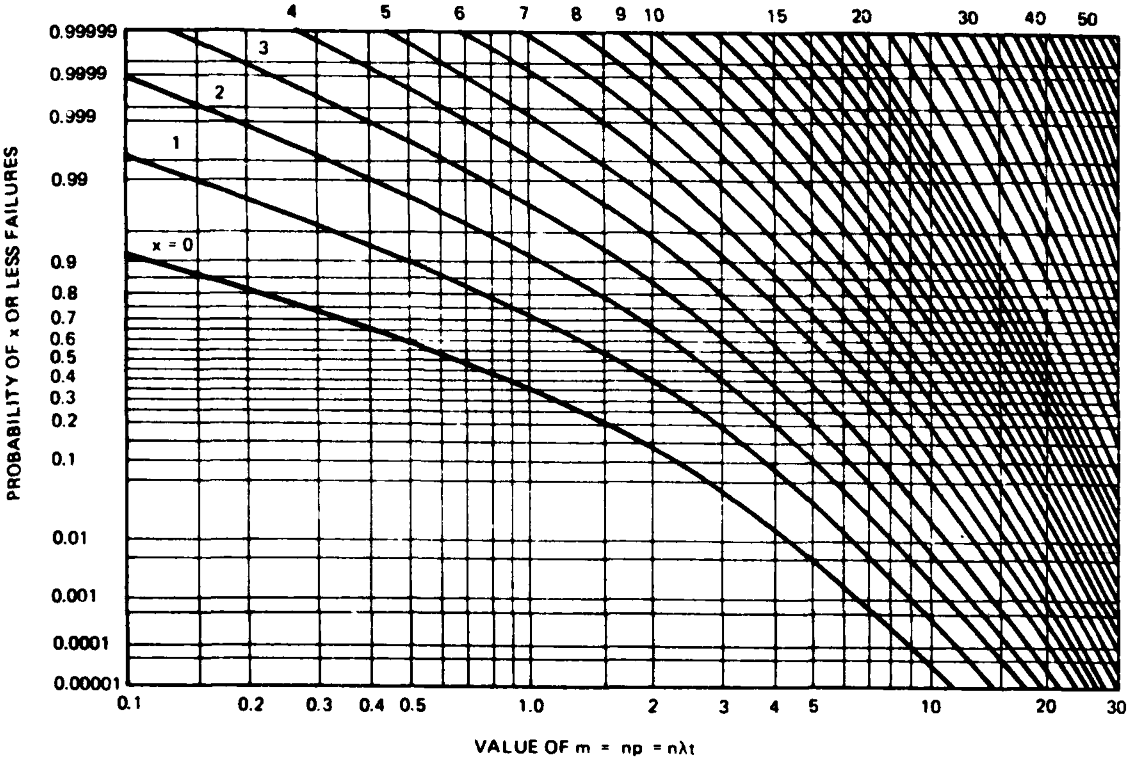


Figure 8-8. Thorndike Chart

MIL-STD-781 does not indicate the procedure used in developing these truncation lines. A method for truncation is provided in NAVORD OD 41146[8] and in § 8.3.1.2.

8.3.1.2 Truncation Criteria

Figure 8-8 is a Thorndike Chart which is used to establish truncation lines according to the following steps:

1. Define "Probability of x or Less Failures" ordinates for Figure 8-8 corresponding to (1-α) and β. For this example the ordinates will be 0.8 and 0.2 respectively.
2. Determine from figure 8-8 abscissa values of np corresponding to the ordinates of step 1 and values of x (number of failures). Enter these values in a table (figure 8-9 for the example) starting with x = 0.
3. Calculate the ratio of $np_{(1-\alpha)}/np_{\beta}$ for each value of x used in step 2 until you find the smallest value that will give a ratio of $np_{(1-\alpha)}/np_{\beta}$ that is greater than θ_1/θ_0 used in the sequential test plan being considered. In this case $\theta_1/\theta_0 = .50$ and the corresponding value of x is 6 failures. (See figure 8-9, value 0.511).
4. Truncate the test at the number of failures (x_0) equal to one plus the value determined in step 3. (In the example $x_0 = 7$ failures).
5. Test time is truncated at x_0 times the slope of the accept/reject lines, in this example 7 x 1.38630_1 or $9.7\theta_1$. Figure 8-7 shows that MIL-STD-781[1] truncated this test at $9.74\theta_1$ and 7.1 failures.

	FAILURES							
	0	1	2	3	4	5	6	7
$m_{(1-\alpha)}$	0.22	0.80	1.50	2.30	3.00	3.80	4.60	5.60
$m_{(\beta)}$	1.50	3.00	4.20	5.40	6.60	8.00	9.00	10.00
$\frac{m_{(1-\alpha)}}{m_{(\beta)}}$	0.146	0.267	0.357	0.426	0.455	0.475	0.511	0.560

Figure 8-9. Failures Required for Truncation

Another criteria sometimes used for truncation is to use three times the number of failures required in a fixed length test (§ 8.3.2) with the same risks.

8.3.2 Chi-Square Method (Fixed Length Tests)

MIL-STD-781[1] has nine fixed lengths and three additional high risk fixed lengths tests. The chi-square method can be used when other fixed length test plans are to be developed for reliability demonstration. Specifications such as those in § 8.1 or figure 8-4 can be used to determine these tests.

Demonstration consists of collecting sufficient test or operational data to accept or reject statements about the levels of the parameters; thus it is basically a hypothesis testing procedure. A demonstration plan can be developed from the applicable statistical formula for interval estimation. The planning task consists of determining combinations of test time, sample size and the maximum number of failures which will satisfy the hypothesis test.

The chi-square method applies when all of three conditions are met:

1) Demonstration is performed at the same assembly level at which interval estimates are to be made (i.e., the entire device is tested as a unit).

2) The device is not cyclic or "one shot", but operates more or less continuously in time, so that failure rate or MTBF is the parameter of interest.

3) The device is described by the exponential failure density function $f(t;\lambda) = \lambda e^{-\lambda t}$, $(t \geq 0, \lambda \geq 0)$ (i.e., the device has a constant failure rate λ).

Given that the foregoing conditions are met, the best (point) estimate of failure rate is $\hat{\lambda} = x/t$ where t is the sum of the operating times accumulated by all devices in the tests and x is the number of failures observed.

For a test with a fixed truncation time, where t units of operating time are accumulated, it can be shown in [6] that the probability of observing x or fewer failures is given by the cumulative Poisson distribution

$$P(x) = \sum_{k=0}^{x} \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

which may be described by the χ^2 distribution:

$$P(x) = P(\chi^2_{2x+2} > 2\lambda t)$$

where 2x+2 is the number of degrees of freedom of the χ^2 variate.

Therefore, if $\lambda \geq \chi^2_{(1-\beta):2x+2}/2t$, then $P(x) \leq \beta$. Here $\chi^2_{1-\beta:2x+2}$ is the 100 β^{th} percentage point of the χ^2 distribution with 2x+2 degrees of freedom and the entire expression is a null hypothesis which we seek to test. Having observed x failures, we may then associate 100 $(1-\beta)$ percent confidence with the alternative hypothesis that

$$\lambda \leq \frac{\chi^2_{\beta:2x+2}}{2t} \qquad (8\text{-}10)$$

Epstein [7] has shown that equation 8-10 is a one-sided confidence interval on λ. It is customary to define

$$\frac{\chi^2_{(1-\beta):2x+2}}{2t} \equiv \lambda_{1-\beta}$$

as a 100 $(1-\beta)$ percent upper confidence limit on λ.

If MTBF (θ) is the parameter of interest, the corresponding one-sided interval is

$$\theta \geq \frac{2t}{\chi^2_{(1-\beta):2x+2}} \qquad (8\text{-}11)$$

and from equation 8-11 we define:

$$\frac{2t}{\chi^2_{(1-\beta):2x+2}} \equiv \theta_{1-\beta}$$

as a 100$(1-\beta)$ percent lower confidence limit on θ.

If reliability for a mission of length T is to be demonstrated, we define

$$R_{(1-\beta)}(T) \equiv e^{-\lambda_{(1-\beta)}T} = e^{-T/\theta_{(1-\beta)}}$$

as a 100$(1-\beta)$ lower confidence limit on R(T).

In computing upper confidence limits on λ, or lower limits on θ, the degrees of freedom are 2(x+1) since the test time is fixed in advance.

The test time t may be accumulated in a replacement or non-replacement test, or from combined, interrupted or sequential tests. Mathematically, sample size and test duration are directly exchangeable. All that is required is that $t = \Sigma t_i$, the sum of operating times of all items tested, and that $x = \Sigma x_i$, the total number of failures observed in the tests. Demonstration is complete when the interval estimate encompasses θ_L^*, the minimum acceptable MTBF, or R_L^*, the minimum acceptable reliability.

From an engineering viewpoint however, it is desirable to test a sample large enough to offer some assurance that the tested items are representative of the population. It is also desirable, though not always feasible, to continue testing long enough to show that wearout (time and environment) effects are not significant in the period during which reliability is of concern. Both wearout and infant mortality effects tend to prevent a system from exhibiting the exponential distribution of times between failures on which the chi-square method depends.

Example, Reliability Demonstration with Interval Estimation, Chi-Square Method

Given $t = 300$ hours, $x = 3$ failures, $\beta = .20$

Find: $\hat{\theta}, \theta_L, \hat{R}(1)$ and $R_L(1)$

$$\hat{\theta} = \frac{t}{x} = \frac{300}{3} = 100 \text{ hours}$$

and using equation 8-11 and figure E-1:

$$\theta_L = \theta_{.80} = \frac{2t}{\chi^2_{.80:8}} = \frac{(2)(300)}{11.030} = 54.397 \text{ hours.}$$

Indicating that the MTBF estimate is 100 hours and its 80% lower bound is 54.397 hours. Continuing,

$$\hat{R}(1) = e^{-\hat{\lambda}(1)} = e^{-.01000} = 0.9900$$

$$R_L(1) = e^{-\lambda_u(1)} = e^{-.01838} = 0.9818.$$

Indicating that the reliability estimate for a one hour mission is 0.9900 and its 80% lower bound is 0.9818.

8.4 NON-EXPONENTIAL RELIABILITY DEMONSTRATION

Two procedures for reliability demonstration are presented in this paragraph; one is taken from NAVORD OD 41146[8] and the other is based on a binomial method.

8.4.1 NAVORD OD 41146 (Sequential Ratio Test) Method

Procedures for demonstration (i.e., acceptance of a hypothesis, H_0, or the alternate hypothesis, H_1) when the exponential assumption is not valid are found in NAVORD OD 41146[8]. Equipment designs that employ redundancy and those which must be judged on an attribute basis are in this class.

These procedures are illustrated by example for a complete reliability specification (figure 8-10) for a hypothetical launcher system.

Step 1. Establish the Hypothesis and the alternate.

In this example the design requirement is 0.98. The hypothesis is therefore

$$H_0 : Q_0 \leqslant 0.02 \text{ at } t = T$$

where

$$Q_0 \leqslant (1-R_0) = (1-.98)$$

and T is the defined mission time.

The alternate hypothesis

$$H_1 : Q_1 \geqslant 0.07 \text{ at } t = T$$

where

$$Q_1 \geqslant (1-R_1) = (1-.93)$$

and T is the defined mission time.

Step 2. Define the Accept/Reject Decision Boundaries.

The accept and reject lines are defined:

Accept Line $x_n = -h_0 + ns$

Reject Line $x_n = +h_1 + ns$

Reliability Specification				
System	Reliability Objective R_0	Producer's Risk α	Minimum Acceptable Reliability R_1	Consumer's Risk β
Launcher	.98	.20	.93	.20

Figure 8-10. Launcher Reliability Objectives

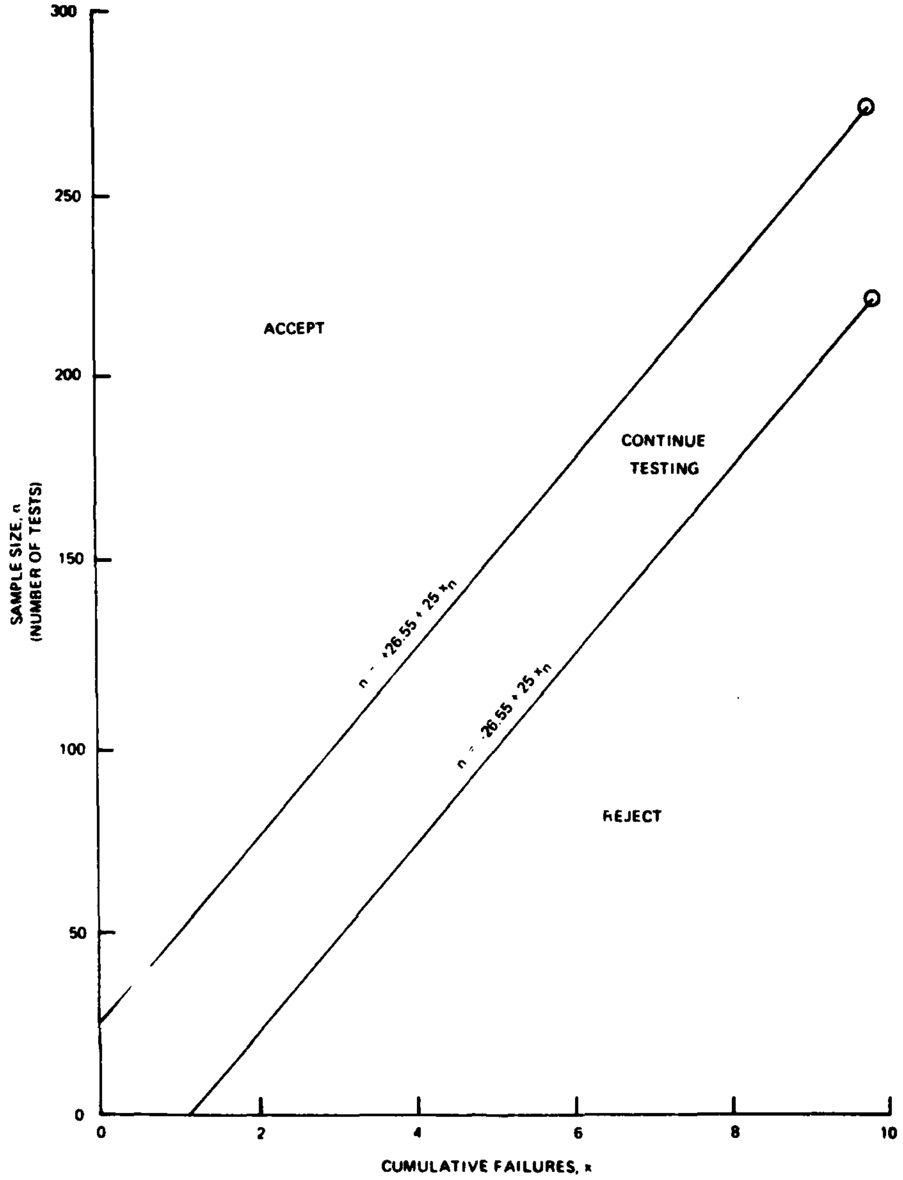


Figure 8-11. Sequential Test Plan (Non-Exponential) $R_0 = .98$ $R_1 = .93$ $\alpha = \beta = 20\%$

where

n = sample size

s = slope

then:

$$h_0 = \frac{\mathrm{Ln}\left[\frac{1-\alpha}{\beta}\right]}{\mathrm{Ln}\left[\frac{Q_1\,(1-Q_0)}{Q_0\,(1-Q_1)}\right]} \qquad (8\text{-}12)$$

$$h_0 = \frac{\mathrm{Ln}\left[\frac{1-.20}{.20}\right]}{\mathrm{Ln}\left[\frac{0.07\,(1-0.02)}{0.02\,(1-0.07)}\right]} = \frac{\mathrm{Ln}\ 4.000}{\mathrm{Ln}\ 3.688}$$

Therefore,

$h_0 = 1.062$

also,

$$h_1 = \frac{\mathrm{Ln}\left[\frac{1-\beta}{\alpha}\right]}{\mathrm{Ln}\left[\frac{Q_1\,(1-Q_0)}{Q_0\,(1-Q_1)}\right]} = \frac{\mathrm{Ln}\left[\frac{1-.20}{.20}\right]}{\mathrm{Ln}\left[\frac{.07\,(1-.02)}{.02\,(1-.07)}\right]} \qquad (8\text{-}13)$$

$h_1 = 1.062$

and,

$$s = \frac{\mathrm{Ln}\left[\frac{1-Q_0}{1-Q_1}\right]}{\mathrm{Ln}\left[\frac{Q_1\,(1-Q_0)}{Q_0\,(1-Q_1)}\right]} = \frac{\mathrm{Ln}\left[\frac{1-.02}{1-.07}\right]}{\mathrm{Ln}\ 3.688} \qquad (8\text{-}14)$$

$s = 0.040$

The Decision boundaries are:

Accept Line $x_n = -1.062 + 0.040\,n$

Reject Line $x_n = +1.062 + 0.040\,n$

These lines can be converted to accept/reject criteria by solving for n:

Accept when $n \geqslant 26.55 + 25\,x_n$

Reject when $n \leqslant -26.55 + 25\,x_n$

These two lines are plotted in figure 8-11 and further illustrated in figure 8-12.

Number of Failures	Minimum Sample to Accept	Maximum Sample to Reject
x	n_A	n_R
0	27	–
1	52	–
2	77	23
3	102	48
4	127	73
5	152	98
6	177	123
7	202	148
8	227	173
9	252	198
10	277	223

Figure 8-12. Acceptance and Rejection Sample Sizes

This test could be terminated (§ 8.3.1.2) at seven failures and 175 samples. (Truncation would change the lower portion of figure 8-12.)

8.4.2 Binomial Interval Estimation Method

If a device is to be used only once (e.g., a missile) or is tested and used in missions of uniform length, and if its performance in any mission may be characterized unambiguously as a success or failure, and if its reliability is stationary over successive missions, then each mission may be viewed as a Bernoulli trial and the device reliability as a binomial parameter.

Terming the reliability R and the failure probability Q = 1-R, the probability of observing x failures in n tests is

$$p_r(x) = \binom{n}{x}(1-R)^x(R)^{n-x} \qquad (8\text{-}15)$$

where

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

If p is the true parameter, the probability of observing as few as x failures in n trials is

$$\sum_{i=0}^{x} p_r(i)$$

Having observed x failures in n trials we may associate 100β% confidence with a statement that the true failure probability is equal to or less than the upper limit p_u, necessary to satisfy the equality

$$\sum_{i=0}^{x} p_r(i) = \beta$$

It has been shown [9] that an upper bound on failure probability is given by

$$Q_u = \frac{1}{1 + \frac{n-x}{x+1} \, \frac{1}{F_{1-\beta:f_1,f_2}}}$$

where $F_{1-\beta:f_1,f_2}$ is the 100 $(1-\beta)$ percentage point of the F distribution with degrees of freedom $f_1 = 2(x+1)$ and $f_2 = 2(n-x)$. The reliability is then bounded by $R_L = 1-Q_U$ with $100(1-\beta)\%$ confidence.

A lower bound on failure probability, usually of less interest in demonstration testing, is given by

$$Q_L = \frac{1}{1 + [(n-x+1)/x] \, F_{1-\alpha:f_1,f_2}}$$

where $f_1 = 2(n-x+1)$ and $f_2 = 2x$. A $100(1-\alpha)\%$ upper bound on reliability is then $R_u = 1-Q_L$.

Using $s = n-x$ as the number of successes, the $(100 \times \beta)\%$ lower limit or bound for reliability is

$$R_L = \frac{1}{1 + [(n-s+1)/s] \, F_{1-\beta:f_1,f_2}} \qquad (8\text{-}16)$$

where $f_1 = 2(n-s+1)$, $f_2 = 2s$

and

$$R_u = \frac{1}{1 + \frac{n-s}{s+1} \, \frac{1}{F_{1-\alpha:f_1,f_2}}}$$

where $f_1 = 2(s+1)$, $f_2 = 2(n-s)$.

Example: Given,

$n = 20 \quad x = 2 \quad s = 18 \quad \beta = .80$

Find: $\hat{R}$ and R_L

$$\hat{R} = \frac{s}{n} = \frac{18}{20} = 0.90$$

$$R_L = \frac{1}{1+[(n-s+1)/s] \, F_{1-\beta:f_1,f_2}}$$

$$f_1 = 2(n-s+1) = 6$$

$$f_2 = 2s = 36$$

$$R_L = \frac{1}{1+[(20-18+1)/18] \, F_{.20:6,36}}$$

From figure E-8; $F = 1.5188$

Therefore: $$R_L = \frac{1}{1+(.1667)(1.5188)}$$

$$R_L = 0.7980$$

8.5 DEMONSTRATION IN THE PRESENCE OF RELIABILITY GROWTH

A complex system as initially assembled will generally contain a number of incipient failure modes. The most serious modes command first attention and, as they are corrected, unmask progressively less significant defects. Reliability grows by this process, which is activated and continued by testing, observing failures and correcting their causes. The failure analysis and corrective action activities required by government development contracts are a closed-loop process designed to ensure that reliability will, in fact, increase with increasing program time. Theoretically, once a failure occurs, testing stops, failure analysis and corrective action is performed and the test continues after corrective action implementation. Monitoring of reliability growth begins early in the development test program. Data to be used in the measurement of reliability growth will be determined from a review of the integrated test program but should start with early engineering evaluation testing.

Reliability theory recognizes early growth but the hypothesis testing methods discussed above treat only mature systems of constant failure rate and constant failure probability. Experience has shown, however, that a large and complex system may require several thousand system-hours of testing and operation before its failure rate stabilizes sufficiently for a static model to represent its reliability accurately.

Under these conditions, the measurement or demonstration problem is one of estimating the current values of changing reliability parameters. Analogously, the prediction problem becomes one of forecasting future values, particularly ultimate static values.

In order to monitor the progress of reliability growth, a model of the growth process is needed. The model proposed by J.T. Duane [2] in 1964, and already described in Section 5, was based on empirical analysis of aerospace equipment failure data. It is applicable to measurement at the same assembly level for which data are collected.

As noted in Section 5, the model is

$$\lambda_\Sigma = KT^{-\alpha} \qquad (8\text{-}17)$$

where λ_Σ is cumulative average failure rate, the ratio of total failures x to total operating time T. In Duane's model the instantaneous failure rate λ changes at the same rate as λ_Σ and is displaced from λ_Σ by the constant factor $(1-\alpha)$.

$$\lambda = \frac{dx}{dT} = \frac{d(\lambda_\Sigma T)}{dT} = \frac{d(KT^{1-\alpha})}{dT} \qquad (8\text{-}18)$$

$$= (1-\alpha)KT^{-\alpha} = (1-\alpha)\,\lambda_\Sigma$$

Plotted on logarithmic scales the curves are linear and parallel to each other (figure 8-13).

K can be interpreted as the initial or time-zero failure rate. It is a function of initial design quality, system complexity, maturity of the system relative to the state-of-the-art, quality of shop operations and other variables. Experience has shown that it is seldom less than 10 times the specified target value. Lacking a better estimate, a safe initial procedure is to assume K at 10 times the predicted failure rate (or 10 percent of the predicted MTBF, since a growth curve can be plotted using either parameter). The curve is more sensitive to the exponent α than to K. The exponent reflects the intensity with which reliability improvement is obtained; it nearly always lies between .2 and .5, the average being close to .3.

The form of presentation shown in figure 8-13 has two practical advantages; it provides a reasonable visual fit of test data to the growth model, and it provides a best estimate line for past, present and future observations with minimum computational effort.

Today, the Duane model is embodied in MIL-STD-1635 [3], the proposed MIL-STD-781D[13], and other military standards.

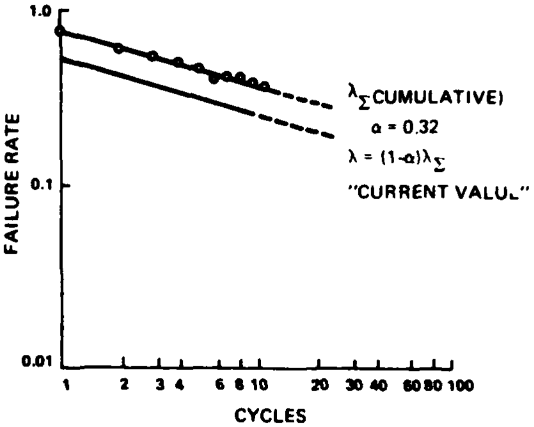


Figure 8-13. Reliability Growth Model

8.6 REFERENCES

1. MIL-STD-781, Reliability Design Qualification and Production Acceptance Tests: Exponential Distribution.

2. Duane, J.T., Learning Curve Approach to Reliability Monitoring, IEEE Transactions on Aerospace, Vol. 2, No. 2, 1964; pp. 563-566.

3. MIL-STD-1635, Reliability Growth Testting.

4. Crow, L.H., On Tracking Reliability Growth, Proceedings 1975 Annual Symposium on Reliability, pp. 438-443.

5. MIL-STD-756, Reliability Prediction.

6. Hald, A., Statistical Theory with Engineering Applications, Wiley, NY, 1952, Chapter 22.

7. Epstein, B., Statistical Techniques in Life Testing, U.S. Dept. of Commerce, PB 171580, Washington, D.C., pp. 3-19.

8. NAVORD OD 41146, Reliability Engineering Handbook, formerly NAVWEPS 00-65-502.

9. Lipson, C. and Sheth, N., Engineering Experiments, McGraw Hill, 1973.

10. Crow, L.H., Reliability Analysis for Complex Repairable Systems Interim Note 26, Jan. 1974, USAMSAA, Aberdeen Proving Ground, MD 21005.

11. Technical Objective and Guidelines Document (WS 14111, WS 13579, WS 20400).

12. CNM Memo, Policy for R&M Requirements in Navy Programs, 10 April 1974.

13. MIL-STD-781D (Proposed), "Reliability Testing for Engineering Development, Qualification and Production", 12/31/80.

Section 9
RMA DATA SYSTEM

NAVSEA OD 21549[1] requires the establishment and maintenance of an Integrated Data System (IDS) whose primary purpose is the implementation of a cost effective and comprehensive data program to support all engineering activities. This section addresses a subset of the IDS, the data which is primarily used for RMA evaluation and the quality of that data.

9.1 INTRODUCTION

The data system is required to collect, control, process, distribute and store essential information. The system must provide for data maintenance and be capable of timely data retrieval. The system must support the needs and objectives of the RMA (hardware and software) evaluation program.

The data system should be flexible in output formats (Sorts) to aid the analysis effort and be able to distinguish between the many types of data inputs required to satisfy data users' needs. Simple analysis assistance (computer) programs such as, time counting, failure counting, repair counting, threshold alarms, point estimates, "white space" analysis,* plotting, etc. can be of great assistance to the analysis effort and can be implemented without difficulty if data entries are standardized.

The RMA data system, as an element of the IDS, must have explicit definitions of tasks, responsibilities and required coordination. Development of the system requires an analysis of program measurement and assessment objectives to establish the program data needs. Extensive interface between the IDS and the integrated test program (ITP) is required to assure that all RMA measurement and assessment objectives are fulfilled, that all data needs are defined, that necessary tests are planned, and that provisions for collection, control, and processing data into useful output reports are properly coordinated. Data control procedures should be prepared to monitor the completeness, conciseness, legibility, accuracy, and validity of reported data, and to format and input the data for computer and/or manual use. Computer programs for data processing are written to enable accurate test history files to be generated and for analysis assistance. Data utilization procedures are developed to make use of the summaries and reports compiled from the history files. Development of data system functions is discussed below as related to the hardware and software portions of systems undergoing development, production, and fleet service.

Once established, the RMA data system should be applicable to any hardware or software system or program (upon definition of requirements and mission information unique to a program) and it should be able to handle multiple programs concurrently. The contractor should strive to make operation of the data system straightforward, self-regulating and largely routine.

Figure 9-1 is an analysis of RMA data needs.

A Flow Chart for RMA data is shown in figure 9-2.

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* "White Space" Analysis – A failure data analysis technique, such that part, equipment, subassembly, etc. failures are sorted by the part number, for example. Rather than listing a part number, such as R1 (for resistor R1), repetitively for each failure of R1, the sort program or manual listing should print only the first R1, blanking the remainder off the listing. The created "white space" between the listing of R1 and the listing of the first R2 alerts the analyst to the magnitude of a problem.

OD 21549A Para #	RMA Requirement	Basic Data Needed to Meet Requirement	Data Source(s)
3.3.1.1	1. Develop system R block diagrams for each applicable mission phase.	System Definition: 1. Description of the system, subsystems, equipment operation by mission phase.	1.(a) Weapon specifications (WSs) for the system (e.g. WS 14111 and WS 13579) and subtier levels. 1.(b) Output of contractor's Analysis for Design (OD 21549A, § 3.2.1.)
		2. System functional block diagram by mission phase.	2. Configuration Baselines (OD 21549A, § 3.5.1.1) • Engineering drawings. • System Tree Charts
3.3.1.2 3.3.2.1	2.(a) Apportion system mission-phase R&M design objectives to system elements. Includes apportionment of system availability (A) design objectives for OR phase.	3. R apportionment model for each mission phase (OD 21549A, § 3.3.1.1).	3. System R block diagrams for each mission phase.
		4. System mission-phase R&A design objectives ($R^*_{OR} = A^*$, R^*_{Launch}, and if applicable, R^*_{Flight}).	4; System weapon specifications, e.g. WS 14111 and WS 13579.
		5. System Maintainability (M) block diagram for Operational Readiness (OR) phase.	5. Output of contractor action to comply with OD 21549A, § 3.3.2.1.
		6. System Availability (A) block diagram for the OR phase.	6. Output of contractor action to blend RMA block diagram (OD 21549A, § 3.3.1.1 and 3.3.2.1) for the OR phase.
		7. Design Requirements for Fault Detection and Isolation Software.	7. Contractor data generated to meet requirements of OD 21549A, § 3.3.2.1.
		8. Relative weights (or scores) for each hardware/software block in the R block diagram indicating its relative complexity, criticality, design maturity, severity of operational demand (e.g. environmental stress profiles, duty cycle profiles, etc.) compared to a familiar, mature design block selected to serve as a design reference standard.	8. No pre-existing source. Program Manager has the basic responsibility for developing and documenting these apportionment factors. See § 4.2.3 of this manual.
	2.(b) Apportion Minimum Acceptable R value for system's overall mission to system equipment requiring formal R demonstration.	(Same data as required for apportionment of system R design objectives, excluding data item #4 above. plus 9. Minimum Acceptable R value for the system's overall mission.	(For additional information see discussion later on R demonstration testing.) 9. CNM R&M Policy Memoranda. See reference [12] at end of Section 8 of this manual.

Note: The term System represents the highest indenture level under development by the Contractor.

Figure 9-1. Analysis of RMA Data Needs

	RMA Requirement	Basic Data Needed to meet Requirement	Data Source(s)
3.3.1.3 3.3.2.2	3. Provide periodic predictions of system reliability by mission phase (i.e. initial, intermediate, and final predictions) and System Maintainability.	10. R prediction model for each mission phase.	10. System R block diagram for each mission phase. (Same as data item #3 above.)
		11. Indentured parts list for each block in the R block diagram.	11. (a) Configuration Baselines (OD 21549A § 3.5.1.1) ● Engineering sketches and drawings. (b) Project Parts, Devices and Materials List (OD 21549A § 3.2.3.2)
		12. Environmental stress profiles (externally and internally induced) for each hardware/software block in the RMA block diagrams; also stress profiles for constituent elements below the block level when different from the block environmental stress profiles.	12. Output of contractor's Mission Analysis (OD 21549A § 3.2.1 a., b.).
		13. Duty cycle profiles for each hardware/software block in the RMA block diagrams (and for constituent elements when different).	13. Same as data item #12 above.
		14. Generic failure rate data for system elements which have unique environmental stress and duty cycle profiles.	14. (a) MIL-HDBK-217: Reliability Prediction of Electronic Equipment (b) NPRD-1: Non-electronic Parts Reliability Data (c) GIDEP: Government Industry Data Exchange Program (d) Contractor's In-House Failure Rate Data Bank
		15. Design Engineering derating selections.	15. (a) Engineering Design Practices and Standards. (b) Project Directives, Procedures and Instructions. (c) Design Engineering Memoranda
		16. Redundancies permitted by system weight and space limitations.	16. Weapon system specifications. (Same as #1 (a).)
		17. Repair/Replacement Times for current hardware software designs.	17. (a) TFR Data Analyses (b) Engineering Analyses (c) MIL-HDBK-472 (d) Contractor In-House Data

Figure 9-1. Analysis of RMA Data Needs (Continued)

OD21549A Para #	RMA Requirement	Basic Data Needed to meet Requirement	Data Source(s)
3.3.1.4	4. Perform Failure Mode, Effects and Criticality Analysis (FMECA)	18. Description of the system, subsystem and equipment operation by mission phase. (Same as #1 above.)	18. (a) **Weapon specifications for system and subtier levels.** (b) Contractor's system Analysis for Design (OD 21549A, § 3.2.1).
		19. System functional block diagram by mission phase. (Same as #2 above.)	19. Configuration Baseline (OD 21549 § 3.2.1) ● Engineering drawings. ● System Tree Chart.
		20. Failure modes encountered in analogous equipment in past programs.	20. FMECA history file for past programs.
		21. Occurrence probabilities for failure modes.	21. Same as 14. (a) through (d) above.
		22. Effectiveness of past measures taken to preclude or minimize failure mode occurrences.	22. Contractor's preventive and corrective action file for past programs.
3.3.1.5 3.3.2.4	5. Record and collect data necessary to support design, reliability and maintainability analysis, and assessment functions.	23. The only specific data enumerated in this paragraph are: ● Equipment operating time and cycle data. ● Number of tests. ● Failures. ● Failure modes. ● Repair time. ● Total down-time. ● Repair frequency. but they actually encompass all of the RMA data listed in this RMA data needs analysis.	23. Composite of all sources listed in this RMA data needs analysis.
3.3.1.6 3.3.2.5	6. The contractor shall assess the reliability and maintainability of the product beginning with the design and test program and continuing through the operational phase.	24. (a) Results of reliability and maintainability analyses, engineering analysis, and valid operating data from previous generations.	24. (a) Contractor's History Files for relevant past programs.
	● Assessments shall include calculations of point estimates and lower confidence limits of achieved reliability and point estimates of maintainability	(b) Applicable (new) test and usage data for the quantitative assessment of product reliability and maintainability.	24. (b) Test Records/Reports and Usage Records/Reports, such as: ● Test Operation Log/SSBN Operations Log. ● Test Reports. ● Elapsed Time Meter Records

Figure 9-1. Analysis of RMA Data Needs (Continued)

OD21549A Para #	RMA Requirement	Basic Data Needed to meet Requirement	Data Source(s)
3.3.1.6 3.3.2.5	6. (Cont'd) ● Measurement data obtained from test and usage shall be used to assure that potential sources of reliability and maintainability programs are identified and evaluated and necessary corrective action initiated.		24.b (Cont'd) ● Failure Reports (FRs, TFRs, . . .). ● Failure Analysis Reports (FARs). ● Fleet Maintenance Records.
3.3.1.7	7. The contractor shall provide evidence that the product meets specified reliability objectives. Reliability . . . demonstration shall be performed on units that represent the production configuration using environmental and other operational stress levels as identified in the product specifications.	25. (a) Description of the production configuration. (b) Operational environments and stress data.	25. (a), (b): Production Design Disclosure Documentation (OD 21549 § 3.5.5.2). ● Design Specifications. ● Product specifications. ● Software design disclosure documents. ● Control drawings ● Index lists that tabulate data lists, parts lists, and top assembly dwgs.
	(a) The contractor shall prepare and submit a Reliability Demonstration Test Plan for approval.	26. (a) Values for test plan parameters. R^*, R_L^*, alpha and beta for each equipment covered by its own R demonstration test plan. (b) Equipment availability for test. (c) Cost and schedule constraints.	26. (a) Contractor records on the apportionment of system level reliabilities to subtier levels IAW OD 21549 § 3.3.1.2. 26. (b), (c): Program Office.
		27. (a) R demonstration test results.	27. (a) R demonstration test operations log and test records.
	(b) A reliability Demonstration Report shall be submitted for approval.	(b) List of failures. (c) Failure analysis results and corrective action recommendations.	(b) Failure reports (FRs, TFRs) (c) Failure analysis reports and corrective action records.
3.3.1.8	8. Periodic Reliability Status Reports (RSRs) shall be submitted for information. Reports shall include	28. (a) Implementation schedules for R evaluation program tasks. (b) Slippage, if any, on contractor's adherence to these implementation schedules; and reasons for any significant slippages.	28. (a) R Evaluation Plan. (b) Milestone Charts maintained by cognizant personnel/office, e.g. R Program Mgt. or Program Office.
	(a) Summary of program status, problems, and corrective actions.	(c) Repetitive failures. (d) Dominant system failure modes.	(c), (d) Failure Reporting Analysis and Corrective Action Records.

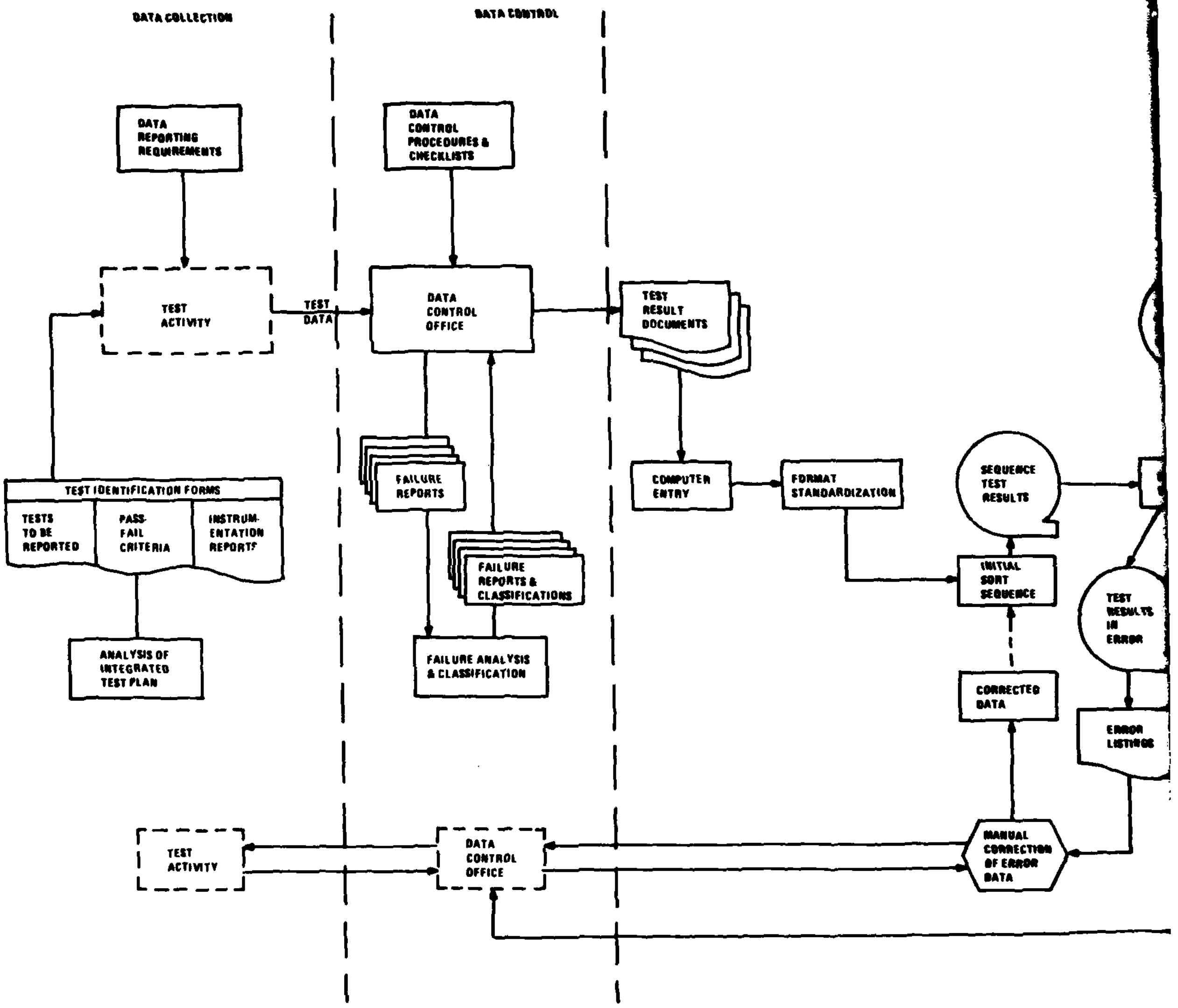
Figure 9-1. Analysis of RMA Data Needs (Continued)

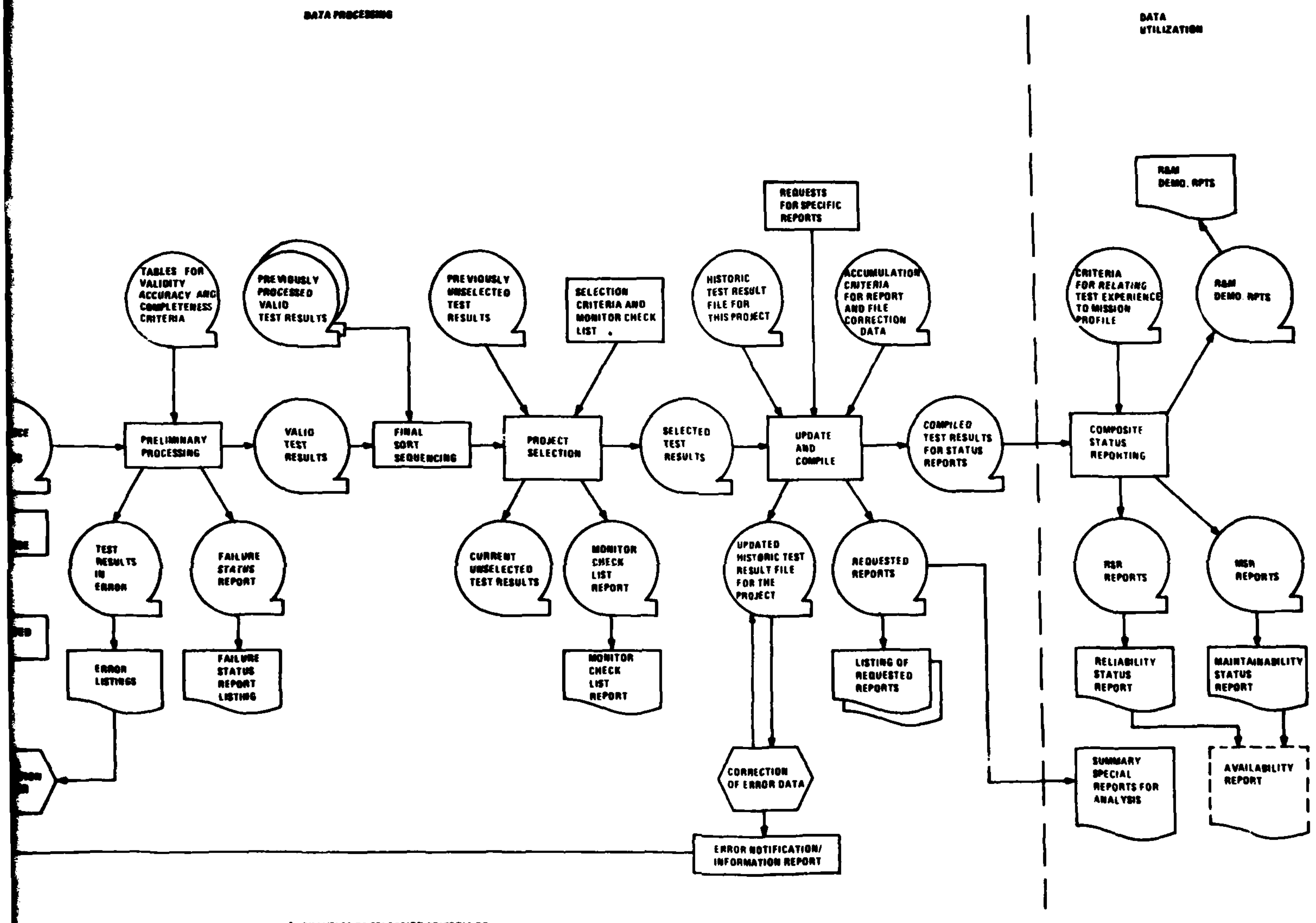
OD21549A Para #	RMA Requirement	Basic Data Needed to meet Requirement	Data Source(s)
3.3.1.8	8 (Cont'd) RSRs shall include	(e) System elements contributing most to system unreliability (beyond apportioned value)	(e) Contractor working papers on which current results of apportioned, predicted, and achieved (system and system element) reliabilities are recorded.
	(b) Summary of significant trends events and achievements.	29. (a) Previous issues of Reliability Status Reports (RSRs)	29. (a) Program History File.
		(b) Current values for apportioned, predicted and achieved reliabilities.	(b) Same as 28. e above.
		(c) R growth data generated since the previous RSR.	(c) R growth test program.
		(d) Results of R improvement activities (e.g. elimination of dominant failure modes)	(d) Failure and Corrective Action Status Summary.
	(c) A discussion of problems that may affect product reliability if not resolved.	30. (a) Results of FMECA. (b) Results of Fault Tree Analysis (if performed). (c) Results of Sneak Circuit Analysis (if performed).	30. (a),(b),(c): Contractor working papers in which results of FMECA, FTA, and SNA are recorded.
		(d) Quality trend data.	(d) Qualify Control Records File.
		(e) Reliability trend data.	(e) Results of R analysis and previous issues of the RSR.
	(d) Data inputs to the R assessment model.	31. (a) New test data generated since the previous RSR at each equipment level, e.g., ● Number of tests. ● Number of successes/failures. ● Operating time. ● Test data sources. ● Test environments.	31. (a) Test and Usage Records and Reports: ● Contractor Test Reports (Development, Production and SITP). ● SSBN Operations Log. ● TFRs. ● Elapsed Time Meter Records. ● Navy assessments of flight test results.
		(b) Previously generated test results	(b) Program History Files at cognizant contractor and Navy offices.
	(e) Apportioned, predicted and estimated (point and confidence limit) values for the system for each mission phase.	32. Same as the data required to fulfill the requirements of OD 21549 § 3.3.1.2, 3.3.1.3, 3.3.1.6 and 3.3.1.7.	32. Same as for Paras. listed at the left.

Figure 9-1. Analysis of RMA Data Needs (Continued)

OD21549A Para #	RMA Requirement	Basic Data Needed to meet Requirement	Data Source(s)
3.3.1.8	8. (Cont'd): RSRs shall include: (f) R Growth curves for the system and new equipments, related to their reliability objectives.	33. R Growth Test Results, such as: (a) For each type of continuous or periodically operating device: ● Name of device. ● Operating time. ● Time to failure (if any). ● Time between failures. ● Number of failures. ● Number of No-Tests. ● Test conditions. (b) For each kind of one-shot devices: ● Name of device. ● Sample Size. ● Number of successes. ● Number of failures. ● Number of No-Tests. ● Test Conditions.	33. (a), (b): R Growth Test Records and Reports.
		34. R objectives for the system and each pertinent equipment.	34. WSs at the system and equipment levels.

Figure 9-1. Analysis of RMA Data Needs (Continued)


DATA COLLECTION
DATA CONTROL
DATA REPORTING REQUIREMENTS
DATA CONTROL PROCEDURES & CHECKLISTS
TEST ACTIVITY
TEST DATA
DATA CONTROL OFFICE
TEST RESULT DOCUMENTS
FAILURE REPORTS
FAILURE REPORTS & CLASSIFICATIONS
TEST IDENTIFICATION FORMS
TESTS TO BE REPORTED
PASS-FAIL CRITERIA
INSTRUMENTATION REPORTS
ANALYSIS OF INTEGRATED TEST PLAN
FAILURE ANALYSIS & CLASSIFICATION
COMPUTER ENTRY
FORMAT STANDARDIZATION
SEQUENCE TEST RESULTS
INITIAL SORT SEQUENCE
TEST RESULTS IN ERROR
CORRECTED DATA
ERROR LISTINGS
MANUAL CORRECTION OF ERROR DATA
TEST ACTIVITY
DATA CONTROL OFFICE



* EXAMPLES OF SELECTION CRITERIA TO PROVIDE THE NEEDED OUTPUTS:

- MONTHLY SUMMARY
- SORT BY DATE
- GRAPHS/PLOTS
- THRESHOLD REPORTS
- TABLES
- SORT BY COMPONENT
- SORT BY TYPES OF TEST

Figure 9-2. RMA Data Flow Chart

2

1

9.2 DATA COLLECTION

Detailed procedures and instructions must be issued to all organizational units and operations personnel who participate in the generation and recording of basic (raw) RMA data. These procedures must define the data to be reported, the forms to be used, the instructions for completing the forms, and the approval process necessary to assure data quality. Reporting forms should include as much preprinted information as possible with emphasis placed on ease of entering data such as "check" box usage. The use of preprinted information leads to the reduction of transcription and recording errors, saves time, and if mechanized data processing techniques are used, standardized forms reduce errors and programming budgets.

The forms should encourage participating personnel to record concise information by providing sufficient choices and avoiding the use of "other", "none of the above", etc. However, while encouraging concise information, the form and instructions should not require information which cannot be provided without guessing, such as, asking for the "reason for failure" (vice symptom of failure) before failure analysis is performed.

Some aspects of and sample forms for hardware and software data collection are discussed below.

9.2.1 Hardware Data Collection

Data collection by means of test and operation logs, test result forms, failure forms or others must be comprehensive and accurate. Development of logs and forms and the instructions and training for proper data recording has a significant impact on the data collection effort and is essential to accomplishing the measurement of RMA. Basic information needed for hardware RMA evaluation is listed in figure 9-3. Information categories A, B and C are needed for reliability evaluation. Categories A through D are needed for maintainability evaluation. Categories E and F provide necessary analytical and reference data, respectively.

9.2.1.1 Attribute Data Collection

The collection of attribute (go/no-go) operational and test data can provide the most error-free recording of information since personnel interpretations of information from meters and other measuring devices are minimized. Figure 9-4 illustrates a sample form for collecting attributes data during a test which was provided to the test operator with the preprinted program standard information. Test personnel would complete the serial number, the date of test, length of test, test result (pass or fail), failure report number (if applicable), calibration dates/status, remarks and the personnel information.

9.2.1.2 Variables Data Collection

Collecting variables data permits the analyst to make more efficient use of the data. For example, data taken on the critical performance parameters of six samples of device A and six samples of device B are shown in figure 9-5.

The Specification limits for the critical performance parameters are provided in figure 9-6.

In both cases (A and B) we have six tests with no failure. Therefore, attribute analysis yields the same result. Variables analysis indicates that design B is superior (and also raises the estimate for design A) as shown in figure 9-7. The result obtained using variables analysis is in agreement with our intuition which tells us that design B is less apt to exceed the specification limits. The attribute analysis indicates the designs are equally good contrary to our intuition and highlighting the value of making efficient use of the data.

Since it is more difficult (probability of collecting erroneous data increases) and costly to collect, process and analyze variables data and to relate probability of operation within specification of a number of performance parameters, the analyst must trade-off the requirements to collect variables data versus the cost of obtaining this information. The evolution of automatic test equipment, with "result" measurements and printout capability, has provided the ability to collect variables data more accurately. Also, the use of preprinted information, especially minimum and maximum limits, is of value in reducing errors in collecting

A. Test Description
  1. Test Report Number
  2. Test Level - Component, Equipment or Subsystem
  3. Test Type - Qualification, Acceptance, etc.
  4. Test Site
  5. Test Environment
  6. Date of Test
  7. Test State - Operating, Non-Operating or Cycling
  8. Test Plan Number
  9. Test Procedure Number

B. Hardware Identification
  1. Hardware Name
  2. Hardware Drawing Number
  3. Hardware Serial Number
  4. Hardware Level
  5. Sub-hardware Actually Involved in Test
  6. Subcontractor
  7. Project

C. Test Results
  1. Sample Size
  2. Operating Time or Cycles
  3. Operating Mode
  4. Test Environment (Temperature, vibration, etc.)
  5. Failures (Number of)
  6. Failure Report Numbers
  7. Failure Classification

D. Maintenance Data
  1. Corrective Maintenance Functions
  2. Corrective Maintenance Time Components
  3. Delay Time Components
  4. Modification Time
  5. Time-to-Failure

E. Relevant Analytical Data
  1. Apportioned Hardware R&M Requirements
  2. Predicted Hardware R&M
  3. Failure Modes Predicted by FMECA
  4. Previously Implemented Corrective Action
  5. Mission Profile (Environment and Duty Cycle Profiles)
  6. Math Model
  7. Generic Failure Rate Data

F. Reference Information
  1. Date of Entry (Data Information Reaches Computer File)
  2. Test Report Number (References a High Level Test from Which Record Was Generated)
  3. Failure Report Number (References a Higher Level Failure Report)
  4. Project Code (Identifies Data Used from Another Contractual Program)

Figure 9-3. Information Useful for Reliability and Maintainability (Availability) Evaluation

PROGRAM Poseidon	TEST REPORT NUMBER P42735	
NAME Recovery Programmer	DRAWING NUMBER 692D912P003	SERIAL NUMBER
LEVEL Component	TYPE FAT	TEST SITE Factory Test Lab
TEST INSTRUCTION (TI) SYS 3793	TEST LEVEL Component	SECURITY CLASSIFICATION Unclassified

TI §	DATE	ENVIRONMENT	STATE *	LENGTH		PASS/ FAIL	FAILURE REPORT NO.
				MINUTES	CYCLES		
3.1.1		Bench	D		—		
3.1.2		Bench	C	—			
3.1.3		High Temp	A		—		
3.1.4		High Temp	D		—		
3.1.5		High Temp	C	—			
3.1.6		Bench	D		—		
3.1.7		Bench	C	—			
3.1.8		Vibration	A		—		
3.1.9		Bench	D		—		
3.1.10		Bench	C	—			

	EQUIPMENT NAME	MODEL	CALIBRATION	
			DATE	STATUS
TEST EQUIPMENT USED	Test Set Brush Recorder Vibrator Temperature Chamber Voltmeter Oscilloscope	TS 4273 BR 418 MB 11 TC 4823 V 234 TECT 4817		
REMARKS				

TESTER	DATE	QC ENGINEER APPROVAL/DATE	DATE TO DATA SYSTEM

*State A – Non-operating but must survive and be operational in a later mission phase.
B – Non-operating but must not operate prematurely and must be operational in a later mission phase.
C – Operating. Duration countable in cycles or discrete events.
D – Operating. Duration measurable in units of time.

Figure 9-4. Performance Data Sheet – Attributes Data

DESIGN	DATA					
	#1	#2	#3	#4	#5	#6
A	1.30	3.55	5.35	7.55	9.50	11.75
B	5.95	6.15	6.40	6.60	6.90	7.20

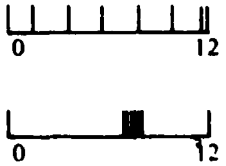


Figure 9-5. Data Taken on Devices A and B

LIMITS	
LSL	0.0
Nominal	7.3
USL	14.6

Figure 9-6. Specification Limits

DESIGN	RELIABILITY			
	ATTRIBUTES		VARIABLES	
	Point Estimate	95% Lower Conf Limit	Point Estimate	95% Lower Conf Limit
A	.8909	.6070	.9355	.8899
B	.8909	.6070	.9999+	.9999+

Figure 9-7. Comparison of Results

variables data. Figure 9-8 is an example of a form that can be used for this purpose.

9.2.1.3 Operating Data Collection

Operating data (time meter readings, operating mode, operating environment, operating results, etc.) at the system and subtier level can be reported by means of test operation records or logs such as figure 9-9. The design and information content of these logs should be tailored to the system under evaluation (Figures 9-9 and 9-10 are examples). Typically, line entries should be made to fully describe each equipment test state and the duration of the state. A new log entry should be made to document each change in conditions of the test such as failure occurrence, state of the equipment under test or change in usage mode. Note, the log should provide for any requirement to record variables data, along with the recorded or measured parameters minimum and maximum values. Responsibility for maintenance of logs should rest with the test supervisor during periods of testing, training, or experimentation, and with the system maintenance supervisor at other times.

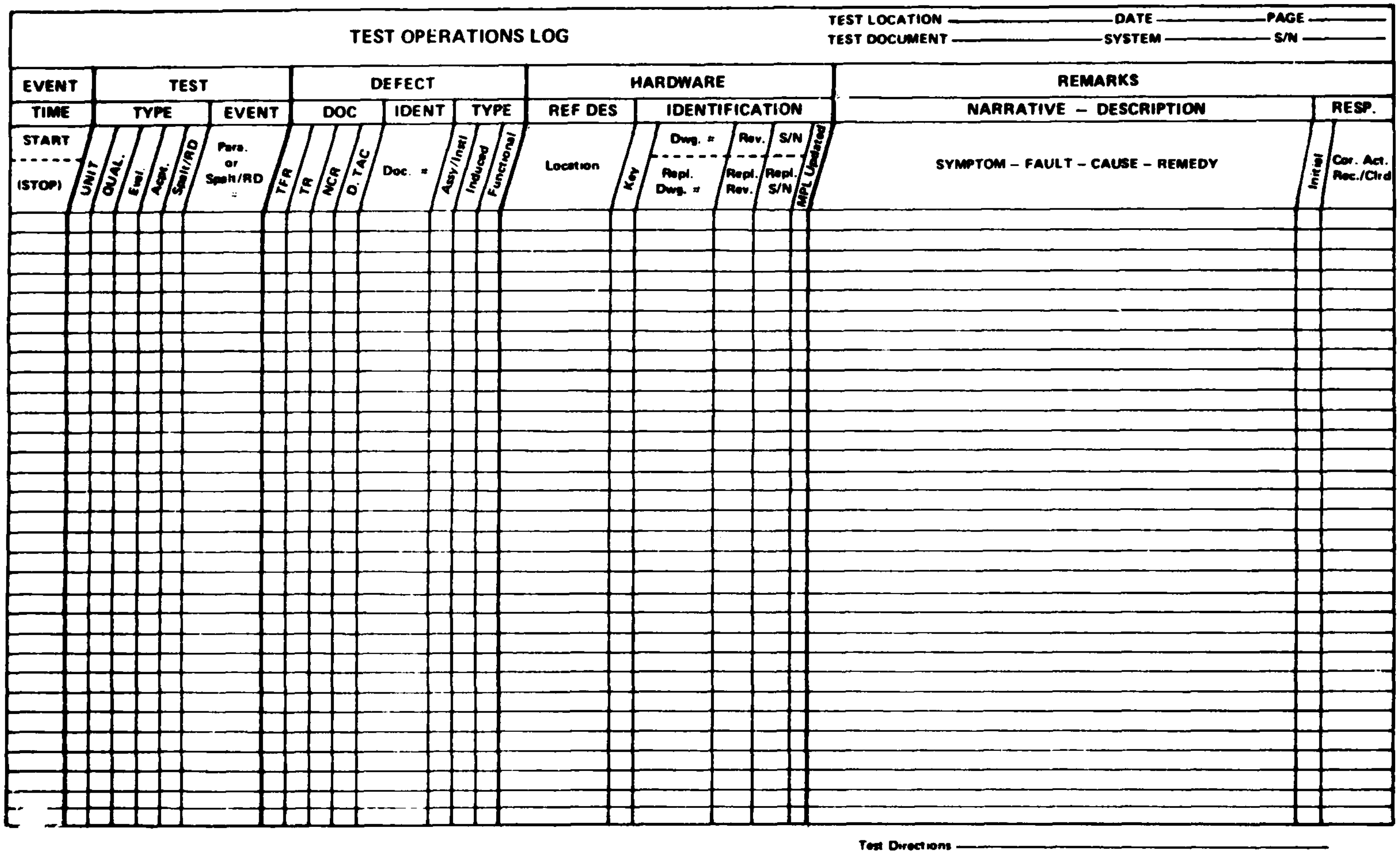
PROGRAM POSEIDON	TEST REPORT NUMBER PV42736	TYPE FAT	LEVEL Component
NAME Recovery Programmer	DRAWING NUMBER 692D912P003	SERIAL NUMBER	SECURITY CLASSIFICATION Unclassified
TEST INSTRUCTION (TI) SYS 3793	TEST LEVEL Component	DATE TO DATA SYSTEM	

TI Paragraph	Description of Test	Environment	Unit of Measure	Class of Character	Specification Limits	Actual* Readings	Date of Test
2.2.1	Visual Inspection	Bench	P/F	M	Pass/Fail		
2.2.2	Light Indication	Bench	P/F	M	Pass/Fail		
2.2.4	Beacon Voltage	Bench	Volts	M	13.2-15.6		
2.2.5	Timer 1 Time	Bench	Seconds	C	0.6- 1.5		
2.3.4	Beacon Voltage	High Temp	Volts	M	13.5-16.0		
2.3.5	Timer 1 Time	High Temp	Seconds	C	0.4- 1.2		

REMARKS			
TESTER	DATE	QC ENGINEER APPROVAL	DATE

*Circle all out or specification readings

Figure 9-8. Performance Data Sheet – Variables Data



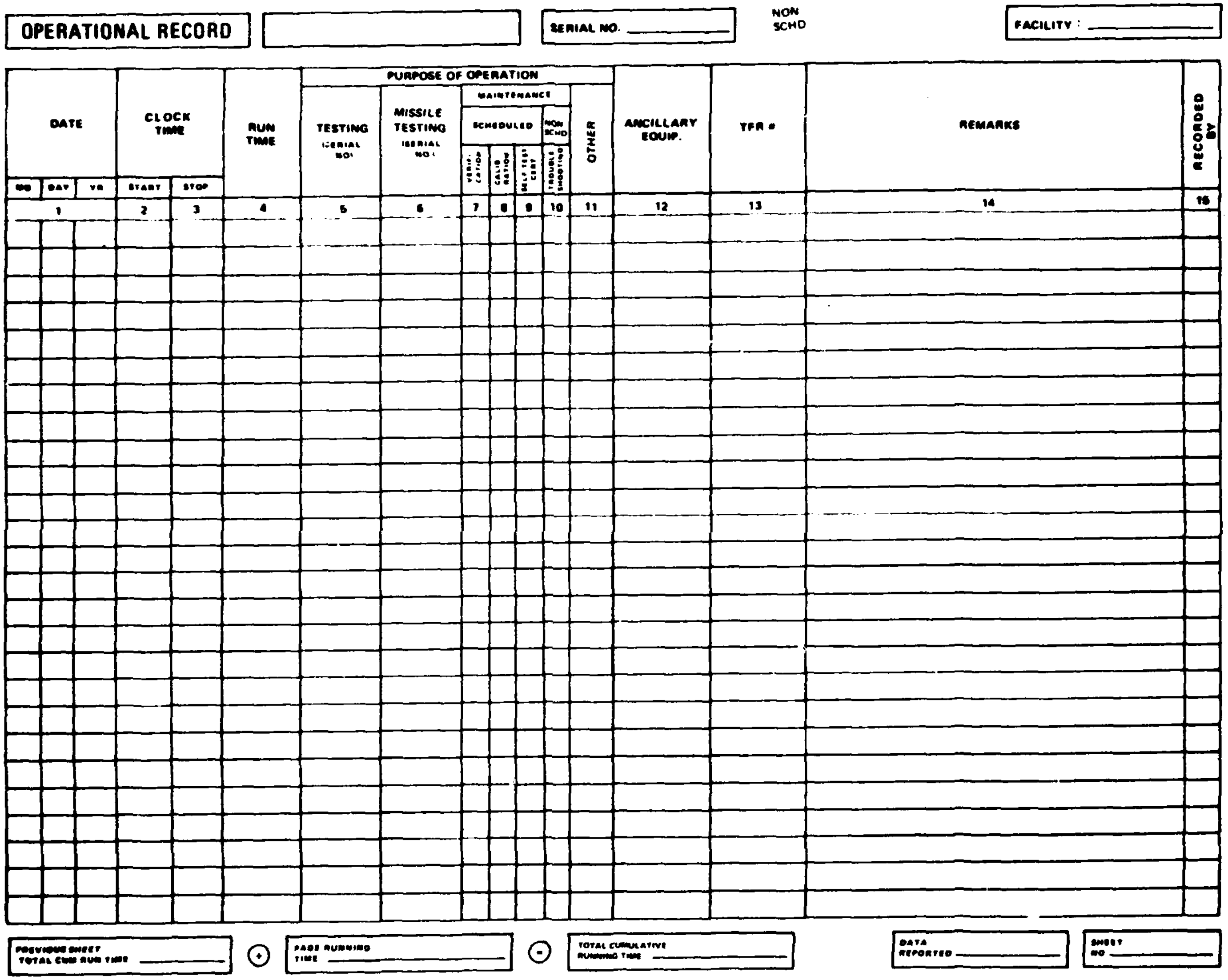
TEST OPERATIONS LOG

TEST LOCATION ―――― DATE ―――― PAGE ――――
TEST DOCUMENT ―――― SYSTEM ―――― S/N ――――

EVENT	TEST						DEFECT								HARDWARE						REMARKS		
TIME	TYPE					EVENT	DOC				IDENT	TYPE			REF DES	IDENTIFICATION					NARRATIVE – DESCRIPTION	RESP.	
START (STOP)	UNIT	QUAL.	Eval.	Acpt.	Spclt/RD	Para. or Spclt/RD #	TFR	TR	NCR	D.TAC	Doc. #	Assy/Instl	Induced	Functional	Location	Key	Dwg. # Repl. Dwg. #	Rev. Repl. Rev.	S/N Repl. S/N	MPL Updated	SYMPTOM – FAULT – CAUSE – REMEDY	Initial	Cor. Act. Rec./Clrd

Test Directions ――――

Figure 9-9. Test Operations Log



OPERATIONAL RECORD

SERIAL NO. ______

NON SCHD

FACILITY : ______

DATE			CLOCK TIME		RUN TIME	PURPOSE OF OPERATION							ANCILLARY EQUIP.	TFR #	REMARKS	RECORDED BY
						TESTING (SERIAL NO)	MISSILE TESTING (SERIAL NO)	MAINTENANCE				OTHER				
								SCHEDULED			NON SCHD					
MO	DAY	YR	START	STOP				VERIFICATION	CALIBRATION	SELF TEST CERT	TROUBLE SHOOTING					
1			2	3	4	5	6	7	8	9	10	11	12	13	14	15

PREVIOUS SHEET TOTAL CUM RUN TIME ______ (+) PAGE RUNNING TIME ______ (=) TOTAL CUMULATIVE RUNNING TIME ______

DATA REPORTED ______

SHEET NO ______

Figure 9-10. Typical Log Form

9.2.1.4 Failure Data Collection

When a failure occurs, a failure report should be generated with the following information recorded: Failure symptoms and circumstances, identity of the failed hardware, conditions at the time of failure, cause of failure (if known) and the disposition recommended or made of the failed hardware. Figures 9-11 and 9-12 are examples of typical failure reports. If the failure occurs during a test, the test document number should be referenced on the failure report and vice versa. Other information will then be added to the report: the failure classification designations (catastrophic, critical, major, minor), relevance classification (include or exclude the failure for reliability measurement), fault isolation data (identification of failed elements), and the environment (the test environment, if applicable) in which the failure occured.

Failure reports and other source documents originating in test or service areas should be routed promptly for processing. Failure reports should go directly to failure investigation for review and classification; then reproduced and distributed. Test data sheets and completed log sheets should go to data control for screening, reproduction, recording in computer files and distribution. Some reports cannot be totally completed by the personnel responsible for detecting a failure. Information, such as "repaired/replaced item" may require another activity's input. Methods for handling these situations should be addressed in the data system procedures. Techniques such as the use of multilayer, multicolor chemically treated forms are available such that additional information can be added to the final copy of the form while all data originating activities can keep a copy of their own inputs.

9.2.2 Software Data Collection

Software is defined as computer programs and data processed in a computer. It is a major element of many current military systems and may be the reliability-limiting element in a system because of its complexity. When software is part of a system development, the contractor's reliability data system should permit software running time and software error data to be collected, evaluated, and reported, so that management can measure the growth of software reliability and forecast the test time needed to reach a satisfactory level of software reliability. Software errors, once detected, are defined as failures. All errors detected after internal release of software modules should be reported, analyzed and classified not only to measure reliability, but to evaluate the software development tools and techniques in use and to provide information for planning and improving future software development projects.

Many organizations that develop large scale military software employ multiple forms to collect software data. However, a single form can be used satisfactorily to collect software reliability data if the form includes information on the occurrence of errors (i.e., detection of software errors and omissions), information gained by subsequent analysis of detected errors and omissions, documentation of corrections and modifications, and verification of them, together with certain category data needed for statistical analysis of software development progress.

Figure 9-13 is an example of a comprehensive software trouble report (TR). Data are reported on the form as indicated below. The form is initiated by test or operating personnel who detect a software error Fields marked with an asterisk are filled in by the analyst or review team who investigate the error.

1. Date – the date the trouble report (TR) form is prepared.

*2. Error Category – Three-character error code from list given on back of TR form.

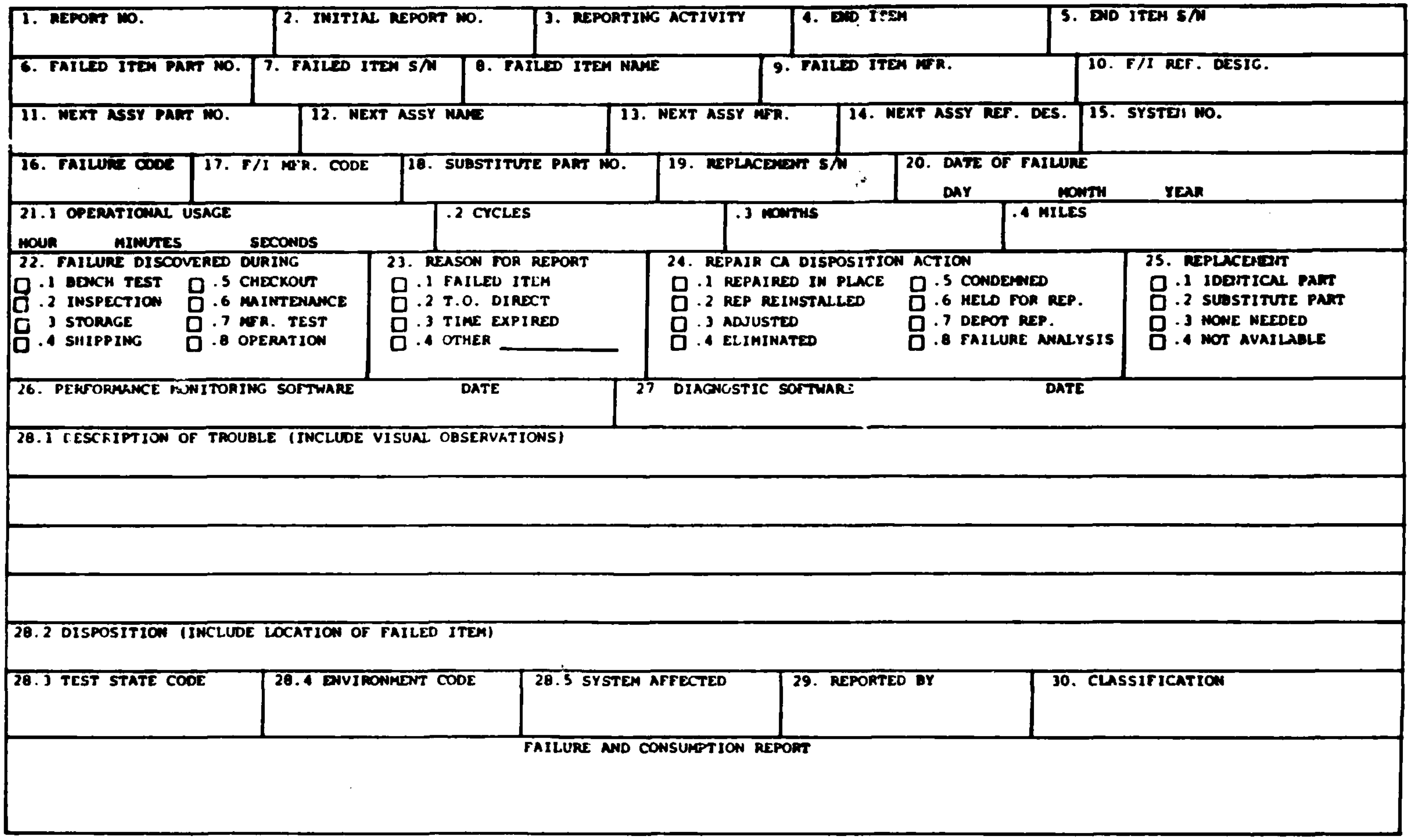
*3. Criticality – Circle appropriate severity code. H = High, M = Medium, L = Low, NA = Not Applicable.

