ATOC 5051 INTRODUCTION TO PHYSICAL OCEANOGRAPHY

Lecture 11

Learning objectives: understand internal waves, explain storm surge, and identify the effect of the Earth's rotation

1. Internal waves; concepts -barotropic and baroclinic modes;
2. Storm surges
3. Effects of rotation, Rossby radius of deformation

Previous classes: we discussed surface gravity waves. What assumptions did we make to isolate "***surface***" gravity waves?

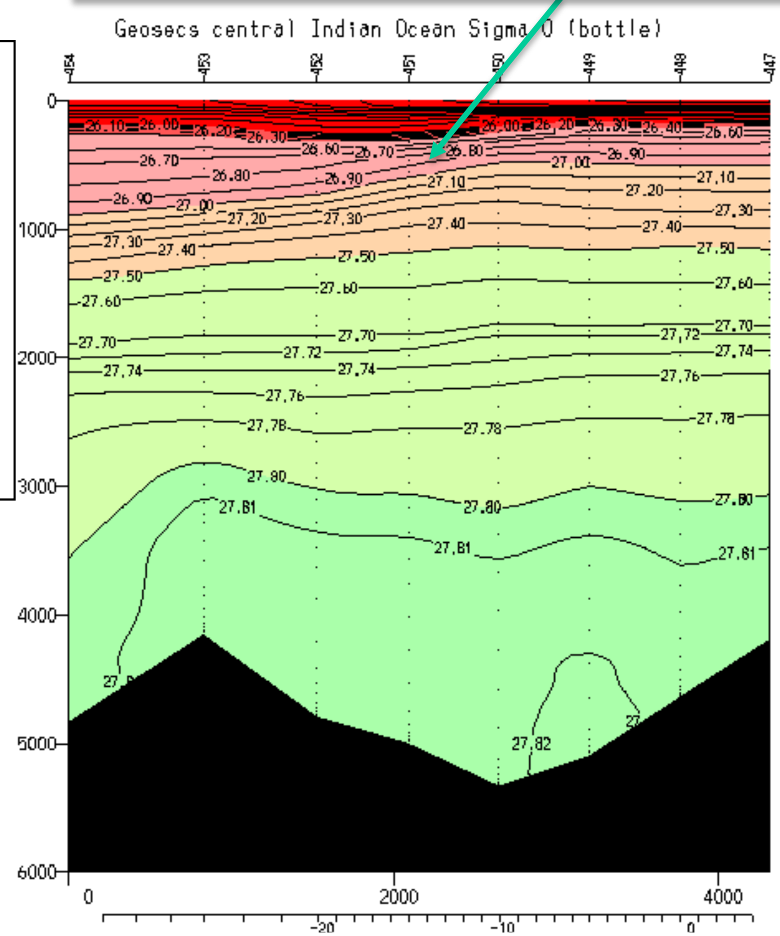
Constant density – homogeneous ocean

Observed ocean density:

