Chapter 6

Previously, we have discussed geostrophy: the motion that large-scale ocean circulation generally satisfies in the ocean interior (away from direct surface forcing and away from side boundaries). Recently, we also introduced instabilities in the ocean that can induce mixing. Today, we will develop an understanding of oceanic boundary layer, which is subject to direct surface forcing.

At the oceanic surface, the ocean is in direct contact with the atmosphere and so is affected by atmospheric winds, heat and salinity fluxes (i.e. buoyancy) flux. Here, we will first introduce forcing due to surface windstress, and will introduce forcing due to heat and salinity fluxes in later classes.

6.1. The Ekman layer

The ocean surface boundary layer is sometimes called the Ekman layer, the layer in which the surface windstress directly acts. The surface boundary layer is also called the mixed layer since temperature and salinity are often observed to be uniform in this layer, thus well mixed.

The surface wind exerts a stress (τ^x, τ^y) on the ocean, which is a force per unit area (N/m^2). Now imagine that the ocean is made up of many layers of thickness δz, subject to the wind force per unit area at the surface (τ^x, τ^y).

Let's first obtain the forcing term due to windstress forcing for a unit mass.

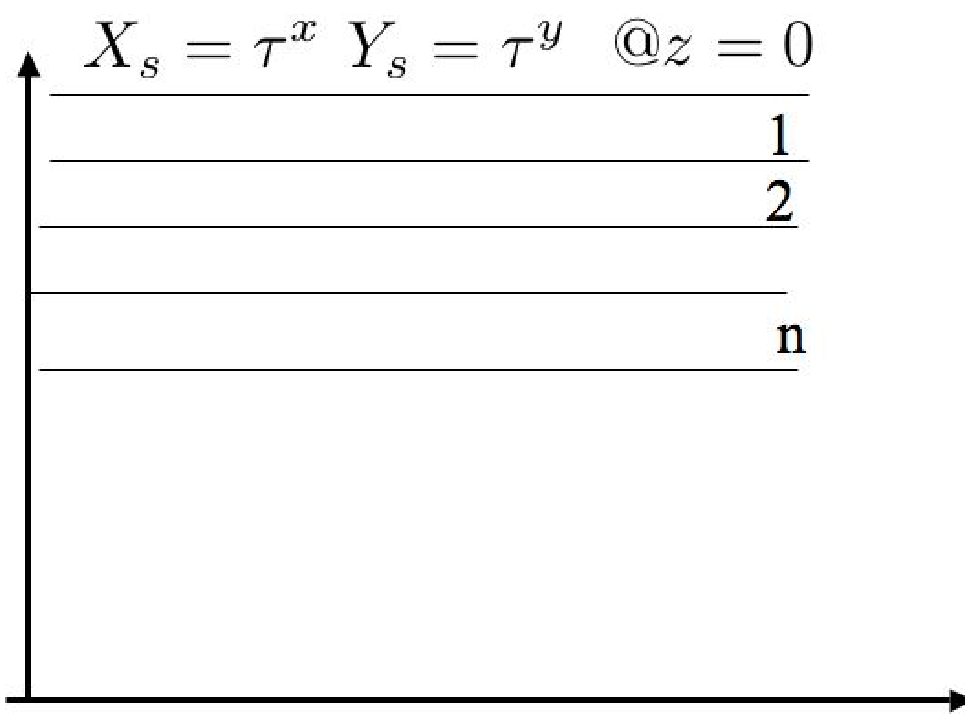

Figure 1: Schematic diagram showing windstress exerting on the Ekman layer.

The upper most layer will be set in motion by the direct action of the surface stress ($X = \tau^x$, $Y = \tau^y$) at $z = 0$. However, **friction or viscosity** between the upper-layer (layer 1) and the next layer (layer 2) will set layer 2 in motion. The stress acting at the top of layer 2 is ($\tau^x - \delta z \frac{\partial X}{\partial z}$, $\tau^y - \delta z \frac{\partial Y}{\partial z}$). This stress sets layer 2 in motion but meanwhile acts to retard layer 1. Because stress energy is used to create motion, stress acts on a layer decreases with the increase of ocean depth. That is, $\frac{\partial X}{\partial z} > 0$, and $\frac{\partial Y}{\partial z} > 0$. The stress acting at the top of layer 3 is ($\tau^x - 2\delta z \frac{\partial X}{\partial z}$, $\tau^y - 2\delta z \frac{\partial Y}{\partial z}$). Applying similar arguments to each layer as we progress down the water column, we can see how the ocean is set in motion by the action of surface windstress.