*4. TR Number – Test report number.

5. Title – A brief description of the problem.

6. Program Designation – The official designation of the computer program against which the TR is written (NA for documentation troubles).

7. Program Document – The official designation of the program document against which the TR is written; include page, paragraph number, etc. (NA for program and logic troubles).



1. REPORT NO.
2. INITIAL REPORT NO.
3. REPORTING ACTIVITY
4. END ITEM
5. END ITEM S/N

6. FAILED ITEM PART NO.
7. FAILED ITEM S/N
8. FAILED ITEM NAME
9. FAILED ITEM MFR.
10. F/I REF. DESIG.

11. NEXT ASSY PART NO.
12. NEXT ASSY NAME
13. NEXT ASSY MFR.
14. NEXT ASSY REF. DES.
15. SYSTEM NO.

16. FAILURE CODE
17. F/I MFR. CODE
18. SUBSTITUTE PART NO.
19. REPLACEMENT S/N
20. DATE OF FAILURE — DAY MONTH YEAR

21.1 OPERATIONAL USAGE — HOUR MINUTES SECONDS
.2 CYCLES
.3 MONTHS
.4 MILES

22. FAILURE DISCOVERED DURING
- ☐ .1 BENCH TEST
- ☐ .2 INSPECTION
- ☐ .3 STORAGE
- ☐ .4 SHIPPING
- ☐ .5 CHECKOUT
- ☐ .6 MAINTENANCE
- ☐ .7 MFR. TEST
- ☐ .8 OPERATION

23. REASON FOR REPORT
- ☐ .1 FAILED ITEM
- ☐ .2 T.O. DIRECT
- ☐ .3 TIME EXPIRED
- ☐ .4 OTHER ______

24. REPAIR CA DISPOSITION ACTION
- ☐ .1 REPAIRED IN PLACE
- ☐ .2 REP REINSTALLED
- ☐ .3 ADJUSTED
- ☐ .4 ELIMINATED
- ☐ .5 CONDEMNED
- ☐ .6 HELD FOR REP.
- ☐ .7 DEPOT REP.
- ☐ .8 FAILURE ANALYSIS

25. REPLACEMENT
- ☐ .1 IDENTICAL PART
- ☐ .2 SUBSTITUTE PART
- ☐ .3 NONE NEEDED
- ☐ .4 NOT AVAILABLE

26. PERFORMANCE MONITORING SOFTWARE DATE
27 DIAGNOSTIC SOFTWARE DATE

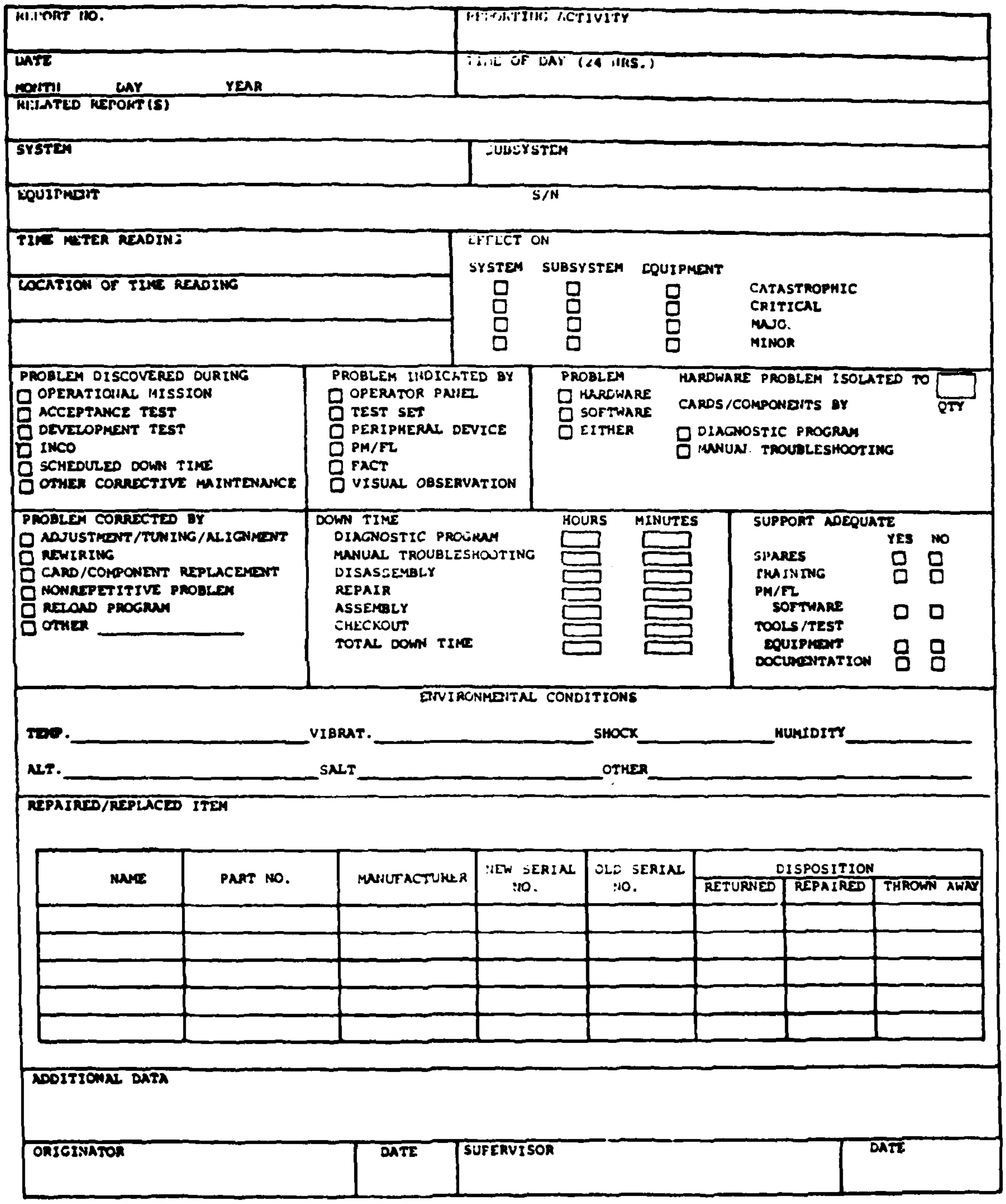
28.1 DESCRIPTION OF TROUBLE (INCLUDE VISUAL OBSERVATIONS)

28.2 DISPOSITION (INCLUDE LOCATION OF FAILED ITEM)

28.3 TEST STATE CODE
28.4 ENVIRONMENT CODE
28.5 SYSTEM AFFECTED
29. REPORTED BY
30. CLASSIFICATION

FAILURE AND CONSUMPTION REPORT

Figure 9-11. Sample Failure Report



REPORT NO.

REPORTING ACTIVITY

DATE

MONTH DAY YEAR

TIME OF DAY (24 HRS.)

RELATED REPORT(S)

SYSTEM

SUBSYSTEM

EQUIPMENT

S/N

TIME METER READING

LOCATION OF TIME READING

EFFECT ON

SYSTEM	SUBSYSTEM	EQUIPMENT	
☐	☐	☐	CATASTROPHIC
☐	☐	☐	CRITICAL
☐	☐	☐	MAJOR
☐	☐	☐	MINOR

PROBLEM DISCOVERED DURING
- ☐ OPERATIONAL MISSION
- ☐ ACCEPTANCE TEST
- ☐ DEVELOPMENT TEST
- ☐ INCO
- ☐ SCHEDULED DOWN TIME
- ☐ OTHER CORRECTIVE MAINTENANCE

PROBLEM INDICATED BY
- ☐ OPERATOR PANEL
- ☐ TEST SET
- ☐ PERIPHERAL DEVICE
- ☐ PM/FL
- ☐ FACT
- ☐ VISUAL OBSERVATION

PROBLEM
- ☐ HARDWARE
- ☐ SOFTWARE
- ☐ EITHER

HARDWARE PROBLEM ISOLATED TO ☐ QTY

CARDS/COMPONENTS BY
- ☐ DIAGNOSTIC PROGRAM
- ☐ MANUAL TROUBLESHOOTING

PROBLEM CORRECTED BY
- ☐ ADJUSTMENT/TUNING/ALIGNMENT
- ☐ REWIRING
- ☐ CARD/COMPONENT REPLACEMENT
- ☐ NONREPETITIVE PROBLEM
- ☐ RELOAD PROGRAM
- ☐ OTHER ____________

DOWN TIME	HOURS	MINUTES
DIAGNOSTIC PROGRAM	☐	☐
MANUAL TROUBLESHOOTING	☐	☐
DISASSEMBLY	☐	☐
REPAIR	☐	☐
ASSEMBLY	☐	☐
CHECKOUT	☐	☐
TOTAL DOWN TIME	☐	☐

SUPPORT ADEQUATE	YES	NO
SPARES	☐	☐
TRAINING	☐	☐
PM/FL SOFTWARE	☐	☐
TOOLS/TEST EQUIPMENT	☐	☐
DOCUMENTATION	☐	☐

ENVIRONMENTAL CONDITIONS

TEMP. __________ VIBRAT. __________ SHOCK __________ HUMIDITY __________

ALT. __________ SALT __________ OTHER __________

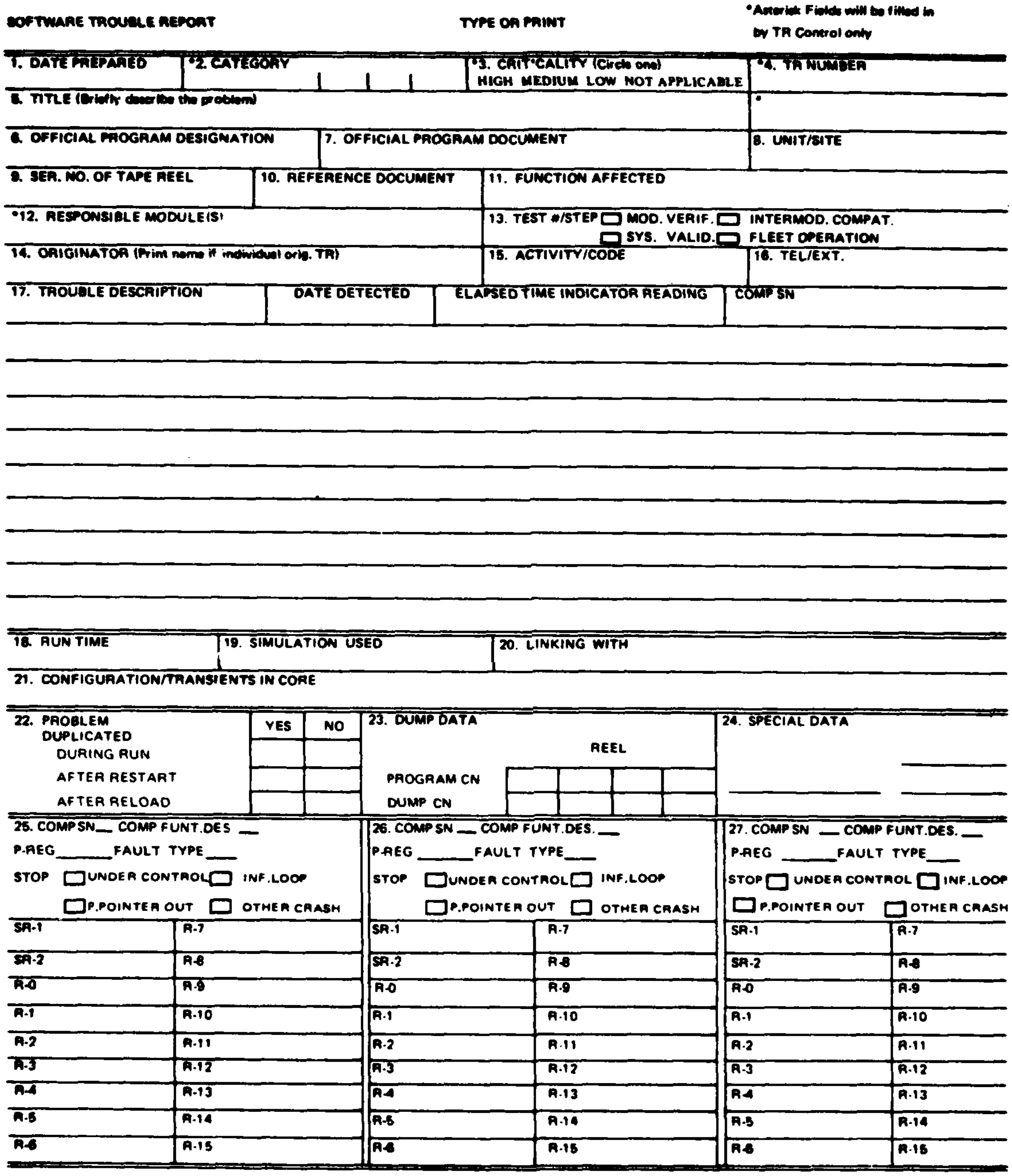
REPAIRED/REPLACED ITEM

NAME	PART NO.	MANUFACTURER	NEW SERIAL NO.	OLD SERIAL NO.	DISPOSITION		
					RETURNED	REPAIRED	THROWN AWAY

ADDITIONAL DATA

ORIGINATOR	DATE	SUPERVISOR	DATE

Figure 9-12. Sample Failure Report



SOFTWARE TROUBLE REPORT

TYPE OR PRINT

*Asterisk Fields will be filled in by TR Control only

1. DATE PREPARED
*2. CATEGORY
*3. CRITICALITY (Circle one) HIGH MEDIUM LOW NOT APPLICABLE
*4. TR NUMBER
5. TITLE (Briefly describe the problem)
*
6. OFFICIAL PROGRAM DESIGNATION
7. OFFICIAL PROGRAM DOCUMENT
8. UNIT/SITE
9. SER. NO. OF TAPE REEL
10. REFERENCE DOCUMENT
11. FUNCTION AFFECTED
*12. RESPONSIBLE MODULE(S)
13. TEST #/STEP ☐ MOD. VERIF. ☐ INTERMOD. COMPAT. ☐ SYS. VALID. ☐ FLEET OPERATION
14. ORIGINATOR (Print name if individual orig. TR)
15. ACTIVITY/CODE
16. TEL/EXT.
17. TROUBLE DESCRIPTION
DATE DETECTED
ELAPSED TIME INDICATOR READING
COMP SN
18. RUN TIME
19. SIMULATION USED
20. LINKING WITH
21. CONFIGURATION/TRANSIENTS IN CORE

22. PROBLEM DUPLICATED	YES	NO
DURING RUN		
AFTER RESTART		
AFTER RELOAD		

23. DUMP DATA

	REEL			
PROGRAM CN				
DUMP CN				

24. SPECIAL DATA

25. COMP SN__ COMP FUNT.DES __
P-REG______FAULT TYPE___
STOP ☐ UNDER CONTROL ☐ INF.LOOP
☐ P.POINTER OUT ☐ OTHER CRASH

SR-1	R-7
SR-2	R-8
R-0	R-9
R-1	R-10
R-2	R-11
R-3	R-12
R-4	R-13
R-5	R-14
R-6	R-15

26. COMP SN __ COMP FUNT.DES. __
P-REG ______FAULT TYPE___
STOP ☐ UNDER CONTROL ☐ INF.LOOP
☐ P.POINTER OUT ☐ OTHER CRASH

SR-1	R-7
SR-2	R-8
R-0	R-9
R-1	R-10
R-2	R-11
R-3	R-12
R-4	R-13
R-5	R-14
R-6	R-15

27. COMP SN __ COMP FUNT.DES. __
P-REG ______FAULT TYPE___
STOP ☐ UNDER CONTROL ☐ INF.LOOP
☐ P.POINTER OUT ☐ OTHER CRASH

SR-1	R-7
SR-2	R-8
R-0	R-9
R-1	R-10
R-2	R-11
R-3	R-12
R-4	R-13
R-5	R-14
R-6	R-15

Figure 9-13. Software Trouble Report Form

Software Trouble Report (Reverse Side)

28. ANALYSIS		DATE RECEIVED
ERROR PREVIOUSLY REPORTED ON TR #(S) or F/M REPORT	SIGNATURE	DATE
29. CORRECTIONS		ECP NO.
CODE CHANGES		
DOCUMENTATION CHANGES		
CORE AND TIMING CHANGES		
CORRECTIONS VERIFIED BY		DATE

ERROR CATEGORIES

A_0	COMPUTATIONAL ERRORS
A_1	Incorrect operand in equation
A_2	Incorrect use of parenthesis
A_3	Sign convention error
A_4	Units or data conversion error
A_5	Computation produces an over/under flow
A_6	Incorrect/inaccurate equation used
A_7	Precision loss due to mixed mode
A_8	Missing computation
A_9	Rounding or truncation error
B_0	LOGIC ERRORS
B_1	Incorrect operand in logical expression
B_2	Logic activities out of sequence
B_3	Wrong variable being checked
B_4	Missing logic or condition tests
B_5	Too many/few statements in loop
B_6	Loop iterated incorrect number of times (including endless loop)
B_7	Duplicate logic
C_0	DATA INPUT ERRORS
C_1	Invalid input read from correct data file
C_2	Input read from incorrect data file
C_3	Incorrect input format
C_4	Incorrect format statement referenced
C_5	End of file encountered prematurely
C_6	End of file missing
D_0	DATA HANDLING ERRORS
D_0	Data file not rewound before reading
D_1	Data initialization not done
D_2	Data initialization done improperly
D_3	Variable used as a flag or index not set properly
D_4	Variable referred to by the wrong name
D_5	Bit manipulation done incorrectly
D_6	Incorrect variable type
D_7	Data packing/unpacking error
D_8	Sort error
D_9	Subscripting error
E_0	DATA OUTPUT ERRORS
E_1	Data written on wrong file
E_2	Data written according to the wrong format statement
E_3	Data written in wrong format
E_4	Data written with wrong carriage control
E_5	Incomplete or missing output
E_6	Output field size too small
E_7	Line count or page eject problem
E_8	Output garbled or misleading

F_0	INTERFACE ERRORS
F_1	Wrong subroutine called
F_2	Call to subroutine not made or made in wrong place
F_3	Subroutine arguments not consistent in type, units, order, etc.
F_4	Subroutine called is nonexistent
F_5	Software/data base interface error
F_6	Software user interface error
F_7	Software/software interface error
G_0	DATA DEFINITION ERRORS
G_1	Data not properly defined/dimensioned
G_2	Data referenced out of bounds
G_3	Data being referenced at incorrect location
G_4	Data pointers not incremented properly
H_0	DATA BASE ERRORS
H_1	Data not initialized in data base
H_2	Data initialized to incorrect value
H_3	Data units are incorrect
I_0	OPERATION ERRORS
I_1	Operating system error (vendor supplied)
I_2	Hardware error
I_3	Operator error
I_4	Test execution error
I_5	User misunderstanding/error
I_6	Configuration control error
J_0	OTHER
J_1	Time limit exceeded
J_2	Core storage limit exceeded
J_3	Output line limit exceeded
J_4	Compilation error
J_5	Code or design inefficient/not necessary
J_6	User/programmer requested enhancement
J_7	Design nonresponsive to requirements
J_8	Code delivery or redelivery
J_9	Software not compatible with project standards
K_0	DOCUMENTATION ERRORS
K_1	User manual
K_2	Interface specification
K_3	Design specification
K_4	Requirements specification
K_5	Test documentation
XX0	PROBLEM REPORT REJECTION
XX1	No problem
XX2	Void/withdrawn
XX3	Out of scope – not part of approved design
XX4	Duplicates another problem report
XX5	Deferred

Figure 9-13. Software Trouble Report Form (Continued)

8. Unit/Site – Ship or test site at which trouble was detected.

9. Reel – Serial number of tape reel used (NA for documentation and logic troubles).

10. Reference Document – The document which provides the basis for determining that trouble exists.

11. Responsible Function – The operational function of the computer program affected by the trouble.

*12. Responsible Module (s) – Designation of module(s) affected.

13. Test # Step – The test designation and step being executed at the time the trouble was discovered (NA for documentation and logic troubles). Check off appropriate box for stage of testing being performed: module verification testing, intermodule compatibility testing, system validation testing, fleet operations.

14. Originator – Printed name of the individual originating the TR.

15. Activity Code – The activity and code name or number of individual originating the TR.

16. Tel/Ext – The office phone number and extension of the individual originating the TR.

17. Trouble Description – Enter date trouble was detected, elapsed time meter reading and equipment having the meter which was read. In absence of elapsed time meters, estimate or reconstruct running time from system logs. Then write a sentence defining the trouble, and develop a word picture of events leading up to and coincident with the problem. Structure statements so that a programmer/test analyst can recreate the situation. Cite equipment being used, unusual cabling, etc. Indicate console on line, modes, etc. if applicable. If continuation sheets are required, fill in page___of___at top of TR form.

18. Run Time – Elapsed time from program start until trouble occurred in hours/ quarter hours (NA for documentation and logic troubles).

19. Simulation Used – Program/equipments used to simulate operational conditions. Indicate tape reel number if applicable (NA for documentation and logic troubles). Indicate test tools used if applicable.

20. Linking With – Write in other link sites/programs (NA for documentation and logic troubles). Indicate interfaces with test programs and tools.

21. Configuration/Transients in Core – Identify configuration/transients loaded when trouble occurred (like patches) (NA for documentation and logic troubles).

22. Problems Duplicated – Check duplication attempts/success/failures for problem troubles (NA for documentation and logic troubles).

23. Dump Data – Write in serial number of reel containing dumped data; enter location of each program call number on dump tape (NA for documentation and logic troubles).

24. Special Data – Enter data from special entrance cells that will aid programmer in isolation of trouble (NA for documentation and logic troubles).

25,26,27. Stop Data – Designate physical computer, function designation, and type of stop and failure when applicable. Check appropriate box to indicate if stop was under normal control or abnormal; if abnormal, indicate whether infinite loop, program pointer out of bounds, or other crash condition. Write in the register content if relevant. (NA for documentation and logic troubles).

*28. Analysis – Indicate date TR received for analysis and write a brief summary of findings. Indicate original TR number if TR is found to be duplicate, that is, if the same error has already been reported. Indicate failure report or failure/maintenance report number if problem first appeared on a hardware reporting form or is associated. Enter printed name of person conducting analysis. Enter data of analysis.

*29. Corrections – Give brief summaries of code changes, documentation changes, core and timing changes resulting from analysis. Enter number of any ECP resulting. Enter printed name of person verifying corrections and date of verification. As a minimum, identify the block (level) of code. Detailed explanation of code changes may be included in the appropriate engineering change proposal.

9.2.2.1 Coding Software Error Source and Type

The error code list given on the reverse of the TR form was recommended by researchers who studied many military software programs. It is reduced from a much longer initial set of categories and is adequate for the majority of programs.

There is a blank character in the 3-character alphanumeric error category designator. This blank is for the error source. Five error source designators are defined.

Error Source Code	Error Source	Description
0	Requirements	Source of problem is changing, ill conceived or poorly stated performance requirement.
1	Coding	Source of problem is an error in implementing the design as code.
2	Design	Source of problem is in preliminary or detailed design.
3	Maintenance	Source of problem is an error introduced in process of trying to fix a previous error.
4	Not Known	Source of error not known.

As an example of categorization using the 3-character designators, A03 would be a sign convention computational error traceable to an origin in the software requirements.

The error categories when completely recorded, define both sources and types of errors. Even if analyses of software errors prove inadequate to support assignment of the final character, a considerable amount of useful statistical analysis can be done using only the first two characters. A category is included for elective enhancements (J_6). making the categories compatible with a single-form system in which not every TR reports an error.

9.2.2.2 Coding Software Error Severity

A general set of software error criticality categories, which can be used as is or as a basis to derive project-specific definitions for use in block 3 of the TR form, are:

HIGH (H): An error which significantly degrades user's mission or prevents its completion.

MEDIUM (M): An error for which a workaround is available, so that mission performance is not significantly degraded.

LOW (L): An error which does not affect performance.

NOT APPLICABLE: A program enhancement or an error which cannot be verified or repeated, or is a secondary error, or is the result of a documentation error or duplication.

9.2.2.3 Collecting Software Development Cycle Data

Block 13 of the TR form provides space for recording the stage of software testing in which an error was detected–module verification testing, intermediate compatibility testing, system validation testing, fleet operation. This information is helpful for analyzing reliability growth during development. Software reliability measurement should properly begin after internal release, when the debugged module is first placed under control of a formal configuration management policy (i.e., prior to module integration).

9.2.2.4 Collecting Software Timing Data

The TR form provides block 18 for run time at error detection. While this information is often of value to the error analyst, the validity of run time or CPU time statistics for real time systems is questionable, since typically all computers are running whenever the system is in use. Moreover, these are not the time data needed for reliability measurement. Cumulative module use time from date of internal release is needed. This information must usually be synthesized from knowledge of internal release date and cumulative system operating time after that date. Thus the time data needed at occurrence of an error is the cumulative system operating time, which is available from elapsed time indicators (ETI) in many computer systems. In the absence of ETIs, cumulative operating time can often be reconstructed roughly from system operating logs.

The TR form provides space adjacent to block 17 to accept ETI readings from a system computer; in military systems having more than one computer, the computer from which the reading is taken should also be completely identified when the reading is recorded. These readings should be capable of rough verification (and resolution of disparities) by reference to the system operating log.

9.2.2.5 Collecting Software Error Analysis Information

A brief narrative explanation of findings about the cause of the error should appear on the TR form. A summary of analysis results is also needed to audit the accuracy of the error category assignment in block 2.

9.2.2.6 Software Error Correction Data

Corrections fall into five general categories: 1) code changes, 2) documentation changes, 3) design changes, 4) core size changes and 5) timing changes. A brief narrative explanation of correction changes, if any, should appear as part of the complete TR after investigation is complete and corrective action determined.

9.2.3 Data Collection from Fleet Service

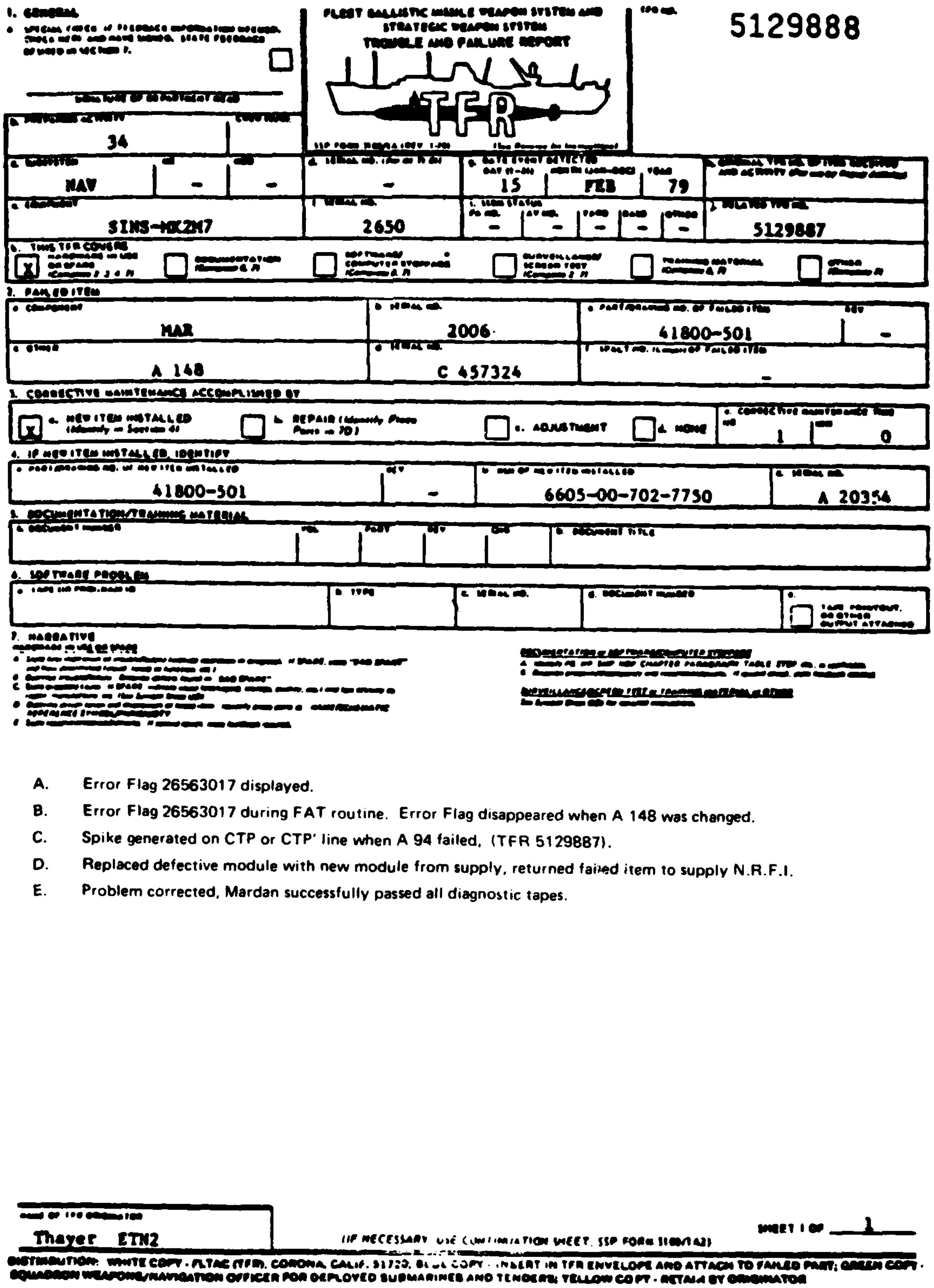
During a development program, interest centers about the inherent reliability and availability of the system, since that is generally the characteristic for which the supplier is accountable. The measurement process is principally concerned with accurate assessment of failure and error rates, but the majority of system problems that engender concern in fleet service center about failure events, particularly repetitive failure events, rather than failure rates. It is a consequence of the definitions of inherent reliability and availability that many of these failures are properly excluded. by reason of their causes, from consideration in computing development type inherent reliability and availability. Their presence in an operational system requires a form of evaluation that 1) centers about discrete failure events, 2) considers all hardware failures and software errors regardless of cause, 3) identifies significant trends, patterns, interactions, 4) focuses attention on repetitive failures, 5) provides measures of relative problem importance, 6) provides measures of maintenance capability or problem resolution, 7) evaluates effectiveness of corrective actions, and 8) responds quickly to current experience.

Contractors' data systems should be structured to support the evaluation of the reliability and availability of systems in fleet use to the extent provided by contract. Such a data system must be able to accept inputs from ship's maintenance and operating logs, and from the Fleet Ballistic Missile Weapon System and Strategic Weapon System Trouble and Failure Report Program (SSPINST 3100. 1F[2]). Data from fleet service (e.g., figures 9-14 and 9-15) can be used directly in the reliability and availability measurement models given herein and can be analyzed and reported by the methods of this manual.

The data collection function must assure that codes developed to represent data elements (failure modes, "when discovered", initiating activities, etc.) are sufficiently comprehensive to represent any permissible information item that may be reported concerning an operational system. This may include information never evolved in the factory, such as failure modes unique to the installed environment, interfaces, utilization of spares, generation of scrap, elapsed time in service, manhours used in maintenance or operation, and compliance status with respect to engineering modifications.

9.2.4 Training for Data Collection

Personnel who test and repair hardware may be reluctant to record all of the information available to them unless they understand the need for the data. Therefore, a program is necessary, however informal, to explain the importance of test data to the program and to motivate personnel at the data source. Test personnel must be helped to realize that their functions are important in fulfilling contractual requirements. To accomplish this, a program should be instituted which trains and motivates test personnel. The training should include a review of all data collection forms



1. GENERAL

☐

FLEET BALLISTIC MISSILE WEAPON SYSTEM AND
STRATEGIC WEAPON SYSTEM
TROUBLE AND FAILURE REPORT

TFR

TFR NO. 5129888

a. 34						
c. SUBSYSTEM NAV	-	-	d. SERIAL NO. -	g. DATE EVENT DETECTED DAY 15	MONTH FEB	YEAR 79
e. COMPONENT SINS-MK2M7			f. SERIAL NO. 2650	i. ITEM STATUS - - - - -		j. RELATED TFR NO. 5129887

k. THIS TFR COVERS ☒ HARDWARE IN USE OR SPARE ☐ DOCUMENTATION ☐ SOFTWARE/COMPUTER SYSTEMS ☐ SURVEILLANCE/SCREEN TEST ☐ TRAINING MATERIAL ☐ OTHER

2. FAILED ITEM

a. COMPONENT MAR	b. SERIAL NO. 2006	c. PART/DRAWING NO. OF FAILED ITEM 41800-501	REV -
e. OTHER A 148	d. SERIAL NO. C 457324	f. -	

3. CORRECTIVE MAINTENANCE ACCOMPLISHED BY

☒ a. NEW ITEM INSTALLED ☐ b. REPAIR ☐ c. ADJUSTMENT ☐ d. NONE e. CORRECTIVE MAINTENANCE TIME HR 1 MIN 0

4. IF NEW ITEM INSTALLED, IDENTIFY

a. 41800-501	REV -	b. 6605-00-702-7750	c. SERIAL NO. A 20354

5. DOCUMENTATION/TRAINING MATERIAL

a. DOCUMENT NUMBER	VOL.	PART	REV	CHG	b. DOCUMENT TITLE

6. SOFTWARE PROBLEM

a.	b. TYPE	c. SERIAL NO.	d. DOCUMENT NUMBER	e. ☐ TAPE PRINTOUT, OR OTHER OUTPUT ATTACHED

7. NARRATIVE

A. Error Flag 26563017 displayed.

B. Error Flag 26563017 during FAT routine. Error Flag disappeared when A 148 was changed.

C. Spike generated on CTP or CTP' line when A 94 failed, (TFR 5129887).

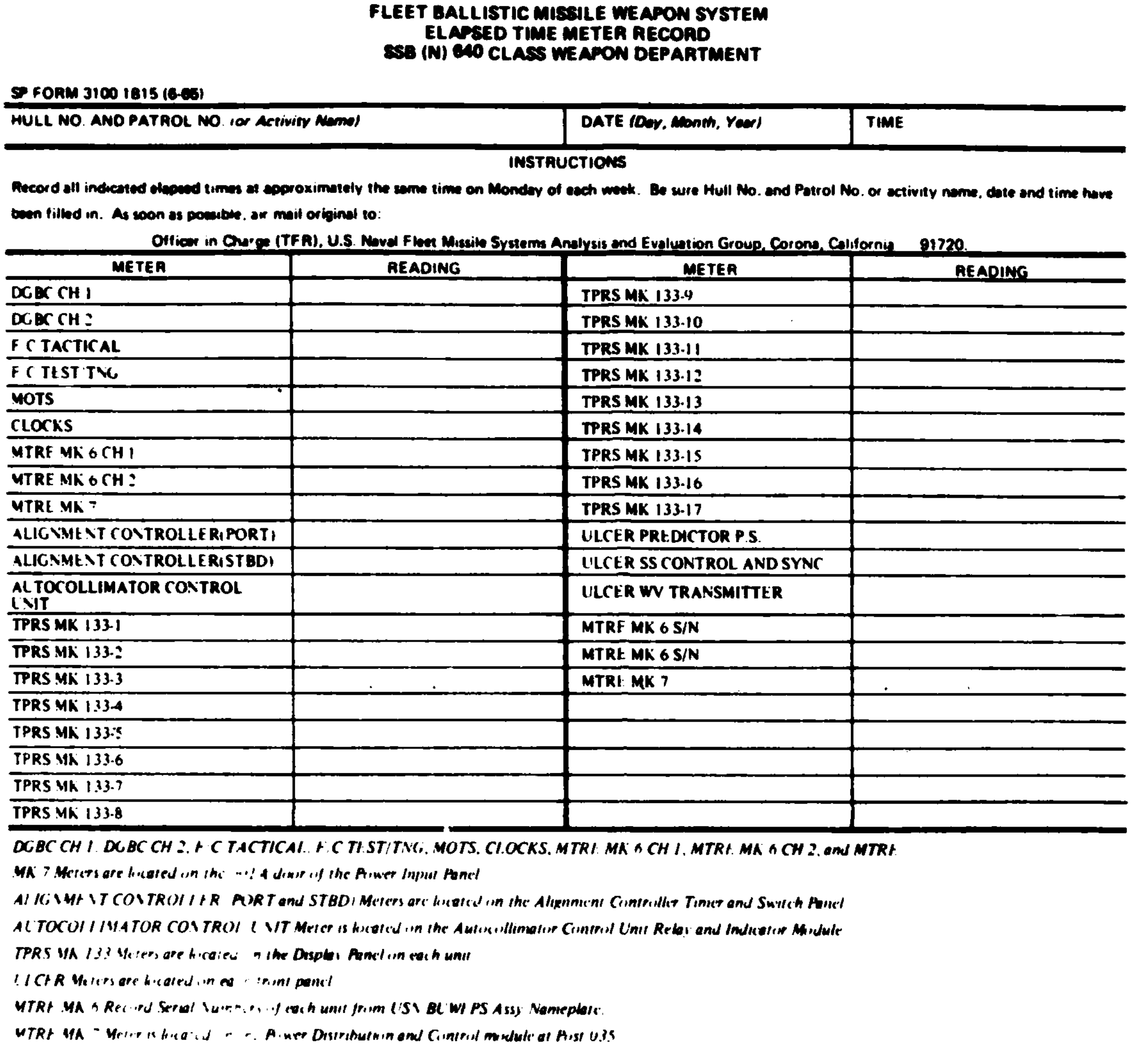
D. Replaced defective module with new module from supply, returned failed item to supply N.R.F.I.

E. Problem corrected, Mardan successfully passed all diagnostic tapes.

Thayer ETN2

SHEET 1 OF 1

Figure 9-14. Trouble and Failure Report Form



FLEET BALLISTIC MISSILE WEAPON SYSTEM
ELAPSED TIME METER RECORD
SSB (N) 640 CLASS WEAPON DEPARTMENT

SP FORM 3100 1815 (6-65)

HULL NO. AND PATROL NO. *(or Activity Name)*	DATE *(Day, Month, Year)*	TIME

INSTRUCTIONS

Record all indicated elapsed times at approximately the same time on Monday of each week. Be sure Hull No. and Patrol No. or activity name, date and time have been filled in. As soon as possible, air mail original to:

Officer in Charge (TFR), U.S. Naval Fleet Missile Systems Analysis and Evaluation Group, Corona, California 91720.

METER	READING	METER	READING
DGBC CH 1		TPRS MK 133-9	
DGBC CH 2		TPRS MK 133-10	
F C TACTICAL		TPRS MK 133-11	
F C TEST TNG		TPRS MK 133-12	
MOTS		TPRS MK 133-13	
CLOCKS		TPRS MK 133-14	
MTRE MK 6 CH 1		TPRS MK 133-15	
MTRE MK 6 CH 2		TPRS MK 133-16	
MTRE MK 7		TPRS MK 133-17	
ALIGNMENT CONTROLLER(PORT)		ULCER PREDICTOR P.S.	
ALIGNMENT CONTROLLER(STBD)		ULCER SS CONTROL AND SYNC	
AUTOCOLLIMATOR CONTROL UNIT		ULCER WV TRANSMITTER	
TPRS MK 133-1		MTRE MK 6 S/N	
TPRS MK 133-2		MTRE MK 6 S/N	
TPRS MK 133-3		MTRE MK 7	
TPRS MK 133-4			
TPRS MK 133-5			
TPRS MK 133-6			
TPRS MK 133-7			
TPRS MK 133-8			

DGBC CH 1, DGBC CH 2, F/C TACTICAL, F/C TESTING, MOTS, CLOCKS, MTRE MK 6 CH 1, MTRE MK 6 CH 2, and MTRE MK 7 Meters are located on the [illegible] 4 door of the Power Input Panel

ALIGNMENT CONTROLLER (PORT and STBD) Meters are located on the Alignment Controller Timer and Switch Panel

AUTOCOLLIMATOR CONTROL UNIT Meter is located on the Autocollimator Control Unit Relay and Indicator Module

TPRS MK 133 Meters are located on the Display Panel on each unit

ULCER Meters are located on each front panel

MTRE MK 6 Record Serial Numbers of each unit from USN BUWEPS Assy Nameplate.

MTRE MK 7 Meter is located on the Power Distribution and Control module at Post 035

Figure 9-15. Elapsed Time Meter Record

and a discussion of the information to be recorded on the forms. To promote the recording of information accurately, completely and legibly, a procedure should be established for immediate review of completed forms for errors, omissions, or poor legibility, and to return forms promptly to the originator for correction with follow-up, and to assure forms are corrected and forwarded to data control within permissible time frames.

9.3 DATA CONTROL

Personnel assigned to data control must have access to test areas, to assure that all testing is recorded and all forms are forwarded to data control. Block-controlled serial numbering of data collection forms is one technique for preventing test personnel from omitting any data (forms) from the data system; another is to submit to the data control activity a checklist containing the drawing numbers and serial numbers of items scheduled to be tested. The checklist is compared against the data subsequently processed. Disparities are referred back to data originators for resolution.

All completed forms should be manually screened for gross errors in the data control area. Screening includes verifying legibility, correct system identification, correct recording of data, and compliance with signature requirements. Errors in these categories are returned to the originator for correction.

In addition to the manual checks made on the data, a computer program to edit the data should be developed. The computer should be programmed to perform accuracy, validity, and completeness tests on the data and to issue an error list. The error list should embody diagnostics to identify the type of error. Errors should be returned to the responsible operation for correction. Experience has shown that this technique provided motivation to personnel involved and that the quantity of errors rapidly diminishes.

Computer tests for accuracy should assure numeric data does not appear in columns reserved for alphabetic characters and vice versa. It should also check ranges where appropriate (e.g., day 1 to 31, month 1 to 12, year 00 to 99, minutes 00 to 60, level 1 to 7, etc.). The computer completeness check should assure that all required data is present for each entry (additional data is required when a failure occurs). The validity checks are used to monitor the test results (time or cycles), permissible ranges for the time for states A, B and D and cycles for state C may be inserted into the computer. Data reported outside these limits would be returned to the test activity to be verified, e.g., test personnel might report four hours for a vibration test when the permissible range for the test is 15 to 30 minutes.

While an error list is capable of detecting errors in reported data, it cannot detect the complete omission of data. The purpose of the check list is to check periodically to determine if all data is being reported. A list of all serialized hardware scheduled for test should be prepared and fed into the computer. The output report indicates the test data which have not been received. A study can then be addressed to determine why the data were omitted. Corrective measures to close the loop can then be instituted.

9.3.1 Hardware Data Control

The general data control function monitors reported data for timeliness, completeness, and accuracy, and then converts the data for machine processing when appropriate. Data control must designate pick-up and delivery points, schedules, distribution and accountability for data received for processing within the data system. Procedures must be established for reproducing, distributing, collecting, editing and filing data forms emanating from test areas and fleet service. These procedures should include the manual and automatic methods for screening the reported data for compliance with requirements and providing for corrections of errors in the reported data.

Working with the data processing activity, data control should determine those data fields that can be checked by computer programs, the applicable checking logic, and information apart from the data itself needed to check for errors. Such information must be furnished by data control, using specific project requirements established by analysis.

Information needed to check for errors in collected data consists of all acceptable hardware identifications and test descriptions, alpha-numeric fields, identification of variables including durations, ranges of variables, and criteria for determining when specific data fields should appear or remain blank on the originating documents (e.g., if a failure is recorded on a test form, it should be accompanied by reference to a failure report number). For externally originated data, complete correction of errors may represent a difficult problem for a contractor's data control office. Thus, attention should be given in data processing to rendering the system outputs as insensitive as possible to data elements having high error rates, and to synthesizing descriptive statistics that are likely to be approximately correct despite undetected errors in the data base.

Source documents should be marked to indicate that essential information has been extracted; this can be accomplished by stamping the source document or punching a hole at a designated location. Data fields or blocks in the documents that are not properly filled in, or that contain information that cannot be interpreted, can be circled in red. Processed documents should then be copied and originals returned to the originating activity for error correction as required. A suspense file must be provided to assure that all detected data errors are corrected prior to final processing and that no data are lost from the system in the error correction loop. The file should contain a record of all errors that have not been corrected. When the data are to be used as inputs to a computerized processing function, formatting instructions and tabular information for automatic error editing and validity checking must be established.

Management provides a key role in the data control process by actively reviewing the data system (at least weekly) and emphasizing the importance of the data. Management should carefully monitor the suspense file and correct personnel deficiencies.

9.3.2 Software Data Control

9.3.2.1 Review of Software Trouble Reports at Close-Out

The process of documenting software errors is itself prone to numerous errors, as is the process of documenting hardware failures and operating time. Thus it is essential that an editorial review of each completed software trouble report (TR) be included as a formal step in the process of closing out the report. Particular attention should be given to assuring the accuracy and completeness of figure 9-13 blocks 2, 3, 12, 13, 17, 25 through 27, if applicable, 28 and 29, which contain data needed for reliability assessment.

9.3.2.2 Editorial Review of Software Error Category and Severity Assignments

The three-character error category code assigned to the error by the analyst or analysis team must be reviewed for correctness at close-out. The criticality score assigned in block 3 of the TR should be reviewed for accuracy. While reliability indices can be computed relative to criticality categories individually or in combination, as a minimum, a single set of indices should be computed using high and medium criticality error events taken together (or a judicious combination of the highest priority levels of MIL-STD-1679[3] if the software errors have been categorized in accordance with that standard). This gives meaningful metrics for characterizing the reliability status and growth of the software relative to its intended mission requirements.

9.3.2.3 Normalization of Software Operating Time Data

Cumulative operating hours following release is also an essential data element for software reliability calculations. In general, it must be computed because not all of the software comprising a system begins testing at the same time. And it has been found that the concepts of mission stress and duty cycle which apply to hardware, have analogs in the progressive testing environments of software development programs. Thus, it is sometimes necessary to apply multipliers to system operating times accrued in various test phases, multipliers that account for the varying effectiveness of the test phases in detecting latent software errors.

Another problem stems from the frequent presence of multiple elapsed-time meters, which in a real-time system should, but do not always, accumulate time at the same rate. Often, too, a meter is not read when a failure occurs, due to oversight or because test personnel consider their reliability data generation function ancillary to their other duties. Sometimes times are not continuous because the meters are an integral part of a removable subassembly (poor design practice). Thus, an important part of the editing and control function is to compute the applicable cumulative operating time when a software error was first detected. Because of the project-unique nature of this computation, and because it may require cross-references to system logs, it is best performed manually by data control personnel editing the TR. In any event, an actual or estimated cumulative time at occurrence must be available for each software error entered in the data system.

As in hardware data control, management must actively evaluate the software data system.

9.4 DATA PROCESSING

The data processing function formats screened data, records it on machine readable media, and generates and maintains a data history file on a storage media for hardware items at component level and above, and for software items at the "module" level and above. It uses the accumulated data to generate summary reports which are used by the reliability analysis activity to evaluate and report the reliability and availability of the system under development or in fleet service.

When errors are detected in the data during processing, listings are prepared explicitly defining the errors and are sent to data control. Data control must then secure correction of the errors, either by making the corrections directly or by forwarding the source documents to their originators. This action must be expeditious so that the originator can accurately resurrect the reported occurence. A suspense file of such errors is maintained in data control to assure that no data are lost from the system in the error-correction loop. Corrected data are again prepared for processing and entered into the next processing cycle.

It is essential that the data processing function provide the capability to sort on all fields of data, the capability to delete old data (sort by date, lot, type of test, etc.), and the capability to correct errors in the data base.

Tasks involved in establishing the data processing function are: 1) determine types and quantities of information to be processed, 2) determine the amount of auditing (computerized and manual error checking) to be performed on data, 3) perform data processing systems analysis, 4) issue computer programming specifications, 5) prepare and verify computer programs, 6) issue operating instructions and procedures. In accomplishing these tasks, the data processing activity should work with groups providing inputs to the data system and with groups that utilize outputs of the data system.

Generation of a history file, formulated and sequenced for use in preparing RMA reports, may require multiple computer runs, because of limitations imposed by the size of the computer or by the number of storage units available. Typically, five runs are needed: 1) format standardization, 2) preliminary processing, 3) project selection, 4) updating and compiling, and 5) a final computer run to provide output reports as defined by the data utilization function. Development of data manipulations to be performed in each of the computer runs is discussed below. Descriptions of typical output reports can be found in section 10.

9.4.1 Format Standardization

The data input formats must be rearranged into a standard record format for future processing. This permits the testing areas to use the forms most suitable to their needs, providing they furnish the common and consistent information required by the data collection activity. A standard fixed-length record format provides ease of processing and eliminates the need to correlate information items later with their locations on the record.

9.4.2 Preliminary Processing

In this step, test and failure data are reviewed to verify the accuracy, conciseness, completeness and validity of the information processed. Review may be manual or by EDIT routines or both. Error listings should be designed to be easily readable by personnel who will do manual error corrections. Because test results collected for a particular system under test may also be applicable to other systems employing the same types of hardware, provisions and criteria should be included in the data system for transferring test data among development programs.

In some instances, reliability or availability assessment may be required on hardware at subcomponent levels, but these subcomponents may never be tested as separate devices, only as portions of larger devices. Therefore, the preliminary processing step may need to provide for estimating test results from component level down to subcomponent level. To accomplish this, the computer must be furnished information on the subcomponent population of the affected components (configuration information).

For example, if component type A contains one each of three types of subcomponents (A1, A2, A3), and if a test is performed involving two of the subcomponents, A1 and A3, but not A2, then only those two subcomponents are assigned operating time or cycles. The third subcomponent, A2, is assigned an amount of non-operating time equal to the operating time assigned to the responsible subcomponent.

9.4.3 Project Selection

In this step the processing program selects and groups data by development project, e.g., POSEIDON, TRIDENT.

9.4.4 Updating and Compiling

Processed test data records will be used to update the existing test history file. Regardless of the effort expended to assure accuracy of test data, errors are to be expected in the history files. It is necessary, therefore, to provide a means for correcting errors in the files during this step.

Since a history file is updated only when reports for its project are required, provisions should be made to generate several optional reports in addition to the composite reliability and availability status report, which is the basic output. Optional reports will usually consist of summaries of test results across or within various categories; examples are summaries within each serial number or across all test environments.

Data processing functions also include computation of summary statistics for software errors, as well as reliability indices for individual modules and for the software system. Summary statistics are of value to management in evaluating the software development process and identifying areas in need of extra attention or control. Analyses of value include frequency plots of software error categories, error sources, and error severities for each software module, intermodule error rate, and percentage of abnormal terminations.

For systems in fleet service, the data processing function should provide certain data manipulations and outputs beyond those provided for systems in development. These additional outputs may include computed operational reliability and availability, listings of repetitive failures, evaluations of the effectiveness of corrective actions, and ordered listings of high failure rate components.

9.4.5 Operational Indices

Operational RMA indices are computed using models selected from Sections 4 through 7.

9.4.6 Failure History

The failure summary report (§ 10.2.2) provides one line of information for each failure that occurs. Special consideration should be given to the following concerns.

9.4.6.1 Repetitive Failures

Contingency tables and other forms of statistical analysis can be programmed into the processing function to evaluate the significance of recurring failures and failure modes. The predicted partial failure rate of a system with respect to a specific mode of failure or a specific location as predicted during system analysis, is an a priori estimate of the relative frequency to be expected of the mode or of the part in that location. By comparing the expected frequency with the observed frequency in a contingency table, the significance of a repetitive mode can be assessed using the chi-square (χ^2) statistic. A listing of repetitive component or parts failures, especially multiple usage components or parts, provides the analyst with a necessary tool to detect reliability problems not necessarily detected by utilizing other printouts.

9.4.6.2 High Failure-Rate Components

The data processing activity should provide programming to highlight components that experience failure rates significantly higher than predicted. Provision should be made to rank these items in order of importance. System performance uses a combination of problem criticality and probability of occurrence ranking, while the maintenance burden is based on cost, spares, maintenance manhours involved, etc., impacts.

9.4.6.3 Effectiveness of Corrective Actions

Availability of a continuous time and failure history enables a cumulative failure incidence curve to be plotted for a system in service. Such a curve is simply a graph of cumulative failures versus cumulative operating time in service—a *non-decreasing* step function. The effective date of specific corrective actions or system modifications can be noted on the curve, permitting ready visual comparison of performance before and after such actions. The height of the risers, or the average slope in various regions of the curve indicates performance trends, an effective corrective action program tending to be reflected in a curve that is concave downward.

9.5 DATA UTILIZATION

9.5.1 Hardware Data Utilization

The data utilization function defines and uses the outputs of the data processing to provide various summary reports and composite reliability and maintainability status reports. Function flow for each type of output is discussed below.

9.5.1.1 Composite RMA Data Output

Data processing must provide outputs (e.g., computer), as requested by the user activity, to support the reliability, maintainability and availability measurement program. The outputs must enable solution of the system statistical models. Reliability, maintainability and availability indices are needed for each type of hardware in each mission environment. Processing must relate test data from the history files to the mission file; this may require use of the "alpha" conversion factors described in § 4.1.2.3 (multipliers, in units of mission per unit time, used to normalize test information to a standard mission as a common time base). Confidence limits on the calculated indices may also be required. Apportioned and predicted reliabilities, maintainabilities and availabilities stored in the computer are recovered for comparison with measured indices.

9.5.1.2 Summary Data Outputs

Data processing must be capable of retrieving data and providing summaries of data in the history files. Figure 9-16 provides a data flow for processing data from the history file to provide summaries. Summaries that use file data directly should be generated during file maintenance and updating, and made available at users' option. At times, special purpose data summaries may be desired. For example, a compendium of failure rates (§ 10.2.7) can be prepared by summarizing failure and test data against general environment categories for specific or general types of hardware. To do this, the data base must be sufficiently extensive, and certain additional information must also be available, in order to consolidate environments and hardware items into valid general categories.

Since management actions by the contractor or user may be major objectives of much of the output of RMA evaluation, the data utilization function should emphasize graphic presentations and other techniques to highlight problems requiring action. In particular, actions requiring long lead-times or having broad system impact (e.g., on logistic support or manning needs) should be singled out and highlighted to data system users at the earliest possible moment.

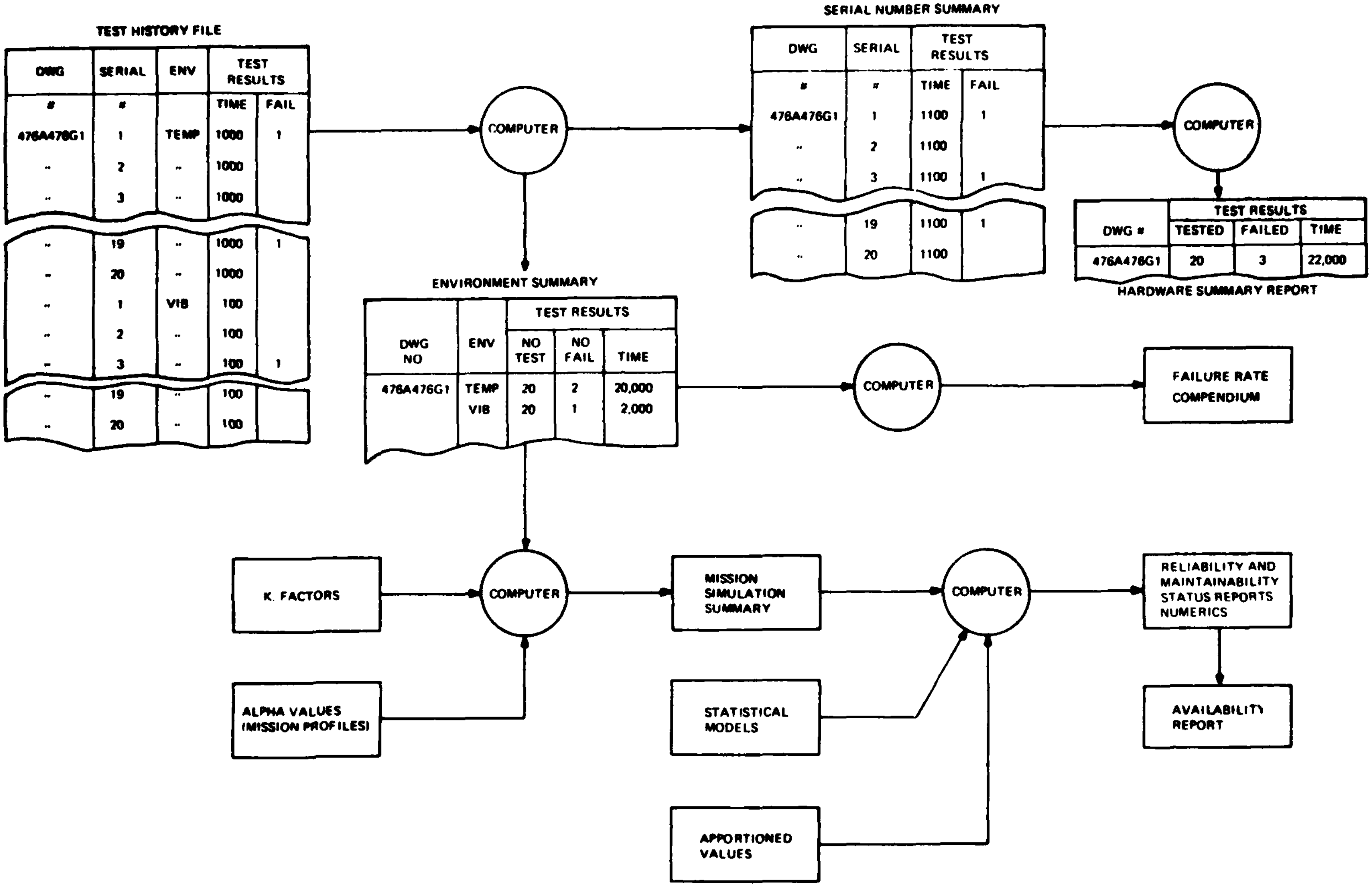
9.5.2 Software Data Utilization

9.5.2.1 Summary Software Error Statistics

Summary statistics should be computed monthly or quarterly, as warranted by the quantity of data processed, for reporting to management. During periods of intense software development activity, weekly reports may be desired. In addition to the summaries discussed below, management may benefit from error breakdowns by software functional area and other categories.

9.5.2.2 Frequency Analysis of Software Error Categories

Errors experienced and reported in development should be summarized by major category (1st character in block 2 of figure 9-13) and reported in tabular form as percentage of total errors or in histogram form. Both forms of presentation are based on the simple statistic $X_i/\Sigma X_i$ where X_i is the number of errors classified in the ith category and ΣX_i is total error events reported. Figures 9-17 and 9-18 provide examples of both forms of summary report. (Statistics presented in figures 9-17 and 9-18 are examples used to illustrate reporting format.)



TEST HISTORY FILE

DWG	SERIAL	ENV	TEST RESULTS	
#	#		TIME	FAIL
476A476G1	1	TEMP	1000	1
"	2	"	1000	
"	3	"	1000	
"	19	"	1000	1
"	20	"	1000	
"	1	VIB	100	
"	2	"	100	
"	3	"	100	1
"	19	"	100	
"	20	"	100	

SERIAL NUMBER SUMMARY

DWG	SERIAL	TEST RESULTS	
#	#	TIME	FAIL
476A476G1	1	1100	1
"	2	1100	
"	3	1100	1
"	19	1100	1
"	20	1100	

ENVIRONMENT SUMMARY

DWG NO	ENV	TEST RESULTS		
		NO TEST	NO FAIL	TIME
476A476G1	TEMP	20	2	20,000
	VIB	20	1	2,000

DWG #	TEST RESULTS		
	TESTED	FAILED	TIME
476A478G1	20	3	22,000

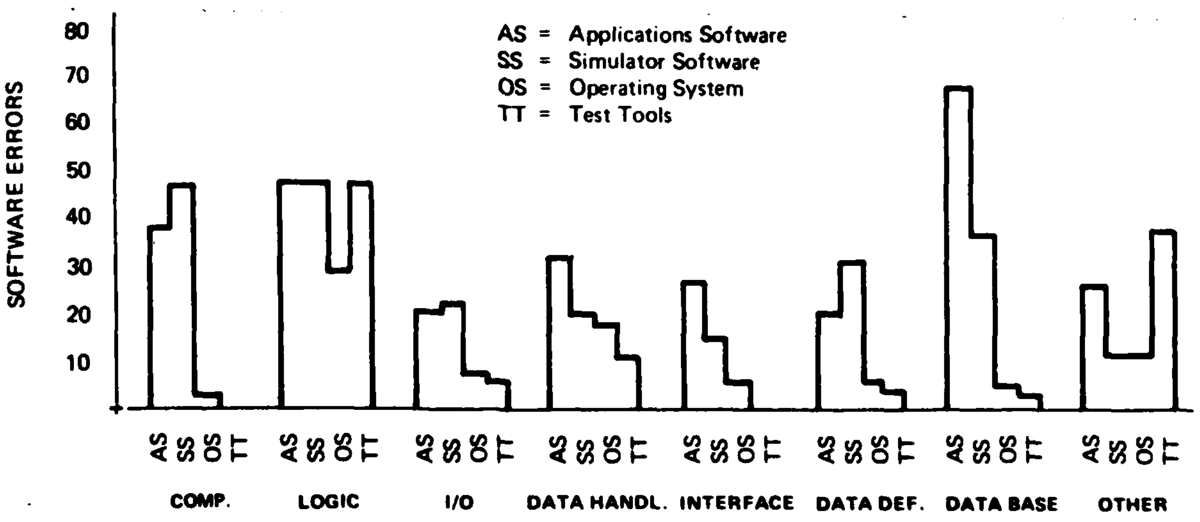
HARDWARE SUMMARY REPORT

Figure 9-16. Data Flow

Major Error Categories		Applications Software (%)	Simulator Software (%)	Operating System (%)	Test Tools (%)	Total
Computational	(A)	13.5	19.6	2.5	0	
Logic	(B)	17.1	20.9	34.6	43.5	
Data Input Data Output	(C) (E)	7.3	9.3	8.6	5.5	
Data Handling	(D)	10.9	8.4	21.0	9.3	
Interface	(F)	9.8	6.7	7.4	0	
Data Definition	(G)	7.3	13.8	7.4	3.7	
Data Base	(H)	24.7	16.4	4.9	2.8	
Other	(J)	9.4	4.9	13.6	35.2	
Code Change Trouble Reports		275	225	81	108	689

FOR ILLUSTRATION ONLY

Figure 9-17. Example of Percentage Breakdown of Major Error Categories



FOR ILLUSTRATION ONLY

Figure 9-18. Example of a Histogram of Major Error Categories

9.5.2.3 Frequency Analysis of Software Error Sources

Similar frequency statistics should be summarized and reported by error source code (2nd character in block 2 of figure 9-13). Figure 9-19 illustrates a typical presentation format for a large software system comprised of four major functional areas–applications software, simulator software, operating system and test tools.

9.5.2.4 Intermodule Error Rate

Intermodule error rate is computed by counting the number of software modules affected by each error, as entered in block 12 of figure 9-13. The summary statistic reported is the percentage of errors involving n modules, where n = 1, 2 . . . Figure 9-20 shows the form of the reported statistics. As shown in figure 9-20, 84.8 percent of the project A errors affected one module, 10.4 percent affected two modules, etc. (The percentages shown are an example computed from actual test data for highly modular military software developed using top down programming techniques.)

9.5.2.5 Termination and Severity Statistics

The percentage of normal versus abnormal terminations should be tabulated and reported, using data from blocks 25, 26, 27 of the TR form (figure 9-13). An example is given in figure 9-21 which shows 90.3% normal and 9.7% abnormal terminations. The level of severity is shown in figure 9-22 which shows 8% high, 31% medium, 41% low and 20% not applicable.

9.5.2.6 Software Reliability Statistics

Figure 9-23 illustrates a report of software reliability growth status. Tabular reports may be supplemented by graphic reporting formats depicting reliability growth history.

9.5.3 Failure Data Utilization

Both hardware and software failure data (a software error when detected is a software failure) are essential inputs to the corrective action system. Each failure must be reported and investigated. The results of the investigation determine the need for analysis and corrective action.

Reports such as those described in § 10.2 are useful in ensuring that failures are properly reported and followed (investigation, analysis, corrective action) until properly closed out.

9.6 REFERENCES

1. NAVSEA OD 21549A: Technical Program Management Requirements for Navy SSPO Acquisition, 1981.

2. SSPINST 3100.1F: Fleet Ballistic Missile Weapon System and Strategic Weapon System Trouble and Failure Report Program.

3. MIL-STD-1679(NAVY), Military Standard Weapon System Software Development, AMSC No. 23033, 1/12/78.

Error Source	Applications Software (AS) %	Simulator Software (SS) %	Operating System (OS) %	Test Tools (TT) %	Total
Requirements	5.1	7.2	7.4	0.9	
Design	41.1	38.3	43.3	32.4	
Code	52.0	47.3	40.7	61.1	
Mainteannce	1.6	3.7	0	2.8	
Not Known	0.2	3.5	8.6	2.8	
Total Errors Reported	280	232	99	131	742
Percentage of Total Errors	37.7	31.3	13.3	17.7	100

FOR ILLUSTRATION ONLY

Figure 9-19. Example of Source Frequency Breakdown

Number of Modules Affected By Each Error	Percentage of Total Errors	
	Project A	Project B
1	84.8	85.9
2	10.4	11.6
3	3.4	1.4
4	1.1	.7
5	.2	.2
6	0	0
7	.1	0
8	.1	.2

Figure 9-20. Cumulative Error Rate Report

Item	Percentage
Normal Termination	90.3
Abnormal Termination	9.7

Figure 9-21. Example of Percentage Breakdown of Software Errors by Type of Termination

Severity	% Errors
High	8
Medium	31
Low	41
Not Applicable	20

Figure 9-22. Example of Percentage Breakdown of Software Errors by Severity

Software Reliability Growth Report

Period of Date Coverage: ____ Date: ____

Software System	Current Version	Cur. Test Phase	Cum. Test (hrs)	Cum. Errors	Cum. MTBF (hrs)	Current MTBF (hrs)		Log. Growth Rate		Predicted MTBF (hrs) at end of development		Spec. Rqmts.	
						$\hat{\theta}$	$\theta_{.90}$	$\hat{\alpha}$	$\alpha_{.90}$	$\hat{\theta}$	$\theta_{.90}$	θ_o	α
Program A	D	IMC	40	11	3.6	4.6	1.8	.31	.16	85	80	85	.3
Executive Module	A	IMC	850	12	70.8	85.2	68.2	.42	.39	1575	1510	1750	.3
Processing Module 1	A	IMC	400	28	14.3	19.5	13.5	.32	.29	360	345	400	.3
Processing Module 2	C	IMC	340	15	22.7	30.3	18.6	.27	.21	560	542	600	.3
Utility Module	B	IMC	120	5	24.0	26.1	18.2	.35	.18	483	465	500	.3
Data Module	D	IMC	75	7	10.7	12.0	8.1	.21	.12	222	200	200	.3

FOR ILLUSTRATION ONLY

Figure 9-23. Software Reliability Growth Report

Section 10
RMA DOCUMENTATION

The documentation described in this section will normally be generated by the prime contractor to fulfill RMA requirements stipulated in NAVSEA OD 21549[1]. This documentation provides for the planning, recording, compiling, analyzing and reporting of pertinent RMA data and information. The development of this documentation should be accomplished in a systematic and timely manner. This means that RMA documentation needed for system design decisions and program management decisions is available in time to facilitate and support those decisions, and that documentation which reports RMA evaluation program results is available at scheduled time points to effectively monitor program progress.

The documentation described in this section is divided into two categories; those prepared by the contractor for submittal to SSPO for approval or information, and those prepared by the contractor for "in-house" use, including those containing information needed to prepare reports for SSPO.

10.1 DOCUMENTS FOR SUBMITTAL TO SSPO

RMA evaluation program documentation and subsequent changes thereto normally submitted to SSPO for information or approval include:

- Plans:
  - Reliability Evaluation Plan
  - Maintainability Program Plan
  - Reliability Demonstration Test Plan
  - Maintainability Demonstration Test Plan
- Test Procedures:
  - Reliability Demonstration Test Procedure
  - Maintainability Demonstration Test Procedure
- Reports:
  - Reliability Prediction Reports (Initial, Intermediate and Final)
  - Reliability Status Reports
  - Maintainability Status Reports

Since Appendix B of NAVSEA OD 21549[1] and the Contract Data Requirements List (CDRL) DD Form 1423 are used to tailor NAVSEA OD 21549 requirements to each specific procurement, contractual requirements for a given project may not require all of the above documents be submitted to SSPO. Nevertheless, the general contents of each of the above documents are described herein to cover the most stringent set of requirements.

10.1.1 Plans

Availability is inherently a function of a system's reliability characteristics, and if the system is repairable, it is also a function of the system's maintainability characteristics. Since SSPO establishes stringent reliability and availability objectives, contractor approaches to the achievement of these objectives are of vital interest and concern to SSPO. Accordingly, SSPO requires the contractor to document his plans for meeting specified RMA system objectives. The content of these plans are described in § 10.1.1.1 and 10.1.1.2 below. The content of contractor plans for verifying achievement of RMA objectives are described in § 10.1.1.3 and 10.1.1.4.