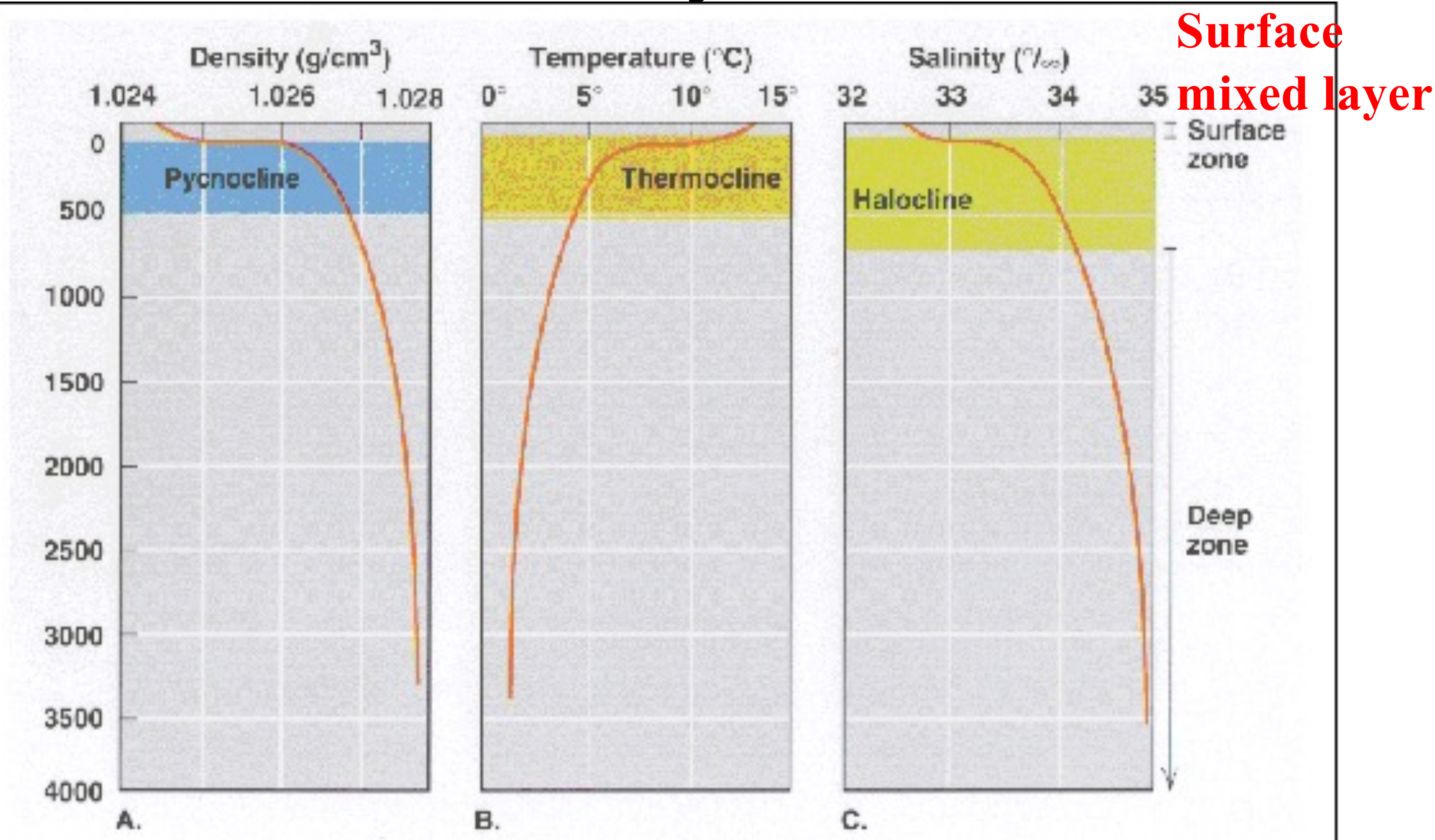
Sigma(0)=(density-1000) kg/m^3

Potential density relative to surface

Sharp vertical density gradient

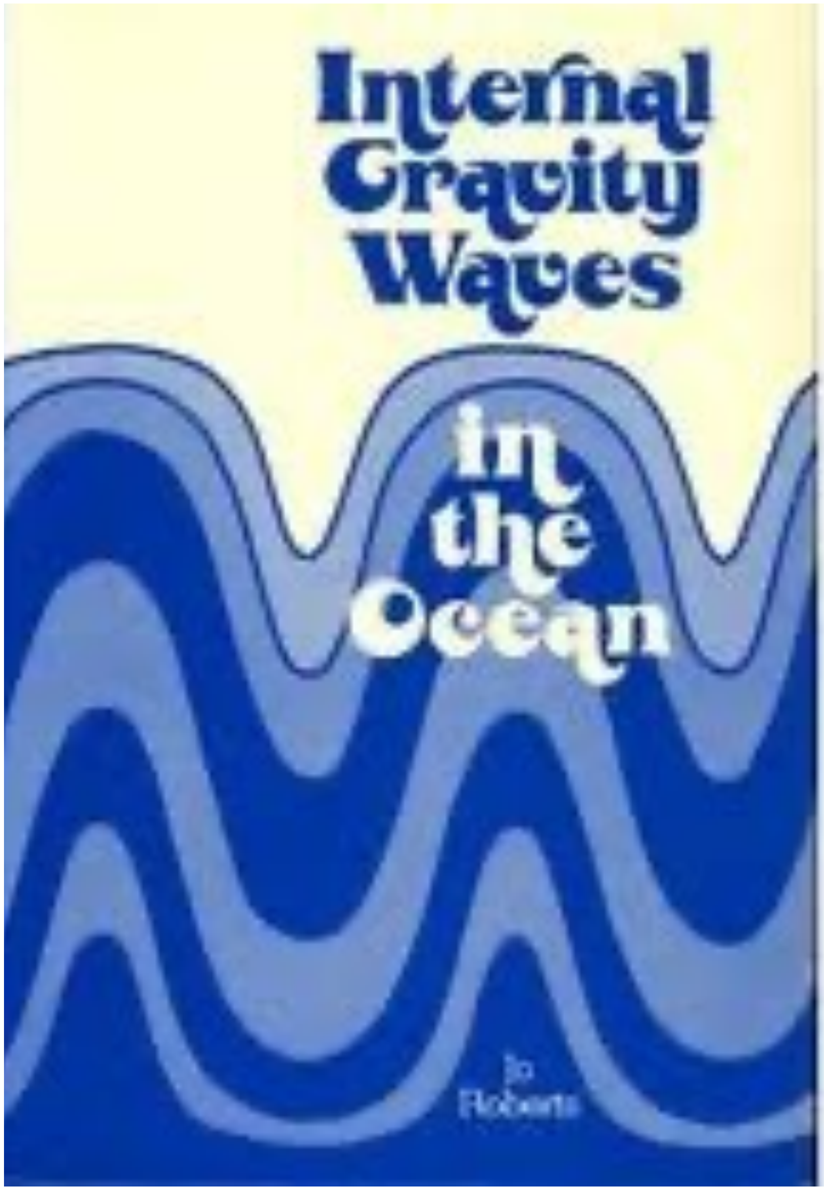


Concepts: Vertical profiles of density, T, S



Thermocline, halocline and pycnocline

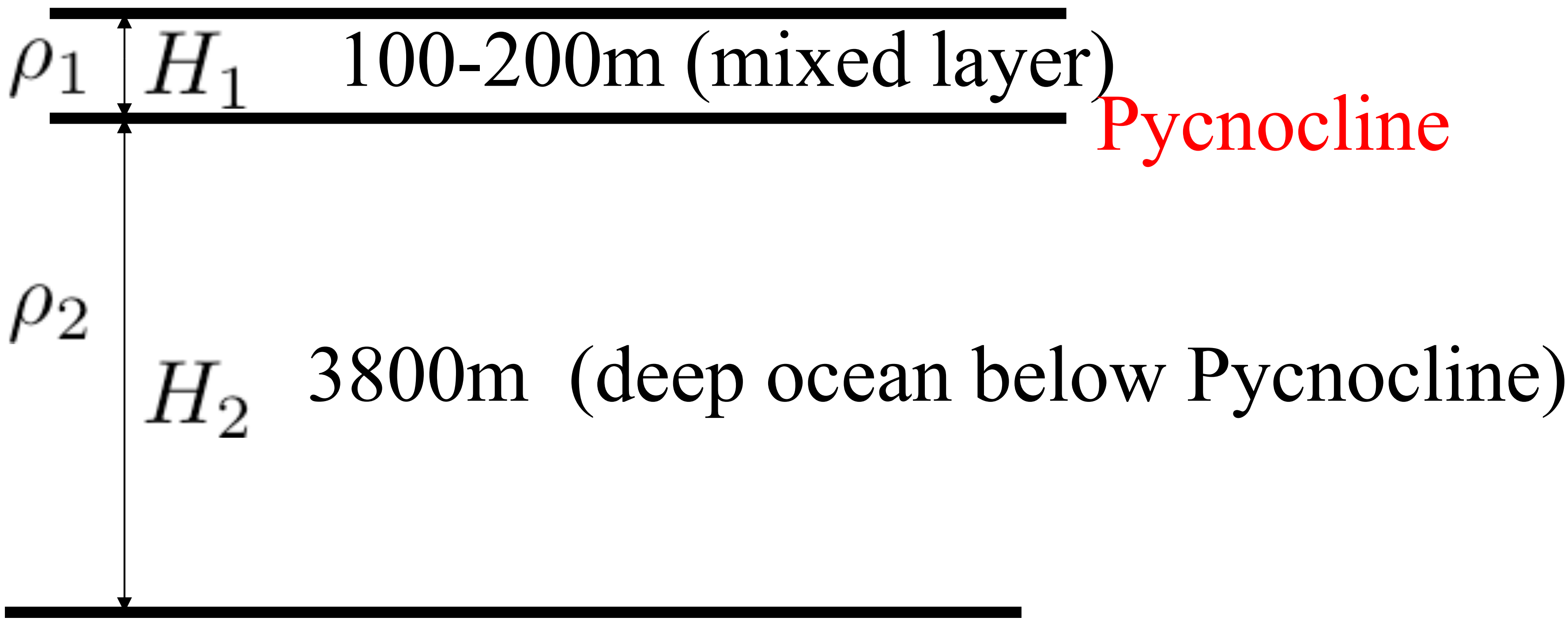
1. Internal gravity waves in density stratified ocean



Largest amplitudes:
in pycnocline

Internal gravity waves in density stratified ocean

a) A 2-layer model



Barotropic and baroclinic modes

The 2-layer system has 2 vertical normal modes.

Total Solution is the superposition of the two modes.

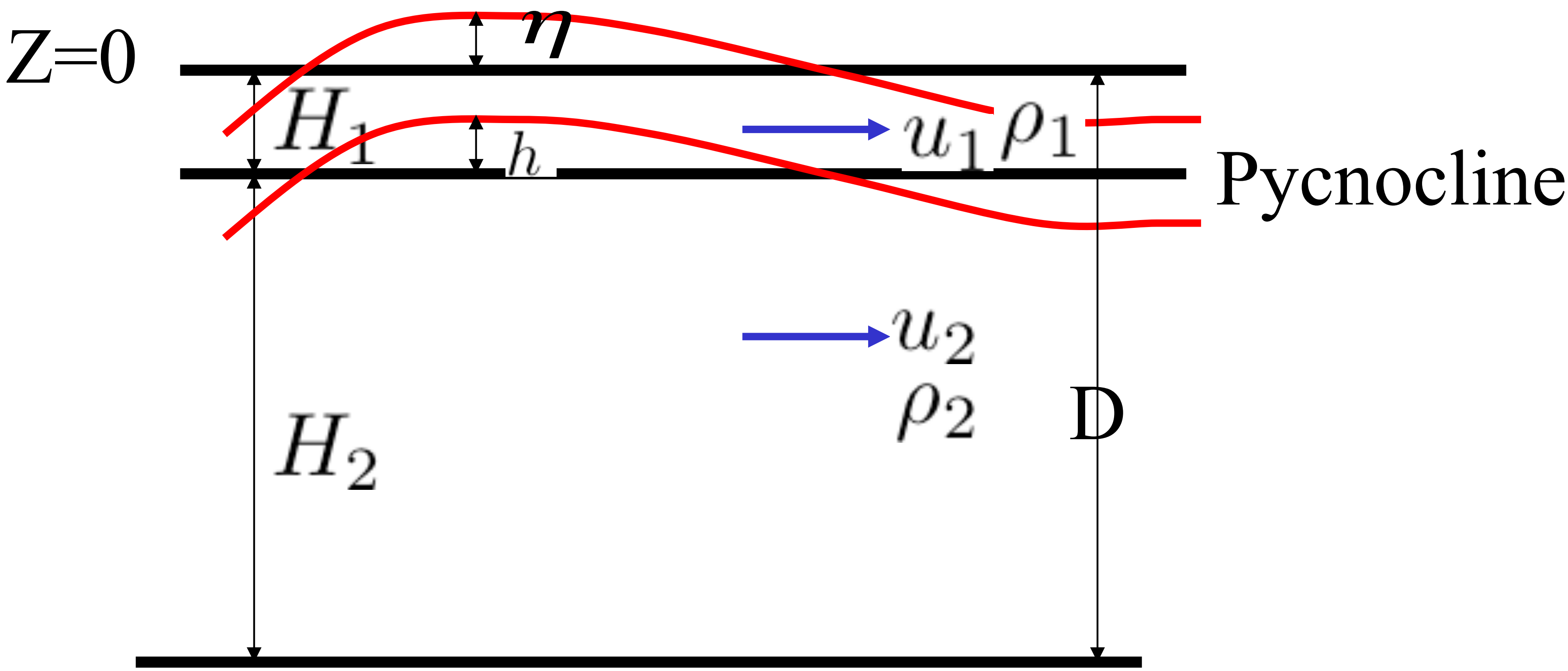
Barotropic mode: independent of z, which represents vertically-averaged motion.

