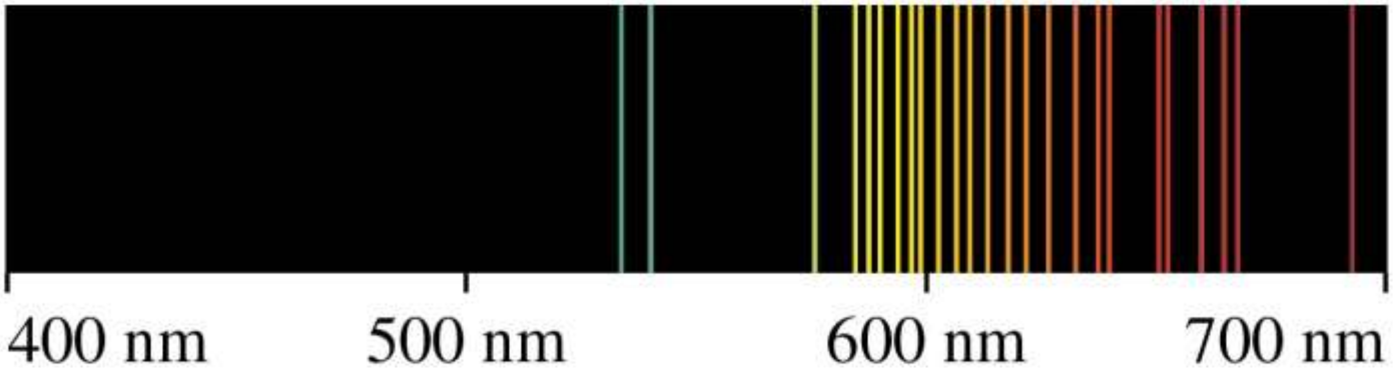
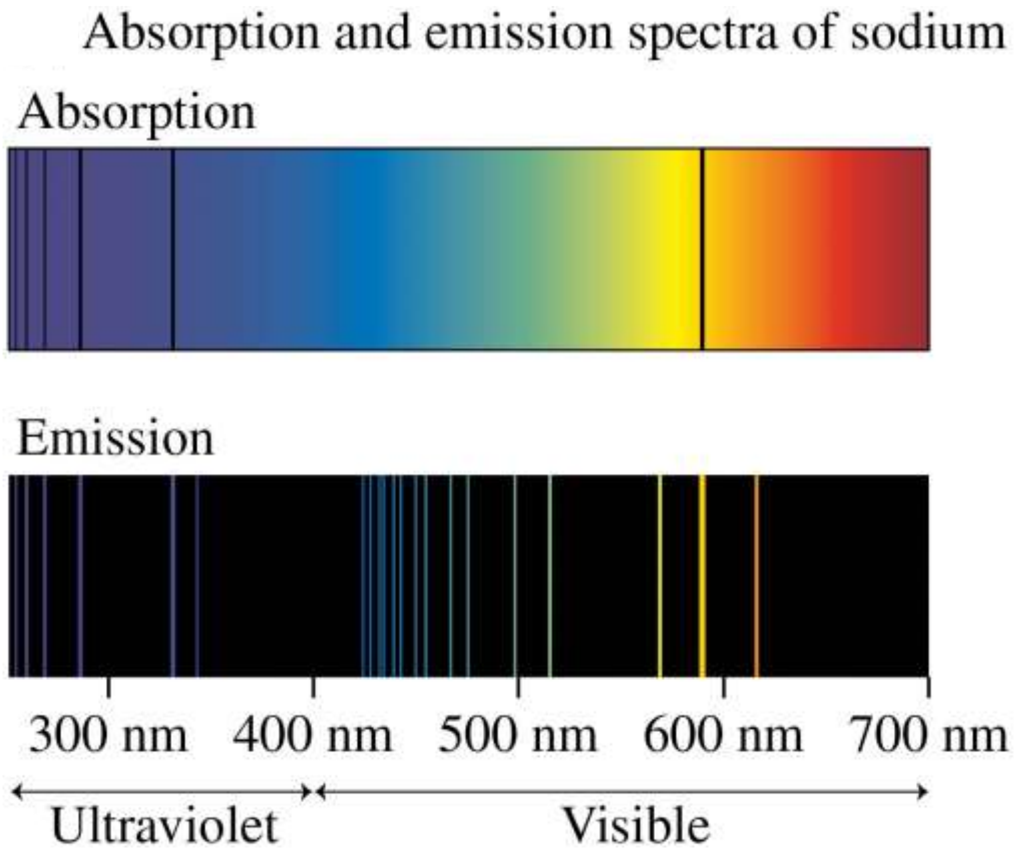
Discrete Spectra

- If light from a gas discharge tube is passed through a spectrometer, it produces a spectrum like the one shown below.
- This is called a **discrete spectrum** because it contains only discrete, individual wavelengths.
- Further, each kind of gas emits a unique spectrum—a spectral fingerprint—that distinguishes it from every other gas.



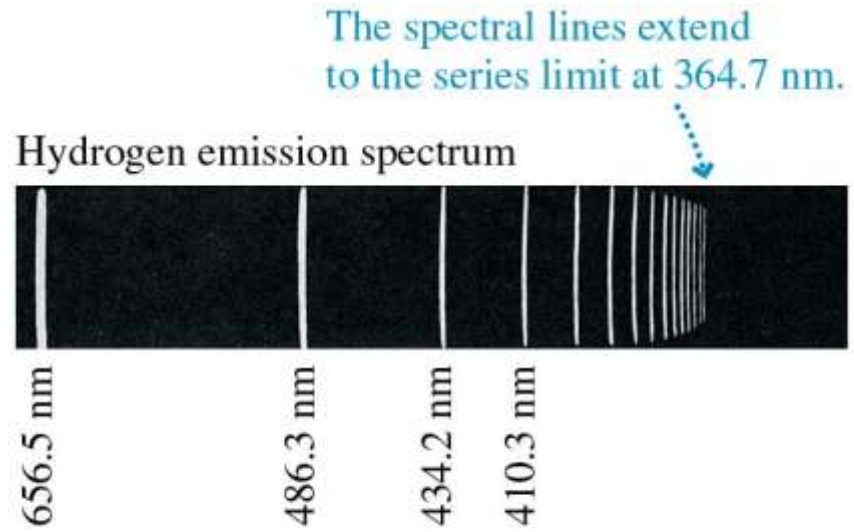
Absorption Spectra

- Not only do gases emit discrete wavelengths, they also absorb discrete wavelengths.
- The top figure shows the spectrum when white light passes through a sample of gas.
- Any wavelengths absorbed by the gas are missing, and the film is dark at that wavelength.



The Hydrogen Emission Spectrum

- The emission spectrum of hydrogen, seen below, is very simple and regular.



