$$B_{21} = \frac{\lambda^3}{8\pi h\tau_{sp}} = \frac{(6000\times10^{-10})^3\,\text{m}^3}{8\times\pi\times6.626\times10^{-34}\,\text{Js}\times10^{-6}\,\text{s}}$$

$$= \frac{216\times10^{-21}}{166.6\times10^{-40}}\cdot\frac{\text{m}^3}{\text{J}\cdot\text{s}^2} = \mathbf{1.3\times10^{19}\ m/kg.}$$

1.11 EINSTEIN RELATIONS

Under thermal equilibrium, the mean population N_1 and N_2 in the lower and upper energy levels respectively must remain constant. This condition requires that the number of transitions from E_2 to E_1 must be equal to the number of transitions from E_1 to E_2. Thus,

$$\left.\begin{array}{r}\text{The number of atoms absorbing}\\ \text{photons per second per unit volume}\end{array}\right\} = \left\{\begin{array}{l}\text{The number of atoms emitting}\\ \text{photons per second per unit}\\ \text{volume}\end{array}\right.$$

$$\left.\begin{array}{r}\text{The number of atoms absorbing}\\ \text{photons per second per unit volume}\end{array}\right\} = B_{12}\,\rho(\nu)\,N_1$$

$$\left.\begin{array}{r}\text{The number of atoms emitting}\\ \text{photons per second per unit volume}\end{array}\right\} = A_{21}\,N_2 + B_{21}\,\rho(\nu)\,N_2$$

In equilibrium condition, the number of transitions from E_2 to E_1 must be equal to the number of transitions from E_1 to E_2. Thus,

$$B_{12}\,\rho(\nu)\,N_1 = A_{21}N_2 + B_{21}\,\rho(\nu)\,N_2 \qquad \text{...(1.27)}$$

$$\rho(\nu)\,(B_{12}\,N_1 - B_{21}N_2) = A_{21}N_2$$

$$\therefore \qquad \rho(\nu) = A_{21}\,N_2/B_{12}N_1 - B_{21}\,N_2$$

On dividing both the numerator and denominator on the right hand side of the above equation with $B_{12}\,N_2$, we get

$$\boxed{\rho(\nu) = \frac{A_{21}/B_{12}}{N_1/N_2 - B_{21}/B_{12}}} \qquad \text{...(1.28)}$$

It follows from eq. (1.13) that

$$N_1/N_2 = e^{(E_2-E_1)/kT}$$

As

$$(E_2 - E_1) = h\nu$$

$$N_1/N_2 = e^{h\nu/kT}$$

$$\therefore \qquad \rho(\nu) = \frac{A_{21}}{B_{12}}\left[\frac{1}{e^{h\nu/kT} - B_{21}/B_{12}}\right] \qquad \text{...(1.29)}$$

To maintain thermal equilibrium, the system must release energy in the form of electromagnetic radiation. It is required that the radiation be identical with black body radiation and be consistent with Planck's radiation law for any value of T. According to Planck's law

$$\rho(\nu) = (8\pi\,h\nu^3\,\mu^3/c^3)\,\frac{1}{e^{h\nu/kT}-1} \qquad \text{...(1.30)}$$

where μ is the refractive index of the medium and c is the velocity of light in free space.

Energy density $\rho(\nu)$ given by Eq. (1.29) will be consistent with Planck's law (Eq. 1.30) only if

$$\boxed{B_{21} = B_{12}} \qquad \text{...(1.31a)}$$

and

$$A_{21}/B_{12} = (8\,\pi\,h\nu^3\,\mu^3/c^3) \qquad \text{...(1.31b)}$$

Therefore,

$$\boxed{B_{12} = B_{21} = \frac{c^3}{8\pi\,h\nu^3\mu^3}A_{21}} \qquad \text{...(1.32)}$$

Equations (1.31*a*) and (1.31*b*) are known as the *Einstein relations*. Equation (1.32) gives the relationship between the *A* and *B* coefficients.

The first relation (1.31*a*) shows that the coefficients for both absorption and stimulated emission are numerically equal. The equality implies the following. When an atom with two energy levels is placed in the radiation field, the probability for an upward (absorption) transition is equal to the probability for a downward (stimulated emission) transition.

The second relation (1.31*b*) shows that the ratio of coefficients of spontaneous versus stimulated emission is proportional to the third power of frequency of the radiation. This is why it is difficult to achieve laser action in higher frequency ranges such as x-rays.