10.1.1.1 Reliability Evaluation Plan

The Reliability Evaluation Plan should delineate those approaches and methods which the contractor proposes to use to fulfill contractual reliability evaluation program requirements. These requirements will normally include: (1) the preparation of reliability block diagrams; (2) apportionment of system reliability objectives and minimum acceptable reliability for mission phases to system elements; (3) during the development phase, periodic predictions of the reliability expected in the system design eventually released for full scale production; (4) the identification of failure modes that could abort the system's mission using the technique of failure mode, effects and criticality analysis (FMECA); (5) provisions for the collection of essential reliability data; (6) assessment of system reliability during the development phase and continuing through subsequent life cycle phases; (7) the provision of evidence by demonstration testing that the system meets specified reliability objectives, and lastly; (8) provisions for the periodic reporting of system reliability status, problems, and trends. The plan should include a description of the system and equipment operation related to the mission phases for which reliability is to be evaluated, and should relate reliability evaluation tasks to system program milestones.

Primary information needed to develop the reliability evaluation plan, such as the results of Mission Analysis and System Analysis (e.g. environmental and duty cycle profiles for system elements, configuration baselines, etc.) must be developed, documented and provided to personnel charged with the development of the plan. This and other essential supporting information are identified in § 10.2.

All information needed to complete the plan is not normally available early in the program, however submission of the plan should not be delayed because it is incomplete in some of its details. Instead, tentative or preliminary information, with a schedule for providing mission details, should be submitted for review. The plan should be completed as early in the development phase as possible. Minor revisions and task schedule updates should normally be done on an annual basis.

10.1.1.2 Maintainability Program Plan

The Maintainability Program Plan should delineate the approach and methods the contractor will use to fulfill contractual maintainability program requirements. The maintainability program should establish realistic maintainability design criteria, provide periodic assessments of achieved maintainability, identify significant maintainability problem areas, and provide evidence of compliance to contractual maintainability program requirements. The plan should relate maintainability program activities to system program milestones. Specific tasks to be discussed in the plan include: (1) apportionment of system maintainability objectives to system elements to be maintained during the system's operational readiness phase; (2) providing periodic predictions of the system's maintainability characteristics under tactical conditions; (3) the examination and evaluation of proposed and actual designs, including software, in order to establish the most effective and efficient design for preventive and corrective maintenance; (4) the provision of a system for the collection of essential maintainability data, i.e., data necessary to support system maintainability design, and maintainability analysis and assessment functions; (5) assessment of maintainability; (6) demonstration of the achievement of maintainability objectives; and (7) the periodic reporting of maintainability program status, problems, and trends. Additional details on the nature of the above tasks are provided in NAVSEA OD 21549[1] paragraphs 3.3.2, 3.3.2.1, . . . , 3.3.2.7.

10.1.1.3 Reliability Demonstration Test Plan

The contractor should provide evidence that the system meets specified reliability objectives. When stipulated in the contract, the contractor should prepare a plan for the formal demonstration of the reliability of the system. System reliability should be demonstrated at the highest assembly level practicable using units that represent the production configuration. Test units should be subjected to environments and operational demands similar to those anticipated in tactical use. The test plan should include, as a minimum:

a. Identification of hardware and quantity to be tested.

b. Identification of software to be tested.

c. Test objectives and type of test plan selected.

d. Test plan criteria for each assembly of hardware/software undergoing demonstration testing:

- Values for R^*, R_L^*, α and β.
- Demonstration pass/fail criteria.
- Character of underlying failure distribution: exponential, binominal, normal, etc.

e. Test requirements including parameters to be measured, environments to be simulated, test time, facilities, test and measuring equipment and related software.

f. Requirements for data collection, analysis and reporting.

g. Criteria for continuing test in the event a failure occurs.

10.1.1.4 Maintainability Demonstration Test Plan

The contractor should demonstrate achievement of system maintainability objectives when required by the contract. This demonstration should be conducted using tools, diagnostic and support equipment, documentation and software as will be used during shipboard maintenance. The approach and the details of demonstration, including the selection of demonstration personnel should be described in a Maintainability Demonstration Test Plan. The plan should also include requirements for data collection, analysis and reporting, and demonstration pass/fail criteria.

10.1.2 Reliability and Maintainability Demonstration Test Procedures

The contractor should prepare detailed test procedures to assure full and controlled implementation of the demonstration plans for system reliability and maintainability described above. These procedures should provide clear and specific instructions on the completion of each step in the testing process. They should be tailored to the special features and elements of each of the demonstration test plans. The following information should be included in the test procedures, as appropriate:

a. Characteristics to be measured, including tolerances.

b. Input and load values, including tolerances.

c. Identification of test and measuring equipment, recording equipment and supporting software.

d. Identification of special equipment or facilities.

e. Method to be used in test performance, including sequential steps. Military standard test methods shall be used when applicable.

f. Verifications to be made before conduct of test.

g. Instructions for data recording.

h. Actions to be taken in the event of test interruptions.

i. Pass or fail criteria.

j. Applicable safety precautions for personnel and facility protection.

k. Diagram or detailed description of the test set-up such as interconnection information, relative equipment placement, mounting of sensors, and grounding points.

l. Identification of calibration and preventive maintenance requirements for items under test or test facility equipment.

m. Descriptions of test conditions; environments, duty cycles, work space constraints, etc.

10.1.3 Reports

10.1.3.1 Prediction Reports

Starting early in the design phase, Reliability and Maintainability* Predictions should be utilized in the design process. These predictions should be updated as the design progresses and continue through the completion of design. Prediction techniques should also be utilized to evaluate the impact on reliability and maintainability of proposed corrective actions to correct deficiencies. Final predictions should be updated to reflect design changes resulting from corrective action and R and M improvement activities.

Formal prediction reports should reflect the design and knowledge available at each report date. However, the working file should be kept current (updated, for example, monthly). The report should provide sufficient information (worksheets, data source, etc.) to permit results to be verified and to

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*Maintainability Predictions are covered in this paragraph, however, Maintainability Prediction Reports are not generally submitted to SSPO for approval or information. Maintainability analysis material is reviewed during program evaluations at contractor facilities.

provide traceability. Figures 10-1 through 10-12 are examples of the content required for various prediction worksheets. Each prediction report should also include a summary of the prediction results, to include as a minimum, a discussion of parts, subassemblies, etc. not meeting the design stress guidelines, repair actions not being within the time constraint allocations, problem areas (such as overly complex circuitry), comparison of results with requirements, any corrective actions required, proposed corrective action, and reason for significant changes in predicted values. When Bayesian evaluation is contemplated and the predictions are to be (or are being) used as prior information, the prediction report should discuss the assumptions made and the procedure used for establishing the strength of the prior and its sensitivity of contravening data.

R and M predictions should be an integral part of the design review file. The contractor may incorporate reliability prediction updates in the Reliability Status Reports (RSRs), if advantageous to do so and if not in conflict with required data item submittals. Maintainability predictions should be a part of the Maintainability Status Reports (MSRs).

10.1.3.2 Status Reports

Reliability and maintainability status reports are submitted periodically, usually quarterly to SSPO throughout the life cycle of the product. The reports should present an assessment of the system reliability and maintainability. An availability assessment, as necessary or as required, should be provided in the reliability status report (RSR). The contractor should utilize qualitative and quantitative techniques in the analysis of the present status of the program. Each report should indicate the current status compared to requirements and projected growth thereby isolating problem areas. The reports should address the problem areas and the corrective actions, both planned and accomplished.

Each report should contain, as appropriate:

a. A management summary, perhaps the most important portion of the report, containing program status, problems, corrective actions and a discussion of significant trends, events and achievements. The discussion of problem areas should include, as a minimum, all items for which the estimated reliability or maintainability are below the required or apportioned values, the cause(s) of the shortfall, the proposed corrective action, the date or serial number effectivity of the corrective action, and the anticipated effectiveness of the corrective action. Subsequent reports should monitor the effectiveness of the corrective actions taken.

b. A discussion of potential problems uncovered by analysis or test, that although not totally verified by test or operation, may degrade product reliability and maintainability if not resolved.

c. The status of the evaluation tasks versus the schedule in the approved Reliability Evaluation Plan or Maintainability Program Plan with a discussion of each slippage, its impact on program schedule and its impact on system reliability and maintainability.

d. A discussion of the data inputs to the R and M models, including the number of successes, the number of failures, the operating time, repair times, downtimes, and the data sources, as applicable. A discussion of the prior distribution should be included in the RSR when Bayesian methods are employed. (The basic description would appear in the reliability evaluation plan or maintainability program plan, however, a discussion of the similarity of the test data and the prior may be appropriate in this report.) In early reports, the data used for prediction is stressed, whereas in later reports the measurement and demonstration data are emphasized. This transition should be acknowledged and discussed.

e. A table of the current apportioned, predicted, and measured (best estimate and 80% lower bound) reliability indices for the subsystem, equipment, components, and software for each mission phase (operational readiness, launch, and flight), as appropriate (e.g., figure 10-13), and an explanation of any significant changes since the last report.

f. Growth curves showing measured reliability versus program time for the subsystem and new equipments. These curves should show the objective, the previous four status points, the present point, and should project at least one year into the future the expected reliability status.

g. A brief description of any changes to the system, subsystem and equipment operation, and of the mission for which reliability and maintainability is being reported.

Assembly A15

Name Mechanical Assembly

Date ________

Failure Rate (λ) Total 29.87/10⁶ hours

Prepared by ________

Sheet 1 of 1

Subassembly	Part Number (Drawing Number)	Base Failure Rate (Failures/ 10^6 hours)	Qty. (N)	Assumed Active Duty Cycle	Assumed Dormant Cycle	Dormant Failure Rate (Failure/ 10^6 hours)	Assumed Environmental Adjustment Factor	Failure Rate (Failure/10^6 hours)			Rated Life	Remarks
								Active	Dormant	Total		
Pump, Hydr.	(E45678)	81.1	1	.10	.90	.811	1.0	8.11	.73	8.84	Repairable; if not repaired, life is 1.0 years.	Based on preliminary parts prediction with assumed stresses (see worksheet*).
Valve, Hydr., Ball	ABC Co. P/N xxyz	8.6	6	.05	.95	.43	1.0	2.58	2.45	5.03	Repairable; if not repaired, life is 1.0 years.	Based on previous experience with similar valve. See cyclic to time failure rate conversion.
Shock Absorber	(D12355)	4.0	4	1.00	.00	N/A	1.0	16.00	N/A	16.00	Non-repairable, 2.2 years	Based on manufacturer's brochure (attached*).

ILLUSTRATION ONLY

*Would expect a data source or worksheet which is part of a prediction to be attached, for this sample that is not necessarily the case.

Figure 10-1. Sample Initial Reliability Prediction

Assembly A15

Failure Rate (λ) Total 19.84/10^6 hours

Name Mechanical Assembly

Prepared by

Date

Sheet 1 of 1

Subassembly	Part Number (Drawing Number)	Pertinent Stress	Part Rating	Design Stress	Stress Ratio	Base Failure Rate (Failures/10^6 hours)	Qty (N)	Assumed Active Duty Cycle	Assumed Dormant Cycle	Dormant Failure Rate (Failures/10^6 hours)	Assumed Environmental Adjustment Factor	Failure Rate (Failures/10^6 hours)			Rated Life	Remarks
												Active	Dormant	Total		
Pump, Hydr.	(E45678)	Pressure RPM Temperature	PSIG RPM °C	PSIG RPM °C	.6 .5 at least 20°C less than rated	6.67	1	.10	.90	.067	1.0	.667	.06	.73	If not repaired, life is .8 yrs (weak point is X seals)	Based on detailed prediction (attached*).
Valve, Hydr., Ball	ABC Co. P/N xxyz	Pressure Temperature	PSIG °C	PSIG °C	.6 at least 20°C less than rated	8.6	6	.05	.95	.43	1.0	2.58	2.45	5.03	If not repaired, life is 2.0 yrs (weak point is Y seals)	Based on detailed prediction (attached*).
		Flow Rate	GPM	GPM	.7											
Absorber, Shock	(D42345)	Weight Torque Vibration Shock	lbs lbs-in g's g's	lbs lbs-in g's g's	.5 .5 .7 .7	4.4	4	1.0	.00	N/A	.8	14.08	N/A	14.08	Nonrepairable, life is 2.5 yrs.	Based on manufacturer's detailed prediction (attached*).

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*Would expect a data source or worksheet which is part of a prediction to be attached, for this sample that is not necessarily the case.

Figure 10-2. Sample Intermediate Reliability Prediction (Assumed Worst Case Design Stress)

Assembly A15

Failure Rate (λ) Total 16.65/10^6 hours

Name Mechanical Assembly

Prepared by ____________

Date ____________

Sheet 1 of 1

Subassembly	Part No. (Dwg No.)	Pertinent Stress	Part Rating	Design Stress	Stress Ratio	Base Failure Rate (Failures/10^6 hours)	Qty. (N)	Active Duty Cycle	Dormant Cycle	Dormant Failure Rate (Failures/10^6 hours)	Environmental Adjustment Factor	Failure Rate (Failure/10^6 hours)			Rated Life	Remarks
												Active	Dormant	Total		
Pump, Hydraulic	(E45678)	Pressure RPM Temperature	750 psig 1000rpm 100°C	450 psig 500rpm 60°C	0.6 0.5 40°C less than rated	6.67	1	0.10	.90	.067	1.0	.667	.067	.73	If not repaired, life is .8 yrs. (Seals)	Based on detailed prediction (attached*).
Valve, Hydraulic, Ball	ABC Co. P/N xxyz	Pressure Flow rate Temperature	750 psig 10GPM 80°C	450 psig 7GPM 60°C	0.6 0.7 20°C less than rated	.5	6	See Remarks	See Remarks	.10 (See Remarks)	See Remarks	3.00	.60	3.60	If not repaired, life is 2.0 yrs. (Seals)	Failure rate from test data. See report 123456. Tests run to simulate equivalent usage and environmental factors. Dormant failure rate from manufacturer: 20% of active failure rate.
Absorber, Shock	(D42345)	Weight Torque Vibration Shock	5000lb 500 lb-in 20 g 20 g	2000lb 250 lb-in 2 g 10 g	0.4 0.5 0.1 0.5	4.4	4	1.0	.00	N/A	0.7	12.32	N/A	12.32	Non-repairable, life is 2.5 yrs.	Based on manufacturer's detailed prediction (attached*).

ILLUSTRATION ONLY

*Would expect a data source or worksheet which is part of a prediction to be attached, for this sample that is not necessarily the case.

Figure 10-3. Sample Final Reliability Prediction (Stress Analysis)

Assembly Pump, Hydraulic

λ Total 6.67/10^6 hours

Drawing Number E45678

Prepared by ____________

Date ____________

Sheet 1 of 1

Components	Part Number (Drawing No.)	Base (λ_b) Failure Rate (Failures/ 10^6 Hours	Quantity (N)	Assumed Duty Cycle (Usage Factor) Π_U	Environmental Adjustment (A_E)	Failure Rate $\lambda_b \times N \times \Pi_U \times A_E$ (Failures/ 10^6 Hours	Rated Life x 10^6 Hr	Remarks
Rotor	E45676.1	5.0	1	.1	1.0	.50	.065	Based on in-house test of rotor E45678.1, see report #A4321D5.
Seals	E45678.2	.7	4	1.0	1.0	2.80	.007	Generic failure rate from Handbook.
Bearings	E45678.3	.5	2	.1	1.0	.10	.052	Manufacturer Supplied data for EFG Bearing Co. Report AD 47396.
Shaft	E45678.4	.35	1	.1	.5	.018	.100	MIL-STD-xxxxx
Strainers	E45678.5	1.62	4	.1	.4	2.59	.032	Manufacturer data - ABC Report #23967.1A.
Gaskets	E45678.6	.02	10	1.0	.5	.10	.035	GDS report AD 49028.
Power Source: Electric Motor	E45678.7	9.36	1	.1	.6	.56	.011	Failure rate for motor see ABC Corporation "Reliability Engineering Data Series, Failure Rates".

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Figure 10-4. Sample Initial Reliability Prediction

Assembly A1A1 Name ABC Assembly λ_T = 13.9562/10^6 hours Date

Prepared by Sheet 1 of 1

Assumptions: Naval Sheltered, High Reliability Parts, Electrical Stress ≤ .5, T Ambient ≤ 85°C

Subassembly	Parts	λ (Failures/ 10^6 Hours)	λ (Qty)	Nλ (Failures/ 10^6 Hours)	Remarks
A1A1PS1	-	10	1	10.0	λ based on previous program experience for a similar subassembly.
A1A1A1	IC 1-20 Bipolar	(.026) (1)	15	.3900	MIL-HDBK-217C Notice 1
	IC 51-100 Bipolar	(.043) (1)	20	.8600	
	RAM 11K-17K, MOS, Static	(2.3) (1)	1	2.3000	
	RCR Level S	(.0046) (.03)	20	.0028	
	CKR Level S	(.0098) (.03)	5	.0015	
A1A1A2	IC 1-20, Bipolar	(.026) (1)	1	.0260	
	RCR Level S	(.0046) (.03)	1	.0001	
	RNR Level S	(.0037) (.03)	1	.0001	
	CSR Level S	(.035) (.03)	1	.0011	
	Q, NPN, TXV	(.26) (.1)	1	.0260	
	CR, G.P., TXV	(.09) (.1)	1	.0090	
	CR, LED, TXV	(.45) (.1)	4	.1800	
	L, R.F. Coil, Fixed	(.0096) (1)	1	.0096	
	SW, Push	(.15) (1)	1	.1500	MIL-HDBK-217C Notice 1

Figure 10-5. Sample Initial Reliability Prediction (Mixed Parts Count)

A1A1A2 Module XX λ_i = 5432/10^6 hours Prepared by ______ Date ______

Page 1 of 2*

$T_A = 80°C$

Environment = Naval Sheltered

									Capacitor	λ_b	Π_E	Π_{SR}	Π_Q	Π_{CV}						λ_P		
									Resistor	λ_b	Π_Q	Π_R	Π_E							λ_P		
									Diode	λ_b	Π_Q	Π_E	Π_R	Π_A	Π_{S2}	Π_C				λ_P		
									Transistor	λ_b	Π_Q	Π_A	Π_R	Π_{S2}	Π_C	Π_E				λ_P		
									I.C.	Number of Trans. or Gates	Π_Q	C_1	C_2	C_3	Π_T	Π_V	Π_E	Π_L	Π_{PT}	λ_P		
Ref. Des.	Part Type	Part Number	MIL Spec	Part Elec. Rating	Design Elec. Stress	Elec. Stress Ratio Note 1	Part Thermal Rating °C	Design Thermal Stress °C	Thermal Stress Ratio												Data Source	Remarks
U1	5404	/00105BCB	M38510	Fan Out	Fan Out	.8	T_j=175	T_j<100	< 100	6	1.0	.0024	.0004	.0048	2.2	1.0	4.0	1	N/A	.0261	217C:2.1.5	Notice 1
R1	RCR	07G103JS	R39008	Watts/ Volts	W/V	.4/.8	105	80	.76	.0025	.03	1.0	3.0							.0002	217C:2.5.1	Notice 1
R2	RNR	55H1333FM	R55182	W/V	W/V	.4/.8	125	80	.64	.0016	1.0	1.0	2.0							.0032	217C:2.5.2	Notice 1
Q1	NPN	JTXV2N2222A	S19500/255	W/V	W/V	.4/.6	T_j=165	T_j<100	< 100	.015	.12	.7	1.0	.88	1.0	10.0				.0111	217C:2.2.1	Notice 1
							T_A=150	T_A=80	.53													
CR1	SWTG	JTX1N4148-1	S19500/116	W/V	W/V	.5/.6	T_j=165	T_j<100	<100	.0095	.3	10	1	.6	.70	1				.0120	217C:2.2.4	Notice 1
							T_A=150	T_A=80	.53													
C1	CSR"R"	39003/ 1-2751	C39003/1	Volts	Volts	.5	125	80	.64	.0270	2.5	.07	.1	1.6						.0008	217C:2.6.5	Notice 1

Note 1: From reference [3].

*Only one page has been prepared for this sample.

Figure 10-6. Sample Intermediate Reliability Prediction (Assumed, Not to Exceed, Electrical and Thermal Stress)

A1A1A2 Module XX λ_T = .4883/10^6 hours Prepared by________ Date________

Page 1 of 2*

T_A = 75°C

Environment = Naval Sheltered

									Capacitor	λ_b	Π_E	Π_{SR}	Π_Q	Π_{CV}						λ_P		
									Resistor	λ_b	Π_Q	Π_R	Π_E							λ_P		
									Diode	λ_b	Π_Q	Π_E	Π_R	Π_A	Π_{S2}	Π_C				λ_P		
									Transistor	λ_b	Π_Q	Π_A	Π_R	Π_{S2}	Π_C	Π_E				λ_P		
									I.C.	Number of Trans. or Gates	Π_Q	C_1	C_2	C_3	Π_T	Π_V	Π_E	Π_L	Π_{PT}	λ_P		
Ref Des	Part Type	Part Number	MIL Spec.	Part Elec. Rating	Design Elec. Stress	Elec. Stress Ratio	Part Thermal Rating °C	Design Thermal Stress °C	Thermal Stress Ratio												Data Source	Remarks
U1	5404	?00105BCB	M38510	Fan Out 10 max	Fan Out 5 max	.5	T_j=175	T_j=81	< 100	6	1.0	.0024	.0004	.0048	1.2	1.0	4.0	1	N/A	.0237	217C:2.1.5	Notice 1
R1	RCR	07G103JS	R39008	.25W/ 250V	.125W/ 10V	.5/ 1	105	75	.71	.0025	.03	1.0	3.0							.0002	217C:2.5.1	Notice 1
R2	RNR	55H1333FM	R55182	.10W/ 200V	<.0001W/ 10V	.1/.1	125	75	.6	.0010	1.0	1.0	2.0							.0020	217C:2.5.2	Notice 1
Q1	NPN	JTXV2N2222A	S19500/255	.5W/ 40V	.2W/4V	.4/.1	T_j=165	T_j=83	< 100	.014	.12	.7	1.0	.30	1.0	10.0				.0035	217C:2.2.1	Notice 1
							T_A=150	T_A=75	.5													
CR1	SWTG	JTX1N4148-1	S19500/116	.2A/ 75V	.02W/ 25V	.1/ 33	T_j=165	T_j=83	<100	.0025	.3	10	1	.6	.70	1				.0032	217C:2.2.4	Notice 1
							T_A=150	T_A=75	.5													
C1	CSR"R"	39003/ 1-2751	C39003/1	15V	5V	33	125	75	.6	.013	2.5	.07	.1	1.6						.0004	217C:2.6.5	Notice 1

*Only one page has been prepared for this sample.

Figure 10-7. Sample Final Reliability Prediction (Stress Analysis)

Item A1A1 Assembly

System MTTR 30 minutes

System MCT_{max} 60 minutes

Apportioned MTTR 15 minutes

Date ____________

Prepared by ____________

Page 1 of 1

Subassembly/Replacement Level Highest Failure Rate First		Estimated Failure Rate (λ) (Failures/10^6 Hours)	Estimated Fixed Repair Time (Minutes)	Estimated Variable Repair Time (Minutes)	Estimated R_p (Minutes)	λR_p	Remarks
Power Supply	A1A1PS1	15	2	15	17	255	(a) Fault Detection, Isolation, Opening Cabinet, Close Cabinet, Check Out = Fixed Time.
ETI	A1A1M3	10	2	35	37	370	
Meter	A1A1M2	10	2	35	37	370	
Meter	A1A1M1	10	2	35	37	370	(b) Most testability points to be used as test points for software diagnostics design on PCB's.
PCB A/D	A1A1A6	4	2	2	4	16	
All other PCB's	(5)	(<1) (5) = 5	2	2	20	100	
Relays	(3)	(<.5) (3) = 1.5	10	45	165	248	(c) Areas of improvement: (1) Connectorize Meters, Relays, Switches. (2) Improve accessibility of PS, Meters (3) Provide Test scheme for relays
Switches	(2)	(<.3) (2) = .6	10	45	110	66	
	Total	56.1				1795	MTTR ≈ 32 minutes: Design requires improvement 60 minutes max.: Design satisfactory

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Figure 10-8. Sample Initial Maintainability Prediction

Item A1A1 Assembly

System MTTR 30 minutes

System MCT_{max} 60 minutes

Apportioned MTTR 13 minutes

Date ____________

Prepared by ____________

Page 1 of 1

Subassembly/Replacement Level Highest Failure Rate First		Estimated Failure Rate (λ) (Failures/10^6 Hours)	Estimated Fixed Repair Time (Minutes)	Estimated Variable Repair Time (Minutes)	Estimated R_p (Minutes)	λR_p	Remarks
Power Supply	A1A1PS1	13	2	8	10	130	Placed on slide, 4 disconnect screws
ETI	A1A1M3	10	2	15	17	170	Connectorized
Meter	A1A1M2	10	2	15	17	170	Connectorized
Meter	A1A1M1	10	2	15	17	170	Connectorized
PCB A/D	A1A1A6	4	2	2	4	16	
All other PCB's	(5)	(<1) (5) = 5	2	2	20	100	
Switches	(2)	(<.3) (2) = .6	10	26	72	43.2	Improved accessibility, look at replacing soldering with connnectors
Relays	(3)	(<.1) (3) = .3	10	40	150	45	Improved accessibility, increased wire length, need to look at terminal blocks
Total		52.9				844.2	MTTR = 15.9: Design still does not meet apportionment 60 minutes max.: Design satisfactory

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Figure 10-9. Sample Intermediate Maintainability Prediction

1. Item A1A1 Assembly

2. Method of Repair Replace-Parts ☒ Subassembly ☒ Assembly ☐ Unit ☐ Sheet 1 of 1

3. Circuit Designation	4. Part Type	5. Failure Rate λ	Maintenance Task Times (Hrs.)							13. R_p	14. λR_p	15. $\log R_p$	16. $\lambda \log R_p$
			6. Localization	7. Isolation	8. Disassembly	9. Interchange	10. Reassembly	11. Alignment	12. Checkout				
A1A1A1	SEM	.154	.001	.005	.0635	.007	.043	.00	.017	.1365	.0210	-1.991	-0.307
A1A1A2	SEM	.261	.001	.005	.0635	.007	.043	.00	.017	.1365	.0356	-1.991	-0.520
A1A1A3	SEM	.454	.001	.005	.0635	.007	.043	.00	.017	.1365	.0620	-1.991	-0.904
A1A1A4	SEM	.055	.001	.005	.0635	.007	.043	.00	.017	.1365	.0075	-1.991	-0.110
A1A1A5	SEM	.105	.001	.005	.0635	.007	.043	.00	.017	.1365	.0143	-1.991	-0.209
A1A1A6	PCB	3.21	.00	.002	.0635	.007	.043	.00	.017	.1325	.4253	-2.021	-6.488
A1A1PS1	PS	11.55	.00	.008	.07	.04	.07	.00	.008	.1960	2.2638	-1.630	-18.822
A1A1SW1	SW	.03	.02	.00	.20	017	.25	.00	.017	.5040	.0151	-0.685	-0.021
A1A1SW2	SW	.03	.02	.00	.20	.017	.25	.00	.017	.5040	.0151	-0.685	-0.021
A1A1K1	REL	.14	.02	.08	.25	.017	.28	.00	.017	.6640	.0930	-0.409	-0.057
A1A1K2	REL	.14	.02	.08	.25	.017	.28	.00	.017	.6640	.0930	-0.409	-0.057
A1A1K3	REL	.28	.02	.08	.25	.017	.28	.00	.017	.6640	.1860	-0.409	-0.115
A1A1M1	METER	7.12	.000	.017	.17	.017	.33	.034	.017	.5850	4.1652	-0.536	-3.817
A1A1M2	METER	5.48	.000	.017	.17	.017	.33	.034	.017	.5850	3.2058	-0.536	-2.938
A1A1M3	ETI	10.00	.000	.017	.17	.017	.33	.000	.017	.5510	5.5100	-0.596	-5.960
17. SUM		39.009								18. SUM	16.1127	19. SUM	-40.346

Note: a) All SEMs are removed and replaced as a set.

b) *Localization/Isolation for PCB/SEM is by PM/FL Software.*

ILLUSTRATION ONLY

Figure 10-10. Sample Final Maintainability Prediction

TASK/INTERCHANGE TIME WORKSHEET

REPLACEABLE MODULE OR COMPONENT	DESIGNATION	FUNCTIONAL LEVEL	NEXT HI ASSY	EQUIPMENT	ENGINEER	DATE
Relay	A1A1K1	Part	A1A1			

DETAILED INTERCHANGE STOPS	ELEMENT TIME	NO. TIMES PERFORMED	TIME FOR INTERCHANGE	COMMENTS
Loosen Securing Screws (non captive)	.0046	2	.0092	
Unsolder Leads	.008	8	.064	.3452
Tag Leads	.034	8	.272	
Orient Replacement Part	.017	1	.017	.017
Solder Leads	.017	8	.136	.1546
Secure Relay by Rescrewing Screws	.0093	2	.0186	

ILLUSTRATION ONLY

Figure 10-11. Sample Initial Task/Interchange Worksheet

TASK/INTERCHANGE TIME WORKSHEET

REPLACEABLE TASK	DESIGNATION	FUNCTIONAL LEVEL	NEXT HI ASSY	EQUIPMENT	ENGINEER	DATE
Cabinet Door	A1	Unit	Subsystem			

DETAILED INTERCHANGE STOPS	ELEMENT TIME	NO. TIMES PERFORMED	TIME FOR INTERCHANGE	COMMENTS
Open				
Loosen Captive Screws	.0046	8	.0368	
Open Door and Secure Latch	.001	1	.001	
			.0378	
Close				
Close Door	.003	1	.003	
Tighten Captive Screws	.0046	8	.0368	
			.0398	

ILLUSTRATION ONLY

Figure 10-12. Sample Final Task/Interchange Worksheet

RELIABILITY STATUS REPORT

Project______________ C_1 = Best Estimate C_2 = 80% Confidence   P = Prediction   A = Apportionment

Nomenclature	Drawing Number	Mission Phase A						Mission Phase B						Mission Phase C					
		Failure Rate		Reliability				Failure Rate		Reliability				Failure Rate		Reliability			
		λ_{C1}	λ_{C2}	R_P	R_A	R_{C1}	R_{C2}	λ_{C1}	λ_{C2}	R_P	R_A	R_{C1}	R_{C2}	λ_{C1}	λ_{C2}	R_P	R_A	R_{C1}	R_{C2}
Reconnaissance Vehicle Subsystem		.0433806	.0755389	.9605	.9340*	.9575	.9273	.0240088	.0418066	.9760	.9684*	.9762	.9591	.0827584	.1441077	.9354	.9007*	.9205	.8658
Telemetry & Command Equipment																			
Transolator	0194C0543G003	.0038418	.0129154	.9962	.9948	.9962	.9872	.0034576	.0116207	.9964	.9968	.9965	.9884	.0061469	.0206359	.9958	.9908	.9939	.9796
Transolator	0194C0543G003	.0038418	.0129154	.9962	.9948	.9962	.9872	.0034576	.0116207	.9964	.9968	.9965	.9884	.0061469	.0206359	.9958	.9908	.9939	.9796
Programmer	0194C0696G001	.0028312	.0116673	.9976	.9960	.9972	.9884	.0017369	.0103553	.9982	.9979	.9983	.9897	.0039563	.0186899	.9973	.9941	.9961	.9815
Amplifier	0604D0147G001	.0033467	.0125532	.9974	.9951	.9967	.9875	.0020166	.0113457	.9979	.9972	.9980	.9887	.0057011	.0199872	.9961	.9915	.9943	.9803
Multicoder	0926B0972G001	.0000000	.0011743	.9965	.9989	.9999	.9988	.0000000	.0006572	.9998	.9965	.9999	.9993	.0000000	.0207191	.9998	.9999	.9999	.9795
Sensors	0926B0975G001	.0003677	.0007143	.9997	.9993	.9996	.9993	.0003993	.0007512	.9996	.9996	.9996	.9992	.0021879	.0175541	.9985	.9967	.9978	.9826
Baroswitch	0863C0799G001	.0110132	.0200054	.9922	.9838	.9890	.9801	.0026063	.0151016	.9973	.9972	.9994	.9850	.0150113	.0289132	.9896	.9777	.9851	.9715
Telemetry & Command Equipment Grouping		.0252424	.0615249	.9760	.9632	.9751	.9404	.0136711	.0333214	.9857	.9821	.9864	.9672	.0391449	.0954103	.9732	.9428	.9617	.9090
Communications Equipment																			
Beacon	0563F0617G001	.0004861	.0016290	.9997	.9991	.9995	.9984	.0003679	.0007145	.9996	.9996	.9996	.9993	.0015500	.0037440	.9987	.9984	.9984	.9963
Pulse Generator	0215E0175G001	.0034357	.0113543	.9975	.9956	.9966	.9876	.0005678	.0095312	.9995	.9993	.9995	.9905	.0110345	.0190534	.9900	.9985	.9890	.9910
Flashing Light	0681C0643G003	.0000000	.0007365	.9994	.9992	.9999	.9993	.0000000	.0004932	.9998	.9992	.9999	.9995	.0000000	.0011223	.9999	.9999	.9999	.9989
Receiver	0194C0796P001	.0009895	.0013563	.9993	.9982	.9991	.9987	.0006863	.0010976	.9993	.9992	.9993	.9989	.0012123	.0012013	.9989	.9987	.9988	.9954
Transmitter	0926B0979P001	.0027573	.0088379	.9980	.9956	.9973	.9912	.0051234	.0065374	.9952	.9938	.9950	.9935	.0102567	.0122667	.9907	.9893	.9898	.9978
Communications Equipment Grouping		.0076686	.0153975	.9939	.9878	.9923	.9847	.0067454	.0135438	.9934	.9911	.9933	.9864	.0240535	.0482961	.9783	.9751	.9762	.9528
Vehicle Equipment																			
Vehicle Controller	0179D0869G003	.0036998	.0101933	.9972	.9940	.9963	.9899	.0025001	.0084567	.9978	.9967	.9975	.9915	.0101023	.0189934	.9910	.9896	.9899	.9812
Propulsion	0215E0168G002	.0060709	.0094637	.9948	.9901	.9939	.9905	.0007011	.0076776	.9994	.9991	.9993	.9923	.0084567	.0136245	.9925	.9913	.9915	.9864
Power Supply	0179D0870P002	.0006989	.0011097	.9993	.9989	.9993	.9988	.0003911	.0010594	.9997	.9994	.9996	.9989	.0010010	.0036364	.9991	.9990	.9990	.9963
Vehicle Equipment Grouping		.0104696	.0164005	.9913	.9830	.9895	.9837	.0035923	.0056271	.9969	.9952	.9964	.9944	.0195600	.0306390	.9827	.9800	.9806	.9699
Payload Equipment																			
TV Camera	0215E0133G001	.0000000	.0017563	.9988	.9986	.9999	.9983	.0000000	.0009911	.9998	.9997	.9999	.9991	.0000000	.0025394	.9998	.9997	.9999	.9975
Payload Equipment Grouping		.0000000	.0017563	.9988	.9986	.9999	.9983	.0000000	.0009911	.9998	.9997	.9999	.9991	.0000000	.0025394	.9998	.9997	.9999	.9975

*Design Requirement (Value Associated with Procedure Risk in MIL-STD-781)

Figure 10-13. Example of Tabular Summary For Reliability Status Report

h. A discussion of any changes to the reliability and maintainability block diagrams and mathematical equations including any assumption changes.

i. A detailed explanation of any changes in methodologies and their impact on calculations.

10.2 CONTRACTOR IN-HOUSE DOCUMENTS

Documentation of the type listed or described in this subsection of the manual should be prepared by the contractor to control and support the overall system design and evaluation effort. Additionally, these documents are used to organize and prepare RMA reports (§ 10.1) submitted to SSPO in accordance with the CDRL. This documentation (indexed to the relevant paragraph of NAVSEA OD 21549[1] includes, as a minimum, information developed for or contained in the following:

a. Program Plan Matrix (3.1.2.1)
b. Internal Audit Reports (3.1.4.2)
c. Integrated Data System Plan (3.1.7)
d. Corrective Action System, Description of (3.1.8)
  - Problem/Failure Reporting, Investigation, Analysis and Corrective Action
  - Trouble Failure Reports (TFRs), contractor use of
e. Government-Industry Data Exchange Program (GIDEP) (3.1.11)
  - Utilization of GIDEP Data, especially GIDEP ALERTS
f. Analysis for Design (3.2.1)
  - Mission Analysis: Environmental and Duty Cycle Profiles
  - Software Functions Analysis
g. Design Practices and Documentation (3.2.2)
h. Parts, Devices, and Material Selection Guide (3.2.3.1)
i. Project Parts, Devices, and Materials List (3.2.3.2)
j. Design Review Plan (3.2.4)
  - Results of Design Reviews
k. Parameter Studies (3.2.5)
  - Parameters Document (3.2.5.b.)
l. Identification and Classification of Characteristics (3.2.6)
  - Identification of Characteristics That Could Affect the Coordination, Life, Interchangeability, Function or Safety (CLIFS) of System Elements
m. Controlled and Limited-Life Items (3.2.7)
  - Determination and Identification of
n. Software Verification (3.2.8)
o. Integrated Test Program (3.4.1)
  - Integrated Test Program Plan (3.4.2)
  - Integrated Test Program Status Reports (3.4.3)
  - Test Reports (3.4.6)
p. Qualification Test Program (3.4.9)
q. Software Tests (3.4.10)
r. Configuration Management Program (3.5)
  - Configuration Identification (3.5.1)
  - Configuration Baselines (3.5.1.1)
  - Design Disclosure Documentation (3.5.5)
    - Development Documentation (3.5.5.1)
    - Production Design Disclosure (3.5.5.2)

For those readers who desire more details on the documentation cited above, these details may be found within the relevant paragraphs of NAVSEA OD 21549[1] identified in parentheses following each type of documentation.

In the paragraphs which follow, additional description and comment are provided in selected documentation areas that relate closely to RMA evaluation program activities. The information documented in these areas have many uses, such as in:

- Spares provisioning (estimating spares usage and logistic support requirements).
- Evaluating the limits for the infant mortality or burn-in period, the useful life period, and the wearout period.
- The development of objective criteria for the removal, replacement, and disposition of Limited-Life items.
- The establishment of a compendium of R and M data based on actual test experience.

10.2.1 Test History File

This file contains an entry (see figure 9-3) for each test conducted. It provides a complete record of all test data in a form that enables various reports to be generated quickly and efficiently (e.g., see figure 9-16, Data Flow).

Data in the test history file may be summarized (sorted) over any or all fields as desired. For example, by using a control on date of test, reports can be generated relative to program periods of interest (e.g., the last two years). By sorting on level of test, reports can be generated which contain only data obtained in equipment and higher level tests. A control on type of test permits reports to be generated using only qualification or flight test data. The flexibility to develop any desired summary should be readily available. Figure 10-14 is an example of a typical test history file format.

It is essential that the data in the test history file be as accurate and complete as possible since it forms the basis of all other reports. See § 9.3 for data control procedures and techniques to improve data accuracy.

The following provides a description of the columns of figure 10-14:

Name

A twenty character field for the name of the item.

Level

A one character field for the indenture level of the item. e.g.,

Hardware	*Software*
1 – system	7 – software
2 – subsystem	note: Contractor may break software down to lower indenture levels when advantageous. The Evaluation Plan must fully identify the categories used.
3 – equipment	
4 – component	
5 – module	
6 – part	

Drawing Number

A fifteen character field for the drawing number. e.g.,

four digit prefix
letter indicating drawing size
four digit suffix
letter P – part or G – group
three digit part or group number
two characters for revision identification

Serial Number

A six character field for the contractor serial number.

Vendor Name Code

A six character field for identifying the vendor. The Federal Stock Code (FSC) number is often used for this purpose.

Vendor Serial Number

A six character field for the vendor serial number.

Level

A one character field for the level of the test being performed as above. 1 – system, 2 – subsystem, etc.

Type

A one character field for the type of test (e.g., acceptance, qualification, demonstration, flight).

Environment

A two character field for the environment of test (e.g., ambient, temperature, vibration, salt spray).

Test Site

A six character field which identifies the location at which the test was performed.

Date of Test

A six character field for the day, month, and year of test.

Test Report Number

A seven character field for the test report number.

State

A one character field for the state of test (A and B – non-operating, C – cycling, and D – operating; see § 4.1.2).

PAGE # ________

DATE ________

PROJECT ________

TEST HISTORY FILE

HARDWARE/SOFTWARE IDENTIFICATION						TEST DESCRIPTION					TEST RESULTS						MAINTENANCE DATA			REFERENCE DATA			
NAME	LEVEL	DRAWING NUMBER	SERIAL NUMBER	VENDOR NAME CODE	VENDOR SERIAL NUMBER	LEVEL	TYPE	ENV	TEST SITE	DATE OF TEST	TEST REPORT NUMBER	STATE	TIME OR CYCLES	FAILURES	FAILURE CLASSIFICATION	FAILURE REPORT NUMBER	SPARES	CORRECTIVE MAINTENANCE TIME	DELAY TIME	PROJECT CODE	ASSOC. TEST REPORT NUMBER	ASSOC. FAILURE REPORT NUMBER	DATE OF ENTRY

Figure 10-14. Test History File

Time or Cycles

A six character field, which for states A, B, and D, the first four characters represent hours, the next two characters represent minutes of test, for state C, the six characters represent cycles of operation. Consideration should be given to including time recording device (ETI, ETM) readings, when available, in an added column.

Failures

A one character field indicating that a failure or discrepancy (such as the failure of one element of a redundant configuration) occurred in the test. (Note a number is entered for all failures or discrepancies, not just reliability failures; applicable failure classifications, failure report numbers, and maintenance data are entered when a failure or discrepancy occurs.)

Failure Classification

A two character field permits classification of failures. The first character defines the relevance of the failure for reliability calculations.

0 – *not relevant*
1 – relevant
2 – previously relevant, corrective action reduced classification to non-relevant

Reliability calculations are based upon the failures classified one in this column (except for reliability growth models which should use failures classified as one or two). If corrective action is not effective on a particular item the two classification must be changed to one for affected failures.

The second character is the FMECA classification (1 – minor, 2 – major, 3 – critical, 4 – catastrophic, see § 4.2.5.2).

Failure Report Number

A field (seven characters) for the failure report number.

Spares

A one character field indicating the status of spares for the particular failure (0 required and available, 1 – required and not available, 2 – not required).

Corrective Maintenance Time

A six character field, the first four are the hours, and the last two the minutes required to perform the corrective maintenance. (Corrective maintenance time includes fault location, isolation, correction, adjustment, calibration, and repair checkout times.)

Delay Time

A six character field, the first four are the hours and the last two the minutes of delay (delay time includes administrative and supply delay times).

Project Code

A four character field which indicates the project on which the test was run (used only when data from another project is being used to supplement data on the current project).

Associated Test Report Number

A seven character field which references a higher level test from which the record was generated.

Associated Failure Report Number

A seven character field which references a higher level failure report.

Date of Entry

A six character field which indicates the day, month, and year that the data is entered into the data system.

10.2.2 Corrective Action System Reports

The contractor must have a closed loop corrective action system. This system must consist of complete problem and failure reporting, investigation, analysis and corrective action. The corrective action process should result in an effective resolution of problems and failures. Failure Summary Reports for hardware and software failures are valuable documents to assure a positive corrective action program. The use of the term failure in the remainder of this discussion should also be taken to include problems.

Each hardware failure summary report should present a complete record of all open hardware failures and those that were closed since the last report. The report provides a tool for assuring; (1) that failures are identified and reported to cognizant engineers, to management, and to SSPO; (2) that significant and *repetitive* failures are analyzed in depth; (3) that the causes and modes of failure are determined correctly and; (4) that effective action is identified and taken.

The report is normally issued monthly and contains hardware identification, including nomenclature, drawing number, serial number, part vendor, and program code; test description including test type, environment, site or reporting activity, and date of test; test results including failure report number, failure classification, and description of failure, to include visual observations; indication of whether formal failure analysis board action is required along with results of the analysis including corrective actions recommended and taken; and responsible personnel including the design and quality control engineers. A separate tabulation of repetitive failure modes should be included and discussed. A sample of Failure Summary Report is included as figure 10-15. Failures can be either hardware or software in origin. Certain failures may be identified immediately as software or as hardware, however some failures may not initially be identified as such. The tracking of this "unknown" type of failure presents problems which the system must handle. When a failure is entered on a summary report, analysis should have identified the type of failure.

A software failure summary report presenting a complete record of all software failures that have occurred provides an analogous report for software reliability.

A Failure Analysis Follow-Up Report should be issued as required (normally monthly) for internal action and information. It should list each action item generated by failure analysis and show the status of action items, cumulatively. This control provides for a closed loop on failures and corrective actions. Figure 10-16 illustrates this report.

10.2.3 Serial Number Summary Report

The serial number summary report contains a one-line entry for each selected serialized piece of hardware tested. The primary purpose of the serial number report is to keep track of time accumulated on critical and time- (or cycle-) sensitive equipment. The accumulated test time is compared with the allowable maximum test time (e.g., the red-time line of figure 2-2). If the test time exceeds the red-time line, a determination should be made if the item should remain in operation or be replaced or overhauled. This determination should be made following an analysis of the reasons for the overtesting (see figure 9-16).

10.2.4 Environmental Summary Report

The environmental summary report contains a one-line summary for each environment within a drawing number. The number of items tested, the number of failures and the test time accumulated in states A, B and D and the test cycles accumulated in test state C are provided. The entries in this report are *not* normalized to equivalent missions. The information in this report is useful in preparing the failure rate compendium (see figure 9-16).

10.2.5 Mission Simulation Report

The mission simulation report is similar to the environmental summary report. The primary differences are that only mission environments are carried and the results are presented in equivalent missions rather than time and cycles.

The mission simulation report arranges test data in a form convenient for calculating reliability. In order to save computation time at this stage, all hardware, software and environmental data which is not required for the reliability status report is eliminated from computation. This report contains one line of information for each mission environment in which the hardware and software are tested. This report is used as an input for preparing the reliability status report (see figure 9-16).

10.2.6 Hardware and Software Summary Report

The hardware and software summary report contains a one-line entry for each drawing number tested. It also contains a summation of total test time and failures accumulated on each drawing number. This report is

FAILURE SUMMARY REPORT

PROGRAM Mark A3	DESIGN ENGINEER I.D. Mann	DRAWING NUMBER 103 C 4277 G1
MANUFACTURER 04615	Q.C. ENGINEER W. E. Build	EQUIPMENT NAME VHF Transmitter

Failure		RPTG ACTVTY	Serial No.	Test		Failure			Failure Investigation/Analysis Conclusion	Corrective Action Recommended/Taken
RPT. No.	Date			Type	Environment	Description	Class	Req.		
39138	10/7/81	Phila.	1	Qual	Humidity	Frequency out of spec. moisture entered transmitter due to faulty test assembly	00	Yes	Human error, assembly screws not properly tightened against component itself, when not attached to base plate. (F.A.R. A-678)	R: Issue AN and SI to eliminate the possibility of recurrence. T: AN 688 F 585-5 and TR 8047-2 to SI 24834
39138	10/7/81	Phila.	2	Qual	Humidity	Frequency out of spec. moisture entered transmitter due to faulty test assembly 20 November 1981	00	Yes	Human error, assembly screws not properly tightened against component itself, when not attached to base plate. (F.A.R. A-678)	R: Issue AN and SI to eliminate the possibility of recurrence. T: AN 688 F 585-5 and TR 8047-2 to SI 24834
352-70	3/14/82	AMR	5476204	Pre-launch	Ambient	Emitting sidebands & noise equal in amplitude to main carrier. Signal strength very low	30	Yes	Improper tuning due to lack of adequate tuning procedure. (F.A.R. A-772)	R. Develop tuning procedure and instruct personnel in use T: Procedure demonstrated and distributed to field personnel.
352-11	12/26/81	AMR	5476043	Hangar	Ambient	Low power output. 20 March 1982	10	Yes	Discrepancy between System and Component requirements. (F.A.R. A-772)	R Change system spec. to conform to component requirements. T: Unable to secure permission to change spec. to 9 watt minimum. Part selection still done for 10 watt minimum.
31176	6/7/82	Phila.	N/A	Syst. O/A	Ambient	Low power output. 30 June 1982	11	No	Cable to power amplifier too long.	
48093	5/31/82	Phila.	Lot 11-1	Comp. O/A	Post Vibration	Power drops intermittently. 20 August 1982	21	No	Defective insulator on Q4 heat sink.	R: Redesign - Use different insulator; Issue AN & SI T AN688B234-1 & SI42357
86720	8/22/82	Phila.	Lot 9-2	Comp O/A	Vibration	Chassis shorted to case	11	No	Insulation shorted.	T AN688B234-1 & SI42357
AC-2257	8/24/82	AMR	5476572	Pre-launch	Ambient	Multiple oscillation above 9.5 watts 20 September 1982	11	No	Unknown	Analysis in progress
02433	9/9/82	Phila.	A30	Comp. O/A	Vibration	Unit broke into oscillation, power output and input current dropped.	11	No	Defective diode, CR6.	Analysis in progress
02436	9/9/82	Phila.	A27	Comp. O/A	Vibration	No output, no oscillations from oscillator. 12 October 1982	11	No	Defective transistors Q1, 7,8,9.	Analysis in progress

Figure 10-15. Failure Summary Report

Page # ________

Date ________

FAILURE ANALYSIS FOLLOW-UP

Project ________

Failure Analysis Number and Date	Component Identification		Action Required By and Date Required	Description of Action (R) Recommended and (T) Taken	Action Documentation Number and Date Issued
	Name	Drawing Number			

Figure 10-16. Failure Analysis Follow-Up Report

useful in logistic (i.e., hardware spares, software maintenance, support personnel, etc.) planning. The report can be expanded to include failure rates for early spares provisioning if these rates are not produced as part of the reliability evaluation plan (see figure 9-16).

10.2.7 Failure Rate Compendium Report

A failure rate compendium report can be a valuable by-product of a reliability data system. Typically, a compendium is a compilation and summary of the hardware test results contained in the test history files of the data system. Data from all projects are summarized by various hardware groupings to provide a reference document for failure rates and failure frequency analysis. The failure rates are based on actual test experience and are valuable in making predictions for new systems. They are also useful in making design decisions for component and vendor selection. The failure frequency data summaries are useful for indicating the relative severity to environments, and the contributions to unreliability from design, manufacturing, and testing activities.

A compendium report can also be used to summarize failure experience due to design, manufacturing, test, handling, or unknown causes; failure rates by criticality of failure (i.e., no failure, catastrophic failure, or out-of-specification failure); failure rate by test type (i.e., qualification, acceptance, field); failure rates by level of test (i.e., component, equipment, or subsystem); and failure rate by test environment (i.e., vibration, bench, high temperatures, etc.). Summaries may be generated for generic component types and broad equipment classes.

A separate compendium report should be developed for software items providing similar analysis.

10.2.8 Test Effectiveness Reports

Since test cost represent a significant percent of project resources, the contractor should be monitoring the effectiveness of the integrated test program. NAVORD OD 42282 [2] discusses the planning, integrating, optimizing, monitoring, control and reporting necessary for this purpose in detail.

Analysis of test effectiveness, and improving it when required, is an essential element of the overall evaluation process. For example, the effectiveness of procedures employed by the contractor to eliminate potential defects from a subsystem can be evaluated using summaries of the experience represented in the data file.

A subsystem can be depicted, as in figure 10-17, in its flow through successive analyses, reviews and tests intended to detect and divert defects from passing downstream to the operational use phase.

Defects that are present in the subsystem and eligible for detection are shown entering the test block. Within the block some defects are generated in the course of the test. Flowing out of the block are those defects that are detected and diverted and those that escape. Defects that will enter the next screen downstream are the sum of the escapes plus any defects that may have completely by-passed the block for reasons of ineligibility (e.g., the test is not designed to detect the failure mode or an equipment containing defects that were detectable was not installed when the test was run) or management decision (e.g., a decision to by-pass the test to meet schedule or other commitments).

Effectiveness of a test block can be characterized by a variety of indices such as:

$$E = 1 - \frac{\text{Escapes}}{\text{Defects Presented}}$$

It should be noted that the ineligible defects reduce the test block efficiency, the management decisions, however, do not enter the test block.

The effectiveness of the test block shown in figure 10-17 is then:

$$E = 1 - \frac{13}{133} = .902.$$

The effectiveness of the screening process (E_p) is less since the management decision permitted ten defects to by-pass the screen. It would be measured as

$$E_p = 1 - \frac{23}{143} = .839.$$

10.2.9 FMECA Summary Reports

As discussed in § 4.2.5, a FMECA shall be performed to identify potential failure modes

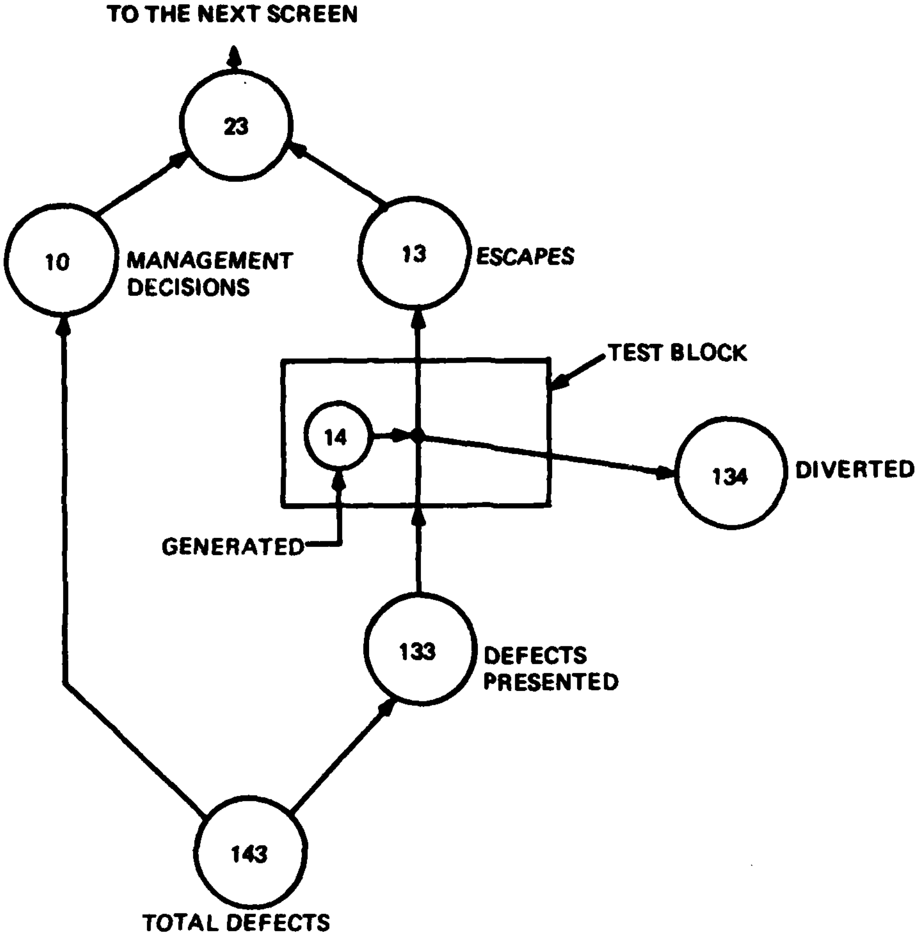

Figure 10-17. Test Screen Model

and assess the effects of these modes on the system, personnel and mission. The results of the engineering analysis, as illustrated in the worksheets in figure 4-27, shall be provided to the design community in a FMECA Summary Report or series of reports. The summary report shall delineate the results of the analysis, to include a discussion of significant findings and a detailed description of design improvement recommendations for precluding or reducing impact of potential failures. Each potential failure being evaluated shall include its criticality, as emphasis shall be directed toward eliminating severe (catastrophic, critical, major) failure modes. The summary report(s) shall be timely and require rapid resolution of design improvement recommendations from responsible management and engineering personnel. Figure 10-18 is an example of a FMECA Summary Report.

10.3 REFERENCES

1. NAVSEA OD 21549A: Technical Program Management Requirements for Navy SSPO Acquisitions.
2. NAVORD OD 42282: Integrated Test Program Manual.
3. NAVSEA 0967-LP-597-1011: Parts Application and Reliability Information Manual for Navy Electronic Equipment.
4. MIL-STD-1629: Procedures for Performing a Failure Mode and Effect Analysis for Shipboard Equipment.
5. MIL-STD-781: Reliability Tests Exponential Distribution.
6. MIL-HDBK-217: Reliability Prediction of Electronic Equipment.

TO: ______________________________ Report Number: ____________

FROM: ____________________________ Date: ________________

Subject: FMECA Summary Report for the Track Signal Controller 6766300.

A failure mode, effects and criticality analysis (FMECA) has been conducted on the Track Signal Controller (TSC) 6766300, two of which are used in PAC VAN M46. Highlights of the analysis are provided below. The analysis was performed by Reliability, Maintainability and Systems Engineering personnel.

1. UNNECESSARY USE OF A 28VDC REGULATOR. The design makes unnecessary use of +28VDC Regulator for energizing relays on the two 8131000 PWAs RA3-108 and RA3-110. This function--the only one it performs in the unit--can be performed by the +26.5VDC input to the unit from the PAC VAN +26.5VDC Power Supplies. Because energization of the relays when required is an essential internal function of the TSC, elimination of the +28VDC Regulator would remove a source of failure from the unit, reduce the heat generated inside the unit, and cut the cost of the unit and its support. It is recommended that the +28VDC Regulator be eliminated from the unit design.

2. DIODE CR1. The design uses a forward biased diode CR1 (on PWA 8229000) in series with the +26.5 VDC power output to the Receiver Frequency Selector unit. The output is returned so as to light the appropriate three lamps of the front panel frequency indicators. In the first place, the diode serves no purpose whatsoever. In the second place, the diode is not correctly rated for this application. Each of the three lamps uses a nominal steady state current of 25 milliamperes, or a total of 75ma for the three lamps in parallel. CR1 has a rating of 75 ma. This is not sufficient to handle current surges which can be up to ten times steady state during lamp turn-on. The misapplication of CR1 can be seen from field data against the TSC 6610800 used in M33 which shows that all four failures of PWA 822900 in that unit were the result of CR1 failing open. This problem is addressed in RFI No. 4730-070. Because CR1 performs no useful function and is a proven source of unreliability, it is strongly urged that it be deleted from the circuit.

3. SIMPLER CIRCUIT NEEDED FOR OUTPUT OF RF RCVD SIGNAL. The relay circuitry used to send contact closures to the Events Processor and Antenna Position Indicator (signifying the event that a suitably strong rf signal is being received) can be made more reliable through simplification. The circuit under discussion here appears in Zone A2, page 2 of 3 of EL 6766301. The recommended change would improve unit reliability by reducing the number of active components involved in the design. Note that the simplified design gets rid of relay K8 (as well as K8's driving and control components) and does away with the 5-VDC pull-up scheme.

The back-up worksheets for this report are on file in Reliability Engineering.

Copies: Systems, Design, Maintainability, Reliability, Components, Software Diagnostics and Test Engineering, Program Manager

Figure 10-18. Example of a FMECA Summary Report

Appendix A
ANALYSIS OF A FIRE CONTROL SYSTEM

This appendix presents an example of the analysis of a hypothetical Fire Control system which is represented by its system block diagram in figure A-1. In the diagram each block represents an equipment or group of equipments. The directions of functional flows are labeled and inputs and outputs are identified. Thus the block diagram is a graphical representation of the dependence of subsystem performance on the operability of its hardware elements. The analysis is simplified for the sake of brevity, however, the procedures used in each step are illustrated.

A.1 MISSION ANALYSIS

The mission consists of two availability phases, designated (a) and (b), and a launch phase. System functions in phase (a) are limited to monitoring and regulation of temperature and electrical power in each of 16 missile guidance systems. Maximum duration of phase (a) is 4,000 hours. Phase (b) functions include those of phase (a) plus functions necessary to control the assignment and erection of the missiles. The system is defined as fully up if it can perform the phase (b) functions and initiate the launch phase for all 16 of the missiles. Maximum duration of phase (b) is 1,500 hours. Functions during the launch phase are those required to control the preparation and firing of the missiles. The launch phase has a maximum length of 3 hours, during which no failures are permitted. Minimum acceptable availability is 0.85 in phase (a) and 0.99 in phase (b). A reliability requirement of 0.95 applies to the launch phase. The system can function in several modes; however, the illustration will be limited to the tactical mode and to availability phase (b). Figure A-2 shows the development of the mission profile.

A.2 SYSTEM ANALYSIS

Returning to the system block diagram of figure A-1, one can construct an idealized reduced block diagram from it which reveals the possible "up" or "down" states of the system. This reduced block diagram is shown in figure A-3. At first it would seem that as many as 2^{19} states can be identified from the possible "up" or "down" combinations of each one of the 19 equipment blocks. If one assumes, however, that all equipments operate, fail, and are repaired independently, and that the minimum system configuration for functional capability are three independent stages in series, S consisting of a single block, C consisting of two blocks in parallel and M consisting of 16 blocks in series, then the problem becomes much simpler.

Corrective maintenance functions are analyzed in figure A-4.

The equation for system availability is derived in accordance with the principles previously set forth in § 4.2.2.2 and Appendix D § D.1 and D.3. The system consists of three independent stages in series and is available when all three stages are up. Thus,

$$A_i = \prod_{j=1}^{3} a_j$$

where a_j is the availability of jth stage. The availability of a stage consisting of n equipment blocks in parallel, any m of which are required to be up for the stage to be up, is given by

$$a_j = \sum_{x=0}^{n-m} \binom{n}{x} a^{n-x}(1-a)^x$$

$$= \sum_{x=0}^{n-m} \binom{n}{x} \frac{\mu^{n-x}\lambda^x}{(\lambda+\mu)^n}$$

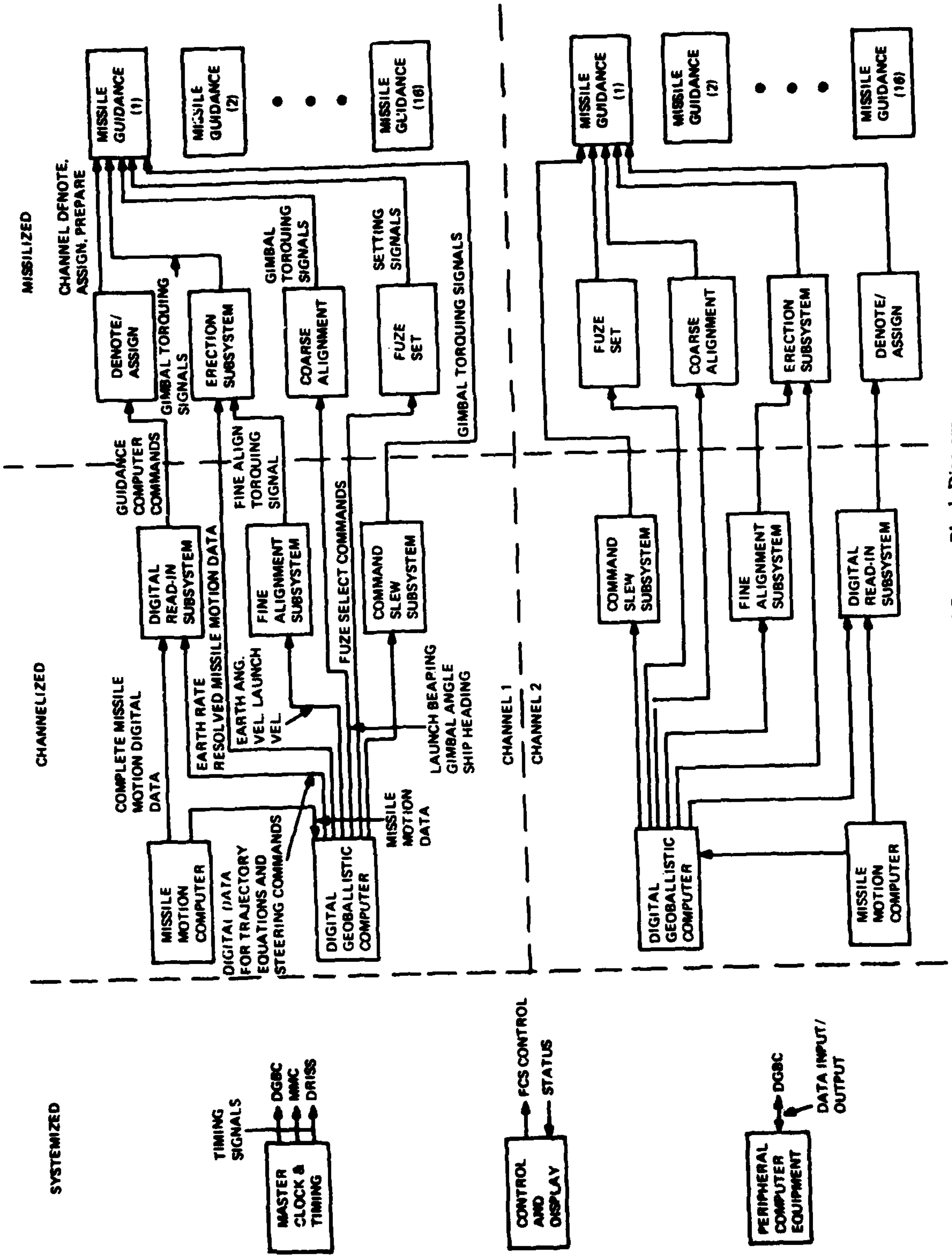

Figure A-1. Fire Control System Block Diagram

System: Fire Control

Mission Phase	Mode	Function	Related Equipments	Success Criteria	Function Time	Env.	Env. Time	Maintenance Constraints
Availability (a)	Standby	1. Temperature regulation	Temp. monitoring power supplies	Repairable failures permitted	4000 hrs	Ambient	4000 hrs	Immediate failure detection and repair
		2. Power to guidance			4000 hrs	Ambient	4000 hrs	
Availability (b)	Tactical	1. Temperature regulation	Temp. monitoring power supplies	Repairable failures permitted	1500 hrs	Ambient	1500 hrs	DCC can be down for maintenance
		2. Power to guidance						
		3. Erect and coarse align all guidance systems	16 erection units 8 coarse alignment units Missile motion computers DGBC, control console					
		4. Denote and assign missiles to each channel	Missile select sw.					
		5. Self-check	Control console ITOP MTRE Module test set	Self-check not essential				
Launch	Tactical	1. Compute range and bearing data and fuze settings for each target	DGBC Fuze set MMC	No failures	3 hrs	Ambient	3 hrs	
		2. Supply digital trajectory parameters	DGBC Digital read-in SS					
		3. Fine alignment of missiles guidance systems	Optical alignment group Servo group SS					
		4. Transmit launch signals to launcher systems	Control and display DRISS Missile select sw.					
Flight	N/A	N/A						

Figure A-2. Development of Mission Profile

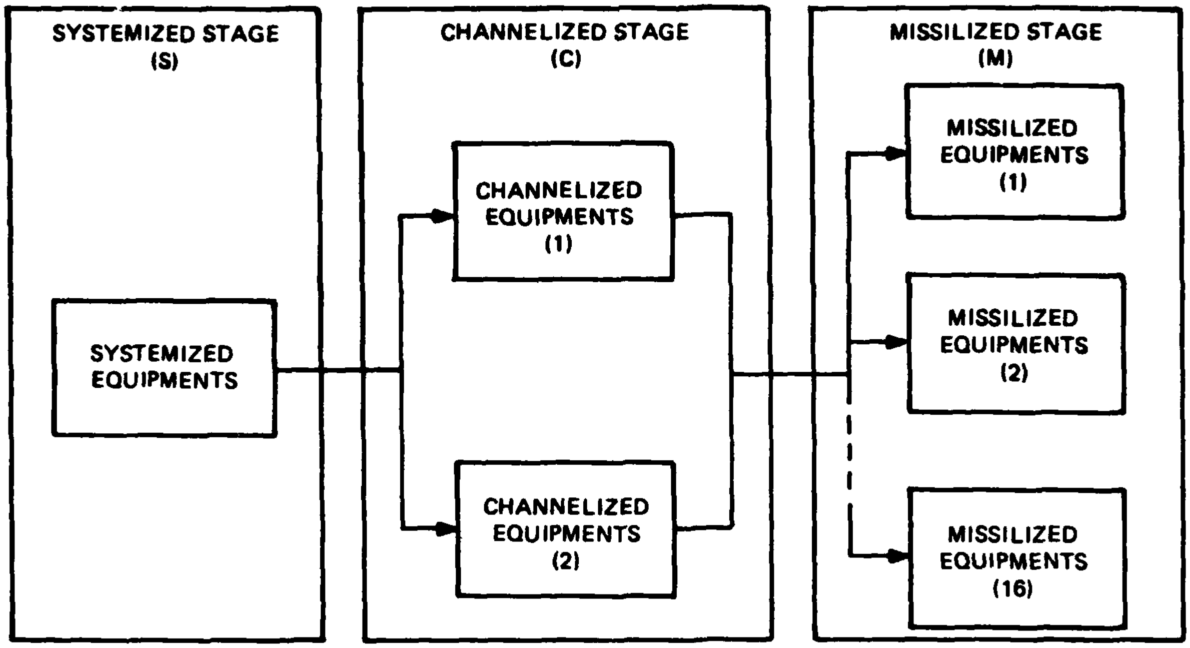

Figure A-3. Fire Control System, Reduced Block Diagram

System: Fire Control

Hardware Block	Equipment	Predicted Partial Fail. Rate λ (Fail/Hr)	Recovery Action		Predicted MTTR $\overline{M}_c$ (Hrs)	$\lambda\overline{M}_c$	$U_i = \frac{\overline{M}_c}{\lambda^{-1} + \overline{M}_c}$	$A_i = 1 - U_i$
			Repair	Replace				
Systemized (S)	Control & Display	.0020	X		1.2	.0024		
	Peripheral Equipment	.0015	X		.8	.0012		
		λ = .0035			$\overline{M}_c$ = 1.03	.0036	.0036	.9964
Channelized (C)	MMC	.0018	X		1.5	.0027		
	DGBC	.0030	X		1.0	.0030		
	DRISS	.0010	X		.5	.0005		
	FASS	.0008	X		2.0	.0016		
	CSS	.0020	X		1.1	.0022		
		λ = .0086			$\overline{M}_c$ = 1.16	.0100	.0099	.9901
Missilized (M)	D/A	.0025	X		.8	.0020		
	ES	.0020	X		.8	.0016		
	CASS	.0016	X		1.5	.0024		
	FSS	.0022	X		1.0	.0022		
		λ = .0083			$\overline{M}_c$ = .99	.0082	.0081	.9919

Figure A-4. Analysis of Corrective Maintenance Tasks

where

$$\binom{n}{x} = n!/[(n-x)!\,x!]$$

and x is the number of failures.

Then stagewise,

$$A_i = A_{i_S} \cdot A_{i_C} \cdot A_{i_M}$$

Blockwise,

$$A_i = \left(\frac{\mu_S}{\lambda_S + \mu_S}\right)\left[\frac{\mu^2{}_C + 2\mu_C\lambda_C}{(\lambda_C + \mu_C)^2}\right]\left[\left(\frac{\mu_M}{\lambda_M + \mu_M}\right)^{16}\right]$$

where

$$\mu = \overline{M}_c^{-1}$$

Data required to support measurements are implicit in the parameters of the equation. They are the uptimes and downtimes of each equipment block from which statistical estimates of the failure rates and repair rates can be obtained.

