EECS 442 Discussion

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October 14, 2015

Announcements

- HW2 due 10/15
- Project Proposals due 10/22
- Jon Beaumont from ETC coming in next week for a Midterm Student Feedback session

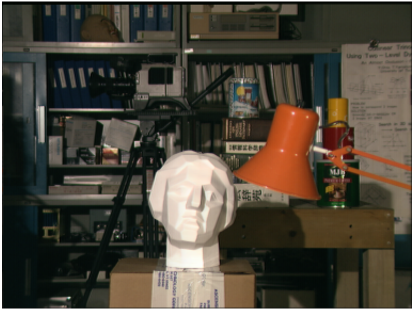
HW2 Problem 1a

$$M = \begin{bmatrix} A & b \end{bmatrix}$$
$$M' = \begin{bmatrix} A' & b' \end{bmatrix}$$
$$\hat{M} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- a_{ij} is not referring to the elements of A
- b_i is not referring to the elements of b

Stereo Cameras

- Why use more than one camera?



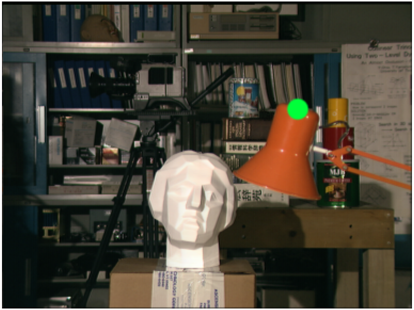
(a) Left

(b) Right

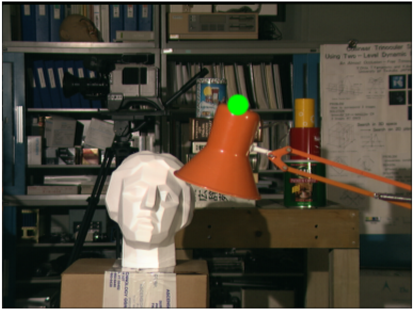
Sample stereo images from OpenCV

Correspondence Problem

- Point in image (a), where is it in image (b)?



(a) Left

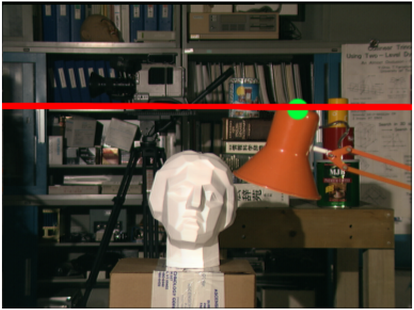


(b) Right

Sample stereo images from OpenCV

Correspondence Problem

- Can't determine exactly, but can constrain with epipolar geometry



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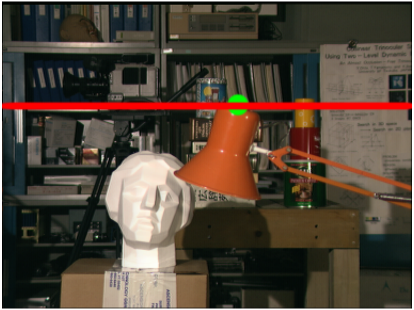
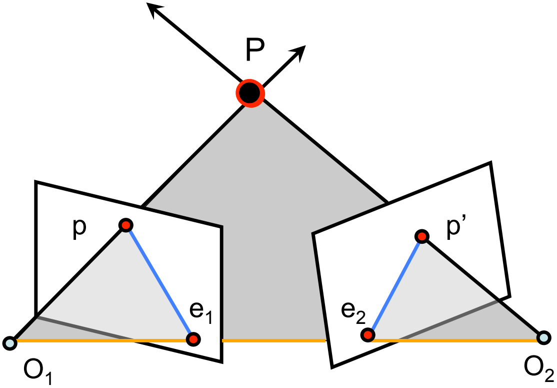


(b) Right

Sample stereo images from OpenCV

Epipolar geometry



- Epipolar Plane
- Baseline
- Epipolar Lines

- Epipoles e_1, e_2

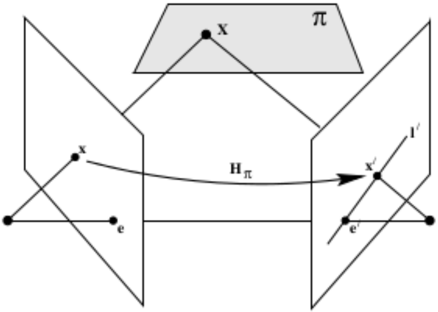
  = intersections of baseline with image planes

  = projections of the other camera center

Epipolar Geometry

- "Point transfer via plane π" (See HZ Chp 9)

$$x = H_1 x_\pi \quad x' = H_2 x_\pi$$
$$x' = H_2 H_1^{-1} x$$
$$x' = H_\pi x$$



Epipolar Geometry

- What is the epipole?

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- Recall that $x' = H_\pi x$

$$l' = [e']_x H_\pi x$$
$$l' = Fx$$

where $F = [e']_x H_\pi$.

Fundamental Matrix: Rank

- $F = [e']_x H_\pi$
- What is the rank of H_π?

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Fundamental Matrix: Rank

- $F = [e']_x H_\pi$
- What is the rank of H_π? 3
- What is the rank of $[e']_x$? 2
- Rank of a product is in the minimum of the ranks of the terms in the product, so rank(F) = min(rank(H), rank($[e']_x$) = 2
- Makes sense, because we're mapping points to lines, and multiple points can end up on the same line

Fundamental Matrix: Degrees of Freedom

- Recall that F is rank 2

$$\begin{bmatrix} a & b & c \\ d & e & f \\ \alpha a + \beta d & \alpha b + \beta e & \alpha c + \beta f \end{bmatrix}$$

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- Everything is in homogeneous coordinates, F only defined up to scale

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- Therefore, F has 7 degrees of freedom

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- Therefore, ${x'}^\top l' = {x'}^\top Fx = 0$
- What about $F^\top$?
- $\left({x'}^\top Fx\right)^\top = x^\top F^\top x' = 0$

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$$e'^{\top} F = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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- So, e' is the left null-vector of F
- Similarly, e is the right null-vector of F