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Baire measurability of (M,N)-Wright convex functions

Abstract. Let $I \subseteq \mathbb{R}$ be an open interval and $M, N : I^2 \longrightarrow I$ be means on I. Let $\varphi : I \longrightarrow \mathbb{R}$ be a solution of the functional equation

$$\varphi(M(x,y)) + \varphi(N(x,y)) = \varphi(x) + \varphi(y), \quad x, y \in I.$$

We give sufficient conditions on M, N and the function φ such that for every Baire measurable solution $f : I \longrightarrow \mathbb{R}$ of the functional inequality

$$f(M(x,y)) + f(N(x,y)) \leq f(x) + f(y), \quad x, y \in I,$$

the function $f \circ \varphi^{-1} : \varphi(I) \longrightarrow \mathbb{R}$ is convex.

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1. Introduction. Let $I \subseteq \mathbb{R}$ be a nonempty open interval. A two place function $M : I^2 \longrightarrow I$ such that

$$\min\{x, y\} \leq M(x,y) \leq \max\{x, y\}, \quad x, y \in I,$$

is called a mean on I. If for all $x, y \in I$, $x \neq y$ these inequalities are sharp, we call M to be a strict mean. In the present paper a function $f : I \longrightarrow \mathbb{R}$ is called to be (M, N)-Wright convex if

$$f(M(x,y)) + f(N(x,y)) \leq f(x) + f(y), \quad x, y \in I, \tag{1}$$

whenever $M, N : I^2 \longrightarrow I$ are means on I such that

$$M(x,y) + N(x,y) = x + y, \quad x, y \in I. \tag{2}$$

The problem of (M, N)-Wright convex functions was considerd by Zs. Páles [12], J. Matkowski and M. Wróbel [9]. In a case of M and N arithmetic means see also Matkowski [7], Gy. Maksa, K. Nikodem and Zs. Pales [10], Kominek [3].

A. Olbryś has shown in [13] that every Baire measurable, t-Wright convex function (see Definition 2.5) is a convex function.

In this connection we deal with the problem of giving the assumption on means M and N, guaranteeing that every Baire measurable, (M, N)-Wright convex function is convex.

Our method is based on the paper M. Lewicki [4].

2. Notions and lemmas. In this section we will recall basic notions.

By $\mathbb{N}_0$ denote the set of all nonnegative integers, i.e. $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. We need the following

Lemma 2.1 *Suppose that $M, N : I^2 \longrightarrow I$ are continuous means on I, and at least one of them is strict. Let the functions $M_k, N_k : I^2 \longrightarrow I$, $k \in \mathbb{N}_0$, be defined by*

$$M_0(x,y) := M(x,y), \quad N_0(x,y) := N(x,y),$$
$$M_{k+1}(x,y) := M(M_k(x,y), N_k(x,y)), \quad N_{k+1}(x,y) := N(M_k(x,y), N_k(x,y)).$$

Then

1° *for every $k \in \mathbb{N}_0$, the functions M_k and N_k are continuous means on I;*

2° *the sequences $\{M_k\}_{k\in\mathbb{N}_0}$ and $\{N_k\}_{k\in\mathbb{N}_0}$ converge on I to the same function $K : I^2 \longrightarrow I$, i.e.*

$$\lim_{k\to+\infty} M_k(x,y) = K(x,y) = \lim_{k\to+\infty} N_k(x,y), \quad x, y \in I, \tag{3}$$

moreover K is a continuous mean on I;

3° *if M and N are strictly increasing in the first (second) variable, then M_k and N_k, for $k \in \mathbb{N}_0$ are strictly increasing in the first (second) variable.*

Proof The parts 1° and 2° has been shown in Matkowski [6] (see Lemma 2, p. 92). The proof of 3° is an easy induction. ■

As an immediate consequence we obtain

Corollary 2.2 *Suppose that $M, N : I^2 \longrightarrow I$ are continuous means on I, at least one of them is strict. Furthermore, assume that* (2) *holds. Let M_k and N_k, $k \in \mathbb{N}_0$, be defined as in* Lemma 2.1. *Then*

$$\lim_{k\to+\infty} M_k(x,y) = \frac{x+y}{2} = \lim_{k\to+\infty} N_k(x,y), \quad x, y \in I.$$

Definition 2.3 The function $f : I \longrightarrow \mathbb{R}$ is called convex if

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y), \quad x, y \in I,$$

for all $t \in [0, 1]$.