1.12 CONDITIONS FOR LARGE STIMULATED EMISSIONS:

The key to laser action is the existence of stimulated emission. In practice, the absorption and spontaneous emissions always occur together with stimulated emission. Let us now study the conditions under which the number of stimulated emissions can be made larger than those of the other two processes.

(*a*). From equations (1.20) and (1.25), we can write for the ratio of the stimulated transitions to spontaneous transitions as

$$R_1 = \frac{\text{Stimulated transitions}}{\text{Spontaneous transitions}}$$

$$= B_{21}\,\rho(\nu)\,N_2/A_{21}\,N_2$$

$$= (B_{21}/A_{21})\,\rho(\nu) \qquad \text{...(1.33)}$$

Using eq.(1.30) for $\rho(\nu)$, we get

$$R_1 = (B_{21}/A_{21})\left[\frac{8\pi h\nu^3\mu^3}{c^3}\,\frac{1}{e^{h\nu/kT}-1}\right] \qquad \text{...(1.34)}$$

From eq.(1.31) and (1.32), we can write

$$\frac{B_{21}}{A_{21}} = \frac{B_{12}}{A_{21}} = \frac{c^3}{8\pi h\nu^3\mu^3} \qquad \text{...(1.35)}$$

Using the eq.(1.35) into eq(1.34), we obtain

$$R_1 = (c^3/8\,\pi\, h\nu^3\,\mu^3)\left[(8\,\pi\, h\nu^3\mu^3/c^3)\frac{1}{e^{h\nu/kT}-1}\right]$$

or

$$\boxed{R_1 = \frac{1}{(e^{h\nu/kT}-1)}} \qquad \ldots(1.36)$$

If we assume $\nu = 5\times10^{14}$ Hz (yellow light) and $T = 2000$ K, the value of $h\nu\,/\,kT$ is 11.99.

$$\therefore \qquad R_1 = \frac{1}{e^{11.99}-1} = 6\times10^{-6}$$

The above result shows that in the optical region spontaneous emissions dominate over the stimulated emissions.

The equation (1.33) suggests that the light field density $\rho(\nu)$ present within the material is required to be enhanced if we want large number of stimulated emissions.

(*b*) The ratio of stimulated transitions to absorption transitions is given by

$$R_2 = \frac{\text{Stimulated transitions}}{\cancel{\text{Spontaneous transitions}}\ \text{absorption}}$$

$$= \frac{B_{21}\,\rho(\nu)\,N_2}{B_{12}\,\rho(\nu)\,N_1} \qquad \ldots(1.37)$$

As $\qquad B_{21} = B_{12}$

$$\boxed{R_2 = N_2/N_1} \qquad \ldots(1.38)$$

At thermodynamic equilibrium,

$$N_2/N_1 \ll 1$$

Therefore at equilibrium, absorption transitions overwhelm stimulated transitions. A photon of the light field may hit an excited atom leading to stimulated emission, or be absorbed on hitting an atom in the ground state. As $N_2 \ll N_1$ at thermodynamic equilibrium, a photon has a much higher probability of being absorbed than of stimulating an excited atom. As a result, the absorption process dominates stimulated emission and the medium will absorb the incident light. If, on the other hand, more atoms are in the excited state, *i.e.* $N_2 > N_1$, photons are more likely to cause stimulated emission than absorption. Therefore, in order to achieve more stimulated emissions, the population N_2 of the excited state should be made larger than the population N_1 of the lower energy state.

To sum up, two conditions are to be satisfied to make stimulated emissions overwhelm the spontaneous emissions. They are that (*i*) the population at excited level should be greater than that at the lower energy level and (*ii*) the radiation density in the medium should be very large.

Example 1.7. *(a) At what temperature are the rates of spontaneous and stimulated emission equal ? Assume λ = 5000 Å.*

(b) At what wavelength are they equal at 300 K ?

