PULL-UP NETS

Liz Meenan describes an easy and exciting way of making 3D solids from their nets without the need for any glue.

Basic instructions for pull-ups:

1 Cut out the net for your pull-up polyhedron.
2 Use a ruler and sharp point to score lightly along the remaining black lines.
3 Make holes at the points A, B, C, D, etc.
4 Thread and weave thin ribbon, string or thread through the holes A, B, C, D, etc to link the shapes together.
5 Gently pull up the net to make your polyhedron.
6 Unfold and decorate the outside of your polyhedron.

When I began teaching I always hit a problem when it came to 3D shapes. I wanted the pupils to get a feel for them, and I would get them to make the shapes from their nets. The pupils would first try to visualise how the 2D nets could become 3D shapes and then they would physically fold the nets into the shapes for themselves. Octahedrons, dodecahedrons, prisms, icosahedrons and even cubes led to a lot of mess and disorder. Pupils would accidentally cut off the tabs or forget or misunderstand which edges had to be glued together. All in all, a lot of hard work went into making very unsatisfactory models instead of things of beauty that we could show off and at the same time use to explore spatial properties.

Then in 1994 I met Bob Vertes at a mathematics conference in Reading. We got talking about teaching 3D shapes and he reassured me that we weren't the first to encounter such problems, and then suggested a solution. "Pull-up nets," he said. "What?" I said. Then he started doodling: he quickly sketched the 'stair net' for a cube and showed on the diagram how to use a loop of thread to 'pull up' the net to make a cube (*figure 1*). Once I saw what he meant, I couldn't wait to have a go.

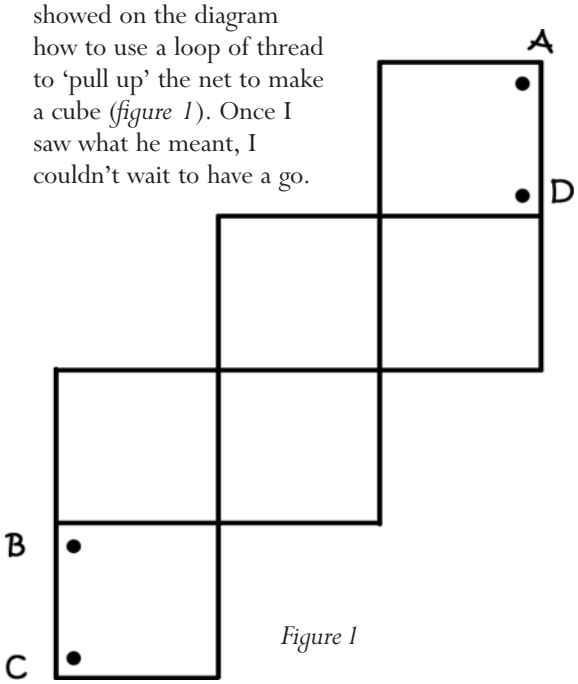

Figure 1

In all figures, the dots in the nets indicate the holes where the string goes.

That night in my room at the conference (whilst the other delegates were letting their hair down at the evening bop), I was using card and thread (with not a tab in sight) to make one. I pulled the thread and the net folded up into a handsome cube! It was one of those '*mathemagical*' experiences. I had to make more: large ones, smaller ones, patterned ones and so on. Exploring the properties of a cube, counting the number of sides, faces and edges, or calculating the area of faces, etc became a cinch. This was real problem solving in 3D; extremely enjoyable, no hassle, very kinaesthetic, and the models were brilliant! They became the must-have conference accessory. Even delegates convalescing from the previous night's bash were folding, threading, pulling![1]

When I got back from the conference I began to ponder various questions:

- Was this the *only* net of a cube which could be pulled up to make a cube? What about the other 10 cube nets? Do any of these pull up?
- How would you modify this pull-up for a cube to make it a pull-up for a *cuboid*?