The Hydrogen Emission Spectrum

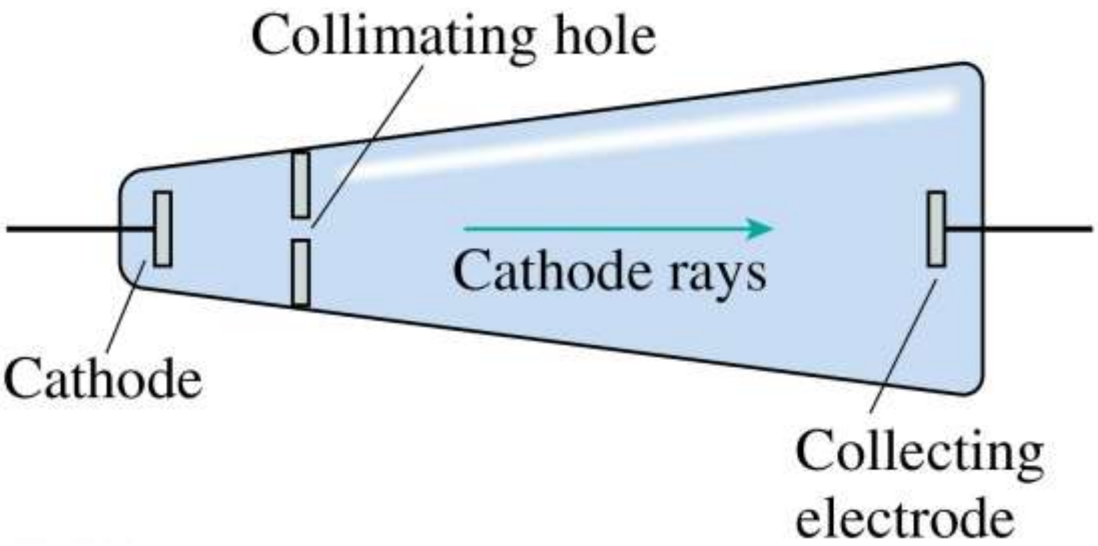
- In 1885 a Swiss schoolteacher named Johann Balmer discovered a formula which accurately describes every wavelength in the emission spectrum of hydrogen:

$$\lambda = \frac{91.18\text{ nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad \begin{aligned} m &= 1, 2, 3, \ldots \\ n &= \text{any integer greater than } m \end{aligned}$$

- This result is called the **Balmer formula**.
- Balmer's original version only included $m = 2$.
- When first discovered, the Balmer formula was *empirical knowledge*: It did not rest on any physical principles or physical laws.

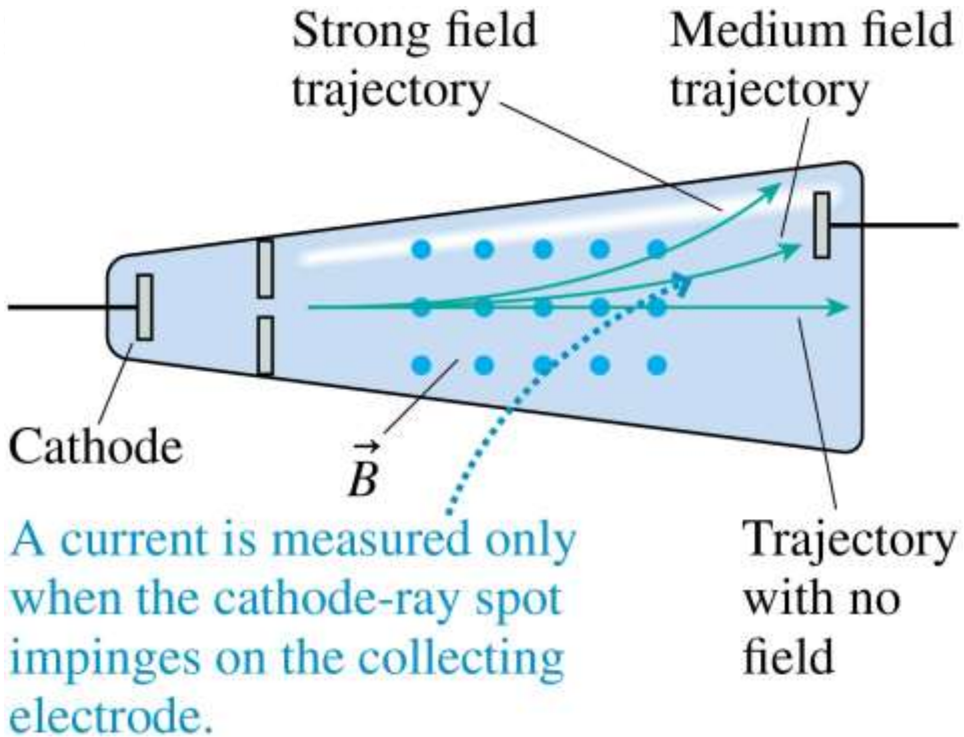
The Discovery of the Electron

- Shortly after the discovery of x rays, J. J. Thomson decided to test whether cathode rays were in fact moving charged particles.
- He started with a Crookes tube like the one shown below.
- An electric current is associated with the cathode rays.



The Discovery of the Electron

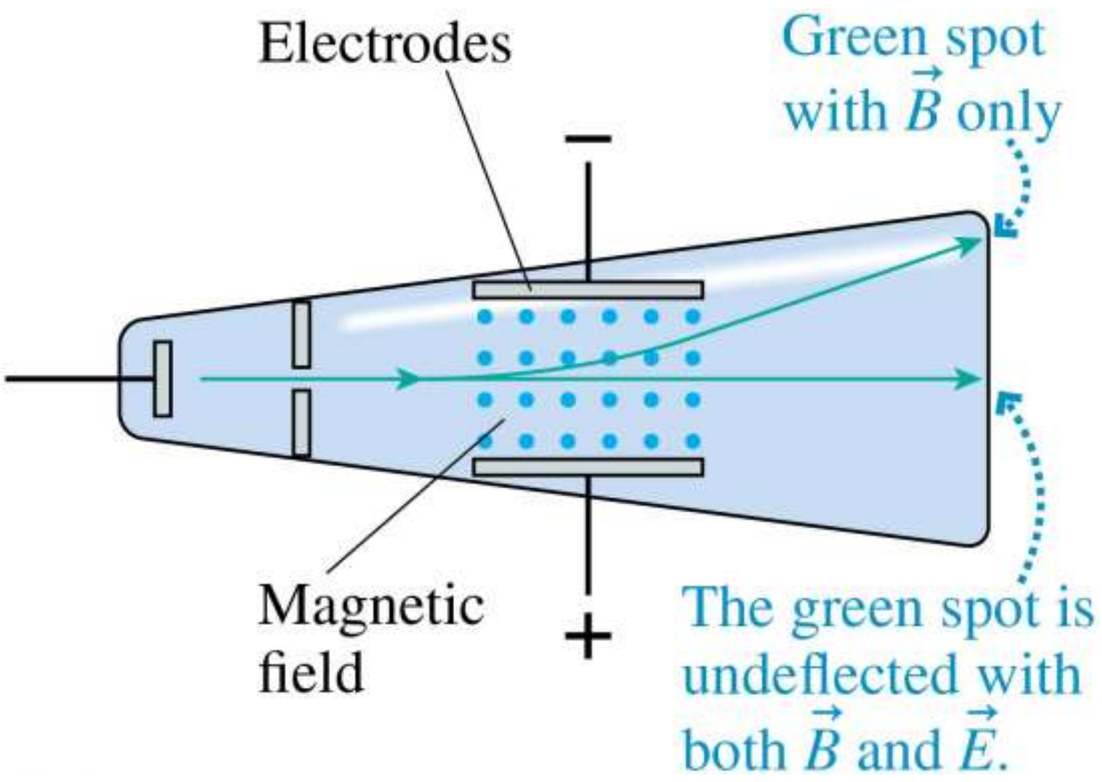
- By applying a magnetic field to the Crookes tube, Thomson could vary the current by adjusting the path of the cathode rays.
- This showed that cathode rays were indeed negatively charged particles.