Restoring force: gravity g

Baroclinic mode: vertical shear flow, and vertically-integrated transport is zero. **Restoring force is reduced gravity,** $g' = \frac{g(\rho_2 - \rho_1)}{\rho_2} \sim 0.03 m/s^2$

Why? At ocean surface – air density is << water density & thus ignored, and restoring force is gravity (g); in the pycnocline, the restoring force is reduced gravity g': due to the small difference between the water density above and below

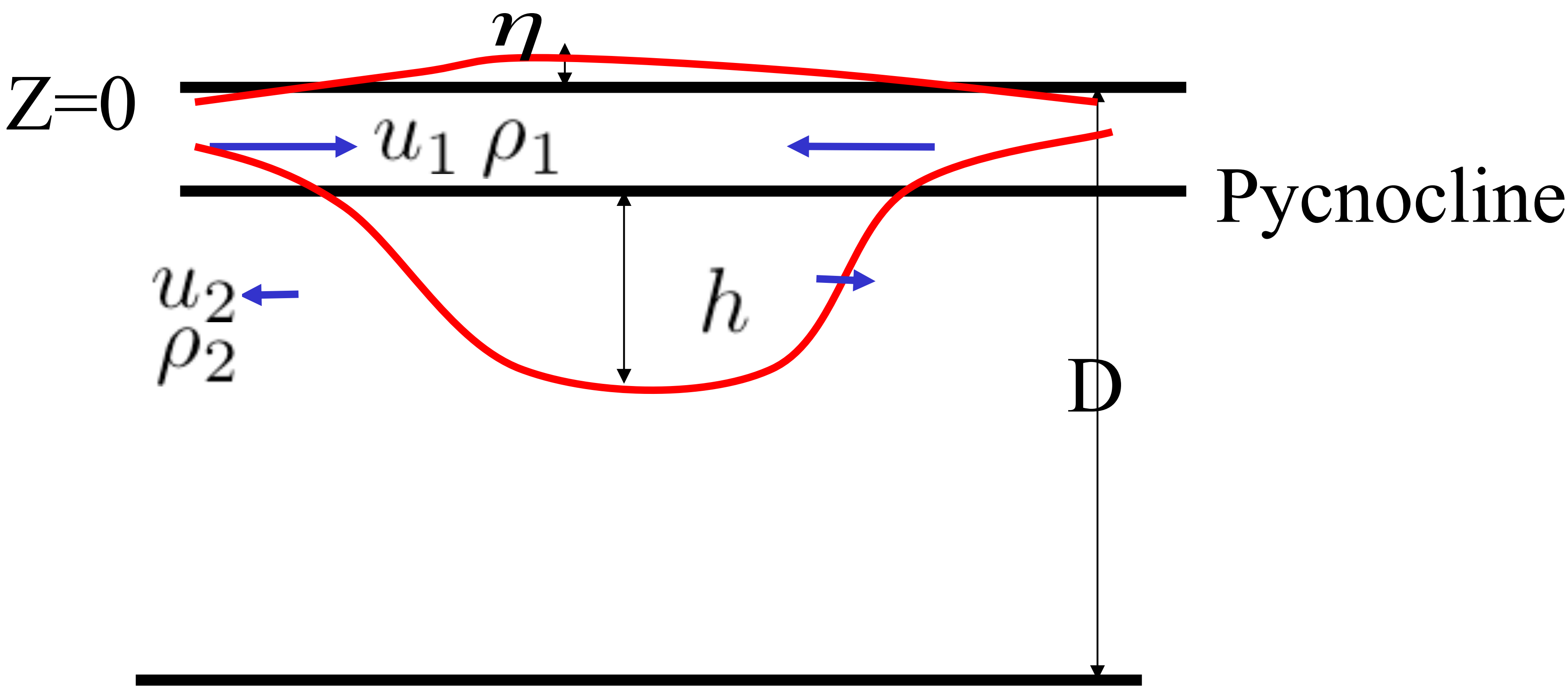
Barotropic mode



$$C_0 = \sqrt{g(H_1 + H_2)} = \sqrt{gD} \sim 200m/s$$

$$\frac{h}{\eta} = \frac{H_2}{H_1 + H_2} \leq 1 \sim 1, \frac{u_2}{u_1} \sim 1$$

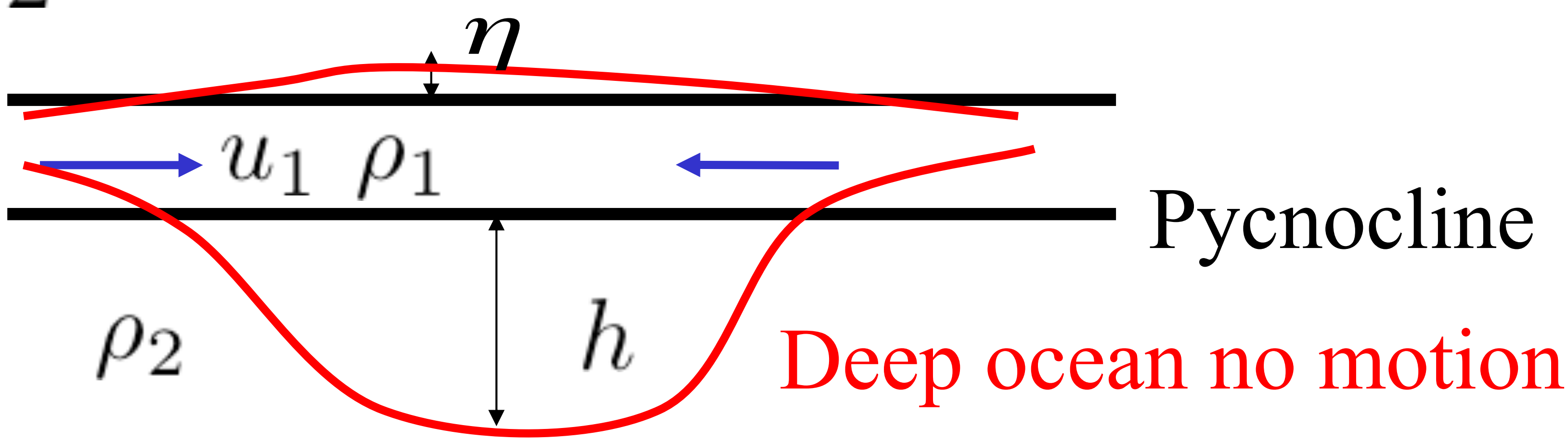
Baroclinic mode



$$C_1 = \sqrt{g' \frac{H_1 H_2}{H_1 + H_2}} \sim 2 - 3m/s$$