The net stress working on each layer is the stress at the top of layer n, ($\tau^x - \delta z \frac{\partial X}{\partial z}(n-1)$), subtracts the stress at the bottom of layer n, ($\tau^x - \delta z \frac{\partial X}{\partial z} n$), which is

$\delta z \frac{\partial X}{\partial z}$ in x direction, and $\delta z \frac{\partial Y}{\partial z}$ in y direction.

Therefore, the force per unit mass is given by:

$\delta x \delta y \delta z (\frac{\partial X}{\partial z}, \frac{\partial Y}{\partial z}) / \rho \delta x \delta y \delta z = \frac{1}{\rho}(\frac{\partial X}{\partial z}, \frac{\partial Y}{\partial z})$.

They represent vertical flux of horizontal momentum. Windstress enters the ocean as "vertical flux of horizontal momentum".

Consider a small Rossby number (nonlinear terms are negligible), steady (time changing term is negligible) and constant density. The equations of motion in the Ekman layer are:

$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\rho_0}\frac{\partial X}{\partial z}, \tag{1a}$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{1}{\rho_0}\frac{\partial Y}{\partial z}. \tag{1b}$$

In the ocean, stresses X, Y decrease rapidly with the increase of depth. So their direct influence is felt only in the surface boundary layer. Typically, this boundary layer is 10-100m thick.

Since

$$-fv_g = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}, \tag{2a}$$

$$fu_g = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}, \tag{2b}$$

We can write equation as:

$$-fv = -fv_g + \frac{1}{\rho_0}\frac{\partial X}{\partial z}, \tag{3a}$$

$$fu = fu_g + \frac{1}{\rho_0}\frac{\partial Y}{\partial z}. \tag{3b}$$

Thus,

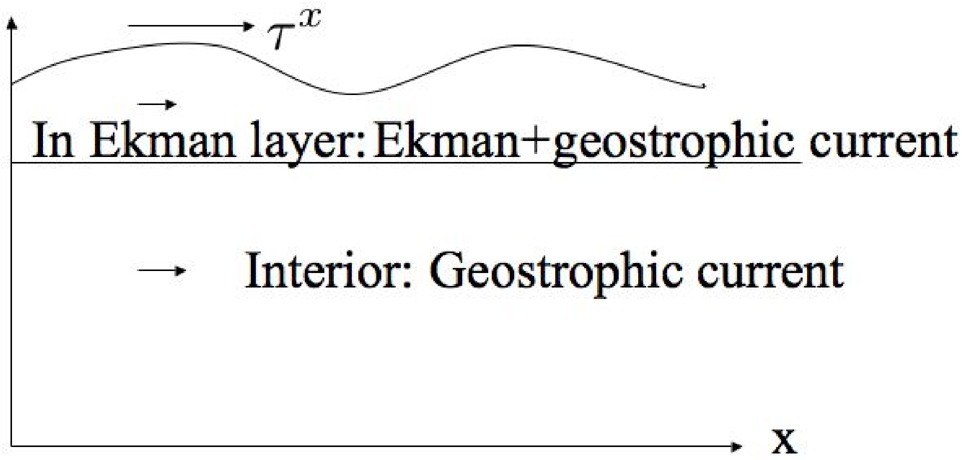

Figure 2: Schematic diagram showing oceanic boundary layer that is on top of the ocean interior and is below the atmospheric boundary layer.

$$-fv_E = -f(v - v_g) = \frac{1}{\rho_0}\frac{\partial X}{\partial z}, \tag{4a}$$

$$fu_E = f(u - u_g) = \frac{1}{\rho_0}\frac{\partial Y}{\partial z}, \tag{4b}$$

where $u_E = u - u_g$, $v_E = v - v_g$.