Thomson's Crossed-Field Experiment

- The magnetic field alone would produce an *upward* force on a negatively charged particle:

$$F_B = qvB$$



Thomson's Crossed-Field Experiment

- Once E and B are set, a charged particle can pass undeflected through the crossed fields only if its speed is

$$v = \frac{E}{B}$$

- Combining this equation with the equation for the radius of the arc of a charged particle in a magnetic field, Thomson was able to solve for and measure the charge-to-mass ratio of the cathode ray particles:

$$\frac{q}{m} = \frac{v}{rB}$$

The Discovery of the Electron

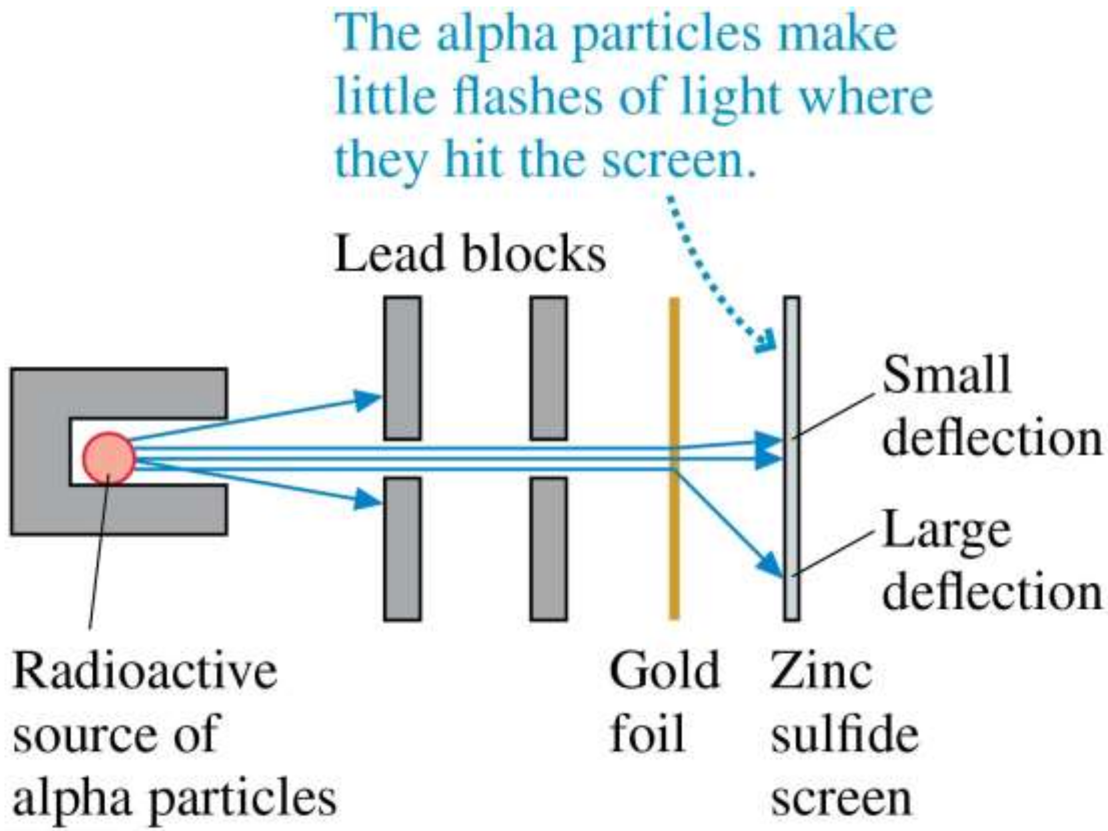
- In a paper published in 1897, Thomson assembled all of the evidence to announce the discovery that cathode rays are negatively charged particles.
- Thomson had discovered the first **subatomic particle**, one of the constituents of which atoms themselves are constructed.

J. J. Thomson

- In recognition of the role this particle plays in electricity, it was later named the **electron**.
- Thomson was awarded the Nobel Prize in Physics in 1906.

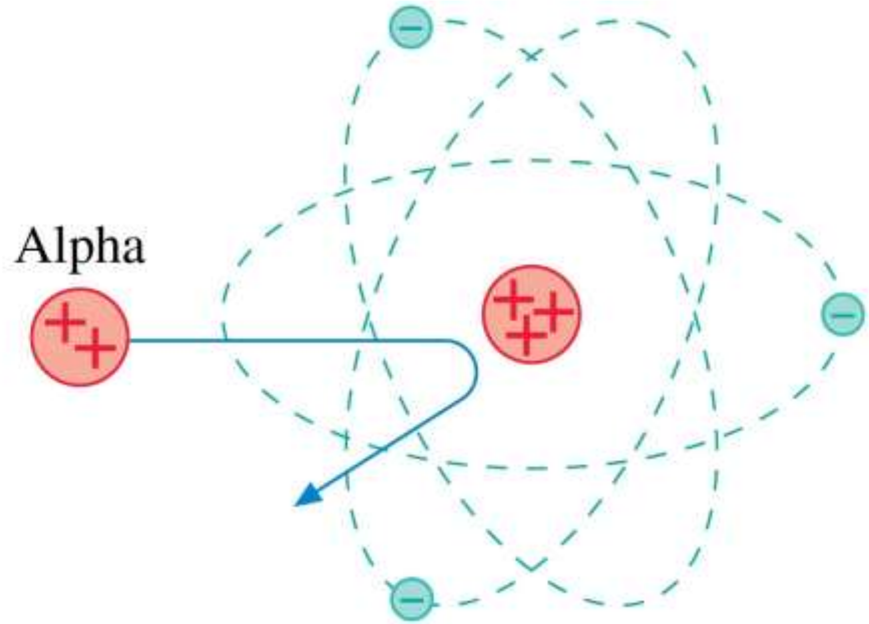
Rutherford's Gold Foil Experiment

- In 1909, Ernest Rutherford set up an experiment to shoot fast-moving alpha particles at very thin metal foils.
- Many alpha particles passed through the foil and were slightly deflected.
- A few alpha particles were deflected at very large angles!



Rutherford's Model of an Atom

- The discovery of large-angle scattering of alpha particles led to Rutherford's **nuclear model of an atom** in which negative electrons orbit an unbelievably small, massive, positive **nucleus**.



If the atom has a concentrated positive nucleus, some alpha particles will be able to come very close to the nucleus and thus feel a very strong repulsive force.

Matter Waves

- In 1924 French graduate student Louis-Victor de Broglie wondered, “If light waves can have a particle-like nature, why shouldn’t material particles have some kind of wave-like nature?”
- In other words, could **matter waves** exist?
- De Broglie thought about an analogy with light, and postulated that *if* a material particle of momentum $p = mv$ has a wave-like nature, then its wavelength must be given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

  where h is Planck’s constant.
- This is called the **de Broglie wavelength**.

QuickCheck

A beam of electrons, and then a beam of protons, are shot through a double slit with a very small slit spacing of $1\ \mu\text{m}$. The electrons and protons travel at the same speed. Which is true?

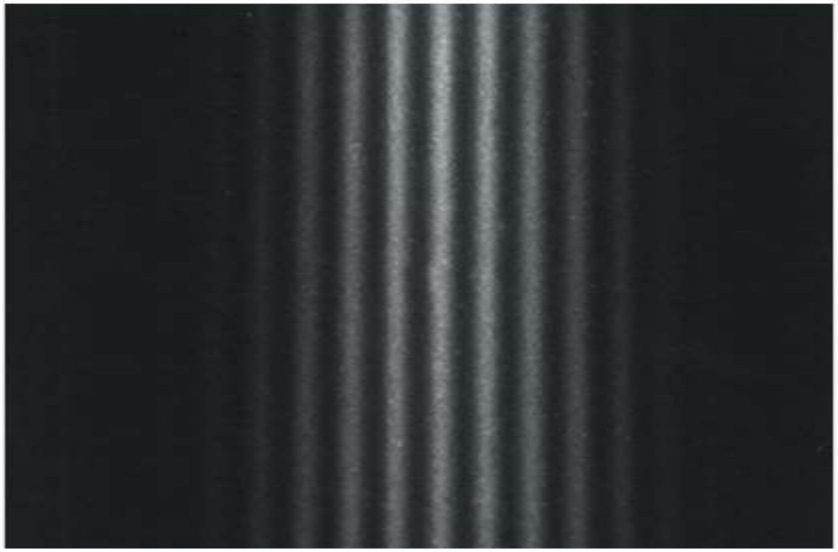
A. They both make interference patterns on a screen. The fringe spacing is wider for the electron interference pattern.