Definition 2.4 The function $f : I \longrightarrow \mathbb{R}$ is called J-convex (Jensen convex) if

$$f\Big(\frac{x+y}{2}\Big) \le \frac{f(x)+f(y)}{2}, \quad x, y \in I.$$

Definition 2.5 Let $t \in (0,1)$ be a fixed number. The function $f : I \longrightarrow \mathbb{R}$ is called t-Wright convex if the following condition

$$f(tx + (1-t)y) + f((1-t)x + ty) \le f(x) + f(y), \quad x, y \in I,$$

is fulfilled.

Remark 2.6 Fix $t \in (0,1)$. Notice that t-Wright convex functions are (M_t, M_{1-t})-Wright convex, where

$$M_t(x,y) := tx + (1-t)y, \quad x, y \in I.$$

In the sequel we will need the following corollary of the theorem of M.R. Mehdi [11] (see M. Kuczma [1], Theorem 2, p. 210),

Theorem 2.7 *Let $I \subseteq \mathbb{R}$ be an open set and $T \subseteq I$ be a second category set with the Baire property. Then every J-convex function, bounded above on T is continuous.*

In Lemma 3.2 we will use the following

Lemma 2.8 (see M. Lewicki [5], Lemma 3.1) *Let $I \subseteq \mathbb{R}$ be an open interval. Assume $M : I^2 \longrightarrow I$ is a strict continuous mean on I, such that, for every fixed $u, v \in I$, functions $M(u,\cdot)$ and $M(\cdot,v)$ are strictly increasing. Let a set $B \subseteq I$ be residual at I. Then, for all $\epsilon > 0$ and $x_0 \in I$ there exist $x, y \in K(x_0,\epsilon) \cap B$ such that $x_0 = M(x,y)$.*

As the Referee noticed, the assumption that the mean M is strict is superfluous in the above lemma. Indeed, if a function $M : I^2 \longrightarrow \mathbb{R}$ is reflexive, i. e. $M(x,x) = x$ for all $x \in I$, and M is strictly increasing with respect to each variable, then M is a strict mean (see J. Matkowski [8]).

3. Main result.

We call a subset $E \subseteq I$ a residual if the set $I \setminus E$ is of the first category.

Lemma 3.1 *Let $I \subseteq \mathbb{R}$ be an open interval if a set $E \subseteq I$ is residual, then the set $\tilde{E} := \{(u,v) \in E^2 : \frac{u+v}{2} \in E\}$ is residual at I^2.*

Proof We show that the set $I^2 \setminus \tilde{E}$ is of the first category. Consider the sequence of equivalent conditions:

$$(s,t) \in I^2 \setminus \tilde{E} \iff \neg((s,t) \in \tilde{E}) \iff \neg(s \in E \wedge t \in E \wedge \frac{s+t}{2} \in E) \iff$$
$$\iff s \in I \setminus E \vee t \in I \setminus E \vee \frac{s+t}{2} \in I \setminus E.$$

This implies that

$$I^2 \setminus \tilde{E} \subseteq (I \setminus E) \times I \cup I \times (I \setminus E) \cup \{(u,v) \in I^2 : \frac{u+v}{2} \in I \setminus E\}.$$

Obviously the sets $(I \setminus E) \times I$ and $I \times (I \setminus E)$ are of the first category. To complete the proof it is enough to show that the set

$$\{(u,v) \in I^2 : \frac{u+v}{2} \in I \setminus E\},$$

is of the first category. As the set $I \setminus E$ is of the first category, there exists a sequence $\{A_n\}_{n\in\mathbb{N}}$ of nowhere dense sets such that $I \setminus E = \bigcup\limits_{n\in\mathbb{N}} A_n$.

Hence

$$\{(u,v) \in I^2 : \frac{u+v}{2} \in I \setminus E\} = \{(u,v) \in I^2 : \frac{u+v}{2} \in \bigcup_{n\in\mathbb{N}} A_n\} =$$
$$= \bigcup_{n\in\mathbb{N}} \{(u,v) \in I^2 : \frac{u+v}{2} \in A_n\}.$$

Therefore, it is enough to show that the set $\{(u,v) \in I^2 : \frac{u+v}{2} \in A_n\}$ is nowhere dense, for every $n \in \mathbb{N}$.