$$\frac{h}{\eta} = -\frac{g}{g'} \sim -300$$ **(internal gravity waves)**

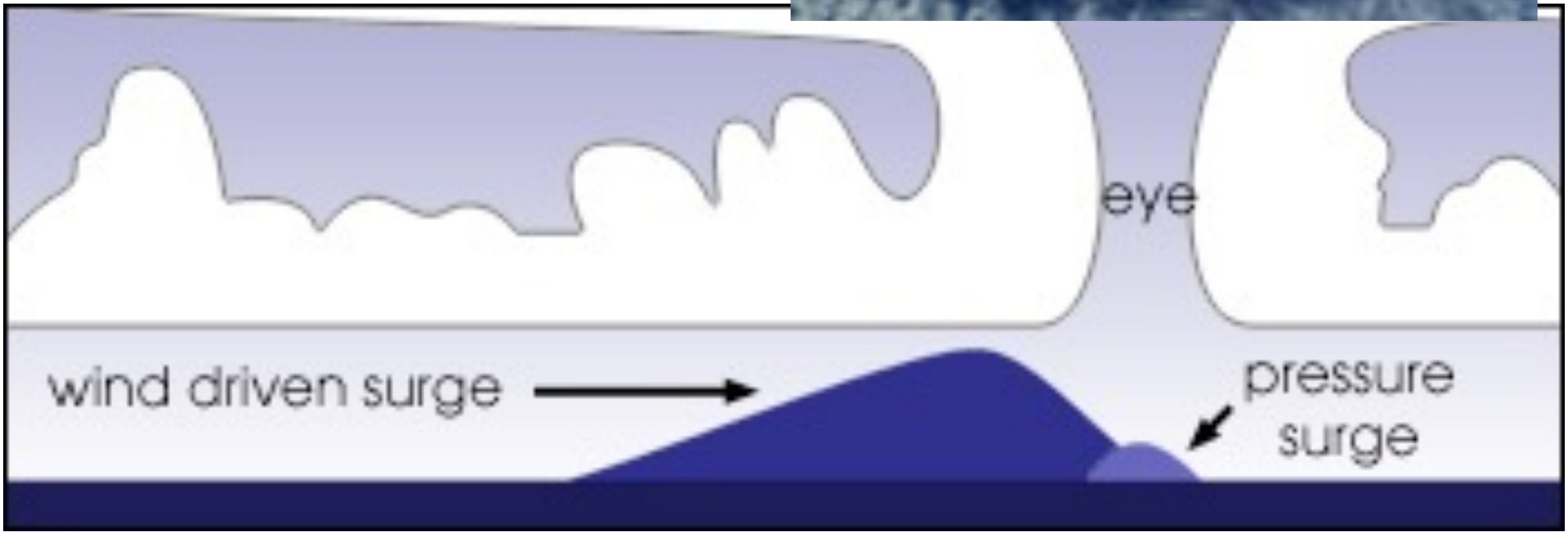
A $1\frac{1}{2} - layer$ model



Deep Ocean: infinitely deep and $\nabla p = 0$

The system has only one baroclinic mode;
No barotropic mode since we assumed $\nabla p = 0$ below the pycnocline. *We can also view it as the deep ocean is infinitely deep.*

2.Storm surge



Coastal shallow water: amplify. Why?

Basic dynamics :

(i) winds associated with storms or hurricanes pile up the water – wind surge;

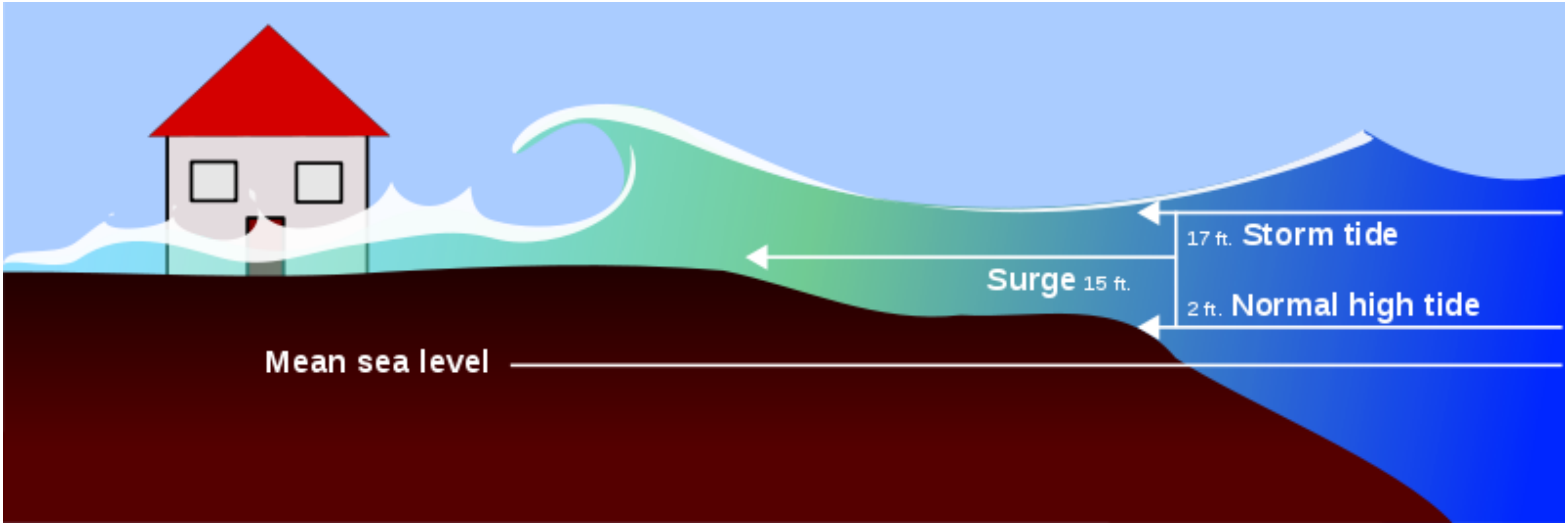
(ii) low sea level pressure at the storm center (minimal comparing with wind) – pressure surge;

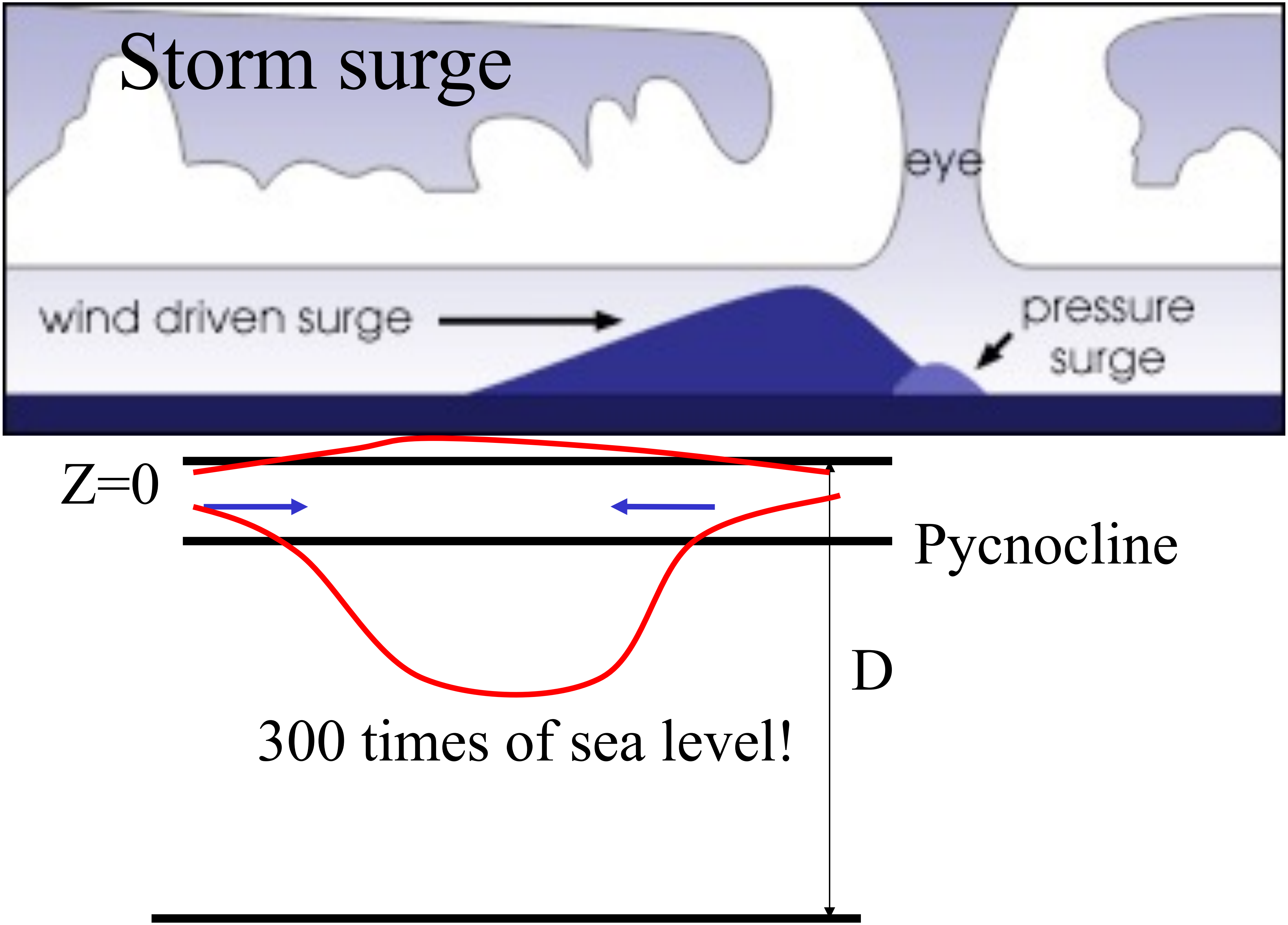
(iii) Shallow, gently sloping coastal region: intensify;

(iv) Overlapping with high tide – more devastating.