6.2. The Ekman transport

Integrating equation over the Ekman layer of depth H_{mix} for a unit length in x or y direction (in fact over a vertical section), subject to the conditions that:
$(X, Y) = (\tau^x, \tau^y)$, at $z = 0$
$(X, Y) = 0$, at $z \leq -H_{mix}$
$(u_E, v_E) = 0$ at $z \leq -H_{mix}$,
we obtain

$$U_E = \int_{-H_{mix}}^{0} u_E dz = \frac{\tau^y}{\rho_0 f}, \tag{5a}$$

$$V_E = \int_{-H_{mix}}^{0} v_E dz = -\frac{\tau^x}{\rho_0 f}. \tag{5b}$$

Therefore, if we only have southerly winds (wind blows from the south), the Ekman transport will be to the east (west) in the NH (SH). *The Ekman mass transport (and volume transport) is at right angles to the windstress due to the action of Coriolis force.*

6.3. The Ekman pumping–interaction of the ocean interior with the Ekman layer

The surface windstress (τ^x, τ^y) varies from place to place, thus produces spatially unevenly distributed Ekman transport, causing convergences and divergences. To conserve mass, these convergences and divergences must be balanced by the vertical movement of fluid into or out of the base of the Ekman layer. This process is called *Ekman pumping*. The Ekman pumping at the base of the Ekman layer pushes isotherms below the Ekman layer up and down, causing pressure gradients in the ocean interior and thus set the interior ocean in motion (geostrophic current). Vertical velocity at the base of the Ekman layer w_E is called Ekman pumping velocity.

This concept can be seen clearly from integrating the continuity equation

$u_x + v_y + w_z = 0$, or $u_{Ex} + u_{gx} + v_{Ey} + v_{gy} + w_z = 0$.

Because $u_{gx} + v_{gy} = 0$, we have

$u_{Ex} + v_{Ey} + w_{Ez} = 0$.

Performing vertical integration within the Ekman layer and use approximation $w = 0$ at $z = 0$ for baroclinic motion, we have:

$U_{Ex} + V_{Ey} - w_E = 0$, and thus

$w_E = U_{Ex} + V_{Ey}$.

6.4. The Ekman spiral

So far we have discussed the vertically integrated feature of the Ekman flow, but have not talked about the vertical structure in the Ekman layer. As discussed earlier, **friction or viscosity** sets the subsurface layers in motion. Therefore, friction or viscosity plays an important role in the Ekman layer.

In 1905, Ekman proposed a model for a frictional, laminar (i.e. non-turbulent) boundary layer, which is still used today. Ekman considered the case in which the internal wind induced stress is balanced by viscous stress within the ocean:

$\frac{1}{\rho}(X, Y) = A_z(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z})$, recall that A_z is vertical viscosity. Assume the winds are spatially uniform so that we do not have convergence and divergence and thus pressure gradient terms are zero and thus $u_g = 0$ and $v_g = 0$. From equation we have

$$-f v_E = A_z \frac{\partial^2 u_E}{\partial z^2}, \tag{6a}$$

$$f u_E = A_z \frac{\partial^2 v_E}{\partial z^2}. \tag{6b}$$

Using boundary conditions

$u_E = v_E = 0$ as z tends to negative infinity (for the deep ocean),

$A_z \frac{\partial u}{\partial z} = \frac{1}{\rho}\tau^x$, and

$A_z \frac{\partial v}{\partial z} = \frac{1}{\rho}\tau^y$ at the surface $z = 0$,

we obtain solutions for Ekman Spiral:

$$u_E = \frac{e^{\sqrt{f/2A_z}z}}{\rho\sqrt{2fA_z}}[(\tau^x + \tau^y)cos(\sqrt{f/2A_z})z + (\tau^x - \tau^y)sin(\sqrt{f/2A_z})z], \tag{7a}$$

$$v_E = \frac{e^{\sqrt{f/2A_z}z}}{\rho\sqrt{2fA_z}}[(\tau^y - \tau^x)cos(\sqrt{f/2A_z})z + (\tau^x + \tau^y)sin(\sqrt{f/2A_z})z]. \tag{7b}$$

If we plot out the solution it will be a clockwise spiral, if we look down from the ocean surface.

Features to note from the Ekman spiral solution:

- the Ekman layer thickness is $H_{mix} = \sqrt{\frac{2A_z}{f}}$, i.e., the e-folding scale. The stronger the viscosity, the thicker the Ekman layer.
- The flow in the Ekman layer is not geostrophic because viscosity is so important.

6.5. Bottom boundary layer

At the bottom of the ocean, the ocean is in contact with the lithosphere. Therefore there will be frictional drag acting on the ocean due to the roughness of the bottom and due to the torques that develop in the presence of bathymetry.

At the bottom, the interior ocean flow plays the role of the windstress at the surface since $u = 0, v = 0$ at the bottom. Bottom boundary layer thickness is: $\delta_e = \sqrt{\frac{2A_z}{f}}$.