Since the prediction does not indicate that the system will initially meet its specified availability (.99 in phase b), it is necessary to apportion requirements to elements of the system. Before this is done, the system should also be analyzed with respect to its reliability requirements since that requirement determines the upper limit of allowable failure rate. Figure A-5 summarizes the system predicted parameters.

The reliability requirement for the system is R(3 hours) ⩾ .95. Then, in terms of blocks (not stage), the system model is

$$\begin{aligned} R_{sys} &= \left[R_S\right]\,[1-(1-R_C)^2]\,\left[R_M{}^{16}\right] \\ &= R_S R_C R_M{}^{16}\,(2-R_C) \\ &= 2e^{-3(\lambda_S + \lambda_C + 16\lambda_M)} - e^{-3(\lambda_S + 2\lambda_C + 16\lambda_M)} \\ &= .95 \end{aligned}$$

		λ	$\overline{M}_c$	A_i
S	BLOCK	.0035	1.03	.9964
	STAGE	.0035	1.03	.9964
C	BLOCK	.0086	1.16	.9901
	STAGE	*	1.16	.9999
M	BLOCK	.0083	0.99	.9919
	STAGE	.1328	0.99	.8780
SYSTEM			0.998	.8750

*The failure rate of an n-block parallel stage is not constant but is a function of time. For small λ it is approximated by $(\lambda t)^n/t$.

Figure A-5. Summary of Predicted Parameters

If the prediction accurately reflects the relative complexities, stress levels, state-of-the-art factors, etc. that characterize each equipment block, it is reasonable to apportion failure rates among the blocks in the same ratios indicated by the prediction. Then the failure rates of the C and M blocks can be expressed in terms of the S failure rate.

$$\lambda_C = \frac{.0086}{.0035}\lambda_S = 2.46\lambda_S$$

$$\lambda_M = \frac{.0083}{.0035}\lambda_S = 2.37\lambda_S$$

Given these relative magnitudes of block failure rates, the system model becomes

$$R(3) = 2e^{-41.38\lambda_S(3)} - e^{-43.84\lambda_S(3)} = .95$$

which is easily solved graphically to yield λ_S = .00044. Then, by the ratios previously stated, λ_C = .00108 and λ_M = .00104. These are the highest permissible block failure rates consistent with the system reliability requirement. They correspond to MTBF's of 2280 hours, 930 hours and 960 hours for the S, C and M blocks respectively, and they represent bounds on the tradeoff regions available for meeting the system availability requirement. The system MTBF is found by integrating the reliability function. It should be noted that the reliability of electronic, mechanical and electro-mechanical devices can usually be characterized in terms of constant failure rates (implying exponentially distributed failure times), provided the devices are complex and consist of parts of varying ages or mixtures of part types having different mean lives and failure distributions [1]. Therefore:

$$\text{MTBF} = \theta = \int_0^\infty 2e^{-.01818t}\,dt - \int_0^\infty e^{-.01926t}\,dt$$

$$= \frac{2}{.01818} - \frac{1}{.01926} = 58 \text{ hours.}$$

Apportionment of availability can be begun by considering the system as a whole, then the minimum acceptable A_i is:

$$A_i = .992$$

An optimum combination of system parameters θ (MTBF) and $\overline{M}_C$ (MTTR) is sought subject to the following constraint:

$$A_i = \frac{1}{1 + \frac{\overline{M}_c}{\theta}}$$

$$\frac{\overline{M}_c}{\theta} = \frac{1 - A_i}{A_i} = \frac{.008}{.992} = .00806$$

Several approaches to apportionment of goals for the improvement of reliability and maintainability are available and are discussed in the literature. One of the simplest is to determine the magnitude of improvement needed in each characteristic alone in order to satisfy the system A_i requirement. The improvements required are:

(a) MTTR required at minimum MTBF:

$$\overline{M}_{c_{required}} = \theta\left(\frac{1-A_i}{A_i}\right) = 58\left(\frac{.008}{.992}\right) \quad \text{(A-1)}$$

$$= .464 \text{ hr.} = 28 \text{ min.}$$

(b) MTBF required at predicted MTTR:

$$\theta_{required} = \overline{M}_c\left(\frac{A_i}{1-A_i}\right) = .998\left(\frac{.992}{.008}\right) = 124 \text{ hrs.}$$

$\overline{M}_c$ in the above equation is computed from equation (A-1). The frequency weighting factor for the C-stage is not really a failure rate but $2\lambda_C/3 = .0057$, the reciprocal of the stage MTBF [1].

Thus, it is apparent that the availability requirement can be met by improving the system MTTR to 28 minutes while meeting the minimum MTBF consistent with the reliability requirement, or the availability requirement can be met with the predicted MTTR of one hour if the system MTBF can be raised to 124 hours. Between these extremes there are an unlimited number of combinations of MTBF and MTTR that will also satisfy the requirement.

In order to apportion reliability and maintainability goals in an optimum manner, it is desirable to predict the relative difficulty of improving each. In this example, the predictions take the form of cost functions, although the actual resources involved may include engineering and manufacturing manhours as the major or sole variables. It is not realistic to formulate reliability and maintainability as deterministic functions of the resources expended for their realization. At best, the analyst can invoke past experience to predict the functional relationships in a largely subjective manner. Feasible improvement actions may be listed, and engineers asked to make optimistic, expected and pessimistic predictions of the costs entailed in each and the degree of change each would produce in the reliability and maintainability of the system. The actions can then be listed as scaled sets and the distributions of the cost versus improvement relationships estimated [2].

For purposes of illustration, let it be assumed that such an analysis is performed for the Fire Control System and that the following expected cost functions are obtained over limited ranges of θ and $\overline{M}_c$. The unit of cost C is dollars X 10^6.

$$C(\overline{M}_c) = \overline{M}_c^{-2}$$

$$C(\theta) = \theta^2/3600$$

The method of Lagrange multipliers is employed to minimize the total cost function, such that:

$$G = C(\overline{M}_c) + C(\theta) + \alpha\left(\frac{\overline{M}_c}{\theta} - \frac{1-A_i}{A_i}\right)$$

where α is a Lagrange multiplier. The partial derivatives of the cost function are set equal to zero:

$$\frac{\partial G}{\partial \overline{M}_C} = C'(\overline{M}_C) + \frac{\alpha}{\theta} = 0$$

$$\frac{\partial G}{\partial \theta} = C'(\theta) - \frac{\alpha M_C}{\theta^2} = 0$$

$$\frac{\partial G}{\partial \alpha} = \frac{\overline{M}_C}{\theta} - \frac{(1-A_i)}{A_i} = 0$$

and the resulting system is solved simultaneously for the optimum values of θ_o and M_{C_o}, yielding:

$$\theta_o = 86.6 \text{ hrs.}$$

$$\overline{M}_{C_o} = .693 \text{ hr.} = 42 \text{ minutes}$$

These optimum system values can then be apportioned in a convenient manner back to block level and ultimately to equipment level. Since all of the blocks have roughly the same predicted maintainability, the apportioned MTTR would be close to 42 minutes for each type. Apportioned MTBF's would be about 3393 hours for the S block, 1379 hours for each C block and 1432 hours for each M block. For purposes of illustration, the apportionments are:

$$\theta_o = 86.6 = \int_0^{\infty} 2e^{-41.38\lambda_s t}\,dt - \int_0^{\infty} e^{-43.84\lambda_s t}\,dt = \frac{2}{41.38\lambda_s} - \frac{1}{43.84\lambda_s}$$

$$\lambda_s = .0002947, \theta_s = 3393 \text{ hr.}$$

$$\lambda_C = 2.46\lambda_s = .0007250, \theta_C = 1379 \text{ hrs.}$$

$$\lambda_M = 2.37\lambda_s = .0006984, \theta_M = 1432 \text{ hrs.}$$

$$\overline{M}_{C_o} = .693 = \left(\lambda_s \overline{M}_{c_S} + \frac{2\lambda_C}{3}\overline{M}_{c_C} + 16\lambda_M \overline{M}_{c_M}\right) \Big/ \left(\lambda_s + \frac{2\lambda_C}{3} + 16\lambda_M\right)$$

$$= \frac{.0002947\overline{M}_{c_S} + \frac{2}{3}(.0007250)(1.13)\overline{M}_{c_S} + 16(.0006984)(0.96)\overline{M}_{c_S}}{.0002947 + \frac{2}{3}(.0007250) + 16(.0006984)}$$

$$\overline{M}_{c_S} = .6707 \text{ hr.} = 40 \text{ minutes}$$

$$\overline{M}_{c_C} = \left(\frac{1.16}{1.03}\right)\overline{M}_{c_S} = 1.13\,\overline{M}_{c_S} = 7579 \text{ hr.} = 45 \text{ minutes}$$

$$\overline{M}_{c_M} = \left(\frac{0.99}{1.03}\right)\overline{M}_{c_S} = 0.96\,\overline{M}_{c_S} = .6439 \text{ hr.} = 39 \text{ minutes}$$

A.3 REFERENCES

1. Bazovsky, I., Reliability Theory and Practice, Prentice-Hall, Inc., 1961, pp. 52-58.

2. Goldman, A. S., and Slattery, T. S., Maintainability, John Wiley and Sons, New York, 1964.

Appendix B
DEMONSTRATED FLIGHT RELIABILITY (DFR) AND RAW SCORE METHODS

This appendix presents two of the methods currently used by the Lockheed Missile and Space Company to assess the flight phase reliability for POSEIDON and TRIDENT missile subsystems.

B.1 RAW SCORE METHOD

The material in this paragraph is taken from [1].

B.1.1 Raw Score

Raw score is a measure of flight success based on the ratio of re-entry bodies (RBs) successful over the number launched. Certain events which interfere with the completion of the flight may cause the exercise to be declared a No-Test (does not count as attempt or outcome).

Raw Score for the missile may be presented based upon many criteria. In the MRR, Raw Score is presented both for the missile system and for the missile body. The missile system Raw Score includes guidance and RB component flight performance, but excludes nontactical hardware and other FBM subsystem problems; the missile body Raw Score additionally excludes guidance and RB problems. The general rule for handling excluded problems is that if the occurrence precluded the completion of the flight it is a No-Test, whereas if it merely perturbated the result the performance is considered successful.

B.1.2 Groundrules for Calculating Raw Score

a. Calculations are based upon the RBs launched and represent a success ratio.

b. Inadvertent actions (such as command destruct) not precipitated by missile malfunction will be counted as No-Test (i.e., neither the attempt nor the result will be counted).

c. Failures of instrumentation and destruct components will be excluded from consideration.

d. The failure/anomaly caused by a non-missile subsystem will be counted on the following basis:

  (1) No-Test if the occurrence precludes the completion of the missile exercise, and

  (2) Success to the extent that the missile performed properly to the input/stimuli.

e. For missile body raw score, additional exclusion of guidance and RB failures are taken on the same basis as described in d. (1) and (2) above.

B.2 DEMONSTRATED FLIGHT RELIABILITY

The material in this paragraph is taken from [2] which was not changed except for paragraph and figure numbering.

B.2.1 Background of Demonstrated Flight Reliability

Reliability assessment models which attempt to integrate data from a number of sources (e.g., flight, system testing, package testing) usually are cumbersome to use and, more importantly, are dependent upon a number of assumptions. For use with flight data alone, a model is needed which is simple to use, is easy to understand, and uses a minimum of assumptions. The Demonstrated

Flight Reliability (DFR) was developed by MARC [2] to provide a method more realistic than Raw Score but still simple to calculate and relatively assumption-free.

Demonstrated Flight Reliability is defined to be the estimate, using flight data only, of the expected percentage of successful re-entry bodies. DFR is based on mission phases rather than missile segments; this reduces the complexity of the calculation and simplifies the process of attributing failures to the proper sources.

The derivation of DFR is simple enough to facilitate modification in several ways. For example, suppose the deployed population consists of two (or more) sub-populations of missiles with presumably different reliabilities (e.g., POMP and pre-POMP missiles). Unless the numbers of flights from each sub-population are roughly in the same proportion as the fleet mix, a DFR computed on the basis of all flight tests combined might not accurately represent the fleet reliability. However, DFR's can be computed for each of the sub-populations on the basis of its flight data alone, and these DFR's can be combined as a weighted average to give a DFR representative of the fleet mix. Another modification is to compute one DFR for the missile system and another for the missile body alone; in the latter case failures of the guidance system, destruct system, and re-entry bodies are not counted against the missile body.

B.2.2 *Definitions*

Mission

The mission profile consists of a first boost phase, a second boost phase, and a deployment phase. The deployment phase in turn consists of the continuous functioning of a portion of the equipment section concurrent with the release, at given intervals, of the re-entry bodies. These constitute the horizontal deployment phase and several individual vertical deployment phases. See figure B-1 for an illustration of this.

N

The number of re-entry bodies in a tactical configuration.

Reliability

It should be noted that the reliabilities defined below are all conditioned by the successful completion of the relevant preceding portions of the mission. For example, the reliability of the horizontal deployment phase between the planned times of the first and second re-entry body releases (R_{12} in figure B-1) assumes the successful completion of the first and second boost phases, and the horizontal deployment phase through the planned

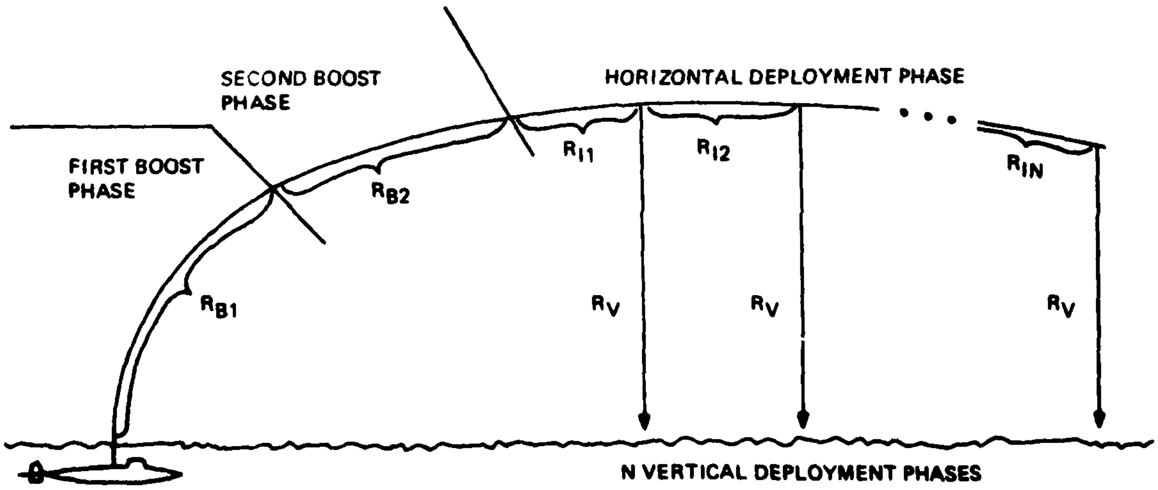

Figure B-1. Mission Profile

time of the first re-entry body release. However, it is independent of the success or failure of the first re-entry body's vertical deployment phases.

It should also be noted that the terminology in the definitions is common to both missile system reliability and missile body reliability. When discussing missile body reliability, the reliabilities of the guidance system, destruct system, and re-entry body are excluded where appropriate.

R_{B1} Reliability of the first boost phase.

This is the probability that a missile which enters the first boost phase will not fail during it.

R_{B2} Reliability of the second boost phase, given successful completion of the first boost phase.

This is the probability that a missile which enters the second boost phase will not fail during it.

R_{I1} Reliability of the horizontal deployment phase from initiation through the planned time of deployment of the first re-entry body, given the successful completion of the first and second boost phases.

This is the probability that a missile which enters the horizontal deployment phase will not fail before the planned time of the first re-entry body release (considering only that portion of the missile common to the functioning of all re-entry bodies).

R_{Ij} for $j = 2, 3, \ldots, N$ this is the reliability of the horizontal deployment phase *between* the planned times of deployment of the (j-1)-st and j-th re-entry bodies, given the successful completion of the two boost phases and all preceding portions of the horizontal deployment phase.

These are the probabilities that a missile which enters the horizontal deployment phase and successfully passes the planned time of a given re-entry body release will not fail before the planned time of the next re-entry body release (considering only that portion of the missile common to the functioning of all re-entry bodies).

$R_{I(N+1)}$

For notational convenience, this is defined to be zero.

Conceptually, this can be interpreted as meaning that there is probability 0 of a tactical missile deploying N+1 or more re-entry bodies.

R_V Reliability of a single vertical deployment phase.

This is the probability of successful operation of that portion of the missile which is unique to the functioning of a single re-entry body. Errors in accuracy (i.e., re-entry bodies off target) are attributed to the phase which was responsible, not to the vertical deployment pnase (unless appropriate). R_V is assumed to be the same for all re-entry bodies on a tactical missile.

B.2.3 Expected Percentage of Successful Re-Entry Bodies

The expected percentage of successful re-entry bodies out of N re-entry bodies on a tactical missile is defined as

$$R \equiv \frac{1}{N} \sum_{k=1}^{N} k \cdot \Pr \left\{ \begin{array}{l} \text{exactly k successful} \\ \text{re-entry bodies} \end{array} \right\} \qquad \text{(B-1)}$$

For exactly k re-entry bodies to be successful, the first and second boost phases must be successful, the horizontal deployment phase must be successful through the planned deployment time of $i \geqslant k$ re-entry bodies, and of these i re-entry bodies exactly k must be successes and i-k must be failures.

The probability of the missile operating through the planned deployment time of the i-th re-entry body and failing prior to the planned deployment time of the (i+1)-st is

$$R_{B1} R_{B2} \left(\prod_{j=1}^{i} R_{Ij} \right) \left[1 - R_{I(i+1)} \right] \qquad \text{(B-2)}$$

for $i = 1, \ldots, N$.

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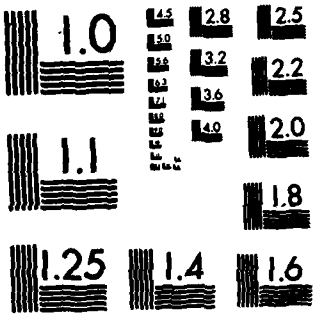

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Also, the probability that exactly k of i re-entry bodies successfully complete their vertical deployment phases is

$$\binom{i}{k} R_V{}^{k}(1-R_V)^{i-k} \quad \text{for } k=0,\ldots,i. \quad \text{(B-3)}$$

Combining (B-2) and (B-3), the probability that exactly k re-entry bodies are successful is

$$R_{B1}R_{B2}\sum_{i=k}^{N}\left(\prod_{j=1}^{i} R_{Ij}\right)\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^{k}(1-R_V)^{i-k} \quad \text{(B-4)}$$

where the summation is needed to take into account all possible missile operating success times long enough to deploy at least k re-entry bodies. Substituting (B-4) into (B-1) yields the expected percentage of re-entry bodies

$$R = \frac{1}{N}R_{B1}R_{B2}\sum_{k=1}^{N} k \sum_{i=k}^{N}\left(\prod_{j=1}^{i} R_{Ij}\right)\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^{k}(1-R_V)^{i-k} \quad \text{(B-5)}$$

Simplifying, we obtain, when $N > 1$,

$$R = \frac{1}{N}R_{B1}R_{B2}\sum_{k=1}^{N-1} k \sum_{i=k}^{N-1}\left(\prod_{j=1}^{i} R_{Ij}\right)\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^{k}(1-R_V)^{i-k} + \frac{1}{N}R_{B1}R_{B2}\left(\prod_{j=1}^{N} R_{Ij}\right)\sum_{k=1}^{N} k \binom{N}{k} R_V{}^{k}(1-R_V)^{N-k}$$

$$R = \frac{1}{N}R_{B1}R_{B2}\sum_{k=1}^{N-1} k \sum_{i=k}^{N-1}\left(\prod_{j=1}^{i} R_{Ij}\right)\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^{k}(1-R_V)^{i-k} + R_{B1}R_{B2}R_V\prod_{j=1}^{N} R_{Ij} \quad \text{(B-6)}$$

where in the first step we used the fact that $R_{I(N+1)} = 0$ by definition, and in the second step we recognized that the summation in the second term was just the definition of the expected value of a binomial random variable with parameters N and R_V.

We will now show that (B-5) and (B-6) are equivalent to

$$R = \frac{1}{N}R_{B1}R_{B2}R_V\sum_{i=1}^{N}\prod_{j=1}^{i} R_{Ij}. \quad \text{(B-7)}$$

First, when N = 1, using (B-5) we obtain $R_{B1}R_{B2}R_{I1}(1-R_{I2})R_V$ and since $R_{I2} = R_{I(N+1)} = 0$ by definition, this in turn becomes $R_{B1}R_{B2}R_V R_{I1}$ which is clearly (B-7) with N = 1.

Next, we will assume that for N = n (B-6) and (B-7) are equivalent, and use this to show that they must be equivalent for N = n + 1. When N = n + 1 (B-6) becomes

$$R = \frac{1}{n+1}R_{B1}R_{B2}\sum_{k=1}^{n} k \sum_{i=k}^{n}\left(\prod_{j=1}^{i} R_{Ij}\right)\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^{k}(1-R_V)^{i-k} + R_{B1}R_{B2}R_V\prod_{j=1}^{n+1} R_{Ij}$$

$$= \frac{1}{n+1}R_{B1}R_{B2}\sum_{k=1}^{n-1} k \sum_{i=k}^{n-1}\left(\prod_{j=1}^{i} R_{Ij}\right)\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^{k}(1-R_V)^{i-k} + \frac{1}{n+1}R_{B1}R_{B2}\left(\prod_{j=1}^{n} R_{Ij}\right)\left[1-R_{I(n+1)}\right]\sum_{k=1}^{n} k\binom{i}{k} R_V{}^{k}(1-R_V)^{i-k} + R_{B1}R_{B2}R_V\prod_{j=1}^{n+1} R_{Ij}$$

time of the first re-entry body release. However, it is independent of the success or failure of the first re-entry body's vertical deployment phases.

It should also be noted that the terminology in the definitions is common to both missile system reliability and missile body reliability. When discussing missile body reliability, the reliabilities of the guidance system, destruct system, and re-entry body are excluded where appropriate.

R_{B1} Reliability of the first boost phase.

This is the probability that a missile which enters the first boost phase will not fail during it.

R_{B2} Reliability of the second boost phase, given successful completion of the first boost phase.

This is the probability that a missile which enters the second boost phase will not fail during it.

R_{I1} Reliability of the horizontal deployment phase from initiation through the planned time of deployment of the first re-entry body, given the successful completion of the first and second boost phases.

This is the probability that a missile which enters the horizontal deployment phase will not fail before the planned time of the first re-entry body release (considering only that portion of the missile common to the functioning of all re-entry bodies).

R_{Ij} for $j = 2, 3, \ldots, N$ this is the reliability of the horizontal deployment phase *between* the planned times of deployment of the (j-1)-st and j-th re-entry bodies, given the successful completion of the two boost phases and all preceding portions of the horizontal deployment phase.

These are the probabilities that a missile which enters the horizontal deployment phase and successfully passes the planned time of a given re-entry body release will not fail before the planned time of the next re-entry body release (considering only that portion of the missile common to the functioning of all re-entry bodies).

$R_{I(N+1)}$

For notational convenience, this is defined to be zero.

Conceptually, this can be interpreted as meaning that there is probability 0 of a tactical missile deploying N+1 or more re-entry bodies.

R_V Reliability of a single vertical deployment phase.

This is the probability of successful operation of that portion of the missile which is unique to the functioning of a single re-entry body. Errors in accuracy (i.e., re-entry bodies off target) are attributed to the phase which was responsible, not to the vertical deployment phase (unless appropriate). R_V is assumed to be the same for all re-entry bodies on a tactical missile.

B.2.3 Expected Percentage of Successful Re-Entry Bodies

The expected percentage of successful re-entry bodies out of N re-entry bodies on a tactical missile is defined as

$$R \equiv \frac{1}{N} \sum_{k=1}^{N} k \cdot \Pr \left\{ \text{exactly } k \text{ successful re-entry bodies} \right\} \qquad \text{(B-1)}$$

For exactly k re-entry bodies to be successful, the first and second boost phases must be successful, the horizontal deployment phase must be successful through the planned deployment time of $i \geq k$ re-entry bodies, and of these i re-entry bodies exactly k must be successes and i-k must be failures.

The probability of the missile operating through the planned deployment time of the i-th re-entry body and failing prior to the planned deployment time of the (i+1)-st is

$$R_{B1} R_{B2} \left(\prod_{j=1}^{i} R_{Ij} \right) \left[1 - R_{I(i+1)} \right] \qquad \text{(B-2)}$$

for $i = 1, \ldots, N$.

Also, the probability that exactly k of i re-entry bodies successfully complete their vertical deployment phases is

$$\binom{i}{k} R_V{}^k(1-R_V)^{i-k} \quad \text{for } k=0,\ldots,i. \quad \text{(B-3)}$$

Combining (B-2) and (B-3), the probability that exactly k re-entry bodies are successful is

$$R_{B1}R_{B2}\sum_{i=k}^{N}\left(\prod_{j=1}^{i} R_{Ij}\right) \quad \text{(B-4)}$$
$$\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^k(1-R_V)^{i-k}$$

where the summation is needed to take into account all possible missile operating success times long enough to deploy at least k re-entry bodies. Substituting (B-4) into (B-1) yields the expected percentage of re-entry bodies

$$R = \frac{1}{N}R_{B1}R_{B2}\sum_{k=1}^{N} k \sum_{i=k}^{N}\left(\prod_{j=1}^{i} R_{Ij}\right) \quad \text{(B-5)}$$
$$\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^k(1-R_V)^{i-k}$$

Simplifying, we obtain, when N > 1,

$$R = \frac{1}{N}R_{B1}R_{B2}\sum_{k=1}^{N-1} k \sum_{i=k}^{N-1}\left(\prod_{j=1}^{i} R_{Ij}\right)$$
$$\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^k(1-R_V)^{i-k}$$
$$+\frac{1}{N}R_{B1}R_{B2}\left(\prod_{j=1}^{N} R_{Ij}\right)$$
$$\sum_{k=1}^{N} k \binom{N}{k} R_V{}^k(1-R_V)^{N-k}$$

$$R = \frac{1}{N}R_{B1}R_{B2}\sum_{k=1}^{N-1} k \sum_{i=k}^{N-1}\left(\prod_{j=1}^{i} R_{Ij}\right) \quad \text{(B-6)}$$
$$\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^k(1-R_V)^{i-k}$$
$$+ R_{B1}R_{B2}R_V\prod_{j=1}^{N} R_{Ij}$$

where in the first step we used the fact that $R_{I(N+1)} = 0$ by definition, and in the second step we recognized that the summation in the second term was just the definition of the expected value of a binomial random variable with parameters N and R_V.

We will now show that (B-5) and (B-6) are equivalent to

$$R = \frac{1}{N}R_{B1}R_{B2}R_V\sum_{i=1}^{N}\prod_{j=1}^{i}R_{Ij}. \quad \text{(B-7)}$$

First, when N = 1, using (B-5) we obtain $R_{B1}R_{B2}R_{I1}(1-R_{I2})R_V$ and since $R_{I2} = R_{I(N+1)} = 0$ by definition, this in turn becomes $R_{B1}R_{B2}R_V R_{I1}$ which is clearly (B-7) with N = 1.

Next, we will assume that for N = n (B-6) and (B-7) are equivalent, and use this to show that they must be equivalent for N = n + 1. When N = n + 1 (B-6) becomes

$$R = \frac{1}{n+1}R_{B1}R_{B2}\sum_{k=1}^{n} k \sum_{i=k}^{n}\left(\prod_{j=1}^{i} R_{Ij}\right)$$
$$\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^k(1-R_V)^{i-k}$$
$$+ R_{B1}R_{B2}R_V\prod_{j=1}^{n+1} R_{Ij}$$
$$= \frac{1}{n+1}R_{B1}R_{B2}\sum_{k=1}^{n-1} k \sum_{i=k}^{n-1}\left(\prod_{j=1}^{i} R_{Ij}\right)$$
$$\left[1-R_{I(i+1)}\right]\binom{i}{k} R_V{}^k(1-R_V)^{i-k}$$
$$+\frac{1}{n+1}R_{B1}R_{B2}\left(\prod_{j=1}^{n} R_{Ij}\right)$$
$$\left[1-R_{I(n+1)}\right]\sum_{k=1}^{n} k\binom{i}{k} R_V{}^k(1-R_V)^{i-k}$$
$$+ R_{B1}R_{B2}R_V\prod_{j=1}^{n+1} R_{Ij}$$

$$= \frac{1}{n+1} R_{B1} R_{B2} R_V \sum_{i=1}^{n} \prod_{j=1}^{n} R_{Ij} - \frac{n}{n+1} R_{B1} R_V \prod_{j=1}^{n} R_{Ij}$$

$$+ \frac{n}{n+1} R_{B1} R_{B2} R_V \left(\prod_{j=1}^{n} R_{Ij}\right)\left[1 - R_{I(n+1)}\right]$$

$$+ R_{B1} R_{B2} R_V \left(\prod_{j=1}^{n+1} R_{Ij}\right)$$

where the first term was simplified using the inductive hypothesis and the second term was simplified in the same fashion as the second term of (B-6). Algebraic simplification of this expression leads to

$$R = \frac{1}{n+1} R_{B1} R_{B2} R_V \sum_{i=1}^{n+1} \prod_{j=1}^{i} R_{Ij}$$

which is (B-7) with N = n + 1.

B.2.4 Simplification for Equal Horizontal Deployment Phase Reliabilities

Next, we make the assumption that $R_{I2} = R_{I3} = \ldots = R_{IN} \equiv R_I$; that is, that the horizontal deployment phase reliability is equal between each pair of successive deployments. In this case, (B-6) becomes

$$R = \frac{1}{N} R_{B1} R_{B2} R_V (R_{I1} + \sum_{i=2}^{N} R_{I1} \prod_{j=2}^{i} R_I)$$

$$= \frac{1}{N} R_{B1} R_{B2} R_V \left[R_{I1} \left(1 + \sum_{i=2}^{N} R_I^{\,i-1}\right)\right]$$

$$= \frac{1}{N} R_{B1} R_{B2} R_V R_{I1} (1 - R_I^{\,N})/(1 - R_I) \quad \text{(B-8)}$$

When R_I is close to 1, which must be the case for the missile to be at all reliable, we can simplify (B-8) further with the aid of a pair of approximations. Setting $Q = 1 - R_I$ for notational ease, we have

$$\frac{1}{N} \left(1 - R_I^{\,N}\right) /(1 - R_I) = [1 - (1-Q)^N]/NQ$$

$$= \left[1 - \left(1 - \binom{N}{1} Q + \binom{N}{2} Q^2 - \binom{N}{3} Q^3 \pm \ldots\right)\right]/NQ$$

$$\simeq \left(NQ - \frac{N(N-1)}{2} Q^2\right) /NQ$$

$$= \left(1 - \frac{N-1}{2} Q\right)$$

$$\simeq (1-Q)^{\frac{N-1}{2}}$$

$$= R_I^{\frac{N-1}{2}} \quad \text{(B-9)}$$

Figure B-2 below summarizes the accuracy of this approximation for selected values of N and R_I. We can see that in general the approximation is accurate to three or more significant figures for realistic values of N and R_I. Substitution of (B-9) into (B-8) yields the approximation for the expected percentage of successful re-entry bodies

$$R \simeq R_{B1} R_{B2} R_V R_{I1} R_I^{\frac{N-1}{2}} \quad \text{(B-10)}$$

R_I	N	$\frac{1-R_I^N}{N(1-R_I)}$	$R_I^{\frac{N-1}{2}}$
.990	8	.9657	.9654
	10	.9652	.9558
	12	.9468	.9462
	14	.9375	.9368
.995	8	.9827	.9826
	10	.9778	.9777
	12	.9730	.9728
	14	.9681	.9679
.999	8	.9965	.9965
	10	.9955	.9955
	12	.9945	.9945
	14	.9935	.9935

Figure B-2. Accuracy of Approximation for Selected N and R_I

B.2.5 Simplification for Constant Failure Rate for Horizontal Deployment Phase

The nature of the horizontal deployment phase makes plausible the assumption of a constant failure rate for the duration of the phase. This means that the probability the phase will not fail before time T+t, given that it has not failed before time T, is $e^{-\lambda t}$, where λ is the failure rate. If the time from thrust termination to the deployment of the first re-entry body is Δ_1, and the time between deployments of any two successive re-entry bodies is Δ, we are led to

$$R_{11} = e^{-\lambda\Delta_1} \text{ and } R_1 = e^{-\lambda\Delta}. \quad \text{(B-11)}$$

Substitution of (B-11) into (B-10) yields the formula used for the expected percentage of successful re-entry bodies,

$$R \simeq R_{B1} R_{B2} R_V e^{-\lambda\left[\Delta_1 + \frac{(N-1)\Delta}{2}\right]}. \quad \text{(B-12)}$$

B.2.6 Calculation of Demonstrated Flight Reliability

To calculate Demonstrated Flight Reliability, we need the following data for each flight test:

- Whether the first boost phase was a success, failure, or no-test.
- Whether the second boost phase was a success, failure, or no-test.
- Whether the horizontal deployment phase was a success, failure, or no-test, and, if a failure, the operating time prior to failure.
- For each re-entry body, whether its vertical deployment phase was a success, failure, or no-test.

When calculating missile body DFR, failures attributed to the guidance system, destruct system, or re-entry body are excluded from consideration. For example, if the missile went off course during the first boost phase due to the guidance system (but otherwise the missile functioned correctly during this phase), and if the second stage motor failed, then the first boost phase would be a success, the second boost phase a failure, and the remaining phases all no-tests.

Let S_{B1} and F_{B1} be the number of first boost phase successes and failures for the flights under consideration. S_{B2} and F_{B2} are defined similarly for the second boost phase, as are S_1 and F_1 for the horizontal deployment phase. We also let T_1 be the total operating time (time until failure) for those horizontal deployment phases which failed. Finally, we let S_V and F_V be the number of vertical deployment phase successes and failures. A value for $S_V + F_V$ other than N is possible for missiles flown in other than the tactical configuration. These numbers are then used to calculate the following estimates.

$$\begin{aligned} \hat{R}_{B1} &= S_{B1}/(S_{B1} + F_{B1}) \\ \hat{R}_V &= S_V/(S_V + F_V) \\ \hat{R}_{B2} &= S_{B2}/(S_{B2} + F_{B2}) \\ \hat{\lambda} &= F_1 / \left\{ S_1 [\Delta_1 + (N-1)\Delta] + T_1 \right\} \end{aligned} \quad \text{(B-13)}$$

Substituting the estimates (B-13) into formula (B-12) for expected percentage of successful re-entry bodies gives the formula defining Demonstrated Flight Reliability:

$$DFR \equiv \hat{R}_{B1} \hat{R}_{B2} \hat{R}_V e^{-\hat{\lambda}\left[\Delta_1 + \frac{(N-1)\Delta}{2}\right]} \quad \text{(B-14)}$$

B.2.7 Example and Comparison with Raw Score

Suppose the tactical configuration has $N = 8$, $\Delta_1 = 1$ minute, and $\Delta = 2$ minutes, and assume the following flight results (Figure B-3):

These results yield the following statistics:

$S_{B1} = 8$, $S_1 = 6$, $F_V = 3$

$S_{B2} = 7$, $F_{B1} = 1$, $F_1 = 1$

$S_V = 49$, $F_{B2} = 1$, $T_1 = 12$

first boost phase	second boost phase	horizontal deployment phase	vertical deployment phase
success	success	success	8 successes
success	success	success	7 successes 1 failure
success	success	success	6 successes
failure	no-test	no-test	4 no-tests
success	success	success	8 successes
success	success	failure at 12 min.	5 successes 1 failure 2 no-tests
success	success	success	7 successes 1 failure
success	failure	no-test	4 no-tests
success	success	success	8 successes

Figure B-3. Flight Results

Substitution of the above into (B-13) yields the estimates:

$$\hat{R}_{B1} = 8/(8+1) = .889$$

$$\hat{R}_{B2} = 7/(7+1) = .875$$

$$\hat{R}_{V} = 49/(49+3) = .942$$

$$\hat{\lambda} = 1/\left\{6 \cdot [1+(8-1)\,2] + 12\right\} = .010$$

Finally, substituting the estimates into (B-14) gives the Demonstrated Flight Reliability for the given data:

$$\text{DFR} = (.889)(.875)(.942)e^{-.010\left[1+\frac{(8-1)2}{2}\right]}$$

$$= .676$$

By comparison, the Raw Score (RS) is defined as the ratio of the number of successful vertical deployment phases to the total number attempted. For this example, we have

$$\text{RS} = \frac{8+7+6+0+8+5+7+0+8}{8+8+6+4+8+8+8+4+8}$$

$$= .790.$$

In this case, RS is higher than DFR. Other examples can be constructed in which the reverse is true. The conclusion is that it is incorrect to attempt to interpret the Raw Score as a measure of the percentage of re-entry bodies which would be expected to be successful.

B.3 REFERENCES

1. LMSC – D368002D Reliability Evaluation Plan for TRIDENT 1 (C4) Missile, May 1980.
2. Methodology for Demonstrated Flight Reliability; Mathematical Analysis Research Corporation (MARC), January 1977.

Appendix C
STATISTICAL TESTS OF HYPOTHESES

This Appendix provides an introduction to some methods of Statistical Tests of Hypotheses which are useful in Reliability and Availability assessment. § C.1 considers the Kolmogorov-Smirnov goodness-of-fit-test, § C.2, the χ^2 test for Compatibility of Bayesian Prior and Posterior Estimates in the Exponential Model; § C.3, Laplace's test for homogeneity; and § C.4 the "label test" for independence.

Very generally, a test of hypothesis is a procedure for deciding whether to accept or reject some assumption or hypothesis about a PDF, CDF or its parameters on the basis of available test data. In goodness-of-fit-tests, for instance, the decision is to accept or reject the hypothesis that a sample originates from a preselected CDF, F(t). A very common type of *simple* hypothesis test is a decision rule to accept or reject an assumed value θ_0 (the *null* hypothesis H_0) of the parameter of a distribution against another specified value θ_1 (the alternate hypothesis H_1). In *composite* hypotheses, not all parameters of a frequency function are specified. In this case, for a two parameter distribution, an example would be: H_0 (the null hypothesis), $\sigma_0 = 1, \mu_0 = 0$; H_1 (the alternate hypothesis), $\sigma_1 = 0.5, \mu_1 > 0$.

Many tests can be constructed for given hypotheses but, just like estimation, tests are expected to display certain desirable qualities. A "best" test for a simple hypothesis, for instance, is one which, for a given type I error (or α, the probability that H_0 is true but will fall in the critical region of the test and be rejected) minimizes the type II error (or β, the probability that H_1 is true but will fall outside the critical region of the test and be rejected). The Neyman-Pearson Lemma [1, p. 214] permits the construction of best tests of simple hypothesis. For composite hypothesis, the choice of a best test is often based on the consideration of the *power* function $P(\theta)$ of the test [1, p. 54] which is the function of the parameter θ that gives the probability that the sample point will fall in the critical region of the test when θ is the true value of the parameter.

Since $P(\theta) = 1 - \beta$, where now β is a function of θ, seeking a test that minimizes the type II error β is equivalent to seeking one that maximizes the power function $P(\theta)$.

C.1 THE KOLMOGOROV-SMIRNOV GOODNESS-OF-FIT-TEST

There are many goodness-of-fit-tests, with the χ^2 [1, p. 347] test one of the most popular. The χ^2 method, however, is not an exact method and requires the classification of data into cells with at least five data points per cell. Thus, the χ^2 test requires fairly large samples. This is not the case with the Kolmogorov-Smirnov test described in this appendix, which is exact and is applicable to small samples. A note of caution, however, is in order. The Kolmogorov-Smirnov test may not always provide accurate results where small samples are involved. It is shown in [2] specifically that at least 40 samples are required to distinguish between fairly close members of the Lognormal, Gamma, and Weibull distributions by means of the Kolmogorov-Smirnov goodness-of-fit-test at the 0.1 level of significance.

Let $t_{(1)}, t_{(2)}, t_{(3)}, \ldots t_{(x)}$ denote an ordered sample of size x from a population with CDF F(t), and let $S_x(t)$ denote the empirical distribution function of $t_{(1)}, t_{(2)}, \ldots t_{(x)}$, defined as follows:

$$S_x(t) = \begin{cases} 0 & t < t_{(1)} \\ k/x & t_{(k)} \leq t < t_{(k+1)} \\ 1 & t \geq t_{(x)} \end{cases}$$

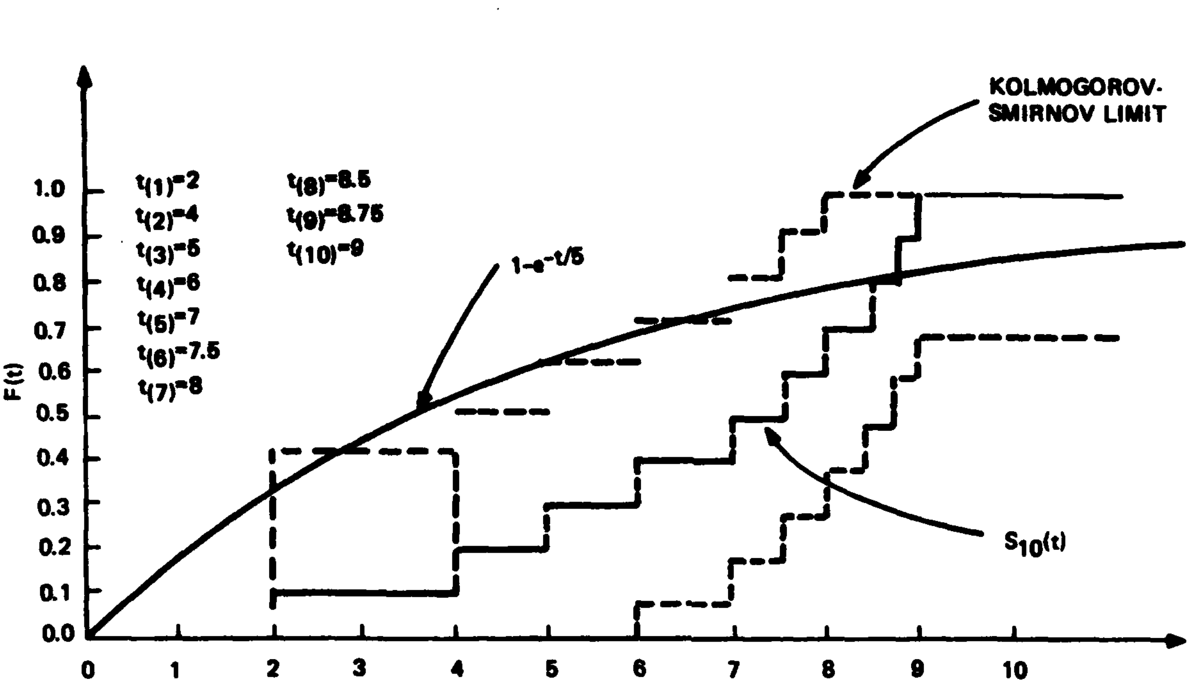

Figure C-1. The Empirical CDF and the Kolmogorov-Smirnov Limits (dashed lines)

Thus, $S_x(t)$ defines a functional ladder as depicted in figure C-1 for a particular sample of size 10.

If F(t) were known, then it would be possible to calculate the value of $|F(t) - S_x(t)|$ for any desired value of t and thus to determine the maximum vertical distance between the graphs of F(t) and $S_x(t)$ over the range of possible t values. Let us denote this maximum distance as:

$$D_x = \max_t |F(t) - S_x(t)|$$

Kolmogorov and Smirnov have shown that the distribution of D_x does not depend on F(t), and have proceeded to tabulate critical values D_x^α of D_x as shown in Figure C-2.

x \ α	Critical Values			
	α = 0.2	α = 0.1	α = 0.05	α = 0.01
5	.45	.51	.56	.67
10	.32	.37	.41	.49
15	.27	.30	.34	.40
20	.23	.26	.29	.36
25	.21	.24	.27	.32
30	.19	.22	.24	.29
35	.18	.20	.23	.27
40	.17	.19	.21	.25
45	.16	.18	.20	.24
50	.15	.17	.19	.23
>50	$1.07/\sqrt{x}$	$1.20/\sqrt{x}$	$1.34/\sqrt{x}$	$1.63/\sqrt{x}$

Figure C-2. Critical Values for D_x in the Kolmogorov-Smirnov Test

Often F(t) is not so much known to the data analyst, as it is selected as a candidate CDF for the data at hand. As such, F(t) often includes one or two (more rarely three) unknown parameters, say θ and γ and is relabeled F(t;θ) or F(t;θ,γ). Such parameters can be estimated from the data itself. If, for instance F(t,θ) is the exponential, that is, $F(t;\theta) = 1-e^{-t/\theta}$, then,

$$\hat{\theta} = \sum_{i=1}^{x} t_{(i)}/x,$$

and the "known" CDF becomes the estimated CDF $F(t;\hat{\theta}) = 1-e^{-t/\hat{\theta}}$. For such estimated CDF's, the Kolmogorov-Smirnov values of D_x^α are no longer strictly valid, but depend on the functional form of the estimated CDF.

The procedure, then, to test the hypothesis of goodness-of-fit of a hypothetical distribution to data at hand is:

a) Select a significance level, α. The lower a significance level (e.g., 0.01) the more likely a good fit to *any* candidate CDF will be demonstrated. The higher a significance level (e.g., 0.20), the less likely *any* CDF will be shown to fit the data, but also the greater the likelihood of rejecting a particular CDF as false when it is actually true.

b) Draw the empirical CDF $S_X(t)$.

c) Draw the values of D_x^{α} about $S_X(t)$ to form an acceptance region.

d) Draw the candidate CDF $F(t)$, $F(t,\hat{\theta})$, or $F(t,\hat{\theta},\hat{\gamma})$, if the appropriate D_x^{α} has been tabulated.

e) If the candidate CDF remains within the acceptance region, the hypothesis of a good fit is accepted at the selected critical value, otherwise it is rejected.

As an example, assume that for the empirical CDF of figure C-1, one shows the Kolmogorov-Smirnov limits corresponding to α = 0.2 at a distance of 0.32 (see figure C-2) of $S_{10}(t)$. These limits are the dashed lines on figure C-1. Assume also that the candidate CDF for the fit is $F(t) = 1\text{-}e^{-t/\theta} = 1\text{-}e^{-t/5}$. In this expression, $\theta = 5$ is *not* estimated from the data. It is assumed to be known. F(t) is drawn in figure C-1 and seen to cross the Kolmogorov-Smirnov acceptance region. Thus, F(t) is rejected as a possible CDF for the given data at the 0.2 level of significance.

The two examples which follow are useful in reliability assessment where it is desirable to know whether times to failure are exponential, and in availability assessment where times to repair are to be tested for lognormality.

Example of Curve Fitting the Exponential Model

The Kolmogorov-Smirnov Goodness of Fit Test specifically applied to the exponential by Lilliefors [3] will be used as an illustration. The model to be fitted is $F(t) = 1\text{-}e^{-\mu t}$ where μ is the repair rate.

It is assumed that the following repair times data are available.

M_{c_1} = 7.0 hrs. M_{c_6} = 2.3 hrs.
M_{c_2} = 5.0 hrs. M_{c_7} = 2.0 hrs.
M_{c_3} = 3.9 hrs. M_{c_8} = 1.7 hrs.
M_{c_4} = 3.0 hrs. M_{c_9} = 1.0 hr.
M_{c_5} = 2.5 hrs. $M_{c_{10}}$ = 0.5 hr.

The times have been ordered in preparation for the goodness of fit test.

An appropriate ML repair rate estimate from the sample is:

$$\hat{\mu} = n \;/\; \sum_{i=1}^{i=n} M_{c_i}$$

where n = 10, the repair sample size.

The sample data is plotted as a cumulative distribution of observed repair times. An expected distribution is also plotted from the relation

$$P(M_c) = 1 - e^{-\hat{\mu} M_c}$$

as shown in Figure C-3.

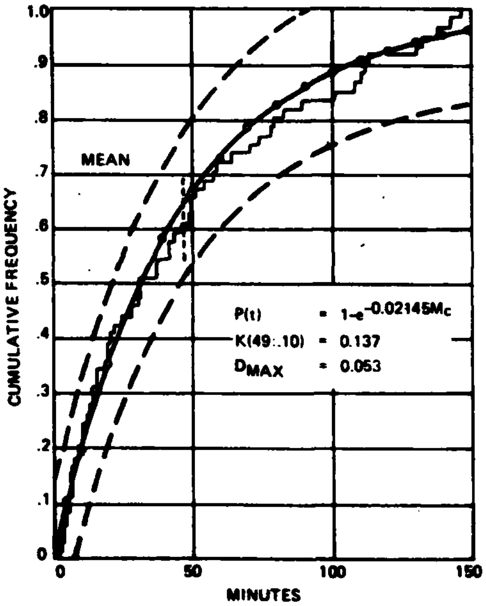

Figure C-3. Cumulative Distribution of Repair Times with Kolmogorov-Smirnov Limits (Exponential)

The Kolmogorov-Smirnov test, as modified by Lilliefors, is then applied to test the hypothesis that the data are from an exponential distribution with mean $\hat{\mu}$. The statistic evaluated is D, the largest absolute deviation between the observed and expected ordinates of the cumulative distribution.

Therefore

$$D = \max \left\{ \left| P(M_c) - S(M_c) \right| \right\}$$

where

M_c = measured repair time

$P(M_c)$ = the computed cumulative frequency

$S(M_c)$ = the observed cumulative frequency

The sampling distribution of D is known (figure C-4). Given a sample of n observed repair times and a significance level α, the result $D > K(n,\alpha)$ supports rejection of the hypothesis with confidence (1-α); otherwise the hypothesis is not rejected.

From our given repair data, the hypothesis of exponential distribution with mean 2.89 hours is not rejected.

Example of Curve Fitting Lognormal (Normal) Data.

Assume ordered observed repair times in hours are as follows:

M_{c_1} = 6.1 $\quad$ M_{c_6} = 2.5

M_{c_2} = 4.6 $\quad$ M_{c_7} = 2.2

M_{c_3} = 3.5 $\quad$ M_{c_8} = 2.0

M_{c_4} = 3.0 $\quad$ M_{c_9} = 2.0

M_{c_5} = 3.0 $\quad$ $M_{c_{10}}$ = 0.6

If the exponentiality goodness-of-fit test of the previous example is applied to the data, the null exponentiality hypothesis would be rejected at the 0.1 significance level.

This is seen in figure C-5. Thus, it becomes necessary to fit a lognormal distribution to the observed data. First the data points are transformed to their logarithms ($x_i = \ln M_{c_i}$), then, in a procedure following exactly the previous example, but using 0.1 level of significance entries from figure C-6 instead of C-4, one sees that the null hypothesis of lognormality is accepted at that level.

ABSOLUTE VALUES OF THE MAXIMUM DIFFERENCE D BETWEEN SAMPLE AND POPULATION CUMULATIVE FRACTIONS SIGNIFICANT AT THE 20, 15, 10, 5 AND 1 PERCENT LEVELS

n = sample size

Sample Size	Level of Significance α				
n	.20	.15	.10	.05	.01
3	.451	.479	.511	.551	.600
4	.396	.422	.449	.487	.548
5	.359	.382	.406	.442	.504
6	.331	.351	.375	.408	.470
7	.309	.327	.350	.382	.442
8	.291	.308	.329	.360	.419
9	.277	.291	.311	.341	.399
10	.263	.277	.295	.325	.380
11	.251	.264	.283	.311	.365
12	.241	.254	.271	.298	.351
13	.232	.245	.261	.287	.338
14	.224	.237	.252	.277	.326
15	.217	.229	.244	.269	.315
16	.211	.222	.236	.261	.306
17	.204	.215	.229	.253	.297
18	.199	.210	.223	.246	.289
19	.193	.204	.218	.239	.283
20	.188	.199	.212	.234	.278
25	.170	.180	.191	.210	.247
30	.155	.164	.174	.192	.226
Over 30	$\frac{.86}{\sqrt{n}}$	$\frac{.91}{\sqrt{n}}$	$\frac{.96}{\sqrt{n}}$	$\frac{1.06}{\sqrt{n}}$	$\frac{1.25}{\sqrt{n}}$

Figure C-4. Kolmogorov-Smirnov Limit Factors K (n,α) for Exponential Distribution with Estimated Mean

C.2 TEST OF HYPOTHESIS FOR COMPATIBILITY OF BAYESIAN PRIOR AND POSTERIOR ESTIMATES IN THE EXPONENTIAL MODEL

In the case of exponential data with gamma conjugate prior (see § 5.4.6.8), the prior estimate of λ is $\hat{\lambda}_0 = \phi/\tau$ where φ are pseudo-failures, and τ are pseudo test hours. The estimate of λ is $\hat{\lambda} = x/T$, where x are actual failures and T is actual test time. The problem is to construct a test-of-hypothesis to accept or reject $H_0 : \hat{\lambda}_0 = \phi/\tau$. The hypothesis test, based on the distribution of $\hat{\lambda}$ for exponential data (See Appendix D § D.1) is as follows:

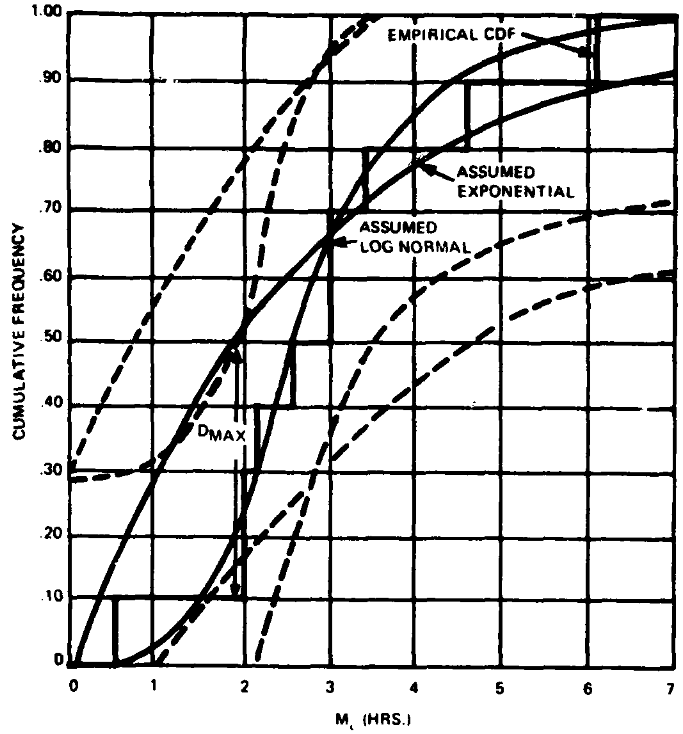

Figure C-5. Cumulative Distribution of Repair Times with Kolmogorov-Smirnov Limits

$$\frac{\chi^2_{(2x,\alpha/2)}}{2T} \leqslant \lambda \leqslant \frac{\chi^2_{(2x+2,\ 1-\alpha/2)}}{2T} \qquad \text{(C-2)}$$

where the χ^2's are the chi-square values at the $(\alpha/2)$ 100% and $(1-\alpha/2)$ 100% percentiles with 2x and 2x+2 degrees of freedom, respectively.

The decision rule is: if $\widehat{\lambda}_0$ is included within the confidence interval given by inequality (equation C-2), accept the hypothesis that λ_0 is compatible with the observed test data. If λ_0 is outside the confidence interval, reject the hypothesis that λ_0 is compatible with the data and consider an appropriate non-Bayesian reliability assessment model.

C.3 LAPLACE'S TEST OF HOMOGENEITY

In a homogeneous poisson process (HPP), successive times, T_i, of failure of the HPP are identical independent uniform random variables (r.v.) over the interval $(0, t_o)$ [4]. Denoting any of the T_i variables as T, $f(T) = 1/t_o$, then the expected value is:

$$E(T) = \int_o^{t_o} (T/t_o)\, dT = t_o/2$$

also

$$\sigma(T) = \left[\int_o^{t_o} \left(T - \frac{t_o}{2} \right)^2 dT/t_o \right]^{1/2}$$

$$= t_o/\sqrt{12}$$

Using the Central Limit Theorem, and theorems on the calculation of moments of r.v.'s which are the sum of x identical independently distributed r.v.'s, then

$$v = \sum_{i=1}^{x} T_i$$

is approximately normally distributed e.g. $x \geqslant 3$, with expectation $xt_o/2$ and standard deviation $t_o/\sqrt{12x}$, and

$$u = \left(\sum_{i=1}^{x} T_i/(xt_o) - \frac{1}{2} \right) \Big/ \sqrt{12x}$$

ABSOLUTE VALUES OF THE MAXIMUM DIFFERENCE BETWEEN SAMPLE AND POPULATION CUMULATIVE FRACTIONS SIGNIFICANT AT THE 20, 15, 10, 5 AND 1 PERCENT LEVELS

Sample Size	Level of Significance				
n	0.20	0.15	0.10	0.05	0.01
4	0.300	0.319	0.352	0.381	0.417
5	0.285	0.299	0.315	0.337	0.405
6	0.265	0.277	0.294	0.319	0.364
7	0.247	0.258	0.276	0.300	0.348
8	0.233	0.244	0.261	0.285	0.331
9	0.223	0.233	0.249	0.271	0.311
10	0.215	0.224	0.239	0.258	0.294
11	0.206	0.217	0.230	0.249	0.284
12	0.199	0.212	0.223	0.242	0.275
13	0.190	0.202	0.214	0.234	0.268
14	0.183	0.194	0.207	0.227	0.261
15	0.177	0.187	0.201	0.220	0.257
16	0.173	0.182	0.195	0.213	0.250
17	0.169	0.177	0.189	0.206	0.245
18	0.166	0.173	0.184	0.200	0.239
19	0.163	0.169	0.179	0.195	0.235
20	0.160	0.166	0.174	0.190	0.231
25	0.142	0.147	0.158	0.173	0.200
30	0.131	0.136	0.144	0.161	0.181
Over 30	$\frac{0.736}{\sqrt{n}}$	$\frac{0.768}{\sqrt{n}}$	$\frac{0.805}{\sqrt{n}}$	$\frac{0.886}{\sqrt{n}}$	$\frac{1.031}{\sqrt{n}}$

Figure C-6. Kolmogorov Limit Factors K(n,α) for the Normal Distribution with Estimated Mean and Variance (Lilliefors)

is approximately normally distributed with mean 0 and standard deviation 1. If the HPP is selected as a null hypothesis, then u can be selected as a statistic to test the hypothesis of homogeneity. The test is best used by considering two-sided critical values on the normal curve since under wearout or growth the T_i will tend to occur after or before, respectively, the midpoint of the observed interval. In other words [4], significantly large or small values of the standardized variate u show significant evidence of wearout or growth, respectively. This test has been shown to be an optimum test against two plausible models by Bates [5] and Cox [6]. As stated in [4] Laplace's Test is not consistent against alternatives where the rate of occurrence of failure is non monotone in such a way that the expected value E is:

$$E\left(\sum_{i=1}^{x} T_i/(t_o)\right) = \frac{1}{2}$$

In this case a test developed by Hollander and Proschan [7] is superior.

C.4 REFERENCES

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Appendix D
DERIVATIONS

This appendix contains:

D.1 Derivation of Availability Confidence Limit Formulae for exponential failure and recovery times, and for exponential failure times and lognormal recovery times.

D.2 Derivation of the Poisson Process of the Exponential Reliability Law.

D.3 Introduction to Birth and Death Processes.

D.4 References.

D.1 DERIVATION OF AVAILABILITY CONFIDENCE LIMIT FORMULAE

The two cases considered under this heading are (a) failure and recovery times are both exponential, and (b) failure times are exponential but recovery times are lognormal.

D.1.1 Confidence Limit for Exponential Failure and Recovery Times[1].

For exponential failure and recovery times the failure PDF of operating times between failures (t) for an item is $f(t) = \lambda e^{-\lambda t}$ and the PDF of times to repair (M_c) is $f(M_c) = \mu e^{-\mu M_c}$. In the exponential model, the instantaneous failure rate λ and the instantaneous repair rate μ are constant.

D.1.1.1 Test Truncated by Failure

In the test truncated by failures case, the number of failures x is *decided in advance.* Several Scenarios are possible.

(a) An item may be placed on test and as soon as it fails it is replaced with a new or good as new item, and so on until x failures are recorded. The operating times between the (i–1)th and the ith failure are denoted by t_i.

The ML estimate $\hat{\lambda}$ of λ is obtained from the likelihood function of the observations:

$$L = f(t_1)\, f(t_2) \,.\,.\,.\,.\,.\, f(t_x) = \lambda^x \, e^{-\lambda \sum_{i=1}^{x} t_i}.$$

Differentiating with respect to λ and setting to 0 yields

$$\hat{\lambda} = x / \sum_{i=1}^{x} t_i \qquad \text{(D-1)}$$

(b) N identical items can be put on test without replacement until x of them fail. If the t_i's represent the times t_i^* between the (i–1)th and the ith failure *multiplied* by the number of operating items between the (i-1)th and the ith failure, that is multipled by N+1–i, then $\hat{\lambda}$ is given either by

$$\hat{\lambda} = x / \sum_{i=1}^{x} (N+1-i)\, t_i^*$$

or

$$\hat{\lambda} = x / \sum_{i=1}^{x} t_i. \qquad \text{(D-1)}$$

These formulas are consistent with the formulae given in section 5

$$\hat{\lambda} = x / \left(\sum_{i=1}^{x} \bar{t}_i + (N-x)\bar{t}_x \right)$$

where $\bar{t}_i$ represents the time of each item on test since the beginning of testing and $\bar{t}_x$ the time to failure of the last item to fail.

(c) N identical items can be put on test with replacement until x failures are observed.

If the t_i's now represent the times t_i^* between the (i-1)th and the ith failure multiplied by the number of operating items between the (i-1)th and the ith failure, that is multiplied by N, then $\hat{\lambda}$ is given either by:

$$\hat{\lambda} = x/\left(N \sum_{i=1}^{x} t_i^*\right)$$

or by
$$\hat{\lambda} = x/\sum_{i=1}^{x} t_i. \qquad \text{(D-1)}$$

This is again consistent with the formula from Section 5

$$\hat{\lambda} = x/N\bar{t}_x$$

where $\bar{t}_x$ represents the time on test of the item failing last.

In all cases, under the appropriate definition of t_i,

$$\hat{\lambda} = x/\sum_{i=1}^{x} t_i \qquad \text{(D-1)}$$

In order to find the PDF of $\hat{\lambda}$, one may first consider the PDF of the variable

$$T = \sum_{i=1}^{x} t_i$$

which, as the sum of x identically distributed exponential r.v.'s of the form $f(t_i) = \lambda e^{-\lambda t_i}$, is gamma:

$$\gamma_x(T) = T^{x-1}\lambda^x \exp(-\lambda T)/(x-1)!\,.$$

The transformation

$$u = \sum_{i=1}^{x} t_i/x = T/x = \frac{1}{\hat{\lambda}} \text{ yields:}$$

$$k_x(u) = x\gamma_x(xu) = \frac{x(xu)^{x-1}\lambda^x \exp(-\lambda xu)}{(x-1)!}$$

and since $\hat{\lambda} = \frac{1}{\mu}$, and

$$g_x(\hat{\lambda}) = k_x\, u(\hat{\lambda}) \left| \frac{du(\hat{\lambda})}{d\hat{\lambda}} \right|, \text{ then}$$

$$g_x(\hat{\lambda}) = x^x \lambda^x \left(\frac{1}{\hat{\lambda}}\right)^{x+1} \exp[-\lambda x/\hat{\lambda}]/(x-1)!\,.$$

The variable of interest in finding confidence limits on availability in § D.1.1.3 is

$$V = 2x\lambda/\hat{\lambda} \text{ or } V = 2\lambda \sum_{i=1}^{x} t_i.$$

Using $k_x(u)$ or $g_x(\hat{\lambda})$, one finds that the PDF of V is:

$$h_x(V) = V^{x-1} \exp(-V/2)/[2^x (x-1)!] \qquad \text{(D-2)}$$

which is a chi-square (χ^2) PDF with 2x degrees of freedom.

D.1.1.2 Test Truncated by Time

In the case of time truncated tests, the total time on test T is decided in advance of testing. The test scenarios of § D.1.1.1 still apply provided the estimator of λ used is

$$\hat{\lambda} = x/T \text{ instead of } x/\sum_{i=1}^{x} t_i.$$

In the expression $\hat{\lambda} = x/T$, x represents the number of failures occurring in T

where
$$T = \sum_{i=1}^{x} t_i + t_{x+1}^*,$$

and t_i represents the operating time between the (i-1)th and the ith failure, and t_{x+1}^* the operating time, not necessarily ending in failure, between the xth failure and the end of test time T.

In § D.1.1.1 the distribution of

$$V = 2\lambda \sum_{i=1}^{x} t_i \text{ or } V = 2x\lambda/\hat{\lambda}$$

was derived. This distribution could be readily obtained under the assumption of times ending in failure. It is not possible to use a parallel argument for $V = 2\lambda T$ in the case of tests truncated by time since both λ and T are constant, and V is *not* a random variable in this case.

For tests truncated by time, x, the number of failures is actually the random variable of interest. One can, nevertheless, arrive at a reasonable treatment of λT as a random variable if one accepts the Bayesian argument that the value of the probability density function for λT is proportional to the probability of observing N failures under that value for λT.

For example, suppose 1 failure was observed.

$$\text{For } \lambda T = 2,\ P(x) = P(1) = \frac{2^1 e^{-2}}{1} = 2e^{-2}.$$

$$\text{For } \lambda T = 3,\ P(x) = P(1) = \frac{3^1 e^{-3}}{1} = 3e^{-3}.$$

Therefore, the ratio of the ordinates of the PDF for λT, which we will call $f(\lambda T)$ is given by the equation

$$\frac{f(2)}{f(3)} = \frac{2e^{-2}}{3e^{-3}} = \frac{2e}{3}$$

$$\text{or } \lambda T \text{ is } \frac{2e}{3}$$

times as likely to be in a small neighborhood near 2 as it is to be in a same size neighborhood near 3. If this reasoning is carried out for all values of λT then for one failure:

$$f(\lambda T) = f(u) \sim \frac{u^1 e^{-u}}{1}$$

and for any number of failures, x,

$$f(\lambda T) = f(u) \sim \frac{u^x e^{-u}}{x!}\ .$$

Since $\int_0^\infty f(u)du \equiv 1$, then

$$f(u) = \frac{u^x e^{-u}/x!}{\int_0^\infty u^x e^{-u} du/x!} = u^x e^{-u}/x!$$

If we substitute $u = 2\lambda T$ for λT, we find:

$$f(2\lambda t) = f(u) = u^x e^{-u/2}/(x!2^{x+1}) \qquad \text{(D-3)}$$

which shows that $2\lambda T$ is χ^2 with $\nu = 2x+2$ degrees of freedom. Also, since

$$\hat{\lambda} = \frac{x}{T},\ 2x\lambda/\hat{\lambda} = 2\lambda T$$

is also distributed as χ^2 with 2x+2 degrees of freedom.

D.1.1.3 Confidence Limit on Availability for Exponential Failure and Exponential Repair Times

For exponential repair times, $f(M_c) = \mu\, e^{-\mu M_c}$ where M_c is repair time and μ is the constant instantaneous repair rate.

The maximum likelihood estimate of μ is

$$\hat{\mu} = m/\sum_{i=1}^{m} M_{c_i}$$

where m is the number of repairs observed and M_{c_i} is the time required for the ith repair. The density function of $\hat{\mu}$ is obtained by a procedure identical to that for the density of $\hat{\lambda}$:

$$g(\hat{\mu}) = (\mu)^m \left(\frac{1}{\hat{\mu}}\right)^{m-1} m^m \exp(-mu/\mu).$$

Let the random variable $v = \dfrac{2m\mu}{\hat{\mu}}$.

It is assumed that each repair time M_{c_i} is terminated by a repair. Then:

$$g(v) = \frac{v^{m-1} e^{\left(-\frac{v}{2}\right)}}{2^m (m-1)!} \qquad \text{(D-4)}$$

is a χ^2 density with 2m degrees of freedom.

For failure truncated tests it was shown in § D.1.1.1 that $u = 2x\lambda/\hat{\lambda}$ is distributed as χ^2 with 2x degrees of freedom for tests truncated by failure. Since u and v are independently distributed with 2x degrees of freedom, the quantity

$$\frac{u/2x}{v/2m}$$

has the variance density ratio F with 2x and 2m degrees of freedom. But

$$\frac{u/2x}{v/2m} = \frac{\lambda/\hat{\lambda}}{\mu/\hat{\mu}} = \frac{\lambda/\mu}{\hat{\lambda}/\hat{\mu}}$$

and since the ith component availability is

$$A_i = \frac{1}{1+(\lambda/\mu)},$$

a 100 (1−γ) percent lower confidence bound on A_i is obtained where testing is terminated by failure from the relation:

$$\left(\frac{\lambda}{\mu}\right)_u = \frac{\hat{\lambda}}{\hat{\mu}} F_{1-\gamma;\,2x,\,2m}. \qquad \text{(D-5)}$$

$F_{1-\gamma;2x,2m}$ is the (1−γ) fractile of the cumulative F distribution with 2x and 2m degrees of freedom. It is tabulated in Appendix E, figure E-6 which gives $F_{.80};f_1,f_2$ for f_1 = 1(1) 80, and f_2 = 1(1)80.

For tests truncated by time, it was shown in § D.1.1.2 that under a particular assumption $2x\lambda/\hat{\lambda}$ was distributed as χ^2_{2x+2}. In this case the 100(1−γ)% lower availability confidence limit is obtained by using

$$\left(\frac{\lambda}{\mu}\right)_u = \left(\frac{x+1}{x}\right)\frac{\hat{\lambda}}{\hat{\mu}} F_{1-\gamma;2x+2,2m} \qquad \text{(D-6)}$$

in

$$A_{i_L} = \frac{1}{1+\left(\frac{\lambda}{\mu}\right)_u}. \qquad \text{(D-7)}$$

A special case is needed for the case x = 0 since

$$\frac{(x+1)\hat{\lambda}}{x}$$

is indeterminate, since $\hat{\lambda} = 0$. One can use:

$$\left(\frac{\lambda}{\mu}\right)_u = \frac{x+1}{\hat{\mu}T} F_{1-\gamma;2x+2,2m} \qquad \text{(D-8)}$$

D.1.2 Estimation of the Parameters of a Lognormal Distribution [2].