Solution : If the rates of spontaneous and stimulated emission are equal, then

$$R = \left[e^{h\nu/kT}-1\right]^{-1}$$

$$\lambda = 5000\ \text{Å},\ \nu = \frac{c}{\lambda} = \frac{3\times10^8\ \text{m/s}}{5000\times10^{-10}\text{m}}$$

$$= 6\times10^{14}\ \text{Hz}$$

$$\therefore \qquad \frac{h\nu}{kT} = \frac{(6.626\times10^{-34}\text{J.s})\,(6\times10^{14}\text{s}^{-1})}{(1.38\times10^{-23}\text{J/K})T}$$

$$= \frac{28.81\times10^3}{T}\text{K}$$

$$(a) \quad \therefore \quad \exp\left[\frac{28.81\times10^3}{T}\text{K}\right] = 2$$

or

$$\frac{28.81\times10^3}{T}\text{K} = \ln 2 = 0.693$$

$$\therefore \qquad T = \frac{28.81\times10^3}{T}\text{K} = \mathbf{41{,}573\ K}$$

$$(b) \quad \therefore \qquad \frac{h\nu}{kT} = \frac{(6.626\times10^{-34}\text{J.s})\ \nu}{(1.38\times10^{-23}\text{J/K})\,(300\ \text{K})}$$

$$= (1.6\times10^{-13}\ \text{s})\nu$$

$$\therefore \qquad \exp\left[(1.6\times10^{-13}\ \text{s})\nu\right] = 2$$

or

$$(1.6\times10^{-13}\ \text{s})\nu = 0.693$$

$$\therefore \qquad \nu = \frac{0.693}{1.6\times10^{-13}}\ \text{Hz} = 4.3\times10^{12}\ \text{Hz}$$

$$\therefore \qquad \lambda = \frac{c}{\nu} = \frac{3\times10^8\text{m/s}}{4.3\times10^{12}\text{s}^{-1}} = \mathbf{69.8\ \mu m}$$

1.13 CONDITION FOR LIGHT AMPLIFICATION

Let us consider a beam of light propagating through a material medium. If the photons strike lower state (unexcited) atoms, they may be absorbed and removed from the stream of photons which therefore loses energy. However, if the photons strike excited atoms, more photons can be produced which are added to the light beam and increase its energy. Since the probabilities of absorption and stimulated emission are the same, both attenuation and amplification of the light beam occur simultaneously. We will now show that amplification can predominate only if there are more atoms in the higher level than in the lower level.

Let there be n photons per unit volume in the light beam. As the beam travels through the medium, some photons are absorbed due to absorption transitions and some photons are generated due to emission transitions. We will not take into account the photons generated by spontaneous emission as these photons go in random directions and do not contribute to the light beam propagating through the medium. Thus, we consider only the photons generated by stimulated emissions.

Let $-(dn/dt)$ be the net rate of loss of photons from the beam as it travels through an elemental volume of the medium having a thickness Δx and an area of unity.

The net rate of loss of photons from the light beam must be equal to the difference between the net rates of absorption and stimulated emission transitions. Thus,

$$-\frac{dn}{dt} = B_{12}\,\rho(\nu)\,N_1 - B_{21}\,\rho(\nu)\,N_2$$

or

$$-\frac{dn}{dt} = (N_1 - N_2)\,\rho(\nu)\,B_{12} \qquad ...(1.39)$$

If the energy density of the light field in the medium is $\rho(\nu)$, then the intensity I is

$$I = \rho(\nu)\,\upsilon \qquad ...(1.40)$$

where υ $(=c/\mu)$ is the velocity of light in the medium.

As $\rho(\nu) = nh\nu$,

$$I = nh\,\nu\upsilon \qquad ...(1.41)$$

The loss of photons, $-dn$ in a small thickness dx of medium may be written as,

$$-dn = \frac{dI(x)}{dx}\cdot\frac{dx}{h\,\nu\upsilon}$$

The net rate of loss during a time interval dt is given by,

$$-\frac{dn}{dt} = \frac{dI(x)}{dx}\cdot\frac{1}{h\nu} \qquad ...(1.42)$$

$dx/dt = \upsilon$ is used in obtaining the above equation.

Using eq.(1.2) into eq.(1.42), we get

$$-\frac{dn}{dt} = +\,\alpha I(x)\cdot\frac{1}{h\nu}$$

Using Eq. (1.40) into the above, we get

$$\therefore \quad -\frac{dn}{dt} = \alpha\rho(\nu)\,\upsilon\cdot\frac{1}{h\nu} \qquad ...(1.43)$$

Comparing equations (1.39) and (1.43), we obtain

$$\alpha\rho(\nu)\,\upsilon\frac{1}{h\nu} = (N_1 - N_2)\,\rho(\nu)\,B_{12}$$

$$\therefore \quad \alpha = (N_1 - N_2)\,\frac{B_{12}\,h\nu}{\upsilon} \qquad ...(1.44)$$

Eq. (1.44) relates the absorption coefficient α to the difference in populations $(N_1 - N_2)$ of the two energy levels. For a material in thermal equilibrium, $N_1 > N_2$ and α is positive.