Ekman spiral is hard to observe in the real ocean, due to the turbulent feature of the mixed layer (not laminar).

6.6. Turbulent mixed layer

The temperature and salinity in the upper 30-100m is very nearly uniform due to the turbulent mixing created by the surface winds; it is therefore often called the mixed layer.

The mixing coefficient or eddy viscosity in the mixed layer is much larger than it is in the Thermocline below the mixed layer. The abrupt transition from the mixed layer value to the thermocline value is actually an observed feature of the ocean since mixing is very much affected by density stratification.

In fact, mixing process is affected by stratification of the ocean was discussed in previous classes. Generally, we can use Richardson number to measure stratification and the level of Kelvin-Helmholtz instability.

$$R_i = \frac{N^2}{(\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial z})^2}. \tag{8}$$

If $R_i < 0$, the ocean is unstably stratified, mixing intensifies. Neutral flow corresponds to $R_i = 0$. When $0 < R_i < 0.25$, K-H instabilities may occur and mixing is strong. Thus, the mixed layer is turbulent rather than laminar.

6.7. Thermocline, halocline, pycnocline, and thermocline theory

Since solar radiation from the sun does not penetrate far into the ocean, temperature of the ocean decreases with depth.

The region below surface mixed layer with a rapid decrease of temperature is called the thermocline.

Seasonal variation of the thermocline. Seasonal thermocline is the region which is part of the thermocline during springtime surface heating when mixed layer retreats, but becomes part of the mixed layer during wintertime surface cooling when mixed layer deepens.

The region of rapid change in salinity with depth is called halocline. It only exists in certain regions, such as the western Pacific warm pool, the eastern Indian Ocean warm pool, and the Bay of Bengal where precipitation or river runoff is strong, causing fresher water (low salinity) near the surface.

The region of rapid change in density with depth is called pycnocline.

Thermocline theory. One may ask: Why is there a thermocline in the ocean and how is it formed?

The key problem is the ocean is in motion, rather than quiescent. The timescale of the advective process is much shorter than the 100,000 years diffusion scale. Therefore, the ocean circulation redistribute the heat.

Existing thermocline theories. (i) Vertical advection of T is balanced by vertical diffusion. As we just discussed, advection scale is much shorter than diffusion and thus these two terms are not likely to be balanced. (ii). Thermocline ventilation and subduction theory [Pedlosky: Ocean circulation theory. ATOC 5061].