B. They both make interference patterns on a screen. The fringe spacing is wider for the proton interference pattern.

C. Neither makes an interference pattern.

The Photon Model of Light

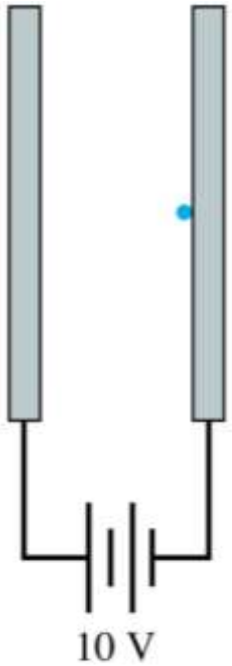
- The image to the right shows the intensity pattern recorded after $50\ \mathrm{keV}$ electrons passed through two slits separated by $1.0\ \mathrm{mm}$.
- The pattern is clearly a double-slit interference pattern.



- Electrons, neutrons, atoms, and even molecules exhibit all the behavior we associate with waves.

QuickCheck

An electron is released from the negative plate. Its de Broglie wavelength upon reaching the positive plate is _____ its de Broglie wavelength at the negative plate.



A. greater than
B. the same as
C. less than

QuickCheck

For a photon, the energy E, frequency f, and wavelength λ are related by the equations $E = hf$, $E = hc/\lambda$, and $f = c/\lambda$. (Here h is Planck's constant and c is the speed of light in vacuum.) Which of these equations also applies to *electrons*?

A. $E = hf$

B. $E = hc/\lambda$

C. $f = c/\lambda$

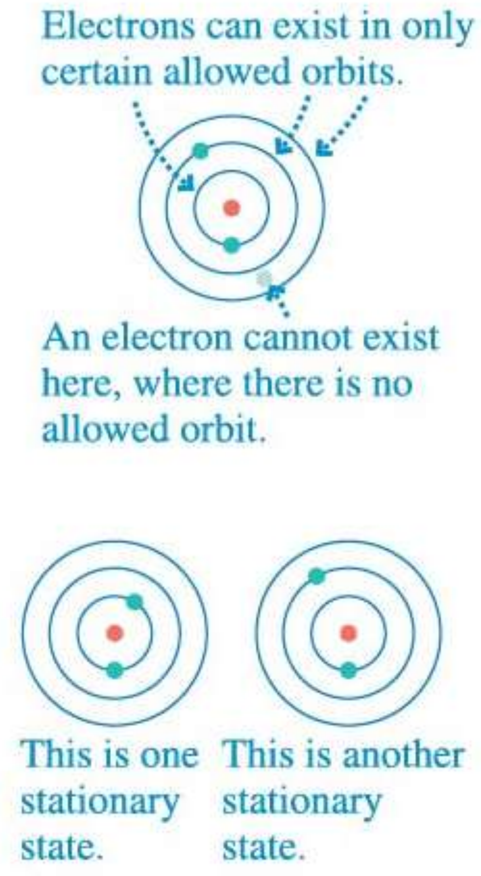
D. all three of A, B, and C apply to electrons.

Example 1 – What is the de Broglie wavelength of an electron with speed (a) $v = 0.480c$ and (b) $v = 0.960c$?

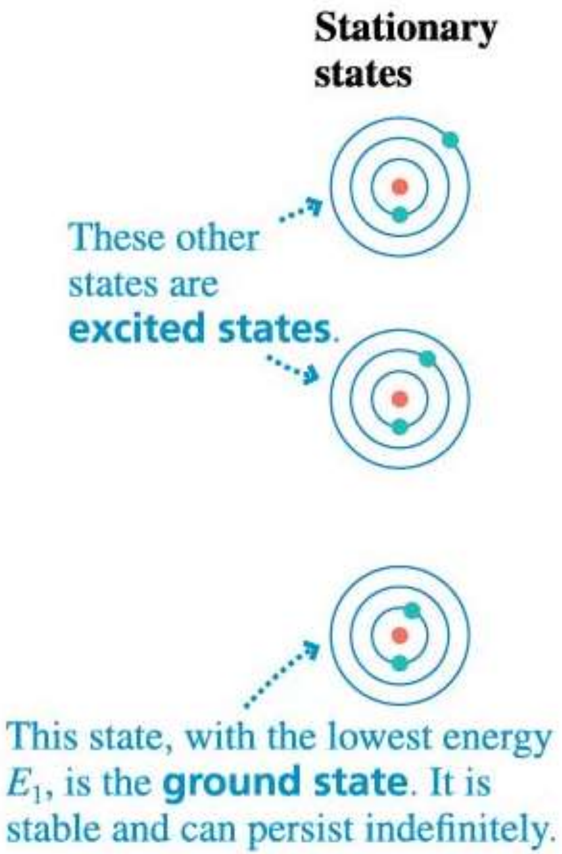
Example 2 – Calculate the de Broglie wavelength of a 5.00-g bullet that is moving at 340 m/s. Will the bullet exhibit wavelike properties?

The Bohr Model of the Atom: Slide 1 of 4

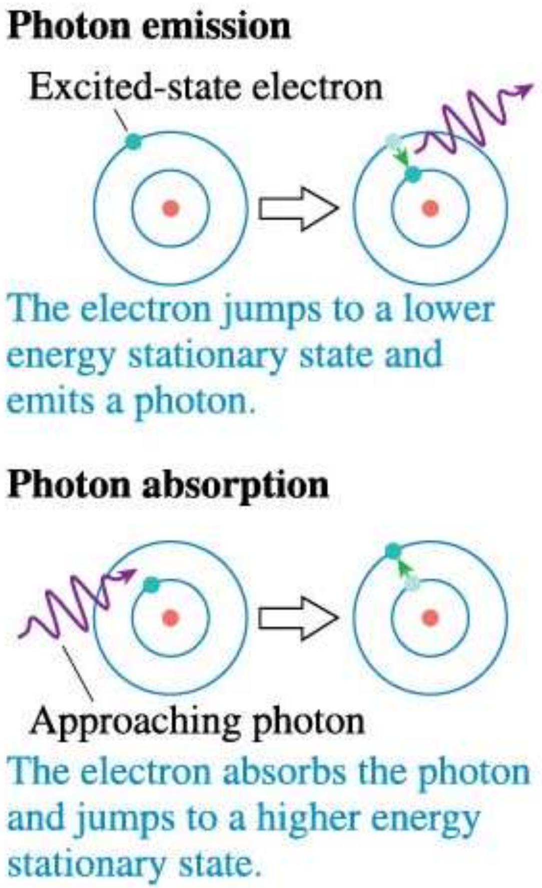
1. The electrons in an atom can exist in only certain allowed orbits. A particular arrangement of electrons in these orbits is called a **stationary state.**



2. Each stationary state has a discrete, well-defined energy E_n. That is, atomic energies are *quantized*. The stationary states are labeled by the *quantum number n* in order of increasing energy: $E_1 < E_2 < E_3 < \cdots$.

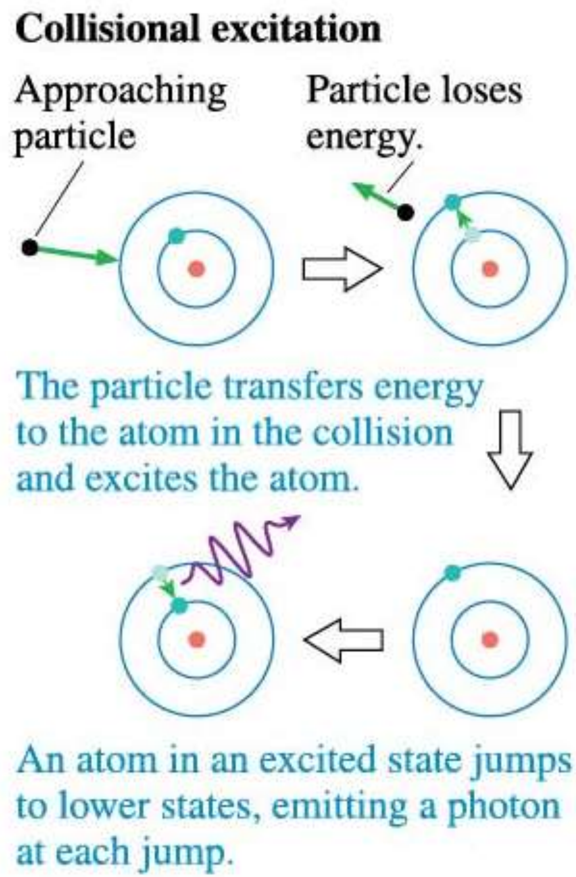


3. An atom can undergo a **transition** or **quantum jump** from one stationary state to another by emitting or absorbing a photon whose energy is exactly equal to the energy *difference* between the two stationary states.