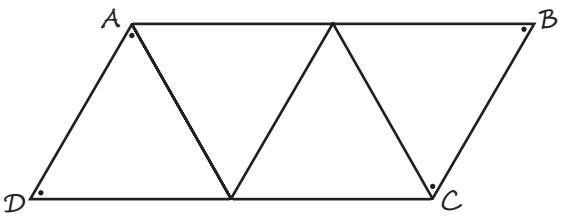

Figure 2

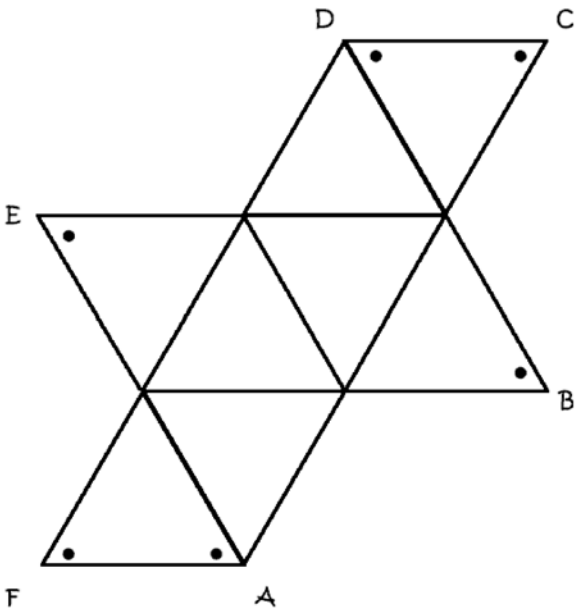

Figure 3

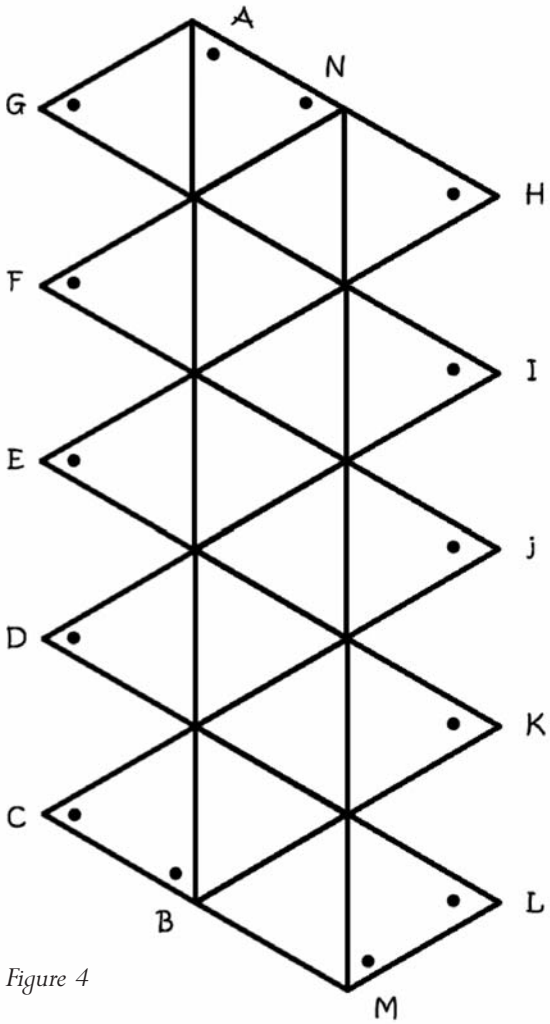

Figure 4

What about other 3D shapes – do they have pull-up nets? Yes indeed; the world is your 3D oyster! The tetrahedron pull-up is simply a parallelogram split into four equilateral triangles (*figure 2*). Or try a pull-up net for an octahedron (*figure 3*) or an icosahedron (*figure 4*).[2] Other pyramids can be made by changing the number of triangles and the shape in the middle, but when I tried to make a pull-up net for a dodecahedron I failed. I really didn't think it was possible, due to the symmetry of the shape, but there is a pop-up net for the dodecahedron which is extremely effective, and again is based on the traditional net (Gardner, 1961).[3]

However, I really wanted a complete set of *pull-up* nets for all five Platonic solids.[4] I had almost given up hope when one afternoon last year, while doing an ICT presentation, Jeff Zhao (a PGCE student) showed us an interactive website that allowed you to click on a net of a 3D shape and then using a slider it folded up to make the 3D shape (www.mathsnet.net/geometry/solid/nets.html). It had the dodecahedron (*figure 5*), so that night at home I proceeded to make the net and put holes near the relevant vertices, and lo and behold, after threading, I pulled up my first (and, dare I say it, the first ever?) dodecahedron into my hand!