A random variable z>0 has a lognormal distribution if it has the density function

$$\lambda(z;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\Pi}\,z}\exp\left[-\frac{1}{2}\left(\frac{\ln z-\mu}{\sigma}\right)^2\right].$$

Then its logarithm y = ln z has a normal distribution

$$n(y;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\Pi}}\, e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$$

The cumulative lognormal distribution is denoted Λ (z; μ, σ²). An expression for it does not exist in terms of elementary functions but by the simple transformation y = ln z it is transformed into the normal integral

$$\Lambda(z;\mu,\sigma^2) = N(\ln z;\mu,\sigma^2)$$

and by a further simple transformation to the standard normal variate

$$x = \frac{y-\mu}{\sigma}.$$

Cumulative probabilities of the lognormal can be found numerically from tabulated values of the standard normal N (x; 0, 1).

$$\Lambda(z;\mu,\sigma^2) = N\left(\frac{\ln z-\mu}{\sigma};0,1\right)$$

The parameters μ and σ can be estimated from lognormal data by means of the transformation

$$\hat{\mu} = \overline{\ln z} = \frac{\sum_{i=1}^{n} \ln z_i}{n}$$

$$\hat{\sigma} = s = \sqrt{\frac{\sum_{i=1}^{n} (\ln z_i - \ln z)^2}{n-1}}$$

D.1.3 Confidence Limit for Exponential Failure Times and Lognormal Recovery Times

Gray and Lewis [3] have shown that if σ² (the variance of ln M_c) is assumed to be known, then for a random sample of m repair times

$$\frac{\left(\prod_{i=1}^{m} M_{c_i}\right)^{1/m}}{e^{\mu}} = \frac{\hat{M}_{c_G}}{e^{\mu}} = \frac{1}{\hat{\nu}e^{\mu}} \sim \Lambda\left(0,\frac{\sigma^2}{m}\right)$$

where $\hat{M}_{c_G}$ is the geometric mean and $\hat{\nu}$ is $1/\hat{M}_{c_G}$. For purposes of analysis σ² is assumed to equal the variance estimated from the sample. For exponentially distributed failure times

$$\frac{2x\lambda}{\widehat{\lambda}} \sim \chi^2\,(2x).$$

Then a confidence interval for λ/μ can be found by finding the distribution of

$$(2x\lambda/\widehat{\lambda})/(1/\widehat{\mu}\,e^{\mu}) = \widehat{\nu}e^{\mu}\frac{2x\lambda}{\widehat{\lambda}}.$$

In general then, letting $U \sim \chi^2\,(k)$ and

$$V \sim \Lambda\left(0, \frac{\sigma^2}{m}\right)$$

$$f(u,v) = \begin{cases} \dfrac{u^{\frac{k}{2}-1}\,e^{-\frac{u}{2}}\,\sqrt{m}\,e^{-m/2\,[(\ln v)/\sigma]^2}}{\Gamma\left(\frac{k}{2}\right)2^{k/2}\,\sigma\,\sqrt{2\pi}\,v} & \begin{matrix}0<u<\infty\\ 0<v<\infty\end{matrix} \\ 0 & \text{elsewhere} \end{cases}$$

Let W = U/V and Z = V; then

$$g(w) = \begin{cases} c\int_0^{\infty} \exp\left[-\dfrac{m}{2\sigma^2}\ln^2 z - \dfrac{1}{2}wz\right](wz)^{\frac{k}{2}-1}\,dz, & 0<w<\infty \\ 0 & \text{elsewhere} \end{cases}$$

where

$$c = \frac{\sqrt{\dfrac{m}{\sigma^2}}}{\Gamma\left(\dfrac{k}{2}\right)2^{k/2}\sqrt{2\pi}}.$$

Hence $P[a<W<b]$ is given by:

$$P\,[a<W<b] = \int_a^b g(w)\,dw = p.$$

Let

$$W = \frac{2\widehat{\nu}\,e^{\mu}x\lambda}{\widehat{\lambda}}$$

then

$$P\,[W<a] = P\left[\frac{\lambda}{\mu} < \frac{\widehat{\lambda}}{\widehat{\mu}} \cdot \frac{a\exp(\sigma^2/2)}{2x}\right] = 1\text{-}\alpha.$$

Then a one-sided 100 $(1-\alpha)$ percent confidence limit for A_1 is

$$A_{i_L} = \frac{1}{1 + \dfrac{\widehat{\lambda}}{\widehat{\nu}}\left(\dfrac{a\,e^{\sigma^2/2}}{2x}\right)} \qquad \text{(D-9)}$$

Figure E-7 gives selected values of the coefficient

$$a_{1-\alpha}\,;\frac{m}{\sigma^2}\,,\,x,\ \text{for}$$

$$\alpha = \frac{\sqrt{\dfrac{m}{\sigma^2}}}{\Gamma(x)\,2^x\sqrt{2\pi}}\int_0^a\int_0^{\infty}\exp\left[-\frac{m}{2\sigma^2}\ln^2 z - \frac{1}{2}wz\right](wz)^{x-1}\,dz\,dw \qquad \text{(D-10)}$$

$$= .20.$$

D.2 DERIVATION OF THE POISSON PROCESS AND OF THE EXPONENTIAL RELIABILITY LAW

D.2.1 Assumptions

The assumptions underlying the Poisson failure process g(x,t) were presented in § 5.1.1 and will not be repeated here.

D.2.2 Derivation of the Poisson Failure Process

From the assumption, the probability of occurrence of 0 failure-inducing shocks prior to the time $t+\Delta t$ is given by:

$$g(0,t+\Delta t) = g(0,t)\,(1-\lambda\Delta t). \qquad \text{(D-11)}$$

The probability of x failure-inducing shocks prior to the time $t+\Delta t$ is given by:

$$g(x,t+\Delta t) = g(x,t)\,(1-\lambda\Delta t) + g(x-1,t)\,(\lambda\Delta t),\ x>0. \qquad \text{(D-12)}$$

From equations (D-11) and (D-12), the following differential equations arise:

$$g'(0,t) = -\lambda g(0,t)$$

$$g'(x,t) = \lambda[g(x-1,t) - g(x,t)], x > 0$$

with $g(0,0) = 1$, and $g(x,0) = 0$ for $x > 0$.

The solution to this set of linear first-order differential equations is:

$$g(x,t) = (\lambda t)^x e^{-\lambda t}/x!, x = 0, 1, 2, \ldots$$

D.2.3 The Exponential Reliability Law

By definition, $R(t) = g(0,t) = e^{-\lambda t}$. The corresponding PDF is

$$f(t) = -\frac{d}{dt}(e^{-\lambda t}) = \lambda e^{-\lambda t}.$$

D.3 INTRODUCTION TO BIRTH AND DEATH PROCESSES

The reliability or availability of many systems, particularly redundant and repairable systems, may be derived by considering Birth and Death processes as expounded initially in [4] and comprehensively developed for RMA application in [5]. A single element is considered first, then, more complex structures are treated.

D.3.1 Reliability of Non-Repairable Element

Figure D-1 shows an example of a death only process as applied to a single non-repairable element. The element can only be in two states, up (U) or down (D). The hazard rate λ is assumed to be constant and is shown directed toward the *absorbing state* D. D is termed an absorbing state because once the element has reached this state, it cannot leave it and remains there with probability 1.0. A graph such as is shown in figure D-1, called a Markov graph, depicts the states and transition probabilities between them.

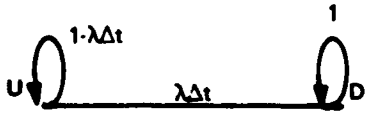

Figure D-1. Markov Graph for a Single Non-Repairable Element

A state transition matrix can be formed from figure D-1. It is shown in figure D-2.

States at time t \ States at time t+Δt	U	D
U	1-λΔt	λΔt
D	0	1

Figure D-2. State Transition Matrix for One Element

In the state transition matrix, the entry λΔt, for instance, represents the probability of going from state U at time t to state D at time t+Δt.

A set of difference equations are readily written for the graph of figure D-1, or for the state transition matrix of figure D-2.

$$P_U(t + \Delta t) = (1-\lambda\Delta t)\, P_U(t) \qquad \text{(D-13)}$$

$$P_D(t + \Delta t) = P_D(t) + \lambda\Delta t\, P_U(t) \qquad \text{(D-14)}$$

In the equations above, P_U and P_D stand for the probability of states U and D, respectively.

Under the assumption that the unit is up at time 0,

$$P_U(0) = 1, P_D(0) = 0.$$

Rewriting (D-13) and (D-14) as:

$$\frac{P_U(t+\Delta t) - P_U(t)}{\Delta t} + \lambda P_U(t) = 0 \qquad \text{(D-15)}$$

$$\frac{P_D(t+\Delta t) - P_D(t)}{\Delta t} = \lambda P_U(t) \qquad \text{(D-16)}$$

Letting $\Delta t \to 0$, the following differential equations are obtained:

$$P'_U(t) + \lambda P_U(t) = 0 \quad , P_U(0) = 1 \qquad \text{(D-17)}$$

$$P'_D(t) = \lambda P_U(t) \quad , P_D(0) = 0. \qquad \text{(D-18)}$$

Solving for $P_U(t)$ in (D-17):

$$P'_U(t)/P_U(t) = -\lambda,\ \frac{d}{dt}\ln(P_U(t)) = -\lambda$$

$$\ln(P_U(t)) = \lambda t + C_1$$

$P_U(t) = Ce^{-\lambda t}$, and the condition $P_U(0) = 1 = C$

leads to $P_U(t) = e^{-\lambda t}$.

Similarly, from equation (D-18):

$$P'_D(t) = \lambda P_U(t) = \lambda e^{-\lambda t}$$

$$P_D(t) = \int \lambda e^{-\lambda t}\,dt = -e^{-\lambda t} + C$$

and since

$$P_D(0) = 0,\ 0 = -e^{-\lambda(0)} + C,\ C = 1$$

then:

$$P_D(t) = 1 - e^{-\lambda t}.$$

D.3.2 Availability of a Repairable Element

Figure D-3 shows a birth and death process as applied to a repairable element. Again, the element can only exist in two states, up (U) or down (D). The hazard rate λ and repair rate $\mu = (\text{MTTR})^{-1}$ are assumed to be constant. State D is no longer absorbing but is a reflecting barrier, because it is possible for the element to go back to state U by repair.

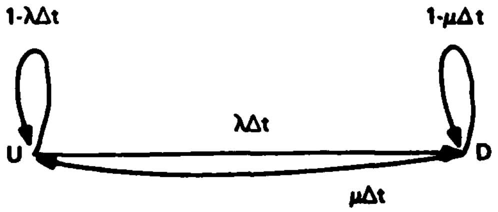

Figure D-3. Markov Graph for a Repairable Single Element

The corresponding State Transition matrix is depicted in figure D-4.

States at time t \ States at time t+Δt	U	D
U	1-λΔt	λΔt
D	μΔt	1-μΔt

Figure D-4. State Transition Matrix for a Repairable Element

The difference equations for the states depicted in figure D-3 and D-4 are:

$$P_U(t+\Delta t) = (1-\lambda\Delta t)P_U(t) + \mu\Delta t\,P_D(t) \quad \text{(D-19)}$$

$$P_D(t+\Delta t) = \lambda\Delta t\,P_U(t) + (1-\mu\Delta t)\,P_D(t) \quad \text{(D-20)}$$

with

$P_U(0) = 1$, $P_D(0) = 0$ as in § D.3.1.

Equations (D-19) and (D-20) lead to:

$$\frac{P_U(t+\Delta t) - P_U(t)}{\Delta t} = -\lambda P_U(t) + \mu\,P_D(t)$$

$$\frac{P_D(t+\Delta t) - P_D(t)}{\Delta t} = \lambda P_U(t) - \mu\,P_D(t)$$

and, letting Δt→0, we have the differential equations:

$$P'_U(t) + \lambda P_U(t) = \mu P_D(t) \quad \text{(D-21)}$$

$$P'_D(t) + \mu P_D(t) = \lambda P_U(t) \quad \text{(D-22)}$$

$$P_U(0) = 1,\quad P_D(0) = 0.$$

Equations (D-21) and (D-22) can be solved a number of different ways, but, choosing the Laplace Transform method, gives

$$s\,L(P_U) - 1 + \lambda\,L(P_U) = \mu\,L(P_D)$$

$$s\,L(P_D) + \mu\,L(P_D) = \lambda\,L(P_U).$$

Solving for $L(P_U)$:

$$L(P_U) = \frac{s+\mu}{s^2 + s(\mu+\lambda)} = \frac{\mu}{(\mu+\lambda)s} + \frac{\lambda}{(\mu+\lambda)(s+\mu+\lambda)}$$

then

$$A(t) = P_U = L^{-1}(L(P_U)) = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda} e^{-(\mu+\lambda)t} .$$

This expression for A(t) is identical with an expression obtained earlier by a different method.

D.3.3 Mean Time Between Failure for a Repairable Redundant System

D.3.3.1 Unrestricted Repair

Consider a parallel active redundant system consisting of six operating elements, four of which must be "up" for the structure to be operational (see figure D-5). The exponential law applies to failures and repairs, that is the failure rate λ and repair rate μ are constant. Repairs are unrestricted, in the sense that there are as many repairmen as needed to repair failed elements. Each repairman works on a single element and repairs it with rate μ.

Figure D-6 shows a Markov Graph for the structure.

Four states are shown in figure D-6; states 6, 5, 4, and 0. State Six is the state: "Six elements are up," state 5 is the state "Five elements are up," state 4 is the state "Four elements are up," and state 0 is the state "Fewer than four elements are up." States 6, 5, and 4 are system success states, state 0 is the system failure state. The 2μ shown between states 4 and 5 reflects the fact that 2 repairmen are available to work on the 2 failed elements.

If only states 4 and 0 existed, then one would be able to write the following difference equation:

$$R_4(t+\Delta t) = R_4(t) - 4\lambda R_4(t)\Delta t.$$

Letting $\Delta t \to 0$, this leads to

$$R_4(t) + 4\lambda R_4(t) = 0,\ R_4(0) = 1$$

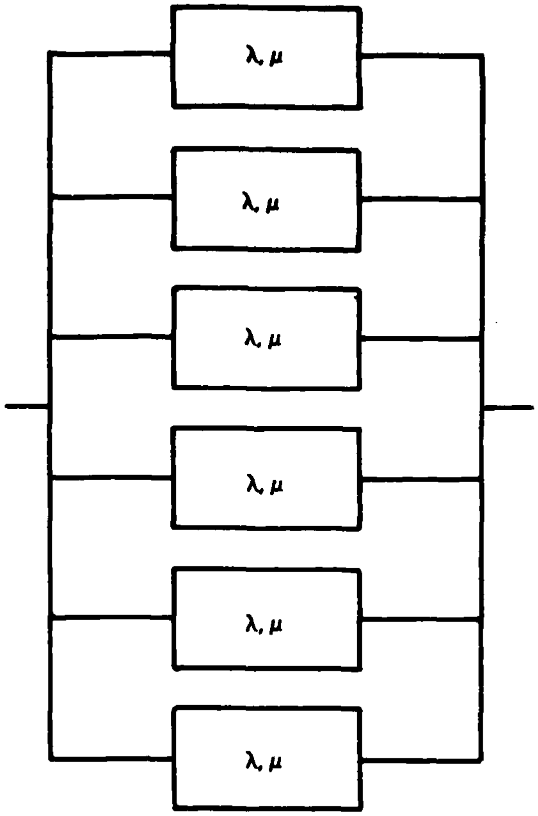

Figure D-5. 4 of 6 Parallel Redundant System

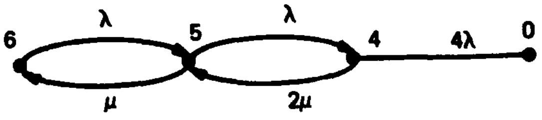

Figure D-6. Markov Graph for 4 of 6 Repairable System with Unrestricted Repair

with the solution $R_4(t) = e^{-4\lambda t}$,

and $\text{MTTF} = \int_0^\infty e^{-4\lambda t}dt = \frac{1}{4\lambda}$.

Assume, however, that R_4 is in a steady-state. Then R_4 is a constant and the failure rate becomes

$$4\lambda R_4 \text{ with MTTF (instantaneous)} = \frac{1}{4\lambda R_4}. \qquad \text{(D-23)}$$

R_4 is actually unknown but can be calculated under steady-state conditions, as follows:

At state 6, $R_6\ (6\lambda) = R_5\ (\mu)$. (D-24)

At state 5, $R_5\ (5\lambda) = R_4\ (2\mu)$. (D-25)

Also $R_0 = 0$ (since this is the failed state)

$$R_4 + R_5 + R_6 = 1.$$

$$R_4 + \frac{2\mu}{5\lambda} R_4 + \frac{2\mu^2}{30\lambda^2} R_4 = 1.$$

$$\frac{1}{R_4} = 1 + \frac{2\mu}{5\lambda} + \frac{\mu^2}{15\lambda^2}\ .$$

$$\text{MTBF} = \frac{1}{4\lambda R_4} = \frac{1}{\lambda}\left[\frac{2\mu^2 + 12\mu\lambda + 30\lambda^2}{120\lambda^2}\right].$$

D.3.3.2 Restricted Repair

Under restricted repair, only a single failed element can be worked on at a time. Figure D-7 shows a Markov Graph for the system.

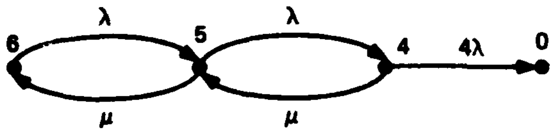

Figure D-7. Markov Graph for 4 of 6 Repairable System with Restricted Repair

To calculate the MTBF,
Equations (D-24) and D-25) are rewritten as:

$$R_6\ (6\lambda) = R_5\ (\mu).$$

$$R_5\ (5\lambda) = R_4\ (\mu).$$

Then: $$R_4 + \frac{\mu}{5\lambda} R_4 + \frac{\mu^2}{30\lambda^2} R_4 = 1.$$

$$\text{MTBF} = \frac{1}{4\lambda R_4} = \frac{1}{\lambda}\left[\frac{\mu^2 + 6\mu\lambda + 30\lambda^2}{120\lambda^2}\right].$$

D.3.3.3 4 of 6 Repairable System—Transient Solution

Thus far the treatment of the 4 of 6 repairable system has assumed that steady state conditions had been attained. This means, in particular, that the MTBF calculated was the MTBF of the structure after the 1st failure and after subsequent failures (when only 4 elements need be up at the start of operation), not the mean time to first failure, or MTTF (when 6 elements are up at the start of operation).

Although it is still possible to derive the MTTF starting with 6 elements up, by a method similar to the one already shown, a full Markov formulation is presented below for illustration. We will consider, however, only the restricted repair case.

From the transition matrix of Figure D-8, the following difference equations can be written.

States at time t \ States at time t+Δt	0	3	4	5	6
0	1	0	0	0	0
3	3λΔt	1 (3λ+μ)Δt	μΔt	0	0
4	0	4λΔt	1 (4λ+μ)Δt	μΔt	0
5	0	0	5λΔt	1 (5λ+μ)Δt	μΔt
6	0	0	0	6λΔt	1-6λΔt

Figure D-8. Markov Graph for 4 of 6 Repairable System with Restricted Repair

$$P_6(t+\Delta t) = (1-6\lambda\Delta t)P_6(t) + \mu\Delta t P_5(t)$$

$$P_5(t+\Delta t) = (1-(5\lambda+\mu)\Delta t)P_5(t) + (6\lambda\Delta t)P_6(t) + \mu\Delta t P_4(t)$$

$$P_4(t+\Delta t) = (1-(4\lambda+\mu)\Delta t)P_4(t) + (5\lambda\Delta t)P_5(t) + \mu\Delta t P_3(t)$$

$$P_3(t+\Delta t) = (1-(3\lambda+\mu)\Delta t)P_3(t) + (4\lambda\Delta t)P_4(t)$$

$$P_0(t+\Delta t) = P_0(t) + 3\lambda\Delta t P_3(t)$$

The initial conditions are: $P_6(0) = 1$, $P_5(0) = P_4(0) = P_3(0) = P_0(0) = 0$. The difference equations lead to the following differential equations:

$$P_6' + 6\lambda P_6 = \mu P_5 \qquad P_6(0) = 1 \qquad \text{(D-26)}$$

$$P_5' + (5\lambda+\mu)P_5 = 6\lambda P_6 + \mu P_4 \qquad P_5(0) = 0 \qquad \text{(D-27)}$$

$$P_4' + (4\lambda+\mu)P_4 = 5\lambda P_5 + \mu P_3 \qquad P_4(0) = 0 \qquad \text{(D-28)}$$

$$P_3' + (3\lambda+\mu)P_3 = 4\lambda P_4 \qquad P_3(0) = 0 \qquad \text{(D-29)}$$

$$P_0' = 3\lambda P_3 \qquad P_0(0) = 0. \qquad \text{(D-30)}$$

Using Laplace transforms to solve (D-26) - (D-30), we have:

$$sL(P_6) + 1 + 6\lambda L(P_6) = \mu L(P_5) \qquad \text{(D-31)}$$

$$sL(P_5) + (5\lambda+\mu)L(P_5) + 6\lambda L(P_6) + \mu L(P_4) \qquad \text{(D-32)}$$

$$sL(P_4) + (4\lambda+\mu)L(P_4) = 5\lambda L(P_5) + \mu L(P_3) \qquad \text{(D-33)}$$

$$sL(P_3) + (3\lambda+\mu)L(P_3) = 4\lambda L(P_4) \qquad \text{(D-34)}$$

$$sL(P_0) = 3\lambda L(P_3) \qquad \text{(D-35)}$$

It is now a simple matter to solve algebraically for $L(P_6)$, $L(P_5)$, $L(P_4)$, $L(P_3)$, and $L(P_0)$, and to find the inverse Laplace transforms P_6, P_5, P_4, P_3, and P_0.

The reliability of the structure at time t is then given by:

$$R(t) = P_6(t) + P_5(t) + P_4(t)$$

while the mean time to first failure or MTTF is given by

$$\int_0^\infty R(t)\,dt$$

D.3.4 Reliability of a 1 of 3 Standby System with a Dormant Hazard Rate and No Repair

In standby redundant systems, standby units may have a positive hazard rate λ_D before they replace a failed operating element, and an operating hazard rate $\lambda > \lambda_D$ after they replace a failed element. Such standby elements are said to display a dormant hazard rate.

Consider a 1 of 3 standby parallel system with perfect switching, such as depicted in figure D-9.

Defining state 0 as "all three units down," state 1 as "the operating unit up," state 2 as "the operating unit and a standby up," and state 3 as "the operating unit and two standby units up," the state transition matrix is as depicted in figure D-10.

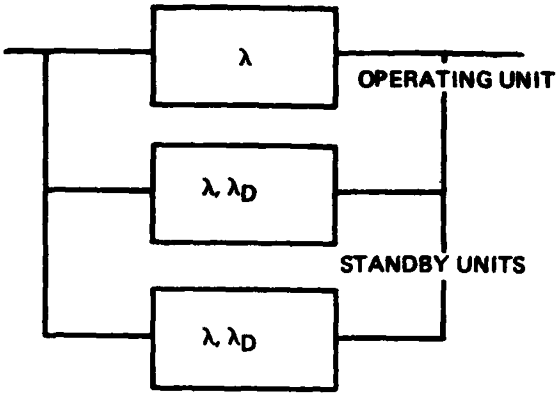

Figure D-9. 1 of 3 Structure: One Operating and Two Standby Elements

time t \ time t+Δt	0	1	2	3
0	1	0	0	0
1	$\lambda\Delta t$	$1-\lambda\Delta t$	0	0
2	0	$(\lambda+\lambda_D)\Delta t$	$1-(\lambda+\lambda_D)\Delta t$	0
3	0	0	$(\lambda+2\lambda_D)\ t$	$1-(\lambda+2\lambda_D)\Delta t$

Figure D-10. State Transition Matrix for 1 of 3 System with Dormant Hazard Rate

The differential equations formed from the state transition matrix are:

$$P_3'(t) = -(\lambda+2\lambda_D)P_3(t) \qquad P_3(0) = 1 \qquad \text{(D-36)}$$

$$P_2'(t) = -(\lambda+\lambda_D)P_2(t) + (\lambda+2\lambda_D)P_3(t) \qquad P_2(0) = 0 \qquad \text{(D-37)}$$

$$P_1'(t) = -\lambda P_1(t) + (\lambda+\lambda_D)P_2(t) \qquad P_1(0) = 0 \qquad \text{(D-38)}$$

$$P_0'(t) = \lambda P_1(t) \qquad P_0(0) = 0 \qquad \text{(D-39)}$$

Solutions of differential equations (D-36 - D-39) are:

$$P_3(t) = e^{-(\lambda+2\lambda_D)t}$$

$$P_2(t) = \frac{\lambda+2\lambda_D}{\lambda_D}\left\{e^{-(\lambda+\lambda_D)t} - e^{-(\lambda+2\lambda_D)t}\right\}$$

$$P_1(t) = \frac{(\lambda+\lambda_D)(\lambda+2\lambda_D)}{\lambda_D}\left\{\frac{e^{-(\lambda+2\lambda_D)t} - e^{-\lambda t}}{2\lambda_D} - \frac{e^{-(\lambda+\lambda_D)t} - e^{-\lambda t}}{\lambda_D}\right\}$$

$$P_0(t) = 1 - P_3(t) - P_2(t) - P_1(t).$$

The reliability of the system, based on the occurrence of either one of the states 1, 2 or 3 is:

$$R(t) = P_3(t) + P_2(t) + P_1(t).$$

D.3.5 MTBF of R of N Identical Repairable Elements in Parallel with Restricted Repair – The Einhorn Equations

The birth and death equations are a natural tool to obtain the MTBF of R out of N identical repairable elements in parallel. A special case (4 of 6 restricted repair) has already been calculated. We develop here the restricted repair solution for the general R of N case.

A Markov graph for this situation is shown in figure D-11.

Under steady state conditions the reliability R_R of state R (R elements up) is constant. The MTBF of state R is

$$\frac{1}{R\lambda R_R}.$$

Since under steady state conditions all reliability rates are zero,

$$\text{at state } N,\ R_N(N\lambda) = R_{N-1}(\mu) \qquad \text{(D-40)}$$

$$\text{at state } N-1,\ R_{N-1}((N-1)\lambda) = R_{N-2}(\mu) \qquad \text{(D-41)}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$R+1,\ R_{R+1}((R+1)\lambda) = R_R(\mu). \qquad \text{(D-42)}$$

Solving these equations recursively and letting $\theta = 1/\lambda$, $M = 1/\mu$ be the mean time to failure and mean time to repair, respectively, of a single element, gives:

$$\text{MTBF} = \left\{\sum_{j=R}^{N}\binom{N}{j}\theta^j M^{N-j}\right\} \Big/ \left\{N\binom{N-1}{R-1}\theta^{R-1}M^{N-R}\right\} \qquad \text{(D-43)}$$

$$\text{MTTR} = \left\{\sum_{j=0}^{R-1}\binom{N}{j}\theta^j M^{N-j}\right\} \Big/ \left\{N\binom{N-1}{R-1}\theta^{R-1}M^{N-R}\right\} \qquad \text{(D-44)}$$

If M is small compared to θ, equations (D-41) and (D-42) can be simplified to:

$$\text{MTBF} = \binom{N}{R-1}\theta^{R-1}M^{N-R+1}/N \qquad \text{(D-45)}$$

$$\text{MTTR} = M/(N-R+1) \qquad \text{(D-46)}$$

Equations (D-45) and (D-46) are the Einhorn [6] approximation which were derived originally by their author from considerations of quorum probabilities.

D.4 REFERENCES

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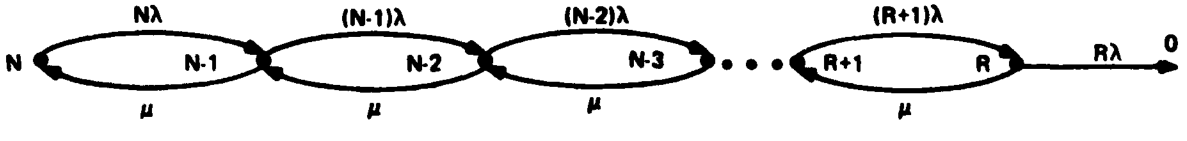

Figure D-11. Markov Graph for R of N Repairable Elements in Parallel, Restricted Repair

Appendix E
STATISTICAL TABLES

This Appendix contains tables useful in reliability and availability evaluation and references to more extensive tables. It is organized:

- E.1 TABLES USEFUL FOR RELIABILITY CALCULATIONS
  - E.1.1 Chi-Square Tables (80% Confidence Level)
  - E.1.2 Reliability Lower Bound Tables (80% Confidence Level)
  - E.1.3 Binomial Tables (80% Confidence Level)
- E.2 TOLERANCE FACTORS FOR THE NORMAL DISTRIBUTION
- E.3 TABLES USEFUL FOR AVAILABILITY CALCULATIONS
  - E.3.1 The F Distribution (80% Confidence Level)
  - E.3.2 The a Distribution (80% Confidence Level)
- E.4 MTBF TABLES FOR m OUT of n WITH AND WITHOUT REPAIR
- E.5 REFERENCES

E.1 TABLES USEFUL FOR RELIABILITY CALCULATIONS

This paragraph provides tables useful in calculating 80% lower bounds on reliability when the underlying process is Poisson, Exponential, or Binomial.

E.1.1 Chi-Square Tables

Figure E-1 presents chi-square tables at the 80% confidence level. These tables may be used to obtain the failure rate or MTBF 80% confidence bound for exponential time to failure data.

Since figure E-1 provides chi-square values at the 80% confidence level, 80% upper bounds on failure rate or 80% lower bounds on MTBF are obtained:

$$\begin{aligned} \lambda_{.80} &= \frac{\chi^2_{.80 f}}{2T} \\ \theta_{.80} &= \frac{2T}{\chi^2_{.80:f}} \end{aligned} \qquad \text{(E-1)}$$

Where f, the degrees of freedom is 2x+2 when the test is truncated by time (Type I life censoring tests § 5.1.1.b and f = 2x when the test is terminated at a predetermined number of failures (Type II life censoring test § 5.1.1.c).

Example

In a Type I test 5 failures are observed in 500 hours of test, find the upper bound on failure rate (80% confidence level)

calculate $f = 2x + 2 = 2(5) + 2$ or $\underline{\underline{12}}$

find $\chi^2_{.80}$ value in figure E-1 = $\underline{\underline{15.812}}$

Solve equation E-1

$$\lambda_{.80} = \frac{\chi^2_{.80:12}}{2T} = \frac{15.812}{2(500)}$$

$$= \underline{\underline{.015812}} \text{ failures/hour}$$

$$\theta_{.80} = \frac{2T}{\chi^2_{.80:12}} = \frac{2(500)}{15.182}$$

$$= \underline{\underline{63.24}} \text{ hours}$$

The reliability lower bound for a one hour mission would be calculated as

f	CHI-SQU	f	CHI-SQU	f	CHI-SQU	f	CHI-SQU
2	3.219	102	113.786	202	218.693	302	322.466
4	5.989	104	115.903	204	220.777	304	324.534
6	8.558	106	118.020	206	222.860	306	326.602
8	11.030	108	120.135	208	224.943	308	328.670
10	13.442	110	122.250	210	227.025	310	330.738
12	15.812	112	124.363	212	229.107	312	332.806
14	18.151	114	126.475	214	231.189	314	334.873
16	20.465	116	128.586	216	233.270	316	336.940
18	22.760	118	130.697	218	235.351	318	339.007
20	25.038	120	132.806	220	237.432	320	341.074
22	27.301	122	134.915	222	239.512	322	343.140
24	29.553	124	137.022	224	241.592	324	345.207
26	31.795	126	139.129	226	243.671	326	347.273
28	34.027	128	141.235	228	245.750	328	349.339
30	36.250	130	143.340	230	247.829	330	351.404
32	38.466	132	145.444	232	249.907	332	353.470
34	40.676	134	147.548	234	251.986	334	355.535
36	42.879	136	149.651	236	254.063	336	357.600
38	45.076	138	151.753	238	256.141	338	359.665
40	47.269	140	153.854	240	258.218	340	361.730
42	49.456	142	155.954	242	260.295	342	363.794
44	51.639	144	158.054	244	262.371	344	365.859
46	53.818	146	160.153	246	264.447	346	367.923
48	55.993	148	162.251	248	266.523	348	369.987
50	58.164	150	164.349	250	268.599	350	372.051
52	60.332	152	166.446	252	270.674	352	374.114
54	62.496	154	168.543	254	272.749	354	376.178
56	64.658	156	170.639	256	274.823	356	378.241
58	66.816	158	172.734	258	276.898	358	380.304
60	68.972	160	174.828	260	278.972	360	382.367
62	71.125	162	176.922	262	281.046	362	384.429
64	73.276	164	179.016	264	283.119	364	386.492
66	75.424	166	181.109	266	285.192	366	388.554
68	77.571	168	183.201	268	287.265	368	390.617
70	79.715	170	185.293	270	289.338	370	392.679
72	81.857	172	187.384	272	291.410	372	394.740
74	83.997	174	189.474	274	293.482	374	396.802
76	86.135	176	191.565	276	295.554	376	398.864
78	88.271	178	193.654	278	297.626	378	400.925
80	90.405	180	195.743	280	299.697	380	402.986
82	92.538	182	197.832	282	301.768	382	405.047
84	94.669	184	199.920	284	303.839	384	407.108
86	96.799	186	202.008	286	305.910	386	409.169
88	98.927	188	204.095	288	307.980	388	411.229
90	101.054	190	206.182	290	310.050	390	413.290
92	103.179	192	208.268	292	312.120	392	415.350
94	105.303	194	210.354	294	314.190	394	417.410
96	107.425	196	212.439	296	316.259	396	419.470
98	109.547	198	214.524	298	318.328	398	421.530
100	111.667	200	216.609	300	320.397	400	423.589

Figure E-1. Chi-Square Distribution at 80% Confidence Level, Degress of Freedom from 2 to 400

$$R_{.80} = e^{-\lambda_{.80} t}$$

$$= e^{-(.015812)(1)}$$

$$= .9843$$

More extensive chi-square tables are available from many sources, e.g., [1].

E.1.2 Reliability Lower Bound Tables

The reliability tables shown in figure E-2 solve the problem illustrated in § E.1.1 directly.

Figure E-2 is entered at 5 failures and a test time of 500 hours. The 80% lower bound on reliability is read directly as:

$$R_{.80} = .9843$$

which agrees with the result obtained in § E.1.1.

A complete discussion of the uses and derivation of the reliability tables and a far more extensive set of tables are available, e.g., [2].

E.1.3 Binomial Tables

Figure E-3 presents binomial tables at the 80% confidence level. The 80% lower bound on reliability is read directly from these tables at the intersection of the sample size (N) and the number of failures (x). For example, if 2 failures are obtained in 40 trials the estimate is $\hat{R} = 38/40 = 0.9500$ and the 80% lower bound $R_{.80} = 0.8960$ (from figure E-3).

Some extensions of the tables are discussed:

(a) For $N > 40$ and $N < 120$ linear interpolation of the table in figure E-3 for odd N will yield a maximum error of 0.0001

(b) For $X = 0$

$$R_{0\,.80} = (0.20)^{1/N} \qquad \text{(E-2)}$$

(c) For $N > 120$ and $X \neq 0$

$$R_{.80} = \frac{S}{S + (X+1)\,e^{2w}} + \epsilon \qquad \text{(E-3)}$$

where:

N = Number of trials
X = Number of failures
S = N-X

$$H = \frac{Z}{\frac{1}{2S-1} + \frac{1}{2X+1}}$$

$$w = \frac{.8416\sqrt{H-.3820}}{H} - \left(\frac{1}{2X+1} - \frac{1}{2S-1}\right)\left(.4514 - \frac{2}{3H}\right)$$

$$|\epsilon| < .0008$$

For example, if equation E-3 is used for N = 120 and X = 5 then

$$R_{.80} = \frac{115}{115 + (5+1)\,e^{2(0.14569)}}$$

$$R_{.80} = .9347 \text{ (figure E-3 gives .9349)}$$

More extensive binomial tables are available, e.g., [1].

E.2 TOLERANCE FACTORS FOR THE NORMAL DISTRIBUTION

Figures E-4 through E-7 present one-sided and two-sided tolerance factors for the Normal Distribution at 50% and 80% confidence levels.

As an example of the use of these tables, the method of § 5.4.3 will be used.

Assume, as a parallel to the example connected with expression (5-41), that $n=5$, $\bar{x}=8$, $s=0.484$, LCLS=6, UCLS=10, then K=4.09 and the two-sided table of figure E-7 yields a reliability of .975 for K=3.8403, and of .99 for K=4.4133. Linear interpolation yields R=0.982 for K=4.09. More extensive tables are available, e.g., [3].

E.3 TABLES USEFUL FOR AVAILABILITY CALCULATIONS

E.3.1 The F Distribution

Figure E-8 presents the F distribution at the 80% confidence level.

80% CONFIDENCE

80% CONFIDENCE

T*	FAILURES											T*	FAILURES										T*
	0	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	16	17	18	19	20	
1	.2000	.0501	.0139	.0040	.0012	.0004	.0001	.0000	.0000	.0000	.0000	1	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	1
2	.4472	.2237	.1177	.0635	.0347	.0192	.0107	.0060	.0034	.0019	.0011	2	.0006	.0004	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	2
3	.5848	.3686	.2402	.1591	.1064	.0717	.0485	.0330	.0225	.0154	.0106	3	.0073	.0050	.0034	.0024	.0016	.0011	.0008	.0005	.0004	.0003	3
4	.6687	.4730	.3431	.2519	.1863	.1386	.1034	.0774	.0581	.0437	.0330	4	.0249	.0188	.0142	.0108	.0082	.0062	.0047	.0036	.0027	.0021	4
5	.7248	.5494	.4249	.3319	.2607	.2057	.1628	.1292	.1027	.0818	.0652	5	.0521	.0416	.0333	.0266	.0214	.0171	.0137	.0110	.0089	.0071	5
6	.7647	.6071	.4901	.3989	.3262	.2678	.2203	.1817	.1501	.1241	.1028	6	.0852	.0707	.0587	.0488	.0405	.0337	.0281	.0234	.0195	.0162	6
7	.7946	.6520	.5427	.4548	.3828	.3232	.2735	.2318	.1968	.1672	.1423	7	.1211	.1032	.0880	.0751	.0641	.0547	.0468	.0400	.0342	.0292	7
8	.8178	.6878	.5857	.5019	.4317	.3722	.3216	.2783	.2411	.2091	.1815	8	.1577	.1371	.1192	.1038	.0903	.0787	.0686	.0598	.0521	.0455	8
9	.8362	.7170	.6216	.5418	.4739	.4154	.3648	.3208	.2824	.2488	.2194	9	.1936	.1709	.1510	.1335	.1180	.1044	.0924	.0817	.0724	.0641	9
10	.8511	.7417	.6519	.5761	.5106	.4536	.4035	.3594	.3205	.2860	.2554	10	.2282	.2040	.1824	.1632	.1461	.1308	.1172	.1050	.0941	.0843	10
11	.8639	.7617	.6777	.6057	.5428	.4874	.4382	.3945	.3554	.3204	.2891	11	.2610	.2357	.2130	.1925	.1740	.1574	.1424	.1289	.1166	.1056	11
12	.8745	.7792	.7001	.6315	.5712	.5175	.4694	.4263	.3874	.3523	.3206	12	.2919	.2659	.2422	.2208	.2013	.1836	.1675	.1529	.1395	.1274	12
13	.8835	.7943	.7195	.6543	.5963	.5444	.4975	.4552	.4167	.3817	.3499	13	.3209	.2944	.2702	.2480	.2278	.2092	.1922	.1766	.1623	.1492	13
14	.8914	.8074	.7366	.6744	.6187	.5685	.5230	.4815	.4436	.4089	.3772	14	.3480	.3212	.2966	.2740	.2531	.2339	.2162	.1999	.1849	.1710	14
15	.8983	.8190	.7518	.6923	.6389	.5903	.5461	.5055	.4683	.4340	.4025	15	.3734	.3465	.3217	.2987	.2774	.2577	.2395	.2226	.2069	.1923	15
16	.9043	.8293	.7653	.7084	.6570	.6101	.5671	.5275	.4910	.4573	.4261	16	.3971	.3702	.3453	.3221	.3006	.2805	.2619	.2445	.2283	.2132	16
17	.9097	.8385	.7775	.7230	.6734	.6281	.5863	.5478	.5120	.4788	.4480	17	.4193	.3925	.3676	.3443	.3226	.3023	.2833	.2656	.2490	.2335	17
18	.9145	.8467	.7884	.7361	.6884	.6445	.6040	.5664	.5314	.4988	.4684	18	.4400	.4135	.3886	.3653	.3435	.3231	.3039	.2859	.2690	.2531	18
19	.9188	.8542	.7983	.7481	.7021	.6596	.6202	.5836	.5494	.5174	.4875	19	.4595	.4331	.4084	.3852	.3634	.3429	.3236	.3054	.2883	.2721	19
20	.9227	.8609	.8074	.7590	.7146	.6735	.6352	.5995	.5661	.5348	.5053	20	.4777	.4516	.4271	.4040	.3823	.3617	.3423	.3240	.3067	.2904	20
21	.9262	.8671	.8157	.7690	.7261	.6863	.6491	.6143	.5816	.5509	.5220	21	.4948	.4691	.4448	.4219	.4002	.3797	.3603	.3419	.3245	.3080	21
22	.9295	.8727	.8232	.7783	.7368	.6981	.6620	.6281	.5961	.5661	.5377	22	.5109	.4855	.4615	.4387	.4172	.3967	.3774	.3590	.3415	.3250	22
23	.9324	.8779	.8302	.7868	.7466	.7091	.6740	.6409	.6097	.5802	.5524	23	.5260	.5010	.4772	.4547	.4333	.4130	.3937	.3753	.3579	.3413	23
24	.9351	.8827	.8367	.7947	.7558	.7193	.6851	.6529	.6224	.5936	.5662	24	.5403	.5156	.4922	.4699	.4487	.4285	.4093	.3910	.3735	.3569	24
25	.9376	.8871	.8427	.8020	.7643	.7289	.6956	.6641	.6343	.6061	.5793	25	.5537	.5295	.5063	.4843	.4633	.4433	.4242	.4060	.3885	.3719	25
26	.9400	.8912	.8483	.8089	.7722	.7378	.7054	.6747	.6455	.6179	.5915	26	.5665	.5426	.5198	.4980	.4772	.4574	.4384	.4203	.4029	.3863	26
27	.9421	.8950	.8534	.8153	.7796	.7462	.7145	.6846	.6561	.6290	.6032	27	.5785	.5550	.5325	.5110	.4905	.4708	.4520	.4340	.4167	.4002	27
28	.9441	.8986	.8583	.8212	.7866	.7540	.7232	.6939	.6660	.6395	.6141	28	.5899	.5668	.5446	.5234	.5031	.4837	.4650	.4471	.4299	.4135	28
29	.9460	.9019	.8628	.8268	.7931	.7614	.7313	.7027	.6754	.6494	.6246	29	.6008	.5780	.5562	.5353	.5152	.4959	.4775	.4597	.4426	.4263	29
30	.9478	.9050	.8671	.8321	.7993	.7683	.7390	.7110	.6843	.6588	.6344	30	.6111	.5887	.5672	.5465	.5267	.5077	.4894	.4718	.4548	.4386	30
31	.9494	.9079	.8711	.8370	.8051	.7749	.7462	.7189	.6927	.6678	.6438	31	.6209	.5988	.5776	.5573	.5377	.5189	.5008	.4833	.4665	.4504	31
32	.9509	.9107	.8748	.8417	.8106	.7811	.7531	.7263	.7007	.6762	.6527	32	.6302	.6085	.5876	.5676	.5482	.5296	.5117	.4944	.4778	.4617	32
33	.9524	.9133	.8784	.8461	.8157	.7670	.7596	.7334	.7083	.6843	.6612	33	.6390	.6177	.5972	.5774	.5583	.5399	.5222	.5051	.4886	.4727	33
34	.9538	.9157	.8817	.8503	.8206	.7925	.7657	.7401	.7155	.6920	.6693	34	.6475	.6265	.6063	.5868	.5680	.5498	.5323	.5154	.4990	.4832	34
35	.9551	.9180	.8849	.8542	.8253	.7978	.7716	.7465	.7224	.6993	.6770	35	.6556	.6349	.6150	.5958	.5772	.5593	.5420	.5252	.5090	.4934	35
36	.9563	.9202	.8879	.8580	.8297	.8028	.7772	.7526	.7290	.7063	.6844	36	.6633	.6430	.6234	.6044	.5861	.5684	.5513	.5347	.5187	.5031	36
37	.9574	.9223	.8908	.8615	.8339	.8076	.7825	.7584	.7352	.7129	.6915	37	.6707	.6507	.6314	.6127	.5946	.5771	.5602	.5438	.5279	.5126	37
38	.9585	.9242	.8935	.8649	.8379	.8122	.7875	.7639	.7412	.7193	.6982	38	.6778	.6581	.6391	.6207	.6028	.5855	.5688	.5526	.5369	.5217	38
39	.9596	.9261	.8961	.8681	.8417	.8165	.7924	.7692	.7469	.7254	.7047	39	.6846	.6652	.6465	.6283	.6107	.5936	.5771	.5611	.5455	.5304	39
40	.9606	.9279	.8985	.8712	.8453	.8207	.7970	.7743	.7524	.7313	.7109	40	.6911	.6720	.6535	.6356	.6183	.6014	.5851	.5692	.5538	.5389	40
41	.9615	.9296	.9004	.8741	.8488	.8246	.8014	.7791	.7576	.7369	.7168	41	.6974	.6786	.6604	.6427	.6256	.6089	.5928	.5771	.5619	.5471	41
42	.9624	.9312	.9031	.8769	.8521	.8284	.8057	.7838	.7627	.7422	.7225	42	.7034	.6849	.6669	.6495	.6326	.6162	.6002	.5847	.5697	.5550	42
43	.9633	.9327	.9053	.8796	.8553	.8321	.8097	.7882	.7675	.7474	.7280	43	.7092	.6909	.6732	.6561	.6394	.6231	.6074	.5921	.5772	.5627	43
44	.9641	.9342	.9073	.8822	.8583	.8355	.8136	.7925	.7721	.7524	.7333	44	.7147	.6968	.6793	.6624	.6459	.6299	.6143	.5992	.5844	.5701	44
45	.9649	.9356	.9093	.8847	.8613	.8389	.8174	.7966	.7766	.7571	.7383	45	.7201	.7024	.6852	.6685	.6522	.6364	.6210	.6060	.5914	.5772	45
46	.9656	.9370	.9112	.8870	.8641	.8421	.8209	.8006	.7808	.7617	.7432	46	.7253	.7078	.6908	.6743	.6583	.6427	.6275	.6127	.5982	.5842	46
47	.9663	.9383	.9130	.8893	.8668	.8452	.8244	.8044	.7850	.7662	.7479	47	.7302	.7130	.6963	.6800	.6642	.6487	.6337	.6191	.6048	.5909	47
48	.9670	.9395	.9147	.8915	.8693	.8481	.8277	.8080	.7889	.7704	.7525	48	.7350	.7181	.7016	.6855	.6699	.6546	.6398	.6253	.6112	.5974	48
49	.9677	.9407	.9164	.8936	.8718	.8510	.8309	.8115	.7928	.7745	.7569	49	.7397	.7229	.7067	.6908	.6754	.6603	.6456	.6313	.6173	.6037	49
50	.9683	.9419	.9180	.8956	.8742	.8537	.8340	.8149	.7964	.7785	.7611	50	.7441	.7276	.7116	.6959	.6807	.6658	.6513	.6371	.6233	.6098	50

T* = NORMALIZED TEST TIME. **If Testing is truncated by failure, enter 1 failure less than those observed to obtain Reliability Lower Bound.

Figure E-2. Reliability Lower Bound for Exponential Components with Failures Truncated by Test Time**

80% CONFIDENCE

80% CONFIDENCE

T*	FAILURES											T*	FAILURES										T*
	0	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	16	17	18	19	20	
51	.9689	.9430	.9195	.8975	.8765	.8564	.8370	.8182	.8000	.7823	.7652	51	.7485	.7322	.7163	.7009	.6858	.6711	.6568	.6428	.6291	.6158	51
52	.9695	.9440	.9210	.8994	.8788	.8590	.8399	.8214	.8034	.7860	.7691	52	.7526	.7366	.7210	.7057	.6908	.6763	.6621	.6483	.6348	.6216	52
53	.9701	.9451	.9224	.9012	.8809	.8614	.8426	.8244	.8068	.7896	.7729	53	.7567	.7409	.7254	.7104	.6957	.6813	.6673	.6536	.6402	.6272	53
54	.9706	.9461	.9238	.9029	.8830	.8638	.8453	.8274	.8100	.7931	.7766	54	.7606	.7450	.7297	.7149	.7004	.6862	.6723	.6585	.6455	.6326	54
55	.9712	.9470	.9251	.9046	.8850	.8661	.8479	.8302	.8131	.7964	.7802	55	.7644	.7490	.7339	.7193	.7049	.6909	.6772	.6638	.6507	.6379	55
56	.9717	.9479	.9264	.9062	.8869	.8683	.8504	.8330	.8161	.7997	.7837	56	.7681	.7529	.7380	.7235	.7093	.6955	.6819	.6687	.6557	.6430	56
57	.9722	.9488	.9277	.9078	.8888	.8705	.8528	.8357	.8190	.8028	.7870	57	.7716	.7566	.7419	.7276	.7136	.6999	.6865	.6734	.6606	.6480	57
58	.9726	.9497	.9289	.9093	.8906	.8726	.8552	.8383	.8218	.8059	.7903	58	.7751	.7603	.7458	.7316	.7178	.7042	.6910	.6780	.6653	.6529	58
59	.9731	.9505	.9300	.9108	.8923	.8746	.8574	.8408	.8246	.8088	.7935	59	.7785	.7638	.7495	.7355	.7218	.7084	.6953	.6825	.6699	.6576	59
60	.9735	.9513	.9312	.9122	.8940	.8765	.8596	.8432	.8272	.8117	.7965	60	.7817	.7672	.7531	.7393	.7257	.7125	.6995	.6869	.6744	.6622	60
61	.9740	.9521	.9323	.9136	.8957	.8784	.8618	.8456	.8298	.8145	.7995	61	.7849	.7706	.7566	.7429	.7296	.7165	.7037	.6911	.6788	.6667	61
62	.9744	.9528	.9333	.9149	.8973	.8803	.8638	.8479	.8323	.8172	.8024	62	.7879	.7738	.7600	.7465	.7333	.7203	.7077	.6952	.6830	.6711	62
63	.9748	.9536	.9343	.9162	.8988	.8821	.8658	.8501	.8347	.8198	.8052	63	.7909	.7770	.7633	.7500	.7369	.7241	.7116	.6993	.6872	.6754	63
64	.9752	.9543	.9353	.9174	.9003	.8838	.8678	.8572	.8371	.8223	.8079	64	.7938	.7800	.7666	.7534	.7404	.7278	.7153	.7032	.6912	.6795	64
65	.9755	.9550	.9363	.9187	.9018	.8855	.8697	.8543	.8394	.8248	.8106	65	.7967	.7830	.7697	.7567	.7439	.7313	.7190	.7070	.6952	.6836	65
66	.9759	.9556	.9372	.9198	.9032	.8871	.8715	.8564	.8416	.8272	.8132	66	.7994	.7859	.7728	.7599	.7472	.7348	.7226	.7107	.6990	.6875	66
67	.9763	.9563	.9381	.9210	.9046	.8887	.8733	.8594	.8438	.8296	.8157	67	.8021	.7888	.7757	.7630	.7505	.7382	.7262	.7143	.7028	.6914	67
68	.9766	.9569	.9390	.9221	.9059	.8902	.8751	.8603	.8459	.8319	.8181	68	.8047	.7915	.7786	.7660	.7536	.7415	.7296	.7179	.7064	.6951	68
69	.9769	.9575	.9399	.9232	.9072	.8917	.8768	.8622	.8480	.8341	.8205	69	.8072	.7942	.7815	.7690	.7567	.7447	.7329	.7213	.7100	.6988	69
70	.9773	.9581	.9407	.9242	.9085	.8932	.8784	.8640	.8500	.8362	.8228	70	.8097	.7968	.7842	.7719	.7598	.7479	.7362	.7247	.7135	.7024	70
71	.9776	.9587	.9415	.9253	.9097	.8946	.8800	.8658	.8519	.8383	.8251	71	.8121	.7994	.7869	.7747	.7627	.7509	.7394	.7280	.7169	.7059	71
72	.9779	.9593	.9423	.9263	.9109	.8960	.8816	.8675	.8538	.8404	.8273	72	.8145	.8019	.7895	.7774	.7656	.7539	.7425	.7312	.7202	.7093	72
73	.9782	.9598	.9431	.9272	.9120	.8974	.8831	.8692	.8557	.8424	.8294	73	.8168	.8043	.7921	.7801	.7684	.7568	.7455	.7344	.7234	.7127	73
74	.9785	.9603	.9438	.9282	.9132	.8987	.8846	.8709	.8575	.8444	.8315	74	.8190	.8067	.7946	.7828	.7711	.7597	.7485	.7374	.7266	.7159	74
75	.9788	.9609	.9445	.9291	.9143	.9000	.8860	.8725	.8592	.8463	.8336	75	.8212	.8090	.7970	.7853	.7738	.7625	.7514	.7404	.7297	.7191	75
76	.9790	.9614	.9453	.9300	.9154	.9012	.8874	.8740	.8609	.8481	.8356	76	.8233	.8113	.7994	.7878	.7764	.7652	.7542	.7434	.7327	.7223	76
77	.9793	.9619	.9459	.9309	.9164	.9024	.8888	.8756	.8626	.8499	.8375	77	.8254	.8135	.8018	.7903	.7790	.7679	.7570	.7462	.7357	.7253	77
78	.9796	.9623	.9466	.9317	.9174	.9036	.8902	.8771	.8642	.8517	.8395	78	.8274	.8156	.8040	.7927	.7815	.7705	.7597	.7491	.7386	.7283	78
79	.9798	.9628	.9473	.9326	.9184	.9048	.8915	.8785	.8658	.8534	.8413	79	.8294	.8177	.8063	.7950	.7839	.7730	.7623	.7518	.7414	.7312	79
80	.9801	.9633	.9479	.9334	.9194	.9059	.8929	.8799	.8674	.8551	.8431	80	.8313	.8198	.8084	.7973	.7863	.7755	.7649	.7545	.7442	.7341	80
81	.9803	.9637	.9485	.9342	.9704	.9070	.8940	.8813	.8689	.8568	.8449	81	.8332	.8218	.8105	.7995	.7886	.7780	.7674	.7571	.7469	.7369	81
82	.9806	.9641	.9492	.9350	.9213	.9081	.8952	.8827	.8704	.8584	.8466	82	.8351	.8238	.8126	.8017	.7909	.7803	.7699	.7597	.7496	.7397	82
83	.9808	.9646	.9498	.9357	.9222	.9091	.8964	.8840	.8719	.8600	.8483	83	.8369	.8257	.8147	.8038	.7932	.7827	.7724	.7622	.7527	.7424	83
84	.9810	.9650	.9503	.9365	.9231	.9102	.8976	.8853	.8733	.8615	.8500	84	.8387	.8276	.8167	.8059	.7954	.7650	.7747	.7647	.7548	.7450	84
85	.9812	.9654	.9509	.9372	.9240	.9112	.8987	.8866	.8747	.8631	.8516	85	.8404	.8294	.8186	.8080	.7975	.7872	.7771	.7671	.7573	.7476	85
86	.9815	.9658	.9515	.9379	.9248	.9122	.8998	.8878	.8761	.8645	.8532	86	.8421	.8312	.8205	.8100	.7996	.7894	.7793	.7695	.7597	.7501	86
87	.9817	.9662	.9520	.9386	.9257	.9131	.9009	.8890	.8774	.8660	.8548	87	.8438	.8330	.8224	.8119	.8017	.7915	.7816	.7718	.7621	.7526	87
88	.9819	.9665	.9525	.9393	.9265	.9141	.9020	.8902	.8787	.8674	.8563	88	.8454	.8347	.8242	.8139	.8037	.7936	.7838	.7741	.7645	.7550	88
89	.9821	.9669	.9531	.9399	.9273	.9150	.9031	.8914	.8800	.8688	.8578	89	.8470	.8364	.8260	.8157	.8057	.7957	.7859	.7763	.7668	.7574	89
90	.9823	.9673	.9536	.9406	.9280	.9159	.9041	.8925	.8812	.8701	.8593	90	.8486	.8381	.8278	.8176	.8076	.7977	.7880	.7785	.7690	.7598	90
91	.9825	.9676	.9541	.9412	.9288	.9168	.9051	.8936	.8824	.8715	.8607	91	.8501	.8397	.8295	.8194	.8095	.7997	.7901	.7806	.7713	.7621	91
92	.9827	.9680	.9546	.9418	.9296	.9177	.9061	.8947	.8836	.8728	.8621	92	.8516	.8413	.8312	.8212	.8114	.8017	.7921	.7827	.7734	.7643	92
93	.9828	.9683	.9550	.9424	.9303	.9185	.9070	.8958	.8848	.8741	.8635	93	.8531	.8429	.8328	.8229	.8132	.8036	.7941	.7848	.7756	.7665	93
94	.9830	.9686	.9555	.9430	.9310	.9193	.9080	.8969	.8860	.8753	.8648	94	.8545	.8444	.8344	.8246	.8150	.8054	.7961	.7868	.7777	.7687	94
95	.9832	.9690	.9560	.9436	.9317	.9201	.9089	.8979	.8871	.8765	.8662	95	.8560	.8459	.8360	.8263	.8167	.8073	.7980	.7888	.7797	.7708	95
96	.9834	.9693	.9564	.9442	.9324	.9209	.9098	.8989	.8882	.8777	.8675	96	.8573	.8474	.8376	.8280	.8184	.8091	.7999	.7908	.7818	.7729	96
97	.9835	.9696	.9568	.9447	.9331	.9217	.9107	.8999	.8893	.8789	.8687	97	.8587	.8488	.8391	.8296	.8201	.8109	.8017	.7927	.7838	.7750	97
98	.9837	.9699	.9573	.9453	.9337	.9225	.9116	.9009	.8904	.8801	.8700	98	.8600	.8503	.8406	.8311	.8218	.8126	.8035	.7945	.7857	.7770	98
99	.9839	.9702	.9577	.9458	.9344	.9232	.9124	.9018	.8914	.8812	.8712	99	.8613	.8516	.8421	.8327	.8234	.8143	.8053	.7964	.7876	.7790	99
100	.9840	.9705	.9581	.9463	.9350	.9240	.9132	.9027	.8924	.8823	.8724	100	.8626	.8530	.8436	.8342	.8250	.8160	.8070	.7982	.7895	.7809	100

T* = NORMALIZED TEST TIME. **If Testing is truncated by failure, enter 1 failure less than those observed to obtain Reliability Lower Bound.

Figure E-2. Reliability Lower Bound for Exponential Components with Failures Truncated by Test Time** (Continued)

80% CONFIDENCE

80% CONFIDENCE

T*	FAILURES											T*	FAILURES										T*
	0	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	16	17	18	19	20	
101	.9842	.9708	.9585	.9469	.9356	.9247	.9141	.9037	.8934	.8834	.8736	101	.8639	.8544	.8450	.8357	.8266	.8176	.8087	.8000	.7914	.7828	101
102	.9843	.9711	.9589	.9474	.9362	.9254	.9149	.9045	.8944	.8845	.8747	102	.8651	.8557	.8464	.8372	.8282	.8192	.8104	.8017	.7932	.7847	102
103	.9845	.9713	.9593	.9479	.9368	.9261	.9157	.9054	.8954	.8856	.8759	103	.8664	.8570	.8477	.8386	.8297	.8208	.8121	.8035	.7950	.7866	103
104	.9846	.9716	.9597	.9484	.9374	.9268	.9164	.9063	.8964	.8866	.8770	104	.8676	.8582	.8491	.8401	.8312	.8224	.8137	.8052	.7967	.7884	104
105	.9848	.9719	.9601	.9488	.9380	.9275	.9172	.9071	.8973	.8876	.8781	105	.8687	.8595	.8504	.8415	.8326	.8239	.8153	.8068	.7984	.7902	105
106	.9849	.9721	.9604	.9493	.9386	.9281	.9179	.9080	.8982	.8886	.8792	106	.8699	.8607	.8517	.8428	.8341	.8254	.8169	.8085	.8001	.7919	106
107	.9851	.9724	.9608	.9498	.9391	.9288	.9187	.9088	.8991	.8896	.8802	107	.8710	.8619	.8530	.8442	.8355	.8269	.8184	.8101	.8018	.7937	107
108	.9852	.9727	.9612	.9502	.9397	.9294	.9194	.9096	.9000	.8905	.8813	108	.8721	.8631	.8542	.8455	.8369	.8284	.8199	.8117	.8035	.7954	108
109	.9853	.9729	.9615	.9507	.9402	.9300	.9201	.9104	.9009	.8915	.8823	109	.8732	.8643	.8555	.8468	.8382	.8298	.8214	.8132	.8051	.7970	109
110	.9855	.9731	.9618	.9511	.9407	.9306	.9208	.9112	.9017	.8924	.8833	110	.8743	.8654	.8567	.8481	.8396	.8312	.8229	.8147	.8067	.7987	110
111	.9856	.9734	.9622	.9515	.9412	.9313	.9215	.9119	.9026	.8933	.8843	111	.8754	.8666	.8579	.8493	.8409	.8326	.8244	.8162	.8082	.8003	111
112	.9857	.9736	.9625	.9520	.9418	.9318	.9222	.9127	.9034	.8942	.8853	112	.8764	.8677	.8591	.8506	.8422	.8339	.8258	.8177	.8098	.8019	112
113	.9859	.9738	.9628	.9524	.9423	.9324	.9228	.9134	.9042	.8951	.8862	113	.8774	.8688	.8602	.8518	.8435	.8353	.8272	.8192	.8113	.8035	113
114	.9860	.9741	.9632	.9528	.9427	.9330	.9235	.9142	.9050	.8960	.8871	114	.8784	.8698	.8614	.8530	.8448	.8366	.8286	.8206	.8128	.8050	114
115	.9861	.9743	.9635	.9532	.9432	.9336	.9241	.9149	.9058	.8969	.8881	115	.8794	.8709	.8625	.8542	.8460	.8379	.9299	.8220	.8142	.8065	115
116	.9862	.9745	.9638	.9536	.9437	.9341	.9247	.9156	.9066	.8977	.8890	116	.8804	.8719	.8636	.8553	.8472	.8392	.8313	.8234	.8157	.8080	116
117	.9863	.9747	.9641	.9540	.9442	.9347	.9254	.9163	.9073	.8985	.8899	117	.8814	.8730	.8647	.8565	.8484	.8404	.8326	.8248	.8171	.8095	117
118	.9865	.9749	.9644	.9543	.9446	.9352	.9260	.9169	.9081	.8993	.8908	118	.8823	.8740	.8657	.8576	.8496	.8417	.8339	.8261	.8185	.8109	118
119	.9866	.9752	.9647	.9547	.9451	.9357	.9266	.9176	.9088	.9001	.8916	119	.8832	.8749	.8668	.8587	.8508	.8429	.8351	.8275	.8199	.8124	119
120	.9867	.9754	.9650	.9551	.9455	.9362	.9272	.9183	.9095	.9009	.8925	120	.8841	.8759	.8678	.8598	.8519	.8441	.8364	.8288	.8212	.8138	120
121	.9868	.9756	.9653	.9554	.9460	.9367	.9277	.9189	.9102	.9017	.8933	121	.8850	.8764	.8688	.8609	.8530	.8453	.8376	.8301	.8226	.8152	121
122	.9869	.9758	.9655	.9558	.9464	.9373	.9283	.9195	.9109	.9025	.8941	122	.8859	.8778	.8698	.8619	.8542	.8464	.8388	.8313	.8239	.8165	122
123	.9870	.9759	.9658	.9562	.9468	.9377	.9289	.9202	.9116	.9032	.8950	123	.8868	.8788	.8708	.8630	.8552	.8476	.8400	.8326	.8252	.8179	123
124	.9871	.9761	.9661	.9565	.9472	.9382	.9294	.9208	.9123	.9040	.8958	124	.8877	.8797	.8718	.8640	.8563	.8487	.8412	.8338	.8265	.8192	124
125	.9872	.9763	.9663	.9568	.9477	.9387	.9300	.9214	.9130	.9047	.8965	125	.8885	.8806	.8727	.8650	.8574	.8498	.8424	.8350	.8277	.8205	125
126	.9873	.9765	.9666	.9572	.9481	.9392	.9305	.9220	.9136	.9054	.8973	126	.8893	.8815	.8737	.8660	.8584	.8509	.8435	.8362	.8290	.8218	126
127	.9874	.9767	.9669	.9575	.9485	.9396	.9310	.9226	.9143	.9061	.8981	127	.8902	.8823	.8746	.8670	.8595	.8520	.8447	.8374	.8302	.8231	127
128	.9875	.9769	.9671	.9578	.9488	.9401	.9316	.9232	.9149	.9068	.8988	128	.8910	.8832	.8755	.8680	.8605	.8531	.8458	.8386	.8314	.8243	128
129	.9876	.9771	.9674	.9581	.9492	.9406	.9321	.9237	.9156	.9075	.8996	129	.8918	.8841	.8764	.8689	.8615	.8541	.8469	.8397	.8326	.8256	129
130	.9877	.9772	.9676	.9585	.9496	.9410	.9326	.9243	.9162	.9082	.9003	130	.8926	.8849	.8773	.8699	.8625	.8552	.8480	.8408	.8338	.8268	130
131	.9878	.9774	.9679	.9588	.9500	.9414	.9331	.9249	.9168	.9089	.9010	131	.8933	.8857	.8782	.8708	.8635	.8562	.8490	.8419	.8349	.8280	131
132	.9879	.9776	.9681	.9591	.9504	.9419	.9336	.9254	.9174	.9095	.9018	132	.8941	.8865	.8791	.8717	.8644	.8572	.8501	.8430	.8361	.8292	132
133	.9880	.9777	.9683	.9594	.9507	.9423	.9340	.9259	.9180	.9102	.9025	133	.8948	.8873	.8799	.8726	.8654	.8582	.8511	.8441	.8372	.8303	133
134	.9881	.9779	.9686	.9597	.9511	.9427	.9345	.9265	.9186	.9108	.9031	134	.8956	.8881	.8808	.8735	.8663	.8592	.8521	.8452	.8383	.8315	134
135	.9881	.9781	.9688	.9600	.9514	.9431	.9350	.9270	.9192	.9114	.9038	135	.8963	.8889	.8816	.8744	.8672	.8601	.8532	.8462	.8394	.8326	135
136	.9882	.9782	.9690	.9603	.9518	.9435	.9354	.9275	.9197	.9121	.9045	136	.8970	.8897	.8824	.8752	.8681	.8611	.8542	.8473	.8405	.8337	136
137	.9883	.9784	.9692	.9605	.9521	.9439	.9359	.9280	.9203	.9127	.9052	137	.8978	.8904	.8832	.8761	.8690	.8620	.8551	.8483	.8415	.8349	137
138	.9884	.9785	.9695	.9608	.9525	.9443	.9364	.9285	.9208	.9133	.9058	138	.8985	.8912	.8940	.8769	.8699	.8630	.8561	.8493	.8426	.8359	138
139	.9885	.9787	.9697	.9611	.9528	.9447	.9368	.9290	.9214	.9139	.9065	139	.8991	.8919	.8848	.8777	.8708	.8639	.8571	.8503	.8436	.8370	139
140	.9886	.9788	.9699	.9614	.9531	.9451	.9372	.9295	.9219	.9145	.9071	140	.8998	.8927	.8856	.8786	.8716	.8648	.8580	.8513	.8447	.8381	140
141	.9887	.9790	.9701	.9616	.9535	.9455	.9377	.9300	.9225	.9150	.9077	141	.9005	.8934	.8863	.8794	.8725	.8657	.8589	.8523	.8457	.8391	141
142	.9887	.9791	.9703	.9619	.9538	.9458	.9381	.9305	.9230	.9156	.9083	142	.9012	.8941	.8871	.8802	.8733	.8666	.8599	.8532	.8467	.8402	142
143	.9888	.9793	.9705	.9622	.9541	.9462	.9385	.9309	.9235	.9162	.9090	143	.9018	.8948	.8878	.8810	.8742	.8674	.8608	.8542	.8477	.8412	143
144	.9889	.9794	.9707	.9624	.9544	.9466	.9389	.9314	.9240	.9167	.9096	144	.9025	.8955	.8886	.8817	.8750	.8683	.8617	.8551	.8486	.8422	144
145	.9890	.9796	.9709	.9627	.9547	.9469	.9393	.9319	.9245	.9173	.9102	145	.9031	.8962	.8893	.8825	.8758	.8691	.8626	.8560	.8496	.8432	145
146	.9890	.9797	.9711	.9629	.9550	.9473	.9397	.9323	.9250	.9178	.9107	146	.9037	.8968	.8900	.8833	.8766	.8700	.8634	.8570	.8505	.8442	146
147	.9891	.9798	.9713	.9632	.9553	.9476	.9401	.9328	.9255	.9184	.9113	147	.9044	.8975	.8907	.8840	.8774	.8708	.8643	.8579	.8515	.8452	147
148	.9892	.9800	.9715	.9634	.9556	.9480	.9405	.9332	.9260	.9189	.9119	148	.9050	.8982	.8914	.8847	.8781	.8716	.8651	.8587	.8524	.8461	148
149	.9893	.9801	.9717	.9637	.9559	.9483	.9409	.9336	.9265	.9194	.9125	149	.9056	.8988	.8921	.8855	.8789	.8724	.8660	.8596	.8533	.8471	149
150	.9893	.9802	.9719	.9639	.9562	.9487	.9413	.9341	.9769	.9199	.9130	150	.9062	.8994	.8928	.8862	.8797	.8732	.8668	.8605	.8542	.8480	150

T* = NORMALIZED TEST TIME. **If Testing is truncated by failure, enter 1 failure less than those observed to obtain Reliability Lower Bound.