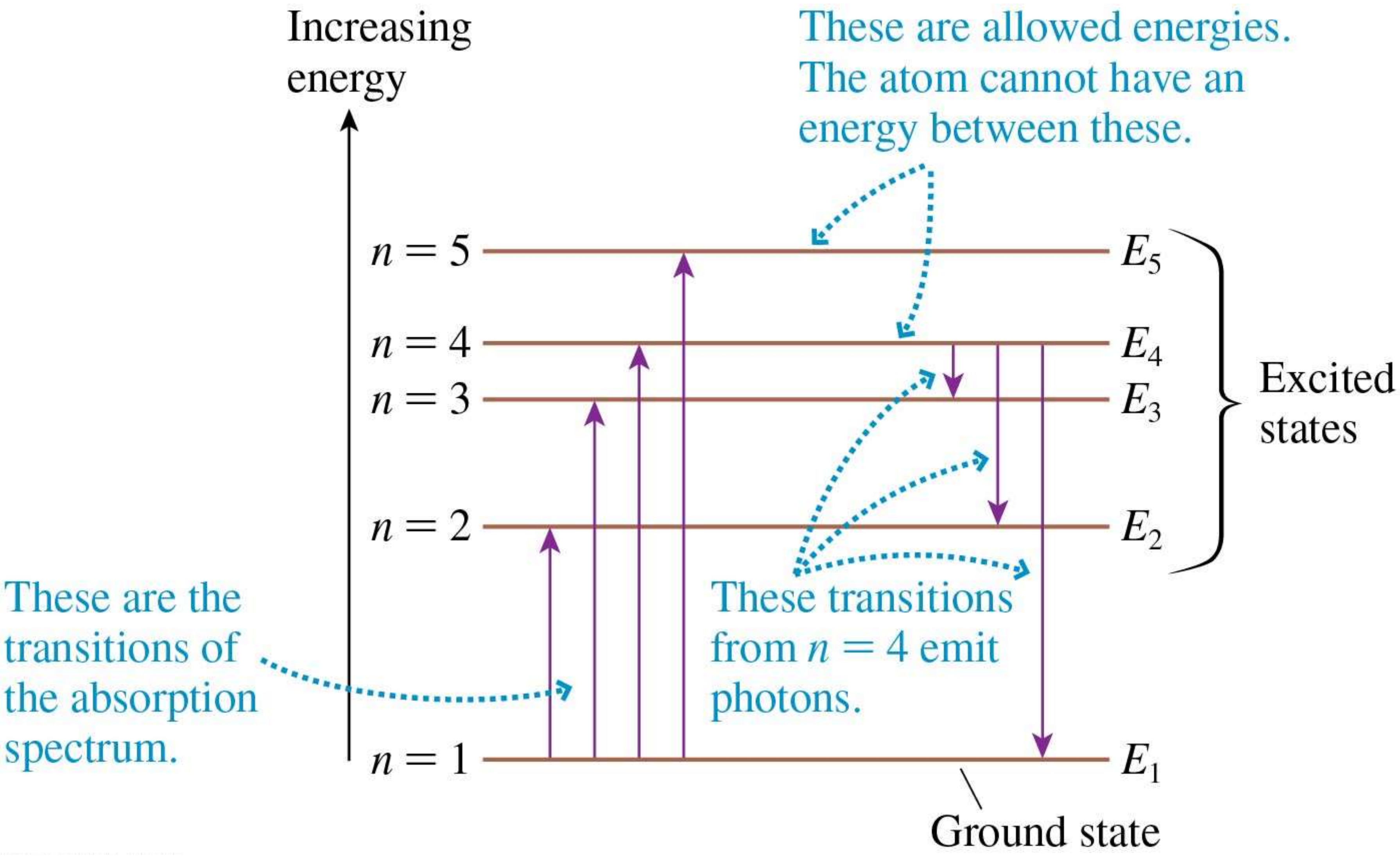


Atoms can also move from a lower energy state to a higher energy state by absorbing energy in a collision with an electron or other atom in a process called **collisional excitation.**

The excited atoms soon jump down to lower states, eventually ending in the stable ground state.



An Energy-Level Diagram



QuickCheck

A certain atom has two energy levels whose energies differ by 2.5 eV. In order for a photon to excite the atom from the lower energy level to the upper energy level, the energy of the photon

A. can have any value greater than or equal to 2.5 eV.

B. must be exactly 2.5 eV.

C. can have any value less than or equal to 2.5 eV.

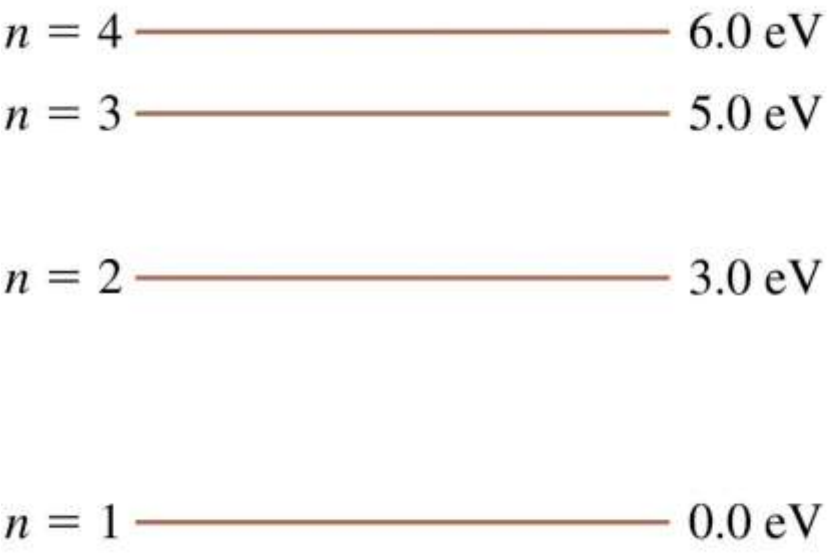
QuickCheck

An atom has the energy levels shown. A photon with a wavelength of 620 nm has an energy of 2.0 eV. Do you expect to see a spectral line with wavelength of 620 nm in this atom's emission spectrum?

A. Yes

B. No

C. There's not enough information to tell.



Example 3 - An atom has stationary states with energies E_1 = 4.00 eV and E_2 = 6.00 eV. What is the wavelength of a photon emitted in a quantum jump from state 1 to 2?

QuickCheck

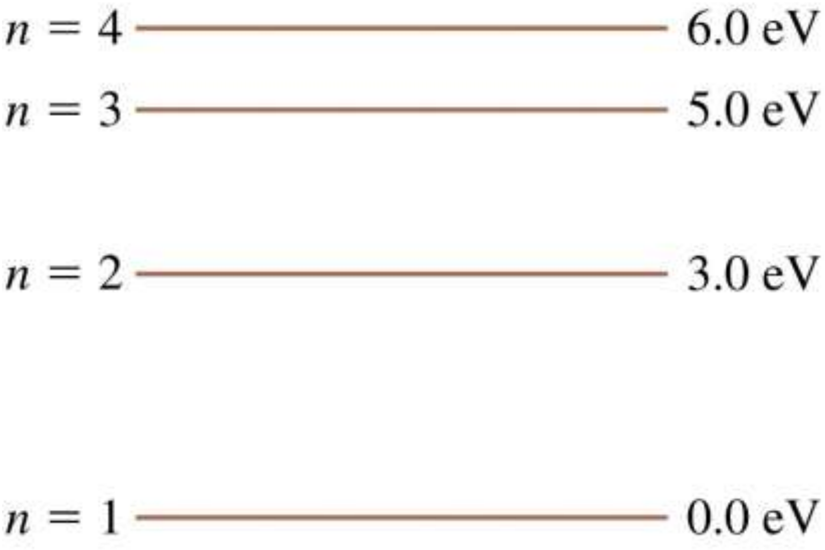
An atom has the energy levels shown. How many spectral lines are in the emission spectrum?