These nets are magical, and using them takes the hassle out of making 3D shapes. They are cheap and easy to make, so have a go yourself!

Liz Meenan works part time at Leeds University as a PGCE tutor and mathematics support tutor.
With thanks to Bob Vertes.

Notes

1 I also indulged in a little mathematical voodoo, making a large pull-up model of a cube and positioning a miniature jack-in-the-box headmaster (not mine of course) on the glued square and pulling the model up round him and letting it down again.

2 This is due to Tom King, a student on the 2004-2005 PGCE course at Leeds University.

3 What amazes me is that very few mathematicians seem ever to have made one. It's ideal to give as a Christmas present, with the calendar months on its faces. And you can even send it through the post. Never mind a lap-dancer from a Christmas cake; start the new year with a dodecahedron popping up from an ordinary A5 envelope! (For other interesting calendar nets go to www.projects.ex.ac.uk/trol/trol/trolqc.htm.)

4 The Platonic solids are the cube, tetrahedron, octahedron, dodecahedron and icosahedron, named after Plato, who wrote about them in a book called *Timaeus*.

Reference

Gardner, M. (1961) 'The Five Platonic Solids', Chapter 1 in *The 2nd Scientific American Book of Mathematical Puzzles and Diversions*, Simon and Schuster

Acknowledgement

An earlier version of this article appeared as Meenan, L. (2005) 'Pull-ups and Pop-ups - 3D shapes made the easy way', *Maths Coordinator*, file 21, Autumn, Optimus Education.

Go to www.atm.org.uk/mt208 for downloadable nets for the pull-ups and an additional article by Liz on the Platonic solids.

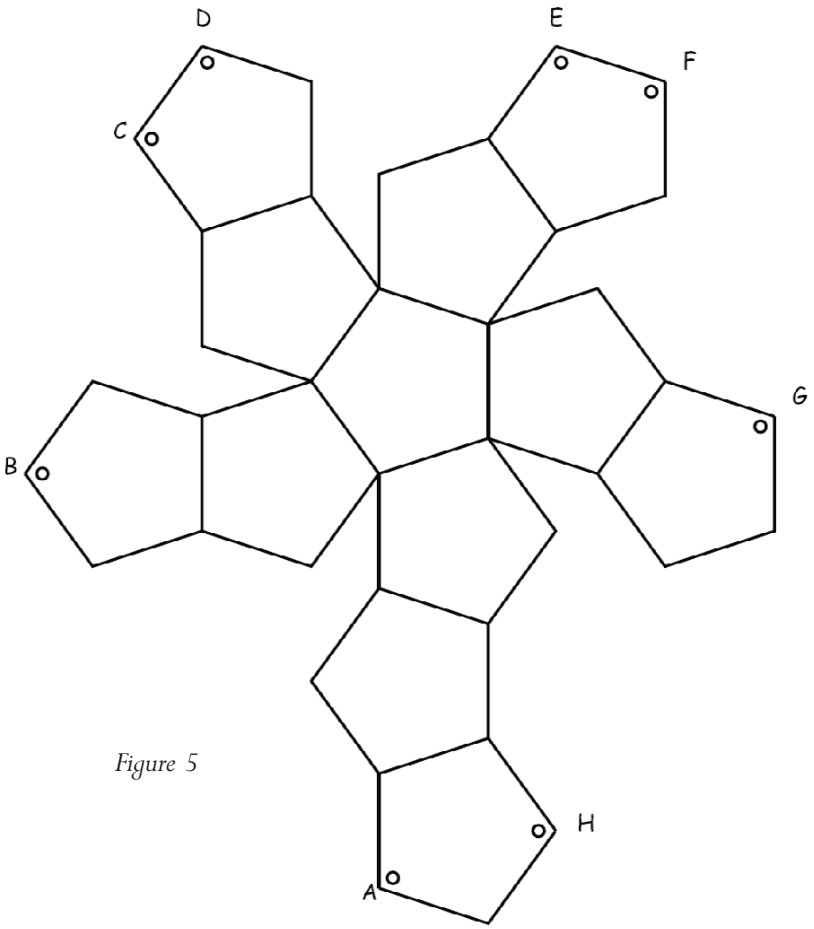

Figure 5

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