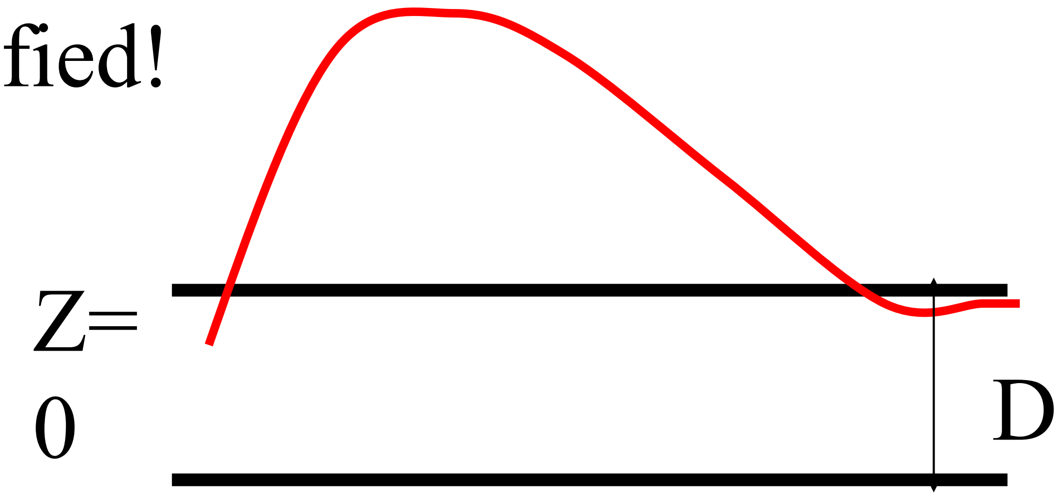
Very complicated, depend on many factors: Storm surge is a very complex phenomenon because it is sensitive to the slightest changes in storm intensity, forward speed, size (radius of maximum winds-RMW), angle of approach to the coast, central pressure (minimal contribution in comparison to the wind), and the shape and characteristics of coastal features such as bays and estuaries.

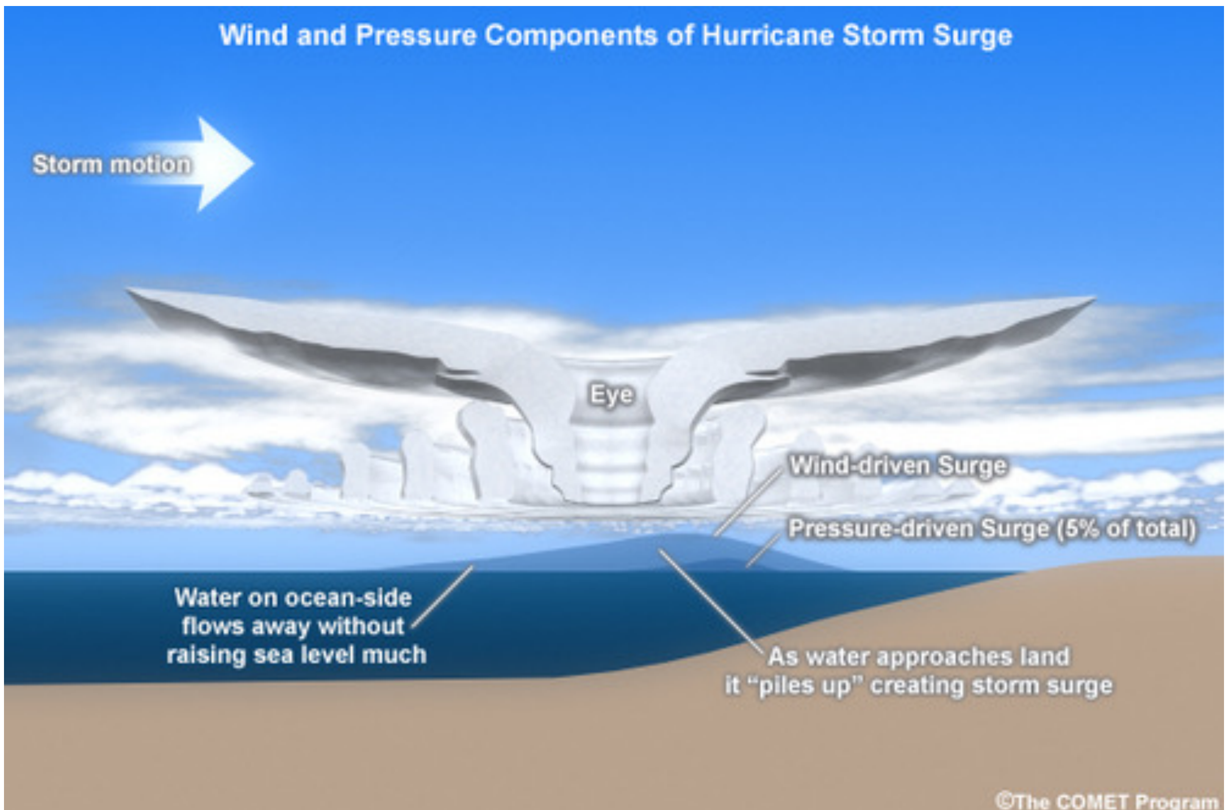
Surge + high tide




Storm surge
eye
wind driven surge
pressure surge
Z=0
Pycnocline
D
300 times of sea level!

Approach the coast, D is shallow: sea level has to go up, surge amplified!





Gentle slope: amplify

Surge examples:

- The highest storm surge in record: 1899 Cyclone Mahina: 13 meters (43 feet) storm surge at Bathurst Bay, Australia (high tide);
- In the U.S., the greatest storm surge was generated by Hurricane Katrina: 9 meters (30feet) high storm surge in Bay St. Louis, Mississippi, and surrounding counties. (Low elevation above sea level, larger impact)

Hurricane Katrina
Near peak strength:
Aug 28, 2005

Formed: Aug 23;
Dissipated: Aug 31

Highest: 175mph
Lowest pressure:
902mbar

- Damages: $81.2 billion (costliest Atlantic hurricane in history), the 6th strongest hurricane;
- Fatalities: greater than 1836 total;
- Areas affected: Bahamas, South Florida, Cuba, Louisiana (especially greater New Orleans), Mississippi, Alabama, Florida Panhandle, most of the eastern North America.

Aftermath of Katrina

Storm Surge video: NOAA National Weather Service:
https://www.youtube.com/watch?v=2GgUn2QTJtE&feature=emb_rel_end
Sea, Lake, and Overland Surges from Hurricanes (SLOSH)

3. Effects of rotation ($f \neq 0$) and Rossby radius of deformation

With f=0, what transient waves are available in the system?

What is the equilibrium state of the ocean after the waves propagation?

Critical thinking: what effects do you think f will have on the transient waves and equilibrium state?