A. 3

B. 4

C. 5

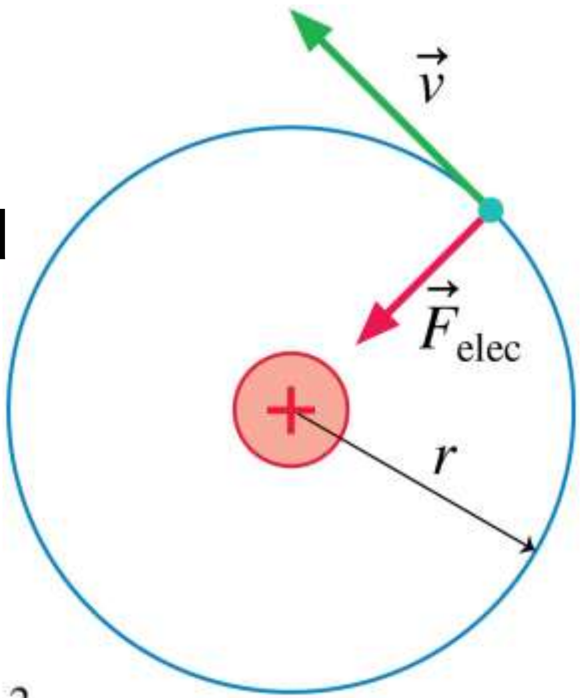
D. 6



Example 4 - An atom has stationary states E_1 = 0.00 eV, E_2 = 3.00 eV, and E_3 = 5.00 eV. What wavelengths are observed in the absorption spectrum and in the emission spectrum of this atom?

The Rutherford Hydrogen Atom

- In the classical model of a hydrogen atom, an electron of charge $-e$ and mass m moves in a circular orbit around the much more massive proton of charge $+e$.
- The centripetal acceleration of the electron is the electric force divided by its mass:

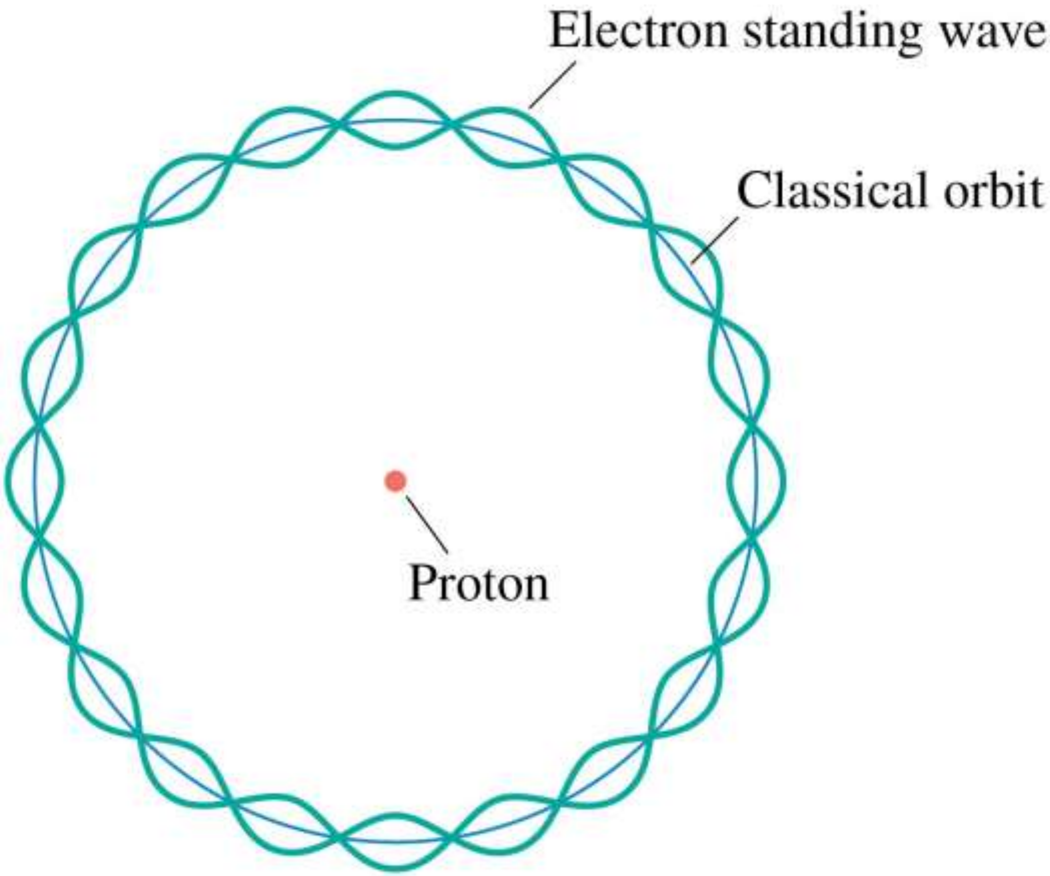


$$a_{elec} = \frac{F_{elec}}{m} = \frac{e^2}{4\pi\epsilon_0 m r^2} = \frac{v^2}{r}$$

- Rearranging, we find that the speed v and the orbital radius r for the electron must be related by

$$v^2 = \frac{e^2}{4\pi\epsilon_0 m r}$$

The Bohr Hydrogen Atom



- In a Bohr hydrogen atom, the wave-like electron sets up a circular standing wave.
- The orbital circumference is an integer number of de Broglie wavelengths.

The Bohr Hydrogen Atom

- In a Bohr hydrogen atom, the electron's orbital circumference is an integer number of de Broglie wavelengths:

$$2\pi r = n\frac{h}{mv}$$

- This is true only if the electron's speed is

$$v_n = \frac{nh}{2\pi mr} \qquad n = 1, 2, 3, \ldots$$

- If we define a new constant "h bar":

$$\hbar \equiv \frac{h}{2\pi} = 1.055 \times 10^{-34}\ \text{J s} = 6.58 \times 10^{-16}\ \text{eV s}$$

$$v_n = \frac{n\hbar}{mr} \qquad n = 1, 2, 3, \ldots$$

The Bohr Hydrogen Atom

- If the electron can act both as a particle *and* a wave, then we can combine our constraint on v and r from the Rutherford atom and our constraint on v and r from requiring the electron to be a standing wave to solve for allowed values of v and r.
- We find the radius of the electron's orbit in a Bohr hydrogen atom must be

$$r_n = n^2 \frac{4\pi\epsilon_0 \hbar^2}{me^2} \qquad n = 1, 2, 3, \ldots$$

- If we define a new constant, the Bohr radius is

$$a_B = \text{Bohr radius} \equiv \frac{4\pi\epsilon_0 \hbar^2}{me^2} = 5.29 \times 10^{-11}\ \text{m} = 0.0529\ \text{nm}$$

$$r_n = n^2 a_B \qquad n = 1, 2, 3, \ldots$$

The Bohr Hydrogen Atom

- Knowing the possible orbital radii, we find the possible electron speeds are

$$v_n = \frac{n\hbar}{mr_n} = \frac{1}{n}\frac{\hbar}{ma_B} = \frac{v_1}{n} \qquad n = 1, 2, 3, \ldots$$

where v_1 is the electron's speed in the $n = 1$ orbit.