Figure E-2. Reliability Lower Bound for Exponential Components with Failures Truncated by Test Time** (Continued)

80% CONFIDENCE

80% CONFIDENCE

T*	FAILURES											T*	FAILURES										T*
	0	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	16	17	18	19	20	
151	.9894	.9804	.9721	.9641	.9565	.9490	.9417	.9345	.9274	.9204	.9136	151	.9068	.9001	.8934	.8869	.8804	.8740	.8676	.8613	.8551	.8489	151
152	.9895	.9805	.9722	.9644	.9567	.9493	.9420	.9349	.9279	.9209	.9141	152	.9074	.9007	.8941	.8876	.8811	.8748	.8684	.8622	.8560	.8499	152
153	.9895	.9806	.9724	.9646	.9570	.9496	.9424	.9353	.9283	.9214	.9146	153	.9079	.9013	.8948	.8883	.8819	.8755	.8692	.8630	.8569	.8508	153
154	.9896	.9807	.9726	.9648	.9573	.9500	.9428	.9357	.9288	.9219	.9152	154	.9085	.9019	.8954	.8890	.8826	.8763	.8700	.8639	.8577	.8517	154
155	.9897	.9809	.9728	.9650	.9576	.9503	.9431	.9361	.9292	.9224	.9157	155	.9091	.9025	.8960	.8896	.8833	.8770	.8708	.8647	.8586	.8525	155
156	.9897	.9810	.9729	.9653	.9578	.9506	.9435	.9365	.9296	.9229	.9162	156	.9096	.9031	.8967	.8903	.8840	.8778	.8716	.8655	.8594	.8534	156
157	.9898	.9811	.9731	.9655	.9581	.9509	.9438	.9369	.9301	.9234	.9167	157	.9102	.9037	.8973	.8910	.8847	.8785	.8724	.8663	.8602	.8543	157
158	.9899	.9812	.9733	.9657	.9584	.9512	.9442	.9373	.9305	.9238	.9172	158	.9107	.9043	.8979	.8916	.8854	.8792	.8731	.8671	.8611	.8551	158
159	.9899	.9813	.9734	.9659	.9586	.9515	.9445	.9377	.9309	.9243	.9177	159	.9113	.9049	.8985	.8923	.8861	.8799	.8739	.8678	.8619	.8560	159
160	.9900	.9815	.9736	.9661	.9589	.9518	.9449	.9380	.9313	.9247	.9182	160	.9118	.9054	.8991	.8929	.8867	.8806	.8746	.8686	.8627	.8568	160
161	.9901	.9816	.9738	.9663	.9591	.9521	.9452	.9384	.9318	.9252	.9187	161	.9123	.9060	.8997	.8935	.8874	.8813	.8753	.8694	.8635	.8576	161
162	.9901	.9817	.9739	.9665	.9594	.9524	.9455	.9388	.9322	.9256	.9192	162	.9128	.9065	.9003	.8941	.8881	.8820	.8760	.8701	.8643	.8584	162
163	.9902	.9818	.9741	.9667	.9596	.9527	.9458	.9392	.9326	.9261	.9197	163	.9133	.9071	.9009	.8948	.8887	.8827	.8768	.8709	.8650	.8592	163
164	.9902	.9819	.9742	.9669	.9598	.9529	.9462	.9395	.9330	.9265	.9201	164	.9138	.9076	.9015	.8954	.8893	.8834	.8775	.8716	.8658	.8600	164
165	.9903	.9820	.9744	.9671	.9601	.9532	.9465	.9399	.9334	.9269	.9206	165	.9143	.9081	.9020	.8960	.8900	.8840	.8782	.8723	.8665	.8608	165
166	.9904	.9821	.9746	.9673	.9603	.9535	.9468	.9402	.9337	.9274	.9211	166	.9148	.9087	.9026	.8966	.8906	.8847	.8788	.8730	.8673	.8616	166
167	.9904	.9822	.9747	.9675	.9606	.9538	.9471	.9406	.9341	.9278	.9215	167	.9153	.9092	.9031	.8971	.8912	.8853	.8795	.8738	.8680	.8624	167
168	.9905	.9823	.9749	.9677	.9608	.9540	.9474	.9409	.9345	.9282	.9220	168	.9158	.9097	.9037	.8977	.8918	.8860	.8802	.8745	.8688	.8631	168
169	.9905	.9824	.9750	.9679	.9610	.9543	.9477	.9412	.9349	.9286	.9224	169	.9163	.9102	.9042	.8983	.8924	.8866	.8809	.8751	.8695	.8639	169
170	.9906	.9825	.9751	.9681	.9612	.9546	.9480	.9416	.9353	.9290	.9228	170	.9167	.9107	.9048	.8989	.8930	.8872	.8815	.8758	.8702	.8646	170
171	.9906	.9826	.9753	.9683	.9615	.9548	.9483	.9419	.9356	.9294	.9233	171	.9172	.9112	.9053	.8994	.8936	.8879	.8822	.8765	.8709	.8654	171
172	.9907	.9827	.9754	.9684	.9617	.9551	.9486	.9422	.9360	.9298	.9237	172	.9177	.9117	.9058	.9000	.8942	.8885	.8828	.8772	.8716	.8661	172
173	.9907	.9828	.9756	.9686	.9619	.9553	.9489	.9426	.9363	.9302	.9241	173	.9181	.9122	.9063	.9005	.8948	.8891	.8834	.8779	.8723	.8668	173
174	.9908	.9829	.9757	.9688	.9621	.9556	.9492	.9429	.9367	.9306	.9245	174	.9186	.9127	.9068	.9011	.89[illegible]	.8897	.8841	.8785	.8730	.8675	174
175	.9908	.9830	.9758	.9690	.9623	.9558	.9495	.9432	.9370	.9310	.9250	175	.9190	.9132	.9074	.9016	.895[illegible]	.8903	.8847	.8792	.8737	.8682	175
176	.9909	.9831	.9760	.9692	.9625	.9561	.9497	.9435	.9374	.9313	.9254	176	.9195	.9136	.9079	.9021	.8965	.8904	.8853	.8798	.8743	.8689	176
177	.9909	.9832	.9761	.9693	.9627	.9563	.9500	.9438	.9377	.9317	.9258	177	.9199	.9141	.9084	.9027	.8970	.8915	.8859	.8804	.8750	.8696	177
178	.9910	.9833	.9762	.9695	.9629	.9566	.9503	.9441	.9381	.9321	.9262	178	.9203	.9146	.9088	.9032	.8976	.8920	.8865	.8811	.8757	.8703	178
179	.9910	.9834	.9764	.9697	.9631	.9568	.9506	.9444	.9384	.9325	.9266	179	.9208	.9150	.9093	.9037	.8981	.8926	.8871	.8817	.8763	.8710	179
180	.9911	.9835	.9765	.9698	.9633	.9570	.9508	.9447	.9387	.9328	.9270	180	.9212	.9155	.9098	.9042	.8987	.8932	.8877	.8823	.8770	.8716	180
181	.9911	.9836	.9766	.9700	.9635	.9573	.9511	.9450	.9391	.9332	.9274	181	.9216	.9159	.9103	.9047	.8992	.8937	.8883	.8829	.8776	.8723	181
182	.9912	.9837	.9768	.9702	.9637	.9575	.9514	.9453	.9394	.9335	.9277	182	.9220	.9164	.9108	.9052	.8997	.8943	.8889	.8835	.8782	.8730	182
183	.9912	.9838	.9769	.9703	.9639	.9577	.9516	.9456	.9397	.9339	.9281	183	.9224	.9168	.9112	.9057	.9002	.8948	.8894	.8841	.8788	.8736	183
184	.9913	.9839	.9770	.9705	.9641	.9579	.9519	.9459	.9400	.9342	.9285	184	.9228	.9172	.9117	.9062	.9008	.8954	.8900	.8847	.8795	.8742	184
185	.9913	.9839	.9771	.9706	.9643	.9582	.9521	.9462	.9403	.9346	.9289	185	.9232	.9177	.9121	.9067	.9013	.8959	.8906	.8853	.8801	.8749	185
186	.9914	.9840	.9773	.9708	.9645	.9584	.9524	.9465	.9407	.9349	.9292	186	.9236	.9181	.9126	.9072	.9018	.8964	.8911	.8859	.8807	.8755	186
187	.9914	.9841	.9774	.9709	.9647	.9586	.9526	.9468	.9410	.9352	.9296	187	.9240	.9185	.9130	.9076	.9023	.8969	.8917	.8865	.8813	.8761	187
188	.9915	.9842	.9775	.9711	.9649	.9588	.9529	.9470	.9413	.9356	.9300	188	.9244	.9189	.9135	.9081	.9028	.8975	.8922	.8870	.8819	.8768	188
189	.9915	.9843	.9776	.9712	.9651	.9590	.9531	.9473	.9416	.9359	.9303	189	.9248	.9193	.9139	.9086	.9032	.8980	.8928	.8876	.8825	.8774	189
190	.9916	.9844	.9777	.9714	.9652	.9592	.9534	.9476	.9419	.9362	.9307	190	.9252	.9197	.9143	.9090	.9037	.8985	.8933	.8881	.8830	.8780	190
191	.9916	.9844	.9778	.9715	.9654	.9595	.9536	.9478	.9422	.9366	.9310	191	.9256	.9201	.9148	.9095	.9042	.8990	.8938	.8887	.8836	.8786	191
192	.9917	.9845	.9780	.9717	.9656	.9597	.9538	.9481	.9425	.9369	.9314	192	.9259	.9205	.9152	.9099	.9047	.8995	.8943	.8892	.8842	.8792	192
193	.9917	.9846	.9781	.9718	.9658	.9599	.9541	.9484	.9427	.9372	.9317	193	.9263	.9209	.9156	.9104	.9052	.9000	.8949	.8898	.8847	.8797	193
194	.9917	.9847	.9782	.9720	.9659	.9601	.9543	.9486	.9430	.9375	.9321	194	.9267	.9213	.9160	.9108	.9056	.9005	.8954	.8903	.8853	.8803	194
195	.9918	.9848	.9783	.9721	.9661	.9603	.9545	.9489	.9433	.9378	.9324	195	.9270	.9217	.9164	.9112	.9061	.9010	.8959	.8908	.8859	.8809	195
196	.9918	.9848	.9784	.9723	.9663	.9605	.9548	.9491	.9436	.9381	.9327	196	.9274	.9221	.9169	.9117	.9065	.9014	.8964	.8914	.8864	.8815	196
197	.9919	.9849	.9785	.9724	.9665	.9607	.95	.9494	.9439	.9384	.9331	197	.9277	.9225	.9173	.9121	.9070	.9019	.8969	.8919	.8869	.8820	197
198	.9919	.9850	.9786	.9725	.9666	.9609	.9552	.9496	.9441	.9387	.9334	198	.9281	.9228	.9177	.9125	.9074	.9024	.8974	.8924	.8875	.8826	198
199	.9919	.9851	.9787	.9727	.9668	.9611	.9554	.9499	.9444	.9390	.9337	199	.9284	.9232	.9181	.9129	.9079	.9028	.8979	.8929	.8880	.8831	199
200	.9920	.9851	.9788	.9728	.9670	.9612	.9556	.9501	.9447	.9393	.9340	200	.9288	.9236	.9185	.9134	.9083	.9033	.8983	.8934	.8885	.8837	200

T* = NORMALIZED TEST TIME. **If Testing is truncated by failure, enter 1 failure less than those observed to obtain Reliability Lower Bound.

Figure E-2. Reliability Lower Bound for Exponential Components with Failures Truncated by Test Time** (Continued)

80% CONFIDENCE

T*	FAILURES											T*	FAILURES										T*
	0	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	16	17	18	19	20	
201	.9920	.9852	.9789	.9729	.9671	.9614	.9559	.9504	.9450	.9396	.9343	201	.9291	.9240	.9188	.9138	.9087	.9038	.8988	.8939	.8891	.8842	201
202	.9921	.9853	.9790	.9731	.9673	.9616	.9561	.9506	.9452	.9399	.9347	202	.9295	.9243	.9192	.9142	.9092	.9042	.8993	.8944	.8896	.8848	202
203	.9921	.9854	.9791	.9732	.9674	.9618	.9563	.9508	.9455	.9402	.9350	203	.9298	.9247	.9196	.9146	.9096	.9047	.8998	.8949	.8901	.8853	203
204	.9921	.9854	.9792	.9733	.9676	.9620	.9565	.9511	.9457	.9405	.9353	204	.9301	.9250	.9200	.9150	.9100	.9051	.9002	.8954	.8906	.8858	204
205	.9922	.9855	.9793	.9735	.9677	.9622	.9567	.9513	.9460	.9408	.9356	205	.9305	.9254	.9204	.9154	.9104	.9056	.9007	.8959	.8911	.8864	205
206	.9922	.9856	.9794	.9736	.9679	.9623	.9569	.9515	.9463	.9410	.9359	206	.9308	.9257	.9207	.9158	.9109	.9060	.9012	.8964	.8916	.8869	206
207	.9923	.9856	.9795	.9737	.9681	.9625	.9571	.9518	.9465	.9413	.9362	207	.9311	.9261	.9211	.9162	.9113	.9064	.9016	.8968	.8921	.8874	207
208	.9923	.9857	.9796	.9738	.9682	.9627	.9573	.9520	.9468	.9416	.9365	208	.9314	.9264	.9215	.9165	.9117	.9068	.9021	.8973	.8926	.8879	208
209	.9923	.9858	.9797	.9740	.9684	.9629	.9575	.9522	.9470	.9419	.9368	209	.9317	.9268	.9218	.9169	.9121	.9073	.9025	.8978	.8931	.8884	209
210	.9924	.9858	.9798	.9741	.9685	.9631	.9577	.9524	.9473	.9421	.9371	210	.9321	.9271	.9222	.9173	.9125	.9077	.9029	.8982	.8936	.8889	210
211	.9924	.9859	.9799	.9742	.9686	.9632	.9579	.9527	.9475	.9424	.9374	211	.9324	.9274	.9225	.9177	.9129	.9081	.9034	.8987	.8940	.8894	211
212	.9924	.9860	.9800	.9743	.9688	.9634	.9581	.9529	.9477	.9427	.9376	212	.9327	.9278	.9229	.9181	.9133	.9085	.9038	.8991	.8945	.8899	212
213	.9925	.9860	.9801	.9744	.9689	.9636	.9583	.9531	.9480	.9429	.9379	213	.9330	.9281	.9232	.9184	.9137	.9089	.9042	.8996	.8950	.8904	213
214	.9925	.9861	.9802	.9746	.9691	.9637	.9585	.9533	.9482	.9432	.9382	214	.9333	.9284	.9236	.9188	.9140	.9093	.9047	.9000	.8954	.8909	214
215	.9925	.9862	.9803	.9747	.9692	.9639	.9587	.9535	.9484	.9434	.9385	215	.9336	.9287	.9239	.9192	.9144	.9097	.9051	.9005	.8959	.8914	215
216	.9926	.9862	.9804	.9748	.9694	.9641	.9589	.9537	.9487	.9437	.9388	216	.9339	.9290	.9243	.9195	.9148	.9101	.9055	.9009	.8964	.8918	216
217	.9926	.9863	.9805	.9749	.9695	.9642	.9590	.9539	.9489	.9439	.9390	217	.9342	.9294	.9246	.9199	.9152	.9105	.9059	.9013	.8968	.8923	217
218	.9926	.9864	.9806	.9750	.9696	.9644	.9592	.9541	.9491	.9442	.9393	218	.9345	.9297	.9249	.9202	.9156	.9109	.9063	.9018	.8973	.8928	218
219	.9927	.9864	.9807	.9751	.9698	.9645	.9594	.9544	.9494	.9444	.9396	219	.9348	.9300	.9253	.9206	.9159	.9113	.9067	.9022	.8977	.8932	219
220	.9927	.9865	.9807	.9752	.9699	.9647	.9596	.9546	.9496	.9447	.9398	220	.9350	.9303	.9256	.9209	.9163	.9117	.9071	.9026	.8981	.8937	220
221	.9927	.9865	.9808	.9754	.9700	.9649	.9598	.9548	.9498	.9449	.9401	221	.9353	.9306	.9259	.9213	.9167	.9121	.9075	.9030	.8986	.8941	221
222	.9928	.9866	.9809	.9755	.9702	.9650	.9599	.9550	.9500	.9452	.9404	222	.9356	.9309	.9262	.9216	.9170	.9125	.9079	.9035	.8990	.8946	222
223	.9928	.9867	.9810	.9756	.9703	.9652	.9601	.9552	.9502	.9454	.9406	223	.9359	.9312	.9265	.9219	.9174	.9128	.9083	.9039	.8994	.8950	223
224	.9928	.9867	.9811	.9757	.9704	.9653	.9603	.9553	.9505	.9456	.9409	224	.9362	.9315	.9269	.9223	.9177	.9132	.9087	.9043	.8999	.8955	224
225	.9929	.9868	.9812	.9758	.9706	.9655	.9605	.9555	.9507	.9459	.9411	225	.9364	.9318	.9272	.9226	.9181	.9136	.9091	.9047	.9003	.8959	225
226	.9929	.9868	.9812	.9759	.9707	.9656	.9606	.9557	.9509	.9461	.9414	226	.9367	.9321	.9275	.9229	.9184	.9139	.9095	.9051	.9007	.8964	226
227	.9929	.9869	.9813	.9760	.9708	.9658	.9608	.9559	.9511	.9463	.9416	227	.9370	.9324	.9278	.9233	.9188	.9143	.9099	.9055	.9011	.8968	227
228	.9930	.9870	.9814	.9761	.9710	.9659	.9610	.9561	.9513	.9466	.9419	228	.9372	.9326	.9281	.9236	.9191	.9147	.9103	.9059	.9015	.8972	228
229	.9930	.9870	.9815	.9762	.9711	.9661	.9611	.9563	.9515	.9468	.9421	229	.9375	.9329	.9284	.9239	.9194	.9150	.9106	.9063	.9019	.8976	229
230	.9930	.9871	.9816	.9763	.9712	.9662	.9613	.9565	.9517	.9470	.9424	230	.9378	.9332	.9287	.9242	.9198	.9154	.9110	.9067	.9023	.8981	230
231	.9931	.9871	.9816	.9764	.9713	.9664	.9615	.9567	.9519	.9472	.9426	231	.9380	.9335	.9290	.9245	.9201	.9157	.9114	.9070	.9027	.8985	231
232	.9931	.9872	.9817	.9765	.9714	.9665	.9616	.9569	.9521	.9475	.9429	232	.9383	.9338	.9293	.9248	.9204	.9161	.9117	.9074	.9031	.8989	232
233	.9931	.9872	.9818	.9766	.9716	.9666	.9618	.9570	.9523	.9477	.9431	233	.9386	.9340	.9296	.9252	.9208	.9164	.9121	.9078	.9035	.8993	233
234	.9931	.9873	.9819	.9767	.9717	.9668	.9620	.9572	.9525	.9479	.9433	234	.9388	.9343	.9299	.9255	.9211	.9168	.9125	.9082	.9039	.8997	234
235	.9932	.9873	.9820	.9768	.9718	.9669	.9621	.9574	.9527	.9481	.9436	235	.9391	.9346	.9302	.9258	.9214	.9171	.9128	.9085	.9043	.9001	235
236	.9932	.9874	.9820	.9769	.9719	.9671	.9623	.9576	.9529	.9483	.9438	236	.9393	.9349	.9304	.9261	.9217	.9174	.9132	.9089	.9047	.9005	236
237	.9932	.9874	.9821	.9770	.9720	.9672	.9624	.9577	.9531	.9485	.9440	237	.9396	.9351	.9307	.9264	.9221	.9178	.9135	.9093	.9051	.9009	237
238	.9933	.9875	.9822	.9771	.9722	.9673	.9626	.9579	.9533	.9488	.9443	238	.9398	.9354	.9310	.9267	.9224	.9181	.9139	.9096	.9055	.9013	238
239	.9933	.9875	.9823	.9772	.9723	.9675	.9627	.9581	.9535	.9490	.9445	239	.9400	.9356	.9313	.9270	.9227	.9184	.9142	.9100	.9058	.9017	239
240	.9933	.9876	.9823	.9773	.9724	.9676	.9629	.9583	.9537	.9492	.9447	240	.9403	.9359	.9316	.9273	.9230	.9187	.9145	.9104	.9062	.9021	240
241	.9933	.9877	.9824	.9774	.9725	.9677	.9630	.9584	.9539	.9494	.9449	241	.9405	.9362	.9318	.9276	.9233	.9191	.9149	.9107	.9066	.9025	241
242	.9934	.9877	.9825	.9775	.9726	.9679	.9632	.9586	.9541	.9496	.9452	242	.9408	.9364	.9321	.9278	.9236	.9194	.9152	.9111	.9070	.9029	242
243	.9934	.9878	.9825	.9776	.9727	.9680	.9633	.9588	.9542	.9498	.9454	243	.9410	.9367	.9324	.9281	.9239	.9197	.9156	.9114	.9073	.9032	243
244	.9934	.9878	.9826	.9777	.9728	.9681	.9635	.9589	.9544	.9500	.9456	244	.9412	.9369	.9326	.9284	.9242	.9200	.9159	.9118	.9077	.9036	244
245	.9935	.9879	.9827	.9777	.9729	.9682	.9636	.9591	.9546	.9502	.9458	245	.9415	.9372	.9329	.9287	.9245	.9203	.9162	.9121	.9080	.9040	245
246	.9935	.9879	.9828	.9778	.9730	.9684	.9638	.9593	.9548	.9504	.9460	246	.9417	.9374	.9332	.9290	.9248	.9207	.9165	.9125	.9084	.9044	246
247	.9935	.9879	.9828	.9779	.9732	.9685	.9639	.9594	.9550	.9506	.9462	247	.9419	.9377	.9334	.9292	.9251	.9210	.9169	.9128	.9087	.9047	247
248	.9935	.9880	.9829	.9780	.9733	.9686	.9641	.9596	.9551	.9508	.9464	248	.9422	.9379	.9337	.9295	.9254	.9213	.9172	.9131	.9091	.9051	248
249	.9936	.9880	.9830	.9781	.9734	.9687	.9642	.9597	.9553	.9510	.9467	249	.9424	.9382	.9340	.9298	.9257	.9216	.9175	.9135	.9094	.9055	249
250	.9936	.9881	.9830	.9782	.9735	.9689	.9643	.9599	.9555	.9512	.9469	250	.9426	.9384	.9342	.9301	.9260	.9219	.9178	.9138	.9098	.9058	250

T* = NORMALIZED TEST TIME. **If Testing is truncated by failure, enter 1 failure less than those observed to attain Reliability Lower Bound.

Figure E-2. Reliability Lower Bound for Exponential Components with Failures Truncated by Test Time** (Continued)

80% CONFIDENCE

80% CONFIDENCE

T*	FAILURES											T*	FAILURES										T*
	0	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	16	17	18	19	20	
251	.9936	.9881	.9831	.9783	.9736	.9690	.9645	.9601	.9557	.9513	.9471	251	.9428	.9386	.9345	.9303	.9262	.9222	.9181	.9141	.9101	.9062	251
252	.9936	.9882	.9832	.9784	.9737	.9691	.9646	.9602	.9558	.9515	.9473	252	.9430	.9389	.9347	.9306	.9265	.9225	.9184	.9144	.9105	.9065	252
253	.9937	.9882	.9832	.9784	.9738	.9692	.9648	.9604	.9560	.9517	.9475	253	.9433	.9391	.9350	.9309	.9268	.9228	.9188	.9148	.9108	.9069	253
254	.9937	.9883	.9833	.9785	.9739	.9694	.9649	.9605	.9562	.9519	.9477	254	.9435	.9393	.9352	.9311	.9271	.9231	.9191	.9151	.9111	.9072	254
255	.9937	.9883	.9834	.9786	.9740	.9695	.9650	.9607	.9564	.9521	.9479	255	.9437	.9396	.9355	.9314	.9274	.9233	.9194	.9154	.9115	.9076	255
256	.9937	.9884	.9834	.9787	.9741	.9696	.9652	.9608	.9565	.9523	.9481	256	.9439	.9398	.9357	.9316	.9276	.9236	.9197	.9157	.9118	.9079	256
257	.9938	.9884	.9835	.9788	.9742	.9697	.9653	.9610	.9567	.9525	.9483	257	.9441	.9400	.9359	.9319	.9279	.9239	.9200	.9160	.9121	.9083	257
258	.9938	.9885	.9836	.9789	.9743	.9698	.9654	.9611	.9569	.9526	.9485	258	.9443	.9402	.9362	.9322	.9282	.9242	.9203	.9164	.9125	.9086	258
259	.9938	.9885	.9836	.9789	.9744	.9699	.9656	.9613	.9570	.9528	.9487	259	.9445	.9405	.9364	.9324	.9284	.9245	.9206	.9167	.9128	.9089	259
260	.9938	.9885	.9837	.9790	.9745	.9700	.9657	.9614	.9572	.9530	.9489	260	.9448	.9407	.9367	.9327	.9287	.9248	.9208	.9170	.9131	.9093	260
261	.9939	.9886	.9837	.9791	.9746	.9702	.9658	.9616	.9573	.9532	.9490	261	.9450	.9409	.9369	.9329	.9290	.9250	.9211	.9173	.9134	.9096	261
262	.9939	.9886	.9838	.9792	.9747	.9703	.9660	.9617	.9575	.9533	.9492	262	.9452	.9411	.9371	.9332	.9292	.9253	.9214	.9176	.9137	.9099	262
263	.9939	.9887	.9839	.9792	.9748	.9704	.9661	.9618	.9577	.9535	.9494	263	.9454	.9413	.9374	.9334	.9295	.9256	.9217	.9179	.9141	.9103	263
264	.9939	.9887	.9839	.9793	.9749	.9705	.9662	.9620	.9578	.9537	.9496	264	.9456	.9416	.9376	.9336	.9297	.9259	.9220	.9182	.9144	.9106	264
265	.9939	.9888	.9840	.9794	.9750	.9706	.9663	.9621	.9580	.9539	.9498	265	.9458	.9418	.9378	.9339	.9300	.9261	.9223	.9185	.9147	.9109	265
266	.9940	.9888	.9840	.9795	.9750	.9707	.9665	.9623	.9581	.9540	.9500	266	.9460	.9420	.9380	.9341	.9302	.9264	.9226	.9188	.9150	.9112	266
267	.9940	.9888	.9841	.9796	.9751	.9708	.9666	.9624	.9583	.9542	.9502	267	.9462	.9422	.9383	.9344	.9305	.9267	.9228	.9191	.9153	.9115	267
268	.9940	.9889	.9842	.9796	.9752	.9709	.9667	.9625	.9584	.9544	.9503	268	.9464	.9424	.9385	.9346	.9307	.9269	.9231	.9193	.9156	.9119	268
269	.9940	.9889	.9842	.9797	.9753	.9710	.9668	.9627	.9586	.9545	.9505	269	.9466	.9426	.9387	.9348	.9310	.9272	.9234	.9196	.9159	.9122	269
270	.9941	.9890	.9843	.9798	.9754	.9711	.9669	.9628	.9587	.9547	.9507	270	.9467	.9428	.9389	.9351	.9312	.9274	.9237	.9199	.9162	.9125	270
271	.9941	.9890	.9843	.9799	.9755	.9712	.9671	.9629	.9589	.9549	.9509	271	.9469	.9430	.9391	.9353	.9315	.9277	.9239	.9202	.9165	.9128	271
272	.9941	.9891	.9844	.9799	.9756	.9714	.9672	.9631	.9590	.9550	.9511	272	.9471	.9432	.9394	.9355	.9317	.9280	.9242	.9205	.9168	.9131	272
273	.9941	.9891	.9844	.9800	.9757	.9715	.9673	.9632	.9592	.9552	.9512	273	.9473	.9434	.9396	.9358	.9320	.9282	.9245	.9208	.9171	.9134	273
274	.9941	.9891	.9845	.9801	.9758	.9716	.9674	.9633	.9593	.9553	.9514	274	.9475	.9436	.9398	.9360	.9322	.9285	.9247	.9210	.9174	.9137	274
275	.9942	.9892	.9846	.9801	.9759	.9717	.9675	.9635	.9595	.9555	.9516	275	.9477	.9438	.9400	.9362	.9325	.9287	.9250	.9213	.9176	.9140	275
276	.9942	.9892	.9846	.9802	.9759	.9718	.9677	.9636	.9596	.9557	.9517	276	.9479	.9440	.9402	.9364	.9327	.9290	.9253	.9216	.9179	.9143	276
277	.9942	.9892	.9847	.9803	.9760	.9719	.9678	.9637	.9597	.9558	.9519	277	.9481	.9442	.9404	.9367	.9329	.9292	.9255	.9219	.9182	.9146	277
278	.9942	.9893	.9847	.9804	.9761	.9720	.9679	.9639	.9599	.9560	.9521	278	.9482	.9444	.9406	.9369	.9332	.9295	.9258	.9221	.9185	.9149	278
279	.9942	.9893	.9848	.9804	.9762	.9721	.9680	.9640	.9600	.9561	.9523	279	.9484	.9446	.9408	.9371	.9334	.9297	.9260	.9224	.9188	.9152	279
280	.9943	.9894	.9848	.9805	.9763	.9722	.9681	.9641	.9602	.9563	.9524	280	.9486	.9448	.9410	.9373	.9336	.9299	.9263	.9227	.9191	.9155	280
281	.9943	.9894	.9849	.9806	.9764	.9723	.9682	.9642	.9603	.9564	.9526	281	.9488	.9450	.9413	.9375	.9338	.9302	.9265	.9229	.9193	.9158	281
282	.9943	.9894	.9849	.9806	.9764	.9724	.9683	.9644	.9604	.9566	.9527	282	.9490	.9452	.9415	.9377	.9341	.9304	.9268	.9232	.9196	.9160	282
283	.9943	.9895	.9850	.9807	.9765	.9725	.9684	.9645	.9606	.9567	.9529	283	.9491	.9454	.9417	.9380	.9343	.9307	.9270	.9234	.9199	.9163	283
284	.9943	.9895	.9850	.9808	.9766	.9725	.9685	.9646	.9607	.9569	.9531	284	.9493	.9456	.9419	.9382	.9345	.9309	.9273	.9237	.9201	.9166	284
285	.9944	.9895	.9851	.9808	.9767	.9726	.9687	.9647	.9608	.9570	.9532	285	.9495	.9457	.9421	.9384	.9347	.9311	.9275	.9240	.9204	.9169	285
286	.9944	.9896	.9851	.9809	.9769	.9727	.9688	.9649	.9610	.9572	.9534	286	.9496	.9459	.9422	.9386	.9350	.9314	.9278	.9242	.9207	.9172	286
287	.9944	.9896	.9852	.9810	.9769	.9728	.9689	.9650	.9611	.9573	.9536	287	.9498	.9461	.9424	.9388	.9352	.9316	.9280	.9245	.9209	.9174	287
288	.9944	.9897	.9853	.9810	.9769	.9729	.9690	.9651	.9613	.9575	.9537	288	.9500	.9463	.9426	.9390	.9354	.9318	.9283	.9247	.9212	.9177	288
289	.9944	.9897	.9853	.9811	.9770	.9730	.9691	.9652	.9614	.9576	.9539	289	.9502	.9465	.9428	.9392	.9356	.9320	.9285	.9250	.9215	.9180	289
290	.9945	.9897	.9854	.9812	.9771	.9731	.9692	.9653	.9615	.9577	.9540	290	.9503	.9467	.9430	.9394	.9358	.9323	.9287	.9252	.9217	.9183	290
291	.9945	.9898	.9854	.9812	.9772	.9732	.9693	.9654	.9616	.9579	.9542	291	.9505	.9468	.9432	.9396	.9360	.9325	.9290	.9255	.9220	.9185	291
292	.9945	.9898	.9855	.9813	.9772	.9733	.9694	.9656	.9618	.9580	.9543	292	.9507	.9470	.9434	.9398	.9363	.9327	.9292	.9257	.9222	.9188	292
293	.9945	.9898	.9855	.9814	.9773	.9734	.9695	.9657	.9619	.9582	.9545	293	.9508	.9472	.9436	.9400	.9365	.9329	.9294	.9260	.9225	.9191	293
294	.9945	.9899	.9856	.9814	.9774	.9735	.9696	.9658	.9620	.9583	.9546	294	.9510	.9474	.9438	.9402	.9367	.9332	.9297	.9262	.9228	.9193	294
295	.9946	.9899	.9856	.9815	.9775	.9736	.9697	.9659	.9622	.9585	.9548	295	.9511	.9475	.9440	.9404	.9369	.9334	.9299	.9264	.9230	.9196	295
296	.9946	.9899	.9856	.9815	.9775	.9736	.9698	.9660	.9623	.9586	.9549	296	.9513	.9477	.9441	.9406	.9371	.9336	.9301	.9267	.9233	.9199	296
297	.9946	.9900	.9857	.9816	.9776	.9737	.9699	.9661	.9624	.9587	.9551	297	.9515	.9479	.9443	.9408	.9373	.9338	.9304	.9269	.9235	.9201	297
298	.9946	.9900	.9857	.9817	.9777	.9738	.9700	.9662	.9625	.9589	.9552	298	.9516	.9481	.9445	.9410	.9375	.9340	.9306	.9272	.9238	.9204	298
299	.9946	.9900	.9858	.9817	.9778	.9739	.9701	.9664	.9627	.9590	.9554	299	.9518	.9482	.9447	.9412	.9377	.9342	.9308	.9274	.9240	.9206	299
300	.9946	.9901	.9858	.9818	.9778	.9740	.9702	.9665	.9628	.9591	.9555	300	.9519	.9484	.9449	.9414	.9379	.9345	.9310	.9276	.9242	.9209	300

T* = NORMALIZED TEST TIME. **If Testing is truncated by failure, enter 1 failure less than those observed to obtain Reliability Lower Bound.

Figure E-2. Reliability Lower Bound for Exponential Components with Failures Truncated by Test Time** (Continued)

80% CONFIDENCE

80% CONFIDENCE

T*	FAILURES											T*	FAILURES										T*
	0	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	16	17	18	19	20	
310	.9949	.9904	.9863	.9824	.9786	.9748	.9711	.9675	.9640	.9604	.9569	310	.9535	.9500	.9466	.9432	.9398	.9365	.9332	.9299	.9266	.9233	310
320	.9950	.9907	.9867	.9829	.9792	.9756	.9720	.9685	.9651	.9616	.9582	320	.9549	.9515	.9482	.9449	.9417	.9384	.9352	.9320	.9288	.9256	320
330	.9951	.9910	.9871	.9834	.9798	.9763	.9729	.9695	.9661	.9628	.9595	330	.9562	.9530	.9498	.9466	.9434	.9402	.9371	.9340	.9309	.9278	330
340	.9953	.9912	.9875	.9839	.9804	.9770	.9737	.9704	.9671	.9638	.9606	340	.9575	.9543	.9512	.9481	.9450	.9419	.9389	.9359	.9328	.9299	340
350	.9954	.9915	.9879	.9844	.9810	.9777	.9744	.9712	.9680	.9649	.9617	350	.9587	.9556	.9526	.9495	.9465	.9435	.9406	.9376	.9347	.9318	350
360	.9955	.9917	.9882	.9848	.9815	.9783	.9751	.9720	.9689	.9658	.9628	360	.9598	.9568	.9538	.9509	.9480	.9451	.9422	.9393	.9365	.9336	360
370	.9957	.9919	.9885	.9852	.9820	.9789	.9758	.9727	.9697	.9667	.9638	370	.9609	.9579	.9551	.9522	.9493	.9465	.9437	.9409	.9381	.9354	370
380	.9958	.9922	.9888	.9856	.9825	.9794	.9764	.9734	.9705	.9676	.9647	380	.9619	.9590	.9562	.9534	.9506	.9479	.9451	.9424	.9397	.9370	380
390	.9959	.9924	.9891	.9860	.9829	.9799	.9770	.9741	.9712	.9684	.9656	390	.9628	.9601	.9573	.9546	.9519	.9492	.9465	.9438	.9412	.9386	390
400	.9960	.9925	.9894	.9863	.9833	.9804	.9776	.9747	.9720	.9692	.9664	400	.9637	.9610	.9584	.9557	.9531	.9504	.9478	.9452	.9426	.9401	400
410	.9961	.9927	.9896	.9866	.9837	.9809	.9781	.9754	.9726	.9699	.9673	410	.9646	.9620	.9594	.9568	.9542	.9516	.9491	.9465	.9440	.9415	410
420	.9962	.9929	.9899	.9870	.9841	.9814	.9786	.9759	.9733	.9706	.9680	420	.9654	.9629	.9603	.9578	.9552	.9527	.9502	.9478	.9453	.9428	420
430	.9963	.9931	.9901	.9873	.9845	.9818	.9791	.9765	.9739	.9713	.9688	430	.9662	.9637	.9612	.9587	.9563	.9538	.9514	.9489	.9465	.9441	430
440	.9963	.9932	.9903	.9875	.9848	.9822	.9796	.9770	.9745	.9719	.9695	440	.9670	.9645	.9621	.9596	.9572	.9548	.9524	.9501	.9477	.9454	440
450	.9964	.9934	.9905	.9878	.9852	.9826	.9800	.9775	.9750	.9726	.9701	450	.9677	.9653	.9629	.9605	.9582	.9558	.9535	.9511	.9488	.9465	450
460	.9965	.9935	.9907	.9881	.9855	.9830	.9805	.9780	.9756	.9732	.9708	460	.9684	.9660	.9637	.9614	.9591	.9568	.9545	.9522	.9499	.9477	460
470	.9966	.9936	.9909	.9883	.9858	.9833	.9809	.9785	.9761	.9737	.9714	470	.9690	.9667	.9644	.9622	.9599	.9577	.9554	.9532	.9510	.9487	470
480	.9967	.9938	.9911	.9886	.9861	.9837	.9813	.9789	.9766	.9743	.9720	480	.9697	.9674	.9652	.9629	.9607	.9585	.9563	.9541	.9520	.9498	480
490	.9967	.9939	.9913	.9888	.9864	.9840	.9816	.9793	.9770	.9748	.9725	490	.9703	.9681	.9659	.9637	.9615	.9593	.9572	.9550	.9529	.9508	490
500	.9968	.9940	.9915	.9890	.9866	.9843	.9820	.9797	.9775	.9753	.9731	500	.9709	.9687	.9665	.9644	.9623	.9601	.9580	.9559	.9538	.9517	500
510	.9968	.9941	.9916	.9892	.9869	.9846	.9824	.9801	.9779	.9758	.9736	510	.9714	.9693	.9672	.9651	.9630	.9609	.9588	.9568	.9547	.9527	510
520	.9969	.9943	.9918	.9895	.9872	.9849	.9827	.9805	.9784	.9762	.9741	520	.9720	.9699	.9678	.9657	.9637	.9616	.9596	.9576	.9556	.9536	520
530	.9970	.9944	.9920	.9896	.9874	.9852	.9830	.9809	.9788	.9767	.9746	530	.9725	.9705	.9684	.9664	.9644	.9624	.9604	.9584	.9564	.9544	530
540	.9970	.9945	.9921	.9898	.9876	.9855	.9833	.9812	.9791	.9771	.9750	540	.9730	.9710	.9690	.9670	.9650	.9630	.9611	.9591	.9572	.9552	540
550	.9971	.9946	.9923	.9900	.9879	.9857	.9836	.9816	.9795	.9775	.9755	550	.9735	.9715	.9695	.9676	.9656	.9637	.9618	.9599	.9579	.9560	550
560	.9971	.9947	.9924	.9902	.9881	.9860	.9839	.9819	.9799	.9779	.9759	560	.9740	.9720	.9701	.9682	.9662	.9643	.9624	.9606	.9587	.9568	560
570	.9972	.9948	.9925	.9904	.9883	.9862	.9842	.9822	.9802	.9783	.9763	570	.9744	.9725	.9706	.9687	.9668	.9649	.9631	.9612	.9594	.9575	570
580	.9972	.9949	.9926	.9905	.9885	.9865	.9845	.9825	.9806	.9786	.9767	580	.9748	.9730	.9711	.9692	.9674	.9655	.9637	.9619	.9601	.9583	580
590	.9973	.9949	.9928	.9907	.9887	.9867	.9847	.9828	.9809	.9790	.9771	590	.9753	.9734	.9716	.9697	.9679	.9661	.9643	.9625	.9607	.9590	590
600	.9973	.9950	.9929	.9909	.9889	.9869	.9850	.9831	.9812	.9794	.9775	600	.9757	.9739	.9720	.9702	.9685	.9667	.9649	.9631	.9614	.9596	600
610	.9974	.9951	.9930	.9910	.9890	.9871	.9852	.9834	.9815	.9797	.9779	610	.9761	.9743	.9725	.9707	.9690	.9672	.9655	.9637	.9620	.9603	610
620	.9974	.9952	.9931	.9911	.9892	.9873	.9855	.9836	.9818	.9800	.9782	620	.9764	.9747	.9729	.9712	.9695	.9677	.9660	.9643	.9626	.9609	620
630	.9974	.9953	.9932	.9913	.9894	.9875	.9857	.9839	.9821	.9803	.9786	630	.9768	.9751	.9734	.9716	.9699	.9682	.9665	.9649	.9632	.9615	630
640	.9975	.9953	.9933	.9914	.9896	.9877	.9859	.9841	.9824	.9806	.9789	640	.9772	.9755	.9738	.9721	.9704	.9687	.9671	.9654	.9637	.9621	640
650	.9975	.9954	.9934	.9916	.9897	.9879	.9861	.9844	.9826	.9809	.9792	650	.9775	.9758	.9742	.9725	.9708	.9692	.9676	.9659	.9643	.9627	650
660	.9976	.9955	.9935	.9917	.9899	.9881	.9863	.9846	.9829	.9812	.9795	660	.9779	.9762	.9746	.9729	.9713	.9697	.9680	.9664	.9648	.9632	660
670	.9976	.9955	.9936	.9918	.9900	.9883	.9865	.9848	.9832	.9815	.9798	670	.9782	.9766	.9749	.9733	.9717	.9701	.9685	.9669	.9653	.9638	670
680	.9976	.9956	.9937	.9919	.9902	.9884	.9867	.9851	.9834	.9818	.9801	680	.9785	.9769	.9754	.9737	.9721	.9705	.9690	.9674	.9658	.9643	680
690	.9977	.9957	.9938	.9920	.9903	.9886	.9869	.9853	.9836	.9820	.9804	690	.9788	.9772	.9756	.9741	.9725	.9710	.9694	.9679	.9663	.9648	690
700	.9977	.9957	.9939	.9922	.9904	.9888	.9871	.9855	.9839	.9823	.9807	700	.9791	.9775	.9760	.9744	.9729	.9714	.9698	.9683	.9668	.9653	700
710	.9977	.9958	.9940	.9923	.9906	.9889	.9873	.9857	.9841	.9825	.9810	710	.9794	.9779	.9763	.9748	.9733	.9718	.9703	.9688	.9673	.9658	710
720	.9978	.9958	.9941	.9924	.9907	.9891	.9875	.9859	.9843	.9828	.9812	720	.9797	.9782	.9766	.9751	.9736	.9721	.9707	.9692	.9677	.9662	720
730	.9978	.9959	.9942	.9925	.9908	.9892	.9876	.9861	.9845	.9830	.9815	730	.9800	.9785	.9770	.9755	.9740	.9725	.9711	.9696	.9681	.9667	730
740	.9978	.9960	.9942	.9926	.9910	.9894	.9878	.9863	.9847	.9832	.9817	740	.9802	.9787	.9773	.9758	.9743	.9729	.9714	.9700	.9686	.9671	740
750	.9979	.9960	.9943	.9927	.9911	.9895	.9880	.9864	.9849	.9834	.9820	750	.9805	.9790	.9776	.9761	.9747	.9732	.9718	.9704	.9690	.9676	750
760	.9979	.9961	.9944	.9928	.9912	.9897	.9881	.9866	.9851	.9837	.9822	760	.9807	.9793	.9779	.9764	.9750	.9736	.9722	.9708	.9694	.9680	760
770	.9979	.9961	.9945	.9929	.9913	.9898	.9883	.9868	.9853	.9839	.9824	770	.9810	.9796	.9781	.9767	.9753	.9739	.9725	.9712	.9698	.9684	770
780	.9979	.9962	.9945	.9930	.9914	.9899	.9884	.9870	.9855	.9841	.9827	780	.9812	.9798	.9784	.9770	.9756	.9743	.9729	.9715	.9702	.9688	780
790	.9980	.9962	.9946	.9930	.9915	.9900	.9886	.9871	.9857	.9843	.9829	790	.9815	.9801	.9787	.9773	.9759	.9746	.9732	.9719	.9705	.9692	790
800	.9980	.9963	.9947	.9931	.9916	.9902	.9887	.9873	.9859	.9845	.9831	800	.9817	.9803	.9790	.9776	.9762	.9749	.9736	.9722	.9709	.9696	800

T* = NORMALIZED TEST TIME. **If Testing is truncated by failure, enter 1 failure less than those observed to obtain Reliability Lower Bound.

Figure E-2. Reliability Lower Bound for Exponential Components with Failures Truncated by Test Time** (Continued)

80% CONFIDENCE

80% CONFIDENCE

T*	FAILURES											T*	FAILURES										T*
	0	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	16	17	18	19	20	
850	.9981	.9965	.9950	.9935	.9921	.9907	.9894	.9880	.9867	.9854	.9841	850	.9828	.9815	.9802	.9789	.9776	.9764	.9751	.9738	.9726	.9713	850
900	.9982	.9967	.9953	.9939	.9926	.9913	.9900	.9887	.9874	.9862	.9849	900	.9837	.9825	.9813	.9801	.9789	.9777	.9765	.9753	.9741	.9729	900
950	.9983	.9969	.9955	.9942	.9930	.9917	.9905	.9893	.9881	.9869	.9857	950	.9846	.9834	.9823	.9811	.9800	.9788	.9777	.9766	.9754	.9743	950
1000	.9984	.9970	.9957	.9945	.9933	.9921	.9910	.9898	.9887	.9876	.9864	1000	.9853	.9842	.9831	.9820	.9810	.9799	.9788	.9777	.9766	.9756	1000
1050	.9985	.9972	.9959	.9948	.9936	.9925	.9914	.9903	.9892	.9881	.9871	1050	.9860	.9850	.9839	.9829	.9818	.9808	.9798	.9788	.9777	.9767	1050
1100	.9985	.9973	.9961	.9950	.9939	.9928	.9918	.9907	.9897	.9887	.9877	1100	.9867	.9857	.9847	.9837	.9827	.9817	.9807	.9797	.9787	.9778	1100
1150	.9986	.9974	.9963	.9952	.9942	.9931	.9921	.9911	.9902	.9892	.9882	1150	.9872	.9863	.9853	.9844	.9834	.9825	.9815	.9806	.9797	.9787	1150
1200	.9987	.9975	.9964	.9954	.9944	.9934	.9925	.9915	.9906	.9896	.9887	1200	.9878	.9868	.9859	.9850	.9841	.9832	.9823	.9814	.9805	.9796	1200
1250	.9987	.9976	.9966	.9956	.9946	.9937	.9928	.9918	.9909	.9900	.9891	1250	.9882	.9874	.9865	.9856	.9847	.9839	.9830	.9821	.9813	.9804	1250
1300	.9988	.9977	.9967	.9958	.9948	.9939	.9930	.9922	.9913	.9904	.9896	1300	.9887	.9878	.9870	.9862	.9853	.9845	.9836	.9828	.9820	.9812	1300
1350	.9988	.9978	.9968	.9959	.9950	.9942	.9933	.9924	.9916	.9908	.9899	1350	.9891	.9883	.9875	.9867	.9859	.9850	.9842	.9834	.9826	.9818	1350
1400	.9989	.9979	.9969	.9961	.9952	.9944	.9935	.9927	.9919	.9911	.9903	1400	.9895	.9887	.9879	.9871	.9864	.9856	.9848	.9840	.9833	.9825	1400
1450	.9989	.9979	.9971	.9962	.9954	.9946	.9938	.9930	.9922	.9914	.9906	1450	.9899	.9891	.9883	.9876	.9868	.9861	.9853	.9846	.9838	.9831	1450
1500	.9989	.9980	.9972	.9963	.9955	.9947	.9940	.9932	.9924	.9917	.9909	1500	.9902	.9895	.9887	.9880	.9873	.9865	.9858	.9851	.9844	.9836	1500
1550	.9990	.9981	.9973	.9964	.9957	.9949	.9942	.9934	.9927	.9920	.9912	1550	.9905	.9898	.9891	.9884	.9877	.9870	.9863	.9856	.9849	.9842	1550
1600	.9990	.9981	.9973	.9966	.9958	.9951	.9943	.9936	.9929	.9922	.9915	1600	.9908	.9901	.9894	.9887	.9881	.9874	.9867	.9860	.9853	.9847	1600
1650	.9990	.9982	.9974	.9967	.9959	.9952	.9945	.9938	.9931	.9924	.9918	1650	.9911	.9904	.9897	.9891	.9884	.9877	.9871	.9864	.9858	.9851	1650
1700	.9991	.9982	.9975	.9968	.9961	.9954	.9947	.9940	.9933	.9927	.9920	1700	.9913	.9907	.9900	.9894	.9888	.9881	.9875	.9868	.9862	.9856	1700
1750	.9991	.9983	.9976	.9969	.9962	.9955	.9948	.9942	.9935	.9929	.9922	1750	.9916	.9910	.9903	.9897	.9891	.9884	.9878	.9872	.9866	.9860	1750
1800	.9991	.9983	.9976	.9969	.9963	.9956	.9950	.9943	.9937	.9931	.9924	1800	.9918	.9912	.9906	.9900	.9894	.9888	.9882	.9876	.9870	.9864	1800
1850	.9991	.9984	.9977	.9970	.9964	.9957	.9951	.9945	.9939	.9933	.9926	1850	.9920	.9914	.9908	.9903	.9897	.9891	.9885	.9879	.9873	.9867	1850
1900	.9992	.9984	.9978	.9971	.9965	.9958	.9952	.9946	.9940	.9934	.9928	1900	.9923	.9917	.9911	.9905	.9899	.9894	.9888	.9882	.9876	.9871	1900
1950	.9992	.9985	.9978	.9972	.9966	.9960	.9954	.9948	.9942	.9936	.9930	1950	.9925	.9919	.9913	.9907	.9902	.9896	.9891	.9885	.9880	.9874	1950
2000	.9992	.9985	.9979	.9972	.9966	.9961	.9955	.9949	.9943	.9938	.9932	2000	.9926	.9921	.9915	.9910	.9904	.9899	.9893	.9888	.9883	.9877	2000
2050	.9992	.9985	.9979	.9973	.9967	.9962	.9956	.9950	.9945	.9939	.9934	2050	.9928	.9923	.9917	.9912	.9907	.9901	.9896	.9891	.9885	.9880	2050
2100	.9992	.9986	.9980	.9974	.9969	.9962	.9957	.9951	.9946	.9941	.9935	2100	.9930	.9925	.9919	.9914	.9909	.9904	.9898	.9893	.9888	.9883	2100
2150	.9993	.9986	.9980	.9974	.9969	.9963	.9958	.9953	.9947	.9942	.9937	2150	.9932	.9926	.9921	.9916	.9911	.9906	.9901	.9896	.9891	.9886	2150
2200	.9993	.9986	.9981	.9975	.9969	.9964	.9959	.9954	.9948	.9943	.9938	2200	.9933	.9928	.9923	.9918	.9913	.9908	.9903	.9898	.9893	.9888	2200
2250	.9993	.9987	.9981	.9976	.9970	.9965	.9960	.9955	.9950	.9945	.9940	2250	.9935	.9930	.9925	.9920	.9915	.9910	.9905	.9900	.9896	.9891	2250
2300	.9993	.9987	.9981	.9976	.9971	.9966	.9961	.9956	.9951	.9946	.9941	2300	.9936	.9931	.9926	.9922	.9917	.9912	.9907	.9902	.9898	.9893	2300
2350	.9993	.9987	.9982	.9977	.9971	.9966	.9961	.9957	.9952	.9947	.9942	2350	.9937	.9933	.9928	.9923	.9918	.9914	.9909	.9905	.9900	.9895	2350
2400	.9993	.9988	.9982	.9977	.9972	.9967	.9962	.9957	.9953	.9948	.9943	2400	.9939	.9934	.9929	.9925	.9920	.9916	.9911	.9907	.9902	.9897	2400
2450	.9993	.9988	.9983	.9978	.9973	.9968	.9963	.9958	.9954	.9949	.9944	2450	.9940	.9935	.9931	.9926	.9922	.9917	.9913	.9908	.9904	.9900	2450
2500	.9994	.9988	.9983	.9978	.9973	.9968	.9964	.9959	.9955	.9950	.9946	2500	.9941	.9937	.9932	.9928	.9923	.9919	.9915	.9910	.9906	.9902	2500
2550	.9994	.9988	.9983	.9978	.9974	.9969	.9954	.9960	.9955	.9951	.9947	2550	.9942	.9938	.9934	.9929	.9925	.9921	.9916	.9912	.9908	.9903	2550
2600	.9994	.9988	.9984	.9979	.9974	.9970	.9965	.9961	.9956	.9952	.9948	2600	.9943	.9939	.9935	.9931	.9926	.9922	.9918	.9914	.9910	.9905	2600
2650	.9994	.9989	.9984	.9979	.9975	.9970	.9966	.9961	.9957	.9953	.9949	2650	.9944	.9940	.9936	.9932	.9928	.9924	.9919	.9915	.9911	.9907	2650
2700	.9994	.9989	.9984	.9980	.9975	.9971	.9966	.9962	.9958	.9954	.9950	2700	.9945	.9941	.9937	.9933	.9929	.9925	.9921	.9917	.9913	.9909	2700
2750	.9994	.9989	.9984	.9980	.9976	.9971	.9967	.9963	.9959	.9955	.9950	2750	.9946	.9942	.9938	.9934	.9930	.9926	.9922	.9918	.9914	.9910	2750
2800	.9994	.9989	.9985	.9980	.9976	.9972	.9968	.9964	.9959	.9955	.9951	2800	.9947	.9943	.9939	.9935	.9932	.9928	.9924	.9920	.9916	.9912	2800
2850	.9994	.9989	.9985	.9981	.9976	.9972	.9968	.9964	.9960	.9956	.9952	2850	.9948	.9944	.9940	.9937	.9933	.9929	.9925	.9921	.9917	.9914	2850
2900	.9994	.9990	.9985	.9981	.9977	.9973	.9969	.9965	.9961	.9957	.9953	2900	.9949	.9945	.9942	.9938	.9934	.9930	.9926	.9923	.9919	.9915	2900
2950	.9995	.9990	.9986	.9981	.9977	.9973	.9969	.9965	.9961	.9958	.9954	2950	.9950	.9946	.9942	.9939	.9935	.9931	.9928	.9924	.9920	.9917	2950
3000	.9995	.9990	.9986	.9982	.9978	.9974	.9970	.9966	.9962	.9958	.9955	3000	.9951	.9947	.9943	.9940	.9936	.9932	.9929	.9925	.9922	.9918	3000
3050	.9995	.9990	.9986	.9982	.9978	.9974	.9970	.9967	.9963	.9959	.9955	3050	.9952	.9948	.9944	.9941	.9937	.9934	.9930	.9926	.9923	.9919	3050
3100	.9995	.9990	.9986	.9982	.9978	.9975	.9971	.9967	.9963	.9960	.9956	3100	.9952	.9949	.9945	.9942	.9938	.9935	.9931	.9928	.9924	.9921	3100
3150	.9995	.9990	.9986	.9983	.9979	.9975	.9971	.9968	.9964	.9960	.9957	3150	.9953	.9950	.9946	.9943	.9939	.9936	.9932	.9929	.9925	.9922	3150
3200	.9995	.9991	.9987	.9983	.9979	.9975	.9972	.9968	.9965	.9961	.9957	3200	.9954	.9950	.9947	.9944	.9940	.9937	.9933	.9930	.9926	.9923	3200
3250	.9995	.9991	.9987	.9983	.9979	.9976	.9972	.9969	.9965	.9962	.9958	3250	.9955	.9951	.9948	.9944	.9941	.9938	.9934	.9931	.9928	.9924	3250
3300	.9995	.9991	.9987	.9983	.9980	.9976	.9973	.9969	.9966	.9962	.9959	3300	.9955	.9952	.9949	.9945	.9942	.9939	.9935	.9932	.9929	.9925	3300

T* = NORMALIZED TEST TIME. **If Testing is truncated by failure, enter 1 failure less than those observed to obtain Reliability Lower Bound.

Figure E-2. Reliability Lower Bound for Exponential Components with Failures Truncated by Test Time** (Continued)

80% CONFIDENCE

80% CONFIDENCE

T*	FAILURES											T*	FAILURES										T*
	0	1	2	3	4	5	6	7	8	9	10		11	12	13	14	15	16	17	18	19	20	
3400	.9995	.9991	.9987	.9984	.9980	.9977	.9973	.9970	.9967	.9963	.9960	3400	.9957	.9953	.9950	.9947	.9944	.9940	.9937	.9934	.9931	.9928	3400
3500	.9995	.9991	.9988	.9984	.9981	.9977	.9974	.9971	.9968	.9964	.9961	3500	.9958	.9955	.9952	.9948	.9945	.9942	.9939	.9936	.9933	.9930	3500
3600	.9996	.9992	.9988	.9985	.9981	.9978	.9975	.9972	.9968	.9965	.9962	3600	.9959	.9956	.9953	.9950	.9947	.9944	.9941	.9938	.9935	.9932	3600
3700	.9996	.9992	.9988	.9985	.9982	.9979	.9976	.9972	.9969	.9966	.9963	3700	.9960	.9957	.9954	.9951	.9948	.9945	.9942	.9939	.9936	.9933	3700
3800	.9996	.9992	.9989	.9985	.9982	.9979	.9976	.9973	.9970	.9967	.9964	3800	.9961	.9958	.9955	.9952	.9950	.9947	.9944	.9941	.9938	.9935	3800
3900	.9996	.9992	.9989	.9986	.9983	.9980	.9977	.9974	.9971	.9968	.9965	3900	.9962	.9959	.9956	.9954	.9951	.9948	.9945	.9942	.9940	.9937	3900
4000	.9996	.9993	.9989	.9986	.9983	.9980	.9977	.9974	.9972	.9969	.9966	4000	.9963	.9960	.9958	.9955	.9952	.9949	.9947	.9944	.9941	.9938	4000
4100	.9996	.9993	.9990	.9987	.9984	.9981	.9978	.9975	.9972	.9970	.9967	4100	.9964	.9961	.9959	.9956	.9953	.9951	.9948	.9945	.9943	.9940	4100
4200	.9996	.9993	.9990	.9987	.9984	.9981	.9978	.9976	.9973	.9970	.9968	4200	.9965	.9962	.9960	.9957	.9954	.9952	.9949	.9946	.9944	.9941	4200
4300	.9996	.9993	.9990	.9987	.9984	.9982	.9979	.9976	.9974	.9971	.9968	4300	.9966	.9963	.9961	.9958	.9955	.9953	.9950	.9948	.9945	.9943	4300
4400	.9996	.9993	.9990	.9987	.9985	.9982	.9979	.9977	.9974	.9972	.9969	4400	.9966	.9964	.9961	.9959	.9956	.9954	.9951	.9949	.9946	.9944	4400
4500	.9996	.9993	.9990	.9988	.9985	.9982	.9980	.9977	.9975	.9972	.9970	4500	.9967	.9965	.9962	.9960	.9957	.9955	.9952	.9950	.9948	.9945	4500
4600	.9997	.9993	.9991	.9988	.9985	.9983	.9980	.9978	.9975	.9973	.9970	4600	.9968	.9965	.9963	.9961	.9958	.9956	.9954	.9951	.9949	.9946	4600
4700	.9997	.9994	.9991	.9988	.9986	.9983	.9981	.9978	.9976	.9973	.9971	4700	.9969	.9966	.9964	.9962	.9959	.9957	.9954	.9952	.9950	.9948	4700
4800	.9997	.9994	.9991	.9989	.9986	.9984	.9981	.9979	.9976	.9974	.9972	4800	.9969	.9967	.9965	.9962	.9960	.9958	.9955	.9953	.9951	.9949	4800
4900	.9997	.9994	.9991	.9989	.9986	.9984	.9981	.9979	.9977	.9974	.9972	4900	.9970	.9968	.9965	.9963	.9961	.9959	.9956	.9954	.9952	.9950	4900
5000	.9997	.9994	.9991	.9989	.9987	.9984	.9982	.9980	.9977	.9975	.9973	5000	.9970	.9968	.9966	.9964	.9962	.9959	.9957	.9955	.9953	.9951	5000
5100	.9997	.9994	.9992	.9989	.9987	.9985	.9982	.9980	.9978	.9975	.9973	5100	.9971	.9969	.9967	.9965	.9962	.9960	.9958	.9956	.9954	.9952	5100
5200	.9997	.9994	.9992	.9989	.9987	.9985	.9983	.9980	.9978	.9976	.9974	5200	.9972	.9969	.9967	.9965	.9963	.9961	.9959	.9957	.9955	.9953	5200
5300	.9997	.9994	.9992	.9990	.9987	.9985	.9983	.9981	.9979	.9976	.9974	5300	.9972	.9970	.9968	.9966	.9964	.9962	.9960	.9958	.9956	.9953	5300
5400	.9997	.9994	.9992	.9990	.9988	.9985	.9983	.9981	.9979	.9977	.9975	5400	.9973	.9971	.9969	.9966	.9964	.9962	.9960	.9958	.9956	.9954	5400
5500	.9997	.9995	.9992	.9990	.9988	.9986	.9984	.9981	.9979	.9977	.9975	5500	.9973	.9971	.9969	.9967	.9965	.9963	.9961	.9959	.9957	.9955	5500
5600	.9997	.9995	.9992	.9990	.9988	.9986	.9984	.9987	.9980	.9978	.9976	5600	.9974	.9972	.9970	.9968	.9966	.9964	.9962	.9960	.9958	.9956	5600
5700	.9997	.9995	.9992	.9990	.9988	.9986	.9984	.9982	.9980	.9978	.9976	5700	.9974	.9972	.9970	.9968	.9966	.9964	.9962	.9961	.9959	.9957	5700
5800	.9997	.9995	.9993	.9990	.9988	.9986	.9984	.9982	.9980	.9978	.9976	5800	.9975	.9973	.9971	.9969	.9967	.9965	.9963	.9961	.9959	.9957	5800
5900	.9997	.9995	.9993	.9991	.9989	.9987	.9985	.9983	.9981	.9979	.9977	5900	.9975	.9973	.9971	.9969	.9967	.9966	.9964	.9962	.9960	.9958	5900
6000	.9997	.9995	.9993	.9991	.9989	.9987	.9985	.9983	.9981	.9979	.9977	6000	.9975	.9974	.9972	.9970	.9968	.9966	.9964	.9963	.9961	.9959	6000
6100	.9997	.9995	.9993	.9991	.9989	.9987	.9985	.9983	.9981	.9979	.9978	6100	.9976	.9974	.9972	.9970	.9969	.9967	.9965	.9963	.9961	.9960	6100
6200	.9997	.9995	.9993	.9991	.9989	.9987	.9985	.9984	.9982	.9980	.9978	6200	.9976	.9974	.9973	.9971	.9969	.9967	.9965	.9964	.9962	.9960	6200
6300	.9997	.9995	.9993	.9991	.9989	.9987	.9986	.9984	.9982	.9980	.9978	6300	.9977	.9975	.9973	.9971	.9970	.9968	.9966	.9964	.9963	.9961	6300
6400	.9997	.9995	.9993	.9991	.9990	.9988	.9986	.9984	.9982	.9980	.9979	6400	.9977	.9975	.9973	.9972	.9970	.9968	.9967	.9965	.9963	.9961	6400
6500	.9998	.9995	.9993	.9992	.9990	.9988	.9986	.9984	.9983	.9981	.9979	6500	.9977	.9976	.9974	.9972	.9970	.9969	.9967	.9965	.9964	.9962	6500
6600	.9998	.9995	.9994	.9992	.9990	.9988	.9986	.9985	.9983	.9981	.9979	6600	.9978	.9976	.9974	.9973	.9971	.9969	.9968	.9966	.9964	.9963	6600
6700	.9998	.9996	.9994	.9992	.9990	.9988	.9986	.9985	.9983	.9981	.9980	6700	.9978	.9976	.9975	.9973	.9971	.9970	.9968	.9966	.9965	.9963	6700
6800	.9998	.9996	.9994	.9992	.9990	.9988	.9987	.9985	.9983	.9982	.9980	6800	.9978	.9977	.9975	.9973	.9972	.9970	.9969	.9967	.9965	.9964	6800
6900	.9998	.9996	.9994	.9992	.9990	.9989	.9987	.9985	.9984	.9982	.9980	6900	.9979	.9977	.9975	.9974	.9972	.9971	.9969	.9967	.9966	.9964	6900
7000	.9998	.9996	.9994	.9992	.9990	.9989	.9987	.9985	.9984	.9982	.9981	7000	.9979	.9977	.9976	.9974	.9973	.9971	.9969	.9968	.9966	.9965	7000
7100	.9998	.9996	.9994	.9992	.9991	.9989	.9987	.9986	.9984	.9982	.9981	7100	.9979	.9978	.9976	.9975	.9973	.9971	.9970	.9968	.9967	.9965	7100
7200	.9998	.9996	.9994	.9992	.9991	.9989	.9987	.9986	.9984	.9983	.9981	7200	.9979	.9978	.9976	.9975	.9973	.9972	.9970	.9969	.9967	.9966	7200
7300	.9998	.9996	.9994	.9992	.9991	.9989	.9988	.9986	.9984	.9983	.9981	7300	.9980	.9978	.9977	.9975	.9974	.9972	.9971	.9969	.9968	.9966	7300
7800	.9998	.9995	.9995	.9993	.9991	.9990	.9988	.9987	.9985	.9984	.9983	7800	.9981	.9980	.9978	.9977	.9975	.9974	.9973	.9971	.9970	.9968	7800
8300	.9998	.9996	.9995	.9993	.9992	.9990	.9989	.9988	.9986	.9985	.9984	8300	.9982	.9981	.9980	.9978	.9977	.9976	.9974	.9973	.9972	.9970	8300
8800	.9998	.9997	.9995	.9994	.9992	.9991	.9990	.9988	.9987	.9986	.9985	8800	.9983	.9982	.9981	.9979	.9978	.9977	.9976	.9974	.9973	.9972	8800
9300	.9998	.9997	.9995	.9994	.9993	.9992	.9990	.9989	.9988	.9987	.9985	9300	.9984	.9983	.9982	.9981	.9979	.9978	.9977	.9976	.9975	.9973	9300
9800	.9998	.9997	.9996	.9994	.9993	.9992	.9991	.9990	.9988	.9987	.9986	9800	.9985	.9984	.9983	.9982	.9980	.9979	.9978	.9977	.9976	.9975	9800
10300	.9998	.9997	.9996	.9995	.9993	.9992	.9991	.9990	.9989	.9988	.9987	10300	.9986	.9985	.9983	.9982	.9981	.9980	.9979	.9978	.9977	.9976	10300
10800	.9999	.9997	.9996	.9995	.9994	.9993	.9992	.9991	.9989	.9988	.9987	10800	.9986	.9985	.9984	.9983	.9982	.9981	.9980	.9979	.9978	.9977	10800
11300	.9999	.9997	.9996	.9995	.9994	.9993	.9992	.9991	.9990	.9989	.9988	11300	.9987	.9986	.9985	.9984	.9983	.9982	.9981	.9980	.9979	.9978	11300
11800	.9999	.9997	.9996	.9995	.9994	.9993	.9992	.9991	.9990	.9989	.9988	11800	.9987	.9987	.9986	.9985	.9984	.9983	.9982	.9981	.9980	.9979	11800
12300	.9999	.9998	.9997	.9996	.9995	.9994	.9993	.9992	.9991	.9990	.9989	12300	.9988	.9987	.9986	.9985	.9984	.9983	.9983	.9982	.9981	.9980	12300

T* = NORMALIZED TEST TIME. **If Testing is truncated by failure, enter 1 failure less than those observed to obtain Reliability Lower Bound.