Effects of rotation ($f \neq 0$) and Rossby radius of deformation

With a uniform rotation (**f is assumed to be a constant**), the equations of motion for the unforced, inviscid ocean are:

$$\frac{dU}{dt} - fV = -\frac{1}{\rho_0}\frac{\partial P}{\partial x},$$

$$\frac{dV}{dt} + fU = -\frac{1}{\rho_0}\frac{\partial P}{\partial y},$$

$$\frac{dW}{dt} = -\frac{1}{\rho_0}\frac{\partial P}{\partial z} - g.$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0.$$

Assumptions:

(i) f constant;

(ii) $\rho = \rho_0 = constant;$

(iii) $\frac{\partial P}{\partial z} = -\rho_0 g.$ Total P

(iv) $W = 0,$ at z=H; bottom

(v) Background state:

$U_0 = V_0 = W_0 = 0$

(Ro<<1, E<<1)

For small perturbations u,v,w,p about the resting state, we have:

$$U = U_0 + u = u,$$
$$V = V_0 + v = v,$$
$$W = W_0 + w = w,$$
$$P = P_0(z) + p, \longrightarrow \frac{\partial P}{\partial z} = -\rho_0 g.$$
$$\rho = \rho_0.$$

The linearized first order equations for perturbation:

$$\frac{\partial u}{\partial t} - fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x},$$
$$\frac{\partial v}{\partial t} + fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y},$$
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

Apply boundary conditions

Z=0, $p = \rho_0 g \eta, w = \frac{\partial \eta}{\partial t}$

Z=H, $w = 0;$

and vertically integrate the perturbation equations:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x},$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y},$$

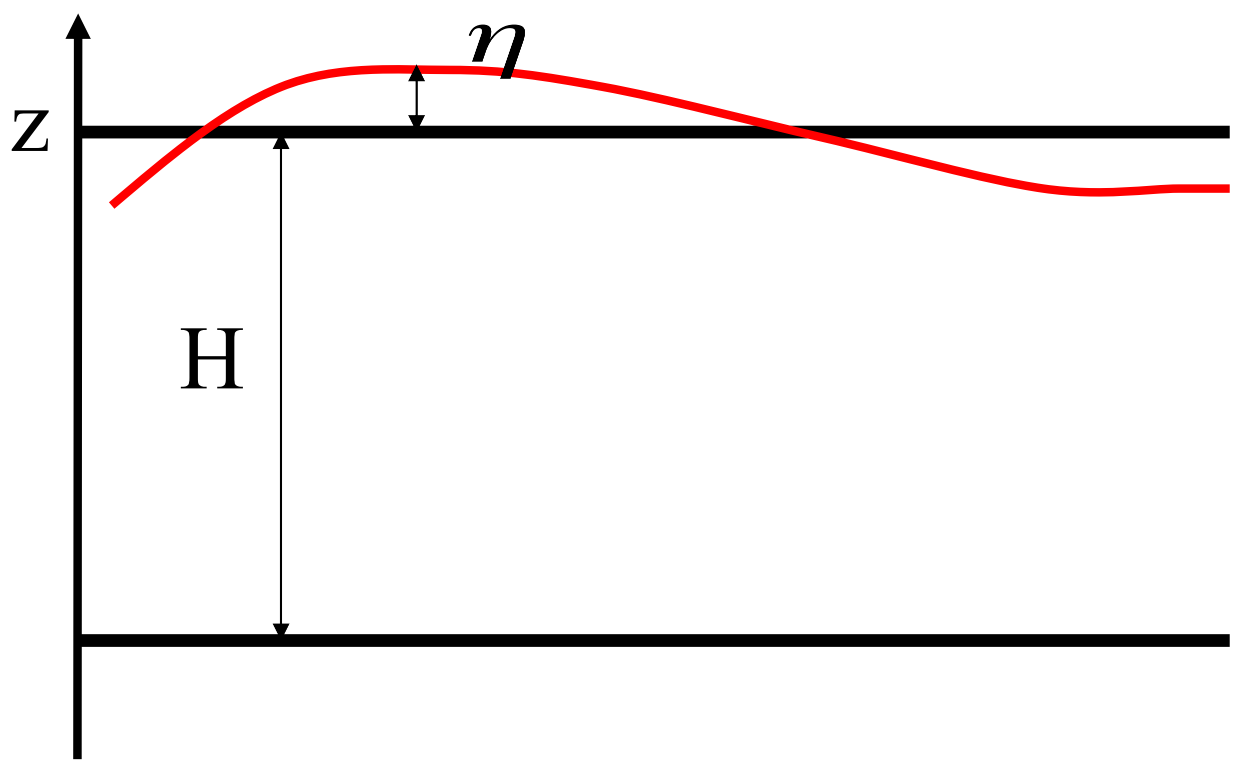
$$\frac{\partial \eta}{\partial t} + H(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0.$$

Following the same procedure as in the non-rotating case, write an equation in η,

$$\frac{\partial^2 \eta}{\partial t^2} - c^2 \nabla^2 \eta + f H \zeta = 0,$$

Where $c^2 = gH, \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

Relative vorticity



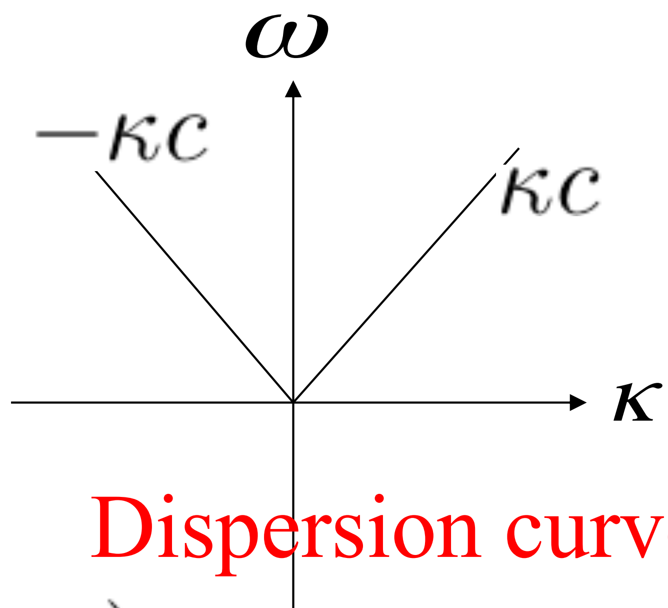
Note: $f + \zeta$ is referred to as absolute vorticity;

f — Planetary vorticity

ζ — Relative vorticity

a) Non-rotating case (f=0):

$$\frac{\partial^2 \eta}{\partial t^2} - c^2 \nabla^2 \eta = 0$$



Dispersion curve

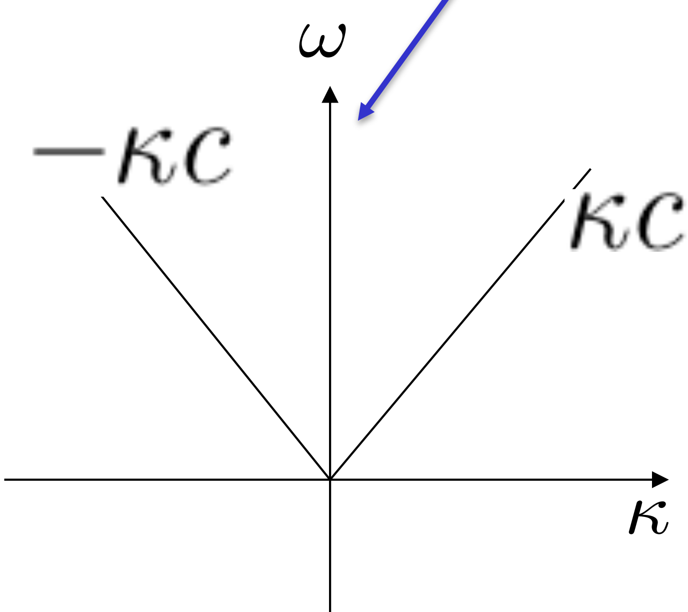
Assume $\eta = \eta_0 cos(kx - \omega t)$ (1-dimensional exp)

$$\omega^2 = c^2 k^2$$

$$c^2 = gH$$

Recall this is the dispersion relation for long-surface gravity waves when f=0!