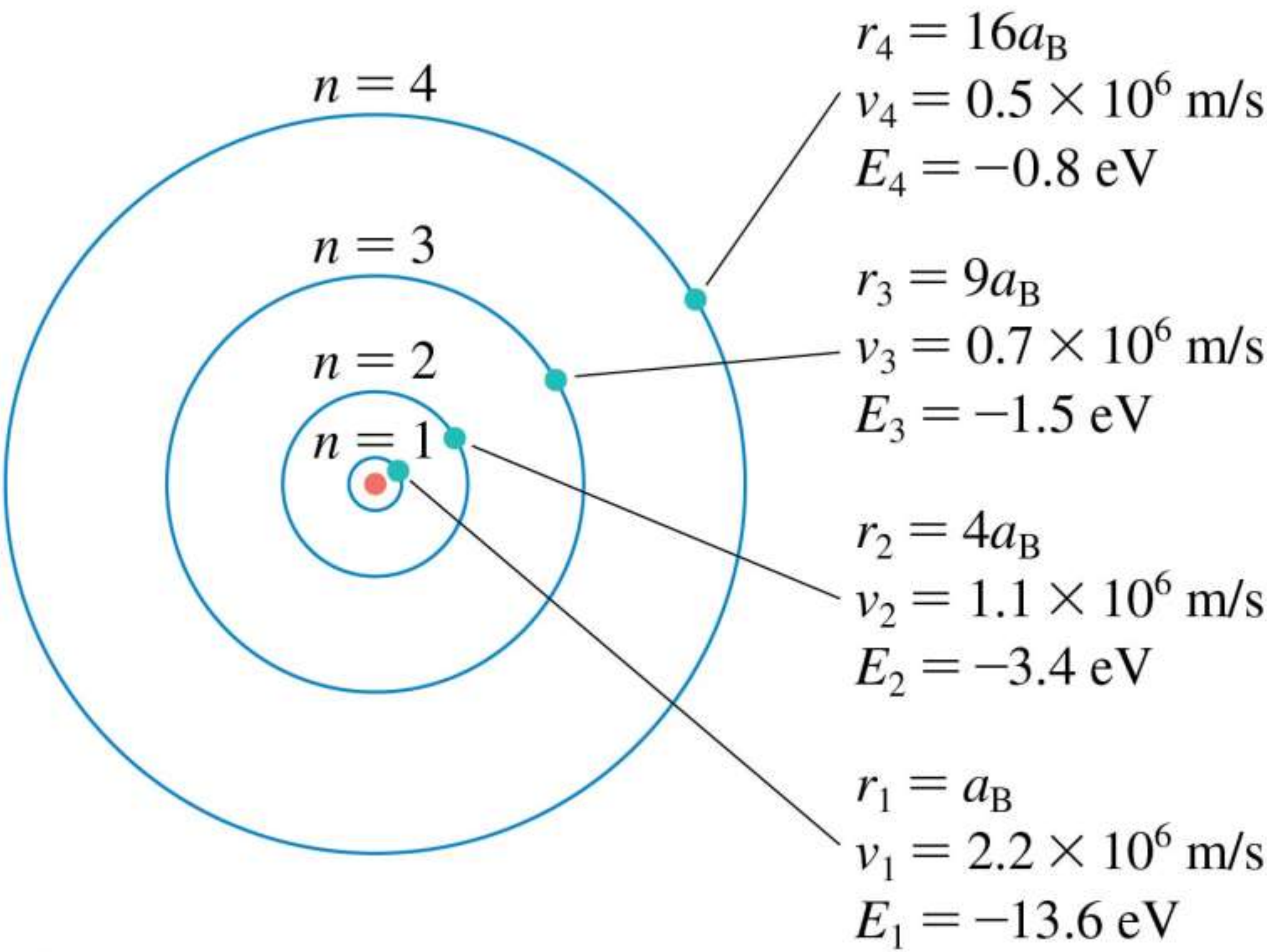
- The total energy of the electron in its orbit is

$$E_n = \tfrac{1}{2}mv_n^2 - \frac{e^2}{4\pi\epsilon_0 r_n} = \tfrac{1}{2}m\left(\frac{\hbar^2}{m^2 a_B^2 n^2}\right) - \frac{e^2}{4\pi\epsilon_0 n^2 a_B}$$

- So the energy levels of the stationary states of the hydrogen atom are

$$E_n = -\frac{13.60\text{ eV}}{n^2} \qquad n = 1, 2, 3, \ldots$$

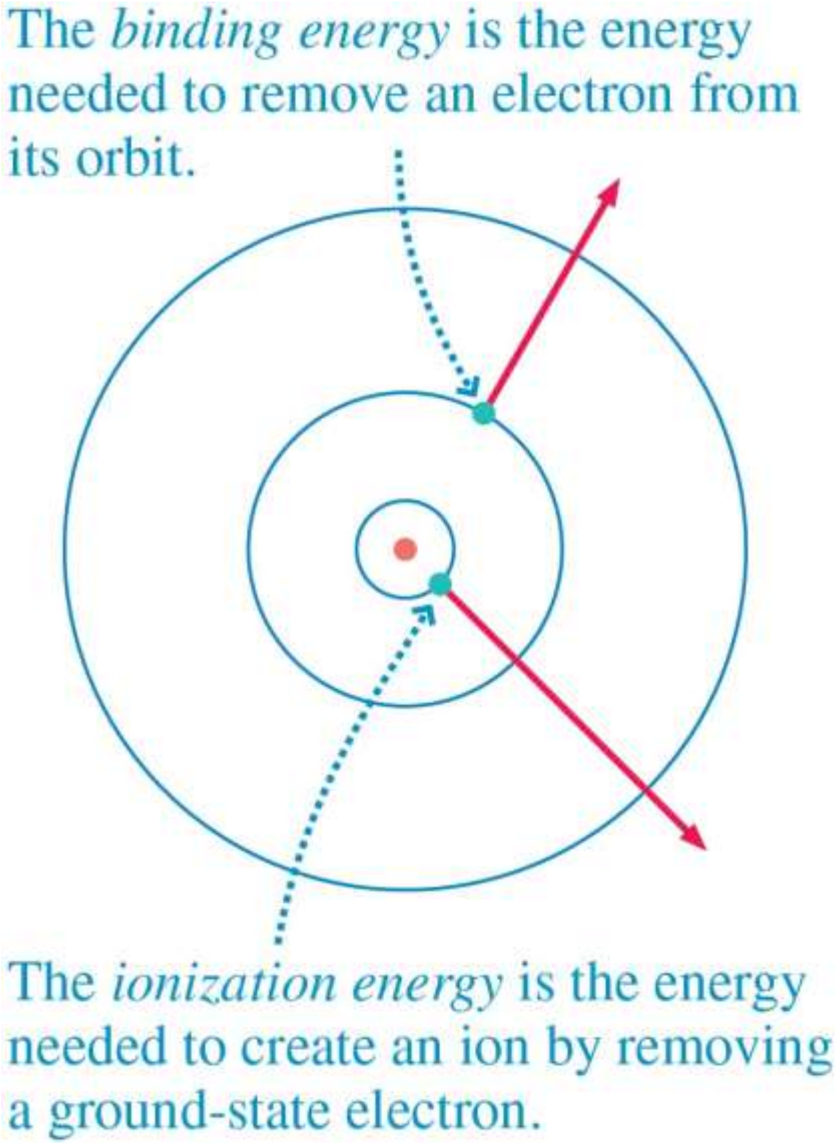
The Bohr Hydrogen Atom



Example 5 - Can an electron in a hydrogen atom have a speed of 3.60×10^5 m/s? If so, what are its energy and the radius of its orbit? What about a speed of 3.65×10^5 m/s?

Binding Energy and Ionization Energy

- The orbital energy E_n of an electron in an atom is always negative.
- $|E_n|$ is called the **binding energy** of an electron in stationary state n.
- Removing an electron from an atom leaves behind a positive ion.
- Since nearly all atoms are in their ground state, $|E_1|$ is called the **ionization energy**.