Fix $n_0 \in \mathbb{N}$. Obviously

$$cl\{(u,v) \in I^2 : \frac{u+v}{2} \in A_{n_0}\} \subseteq \{(u,v) \in I^2 : \frac{u+v}{2} \in cl\, A_{n_0}\}. \tag{4}$$

We show that $int\, cl\{(u,v) \in I^2 : \frac{u+v}{2} \in A_{n_0}\} = \emptyset$. For an indirect argument assume that $int\, cl\{(u,v) \in I^2 : \frac{u+v}{2} \in A_{n_0}\} \neq \emptyset$. Then there exist non-degenerated open intervals $(a,b) \subseteq I$ and $(c,d) \subseteq I$ such that

$$(a,b) \times (c,d) \subseteq cl\{(u,v) \in I^2 : \frac{u+v}{2} \in A_{n_0}\}.$$

By inclusion (4)

$$(a,b) \times (c,d) \subseteq \{(u,v) \in I^2 : \frac{u+v}{2} \in cl\, A_{n_0}\}.$$

Hence

$$\frac{(a,b) + (c,d)}{2} \subseteq cl\, A_{n_0}.$$

Therefore $int\, cl A_{n_0} \neq \emptyset$, contrary to the definition of A_{n_0}. This completes the proof. ■

In the sequel we will need the following lemma. The proof is analogous to the proof of Theorem 2.3 the paper M. Lewicki [4].

Lemma 3.2 *Let $I \subseteq \mathbb{R}$ be an open interval and $M, N : I^2 \longrightarrow I$ be means on I, strictly increasing in each variable. Furthermore, suppose that M is continuous. Let $f : I \longrightarrow \mathbb{R}$ be (M,N)-Wright convex function. If $g : I \longrightarrow \mathbb{R}$ is a continuous function such that the set $R := \{x \in I : f(x) = g(x)\}$ is residual, then $f(x) = g(x)$ for all $x \in I$.*

Proof Fix $x_0 \in I$ and $\xi > 0$. We proof that

$$g(x_0) \le f(x_0). \tag{5}$$

By the continuity of g there exists $\delta > 0$ such that

$$|g(x_0) - g(x)| \le \xi, \quad x \in (x_0 - \delta, x_0 + \delta). \tag{6}$$

Define functions $M^{x_0}, N^{x_0} : I \longrightarrow I$ as

$$M^{x_0}(x) := M(x_0, x), \quad x \in I,$$

and

$$N^{x_0}(x) := N(x_0, x), \quad x \in I.$$

respectively.

Due to assumptions, the functions M^{x_0} and N^{x_0} are strictly increasing and continuous. It implies that functions $(M^{x_0})^{-1} : M^{x_0}((x_0-\delta, x_0+\delta)) \longrightarrow (x_0-\delta, x_0+\delta)$ and $(N^{x_0})^{-1} : N^{x_0}((x_0 - \delta, x_0 + \delta)) \longrightarrow (x_0 - \delta, x_0 + \delta)$ are strictly increasing and continuous. Define a set $B := M^{x_0}((x_0 - \delta, x_0 + \delta)) \cap N^{x_0}((x_0 - \delta, x_0 + \delta)) \cap (x_0 - \delta, x_0 + \delta)$. Obviously B is an open neighbourhood of the point x_0. The set $B \cap R$ is residual in B, so the set $(M^{x_0})^{-1}(B \cap R) \cap (N^{x_0})^{-1}(B \cap R)$ is residual in $A := (M^{x_0})^{-1}(B) \cap (N^{x_0})^{-1}(B)$. As A is an open neighbourhood of the point x_0, the set $(M^{x_0})^{-1}(B \cap R) \cap (N^{x_0})^{-1}(B \cap R) \cap R$ is nonempty.

Taking $x \in (M^{x_0})^{-1}(B \cap R) \cap (N^{x_0})^{-1}(B \cap R) \cap R$ we get $x \in R$ and $M^{x_0}(x), N^{x_0}(x) \in B \cap R \subseteq (x_0 - \delta, x_0 + \delta)$.

Now, by (1) and by the definition of the set R

$$\begin{aligned} g(M(x_0, x)) + g(N(x_0, x)) = f(M(x_0, x)) + f(N(x_0, x)) \le \\ \le f(x_0) + f(x). \end{aligned}$$

Taking into account (6) we get

$$\begin{aligned} g(x_0) - \xi + g(x_0) - \xi \le g(M(x_0, x)) + g(N(x_0, x)) \le \\ f(x_0) + g(x) \le f(x_0) + g(x_0) + \xi. \end{aligned}$$

Finally we have

$$g(x_0) \le f(x_0) + 3\xi.$$

Letting $\xi \longrightarrow 0+$ we get (5).