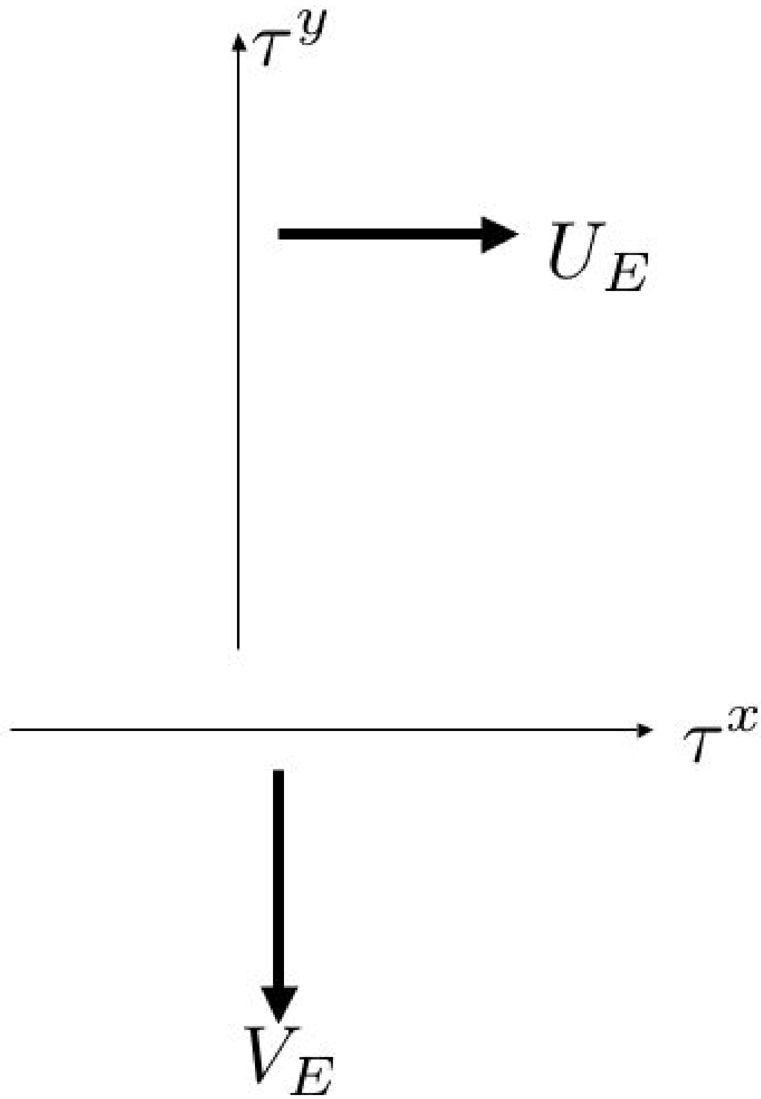

Figure 3: Schematic diagram showing Ekman transport (sometimes called Ekman drift) in the NH.

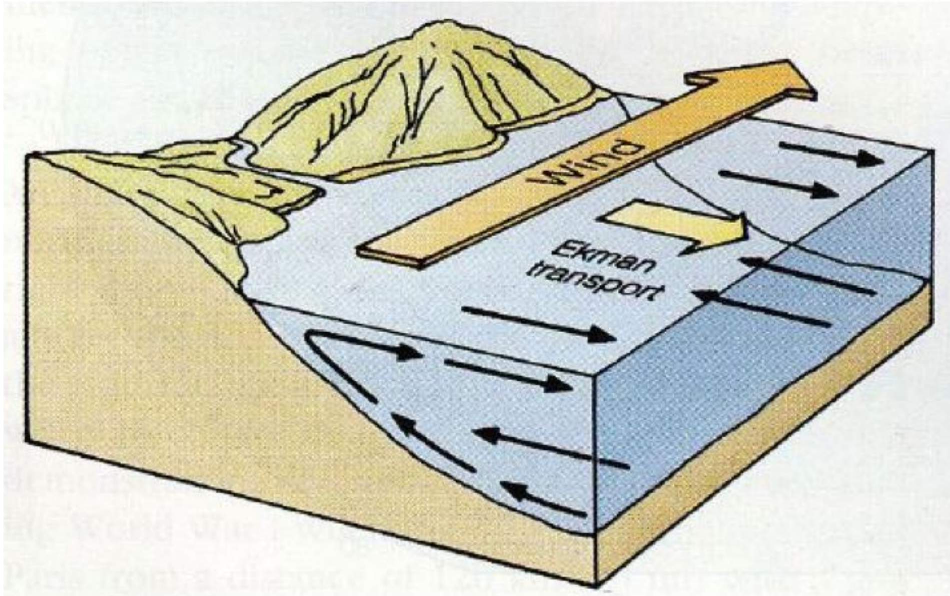

Figure 4a: Schematic diagram showing Ekman transport and upwelling along Somali coast.

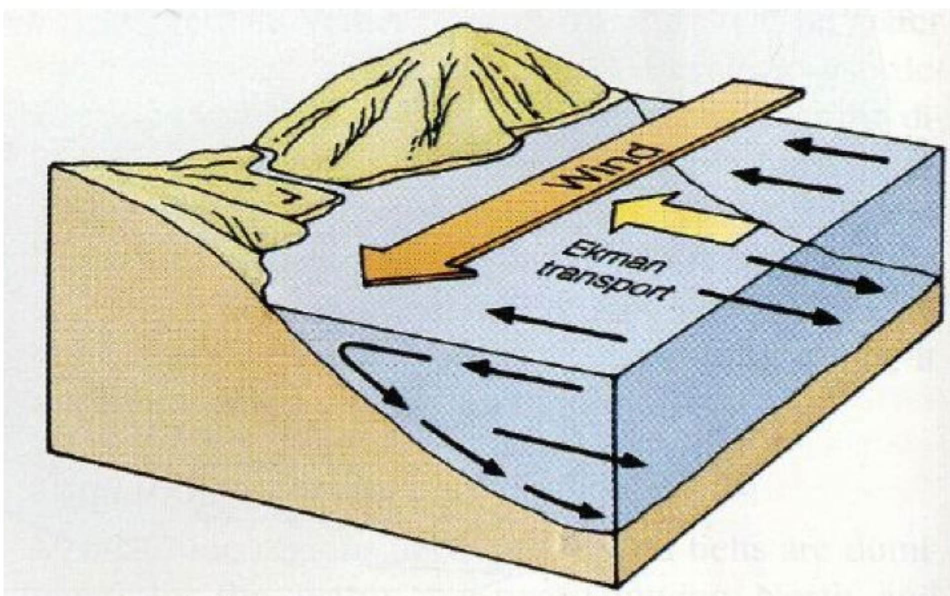

Figure 4b: Schematic diagram showing Ekman transport and downwelling along Somali coast.