If N_2 is somehow made greater than N_1, then α becomes a negative quantity and the relation (1.4) takes the following form.

$$I = I_0 e^{(-\alpha)x}$$

or

$$I = I_0 e^{\gamma x} \qquad ...(1.45)$$

where γ $(= -\alpha)$ is referred to as the *gain coefficient* per unit length. As the gain coefficient γ is a positive quantity, the equation (1.45) implies that the intensity of light grows exponentially as the light beam travels through the medium. This is clearly amplification of light. Incorporating γ, we rewrite eq.(1.44) as,

$$\boxed{\gamma = (N_2 - N_1)\,\frac{B_{12}\,h\nu}{\upsilon}\ \ \text{Condition for amplification}} \qquad ...(1.46)$$

γ will be positive if $(N_2 - N_1) > 0$, that is,

$$\boxed{N_2 > N_1} \qquad ...(1.47)$$

The condition (1.47) is known as *population inversion*, because it is the inverse of the normal situation. Eq. (1.46) thus indicates that population inversion is a necessary condition to be satisfied for causing the amplification of incident light.

Using the relations (1.32) for B_{12}, (1.21) for A_{21} and $c = \mu\,\upsilon$, we can rewrite equation (1.46) as

$$\boxed{\gamma = (N_2 - N_1).\,\frac{\nu^2}{8\,\pi\,\nu^2\tau_{sp}}} \qquad ...(1.48)$$

1.14 LINESHAPE FUNCTION

For the sake of simplicity, we have assumed till now that the emission of light by all atoms in the specimen is strictly monochromatic and the frequency is given by

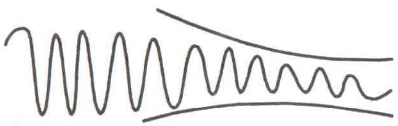
$$\nu_0 = \frac{E_2 - E_1}{h}$$

However, this is not true. The light emitted by an atom is in reality a burst of decaying exponential, as shown in Fig.1.14. The actual frequency of the light, although centered at ν_o, has a spread over a certain range $\Delta\nu$, as depicted in Fig. 1.15. Fourier analysis of the decaying light shows that

$$\boxed{\Delta\nu \approx \frac{1}{\tau_{sp}}} \qquad ...(1.49)$$

$\Delta\nu$ is known as the *linewidth* and is defined as the frequency difference corresponding to the intensity $\frac{1}{2}I$.

Similarly, atoms are capable of absorbing light not just at a single frequency but over a band of frequencies. It implies that atoms interact with light over a range of frequencies. If we plot the intensity of radiation as a function of frequency, we would again obtain the bell-shaped curve shown in Fig. 1.14.



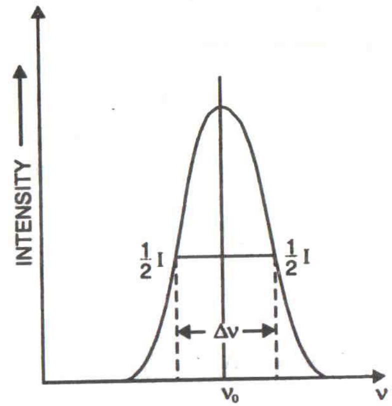


Fig. 1.14. A decaying electric field of light burst. **Fig. 1.15.** A Lineshape function.

The shape of the curves is described by the *lineshape function* $g(\nu)$. The function $g(\nu)$ is usually normalized according to

$$\int_{-\infty}^{+\infty} g(\nu)\, d\nu = 1 \qquad ...(1.50)$$

We may consequently view $g(\nu)\, d\nu$ as the probability that a given transition between the energy levels E_2 and E_1 will result in the emission (or absorption) of a photon whose frequency lies between ν and $\nu + d\nu$. Hence a photon of energy $h\nu$ may not necessarily stimulate another photon of energy $h\nu$, but it, may stimulate a photon which has an energy between $h\nu$ and $h(\nu + d\nu)$.