Figure E-2. Reliability Lower Bound for Exponential Components with Failures Truncated by Test Time** (Continued)

Number of Trials, N	Number of Failures, X									
	0	1	2	3	4	5	6	7	8	9
1	.2000									
2	.4472	.1056								
3	.5848	.2872	.0717							
4	.6687	.4175	.2123	.0542						
5	.7248	.5098	.3266	.1686	.0436					
6	.7647	.5776	.4146	.2686	.1399	.0365				
7	.7946	.6291	.4833	.3501	.2283	.1195	.0314			
8	.8178	.6696	.5379	.4164	.3033	.1986	.1044	.0275		
9	.8363	.7022	.5823	.4708	.3661	.2676	.1757	.0926	.0245	
10	.8513	.7290	.6191	.5163	.4191	.3268	.2394	.1577	.0833	.0220
11	.8639	.7514	.6499	.5548	.4643	.3779	.2953	.2168	.1429	.0756
12	.8745	.7704	.6762	.5876	.5031	.4221	.3441	.2693	.1980	.1307
13	.8836	.7867	.6988	.6160	.5369	.4606	.3870	.3160	.2476	.1822
14	.8914	.8009	.7186	.6408	.5664	.4944	.4248	.3574	.2921	.2291
15	.8983	.8132	.7358	.6626	.5924	.5244	.4585	.3944	.3321	.2716
16	.9043	.8242	.7511	.6820	.6155	.5511	.4885	.4275	.3680	.3101
17	.9097	.8339	.7648	.6993	.6361	.5750	.5153	.4572	.4005	.3450
18	.9145	.8426	.7770	.7147	.6547	.5965	.5396	.4841	.4299	.3767
19	.9188	.8505	.7880	.7287	.6715	.6159	.5616	.5086	.4566	.4056
20	.9227	.8575	.7980	.7414	.6867	.6335	.5816	.5308	.4809	.4320
21	.9262	.8641	.8071	.7529	.7006	.6497	.5999	.5511	.5032	.4562
22	.9295	.8699	.8154	.7635	.7133	.6645	.6167	.5698	.5238	.4786
23	.9324	.8753	.8230	.7732	.7250	.6780	.6321	.5870	.5427	.4991
24	.9352	.8803	.8300	.7821	.7358	.6906	.6463	.6029	.5602	.5182
25	.9377	.8849	.8365	.7904	.7457	.7022	.6596	.6177	.5764	.5358
26	.9400	.8891	.8425	.7980	.7550	.7130	.6718	.6314	.5915	.5523
27	.9421	.8931	.8481	.8051	.7635	.7230	.6832	.6441	.6056	.5676
28	.9441	.8968	.8533	.8118	.7716	.7323	.6939	.6560	.6187	.5819
29	.9460	.9003	.8582	.8180	.7791	.7411	.7038	.6671	.6310	.5953
30	.9478	.9034	.8627	.8238	.7861	.7492	.7131	.6775	.6425	.6079
31	.9494	.9065	.8670	.8293	.7927	.7569	.7218	.6874	.6533	.6197
32	.9510	.9093	.8710	.8344	.7989	.7642	.7301	.6966	.6635	.6309
33	.9524	.9120	.8747	.8392	.8047	.7710	.7379	.7053	.6732	.6414
34	.9538	.9145	.8783	.8437	.8102	.7774	.7452	.7135	.6822	.6513
35	.9550	.9169	.8817	.8480	.8154	.7835	.7522	.7213	.6909	.6607
36	.9563	.9191	.8849	.8521	.8203	.7892	.7587	.7287	.6990	.6697
37	.9574	.9212	.8879	.8560	.8250	.7947	.7650	.7356	.7067	.6781
38	.9585	.9233	.8907	.8596	.8295	.7999	.7709	.7423	.7141	.6862
39	.9596	.9251	.8934	.8631	.8337	.8048	.7765	.7486	.7211	.6938
40	.9605	.9270	.8960	.8664	.8377	.8096	.7819	.7546	.7277	.7011

Figure E-3. Binominal Tables (80% confidence)

	Number of Failures, X									
Number of Trials, N	0	1	2	3	4	5	6	7	8	9
42	.9624	.9304	.9008	.8726	.8451	.8183	.7919	.7658	.7401	.7147
44	.9641	.9335	.9052	.8782	.8520	.8262	.8010	.7761	.7514	.7270
46	.9656	.9363	.9093	.8833	.8582	.8336	.8094	.7854	.7618	.7384
48	.9670	.9389	.9130	.8881	.8639	.8403	.8170	.7941	.7714	.7489
50	.9684	.9413	.9164	.8925	.8692	.8465	.8241	.8020	.7802	.7586
52	.9695	.9435	.9195	.8965	.8741	.8522	.8306	.8094	.7883	.7675
54	.9706	.9456	.9224	.9003	.8787	.8575	.8367	.8162	.7959	.7758
56	.9716	.9475	.9251	.9037	.8829	.8625	.8424	.8226	.8030	.7836
58	.9726	.9493	.9277	.9070	.8869	.8671	.8477	.8285	.8096	.7908
60	.9735	.9509	.9301	.9100	.8905	.8714	.8526	.8341	.8158	.7975
62	.9744	.9525	.9323	.9128	.8939	.8754	.8573	.8393	.8215	.8039
64	.9752	.9539	.9343	.9155	.8972	.8793	.8616	.8442	.8269	.8098
66	.9759	.9553	.9363	.9180	.9003	.8829	.8657	.8488	.8320	.8154
68	.9766	.9566	.9381	.9204	.9031	.8862	.8696	.8531	.8368	.8207
70	.9773	.9578	.9399	.9226	.9058	.8894	.8732	.8572	.8414	.8257
72	.9779	.9590	.9415	.9247	.9084	.8924	.8766	.8611	.8457	.8304
74	.9785	.9601	.9431	.9268	.9109	.8952	.8799	.8648	.8497	.8349
76	.9790	.9611	.9445	.9286	.9132	.8980	.8830	.8682	.8536	.8392
78	.9796	.9621	.9460	.9304	.9153	.9005	.8859	.8716	.8573	.8432
80	.9801	.9631	.9473	.9322	.9174	.9030	.8887	.8747	.8608	.8470
82	.9806	.9639	.9485	.9337	.9194	.9053	.8914	.8777	.8641	.8507
84	.9810	.9648	.9498	.9353	.9213	.9075	.8939	.8806	.8673	.8541
86	.9815	.9656	.9509	.9368	.9231	.9096	.8964	.8833	.8703	.8575
88	.9819	.9663	.9520	.9382	.9248	.9117	.8987	.8859	.8732	.8606
90	.9823	.9671	.9530	.9396	.9264	.9136	.9009	.8884	.8759	.8637
92	.9826	.9678	.9540	.9409	.9280	.9154	.9030	.8907	.8786	.8666
94	.9830	.9685	.9550	.9421	.9295	.9172	.9051	.8930	.8811	.8693
96	.9834	.9691	.9559	.9433	.9310	.9189	.9070	.8952	.8836	.8720
98	.9837	.9698	.9568	.9444	.9323	.9205	.9088	.8973	.8859	.8745
100	.9840	.9703	.9577	.9455	.9337	.9221	.9107	.8993	.8881	.8770
102	.9843	.9709	.9585	.9466	.9350	.9236	.9124	.9013	.8903	.8794
104	.9846	.9715	.9593	.9476	.9362	.9251	.9140	.9031	.8923	.8817
106	.9850	.9720	.9601	.9486	.9374	.9265	.9156	.9049	.8944	.8838
108	.9852	.9725	.9608	.9495	.9385	.9278	.9171	.9067	.8963	.8860
110	.9855	.9730	.9615	.9504	.9397	.9291	.9187	.9083	.8982	.8880
112	.9857	.9735	.9622	.9513	.9407	.9303	.9201	.9099	.8999	.8900
114	.9860	.9740	.9628	.9521	.9417	.9315	.9215	.9115	.9016	.8919
116	.9862	.9744	.9635	.9529	.9427	.9327	.9228	.9130	.9033	.8937
118	.9865	.9749	.9641	.9538	.9437	.9338	.9241	.9145	.9049	.8955
120	.9867	.9752	.9647	.9545	.9446	.9349	.9253	.9159	.9065	.8972

Figure E-3. Binomial Tables (80% confidence) (Continued)

	Reliability							
n	.75	.90	.95	.975	.99	.999	.9999	.99999
all	0.6745	1.2816	1.6449	1.9600	2.3263	3.0902	3.7190	4.2648

Figure E-4. One-Sided Tolerance Factors for the Normal Distribution at 50% Confidence

	RELIABILITY							
N	.75	.90	.95	.975	.99	.999	.9999	.99999
2	2.0111	3.3782	4.2338	4.9867	5.8700	7.7271	9.2645	10.6026
3	1.4567	8.4037	2.9960	3.5181	4.1316	5.4238	6.4951	7.4283
4	1.2842	2.1240	2.6470	3.1077	3.6488	4.7885	5.7334	6.5566
5	1.1916	1.9809	2.4708	2.9019	3.4080	4.4736	5.3568	6.1262
6	1.1313	1.8909	2.3610	2.7744	3.2594	4.2800	5.1258	5.8625
7	1.0881	1.8278	2.2846	2.6859	3.1566	4.1467	4.9670	5.6813
8	1.0551	1.7805	2.2276	2.6202	3.0804	4.0482	4.8497	5.5477
9	1.0288	1.7433	2.1831	2.5690	3.0212	3.9717	4.7589	5.4442
10	1.0073	1.7132	2.1471	2.5276	2.9735	3.9103	4.6859	5.3611
11	0.9892	1.6882	2.1172	2.4934	2.9340	3.8596	4.6258	5.2927
12	0.9737	1.6669	2.0920	2.4645	2.9008	3.8169	4.5752	5.2351
13	0.9603	1.6486	2.0703	2.4397	2.8722	3.7804	4.5318	5.1858
14	0.9485	1.6326	2.0513	2.4181	2.8474	3.7486	4.4942	5.1430
15	0.9381	1.6184	2.0347	2.3991	2.8255	3.7206	4.4611	5.1054
16	0.9287	1.6058	2.0198	2.3822	2.8061	3.6959	4.4318	5.0721
17	0.9203	1.5945	2.0065	2.3670	2.7887	3.6737	4.4055	5.0423
18	0.9126	1.5843	1.9945	2.3533	2.7731	3.6537	4.3819	5.0154
19	0.9056	1.5750	1.9835	2.3409	2.7588	3.6355	4.3604	4.9911
20	0.8992	1.5664	1.9735	2.3295	2.7458	3.6190	4.3408	4.9688
21	0.8933	1.5586	1.9643	2.3191	2.7339	3.6038	4.3229	4.9485
22	0.8878	1.5513	1.9558	2.3095	2.7229	3.5898	4.3063	4.9297
23	0.8827	1.5446	1.9480	2.3005	2.7127	3.5768	4.2910	4.9123
24	0.8779	1.5384	1.9407	2.2923	2.7032	3.5648	4.2768	4.8962
25	0.8735	1.5325	1.9339	2.2845	2.6944	3.5536	4.2636	4.8812
26	0.8693	1.5271	1.9275	2.2773	2.6862	3.5431	4.2513	4.8672
27	0.8654	1.5219	1.9215	2.2705	2.6784	3.5333	4.2397	4.8541
28	0.8617	1.5171	1.9159	2.2642	2.6712	3.5241	4.2288	4.8417
29	0.8582	1.5126	1.9106	2.2582	2.6643	3.5154	4.2185	4.8301
30	0.8549	1.5082	1.9055	2.2525	2.6578	3.5072	4.2089	4.8191
31	0.8517	1.5042	1.9008	2.2471	2.6517	3.4994	4.1997	4.8087
32	0.8487	1.5003	1.8963	2.2420	2.6459	3.4920	4.1910	4.7989
34	0.8432	1.4931	1.8879	2.2326	2.6351	3.4784	4.1750	4.7807
36	0.8381	1.4866	1.8803	2.2240	2.6254	3.4661	4.1604	4.7642
38	0.8335	1.4806	1.8734	2.2162	2.6165	3.4548	4.1471	4.7491
40	0.8293	1.4751	1.8670	2.2090	2.6083	3.4444	4.1349	4.7353
42	0.8253	1.4701	1.8612	2.2024	2.6007	3.4349	4.1237	4.7226
44	0.8217	1.4654	1.8557	2.1962	2.5938	3.4261	4.1133	4.7108
46	0.8183	1.4610	1.8507	2.1905	2.5873	3.4179	4.1036	4.6998
48	0.8151	1.4569	1.8459	2.1852	2.5812	3.4102	4.0946	4.6897
50	0.8121	1.4531	1.8415	2.1802	2.5756	3.4031	4.0862	4.6801
52	0.8093	1.4495	1.8374	2.1756	2.5703	3.3964	4.0784	4.6712
54	0.8066	1.4461	1.8335	2.1712	2.5653	3.3901	4.0710	4.6628
56	0.8042	1.4430	1.8298	2.1671	2.5606	3.3842	4.0640	4.6549
58	0.8018	1.4400	1.8263	2.1632	2.5562	3.3786	4.0574	4.6475
60	0.7996	1.4371	1.8231	2.1595	2.5520	3.3733	4.0512	4.6404

Figure E-5. One-Sided Tolerance Factors for the Normal Distribution at 80% Confidence with Sample Size n from 2 to 10,000

N	.75	.90	.95	.975	.99	.999	.9999	.99999
	RELIABILITY							
62	0.7974	1.4344	1.8199	2.1560	2.5480	3.3682	4.0453	4.6337
64	0.7954	1.4319	1.8170	2.1526	2.5442	3.3635	4.0396	4.6274
66	0.7935	1.4294	1.8142	2.1495	2.5406	3.3589	4.0343	4.6213
68	0.7917	1.4271	1.8115	2.1464	2.5372	3.3546	4.0292	4.6156
70	0.7899	1.4249	1.8089	2.1435	2.5339	3.3505	4.0244	4.6101
80	0.7822	1.4151	1.7976	2.1309	2.5195	3.3324	4.0031	4.5860
90	0.7758	1.4070	1.7884	2.1205	2.5077	3.3175	3.9856	4.5662
100	0.7704	1.4003	1.7806	2.1117	2.4978	3.3051	3.9710	4.5497
125	0.7600	1.3872	1.7656	2.0949	2.4787	3.2811	3.9428	4.5177
150	0.7524	1.3776	1.7546	2.0826	2.4648	3.2635	3.9222	4.4945
175	0.7465	1.3703	1.7461	2.0731	2.4540	3.2501	3.9064	4.4766
200	0.7417	1.3643	1.7393	2.0655	2.4454	3.2393	3.8937	4.4623
225	0.7378	1.3595	1.7337	2.0592	2.4384	3.2304	3.8833	4.4505
250	0.7345	1.3553	1.7290	2.0540	2.4324	3.2229	3.8745	4.4406
275	0.7317	1.3518	1.7250	2.0494	2.4273	3.2165	3.8670	4.4321
300	0.7292	1.3488	1.7215	2.0455	2.4229	3.2110	3.8605	4.4247
325	0.7270	1.3460	1.7184	2.0420	2.4190	3.2061	3.8547	4.4182
350	0.7251	1.3436	1.7156	2.0390	2.4155	3.2017	3.8496	4.4124
375	0.7233	1.3415	1.7132	2.0362	2.4124	3.1978	3.8451	4.4072
400	0.7217	1.3395	1.7110	2.0337	2.4096	3.1943	3.8409	4.4026
425	0.7203	1.3378	1.7089	2.0315	2.4070	3.1911	3.8372	4.3983
450	0.7190	1.3362	1.7071	2.0294	2.4047	3.1882	3.8338	4.3945
475	0.7178	1.3347	1.7054	2.0275	2.4025	3.1855	3.8306	4.3909
500	0.7167	1.3333	1.7038	2.0258	2.4006	3.1830	3.8277	4.3876
525	0.7157	1.3320	1.7024	2.0241	2.3987	3.1807	3.8250	4.3846
550	0.7147	1.3309	1.7010	2.0226	2.3970	3.1786	3.8225	4.3818
575	0.7138	1.3297	1.6998	2.0212	2.3954	3.1766	3.8202	4.3791
600	0.7130	1.3287	1.6986	2.0199	2.3939	3.1747	3.8180	4.3767
625	0.7122	1.3277	1.6975	2.0187	2.3925	3.1730	3.8160	4.3743
650	0.7114	1.3268	1.6964	2.0175	2.3912	3.1713	3.8140	4.3722
675	0.7108	1.3260	1.6954	2.0164	2.3900	3.1698	3.8122	4.3701
700	0.7101	1.3252	1.6945	2.0154	0.3888	3.1683	3.8105	4.3682
725	0.7095	1.3244	1.6936	2.0144	2.3877	3.1669	3.8089	4.3663
750	0.7089	1.3237	1.6928	2.0135	2.3867	3.1656	3.8073	4.3646
800	0.7078	1.3223	1.6913	2.0117	2.3847	3.1632	3.8045	4.3614
850	0.7068	1.3211	1.6898	2.0102	2.3829	3.1610	3.8019	4.3584
900	0.7058	1.3199	1.6886	2.0087	2.3813	3.1589	3.7995	4.3557
950	0.7050	1.3189	1.6874	2.0074	2.3798	3.1571	3.7973	4.3532
1000	0.7042	1.3179	1.6863	2.0062	2.3784	3.1553	3.7953	4.3510
1500	0.6987	1.3112	1.6786	1.9976	2.3687	3.1432	3.7811	4.3349
2000	0.6955	1.3072	1.6740	1.9925	2.3630	3.1360	3.7726	4.3254
3000	0.6916	1.3024	1.6686	1.9865	2.3562	3.1275	3.7627	4.3142
4000	0.6893	1.2996	1.6654	1.9829	2.3522	3.1225	3.7568	4.3075
5000	0.6877	1.2977	1.6632	1.9804	2.3494	3.1191	3.7528	4.3030
11000	0.6838	1.2930	1.6578	1.9744	2.3426	3.1106	3.7428	4.2917

Figure E-5. One-Sided Tolerance Factors for the Normal Distribution at 80% Confidence with Sample Size n from 2 to 10,000 (Continued)

	RELIABILITY							
N	.75	.90	.95	.975	.99	.999	.9999	.99999
2	2.1319	3.0483	3.6323	4.1539	4.7737	6.0982	7.2103	8.1858
3	1.6120	2.3049	2.7465	3.1409	3.6095	4.6110	5.4519	6.1896
4	1.4573	2.0837	2.4829	2.8394	3.2631	4.1684	4.9286	5.5954
5	1.3813	1.9751	2.3535	2.6916	3.0930	3.9512	4.6718	5.3039
6	1.3359	1.9101	2.2760	2.6029	2.9912	3.8212	4.5180	5.1293
7	1.3055	1.8667	2.2243	2.5437	2.9232	3.7343	4.4152	5.0126
8	1.2837	1.8355	2.1872	2.5012	2.8744	3.6720	4.3416	4.9290
9	1.2673	1.8121	2.1593	2.4693	2.8377	3.6251	4.2862	4.8661
10	1.2545	1.7938	2.1375	2.4444	2.8091	3.5886	4.2430	4.8170
11	1.2443	1.7792	2.1200	2.4244	2.7862	3.5592	4.2083	4.7777
12	1.2359	1.7671	2.1057	2.4080	2.7673	3.5352	4.1798	4.7454
13	1.2288	1.7571	2.0937	2.3944	2.7516	3.5151	4.1561	4.7184
14	1.2229	1.7486	2.0836	2.3827	2.7383	3.4980	4.1359	4.6955
15	1.2178	1.7413	2.0748	2.3728	2.7268	3.4834	4.1186	4.6759
16	1.2133	1.7349	2.0673	3.3641	2.7169	3.4707	4.1036	4.6588
17	1.2094	1.7294	2.0607	2.3566	2.7082	3.4596	4.0905	4.6439
18	1.2060	1.7244	2.0548	2.3499	2.7005	3.4497	4.0789	4.6307
19	1.2030	1.7201	2.0496	2.3439	2.6936	3.4410	4.0685	4.6190
20	1.2002	1.7162	2.0449	2.3386	2.6875	3.4332	4.0592	4.6084
21	1.1977	1.7126	2.0407	2.3338	2.6820	3.4261	4.0509	4.5990
22	1.1955	1.7094	2.0369	2.3294	2.6770	3.4197	4.0433	4.5904
23	1.1935	1.7065	2.0334	2.3254	2.6724	3.4139	4.0365	4.5826
24	1.1916	1.7039	2.0303	2.3216	2.6682	3.4086	4.0302	4.5754
25	1.1899	1.7014	2.0274	2.3185	2.6644	3.4037	4.0244	4.5689
26	1.1883	1.6992	2.0247	2.3154	2.6609	3.3992	4.0191	4.5629
27	1.1869	1.6971	2.0222	2.3126	2.6577	3.3951	4.0142	4.5573
28	1.1856	1.6952	2.0199	2.3100	2.6547	3.3912	4.0097	4.5521
29	1.1843	1.6934	2.0178	2.3076	2.6519	3.3877	4.0054	4.5474
30	1.1831	1.6917	2.0158	2.3053	2.6493	3.3843	4.0015	4.5429
31	1.1821	1.6902	2.0140	2.3032	2.6468	3.3812	3.9978	4.5387
32	1.1810	1.6887	2.0123	2.3012	2.6446	3.3783	3.9944	4.5348
34	1.1792	1.6861	2.0091	2.2976	2.6404	3.3730	3.9882	4.5277
36	1.1776	1.6838	2.0063	2.2944	2.6367	3.3684	3.9826	4.5214
38	1.1761	1.6817	2.0038	2.2916	2.6335	3.3642	3.9777	4.5158
40	1.1748	1.6798	2.0016	2.2890	2.6305	3.3604	3.9732	4.5108
42	1.1736	1.6781	1.9996	2.2867	2.6279	3.3570	3.9692	4.5062
44	1.1725	1.6766	1.9977	2.2846	2.6255	3.3539	3.9656	4.5021
46	1.1715	1.6752	1.9961	2.2827	2.6233	3.3511	3.9623	4.4983
48	1.1706	1.6739	1.9945	2.2809	2.6213	3.3486	3.9592	4.4949
50	1.1698	1.6727	1.9931	2.2793	2.6194	3.3462	3.9564	4.4917
52	1.1691	1.6716	1.9918	2.2779	2.6177	3.3440	3.9539	4.4888
54	1.1684	1.6706	1.9906	2.2765	2.6161	3.3420	3.9515	4.4861
56	1.1677	1.6697	1.9895	2.2752	2.6147	3.3402	3.9493	4.4836
58	1.1671	1.6688	1.9885	2.2740	2.6133	3.3384	3.9472	4.4813
60	1.1665	1.6680	1.9875	2.2729	2.6121	3.3368	3.9453	4.4791

Figure E-6. Two-Sided Tolerance Factors for the Normal Distribution at 50% Confidence with Sample Size n from 2 to 10,000

	RELIABILITY							
N	.75	.90	.95	.975	.99	.999	.9999	.99999
62	1.1660	1.6672	1.9866	2.2719	2.6109	3.3353	3.9435	4.4771
64	1.1655	1.6665	1.9858	2.2709	2.6098	3.3339	3.9419	4.4752
66	1.1650	1.6659	1.9850	2.2700	2.6087	3.3326	3.9403	4.4734
68	1.1646	1.6652	1.9843	2.2692	2.6078	3.3313	3.9388	4.4717
70	1.1642	1.6646	1.9836	2.2684	2.6068	3.3301	3.9374	4.4701
80	1.1624	1.6622	1.9806	2.2650	2.6029	3.3251	3.9315	4.4634
90	1.1611	1.6602	1.9783	2.2623	2.5999	3.3213	3.9269	4.4582
100	1.1600	1.6587	1.9764	2.2602	2.5975	3.3182	3.9233	4.4541
125	1.1581	1.6559	1.9731	2.2564	2.5931	3.3126	3.9167	4.4466
150	1.1568	1.6540	1.9709	2.2539	2.5902	3.3089	3.9123	4.4416
175	1.1559	1.6527	1.9693	2.2521	2.5881	3.3063	3.9092	4.4381
200	1.1552	1.6517	1.9682	2.2508	2.5866	3.3043	3.9069	4.4355
225	1.1546	1.6510	1.9672	2.2497	2.5854	3.3028	3.9051	4.4334
250	1.1542	1.6504	1.9665	2.2489	2.5844	3.3015	3.9036	4.4317
275	1.1538	1.6499	1.9659	2.2482	2.5837	3.3005	3.9024	4.4304
300	1.1536	1.6494	1.9654	2.2476	2.5830	3.2997	3.9014	4.4293
325	1.1533	1.6491	1.9650	2.2472	2.5824	3.2990	3.9006	4.4283
350	1.1531	1.6488	1.9646	2.2468	2.5820	3.2984	3.8999	4.4275
375	1.1529	1.6485	1.9643	2.2464	2.5816	3.2979	3.8993	4.4268
400	1.1528	1.6483	1.9641	2.2461	2.5812	3.2974	3.8997	4.4262
425	1.1526	1.6481	1.9638	2.2458	2.5809	3.2970	3.8982	4.4257
450	1.1525	1.6479	1.9636	2.2456	2.5806	3.2966	3.8978	4.4252
475	1.1524	1.6477	1.9634	2.2453	2.5804	3.2963	3.8974	4.4247
500	1.1523	1.6476	1.9632	2.2451	2.5801	3.2960	3.8971	4.4244
525	1.1522	1.6475	1.9631	2.2450	2.5799	3.2958	3.8968	4.4240
550	1.1521	1.6473	1.9629	2.2448	2.5797	3.2955	3.8965	4.4237
575	1.1520	1.6472	1.9628	2.2447	2.5796	3.2953	3.8962	4.4234
600	1.1519	1.6471	1.9627	2.2445	2.5794	3.2951	3.8960	4.4231
625	1.1519	1.6470	1.9626	2.2444	2.5793	3.2949	3.8958	4.4229
650	1.1518	1.6470	1.9625	2.2443	2.5791	3.2948	3.8956	4.4226
675	1.1518	1.6469	1.9624	2.2442	2.5790	3.2946	3.8954	4.4224
700	1.1517	1.6468	1.9623	2.2441	2.5789	3.2944	3.8952	4.4222
725	1.1517	1.6467	1.9622	2.2440	2.5788	3.2943	3.8951	4.4221
750	1.1516	1.6467	1.9621	2.2439	2.5787	3.2942	3.8949	4.4219
800	1.1515	1.6466	1.9620	2.2437	2.5785	3.2940	3.8947	4.4216
850	1.1515	1.6465	1.9619	2.2436	2.5784	3.2938	3.8944	4.4213
900	1.1514	1.6464	1.9618	2.2435	2.5782	3.2936	3.8942	4.4211
950	1.1514	1.6463	1.9617	2.2434	2.5781	3.2934	3.8940	4.4209
1000	1.1513	1.6462	1.9616	2.2433	2.5780	3.2933	3.8938	4.4207
1500	1.1510	1.6458	1.9611	2.2426	2.5773	3.2924	3.8928	4.4194
2000	1.1508	1.6455	1.9608	2.2423	2.5769	3.2919	3.8922	4.4188
3000	1.1507	1.6453	1.9605	2.2420	2.5765	3.2914	3.8917	4.4182
4000	1.1506	1.6452	1.9604	2.2419	2.5764	3.2912	3.8914	4.4179
5000	1.1505	1.6451	1.9603	2.2418	2.5763	3.2911	3.8912	4.4177
10000	1.1504	1.6450	1.9601	2.2416	2.5760	3.2908	3.8909	4.4173

Figure E-6. Two-Sided Tolerance Factors for the Normal Distribution at 50% Confidence with Sample Size n from 2 to 10,000 (Continued)

	RELIABILITY							
N	.75	.90	.95	.975	.99	.999	.9999	.99999
2	5.6758	8.1156	9.6703	11.0590	12.7090	16.2353	19.1960	21.9731
3	2.8411	4.0624	4.8406	5.5357	6.3617	8.1268	9.6088	10.9089
4	2.2357	3.1968	3.8093	4.3562	5.0062	6.3953	7.5615	8.5846
5	1.9709	2.8182	3.3581	3.8403	4.4133	5.6378	6.6659	7.5678
6	1.8207	2.6033	3.1021	3.5475	4.0768	5.2080	6.1577	6.9908
7	1.7230	2.4637	2.9357	3.3572	3.8582	4.9287	5.8275	6.6159
8	1.6540	2.3651	2.8181	3.2228	3.7037	4.7313	5.5941	6.3510
9	1.6024	2.2913	2.7302	3.1223	3.5881	4.5837	5.4196	6.1528
10	1.5622	2.2338	2.6617	3.0439	3.4981	4.4687	5.2836	5.9985
11	1.5299	2.1876	2.6067	2.9810	3.4258	4.3763	5.1744	5.8745
12	1.5033	2.1496	2.5614	2.9292	3.3662	4.3002	5.0845	5.7724
13	1.4810	2.1177	2.5234	2.8857	3.3162	4.2364	5.0089	5.6866
14	1.4620	2.0904	2.4909	2.8486	3.2736	4.1819	4.9445	5.6135
15	1.4455	2.0669	2.4629	2.8165	3.2367	4.1348	4.8889	5.5503
16	1.4311	2.0463	2.4383	2.7885	3.2045	4.0936	4.8402	5.4950
17	1.4184	2.0281	2.4167	2.7637	3.1761	4.0573	4.7972	5.4462
18	1.4071	2.0120	2.3974	2.7416	3.1507	4.0249	4.7589	5.4028
19	1.3969	1.9974	2.3801	2.7219	3.1280	3.9959	4.7246	5.3638
20	1.3878	1.9843	2.3645	2.7040	3.1075	3.9697	4.6936	5.3286
21	1.3795	1.9724	2.3503	2.6878	3.0888	3.9459	4.6655	5.2967
22	1.3719	1.9616	2.3374	2.6730	3.0718	3.9242	4.6398	5.2675
23	1.3649	1.9516	2.3255	2.6594	3.0562	3.9042	4.6162	5.2408
24	1.3585	1.9425	2.3146	2.6469	3.0419	3.8859	4.5945	5.2162
25	1.3526	1.9340	2.3045	2.6354	3.0286	3.8689	4.5745	5.1934
26	1.3471	1.9261	2.2951	2.6247	3.0163	3.8532	4.5559	5.1723
27	1.3419	1.9188	2.2864	2.6147	3.0048	3.8386	4.5386	5.1526
28	1.3372	1.9120	2.2782	2.6054	2.9941	3.8249	4.5224	5.1343
29	1.3327	1.9056	2.2706	2.5967	2.9841	3.8121	4.5073	5.1171
30	1.3285	1.8996	2.2635	2.5885	2.9747	3.8001	4.4931	5.1010
31	1.3245	1.8939	2.2567	2.5808	2.9659	3.7888	4.4797	5.0858
32	1.3208	1.8886	2.2504	2.5736	2.9575	3.7781	4.4671	5.0715
34	1.3140	1.8766	2.2388	2.5602	2.9422	3.7586	4.4440	5.0453
36	1.3078	1.8700	2.2283	2.5482	2.9285	3.7410	4.4232	5.0217
38	1.3023	1.8621	2.2188	2.5374	2.9160	3.7251	4.4044	5.0003
40	1.2972	1.8549	2.2102	2.5276	2.9047	3.7106	4.3873	4.9809
42	1.2926	1.8482	2.2023	2.5186	2.8943	3.6974	4.3717	4.9631
44	1.2883	1.8422	2.1951	2.5103	2.8848	3.6853	4.3573	4.9468
46	1.2844	1.8366	2.1884	2.5026	2.8761	3.6741	4.3441	4.9318
48	1.2808	1.8314	2.1822	2.4956	2.8679	3.6637	4.3318	4.9179
50	1.2774	1.8266	2.1765	2.4890	2.8604	3.6540	4.3204	4.9049
52	1.2743	1.8221	2.1711	2.4829	2.8534	3.6451	4.3098	4.8929
54	1.2714	1.8179	2.1661	2.4772	2.8468	3.6367	4.2998	4.8816
56	1.2686	1.8139	2.1614	2.4718	2.8406	3.6288	4.2905	4.8710
58	1.2660	1.8102	2.1570	2.4668	2.8348	3.6214	4.2818	4.8611
60	1.2636	1.8068	2.1529	2.4620	2.8294	3.6144	4.2735	4.8517

Figure E-7. Two-Sided Tolerance Factors for the Normal Distribution at 80% Confidence with Sample Size n from 2 to 10,000

	RELIABILITY							
N	.75	.90	.95	.975	.99	.999	.9999	.99999
62	1.2613	1.8035	2.1490	2.4575	2.8242	3.6078	4.2658	4.8429
64	1.2591	1.8004	2.1453	2.4533	2.8194	3.6016	4.2584	4.8346
66	1.2570	1.7974	2.1418	2.4493	2.8147	3.5957	4.2515	4.8267
68	1.2551	1.7946	2.1384	2.4455	2.8104	3.5901	4.2448	4.8192
70	1.2532	1.7920	2.1353	2.4419	2.8062	3.5848	4.2386	4.8120
80	1.2451	1.7804	2.1215	2.4261	2.7881	3.5617	4.2112	4.7810
90	1.2386	1.7711	2.1104	2.4134	2.7735	3.5430	4.1892	4.7559
100	1.2332	1.7633	2.1012	2.4029	2.7614	3.5276	4.1709	4.7352
125	1.2230	1.7487	2.0837	2.3829	2.7384	3.4983	4.1362	4.6958
150	1.2157	1.7382	2.0712	2.3687	2.7221	3.4773	4.1115	4.6677
175	1.2101	1.7303	2.0618	2.3579	2.7097	3.4615	4.0927	4.6465
200	1.2057	1.7240	2.0543	2.3493	2.6998	3.4490	4.0779	4.6296
225	1.2022	1.7189	2.0482	2.3424	2.6918	3.4387	4.0658	4.6159
250	1.1992	1.7147	2.0432	2.3366	2.6852	3.4302	4.0558	4.6045
275	1.1966	1.7111	2.0388	2.3316	2.6795	3.4230	4.0472	4.5948
300	1.1945	1.7079	2.0351	2.3274	2.6746	3.4167	4.0398	4.5864
325	1.1925	1.7052	2.0319	2.3236	2.6703	3.4112	4.0333	4.5790
350	1.1909	1.7028	2.0290	2.3203	2.6665	3.4064	4.0276	4.5725
375	1.1893	1.7006	2.0264	2.3174	2.6632	3.4021	4.0225	4.5667
400	1.1880	1.6987	2.0241	2.3147	2.6601	3.3982	4.0179	4.5615
425	1.1868	1.6969	2.0220	2.3124	2.6574	3.3947	4.0138	4.5568
450	1.1856	1.6953	2.0201	2.3102	2.6549	3.3915	4.0100	4.5525
475	1.1846	1.6939	2.0184	2.3082	2.6526	3.3886	4.0065	4.5486
500	1.1837	1.6925	2.0168	2.3063	2.6505	3.3859	4.0033	4.5450
525	1.1828	1.6913	2.0153	2.3047	2.6485	3.3834	4.0004	4.5416
550	1.1820	1.6901	2.0139	2.3031	2.6467	3.3811	3.9977	4.5385
575	1.1813	1.6890	2.0126	2.3016	2.6450	3.3789	3.9951	4.5356
600	1.1806	1.6880	2.0114	2.3002	2.6435	3.3769	3.9927	4.5329
625	1.1799	1.6871	2.0103	2.2990	2.6420	3.3750	3.9905	4.5304
650	1.1793	1.6862	2.0093	2.2978	2.6406	3.3733	3.9884	4.5281
675	1.1787	1.6854	2.0083	2.2966	2.6393	3.3716	3.9865	4.5258
700	1.1781	1.6846	2.0073	2.2956	2.6381	3.3700	3.9846	4.5237
725	1.1776	1.6839	2.0064	2.2946	2.6369	3.3686	3.9829	4.5217
750	1.1771	1.6832	2.0056	2.2936	2.6358	3.3672	3.9812	4.5199
800	1.1762	1.6819	2.0041	2.2918	2.6338	3.3646	3.9781	4.5164
850	1.1754	1.6807	2.0027	2.2902	2.6319	3.3622	3.9754	4.5132
900	1.1747	1.6796	2.0014	2.2888	2.6303	3.3601	3.9728	4.5103
950	1.1740	1.6786	2.0002	2.2874	2.6287	3.3581	3.9705	4.5077
1000	1.1733	1.6777	1.9991	2.2862	2.6273	3.3563	3.9683	4.5052
1500	1.1689	1.6714	1.9916	2.2776	2.6174	3.3436	3.9534	4.4883
2000	1.1663	1.6677	1.9872	2.2725	2.6116	3.3362	3.9446	4.4783
3000	1.1633	1.6634	1.9820	2.2666	2.6048	3.3275	3.9344	4.4667
4000	1.1615	1.6608	1.9790	2.2631	2.6008	3.3224	3.9283	4.4598
5000	1.1603	1.6591	1.9769	2.2608	2.5981	3.3190	3.9242	4.4552
10000	1.1573	1.6548	1.9719	2.2550	2.5915	3.3105	3.9142	4.4438

Figure E-7. Two-Sided Tolerance Factors for the Normal Distribution at 80% Confidence with Sample Size n from 2 to 10,000 (Continued)

VALUES OF F1

VALUES OF F2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
2	4.0000	4.2360	4.3168	4.3576	4.3822	4.3986	4.4104	4.4192	4.4261	4.4316	4.4361	4.4399	4.4430	4.4458	4.4482
4	2.4721	2.4826	2.4733	2.4654	2.4596	2.4552	2.4517	2.4490	2.4468	2.4450	2.4434	2.4421	2.4410	2.4400	2.4392
6	2.1299	2.0924	2.0619	2.0417	2.0278	2.0177	2.0100	2.0039	1.9990	1.9951	1.9917	1.9889	1.9864	1.9843	1.9825
8	1.9814	1.9230	1.8826	1.8564	1.8383	1.8251	1.8151	1.8072	1.8008	1.7956	1.7912	1.7875	1.7843	1.7815	1.7791
10	1.8986	1.8286	1.7823	1.7523	1.7316	1.7164	1.7048	1.6957	1.6883	1.6822	1.6771	1.6728	1.6690	1.6658	1.6629
12	1.8460	1.7684	1.7181	1.6856	1.6630	1.6464	1.6337	1.6237	1.6155	1.6088	1.6031	1.5983	1.5941	1.5905	1.5873
14	1.8095	1.7767	1.6736	1.6392	1.6152	1.5975	1.5840	1.5732	1.5644	1.5572	1.5511	1.5459	1.5413	1.5374	1.5339
16	1.7828	1.6961	1.6409	1.6050	1.5800	1.5614	1.5471	1.5358	1.5265	1.5189	1.5124	1.5068	1.5020	1.4978	1.4941
18	1.7623	1.6727	1.6159	1.5788	1.5528	1.5336	1.5187	1.5069	1.4972	1.4892	1.4824	1.4766	1.4716	1.4671	1.4632
20	1.7462	1.6543	1.5960	1.5581	1.5313	1.5115	1.4962	1.4839	1.4739	1.4656	1.4585	1.4525	1.4472	1.4426	1.4385
22	1.7331	1.6393	1.5800	1.5412	1.5139	1.4936	1.4778	1.4652	1.4549	1.4463	1.4390	1.4327	1.4273	1.4225	1.4183
24	1.7224	1.6269	1.5667	1.5272	1.4994	1.4787	1.4625	1.4496	1.4391	1.4302	1.4227	1.4163	1.4107	1.4058	1.4014
26	1.7133	1.6166	1.5555	1.5155	1.4872	1.4661	1.4497	1.4365	1.4257	1.4167	1.4090	1.4024	1.3966	1.3916	1.3871
28	1.7056	1.6077	1.5460	1.5055	1.4768	1.4554	1.4387	1.4253	1.4142	1.4050	1.3972	1.3904	1.3846	1.3794	1.3748
30	1.6990	1.6001	1.5378	1.4968	1.4678	1.4461	1.4291	1.4155	1.4043	1.3949	1.3870	1.3801	1.3741	1.3688	1.3641
32	1.6932	1.5935	1.5306	1.4893	1.4600	1.4380	1.4208	1.4070	1.3956	1.3861	1.3780	1.3710	1.3649	1.3595	1.3547
34	1.6881	1.5876	1.5243	1.4827	1.4531	1.4308	1.4135	1.3995	1.3880	1.3783	1.3701	1.3630	1.3568	1.3513	1.3464
36	1.6836	1.5825	1.5188	1.4768	1.4469	1.4245	1.4070	1.3928	1.3812	1.3714	1.3630	1.3558	1.3495	1.3440	1.3370
38	1.6796	1.5778	1.5138	1.4715	1.4415	1.4188	1.4011	1.3869	1.3751	1.3652	1.3567	1.3494	1.3430	1.3374	1.3324
40	1.6760	1.5737	1.5093	1.4668	1.4365	1.4137	1.3959	1.3815	1.3696	1.3596	1.3510	1.3437	1.3372	1.3315	1.3264
42	1.6727	1.5700	1.5053	1.4626	1.4321	1.4091	1.3911	1.3766	1.3646	1.3545	1.3459	1.3384	1.3319	1.3261	1.3210
44	1.6698	1.5666	1.5016	1.4587	1.4280	1.4050	1.3868	1.3722	1.3601	1.3499	1.3412	1.3337	1.3271	1.3212	1.3161
46	1.6671	1.5635	1.4983	1.4552	1.4244	1.4011	1.3829	1.3682	1.3560	1.3457	1.3369	1.3293	1.3227	1.3168	1.3115
48	1.6646	1.5607	1.4953	1.4519	1.4210	1.3976	1.3793	1.3645	1.3522	1.3419	1.3330	1.3253	1.3186	1.3127	1.3074
50	1.6624	1.5581	1.4925	1.4490	1.4179	1.3944	1.3760	1.3611	1.3487	1.3383	1.3294	1.3217	1.3149	1.3089	1.3035
52	1.6603	1.5557	1.4899	1.4462	1.4150	1.3915	1.3729	1.3579	1.3455	1.3350	1.3260	1.3183	1.3114	1.3054	1.3000
54	1.6584	1.5535	1.4875	1.4437	1.4124	1.3887	1.3701	1.3550	1.3425	1.3320	1.3230	1.3151	1.3082	1.3021	1.2967
56	1.6566	1.5514	1.4853	1.4414	1.4099	1.3862	1.3675	1.3523	1.3398	1.3292	1.3201	1.3122	1.3053	1.2991	1.2937
58	1.6547	1.5495	1.4832	1.4392	1.4077	1.3838	1.3650	1.3498	1.3372	1.3265	1.3174	1.3094	1.3025	1.2963	1.2908
60	1.6534	1.5478	1.4813	1.4371	1.4055	1.3816	1.3628	1.3475	1.3348	1.3241	1.3149	1.3069	1.2999	1.2937	1.2881
62	1.6520	1.5461	1.4795	1.4352	1.4035	1.3795	1.3606	1.3453	1.3325	1.3218	1.3125	1.3045	1.2975	1.2912	1.2856
64	1.6506	1.5445	1.4778	1.4335	1.4017	1.3776	1.3586	1.3432	1.3304	1.3196	1.3103	1.3023	1.2952	1.2889	1.2833
66	1.6493	1.5431	1.4762	1.4318	1.3999	1.3758	1.3567	1.3413	1.3284	1.3176	1.3083	1.3002	1.2930	1.2867	1.2811
68	1.6481	1.5417	1.4747	1.4302	1.3983	1.3741	1.35[illegible]	1.3395	1.3266	1.3157	1.3063	1.2982	1.2910	1.2847	1.2790
70	1.6470	1.5404	1.4733	1.4287	1.3967	1.3724	1.3533	1.3377	1.3248	1.3139	1.3045	1.2963	1.2891	1.2827	1.2771
72	1.6460	1.5392	1.4720	1.4273	1.3952	1.3709	1.3517	1.3361	1.3232	1.3122	1.3027	1.2945	1.2873	1.2809	1.2752
74	1.6450	1.5381	1.4708	1.4260	1.3939	1.3695	1.3502	1.3346	1.3216	1.3106	1.3011	1.2929	1.2856	1.2792	1.2734
76	1.6440	1.5370	1.4696	1.4248	1.3925	1.3681	1.3488	1.3331	1.3201	1.3090	1.2995	1.2913	1.2840	1.2775	1.2718
78	1.6431	1.5359	1.4685	1.4236	1.3913	1.3668	1.3475	1.3318	1.3187	1.3076	1.2981	1.2898	1.2825	1.2760	1.2702
80	1.6423	1.5349	1.4674	1.4224	1.3901	1.3656	1.3462	1.3304	1.3173	1.3062	1.2966	1.2883	1.2810	1.2745	1.2687
82	1.6414	1.5340	1.4664	1.4214	1.3890	1.3644	1.3450	1.3292	1.3160	1.3049	1.2953	1.2870	1.2796	1.2731	1.2673
84	1.6407	1.5331	1.4654	1.4203	1.3879	1.3633	1.3438	1.3280	1.3148	1.3036	1.2940	1.2857	1.2783	1.2717	1.2659
86	1.6399	1.5323	1.4645	1.4194	1.3869	1.3622	1.3427	1.3269	1.3137	1.3025	1.2928	1.2844	1.2770	1.2705	1.2646
88	1.6392	1.5315	1.4636	1.4184	1.3859	1.3612	1.3417	1.3258	1.3125	1.3013	1.2917	1.2832	1.2758	1.2692	1.2633
90	1.6386	1.5307	1.4628	1.4175	1.3850	1.3602	1.3407	1.3248	1.3115	1.3002	1.2905	1.2821	1.2747	1.2681	1.2621

Figure E-8. F Distribution (80% Confidence)

VALUES OF F1

VALUES OF F2	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60
2	4.4503	4.4521	4.4537	4.4552	4.4565	4.4577	4.4587	4.4597	4.4606	4.4614	4.4622	4.4629	4.4636	4.4642	4.4648
4	2.4384	2.4377	2.4371	2.4366	2.4361	2.4356	2.4352	2.4348	2.4345	2.4342	2.4339	2.4336	2.4333	2.4331	2.4329
6	1.9808	1.9794	1.9781	1.9769	1.9758	1.9749	1.9740	1.9732	1.9724	1.9717	1.9711	1.9705	1.9699	1.9694	1.9689
8	1.7767	1.7750	1.7732	1.7717	1.7703	1.7690	1.7678	1.7667	1.7657	1.7648	1.7640	1.7632	1.7624	1.7617	1.7611
10	1.6604	1.6581	1.6560	1.6542	1.6525	1.6510	1.6495	1.6484	1.6472	1.6461	1.6451	1.6442	1.6433	1.6425	1.6417
12	1.5844	1.5819	1.5796	1.5776	1.5757	1.5740	1.5725	1.5710	1.5697	1.5685	1.5673	1.5663	1.5653	1.5644	1.5635
14	1.5308	1.5281	1.5256	1.5234	1.5213	1.5195	1.5178	1.5162	1.5148	1.5134	1.5122	1.5110	1.5100	1.5090	1.5080
16	1.4908	1.4879	1.4853	1.4829	1.4807	1.4787	1.4769	1.4752	1.4736	1.4722	1.4709	1.4696	1.4685	1.4674	1.4664
18	1.4598	1.4567	1.4539	1.4514	1.4490	1.4469	1.4450	1.4432	1.4416	1.4401	1.4387	1.4373	1.4361	1.4350	1.4339
20	1.4349	1.4317	1.4288	1.4261	1.4237	1.4215	1.4195	1.4176	1.4159	1.4143	1.4128	1.4114	1.4101	1.4089	1.4078
22	1.4146	1.4113	1.4083	1.4055	1.4031	1.4008	1.3987	1.3968	1.3950	1.3934	1.3919	1.3905	1.3891	1.3879	1.3867
24	1.3976	1.3941	1.3910	1.3882	1.3856	1.3832	1.3811	1.3791	1.3772	1.3755	1.3739	1.3725	1.3711	1.3698	1.3686
26	1.3831	1.3796	1.3763	1.3734	1.3708	1.3683	1.3661	1.3640	1.3621	1.3603	1.3587	1.3572	1.3557	1.3544	1.3531
28	1.3707	1.3670	1.3637	1.3607	1.3580	1.3555	1.3532	1.3511	1.3491	1.3473	1.3456	1.3440	1.3425	1.3411	1.3398
30	1.3599	1.3562	1.3528	1.3497	1.3469	1.3443	1.3419	1.3398	1.3377	1.3359	1.3341	1.3325	1.3310	1.3295	1.3282
32	1.3505	1.3466	1.3432	1.3400	1.3371	1.3345	1.3321	1.3298	1.3278	1.3258	1.3241	1.3224	1.3208	1.3194	1.3180
34	1.3421	1.3382	1.3346	1.3314	1.3285	1.3258	1.3233	1.3210	1.3189	1.3170	1.3151	1.3134	1.3118	1.3103	1.3089
36	1.3346	1.3307	1.3271	1.3238	1.3208	1.3180	1.3155	1.3132	1.3110	1.3090	1.3072	1.3054	1.3038	1.3022	1.3008
38	1.3280	1.3239	1.3202	1.3169	1.3139	1.3111	1.3085	1.3061	1.3039	1.3019	1.3000	1.2982	1.2965	1.2950	1.2935
40	1.3219	1.3178	1.3141	1.3107	1.3076	1.3048	1.3022	1.2998	1.2975	1.2954	1.2935	1.2917	1.2900	1.2884	1.2869
42	1.3165	1.3123	1.3085	1.3051	1.3020	1.2991	1.2964	1.2940	1.2917	1.2896	1.2876	1.2858	1.2840	1.2824	1.2809
44	1.3115	1.3073	1.3034	1.3000	1.2968	1.2939	1.2912	1.2887	1.2864	1.2842	1.2822	1.2804	1.2786	1.2770	1.2754
46	1.3069	1.3026	1.2988	1.2953	1.2920	1.2891	1.2864	1.2838	1.2815	1.2793	1.2773	1.2754	1.2736	1.2719	1.2704
48	1.3027	1.2984	1.2945	1.2909	1.2877	1.2847	1.2819	1.2794	1.2770	1.2748	1.2728	1.2708	1.2690	1.2673	1.2657
50	1.2989	1.2945	1.2906	1.2870	1.2837	1.2806	1.2779	1.2753	1.2729	1.2706	1.2686	1.2666	1.2648	1.2631	1.2615
52	1.2953	1.2909	1.2869	1.2833	1.2800	1.2769	1.2741	1.2715	1.2690	1.2668	1.2647	1.2627	1.2609	1.2591	1.2575
54	1.2920	1.2875	1.2835	1.2799	1.2765	1.2734	1.2706	1.2679	1.2655	1.2632	1.2611	1.2591	1.2572	1.2554	1.2538
56	1.2889	1.2844	1.2804	1.2767	1.2733	1.2702	1.2673	1.2646	1.2622	1.2599	1.2577	1.2557	1.2538	1.2520	1.2503
58	1.2860	1.2815	1.2774	1.2737	1.2703	1.2671	1.2642	1.2616	1.2591	1.2567	1.2546	1.2525	1.2506	1.2488	1.2471
60	1.2833	1.2788	1.2747	1.2709	1.2675	1.2643	1.2614	1.2587	1.2562	1.2538	1.2516	1.2496	1.2476	1.2458	1.2441
62	1.2808	1.2763	1.2721	1.2683	1.2649	1.2617	1.2587	1.2560	1.2534	1.2511	1.2489	1.2468	1.2448	1.2430	1.2413
64	1.2785	1.2739	1.2697	1.2659	1.2624	1.2592	1.2562	1.2534	1.2509	1.2485	1.2463	1.2442	1.2422	1.2404	1.2386
66	1.2762	1.2716	1.2674	1.2636	1.2601	1.2568	1.2538	1.2511	1.2485	1.2461	1.2438	1.2417	1.2397	1.2379	1.2361
68	1.2741	1.2695	1.2653	1.2614	1.2579	1.2546	1.2516	1.2488	1.2462	1.2438	1.2415	1.2394	1.2374	1.2355	1.2338
70	1.2722	1.2675	1.2632	1.2593	1.2558	1.2525	1.2495	1.2467	1.2441	1.2416	1.2393	1.2372	1.2352	1.2333	1.2315
72	1.2703	1.2656	1.2613	1.2574	1.2538	1.2505	1.2475	1.2446	1.2420	1.2396	1.2373	1.2351	1.2331	1.2312	1.2294
74	1.2685	1.2638	1.2595	1.2556	1.2520	1.2486	1.2456	1.2427	1.2401	1.2376	1.2353	1.2331	1.2311	1.2292	1.2274
76	1.2668	1.2621	1.2578	1.2538	1.2502	1.2469	1.2438	1.2409	1.2383	1.2358	1.2335	1.2313	1.2292	1.2273	1.2255
78	1.2652	1.2605	1.2561	1.2522	1.2485	1.2452	1.2421	1.2392	1.2365	1.2340	1.2317	1.2295	1.2274	1.2255	1.2237
80	1.2637	1.2589	1.2546	1.2506	1.2469	1.2436	1.2404	1.2376	1.2349	1.2324	1.2300	1.2278	1.2257	1.2238	1.2219
82	1.2623	1.2575	1.2531	1.2491	1.2454	1.2420	1.2389	1.2360	1.2333	1.2308	1.2284	1.2262	1.2241	1.2221	1.2203
84	1.2609	1.2561	1.2517	1.2477	1.2440	1.2406	1.2374	1.2345	1.2318	1.2292	1.2269	1.2246	1.2225	1.2206	1.2187
86	1.2596	1.2547	1.2503	1.2463	1.2426	1.2392	1.2360	1.2331	1.2303	1.2278	1.2254	1.2232	1.2211	1.2191	1.2172
88	1.2583	1.2535	1.2490	1.2450	1.2413	1.2378	1.2347	1.2317	1.2290	1.2264	1.2240	1.2218	1.2196	1.2176	1.2158
90	1.2571	1.2522	1.2478	1.2437	1.2400	1.2366	1.2334	1.2304	1.2276	1.2251	1.2227	1.2204	1.2183	1.2163	1.2144

Figure E-8. F Distribution (80% Confidence) (Continued)

The F Distribution, figure E-8 is the distribution of the quotient of two chi-square distributions.

Its use has already been illustrated in § 7.2.6 in the determination of availability bounds when both times to failure and times to repair are exponentially distributed.

Another use, illustrated in § 5.4.6.8, is in the acceptance or rejection of the Bayesian λ prior when both prior and posterior data arise both from frequency data to be tested for the hypothesis that they are from the same exponential distribution.

Yet a third use of the F distribution not illustrated in the manual is in testing the hypothesis that the 1st time to failure of an item is consistent with subsequent time to failure. This is important to know for repairable items since a new item could be very unreliable, for instance, until repaired at least once.

The test statistic is

$$u = \sum_{i=2}^{x} t_i/[(x-1)t_1]$$

If $F_{\alpha,2,2x-2} < u$ then there is evidence that t_1 represents an abnormally short time to 1st failure.

In the expression $F_{\alpha,2,2x-2}$, α represents the critical region (always 20% for figure E-8), 2 represents f_1, the number of degrees of freedom associated with t_1 (one failure), and 2x-2 represents f_2, the number of degrees of freedom associated with t_2 through t_x or x-1 failures.

If, for instance, t_1 = 20 hours, and

$$\sum_{i=2}^{12} t_i = 980 \text{ hours,}$$

then:

$$u = \frac{980}{(11)(20)} = 4.455$$

But $F_{0.20,2,22} = 1.7331$, and one must conclude that the time to first failure is inconsistent with subsequent times to failure at the 20% level of significance.

The F distribution could similarly be used to test the hypothesis of abnormally long times to first failure.

More extensive F tables are found in [1].

E.3.2 The a Distribution

The a distribution, figure E-9, is useful in calculating availability bounds when times to failure are exponentially distributed, and times to repair are lognormally distributed.

Example of Lower Availability Bound Computation

It is shown in [4] and [5] that a lower bound on availability for a component exhibiting exponential times to failure and lognormal times to repair is given by:

$$A_L = 1/[1+(\lambda/\mu)_u] \qquad \text{(E-4)}$$

where

$$(\lambda/\mu)_u = (\widehat{\lambda}/\widehat{\mu})\frac{e^{\sigma^2/2}}{2x}a_{1-\alpha;\,m/\sigma^2;\,x} \qquad \text{(E-5)}$$

The variance σ^2 of Naperian logarithms of repair times is assumed to be *known*. When this variance is unknown, it can be estimated directly for repair times M_{c_i}, i=1, 2...m; $m \geqslant 2$ from formula (E-6):

$$\widehat{\sigma^2} = \frac{1}{m-1}\left\{\sum_{m-1}^{m} \ell n^2(M_{c_i}) - \frac{1}{m}\left(\sum_{i=1}^{m} \ell n(M_{c_i})\right)^2\right\} \qquad \text{(E-6)}$$

The penalty for using the estimated variance $\widehat{\sigma^2}$ rather than the true variance σ^2 is that the lower bound on availability tends to be optimistic (too high) if repair data are few.

In [E-5] $\widehat{\lambda}$ represents a failure rate estimate x/T from exponential components tested to failure with x the total number of failures observed and T the cumulative test time of all units on test. $\widehat{\mu}$ represents a repair rate estimated from

$$\widehat{\mu} = 1/\left(\prod_{i=1}^{m} M_{c_i}\right)^{1/m}$$

where the M_{c_i}s are the m observed repair times. σ^2, the variance of the Naperian logarithms of repair times, is assumed to be

N	m/σ^2														
	1	2	3	4	5	6	7	8	9	10	20	50	100	200	500
1	4.330	3.757	3.572	3.480	3.426	3.391	3.365	3.346	3.332	3.320	3.260	3.237	3.226	3.221	3.218
2	8.978	7.496	6.998	6.746	6.595	6.494	6.421	6.367	6.324	6.291	6.139	6.048	6.017	6.001	5.992
3	13.612	11.166	10.327	9.898	9.637	9.461	9.334	9.238	9.164	9.104	8.832	8.667	8.612	8.583	8.567
4	18.245	14.814	13.621	13.006	12.629	12.373	12.188	12.047	11.937	11.849	11.444	11.197	11.113	11.070	11.044
5	22.879	18.452	16.900	16.093	15.595	15.256	15.010	14.823	14.676	14.557	14.010	13.672	13.557	13.498	13.463
6	27.514	22.086	20.170	19.168	18.547	18.123	17.814	17.578	17.392	17.242	16.546	16.110	15.961	15.886	15.840
7	32.150	25.717	23.434	22.236	21.490	20.979	20.605	20.319	20.094	19.911	19.060	18.522	18.337	18.243	18.186
8	36.788	29.347	26.695	25.299	24.427	23.827	23.388	23.052	22.786	22.570	21.559	20.914	20.691	20.578	20.508
9	41.425	32.976	29.954	28.358	27.358	26.670	26.164	25.777	25.469	25.220	24.045	23.290	23.027	22.893	[illegible]
10	46.063	36.604	33.211	31.414	30.287	29.508	28.936	28.496	28.148	27.864	26.522	25.652	25.349	25.193	25.098
11	50.700	40.232	36.468	34.469	33.213	32.344	31.704	31.212	30.821	30.503	28.992	28.005	27.658	27.480	27.371
12	55.339	43.859	39.723	37.523	36.137	35.177	34.470	33.925	33.491	33.130	31.454	30.347	29.957	29.756	29.633
13	59.978	47.486	42.978	40.575	39.060	38.009	37.233	36.635	36.158	35.769	33.912	32.683	32.247	32.022	31.834
14	64.615	51.112	46.231	43.626	41.981	40.838	39.994	39.342	38.823	38.398	36.365	35.011	34.528	34.279	34.126
15	69.252	54.738	49.484	46.674	44.901	43.667	42.754	42.048	41.485	41.025	38.815	37.334	36.803	36.529	36.341
16	73.895	58.367	52.739	49.728	47.822	46.496	45.513	44.754	44.147	43.651	41.262	39.653	39.073	38.773	[illegible]
17	78.534	61.994	55.992	52.778	50.742	49.322	48.271	47.457	46.807	46.275	43.706	41.966	41.337	41.010	40.827
18	83.174	65.620	59.245	55.827	53.660	52.149	51.028	50.160	49.466	48.890	46.147	44.275	43.595	43.242	43.024
19	87.813	69.247	62.498	58.876	56.578	54.974	53.704	52.861	52.123	51.517	48.586	46.581	45.850	45.448	45.233
20	92.453	72.874	65.750	61.925	59.496	57.799	56.539	55.562	54.788	54.139	51.073	48.884	48.108	47.691	47.438
22	101.732	80.128	72.255	68.022	65.330	63.448	62.048	60.962	60.092	59.377	55.893	53.481	52.590	52.123	51.833
24	111.008	87.379	78.758	74.118	71.163	69.095	67.555	66.359	65.400	64.612	60.756	58.067	57.067	56.540	56.213
26	120.290	94.632	85.263	80.214	76.996	74.741	73.062	71.756	70.707	69.846	65.617	62.646	61.534	60.945	60.579
28	129.56	101.88	91.76	86.31	82.83	80.38	78.56	77.15	76.01	75.08	70.47	67.22	65.99	65.34	64.93
30	138.84	109.13	98.27	92.40	88.66	86.03	84.07	82.54	81.31	80.30	75.32	71.78	70.44	69.72	69.28
32	148.13	116.39	104.77	98.50	94.49	91.68	89.57	87.94	86.62	85.54	80.18	76.35	74.88	74.10	73.41
34	157.41	123.64	111.28	104.59	100.32	97.32	95.08	93.33	91.92	90.76	85.02	80.90	79.32	78.47	77.94
36	166.69	130.90	117.78	110.69	106.15	102.96	100.58	98.72	97.22	95.99	89.87	85.46	83.75	82.83	82.25
38	175.97	138.15	124.28	116.78	111.98	108.60	106.08	104.11	102.53	101.22	94.72	90.00	88.18	87.19	86.56
40	185.25	145.40	130.79	122.88	117.81	114.25	111.58	109.50	107.83	106.44	99.56	94.55	92.60	91.54	90.87
42	194.52	152.65	137.29	128.97	123.64	119.89	117.08	114.89	113.12	111.67	104.40	99.09	97.01	95.88	95.16
44	203.81	159.91	143.79	135.06	129.47	125.53	122.58	120.28	118.42	116.89	109.24	103.63	101.43	100.22	99.46
46	213.09	167.16	150.29	141.16	135.30	131.17	128.08	125.67	123.72	122.11	114.08	108.17	105.84	104.56	103.74
48	222.36	174.41	156.79	147.25	141.13	136.81	133.58	131.06	129.02	127.34	118.92	112.71	110.25	108.89	108.03
50	231.64	181.66	163.30	153.34	146.95	142.45	139.08	136.45	134.32	132.56	123.76	117.24	114.65	113.22	112.30
55	254.83	199.79	179.55	168.57	161.52	156.54	152.83	149.92	147.56	145.62	135.85	128.57	125.65	124.02	122.98
60	278.00	217.90	195.79	183.79	176.09	170.65	166.57	163.38	160.80	158.67	147.94	139.89	136.63	134.81	133.63
65	301.25	236.07	212.07	199.04	190.67	184.77	180.33	176.86	174.05	171.73	160.03	151.21	147.61	145.59	144.27
70	324.45	254.20	228.33	214.28	205.25	198.87	194.08	190.33	187.30	184.79	172.12	162.52	158.58	156.35	154.89
75	347.64	272.32	244.58	229.51	219.81	212.97	207.83	203.88	200.54	197.84	184.20	173.82	169.53	167.10	165.49
80	370.85	290.46	260.84	244.74	234.39	227.07	221.57	217.27	213.78	210.89	196.28	185.12	180.48	177.83	176.08
85	394.04	308.59	277.09	259.97	248.96	241.17	235.32	230.73	227.82	223.94	208.36	196.42	191.43	188.56	186.66
90	417.25	326.72	293.35	275.20	263.53	255.27	249.07	244.20	240.26	237.00	220.44	207.71	202.37	199.28	197.23
95	440.43	344.84	309.60	290.43	278.09	269.37	262.81	257.66	253.50	250.04	232.52	219.00	213.30	210.00	207.78
100	463.62	362.97	325.85	305.66	292.66	283.47	276.85	271.13	266.74	263.10	244.59	230.29	224.23	220.70	218.33

Figure E-9. The a Distribution (80% Confidence) known or is estimated from expression [E-6] and $a_{1-\alpha;m/\sigma^2;x}$ is the "a" distribution at confidence $\gamma = 1-\alpha$ considered in this appendix.

As an example of the application of [E-4], assume that the following parameters are known or estimated:

$$\hat{\lambda} = 0.005, x = 10, \hat{\mu} = 0.393, \sigma = 0.413,$$

$$m = 10$$

then,

$$a_{1-\alpha;m/\sigma^2;x} = a_{0.80;60;10}$$

which is *not* tabulated in figure E-9 of this report.

One can also interpolate (harmonically for good results) between the tabular entries $a_{0.80;50;10}$ and $a_{0.80;100;10}$ to obtain $a_{0.80;60;10} = 25.55$.

A harmonic interpolate is obtained by using the linear interpolation technique, after the marginal values are changed to their reciprocals. For this example, the endpoints (50 and 100) become .02 and .01. The desired midpoint (60) becomes .01666.

Both the original margins and their reciprocals correspond to the same tabular entries.

Note that interpolation between values of N = x (rows of the table) is possible too. Linear interpolation works well without any transformation.

Using the value of $a_{0.80;60;10}$ in [E-5], one obtains:

$$(\lambda/\mu)_u = \frac{(0.005)}{0.393}\,\frac{(1.089)}{20}\,(25.55) = 0.017$$

and

$$A_L = \frac{1}{1+(\lambda/\mu)_u} = 0.982$$

Notice that this 80% lower confidence bound is lower than the point estimate for component availability which is:

$$\hat{A} = \hat{\mu}/(\hat{\mu}+\hat{\lambda}) = 0.393/(0.393 + 0.005) = 0.987$$

More extensive tables are found in [4].

E.4 MTBF TABLES (m of n)

Figure E-11 provides tables of the MTBF for various m-out-of-n configurations of *identical* items. The MTBF is provided under two options. The first is to permit the system to degrade until it fails (i.e., until less than the required m-out-of-n are operating). The second is to repair the first failure immediately. The basic configuration is shown in figure E-10.