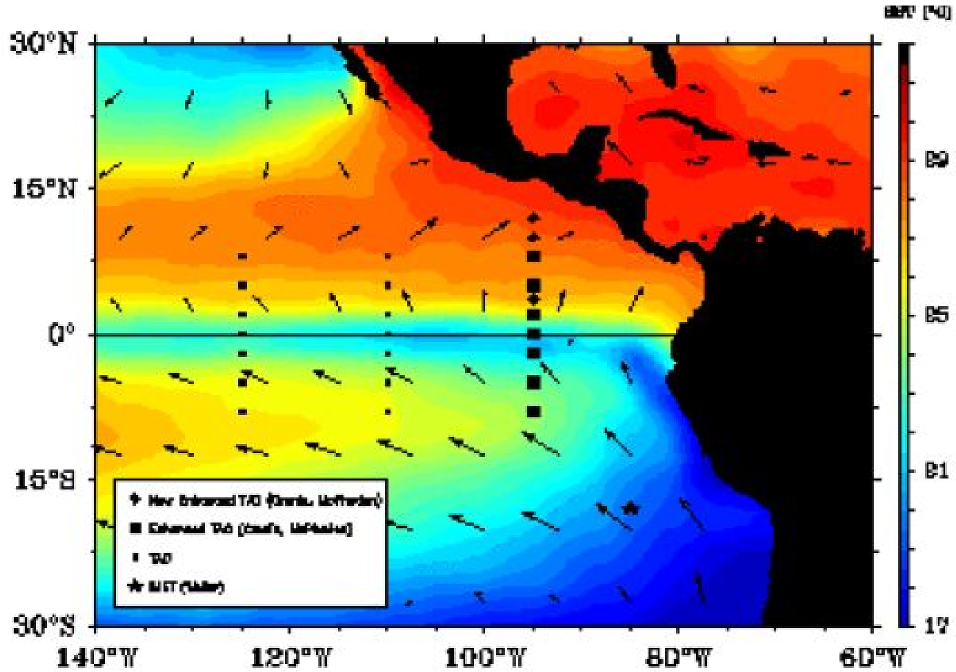

Figure 4c: Ekman transport induced equatorial upwelling in the eastern Pacific Ocean.

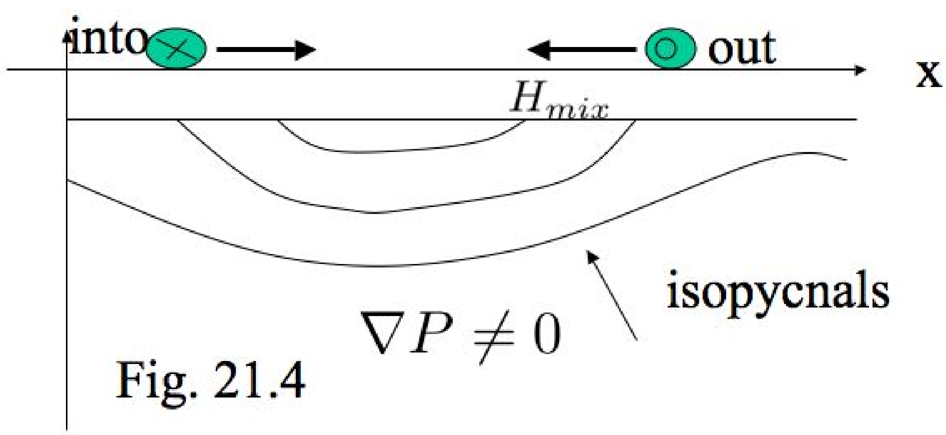

Figure 5: Schematic diagram showing Ekman pumping.