Today’s class: dispersion relation

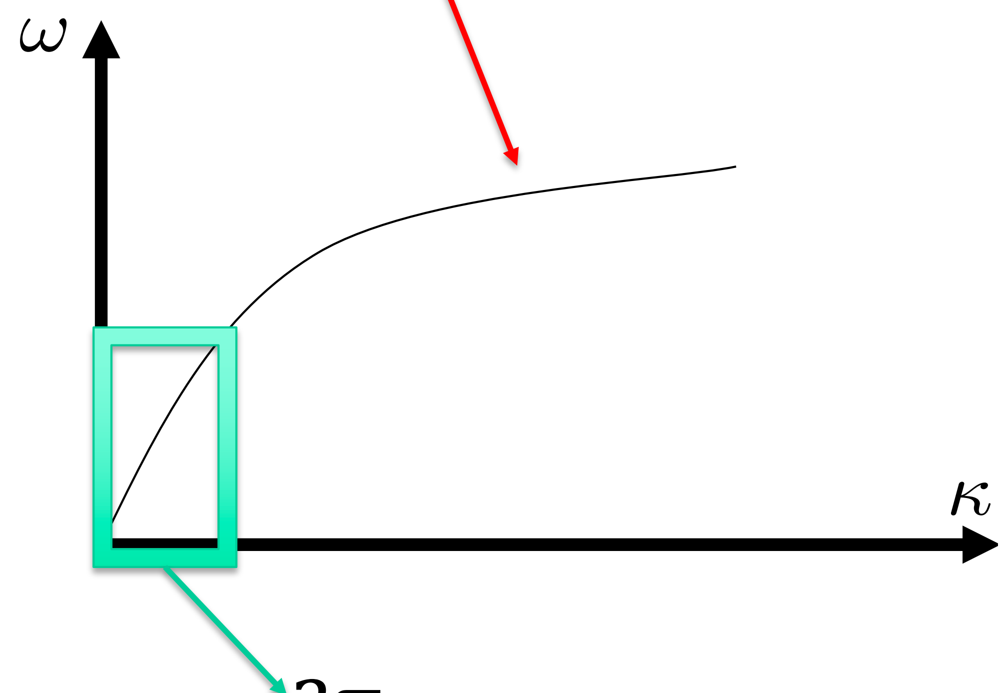


Only has long surface gravity waves;

short waves: distorted by hydrostatic approximation

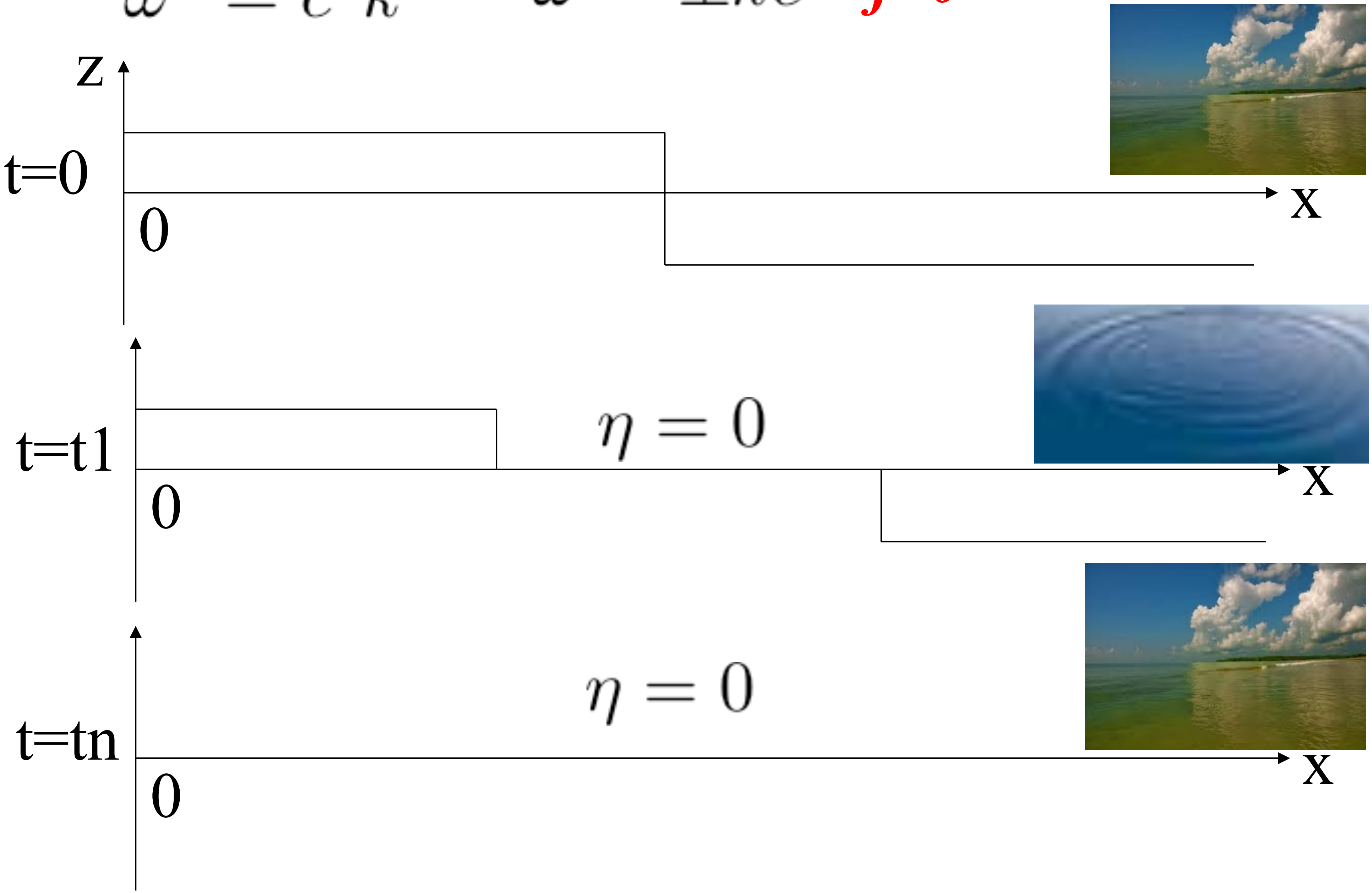
$$\frac{\partial P}{\partial z} = -\rho_0 g.$$

Previous: dispersion relation



$L = \frac{2\pi}{\kappa}$: **long surface gravity wave with *f=0***

$\omega^2 = c^2k^2 \qquad \omega = \pm kc \qquad$ ***f=0***



b) Rotating case ($f \neq 0$)

Assume wave form of solution

$$\eta = \eta_0 e^{i(kx+ly-\omega t)},$$

$$u = u_0 e^{i(kx+ly-\omega t)},$$

$$v = v_0 e^{i(kx+ly-\omega t)},$$

Substitute into the vertically-integrated perturbation equation for η, u, v

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x},$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y},$$

$$\frac{\partial \eta}{\partial t} + H(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0.$$