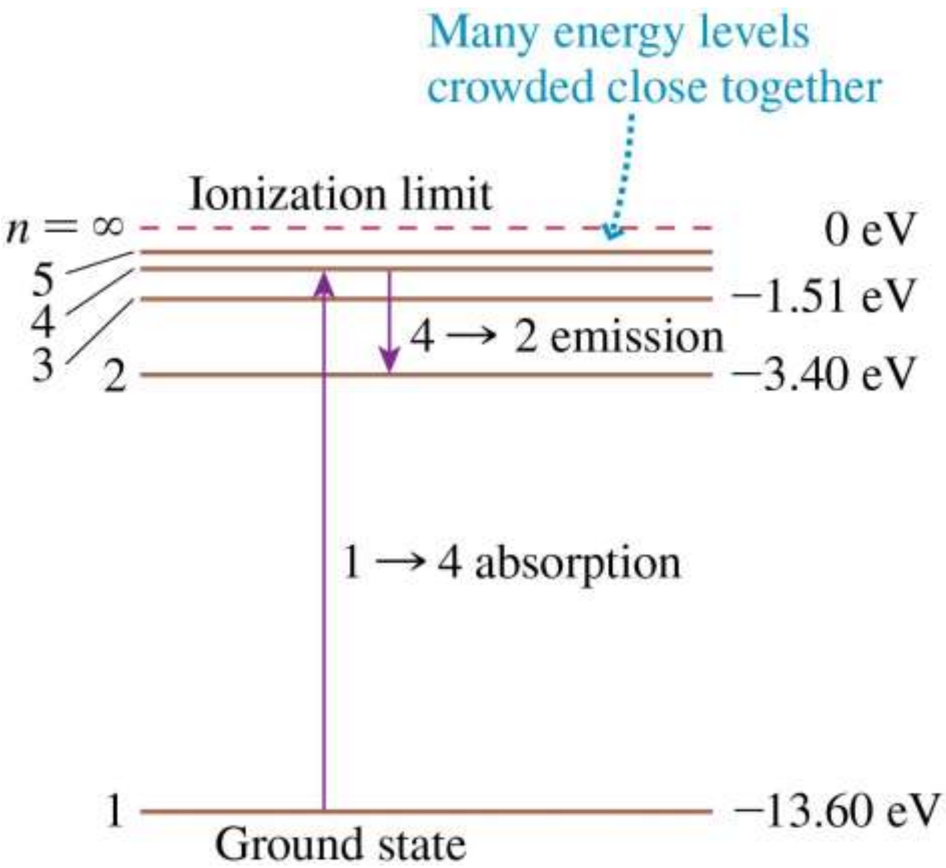
Quantization of Angular Momentum

- The angular momentum of a particle in circular motion, whether it is a planet or an electron, is $L = mvr$.
- Since v and r can only have certain values for an electron standing wave, the angular momentum is also quantized:

$$L = n\hbar \qquad n = 1, 2, 3, \ldots$$

- Quantization of angular momentum plays a major role in the behavior of more complex atoms, leading to the idea of electron shells that you likely have studied in chemistry.

The Hydrogen Energy-Level Diagram



- Shown is an energy-level diagram for the hydrogen atom.
- If $n > m$, an atom can emit a photon in an $n \rightarrow m$ transition or absorb a photon in an $m \rightarrow n$ transition.

The Hydrogen Spectrum

- The frequency of the photon emitted by an atom when its electron goes from the n to the m state is

$$f = \frac{\Delta E_{\text{atom}}}{h} = \frac{E_n - E_m}{h}$$

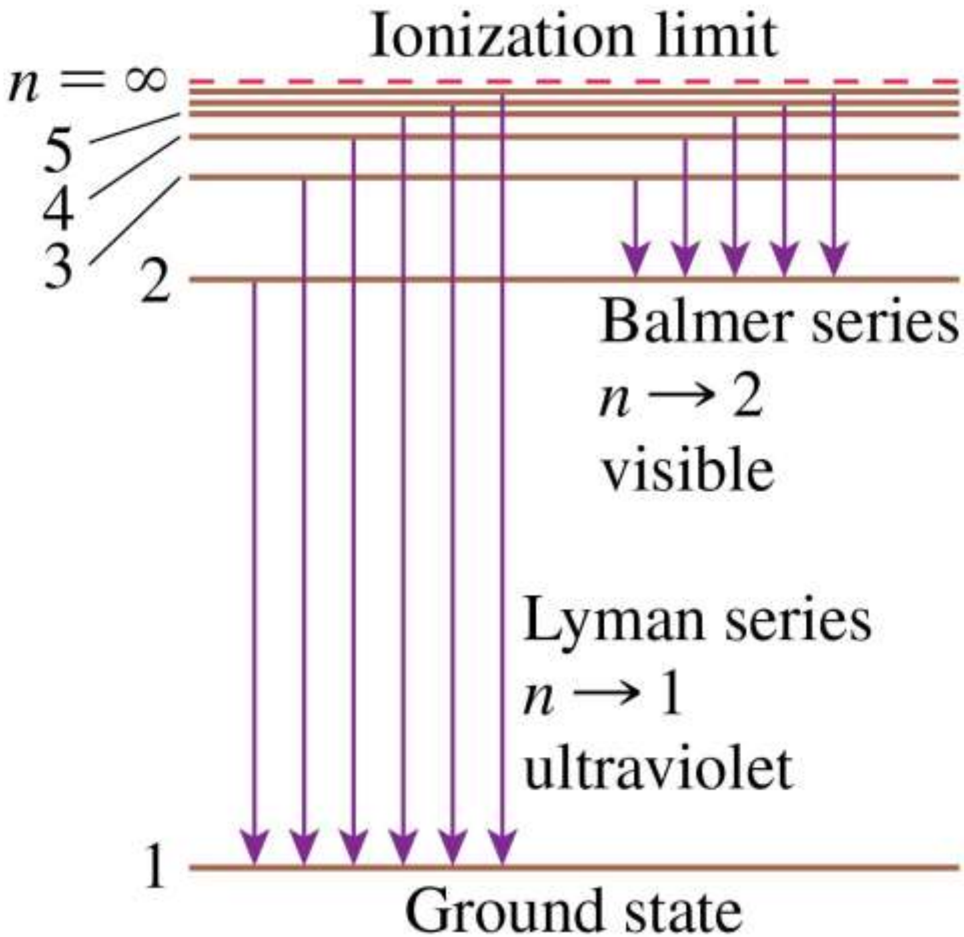
- We have solved for the allowed energies of the hydrogen atom, so we can predict all the possible wavelengths of the hydrogen spectrum:

$$\lambda_{n \to m} = \frac{\lambda_0}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad \begin{array}{l} m = 1, 2, 3, \ldots \\ n = \text{any integer greater than } m \end{array}$$

where

$$\lambda_0 = \frac{8\pi\epsilon_0 h c a_B}{e^2} = 9.112 \times 10^{-8}\text{ m} = 91.12\text{ nm}$$

The Hydrogen Spectrum



- The Bohr hydrogen atom correctly predicts the discrete spectrum of the hydrogen atom.
- The figure shows the *Balmer series* and the *Lyman series* transitions on an energy-level diagram.

Example 6 - Fluorescence is the absorption of light at one wavelength followed by emission at a longer wavelength. Suppose a hydrogen atom in its ground state absorbs an ultraviolet photon with a wavelength of 95.10 nm. Immediately after the absorption, the atom undergoes a quantum jump with $\Delta n=3$. What is the wavelength of the photon emitted in this quantum jump?

In-class Activity #1 - Whenever astronomers look at distant galaxies, they find that the light has been strongly absorbed at the wavelength of the $1 \rightarrow 2$ transition in the Lyman series of hydrogen. This absorption tells us that interstellar space is filled with vast clouds of hydrogen left over from the Big Bang. What is the wavelength of the $1 \rightarrow 2$ absorption in hydrogen?