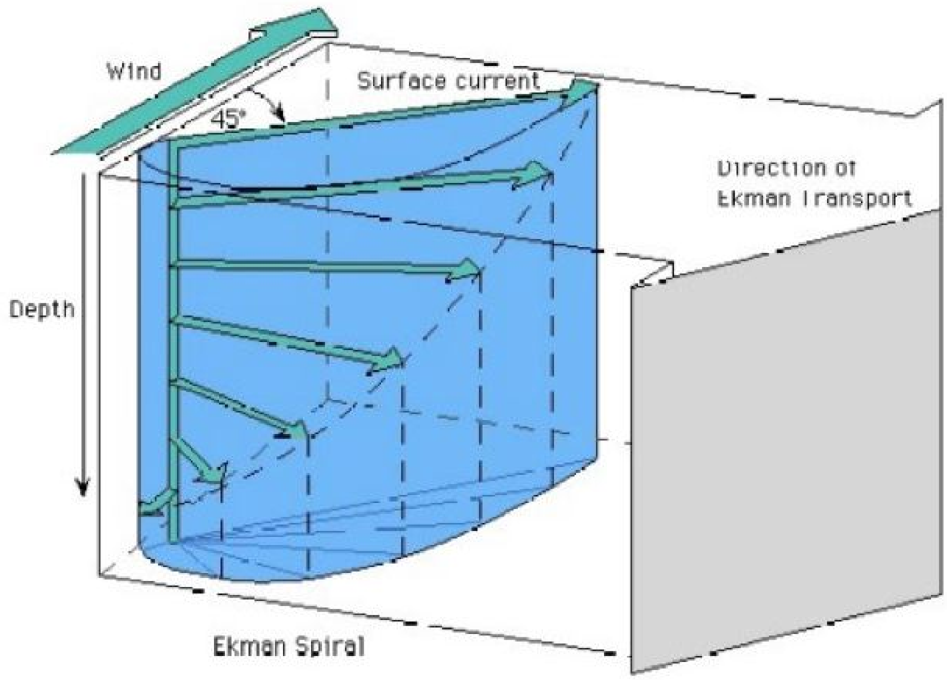

Figure 6: Ekman spiral.

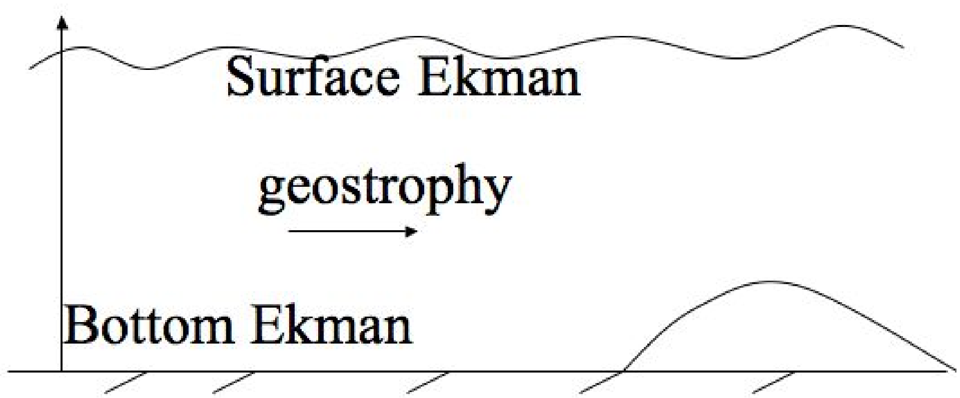

Figure 7: Bottom Ekman layer.

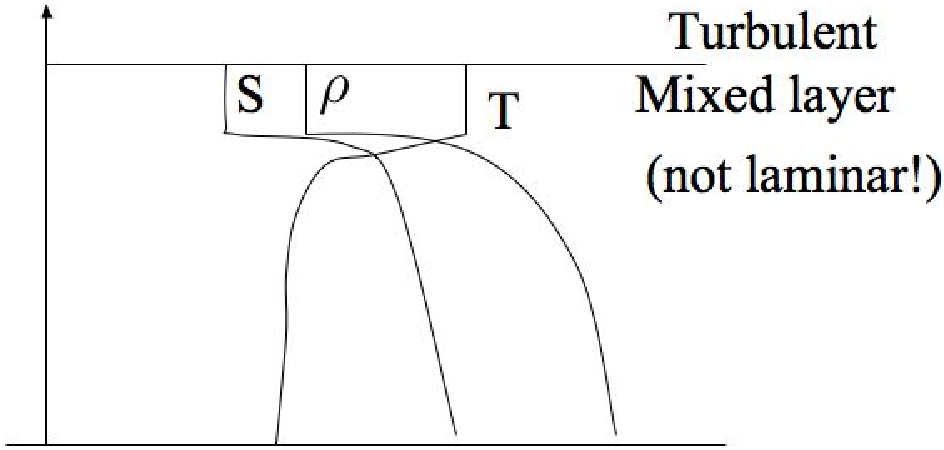

Figure 8: Schematic diagram showing surface mixed layer.