let coefficient matrix = 0,

$\longrightarrow$ $\omega^2 = \kappa^2 c^2 + f^2,$ where $\kappa^2 = k^2 + l^2$

$$\begin{vmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{vmatrix} =$$

$$a11a22a33 + a12a23a31 + a13a21a32$$

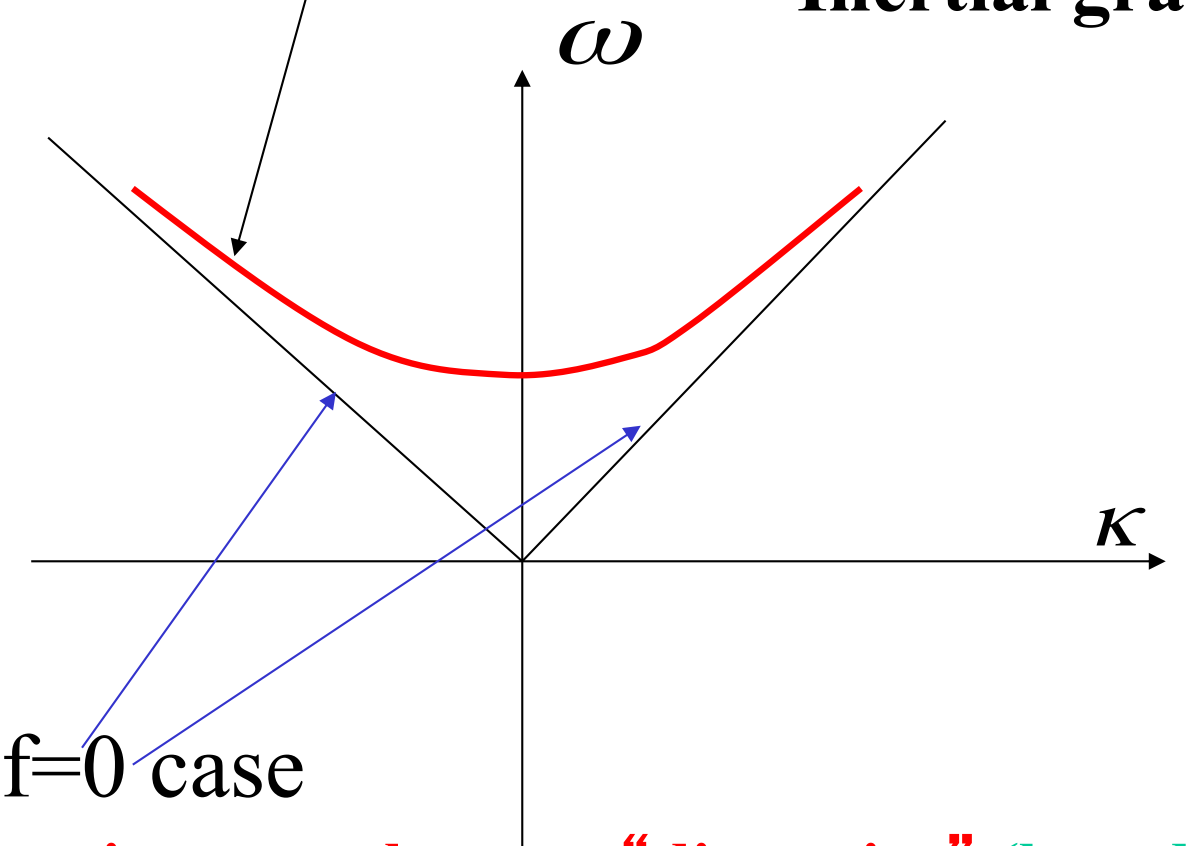
$$-a11a23a32 - a12a21a33 - a13a22a31$$

$$\omega^2 = \kappa^2 c^2 + f^2,$$

What does f do to the system?

$$\omega^2 = \kappa^2 c^2 + f^2,$$

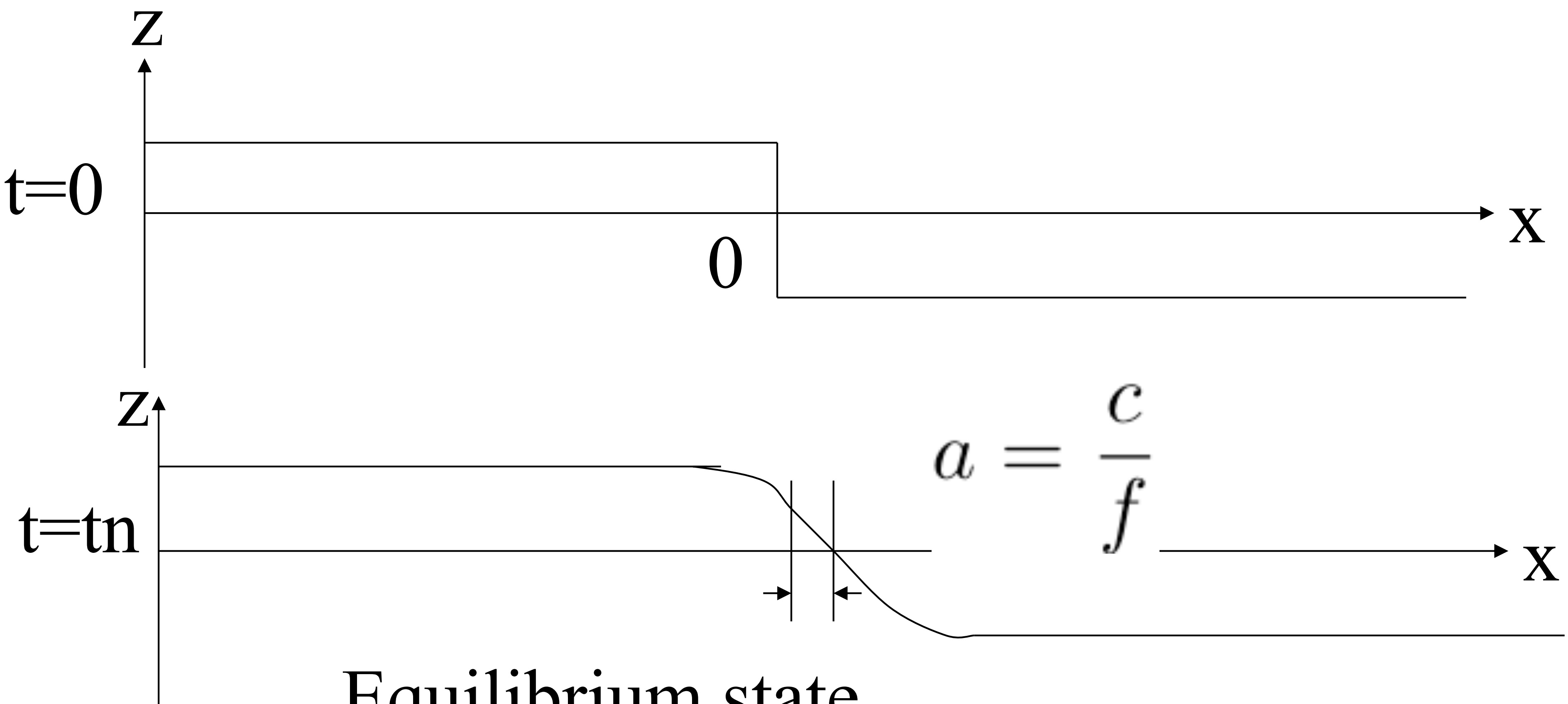
Inertial gravity waves



(i) Long gravity waves become "dispersive" **(board demo)**

(ii) Long gravity waves do not have "zero" frequency anymore. Their lowest frequency is "f", which has a period of a few days in mid latitude.

Adjustment with f (1-dimensional in x)



Equilibrium state

$$\frac{\partial^2 \eta}{\partial t^2} - c^2 \frac{\partial^2 \eta}{\partial x^2} + fH \frac{\partial v}{\partial x} = 0 \longrightarrow$$

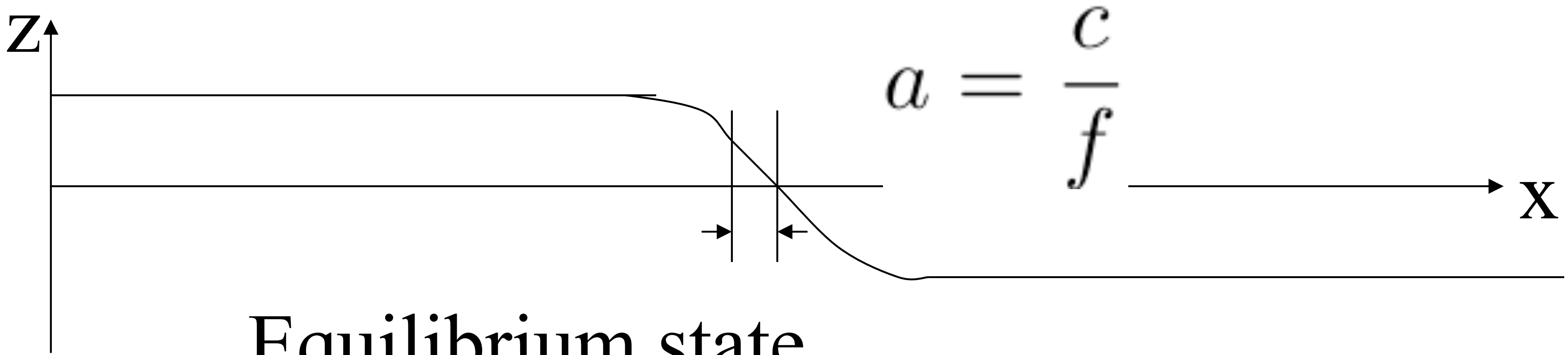
Geostrophic balance $$fv = g \frac{\partial \eta}{\partial x}$$

Solutions:

$$\eta = \eta_0[-1 + exp(-\frac{x}{a})], for x > 0,$$

$$\eta = \eta_0[1 - exp(\frac{x}{a})], for x < 0,$$

where $a = \frac{c}{f}$ is Rossby radius of deformation.



$a = \frac{c}{f}$

Equilibrium state