We now assume that radiation density $\rho(\nu)\, d\nu$ incident on the collection of atoms has frequencies between ν and $\nu + d\nu$. Further, out of the total N_1 and N_2 atoms per unit volume only $N_1\, g(\nu)\, d\nu$ and $N_2\, g(\nu)\, d\nu$ atoms per unit volume are capable of interacting with radiation of frequencies lying between ν and $\nu + d\nu$. Incorporating these ideas, Eq. (1.46) takes the form

$$\boxed{\gamma = (N_2 - N_1)\,\frac{B_{21}\, h\nu_o}{\upsilon}\, g(\nu) = (N_2 - N_1)\,\frac{\upsilon^2\, g(\nu)}{8\pi\, \nu^2\, \tau_{sp}}} \qquad ...(1.51)$$

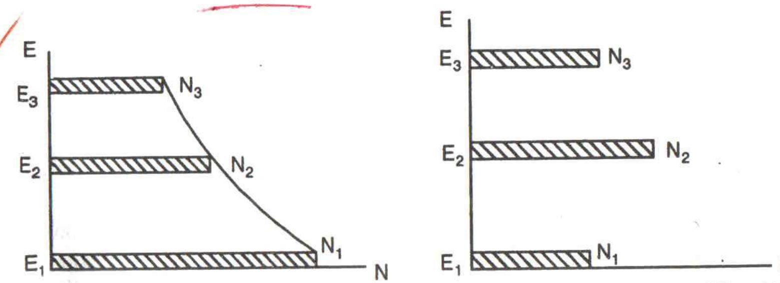
where ν_o is the central frequency.

1.15 POPULATION INVERSION

When an atomic system is in thermal equilibrium, photon absorption and emission processes take place side by side, but because $N_1 > N_2$, absorption dominates. However, laser operation requires obtaining stimulated emission exclusively. To achieve a high percentage of stimulated emission, a majority of atoms should be at the higher energy level than at the lower level. *The non-equilibrium state in which the population N_2 of the upper energy level exceeds to a large extent the population N_1 of the lower energy level is known as the state of population inversion.*

Extending the Boltzmann distribution, Eq. (1.13), to this non-equilibrium state of population inversion, it is seen that N_2 can exceed N_1 only if the temperature *were* negative. In view of this, the state of population inversion is sometimes referred to as a *negative temperature state.* It does not mean that we can attain temperatures below absolute zero. The terminology underlines the fact that the state of population inversion is a non-equilibrium state. It should be borne in mind that the population inversion state is attained at normal temperatures.

Let us consider for the moment a system that has three energy states E_1, E_2 and E_3. With the system in equilibrium, the uppermost level E_3 is populated least and the lowest level E_1 is populated most, as shown in Fig.1.16(*a*). The dotted curve shown in Fig. 1.16(*a*) represents a normal Boltzmann distribution. Since the population in the three states is such that $N_3 < N_2 < N_1$, the system absorbs photons rather than emit photons. However, if the system is supplied with external energy such that N_2 exceeds N_1, we say that the system reached population inversion, the population inversion having taken place between the levels E_2 and E_1, as shown in Fig. 1.16(*b*).



(*a*) Population in equilibrium. (*b*) Population inversion of E_2 with respect to E_1.

Fig. 1.16. Three level system.

Under the population inversion condition, the stimulated emission can produce a cascade of light. The first few randomly emitted spontaneous photons trigger stimulated emission of more photons and those stimulated photons induce still more stimulated emissions and so on. As long as the excited state population is more than the lower level population, stimulated emissions are more likely than absorption ; and consequently, light gets amplified as shown in Fig. 1.17. As soon as the population at lower level becomes equal to or larger than that at the excited state, population inversion ends, stimulated emissions diminish and amplification of light ceases.

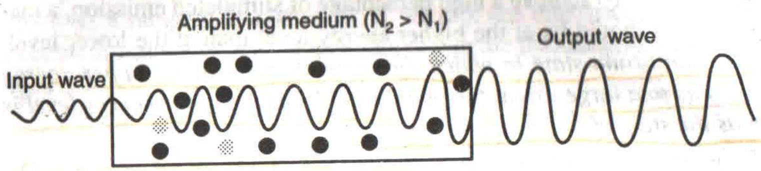


Fig. 1.17. Amplification of a light wave in a medium with population inversion.

1.16 PUMPING

In order to realize and maintain the state of population inversion, it is necessary that atoms must be continuously promoted from the lower level to the excited level. Energy is to be supplied somehow to the laser medium to raise atoms from the lower level to the excited level and for maintaining population at the excited level at a value greater than that of the lower energy. The process by which atoms are raised from the lower level to the upper level is called *pumping*.

The usual method of exciting atoms to higher energy levels is to heat the material. From Eq. (1.13) it is seen that as long as $E_2 > E_1$, $N_2 < N_1$. Heating the material only increases the average energy of atoms but does not make N_2 greater than N_1. Therefore, population inversion cannot be achieved by heating the material.