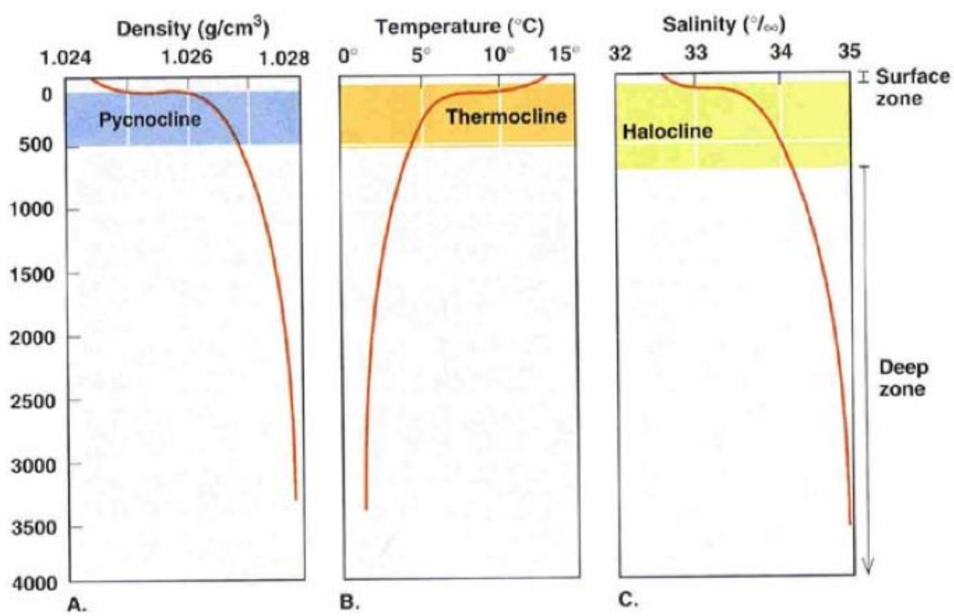

Figure 9: Pycnocline, thermocline and halocline

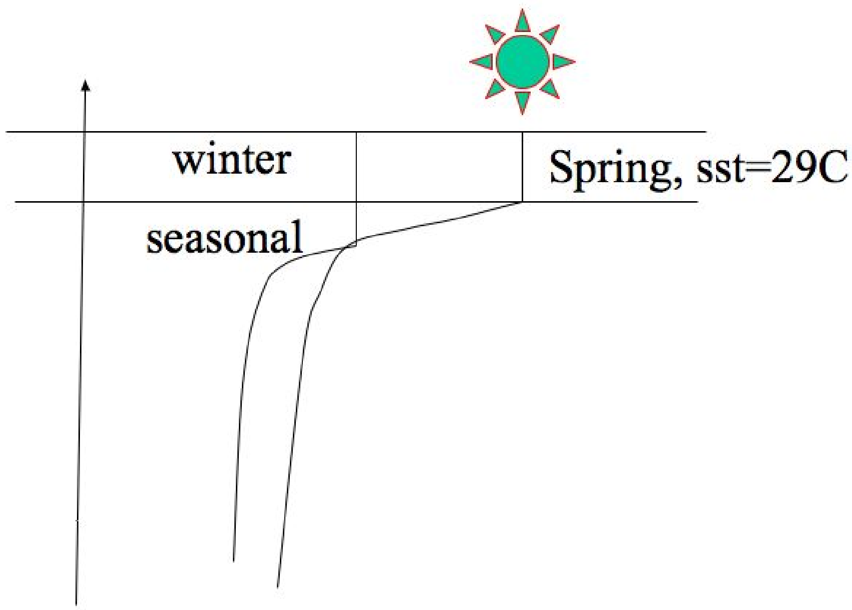

Figure 10: Schematic diagram showing seasonal change of mixed layer and thermocline.