The MTBF for the first case is obtained by integration of the reliability function from zero to infinity.

$$\text{MTBF} = \int_0^{\infty} R(t)dt$$

For example, in the case of 5-out-of-8 required we would have

$$\text{MTBF} = \int_0^{\infty} [56R(t)^5 - 140R(t)^6 + 120R(t)^7 - 35R(t)^8]\,dt$$

Assuming a single item has a failure rate λ and obeys the exponential failure law, we have:

$$\text{MTBF} = \int_0^{\infty} [56e^{-5\lambda t} - 140e^{-6\lambda t} + 120e^{-7\lambda t} - 35e^{-8\lambda t}]\,dt$$

$$= \frac{56}{5\lambda} - \frac{140}{6\lambda} + \frac{120}{7\lambda} - \frac{35}{8\lambda}$$

$$= \frac{+9{,}408 - 19{,}600 + 14{,}400 - 3{,}675}{840\lambda}$$

$$= \frac{533}{840\lambda}$$

$$\text{MTBF} = 0.6345/\lambda$$

This result can also be obtained using:

$$\text{MTBF} = \sum_{S=m}^{n} \frac{1}{S\lambda}$$

for 5 of 8

$$\text{MTBF} = \sum_{S=5}^{8} \frac{1}{S\lambda}$$

and

$$\text{MTBF} = \frac{1}{5\lambda} + \frac{1}{6\lambda} + \frac{1}{7\lambda} + \frac{1}{8\lambda}$$

$$\text{MTBF} = 0.6345/\lambda$$

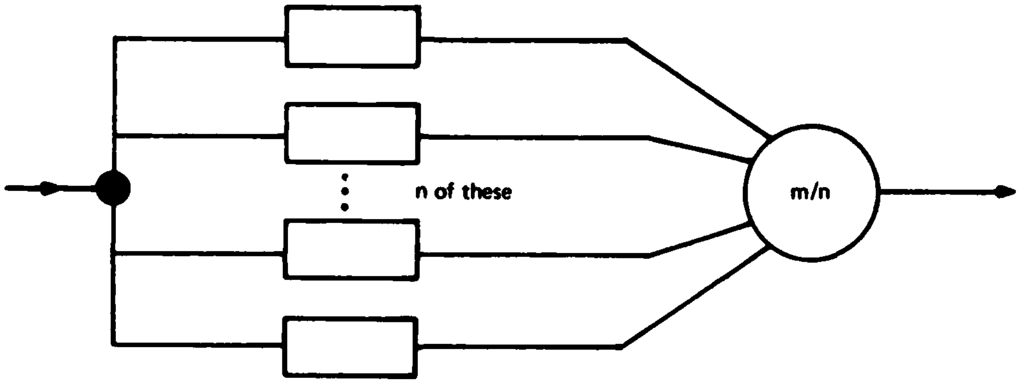

Figure E-10. Redundant Configuration, m-out-of-n

M/1	MTBF	MTBF*
1	$1.000000/\lambda$	$1.000000/\lambda$

M/2	MTBF	MTBF*
2	$0.500000/\lambda$	$0.500000/\lambda$
1	$1.500000/\lambda$	$0.500000\mu/\lambda^2$

M/3	MTBF	MTBF*
3	$0.333333/\lambda$	$0.333333/\lambda$
2	$0.833333/\lambda$	$0.166667\mu/\lambda^2$
1	$1.833333/\lambda$	$0.333333\mu^2/\lambda^3$

M/4	MTBF	MTBF*
4	$0.250000/\lambda$	$0.250000/\lambda$
3	$0.583333/\lambda$	$0.083333\mu/\lambda^2$
2	$1.083333/\lambda$	$0.083333\mu^2/\lambda^3$
1	$2.083333/\lambda$	$0.250000\mu^3/\lambda^4$

M/5	MTBF	MTBF*
5	$0.200000/\lambda$	$0.200000/\lambda$
4	$0.450000/\lambda$	$0.050000\mu/\lambda^2$
3	$0.783333/\lambda$	$0.033333\mu^2/\lambda^3$
2	$1.283333/\lambda$	$0.050000\mu^3/\lambda^4$
1	$2.283333/\lambda$	$0.200000\mu^4/\lambda^5$

M/6	MTBF	MTBF*
6	$0.166667/\lambda$	$0.166667/\lambda$
5	$0.366667/\lambda$	$0.033333\mu/\lambda^2$
4	$0.616667/\lambda$	$0.016667\mu^2/\lambda^3$
3	$0.950000/\lambda$	$0.016667\mu^3/\lambda^4$
2	$1.450000/\lambda$	$0.033333\mu^4/\lambda^5$
1	$2.450000/\lambda$	$0.166667\mu^5/\lambda^6$

M/7	MTBF	MTBF*
7	$0.142857/\lambda$	$0.142857/\lambda$
6	$0.309524/\lambda$	$0.023810\mu/\lambda^2$
5	$0.509524/\lambda$	$0.009524\mu^2/\lambda^3$
4	$0.759524/\lambda$	$0.007143\mu^3/\lambda^4$
3	$1.092857/\lambda$	$0.009524\mu^4/\lambda^5$
2	$1.592857/\lambda$	$0.023810\mu^5/\lambda^6$
1	$2.592857/\lambda$	$0.142857\mu^6/\lambda^7$

M/8	MTBF	MTBF*
8	$0.125000/\lambda$	$0.125000/\lambda$
7	$0.267857/\lambda$	$0.017857\mu/\lambda^2$
6	$0.434524/\lambda$	$0.005952\mu^2/\lambda^3$
5	$0.634524/\lambda$	$0.003571\mu^3/\lambda^4$
4	$0.884524/\lambda$	$0.003571\mu^4/\lambda^5$
3	$1.217857/\lambda$	$0.005952\mu^5/\lambda^6$
2	$1.717857/\lambda$	$0.017857\mu^6/\lambda^7$
1	$2.717857/\lambda$	$0.125000\mu^7/\lambda^8$

M/9	MTBF	MTBF*
9	$0.111111/\lambda$	$0.111111/\lambda$
8	$0.236111/\lambda$	$0.013889\mu/\lambda^2$
7	$0.378968/\lambda$	$0.003968\mu^2/\lambda^3$
6	$0.545635/\lambda$	$0.001984\mu^3/\lambda^4$
5	$0.745635/\lambda$	$0.001587\mu^4/\lambda^5$
4	$0.995635/\lambda$	$0.001984\mu^5/\lambda^6$
3	$1.328968/\lambda$	$0.003968\mu^6/\lambda^7$
2	$1.828968/\lambda$	$0.013889\mu^7/\lambda^8$
1	$2.828968/\lambda$	$0.111111\mu^8/\lambda^9$

M/10	MTBF	MTBF*
10	$0.100000/\lambda$	$0.100000/\lambda$
9	$0.211111/\lambda$	$0.011111\mu/\lambda^2$
8	$0.336111/\lambda$	$0.002778\mu^2/\lambda^3$
7	$0.478968/\lambda$	$0.001190\mu^3/\lambda^4$
6	$0.645635/\lambda$	$0.000794\mu^4/\lambda^5$
5	$0.845635/\lambda$	$0.000794\mu^5/\lambda^6$
4	$1.095635/\lambda$	$0.001190\mu^6/\lambda^7$
3	$1.428968/\lambda$	$0.002778\mu^7/\lambda^8$
2	$1.928968/\lambda$	$0.011111\mu^8/\lambda^9$
1	$2.928968/\lambda$	$0.100000\mu^9/\lambda^{10}$

M/11	MTBF	MTBF*
11	$0.090909/\lambda$	$0.090909/\lambda$
10	$0.190909/\lambda$	$0.009091\mu/\lambda^2$
9	$0.302020/\lambda$	$0.002020\mu^2/\lambda^3$
8	$0.427020/\lambda$	$0.000758\mu^3/\lambda^4$
7	$0.569877/\lambda$	$0.000433\mu^4/\lambda^5$
6	$0.736544/\lambda$	$0.000361\mu^5/\lambda^6$
5	$0.936544/\lambda$	$0.000433\mu^6/\lambda^7$
4	$1.186544/\lambda$	$0.000758\mu^7/\lambda^8$
3	$1.519877/\lambda$	$0.002020\mu^8/\lambda^9$
2	$2.019877/\lambda$	$0.009091\mu^9/\lambda^{10}$
1	$3.019877/\lambda$	$0.090909\mu^{10}/\lambda^{11}$

M/12	MTBF	MTBF*
12	$0.083333/\lambda$	$0.083333/\lambda$
11	$0.174242/\lambda$	$0.007576\mu/\lambda^2$
10	$0.274242/\lambda$	$0.001515\mu^2/\lambda^3$
9	$0.385354/\lambda$	$0.000505\mu^3/\lambda^4$
8	$0.510354/\lambda$	$0.000253\mu^4/\lambda^5$
7	$0.653211/\lambda$	$0.000180\mu^5/\lambda^6$
6	$0.819877/\lambda$	$0.000180\mu^6/\lambda^7$
5	$1.019877/\lambda$	$0.000253\mu^7/\lambda^8$
4	$1.269877/\lambda$	$0.000505\mu^8/\lambda^9$
3	$1.603211/\lambda$	$0.001515\mu^9/\lambda^{10}$
2	$2.103211/\lambda$	$0.007576\mu^{10}/\lambda^{11}$
1	$3.103211/\lambda$	$0.083333\mu^{11}/\lambda^{12}$

M/13	MTBF	MTBF*
13	$0.076923/\lambda$	$0.076923/\lambda$
12	$0.160256/\lambda$	$0.006410\mu/\lambda^2$
11	$0.251166/\lambda$	$0.001166\mu^2/\lambda^3$
10	$0.351166/\lambda$	$0.000350\mu^3/\lambda^4$
9	$0.462277/\lambda$	$0.000155\mu^4/\lambda^5$
8	$0.587277/\lambda$	$9.71E\text{-}05\mu^5/\lambda^6$
7	$0.730134/\lambda$	$8.33E\text{-}05\mu^6/\lambda^7$
6	$0.896800/\lambda$	$9.71E\text{-}05\mu^7/\lambda^8$
5	$1.096800/\lambda$	$0.000155\mu^8/\lambda^9$
4	$1.346800/\lambda$	$0.000350\mu^9/\lambda^{10}$
3	$1.680134/\lambda$	$0.001166\eta^{10}/\lambda^{11}$
2	$2.180134/\lambda$	$0.006410\mu^{11}/\lambda^{12}$
1	$3.180134/\lambda$	$0.076923\mu^{12}/\lambda^{13}$

M/14	MTBF	MTBF*
14	$0.071429/\lambda$	$0.071429/\lambda$
13	$0.148352/\lambda$	$0.005495\mu/\lambda^2$
12	$0.231685/\lambda$	$0.000916\mu^2/\lambda^3$
11	$0.322594/\lambda$	$0.000250\mu^3/\lambda^4$
10	$0.422594/\lambda$	$9.99E\text{-}05\mu^4/\lambda^5$
9	$0.533705/\lambda$	$5.55E\text{-}05\mu^5/\lambda^6$
8	$0.658705/\lambda$	$4.16E\text{-}05\mu^6/\lambda^7$
7	$0.801562/\lambda$	$4.16E\text{-}05\mu^7/\lambda^8$
6	$0.968229/\lambda$	$5.55E\text{-}05\mu^8/\lambda^9$
5	$1.168229/\lambda$	$9.99E\text{-}05\mu^9/\lambda^{10}$
4	$1.418229/\lambda$	$0.000250\mu^{10}/\lambda^{11}$
3	$1.751562/\lambda$	$0.000916\mu^{11}/\lambda^{12}$
2	$2.251562/\lambda$	$0.005495\mu^{12}/\lambda^{13}$
1	$3.251562/\lambda$	$0.071429\mu^{13}/\lambda^{14}$

M/15	MTBF	MTBF*
15	$0.066667/\lambda$	$0.066667/\lambda$
14	$0.138095/\lambda$	$0.004762\mu/\lambda^2$
13	$0.215018/\lambda$	$0.000733\mu^2/\lambda^3$
12	$0.298352/\lambda$	$0.000183\mu^3/\lambda^4$
11	$0.389261/\lambda$	$6.66E\text{-}05\mu^4/\lambda^5$
10	$0.489261/\lambda$	$3.33E\text{-}05\mu^5/\lambda^6$
9	$0.600372/\lambda$	$2.22E\text{-}05\mu^6/\lambda^7$
8	$0.725372/\lambda$	$1.94E\text{-}05\mu^7/\lambda^8$
7	$0.868229/\lambda$	$2.22E\text{-}05\mu^8/\lambda^9$
6	$1.034896/\lambda$	$3.33E\text{-}05\mu^9/\lambda^{10}$
5	$1.234896/\lambda$	$6.66E\text{-}05\mu^{10}/\lambda^{11}$
4	$1.484896/\lambda$	$0.000183\mu^{11}/\lambda^{12}$
3	$1.818229/\lambda$	$0.000733\mu^{12}/\lambda^{13}$
2	$2.318229/\lambda$	$0.004762\mu^{13}/\lambda^{14}$
1	$3.318229/\lambda$	$0.066667\mu^{14}/\lambda^{15}$

M/16	MTBF	MTBF*
16	$0.062500/\lambda$	$0.062500/\lambda$
15	$0.129167/\lambda$	$0.004167\mu/\lambda^2$
14	$0.200595/\lambda$	$0.000595\mu^2/\lambda^3$
13	$0.277518/\lambda$	$0.000137\mu^3/\lambda^4$
12	$0.360852/\lambda$	$4.58E\text{-}05\mu^4/\lambda^5$
11	$0.451761/\lambda$	$2.08E\text{-}05\mu^5/\lambda^6$
10	$0.551761/\lambda$	$1.25E\text{-}05\mu^6/\lambda^7$
9	$0.662872/\lambda$	$9.71E\text{-}06\mu^7/\lambda^8$
8	$0.787872/\lambda$	$9.71E\text{-}06\mu^8/\lambda^9$
7	$0.930729/\lambda$	$1.25E\text{-}05\mu^9/\lambda^{10}$
6	$1.097396/\lambda$	$2.08E\text{-}05\mu^{10}/\lambda^{11}$
5	$1.297396/\lambda$	$4.58E\text{-}05\mu^{11}/\lambda^{12}$
4	$1.547396/\lambda$	$0.000137\mu^{12}/\lambda^{13}$
3	$1.880729/\lambda$	$0.000595\mu^{13}/\lambda^{14}$
2	$2.380729/\lambda$	$0.004167\mu^{14}/\lambda^{15}$
1	$3.380729/\lambda$	$0.062500\mu^{15}/^{16}$

Figure E-11. MTBF for M out of N Redundant Configuration Without and With* Repair

M/17	MTBF	MTBF*
17	$0.058824/\lambda$	$0.058824/\lambda$
16	$0.121324/\lambda$	$0.003676\mu/\lambda^2$
15	$0.187990/\lambda$	$0.000490\mu^2/\lambda^3$
14	$0.259419/\lambda$	$0.000105\mu^3/\lambda^4$
13	$0.336342/\lambda$	$3.23E\text{-}05\mu^4/\lambda^5$
12	$0.419675/\lambda$	$1.35E\text{-}05\mu^5/\lambda^6$
11	$0.510584/\lambda$	$7.35E\text{-}06\mu^6/\lambda^7$
10	$0.610584/\lambda$	$5.14E\text{-}06\mu^7/\lambda^8$
9	$0.721695/\lambda$	$4.57E\text{-}06\mu^8/\lambda^9$
8	$0.846695/\lambda$	$5.14E\text{-}06\mu^9/\lambda^{10}$
7	$0.989553/\lambda$	$7.35E\text{-}06\mu^{10}/\lambda^{11}$
6	$1.156219/\lambda$	$1.35E\text{-}05\mu^{11}/\lambda^{12}$
5	$1.356219/\lambda$	$3.23E\text{-}05\mu^{12}/\lambda^{13}$
4	$1.606219/\lambda$	$0.000105\mu^{13}/\lambda^{14}$
3	$1.939552/\lambda$	$0.000490\mu^{14}/\lambda^{15}$
2	$2.439552/\lambda$	$0.003676\mu^{15}/\lambda^{16}$
1	$3.439552/\lambda$	$0.058824\mu^{16}/\lambda^{17}$

M/18	MTBF	MTBF*
18	$0.055556/\lambda$	$0.055556/\lambda$
17	$0.114379/\lambda$	$0.003268\mu/\lambda^2$
16	$0.176879/\lambda$	$0.000408\mu^2/\lambda^3$
15	$0.243546/\lambda$	$8.17E\text{-}05\mu^3/\lambda^4$
14	$0.314974/\lambda$	$2.33E\text{-}05\mu^4/\lambda^5$
13	$0.391897/\lambda$	$8.98E\text{-}06\mu^5/\lambda^6$
12	$0.475231/\lambda$	$4.49E\text{-}06\mu^6/\lambda^7$
11	$0.566140/\lambda$	$2.86E\text{-}06\mu^7/\lambda^8$
10	$0.666140/\lambda$	$2.29E\text{-}06\mu^8/\lambda^9$
9	$0.777251/\lambda$	$2.29E\text{-}06\mu^9/\lambda^{10}$
8	$0.902251/\lambda$	$2.86E\text{-}06\mu^{10}/\lambda^{11}$
7	$1.045108/\lambda$	$4.49E\text{-}06\mu^{11}/\lambda^{12}$
6	$1.211775/\lambda$	$8.98E\text{-}06\mu^{12}/\lambda^{13}$
5	$1.411775/\lambda$	$2.33E\text{-}05\mu^{13}/\lambda^{14}$
4	$1.661775/\lambda$	$8.17E\text{-}05\mu^{14}/\lambda^{15}$
3	$1.995108/\lambda$	$0.000408\mu^{15}/\lambda^{16}$
2	$2.495108/\lambda$	$0.003268\mu^{16}/\lambda^{17}$
1	$3.495108/\lambda$	$0.055556\mu^{17}/\lambda^{18}$

M/19	MTBF	MTBF*
19	$0.052632/\lambda$	$0.052632/\lambda$
18	$0.108187/\lambda$	$0.002924\mu/\lambda^2$
17	$0.167011/\lambda$	$0.000344\mu^2/\lambda^3$
16	$0.229511/\lambda$	$6.45E\text{-}05\mu^3/\lambda^4$
15	$0.296177/\lambda$	$1.72E\text{-}05\mu^4/\lambda^5$
14	$0.367606/\lambda$	$6.14E\text{-}06\mu^5/\lambda^6$
13	$0.444529/\lambda$	$2.84E\text{-}06\mu^6/\lambda^7$
12	$0.527862/\lambda$	$1.65E\text{-}06\mu^7/\lambda^8$
11	$0.618771/\lambda$	$1.20E\text{-}06\mu^8/\lambda^9$
10	$0.718771/\lambda$	$1.08E\text{-}06\mu^9/\lambda^{10}$
9	$0.829883/\lambda$	$1.20E\text{-}06\mu^{10}/\lambda^{11}$
8	$0.954883/\lambda$	$1.65E\text{-}06\mu^{11}/\lambda^{12}$
7	$1.097740/\lambda$	$2.84E\text{-}06\mu^{12}/\lambda^{13}$
6	$1.264406/\lambda$	$6.14E\text{-}06\mu^{13}/\lambda^{14}$
5	$1.464406/\lambda$	$1.72E\text{-}05\mu^{14}/\lambda^{15}$
4	$1.714406/\lambda$	$6.45E\text{-}05\mu^{15}/\lambda^{16}$
3	$2.047740/\lambda$	$0.000344\mu^{16}/\lambda^{17}$
2	$2.547740/\lambda$	$0.002924\mu^{17}/\lambda^{18}$
1	$3.547740/\lambda$	$0.052632\mu^{18}/\lambda^{19}$

M/20	MTBF	MTBF*
20	$0.050000/\lambda$	$0.050000/\lambda$
19	$0.102632/\lambda$	$0.002632\mu/\lambda^2$
18	$0.158187/\lambda$	$0.000292\mu^2/\lambda^3$
17	$0.217011/\lambda$	$5.16E\text{-}05\mu^3/\lambda^4$
16	$0.279511/\lambda$	$1.29E\text{-}05\mu^4/\lambda^5$
15	$0.346177/\lambda$	$4.30E\text{-}06\mu^5/\lambda^6$
14	$0.417606/\lambda$	$1.84E\text{-}06\mu^6/\lambda^7$
13	$0.494529/\lambda$	$9.92E\text{-}07\mu^7/\lambda^8$
12	$0.577862/\lambda$	$6.62E\text{-}07\mu^8/\lambda^9$
11	$0.668771/\lambda$	$5.41E\text{-}07\mu^9/\lambda^{10}$
10	$0.768771/\lambda$	$5.41E\text{-}07\mu^{10}/\lambda^{11}$
9	$0.879883/\lambda$	$6.62E\text{-}07\mu^{11}/\lambda^{12}$
8	$1.004883/\lambda$	$9.92E\text{-}07\mu^{12}/\lambda^{13}$
7	$1.147740/\lambda$	$1.84E\text{-}06\mu^{13}/\lambda^{14}$
6	$1.314406/\lambda$	$4.30E\text{-}06\mu^{14}/\lambda^{15}$
5	$1.514406/\lambda$	$1.29E\text{-}05\mu^{15}/\lambda^{16}$
4	$1.764406/\lambda$	$5.16E\text{-}05\mu^{16}/\lambda^{17}$
3	$2.097740/\lambda$	$0.000292\mu^{17}/\lambda^{18}$
2	$2.597740/\lambda$	$0.002632\mu^{18}/\lambda^{19}$
1	$3.597740/\lambda$	$0.050000\mu^{19}/\lambda^{20}$

M/21	MTBF	MTBF*
21	$0.047619/\lambda$	$0.047619/\lambda$
20	$0.097619/\lambda$	$0.002381\mu/\lambda^2$
19	$0.150251/\lambda$	$0.000251\mu^2/\lambda^3$
18	$0.205806/\lambda$	$4.18E\text{-}05\mu^3/\lambda^4$
17	$0.264630/\lambda$	$9.83E\text{-}06\mu^4/\lambda^5$
16	$0.327130/\lambda$	$3.07E\text{-}06\mu^5/\lambda^6$
15	$0.393796/\lambda$	$1.23E\text{-}06\mu^6/\lambda^7$
14	$0.465225/\lambda$	$6.14E\text{-}07\mu^7/\lambda^8$
13	$0.542148/\lambda$	$3.78E\text{-}07\mu^8/\lambda^9$
12	$0.625481/\lambda$	$2.84E\text{-}07\mu^9/\lambda^{10}$
11	$0.716390/\lambda$	$2.58E\text{-}07\mu^{10}/\lambda^{11}$
10	$0.816390/\lambda$	$2.84E\text{-}07\mu^{11}/\lambda^{12}$
9	$0.927502/\lambda$	$3.78E\text{-}07\mu^{12}/\lambda^{13}$
8	$1.052502/\lambda$	$6.14E\text{-}07\mu^{13}/\lambda^{14}$
7	$1.195359/\lambda$	$1.23E\text{-}06\mu^{14}/\lambda^{15}$
6	$1.362025/\lambda$	$3.07E\text{-}06\mu^{15}/\lambda^{16}$
5	$1.562025/\lambda$	$9.83E\text{-}06\mu^{16}/\lambda^{17}$
4	$1.812025/\lambda$	$4.18E\text{-}05\mu^{17}/\lambda^{18}$
3	$2.145359/\lambda$	$0.000251\mu^{18}/\lambda^{19}$
2	$2.645359/\lambda$	$0.002381\mu^{19}/\lambda^{20}$
1	$3.645359/\lambda$	$0.047619\mu^{20}/\lambda^{21}$

M/22	MTBF	MTBF*
22	$0.045455/\lambda$	$0.045455/\lambda$
21	$0.093074/\lambda$	$0.002165\mu/\lambda^2$
20	$0.143074/\lambda$	$0.000216\mu^2/\lambda^3$
19	$0.195705/\lambda$	$3.42E\text{-}05\mu^3/\lambda^4$
18	$0.251261/\lambda$	$7.59E\text{-}06\mu^4/\lambda^5$
17	$0.310084/\lambda$	$2.23E\text{-}06\mu^5/\lambda^6$
16	$0.372584/\lambda$	$8.38E\text{-}07\mu^6/\lambda^7$
15	$0.439251/\lambda$	$3.91E\text{-}07\mu^7/\lambda^8$
14	$0.510679/\lambda$	$2.23E\text{-}07\mu^8/\lambda^9$
13	$0.587603/\lambda$	$1.55E\text{-}07\mu^9/\lambda^{10}$
12	$0.670936/\lambda$	$1.29E\text{-}07\mu^{10}/\lambda^{11}$
11	$0.761845/\lambda$	$1.29E\text{-}07\mu^{11}/\lambda^{12}$
10	$0.861845/\lambda$	$1.55E\text{-}07\mu^{12}/\lambda^{13}$
9	$0.972956/\lambda$	$2.23E\text{-}07\mu^{13}/\lambda^{14}$
8	$1.097956/\lambda$	$3.91E\text{-}07\mu^{14}/\lambda^{15}$
7	$1.240813/\lambda$	$8.38E\text{-}07\mu^{15}/\lambda^{16}$
6	$1.407480/\lambda$	$2.23E\text{-}06\mu^{16}/\lambda^{17}$
5	$1.607480/\lambda$	$7.59E\text{-}06\mu^{17}/\lambda^{18}$
4	$1.857480/\lambda$	$3.42E\text{-}05\mu^{18}/\lambda^{19}$
3	$2.190813/\lambda$	$0.000216\mu^{19}/\lambda^{20}$
2	$2.690813/\lambda$	$0.002165\mu^{20}/\lambda^{21}$
1	$3.690813/\lambda$	$0.045455\mu^{21}/\lambda^{22}$

M/23	MTBF	MTBF*
23	$0.043478/\lambda$	$0.043478/\lambda$
22	$0.088933/\lambda$	$0.001976\mu/\lambda^2$
21	$0.136552/\lambda$	$0.000188\mu^2/\lambda^3$
20	$0.186552/\lambda$	$2.82E\text{-}05\mu^3/\lambda^4$
19	$0.239183/\lambda$	$5.94E\text{-}06\mu^4/\lambda^5$
18	$0.294739/\lambda$	$1.65E\text{-}06\mu^5/\lambda^6$
17	$0.353563/\lambda$	$5.83E\text{-}07\mu^6/\lambda^7$
16	$0.416063/\lambda$	$2.55E\text{-}07\mu^7/\lambda^8$
15	$0.482729/\lambda$	$1.36E\text{-}07\mu^8/\lambda^9$
14	$0.554158/\lambda$	$8.74E\text{-}08\mu^9/\lambda^{10}$
13	$0.631081/\lambda$	$6.72E\text{-}08\mu^{10}/\lambda^{11}$
12	$0.714414/\lambda$	$6.16E\text{-}08\mu^{11}/\lambda^{12}$
11	$0.805323/\lambda$	$6.72E\text{-}08\mu^{12}/\lambda^{13}$
10	$0.905323/\lambda$	$8.74E\text{-}08\mu^{13}/\lambda^{14}$
9	$1.016434/\lambda$	$1.36E\text{-}07\mu^{14}/\lambda^{15}$
8	$1.141434/\lambda$	$2.55E\text{-}07\mu^{15}/\lambda^{16}$
7	$1.284292/\lambda$	$5.83E\text{-}07\mu^{16}/\lambda^{17}$
6	$1.450958/\lambda$	$1.65E\text{-}06\mu^{17}/\lambda^{18}$
5	$1.650958/\lambda$	$5.94E\text{-}06\mu^{18}/\lambda^{19}$
4	$1.900958/\lambda$	$2.82E\text{-}05\mu^{19}/\lambda^{20}$
3	$2.234292/\lambda$	$0.000188\mu^{20}/\lambda^{21}$
2	$2.734292/\lambda$	$0.001976\mu^{21}/\lambda^{22}$
1	$3.734292/\lambda$	$0.043478\mu^{22}/\lambda^{23}$

M/24	MTBF	MTBF*
24	$0.041667/\lambda$	$0.041667/\lambda$
23	$0.085145/\lambda$	$0.001812\mu/\lambda^2$
22	$0.130599/\lambda$	$0.000165\mu^2/\lambda^3$
21	$0.178219/\lambda$	$2.35E\text{-}05\mu^3/\lambda^4$
20	$0.228219/\lambda$	$4.71E\text{-}06\mu^4/\lambda^5$
19	$0.280850/\lambda$	$1.24E\text{-}06\mu^5/\lambda^6$
18	$0.336406/\lambda$	$4.13E\text{-}07\mu^6/\lambda^7$
17	$0.395229/\lambda$	$1.70E\text{-}07\mu^7/\lambda^8$
16	$0.457729/\lambda$	$8.50E\text{-}08\mu^8/\lambda^9$
15	$0.524396/\lambda$	$5.10E\text{-}08\mu^9/\lambda^{10}$
14	$0.595824/\lambda$	$3.64E\text{-}08\mu^{10}/\lambda^{11}$
13	$0.672747/\lambda$	$3.08E\text{-}08\mu^{11}/\lambda^{12}$
12	$0.756081/\lambda$	$3.08E\text{-}08\mu^{12}/\lambda^{13}$
11	$0.846880/\lambda$	$3.64E\text{-}06\mu^{13}/\lambda^{14}$
10	$0.946990/\lambda$	$5.10E\text{-}08\mu^{14}/\lambda^{15}$
9	$1.058101/\lambda$	$8.50E\text{-}08\mu^{15}/\lambda^{16}$
8	$1.183101/\lambda$	$1.70E\text{-}07\mu^{16}/\lambda^{17}$
7	$1.325958/\lambda$	$4.13E\text{-}07\mu^{17}/\lambda^{18}$
6	$1.492625/\lambda$	$1.24E\text{-}06\mu^{18}/\lambda^{19}$
5	$1.692825/\lambda$	$4.71E\text{-}06\mu^{19}/\lambda^{20}$
4	$1.942625/\lambda$	$2.35E\text{-}05\mu^{20}/\lambda^{21}$
3	$2.275958/\lambda$	$0.000165\mu^{21}/\lambda^{22}$
2	$2.775958/\lambda$	$0.001812\mu^{22}/\lambda^{23}$
1	$3.775958/\lambda$	$0.041667\mu^{23}/\lambda^{24}$

Figure E-11. MTBF for M out of N Redundant Configuration Without and With* Repair (Continued)

M/25	MTBF	MTBF*
25	$0.040000/\lambda$	$0.040000/\lambda$
24	$0.081667/\lambda$	$0.001667\mu/\lambda^{2}$
23	$0.125145/\lambda$	$0.000145\mu^{2}/\lambda^{3}$
22	$0.170599/\lambda$	$1.98E\text{-}05\mu^{3}/\lambda^{4}$
21	$0.218219/\lambda$	$3.76E\text{-}06\mu^{4}/\lambda^{5}$
20	$0.268219/\lambda$	$9.41E\text{-}07\mu^{5}/\lambda^{6}$
19	$0.320850/\lambda$	$2.97E\text{-}07\mu^{6}/\lambda^{7}$
18	$0.376406/\lambda$	$1.16E\text{-}07\mu^{7}/\lambda^{8}$
17	$0.435229/\lambda$	$5.44E\text{-}08\mu^{8}/\lambda^{9}$
16	$0.497729/\lambda$	$3.06E\text{-}08\mu^{9}/\lambda^{10}$
15	$0.564396/\lambda$	$2.04E\text{-}08\mu^{10}/\lambda^{11}$
14	$0.635824/\lambda$	$1.602\text{-}08\mu^{11}/\lambda^{12}$
13	$0.712747/\lambda$	$1.48E\text{-}08\mu^{12}/\lambda^{13}$
12	$0.796081/\lambda$	$1.60E\text{-}08\mu^{13}/\lambda^{14}$
11	$0.886990/\lambda$	$2.04E\text{-}08\mu^{14}/\lambda^{15}$
10	$0.986990/\lambda$	$3.06E\text{-}08\mu^{15}/\lambda^{16}$
9	$1.098101/\lambda$	$5.44E\text{-}08\mu^{16}/\lambda^{17}$
8	$1.223101/\lambda$	$1.16E\text{-}07\mu^{17}/\lambda^{18}$
7	$1.365958/\lambda$	$2.97E\text{-}07\mu^{18}/\lambda^{19}$
6	$1.532625/\lambda$	$9.41E\text{-}07\mu^{19}/\lambda^{20}$
5	$1.732625/\lambda$	$3.76E\text{-}06\mu^{20}/\lambda^{21}$
4	$1.982625/\lambda$	$1.98E\text{-}05\mu^{21}/\lambda^{22}$
3	$2.315958/\lambda$	$0.000145\mu^{22}/\lambda^{23}$
2	$2.815958/\lambda$	$0.001667\mu^{23}/\lambda^{24}$
1	$3.815958/\lambda$	$0.040000\mu^{24}/\lambda^{25}$

M/26	MTBF	MTBF*
26	$0.038462/\lambda$	$0.038462/\lambda$
25	$0.078462/\lambda$	$0.001538\mu/\lambda^{2}$
24	$0.120128/\lambda$	$0.000128\mu^{2}/\lambda^{3}$
23	$0.163606/\lambda$	$1.67E\text{-}05\mu^{3}/\lambda^{4}$
22	$0.209061/\lambda$	$3.04E\text{-}06\mu^{4}/\lambda^{5}$
21	$0.256680/\lambda$	$7.24E\text{-}07\mu^{5}/\lambda^{6}$
20	$0.306680/\lambda$	$2.17E\text{-}07\mu^{6}/\lambda^{7}$
19	$0.359312/\lambda$	$8.00E\text{-}08\mu^{7}/\lambda^{8}$
18	$0.414867/\lambda$	$3.56E\text{-}08\mu^{8}/\lambda^{9}$
17	$0.473691/\lambda$	$1.88E\text{-}08\mu^{9}/\lambda^{10}$
16	$0.536191/\lambda$	$1.18E\text{-}08\mu^{10}/\lambda^{11}$
15	$0.602857/\lambda$	$8.63E\text{-}09\mu^{11}/\lambda^{12}$
14	$0.674286/\lambda$	$7.40E\text{-}09\mu^{12}/\lambda^{13}$
13	$0.751209/\lambda$	$7.40E\text{-}09\mu^{13}/\lambda^{14}$
12	$0.834542/\lambda$	$8.63E\text{-}09\mu^{14}/\lambda^{15}$
11	$0.925451/\lambda$	$1.18E\text{-}08\mu^{15}/\lambda^{16}$
10	$1.025431/\lambda$	$1.88E\text{-}08\mu^{16}/\lambda^{17}$
9	$1.136563/\lambda$	$3.56E\text{-}08\mu^{17}/\lambda^{18}$
8	$1.261563/\lambda$	$8.00E\text{-}08\mu^{18}/\lambda^{19}$
7	$1.404420/\lambda$	$2.17E\text{-}07\mu^{19}/\lambda^{20}$
6	$1.571086/\lambda$	$7.24E\text{-}07\mu^{20}/\lambda^{21}$
5	$1.771086/\lambda$	$3.04E\text{-}06\mu^{21}/\lambda^{22}$
4	$2.021086/\lambda$	$1.67E\text{-}05\mu^{22}/\lambda^{23}$
3	$2.354420/\lambda$	$0.000128\mu^{23}/\lambda^{24}$
2	$2.854420/\lambda$	$0.001538\mu^{24}/\lambda^{25}$
1	$3.854420/\lambda$	$0.038462\mu^{25}/\lambda^{26}$

M/27	MTBF	MTBF*
27	$0.037037/\lambda$	$0.037037/\lambda$
26	$0.075499/\lambda$	$0.001425\mu/\lambda^{2}$
25	$0.115499/\lambda$	$0.000114\mu^{2}/\lambda^{3}$
24	$0.157165/\lambda$	$1.42E\text{-}05\mu^{3}/\lambda^{4}$
23	$0.200644/\lambda$	$2.48E\text{-}06\mu^{4}/\lambda^{5}$
22	$0.246098/\lambda$	$5.63E\text{-}07\mu^{5}/\lambda^{6}$
21	$0.293717/\lambda$	$1.61E\text{-}07\mu^{6}/\lambda^{7}$
20	$0.343717/\lambda$	$5.63E\text{-}08\mu^{7}/\lambda^{8}$
19	$0.396349/\lambda$	$2.37E\text{-}08\mu^{8}/\lambda^{9}$
18	$0.451904/\lambda$	$1.19E\text{-}08\mu^{9}/\lambda^{10}$
17	$0.510728/\lambda$	$6.97E\text{-}09\mu^{10}/\lambda^{11}$
16	$0.573228/\lambda$	$4.79E\text{-}09\mu^{11}/\lambda^{12}$
15	$0.639894/\lambda$	$3.83E\text{-}09\mu^{12}/\lambda^{13}$
14	$0.711323/\lambda$	$3.56E\text{-}09\mu^{13}/\lambda^{14}$
13	$0.788246/\lambda$	$3.83E\text{-}09\mu^{14}/\lambda^{15}$
12	$0.871579/\lambda$	$4.79E\text{-}09\mu^{15}/\lambda^{16}$
11	$0.962488/\lambda$	$6.97E\text{-}09\mu^{16}/\lambda^{17}$
10	$1.062488/\lambda$	$1.19E\text{-}08\mu^{17}/\lambda^{18}$
9	$1.173600/\lambda$	$2.37E\text{-}08\mu^{18}/\lambda^{19}$
8	$1.298600/\lambda$	$5.63E\text{-}08\mu^{19}/\lambda^{20}$
7	$1.441457/\lambda$	$1.61E\text{-}07\mu^{20}/\lambda^{21}$
6	$1.608123/\lambda$	$5.63E\text{-}07\mu^{21}/\lambda^{22}$
5	$1.808123/\lambda$	$2.48E\text{-}06\mu^{22}/\lambda^{23}$
4	$2.058123/\lambda$	$1.42E\text{-}05\mu^{23}/\lambda^{24}$
3	$2.391457/\lambda$	$0.000114\mu^{24}/\lambda^{25}$
2	$2.891457/\lambda$	$0.001425\mu^{25}/\lambda^{26}$
1	$3.891457/\lambda$	$0.037037\mu^{26}/\lambda^{27}$

M/28	MTBF	MTBF*
28	$0.035714/\lambda$	$0.035714/\lambda$
27	$0.072751/\lambda$	$0.001323\mu/\lambda^{2}$
26	$0.111213/\lambda$	$0.000102\mu^{2}/\lambda^{3}$
25	$0.151213/\lambda$	$1.22E\text{-}05\mu^{3}/\lambda^{4}$
24	$0.192880/\lambda$	$2.04E\text{-}05\mu^{4}/\lambda^{5}$
23	$0.236358/\lambda$	$4.42E\text{-}07\mu^{5}/\lambda^{6}$
22	$0.281812/\lambda$	$1.21E\text{-}07\mu^{6}/\lambda^{7}$
21	$0.329431/\lambda$	$4.02E\text{-}08\mu^{7}/\lambda^{8}$
20	$0.379431/\lambda$	$1.61E\text{-}08\mu^{8}/\lambda^{9}$
19	$0.432063/\lambda$	$7.62E\text{-}09\mu^{9}/\lambda^{10}$
18	$0.487619/\lambda$	$4.23E\text{-}09\mu^{10}/\lambda^{11}$
17	$0.546442/\lambda$	$2.74E\text{-}09\mu^{11}/\lambda^{12}$
16	$0.608942/\lambda$	$2.05E\text{-}09\mu^{12}/\lambda^{13}$
15	$0.675609/\lambda$	$1.78E\text{-}09\mu^{13}/\lambda^{14}$
14	$0.747037/\lambda$	$1.78E\text{-}09\mu^{14}/\lambda^{15}$
13	$0.823960/\lambda$	$2.05E\text{-}09\mu^{15}/\lambda^{16}$
12	$0.907294/\lambda$	$2.74E\text{-}09\mu^{16}/\lambda^{17}$
11	$0.998203/\lambda$	$4.23E\text{-}09\mu^{17}/\lambda^{18}$
10	$1.098203/\lambda$	$7.62E\text{-}09\mu^{18}/\lambda^{19}$
9	$1.209304/\lambda$	$1.61E\text{-}08\mu^{19}/\lambda^{20}$
8	$1.334314/\lambda$	$4.02E\text{-}08\mu^{20}/\lambda^{21}$
7	$1.477171/\lambda$	$1.21E\text{-}07\mu^{21}/\lambda^{22}$
6	$1.643838/\lambda$	$4.42E\text{-}07\mu^{22}/\lambda^{23}$
5	$1.843838/\lambda$	$2.04E\text{-}06\mu^{23}/\lambda^{24}$
4	$2.093838/\lambda$	$1.22E\text{-}06\mu^{24}/\lambda^{25}$
3	$2.427171/\lambda$	$0.000102\mu^{25}/\lambda^{26}$
2	$2.927171/\lambda$	$0.001323\mu^{26}/\lambda^{27}$
1	$3.927171/\lambda$	$0.035714\mu^{27}/\lambda^{28}$

Figure E-11. MTBF for M out of N Redundant Configuration Without and With* Repair (Continued)

M/29	MTBF	MTBF*
29	$0.034483/\lambda$	$0.034483\ \lambda$
28	$0.070197/\lambda$	$0.001232\mu/\lambda^2$
27	$0.107234/\lambda$	$9.12E\text{-}05\mu^2/\lambda^3$
26	$0.145696/\lambda$	$1.05E\text{-}05\mu^3/\lambda^4$
25	$0.185696/\lambda$	$1.68E\text{-}06\mu^4/\lambda^5$
24	$0.227362/\lambda$	$3.51E\text{-}07\mu^5/\lambda^6$
23	$0.270841/\lambda$	$9.15E\text{-}08\mu^6/\lambda^7$
22	$0.316295/\lambda$	$2.91E\text{-}08\mu^7/\lambda^8$
21	$0.363914/\lambda$	$1.11E\text{-}08\mu^8/\lambda^9$
20	$0.413914/\lambda$	$4.99E\text{-}09\mu^9/\lambda^{10}$
19	$0.466546/\lambda$	$2.63E\text{-}09\mu^{10}/\lambda^{11}$
18	$0.522101/\lambda$	$1.61E\text{-}09\mu^{11}/\lambda^{12}$
17	$0.580925/\lambda$	$1.13E\text{-}09\mu^{12}/\lambda^{13}$
16	$0.643425/\lambda$	$9.21E\text{-}10\mu^{13}/\lambda^{14}$
15	$0.710091/\lambda$	$8.60E\text{-}10\mu^{14}/\lambda^{15}$
14	$0.781520/\lambda$	$9.21E\text{-}10\mu^{15}/\lambda^{16}$
13	$0.858443/\lambda$	$1.13E\text{-}09\mu^{16}/\lambda^{17}$
12	$0.941776/\lambda$	$1.61E\text{-}09\mu^{17}/\lambda^{18}$
11	$1.032685/\lambda$	$2.63E\text{-}09\mu^{18}/\lambda^{19}$
10	$1.132685/\lambda$	$4.99E\text{-}09\mu^{19}/\lambda^{20}$
9	$1.243797/\lambda$	$1.11E\text{-}08\mu^{20}/\lambda^{21}$
8	$1.368797/\lambda$	$2.91E\text{-}08\mu^{21}/\lambda^{22}$
7	$1.511654/\lambda$	$9.15E\text{-}08\mu^{22}/\lambda^{23}$
6	$1.678320/\lambda$	$3.51E\text{-}07\mu^{23}/\lambda^{24}$
5	$1.878320/\lambda$	$1.68E\text{-}06\mu^{24}/\lambda^{25}$
4	$2.128320/\lambda$	$1.05E\text{-}05\mu^{25}/\lambda^{26}$
3	$2.461654/\lambda$	$9.12E\text{-}05\mu^{26}/\lambda^{27}$
2	$2.961654/\lambda$	$0.001232\mu^{27}/\lambda^{28}$
1	$3.961654/\lambda$	$0.034483\mu^{28}/\lambda^{29}$

M/30	MTBF	MTBF*
30	$0.033333/\lambda$	$0.033333\ /\lambda$
29	$0.067816/\lambda$	$0.001149\mu\ /\lambda^2$
28	$0.103530/\lambda$	$8.21E\text{-}05\mu^2/\lambda^3$
27	$0.140567/\lambda$	$9.12E\text{-}06\mu^3/\lambda^4$
26	$0.179029/\lambda$	$1.40E\text{-}06\mu^4/\lambda^5$
25	$0.219029/\lambda$	$2.81E\text{-}07\mu^5/\lambda^6$
24	$0.260696/\lambda$	$7.02E\text{-}08\mu^6/\lambda^7$
23	$0.304174/\lambda$	$2.14E\text{-}08\mu^7/\lambda^8$
22	$0.349628/\lambda$	$7.77E\text{-}09\mu^8/\lambda^9$
21	$0.397247/\lambda$	$3.33E\text{-}09\mu^9/\lambda^{10}$
20	$0.447247/\lambda$	$1.66E\text{-}09\mu^{10}/\lambda^{11}$
19	$0.499879/\lambda$	$9.63E\text{-}10\mu^{11}/\lambda^{12}$
18	$0.555435/\lambda$	$6.42E\text{-}10\mu^{12}/\lambda^{13}$
17	$0.614258/\lambda$	$4.91E\text{-}10\mu^{13}/\lambda^{14}$
16	$0.676758/\lambda$	$4.30E\text{-}10\mu^{14}/\lambda^{15}$
15	$0.743425/\lambda$	$4.30E\text{-}10\mu^{15}/\lambda^{16}$
14	$0.814853/\lambda$	$4.91E\text{-}10\mu^{16}/\lambda^{17}$
13	$0.891776/\lambda$	$6.42E\text{-}10\mu^{17}/\lambda^{18}$
12	$0.975110/\lambda$	$9.63E\text{-}10\mu^{18}/\lambda^{19}$
11	$1.066019/\lambda$	$1.66E\text{-}09\mu^{19}/\lambda^{20}$
10	$1.166019/\lambda$	$3.33E\text{-}09\mu^{20}/\lambda^{21}$
9	$1.277130/\lambda$	$7.77E\text{-}09\mu^{21}/\lambda^{22}$
8	$1.402130/\lambda$	$2.14E\text{-}08\mu^{22}/\lambda^{23}$
7	$1.544987/\lambda$	$7.02E\text{-}08\mu^{23}/\lambda^{24}$
6	$1.711654/\lambda$	$2.81E\text{-}07\mu^{24}/\lambda^{25}$
5	$1.911654/\lambda$	$1.40E\text{-}06\mu^{25}/\lambda^{26}$
4	$2.161654/\lambda$	$9.12E\text{-}06\mu^{26}/\lambda^{27}$
3	$2.494987/\lambda$	$8.21E\text{-}05\mu^{27}/\lambda^{28}$
2	$2.994987/\lambda$	$0.001149\mu^{28}/\lambda^{29}$
1	$3.994987/\lambda$	$0.033333\mu^{29}/\lambda^{30}$

M/31	MTBF	MTBF*
31	$0.032258/\lambda$	$0.032258\ /\lambda$
30	$0.065591/\lambda$	$0.001075\mu\ /\lambda^2$
29	$0.100074/\lambda$	$7.42E\text{-}05\mu^2/\lambda^3$
28	$0.135788/\lambda$	$7.95E\text{-}06\mu^3/\lambda^4$
27	$0.172825/\lambda$	$1.18E\text{-}06\mu^4/\lambda^5$
26	$0.211287/\lambda$	$2.26E\text{-}07\mu^5/\lambda^6$
25	$0.251287/\lambda$	$5.43E\text{-}08\mu^6/\lambda^7$
24	$0.292954/\lambda$	$1.58E\text{-}08\mu^7/\lambda^8$
23	$0.336432/\lambda$	$5.51E\text{-}09\mu^8/\lambda^9$
22	$0.381886/\lambda$	$2.25E\text{-}09\mu^9/\lambda^{10}$
21	$0.429506/\lambda$	$1.07E\text{-}09\mu^{10}/\lambda^{11}$
20	$0.479506/\lambda$	$5.91E\text{-}10\mu^{11}/\lambda^{12}$
19	$0.532137/\lambda$	$3.73E\text{-}10\mu^{12}/\lambda^{13}$
18	$0.587693/\lambda$	$2.69E\text{-}10\mu^{13}/\lambda^{14}$
17	$0.646516/\lambda$	$2.22E\text{-}10\mu^{14}/\lambda^{15}$
16	$0.709016/\lambda$	$2.08E\text{-}10\mu^{15}/\lambda^{16}$
15	$0.775683/\lambda$	$2.22E\text{-}10\mu^{16}/\lambda^{17}$
14	$0.847111/\lambda$	$2.69E\text{-}10\mu^{17}/\lambda^{18}$
13	$0.924035/\lambda$	$3.73E\text{-}10\mu^{18}/\lambda^{19}$
12	$1.007368/\lambda$	$5.91E\text{-}10\mu^{19}/\lambda^{20}$
11	$1.098277/\lambda$	$1.07E\text{-}09\mu^{20}/\lambda^{21}$
10	$1.198277/\lambda$	$2.25E\text{-}09\mu^{21}/\lambda^{22}$
9	$1.309388/\lambda$	$5.51E\text{-}09\mu^{22}/\lambda^{23}$
8	$1.434388/\lambda$	$1.58E\text{-}08\mu^{23}/\lambda^{24}$
7	$1.577245/\lambda$	$5.43E\text{-}08\mu^{24}/\lambda^{25}$
6	$1.743912/\lambda$	$2.26E\text{-}07\mu^{25}/\lambda^{26}$
5	$1.943912/\lambda$	$1.18E\text{-}06\mu^{26}/\lambda^{27}$
4	$2.193912/\lambda$	$7.95E\text{-}06\mu^{27}/\lambda^{28}$
3	$2.527245/\lambda$	$7.42E\text{-}05\mu^{28}/\lambda^{29}$
2	$3.027245/\lambda$	$0.001075\mu^{29}/\lambda^{30}$
1	$4.027245/\lambda$	$0.032258\mu^{30}/\lambda^{31}$

M/32	MTBF	MTBF*
32	$0.031250/\lambda$	$0.031250/\lambda$
31	$0.063508/\lambda$	$0.001005\mu/\lambda^2$
30	$0.096841/\lambda$	$6.72E\text{-}05\mu^2/\lambda^3$
29	$0.131324/\lambda$	$6.95E\text{-}05\mu^3/\lambda^4$
28	$0.167038/\lambda$	$9.93E\text{-}07\mu^4/\lambda^5$
27	$0.204075/\lambda$	$1.84E\text{-}07\mu^5/\lambda^6$
26	$0.242537/\lambda$	$4.24E\text{-}08\mu^6/\lambda^7$
25	$0.282537/\lambda$	$1.19E\text{-}08\mu^7/\lambda^8$
24	$0.324204/\lambda$	$3.88E\text{-}08\mu^8/\lambda^9$
23	$0.367682/\lambda$	$1.55E\text{-}08\mu^9/\lambda^{10}$
22	$0.413136/\lambda$	$7.05E\text{-}10\mu^{10}/\lambda^{11}$
21	$0.460756/\lambda$	$3.69E\text{-}10\mu^{11}/\lambda^{12}$
20	$0.510756/\lambda$	$2.21E\text{-}10\mu^{12}/\lambda^{13}$
19	$0.563387/\lambda$	$1.52E\text{-}10\mu^{13}/\lambda^{14}$
18	$0.618943/\lambda$	$1.18E\text{-}10\mu^{14}/\lambda^{15}$
17	$0.677766/\lambda$	$1.04E\text{-}10\mu^{15}/\lambda^{16}$
16	$0.740266/\lambda$	$1.04E\text{-}10\mu^{16}/\lambda^{17}$
15	$0.806933/\lambda$	$1.18E\text{-}10\mu^{17}/\lambda^{18}$
14	$0.878361/\lambda$	$1.52E\text{-}10\mu^{18}/\lambda^{19}$
13	$0.955285/\lambda$	$2.21E\text{-}10\mu^{19}/\lambda^{20}$
12	$1.038618/\lambda$	$3.69E\text{-}10\mu^{20}/\lambda^{21}$
11	$1.129527/\lambda$	$7.05E\text{-}10\mu^{21}/\lambda^{22}$
10	$1.229527/\lambda$	$1.55E\text{-}08\mu^{22}/\lambda^{23}$
9	$1.340638/\lambda$	$3.88E\text{-}08\mu^{23}/\lambda^{24}$
8	$1.465638/\lambda$	$1.19E\text{-}08\mu^{24}/\lambda^{25}$
7	$1.608495/\lambda$	$4.24E\text{-}08\mu^{25}/\lambda^{26}$
6	$1.775162/\lambda$	$1.84E\text{-}07\mu^{26}/\lambda^{27}$
5	$1.975162/\lambda$	$9.93E\text{-}07\mu^{27}/\lambda^{28}$
4	$2.225162/\lambda$	$6.95E\text{-}05\mu^{28}/\lambda^{29}$
3	$2.558495/\lambda$	$6.72E\text{-}05\mu^{29}/\lambda^{30}$
2	$3.058495/\lambda$	$0.001005\mu^{30}/\lambda^{31}$
1	$4.058495/\lambda$	$0.031250\mu^{31}/\lambda^{32}$

Figure E-11. MTBF for M out of N Redundant Configuration Without and With* Repair (Continued)

M/33	MTBF	MTBF*
33	$0.030303/\lambda$	$0.030303/\lambda$
32	$0.061553/\lambda$	$0.000947\mu/\lambda^{2}$
31	$0.093811/\lambda$	$6.11\text{E-}05\mu^{2}/\lambda^{3}$
30	$0.127144/\lambda$	$6.11\text{E-}06\mu^{3}/\lambda^{4}$
29	$0.161627/\lambda$	$8.43\text{E-}07\mu^{4}/\lambda^{5}$
28	$0.197341/\lambda$	$1.50\text{E-}07\mu^{5}/\lambda^{6}$
27	$0.234379/\lambda$	$3.43\text{E-}08\mu^{6}/\lambda^{7}$
26	$0.272840/\lambda$	$9.00\text{E-}09\mu^{7}/\lambda^{8}$
25	$0.312840/\lambda$	$2.88\text{E-}09\mu^{8}/\lambda^{9}$
24	$0.354507/\lambda$	$1.08\text{E-}09\mu^{9}/\lambda^{10}$
23	$0.397985/\lambda$	$4.70\text{E-}10\mu^{10}/\lambda^{11}$
22	$0.443440/\lambda$	$2.35\text{E-}10\mu^{11}/\lambda^{12}$
21	$0.491059/\lambda$	$1.34\text{E-}10\mu^{12}/\lambda^{13}$
20	$0.541059/\lambda$	$8.72\text{E-}11\mu^{13}/\lambda^{14}$
19	$0.593690/\lambda$	$6.43\text{E-}11\mu^{14}/\lambda^{15}$
18	$0.649246/\lambda$	$5.36\text{E-}11\mu^{15}/\lambda^{16}$
17	$0.708069/\lambda$	$5.04\text{E-}11\mu^{16}/\lambda^{17}$
16	$0.770569/\lambda$	$5.36\text{E-}11\mu^{17}/\lambda^{18}$
15	$0.837236/\lambda$	$6.43\text{E-}11\mu^{18}/\lambda^{19}$
14	$0.908664/\lambda$	$8.72\text{E-}11\mu^{19}/\lambda^{20}$
13	$0.985588/\lambda$	$1.34\text{E-}10\mu^{20}/\lambda^{21}$
12	$1.068921/\lambda$	$2.35\text{E-}10\mu^{21}/\lambda^{22}$
11	$1.159830/\lambda$	$4.70\text{E-}10\mu^{22}/\lambda^{23}$
10	$1.259830/\lambda$	$1.08\text{E-}09\mu^{23}/\lambda^{24}$
9	$1.370941/\lambda$	$2.88\text{E-}09\mu^{24}/\lambda^{25}$
8	$1.495941/\lambda$	$9.00\text{E-}09\mu^{25}/\lambda^{26}$
7	$1.638798/\lambda$	$3.34\text{E-}08\mu^{26}/\lambda^{27}$
6	$1.805465/\lambda$	$1.50\text{E-}07\mu^{27}/\lambda^{28}$
5	$2.005465/\lambda$	$8.43\text{E-}07\mu^{28}/\lambda^{29}$
4	$2.255465/\lambda$	$6.11\text{E-}06\mu^{29}/\lambda^{30}$
3	$2.588798/\lambda$	$6.11\text{E-}05\mu^{30}/\lambda^{31}$
2	$3.088798/\lambda$	$0.000947\mu^{31}/\lambda^{32}$
1	$4.088798/\lambda$	$0.030303\mu^{32}/\lambda^{33}$

M/34	MTBF	MTBF*
34	$0.029412/\lambda$	$0.029412/\lambda$
33	$0.059715/\lambda$	$0.000891\mu/\lambda^{2}$
32	$0.090965/\lambda$	$5.57\text{E-}05\mu^{2}/\lambda^{3}$
31	$0.123223/\lambda$	$5.39\text{E-}06\mu^{3}/\lambda^{4}$
30	$0.156556/\lambda$	$7.19\text{E-}07\mu^{4}/\lambda^{5}$
29	$0.191039/\lambda$	$1.24\text{E-}07\mu^{5}/\lambda^{6}$
28	$0.226753/\lambda$	$2.66\text{E-}08\mu^{6}/\lambda^{7}$
27	$0.263790/\lambda$	$6.88\text{E-}09\mu^{7}/\lambda^{8}$
26	$0.302252/\lambda$	$2.12\text{E-}09\mu^{8}/\lambda^{9}$
25	$0.342252/\lambda$	$7.63\text{E-}10\mu^{9}/\lambda^{10}$
24	$0.383918/\lambda$	$3.18\text{E-}10\mu^{10}/\lambda^{11}$
23	$0.427397/\lambda$	$1.52\text{E-}10\mu^{11}/\lambda^{12}$
22	$0.472851/\lambda$	$8.29\text{E-}11\mu^{12}/\lambda^{13}$
21	$0.520470/\lambda$	$5.13\text{E-}11\mu^{13}/\lambda^{14}$
20	$0.570470/\lambda$	$3.59\text{E-}11\mu^{14}/\lambda^{15}$
19	$0.623103/\lambda$	$2.84\text{E-}11\mu^{15}/\lambda^{16}$
18	$0.678657/\lambda$	$2.52\text{E-}11\mu^{16}/\lambda^{17}$
17	$0.737481/\lambda$	$2.52\text{E-}11\mu^{17}/\lambda^{18}$
16	$0.799981/\lambda$	$2.84\text{E-}11\mu^{18}/\lambda^{19}$
15	$0.866648/\lambda$	$3.59\text{E-}11\mu^{19}/\lambda^{20}$
14	$0.938076/\lambda$	$5.13\text{E-}11\mu^{20}/\lambda^{21}$
13	$1.014999/\lambda$	$8.29\text{E-}11\mu^{21}/\lambda^{22}$
12	$1.098333/\lambda$	$1.52\text{E-}10\mu^{22}/\lambda^{23}$
11	$1.189242/\lambda$	$3.18\text{E-}10\mu^{23}/\lambda^{24}$
10	$1.289242/\lambda$	$7.63\text{E-}10\mu^{24}/\lambda^{25}$
9	$1.400353/\lambda$	$2.12\text{E-}09\mu^{25}/\lambda^{26}$
8	$1.525353/\lambda$	$6.88\text{E-}09\mu^{26}/\lambda^{27}$
7	$1.668210/\lambda$	$2.66\text{E-}08\mu^{27}/\lambda^{28}$
6	$1.834877/\lambda$	$1.24\text{E-}07\mu^{28}/\lambda^{29}$
5	$2.034877/\lambda$	$7.19\text{E-}07\mu^{29}/\lambda^{30}$
4	$2.284877/\lambda$	$5.39\text{E-}06\mu^{30}/\lambda^{31}$
3	$2.618210/\lambda$	$5.57\text{E-}05\mu^{31}/\lambda^{32}$
2	$3.118210/\lambda$	$0.000891\mu^{32}/\lambda^{33}$
1	$4.118210/\lambda$	$0.029412\mu^{33}/\lambda^{34}$

M/35	MTBF	MTBF*
35	$0.028571/\lambda$	$0.028571/\lambda$
34	$0.057982/\lambda$	$0.000840\mu/\lambda^{2}$
33	$0.088286/\lambda$	$5.09\text{E-}05\mu^{2}/\lambda^{3}$
32	$0.119536/\lambda$	$4.77\text{E-}06\mu^{3}/\lambda^{4}$
31	$0.151794/\lambda$	$6.16\text{E-}07\mu^{4}/\lambda^{5}$
30	$0.185128/\lambda$	$1.03\text{E-}07\mu^{5}/\lambda^{6}$
29	$0.219610/\lambda$	$2.12\text{E-}08\mu^{6}/\lambda^{7}$
28	$0.255325/\lambda$	$5.31\text{E-}09\mu^{7}/\lambda^{8}$
27	$0.292362/\lambda$	$1.57\text{E-}09\mu^{8}/\lambda^{9}$
26	$0.330823/\lambda$	$5.45\text{E-}10\mu^{9}/\lambda^{10}$
25	$0.370823/\lambda$	$2.18\text{E-}10\mu^{10}/\lambda^{11}$
24	$0.412490/\lambda$	$9.99\text{E-}11\mu^{11}/\lambda^{12}$
23	$0.455968/\lambda$	$5.21\text{E-}11\mu^{12}/\lambda^{13}$
22	$0.501423/\lambda$	$3.08\text{E-}11\mu^{13}/\lambda^{14}$
21	$0.549042/\lambda$	$2.05\text{E-}11\mu^{14}/\lambda^{15}$
20	$0.599042/\lambda$	$1.54\text{E-}11\mu^{15}/\lambda^{16}$
19	$0.651673/\lambda$	$1.30\text{E-}11\mu^{16}/\lambda^{17}$
18	$0.707229/\lambda$	$1.22\text{E-}11\mu^{17}/\lambda^{18}$
17	$0.766052/\lambda$	$1.30\text{E-}11\mu^{18}/\lambda^{19}$
16	$0.828552/\lambda$	$1.54\text{E-}11\mu^{19}/\lambda^{20}$
15	$0.895218/\lambda$	$2.05\text{E-}11\mu^{20}/\lambda^{21}$
14	$0.966648/\lambda$	$3.08\text{E-}11\mu^{21}/\lambda^{22}$
13	$1.043571/\lambda$	$5.21\text{E-}11\mu^{22}/\lambda^{23}$
12	$1.126904/\lambda$	$9.99\text{E-}11\mu^{23}/\lambda^{24}$
11	$1.217813/\lambda$	$2.18\text{E-}10\mu^{24}/\lambda^{25}$
10	$1.317813/\lambda$	$5.45\text{E-}10\mu^{25}/\lambda^{26}$
9	$1.428924/\lambda$	$1.57\text{E-}09\mu^{26}/\lambda^{27}$
8	$1.553924/\lambda$	$5.31\text{E-}09\mu^{27}/\lambda^{28}$
7	$1.696781/\lambda$	$2.12\text{E-}08\mu^{28}/\lambda^{29}$
6	$1.863448/\lambda$	$1.03\text{E-}07\mu^{29}/\lambda^{30}$
5	$2.063448/\lambda$	$6.16\text{E-}07\mu^{30}/\lambda^{31}$
4	$2.313448/\lambda$	$4.77\text{E-}06\mu^{31}/\lambda^{32}$
3	$2.646781/\lambda$	$5.09\text{E-}08\mu^{32}/\lambda^{33}$
2	$3.146781/\lambda$	$0.000840\mu^{33}/\lambda^{34}$
1	$4.146781/\lambda$	$0.028571\mu^{34}/\lambda^{35}$

M/36	MTBF	MTBF*
36	$0.027778/\lambda$	$0.027778/\lambda$
35	$0.056349/\lambda$	$0.000794\mu/\lambda^{2}$
34	$0.085761/\lambda$	$4.67\text{E-}05\mu^{2}/\lambda^{3}$
33	$0.116064/\lambda$	$4.24\text{E-}06\mu^{3}/\lambda^{4}$
32	$0.147314/\lambda$	$5.31\text{E-}07\mu^{4}/\lambda^{5}$
31	$0.179572/\lambda$	$8.56\text{E-}08\mu^{5}/\lambda^{6}$
30	$0.212905/\lambda$	$1.71\text{E-}08\mu^{6}/\lambda^{7}$
29	$0.247388/\lambda$	$4.13\text{E-}09\mu^{7}/\lambda^{8}$
28	$0.283102/\lambda$	$1.18\text{E-}09\mu^{8}/\lambda^{9}$
27	$0.320139/\lambda$	$3.93\text{E-}10\mu^{9}/\lambda^{10}$
26	$0.358601/\lambda$	$1.51\text{E-}10\mu^{10}/\lambda^{11}$
25	$0.398601/\lambda$	$6.66\text{E-}11\mu^{11}/\lambda^{12}$
24	$0.440268/\lambda$	$3.33\text{E-}11\mu^{12}/\lambda^{13}$
23	$0.483745/\lambda$	$1.88\text{E-}11\mu^{13}/\lambda^{14}$
22	$0.529200/\lambda$	$1.20\text{E-}11\mu^{14}/\lambda^{15}$
21	$0.576820/\lambda$	$8.55\text{E-}12\mu^{15}/\lambda^{16}$
20	$0.626820/\lambda$	$6.84\text{E-}12\mu^{16}/\lambda^{17}$
19	$0.679451/\lambda$	$6.12\text{E-}12\mu^{17}/\lambda^{18}$
18	$0.735007/\lambda$	$6.12\text{E-}12\mu^{18}/\lambda^{19}$
17	$0.783830/\lambda$	$6.84\text{E-}12\mu^{19}/\lambda^{20}$
16	$0.856330/\lambda$	$8.55\text{E-}12\mu^{20}/\lambda^{21}$
15	$0.922997/\lambda$	$1.20\text{E-}11\mu^{21}/\lambda^{22}$
14	$0.994425/\lambda$	$1.88\text{E-}11\mu^{22}/\lambda^{23}$
13	$1.071349/\lambda$	$3.33\text{E-}11\mu^{23}/\lambda^{24}$
12	$1.154682/\lambda$	$6.66\text{E-}11\mu^{24}/\lambda^{25}$
11	$1.245591/\lambda$	$1.51\text{E-}10\mu^{25}/\lambda^{26}$
10	$1.345591/\lambda$	$3.93\text{E-}10\mu^{26}/\lambda^{27}$
9	$1.456702/\lambda$	$1.18\text{E-}09\mu^{27}/\lambda^{28}$
8	$1.581702/\lambda$	$4.13\text{E-}09\mu^{28}/\lambda^{29}$
7	$1.724558/\lambda$	$1.71\text{E-}08\mu^{29}/\lambda^{30}$
6	$1.891226/\lambda$	$8.56\text{E-}08\mu^{30}/\lambda^{31}$
5	$2.091226/\lambda$	$5.31\text{E-}07\mu^{31}/\lambda^{32}$
4	$2.341226/\lambda$	$4.24\text{E-}06\mu^{32}/\lambda^{33}$
3	$2.674559/\lambda$	$4.67\text{E-}05\mu^{33}/\lambda^{34}$
2	$3.174559/\lambda$	$0.000794\mu^{34}/\lambda^{35}$
1	$4.174559/\lambda$	$0.027778\mu^{35}/\lambda^{36}$

Figure E-11. MTBF for M out of N Redundant Configuration Without and With* Repair (Continued)

M/37	MTBF	MTBF*
37	0.027027/λ	0.027027/λ
36	0.054805/λ	0.00751μ /λ^{2}
35	0.083376/λ	4.29E-05μ^{2}/λ^{3}
34	0.112788/λ	3.79E-06μ^{3}/λ^{4}
33	0.143091/λ	4.59E-07μ^{4}/λ^{5}
32	0.174341/λ	7.17E-08μ^{5}/λ^{6}
31	0.206599/λ	1.39E-08μ^{6}/λ^{7}
30	0.239932/λ	3.24E-09μ^{7}/λ^{8}
29	0.274415/λ	8.93E-10μ^{8}/λ^{9}
28	0.310129/λ	2.87E-10μ^{9}/λ^{10}
27	0.347167/λ	1.06E-10μ^{10}/λ^{11}
26	0.385628/λ	4.50E-11μ^{11}/λ^{12}
25	0.425628/λ	2.16E-11μ^{12}/λ^{13}
24	0.467295/λ	1.17E-11μ^{13}/λ^{14}
23	0.510773/λ	7.12E-12μ^{14}/λ^{15}
22	0.556228/λ	4.85E-12μ^{15}/λ^{16}
21	0.603847/λ	3.70E-12μ^{16}/λ^{17}
20	0.653847/λ	3.14E-12μ^{17}/λ^{18}
19	0.706478/λ	2.98E-12μ^{18}/λ^{19}
18	0.762034/λ	3.14E-12μ^{19}/λ^{20}
17	0.820857/λ	3.70E-12μ^{20}/λ^{21}
16	0.883357/λ	4.85E-12μ^{21}/λ^{22}
15	0.950024/λ	7.12E-12μ^{22}/λ^{23}
14	1.021452/λ	1.17E-11μ^{23}/λ^{24}
13	1.098376/λ	2.16E-11μ^{24}/λ^{25}
12	1.181709/λ	4.50E-11μ^{25}/λ^{26}
11	1.272618/λ	1.06E-10μ^{26}/λ^{27}
10	1.372618/λ	2.87E-10μ^{27}/λ^{28}
9	1.483729/λ	8.93E-10μ^{28}/λ^{29}
8	1.608729/λ	3.24E-09μ^{29}/λ^{30}
7	1.751586/λ	1.39E-08μ^{30}/λ^{31}
6	1.918253/λ	7.17E-08μ^{31}/λ^{32}
5	2.118253/λ	4.59E-07μ^{32}/λ^{33}
4	2.368253/λ	3.79E-06μ^{33}/λ^{34}
3	2.701586/λ	4.29E-05μ^{34}/λ^{35}
2	3.201586/λ	0.000751μ^{35}/λ^{36}
1	4.201586/λ	0.027027μ^{36}/λ^{37}

M/38	MTBF	MTBF*
38	0.026316/λ	0.026316/λ
37	0.053343/λ	0.000711μ /λ^{2}
36	0.081121/λ	3.95E-05μ^{2}/λ^{3}
35	0.109692/λ	3.39E-06μ^{3}/λ^{4}
34	0.139104/λ	3.98E-07μ^{4}/λ^{5}
33	0.169407/λ	6.04E-08μ^{5}/λ^{6}
32	0.200657/λ	1.13E-08μ^{6}/λ^{7}
31	0.232915/λ	2.56E-09μ^{7}/λ^{8}
30	0.266248/λ	6.82E-10μ^{8}/λ^{9}
29	0.300731/λ	2.12E-10μ^{9}/λ^{10}
28	0.336445/λ	7.55E-11μ^{10}/λ^{11}
27	0.373482/λ	3.08E-11μ^{11}/λ^{12}
26	0.411944/λ	1.42E-11μ^{12}/λ^{13}
25	0.451944/λ	7.39E-12μ^{13}/λ^{14}
24	0.493611/λ	4.31E-12μ^{14}/λ^{15}
23	0.537089/λ	2.81E-12μ^{15}/λ^{16}
22	0.582543/λ	2.04E-12μ^{16}/λ^{17}
21	0.630162/λ	1.65E-12μ^{17}/λ^{18}
20	0.680162/λ	1.49E-12μ^{18}/λ^{19}
19	0.732794/λ	1.49E-12μ^{19}/λ^{20}
18	0.788349/λ	1.65E-12μ^{20}/λ^{21}
17	0.847173/λ	2.04E-12μ^{21}/λ^{22}
16	0.909673/λ	2.81E-12μ^{22}/λ^{23}
15	0.976340/λ	4.31E-12μ^{23}/λ^{24}
14	1.047768/λ	7.39E-12μ^{24}/λ^{25}
13	1.124691/λ	1.42E-11μ^{25}/λ^{26}
12	1.208024/λ	3.08E-11μ^{26}/λ^{27}
11	1.298934/λ	7.55E-11μ^{27}/λ^{28}
10	1.398934/λ	2.12E-10μ^{28}/λ^{29}
9	1.510045/λ	6.82E-10μ^{29}/λ^{30}
8	1.635045/λ	2.56E-09μ^{30}/λ^{31}
7	1.777902/λ	1.13E-08μ^{31}/λ^{32}
6	1.944569/λ	6.04E-08μ^{32}/λ^{33}
5	2.144569/λ	3.98E-07μ^{33}/λ^{34}
4	2.394569/λ	3.39E-06μ^{34}/λ^{35}
3	2.727902/λ	3.95E-05μ^{35}/λ^{36}
2	3.227902/λ	0.000711μ^{36}/λ^{37}
1	4.227902/λ	0.026316μ^{37}/λ^{38}

M/39	MTBF	MTBF*
39	0.025641/λ	0.025641 /λ
38	0.051957/λ	0.000675μ /λ^{2}
37	0 078984/λ	3.65E-05μ^{2}/λ^{3}
36	0.106762/λ	3.04E-06μ^{3}/λ^{4}
35	0.135333/λ	3.47E-07μ^{4}/λ^{5}
34	0.164745/λ	5.11E-08μ^{5}/λ^{6}
33	0.195048/λ	9.29E-09μ^{6}/λ^{7}
32	0.226298/λ	2.03E-09μ^{7}/λ^{8}
31	0.258556/λ	5.24E-10μ^{8}/λ^{9}
30	0.291889/λ	1.57E-10μ^{9}/λ^{10}
29	0.326372/λ	5.42E-11μ^{10}/λ^{11}
28	0.362086/λ	2.13E-11μ^{11}/λ^{12}
27	0.399123/λ	9.47E-12μ^{12}/λ^{13}
26	0.437585/λ	4.47E-12μ^{13}/λ^{14}
25	0.477585/λ	2.65E-12μ^{14}/λ^{15}
24	0.519252/μ	1.66E-12μ^{15}/λ^{16}
23	0.562730/λ	1.15E-12μ^{16}/λ^{17}
22	0.608184/λ	8.91E-13μ^{17}/λ^{18}
21	0.655803/λ	7.64E-13μ^{18}/λ^{19}
20	0.705803/λ	7.25E-13μ^{19}/λ^{20}
19	0.758435/λ	7.64E-13μ^{20}/λ^{21}
18	0.813991/λ	8.91E-13μ^{21}/λ^{22}
17	0.872814/λ	1.15E-12μ^{22}/λ^{23}
16	0.935314/λ	1.66E-12μ^{23}/λ^{24}
15	1.001981/λ	2.65E-12μ^{24}/λ^{25}
14	1.073409/λ	4.74E-12μ^{25}/λ^{26}
13	1.150332/λ	9.47E-12μ^{26}/λ^{27}
12	1.233666/λ	2.13E-11μ^{27}/λ^{28}
11	1.324575/λ	5.42E-11μ^{28}/λ^{29}
10	1.424575/λ	1.57E-10μ^{29}/λ^{30}
9	1.535686/λ	5.24E-10μ^{30}/λ^{31}
8	1.660686/λ	2.03E-09μ^{31}/λ^{32}
7	1.803543/λ	9.29E-09μ^{32}/λ^{33}
6	1.970210/λ	5.11E-08μ^{33}/λ^{34}
5	2.170210/λ	3.47E-07μ^{34}/λ^{35}
4	2.420210/λ	3.04E-06μ^{35}/λ^{36}
3	2.753543/λ	3.65E-05μ^{36}/λ^{37}
2	3.253543/λ	0.000675μ^{37}/λ^{38}
1	4.253543/λ	0 025641μ^{38}/λ^{39}

M/40	MTBF	MTBF*
40	0.025000/λ	0.025000 /λ
39	0.050641/λ	0.000641μ /λ^{2}
38	0.076957/λ	3.37E-05μ^{2}/λ^{3}
37	0.103984/λ	2.74E-06μ^{3}/λ^{4}
36	0.131762/λ	3.04E-07μ^{4}/λ^{5}
35	0.160333/λ	4.34E-08μ^{5}/λ^{6}
34	0.189745/λ	7.66E-09μ^{6}/λ^{7}
33	0.220048/λ	1.63E-09μ^{7}/λ^{8}
32	0.251298/λ	4.06E-10μ^{8}/λ^{9}
31	0.283556/λ	1.18E-10μ^{9}/λ^{10}
30	0.316889/λ	3.93E-11μ^{10}/λ^{11}
29	0.351372/λ	1.49E-11μ^{11}/λ^{12}
28	0.387086/λ	6.39E-12μ^{12}/λ^{13}
27	0.424123/λ	3.08E-12μ^{13}/λ^{14}
26	0.462585/λ	1.66E-12μ^{14}/λ^{15}
25	0.502585/λ	9.94E-13μ^{15}/λ^{16}
24	0.544252/λ	6.63E-13μ^{16}/λ^{17}
23	0.587730/λ	4.90E-13μ^{17}/λ^{18}
22	0.633184/λ	4.01E-13μ^{18}/λ^{19}
21	0.680803/λ	3.63E-13μ^{19}/λ^{20}
20	0.730803/λ	3.63E-13μ^{20}/λ^{21}
19	0.783435/λ	4.01E-13μ^{21}/λ^{22}
18	0.838991/λ	4.90E-13μ^{22}/λ^{23}
17	0.897814/λ	6.63E-13μ^{23}/λ^{24}
16	0.960314/λ	9.94E-13μ^{24}/λ^{25}
15	1.026981/λ	1.66E-12μ^{25}/λ^{26}
14	1.098409/λ	3.08E-12μ^{26}/λ^{27}
13	1.175332/λ	6.39E-12μ^{27}/λ^{28}
12	1.258666/λ	1.49E-11μ^{28}/λ^{29}
11	1.349575/λ	3.93E-11μ^{29}/λ^{30}
10	1.449575/λ	1.18E-10μ^{30}/λ^{31}
9	1.560686/λ	4.06E-10μ^{31}/λ^{32}
8	1.685686/λ	1.63E-09μ^{32}/λ^{33}
7	1.828543/λ	7.66E-09μ^{33}/λ^{34}
6	1.995210/λ	4.34E-08μ^{34}/λ^{35}
5	2.195210/λ	3.04E-07μ^{35}/λ^{36}
4	2.445210/λ	2.74E-06μ^{36}/λ^{37}
3	2.778543/λ	3.37E-05μ^{37}/λ^{38}
2	3.278543/λ	0.000641μ^{38}/λ^{39}
1	4.278543/λ	0.025000μ^{39}/λ^{40}

Figure E-11. MTBF for M out of N Redundant Configuration Without and With* Repair (Continued)

The approximation:

$$\mathrm{MTBF} = \frac{\frac{1}{\lambda}\left(\frac{u}{\lambda}\right)^{n-m}}{n\left[\binom{n-1}{m-1}\right]} \qquad \text{(E-7)}$$

is used when the first failed item is repaired immediately. u, the repair rate or reciprocal of MTTR, is normally much greater than the failure rate. When this maintenance strategy is used the major gain comes from the first redundant unit [i.e., (n-1)-out-of-n]. In cases where the repair rate, u, is much greater than the failure rate, λ, it is rare to require more than two excess units as is shown by extending our example where 5-out-of-8 are required (i.e., n-m = 8-5 = 3 excess units).

$$\mathrm{MTBF} = \frac{\left(\frac{1}{\lambda}\right)\left(\frac{u}{\lambda}\right)^{8-5}}{8\left[\binom{7}{4}\right]}$$

$$= \frac{1}{8\,\frac{7!}{4!3!}}\left(\frac{u^3}{\lambda^4}\right)$$

$$= \frac{1}{280}\,\frac{u^3}{\lambda^4}$$

which for a case where $\lambda = 1 \times 10^{-6}$ and MTTR = 1 hour would yield an MTBF of 3.57×10^{21} hours.

The tabular information is based upon the assumption that the failure rate of a single unit obeys the exponential failure law (the parallel combination does *not*).

A refined result for the MTBF in the one-out-of-two case with repair is obtained:

$$\mathrm{MTBF} = \frac{3\lambda + u}{2\lambda^2}$$

The tabular value uses

$$\mathrm{MTBF} = \frac{u}{2\lambda^2}$$

in accordance with the approximate formula (Equation E-7).

MTBF tables are presented in figure E-11.

Example

Use the tables in figure E-11 to calculate MTBF when 5 of 8 units are required for success and $\lambda = 1 \times 10^{-3}$ failures/hour and MTTR = 1 hour (μ = 1 repair/hour)

From figure E-10

Without repair
$\mathrm{MTBF} = 0.634524/\lambda$
MTBF = 634.524 hours

With repair (unrestricted)
$\mathrm{MTBF} = 0.003571\ \mu^3/\lambda^4$
$\mathrm{MTBF} = 3{,}571 \times 10^{+6}$ hours

E.5 REFERENCES

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