1.17 PUMPING METHODS

In order to create the state of population inversion, we have to selectively excite the atoms in the material to particular energy levels. There are a variety of methods employed for this purpose. The most common methods of pumping make use of light and electrons. We confine our study here only to these techniques.

(a) Optical Pumping :

Optical pumping is the use of photons to excite the atoms. A light source such as a flash discharge tube is used to illuminate the laser medium and the photons of appropriate frequency excite the atoms to an uppermost level. From there, they drop to the metastable upper laser level to create the state of population inversion. Optical pump sources are flash discharge tubes, continuously operating lamps, spark gaps or an auxiliary laser is sometimes used as the pump source.

The pump photon must have higher frequency than the emitted photon. This is because the atoms are to be raised above the upper laser level from a lower level which is below or at the lower laser level. This is one of the factors which reduces the laser efficiency.

The pumping level of the atom must not be a narrow level. It should be sufficiently wide, spanning a range of energies. If it is narrow, one can use a

pump photon of only a specific frequency. Such a situation severely restricts the choice of sources and a large portion of the source power would go wasted. Fortunately, in a majority of cases the upper levels are wide bands and atoms can be excited to many of the upper levels. Therefore, light sources emitting a broad range of wavelengths like a flash lamp can be used to excite atoms.

Optical pumping is suitable for any laser medium which is transparent to pump light. Optical pumping is used for solid state crystalline lasers and liquid tunable dye lasers.

(b) Electrical Pumping :

Electrical pumping can be used only in case of laser materials that can conduct electricity without destroying lasing activity. This method is limited to gases. In case of a gas laser, a high voltage pulse initially ionizes the gas so that it conducts electricity. An electric current flowing through the gas excites atoms to the excited level from where they drop to the metastable upper laser level leading to population inversion.

(c) Direct Conversion :

In semiconductor lasers also electrical pumping is used, but here it is not the atoms that are excited. It is the current carriers namely electrons and holes which are excited and a population inversion is achieved in the junction region. The electrons recombine with holes in the junction regions producing laser light. Thus, in semiconductor lasers, a direct conversion of electrical energy into light energy takes place.

1.18 ACTIVE MEDIUM

Atoms in general are characterized by a large number of energy levels. However, all types of atoms are not suitable for laser operation. Even in a medium consisting of different species of atoms, only a small fraction of atoms of a particular species are suitable for stimulated emission and laser action. Those atoms which cause light amplification are called *active centers*. The rest of the medium acts as host and supports active centers. The medium hosting the active centers is called an *active medium*. *An active medium is thus a medium which, when excited, reaches the state of population inversion, and eventually causes light amplification.* The active medium may be a solid, a liquid or a gas.

1.19 METASTABLE STATES

An atom can be excited to a higher level by supplying energy to it. Normally, excited states have short lifetimes and release their excess energy in a matter of nanoseconds (10^{-9} sec) by spontaneous emission. Atoms do not stay at such excited states long enough to be stimulated to emit their energy. Though, the pumping agent continuously raises the atoms to the excited level, many of them rapidly undergo spontaneous transitions to the lower energy level. Population inversion cannot be therefore established. For establishing population inversion, the excited atoms are required to " wait" at the upper lasing level till a large number of atoms accumulate at that level. Thus, what is needed is an excited state with a longer lifetime. Such longer-lived upper levels from where an excited atom does not return to lower level at once, but remains excited for an appreciable time, are known as *metastable states*. Phosphors are an example of materials having metastable states. They emit persistent light called *phosphorescence* because of metastable states existent in them.

Atoms stay in metastable states for about 10^{-6} to 10^{-3} s. This is 10^3 to 10^6 times longer than the time of stay of atom at excited levels. Therefore , it is possible for a large number of atoms to accumulate at a metastable level. The metastable state population can exceed the population of a lower level and lead to the state of population inversion.

If the metastable states do not exist, there could be no population inversion, no stimulated emission and hence no laser operation.

Thus, the foundation to the laser operation is the existence of metastable states.

1.20 PUMPING SCHEMES

Atoms in general are characterized by a large number of energy levels. Among these energy levels, two, three or four levels will be pertinent to the pumping process. Accordingly, pumping schemes are classified as two-level, three-level and four-level schemes. Among them, the two-level scheme will not lead to laser action. The three-level and four-level schemes are important and are widely employed.

(a) Two Level Pumping Scheme :

It appears that the most simple and straight-forward method to establish population inversion is to pump an excess of atoms into the higher energy state by applying intense radiation. But basically, a two-level pumping scheme, shown in Fig. 1.5, is not suitable for attaining population inversion. We can establish it through the following considerations.

Population inversion requires a build-up of population in the upper laser level. It is possible only if the upper level is populated faster than it decays, so that the population of the upper level exceeds that of the lower level. It, in turn, is possible only if the lifetime of spontaneous emission is longer. Thus, it is required that the lifetime Δt at upper level E_2 must be longer. According to Heisenberg uncertainty principle, the linewidth ΔE_2 of level E_2 and the lifetime Δt are related by.

$$\Delta E_2 \,.\, \Delta t \geq \hbar \qquad \ldots(1.52)$$

Δt will be longer if ΔE_2 is smaller which implies that the upper energy level E_2 must be narrow. However, if E_2 is narrow, we have to use only a specific frequency photon $(\nu = E_2 - E_1 / \hbar)$ to pump atoms. It means that the pump source should be highly monochromatic. In practice, monochromatic source of required frequency may not exist. Even if it exists, the pumping efficiency would be very low. The result is that enough population cannot be excited to level E_2. Further, pumping radiation on one hand excites the ground state atoms and on the other hand induces transitions from the upper level to the lower level. It means that pumping operation simultaneously populates and depopulates the upper level. Hence, population inversion cannot be attained in a two-level scheme. All that we may achieve at best is a system of equally populated levels.

One must therefore look for materials with either three or four energy level systems, which operate with *different frequencies* of pumping and lasing transitions.

The transition between the two levels that generate stimulated emission is called a *lasing transition*. The terminal level is called the *lower lasing level* and the upper level as *upper lasing level*. The upper lasing level should be a metastable level. The uppermost level to which atoms are excited is known as the *pumping level*. The transition between the ground level and the pump level is called the *pump transition*.

(b) Three Level Pumping Scheme :

A model of a three level pumping scheme is illustrated in Fig. 1.18. A three level scheme is one in which the lower laser level is either the ground state or a level whose separation from the ground state is small compared to *kT*.

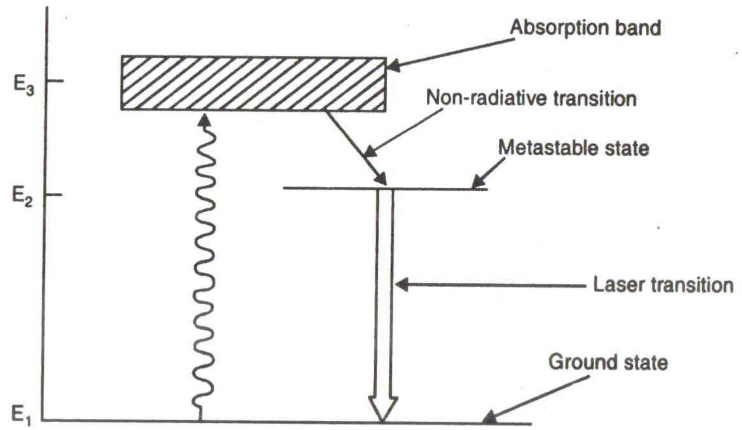


Fig. 1.18. Three-level energy diagram. Pumping transition is shown by wavy line, nonradiative transition by simple arrow, laser transition by hollow arrow.

Initially, the population distribution among the three levels obeys the Boltzmann law. When the atoms are subjected to an intense radiation of pumping frequency $\nu_p = (E_3 - E_1 / \hbar)$, the atoms are pumped to the higher level E_3.

Some of the excited atoms make spontaneous transitions to the ground state but many of them undergo spontaneous non-radiative transitions to the meta-stable level E_2. As spontaneous transitions from E_2 to E_1 do not occur often, the atoms accumulate at the metastable level E_2. The build-up of atoms at E_2 continues because of pumping process. Eventually, the population N_2 at E_2 exceeds the population N_1 at E_1 and population inversion is attained. Now, a photon of $h\nu$ ($= E_2 - E_1$) can induce stimulated emission and laser action.

For better pumping efficiency, the level E_2 should be a band of energy levels instead of being a single narrow line. It allows use of a pumping radiation of wider bandwidth to excite more atoms. However, the major disadvantage of a three level scheme is that it requires very high pump powers. In this scheme, the terminal level of the laser transition is simultaneously the ground state. Therefore, inversion of population requires more than half of the ground state atoms to be lifted to the higher energy level. As the ground state is heavily populated, large pumping power is to be used to depopulate the ground level to the required extent.

The three level scheme can produce light only in pulses. Once stimulated emission commences, the metastable state E_2 gets depopulated very rapidly and the population of the ground state increases quickly. As a result the population inversion ends. One has to wait till the population inversion is again established. Thus, three level lasers operate in *pulsed mode.*

(c) Four Level Pumping Scheme

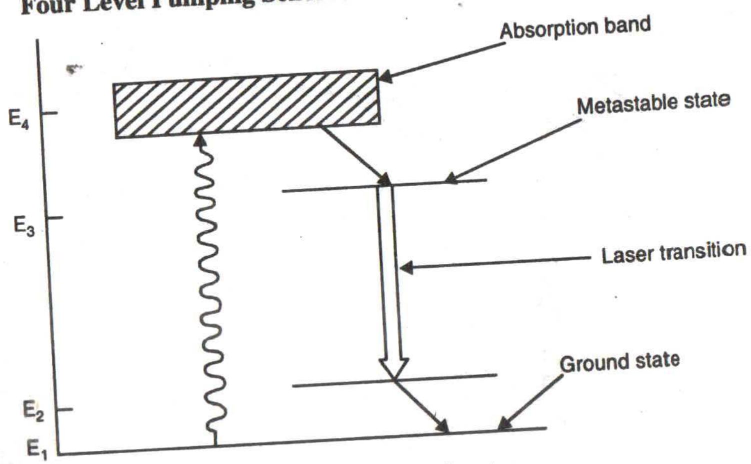


Fig. 1.19. Four-level system.

A typical four level pumping scheme is depicted in Fig. 1.19. In this scheme, the terminal laser level E_2 is well above the ground level such that $(E_2 - E_1) >> kT$. It guarantees that the thermal equilibrium population of E_2 level is negligible. As in the three level pumping scheme, the pump energy elevates the atoms to a short lived uppermost level E_4. The atoms then drop spontaneously to a metastable upper laser level E_3. As the terminal laser level

E_2 is virtually vacant, population inversion between the states E_3 and E_2 is quickly established. A spontaneous photon of energy $h\nu = E_3 - E_2$ can initiate a chain of stimulated emissions culminating in lasing. The laser transition takes the atoms to the level E_2. From there, the atoms lose the rest of their excess energy by radiative or non-radiative transitions and finally reach the ground state E_1. Atoms are once again available for excitation.

In contrast to three level scheme, the lower laser transition level in four level scheme is not the ground state and is virtually vacant. As soon as some atoms are pumped to the upper laser level, population inversion is achieved. Thus, it requires less pumping energy than does a three level laser. This is the major advantage of this scheme. Further, the lifetime of the lower laser transition level E_2 is much shorter as it is not a metastable state. Hence atoms in level E_2 quickly drop to the ground state. This steady depletion of E_2 level helps sustain the population inversion by avoiding an accumulation of atoms in the lower lasing level. Therefore, four level lasers can operate in a *continuous wave (cw) mode.*

1.21 AMPLIFICATION AND GAIN

When an active medium is in the inverted state, a photon of appropriate energy can stimulate the emission of a cascade of photons. This is the process of amplification. The initial photon may be looked upon as the input signal, the active medium as the quantum optical amplifier and the emerging light as the amplified output. The degree of amplification is measured as gain which is the increase in intensity when a light beam passes through an active medium. The gain may be expressed as

$$G = \frac{1}{I} \cdot \frac{dI}{dx} \qquad \text{...(1.53)}$$

The *gain* may be otherwise defined as the amount of stimulated emission which a photon can generate as it travels a given distance. For example, if $G = 4$ per cm, it means that one photon produces four photons per each centimeter it travels in the medium. Unfortunately, laser materials have a very low gain, of the order of 0.0001/cm to 0.01/cm It means that the photon has to travel a long length of the laser material for producing large amplification. As an example, if we have a material whose gain is 0.001/cm and if we wish to achieve a light amplification of 1000 times, it is calculated that the light has to travel about 69 meters in the medium. Such long distances are obviously not practical. However, the important point to note here is that the amount of amplification increases rapidly with the distance.

In practice, laser materials are not used to amplify light from some outside source. Laser, despite its name, is basically a generator of light.

One of the ways of making light to pass through a long length of laser medium is by keeping mirrors on both sides of a short laser rod or tube. The light bounces back and forth between the mirrors and makes many passes