Now we show that the inequality

$$f(x_0) \le g(x_0), \tag{7}$$

holds.

By Lemma 2.8 for arbitrary $n \in \mathbb{N}$ there exist $x_n, y_n \in R \cap (x_0 - \frac{1}{n}, x_0 + \frac{1}{n})$ such that $M(x_n, y_n) = x_0$. Now, due to (5), (1) and definition of the set R, we get

$$f(x_0) + g(N(x_n, y_n)) \leq f(M(x_n, y_n)) + f(N(x_n, y_n)) \leq \\ f(x_n) + f(y_n) = g(x_n) + g(y_n).$$

Hence, by continuity of functions g and N we have

$$f(x_0) + g(x_0) = f(x_0) + \lim_{n \to +\infty} g(N(x_n, y_n)) \leq \\ \lim_{n \to +\infty} g(x_n) + \lim_{n \to +\infty} g(y_n) = 2g(x_0).$$

That completes the proof of (7), and the proof of the Lemma. ∎

Theorem 3.3 *Let $I \subseteq \mathbb{R}$ be an open interval and $M, N : I^2 \longrightarrow I$ be means on I, strictly increasing in each variable. Furthermore, suppose that M is continuous. If $f : I \longrightarrow \mathbb{R}$ is an (M, N)-Wright convex and Baire measurable function, then f is convex.*

Proof By Nikodym's Theorem there exists a residual set $E \subseteq I$, such that $f|_E$ is continuous. Let set $\tilde{E}$ be defined as in Lemma 3.1. We will show that

$$(u, v) \in \tilde{E} \Longrightarrow f\Big(\frac{u+v}{2}\Big) \leq \frac{f(u) + f(v)}{2}. \tag{8}$$

Fix $(s, t) \in \tilde{E}$ and $\epsilon > 0$. By the continuity of the function $f|_E$, there exist $U(t, \delta) = (t - \delta, t + \delta)$ and $U(\frac{s+t}{2}, \delta) = (\frac{s+t}{2} - \delta, \frac{s+t}{2} + \delta)$ such that

$$|f(t) - f(x)| < \frac{2}{3}\epsilon, \quad x \in E \cap U(t, \delta), \tag{9}$$

and

$$\Big|f\Big(\frac{s+t}{2}\Big) - f(x)\Big| < \frac{2}{3}\epsilon, \quad x \in E \cap U\Big(\frac{s+t}{2}, \delta\Big). \tag{10}$$

Let M_k and N_k, for $k \in \mathbb{N}_0$ be defined as in Lemma 2.1. Due to Corollary 2.2 we have

$$\lim_{n \to +\infty} M_n(s, t) = \frac{s+t}{2} = \lim_{n \to +\infty} N_n(s, t)$$

Hence, there exists $n_0 \in \mathbb{N}$ such that $M_{n_0}(s, t) \in U(\frac{s+t}{2}, \delta)$ and $N_{n_0}(s, t) \in U(\frac{s+t}{2}, \delta)$. By the continuity of the functions M_{n_0} and N_{n_0} (see Lemma 2.1, p. 1°) there exists $0 < \eta < \delta$ such that

$$M_{n_0}(s, U(t, \eta)) \in U\Big(\frac{s+t}{2}, \delta\Big), \qquad N_{n_0}(s, U(t, \eta)) \in U\Big(\frac{s+t}{2}, \delta\Big). \tag{11}$$

Now, by Lemma 2.1 (p. 3°), the functons $M_{n_0}(s,\cdot) =: M_{n_0}$ and $N_{n_0}(s,\cdot) =: N_{n_0}$ are strictly increasing and continuous. Consider the inverse functions $(M_{n_0})^{-1} : M_{n_0}(s, U(t,\eta)) \longrightarrow U(t,\eta)$ and $(N_{n_0})^{-1} : N_{n_0}(s, U(t,\eta)) \longrightarrow U(t,\eta)$, respectively. Obviously both functions are strictly increasing and continuous.

Since E is residual in the set $M_{n_0}(s, U(t,\eta))$, the set $M_{n_0}^{-1}(E)$ is residual in $U(t,\eta)$. Analogously, $N_{n_0}^{-1}(E)$ is residual in $U(t,\eta)$. Consequently, a set $B := M_{n_0}^{-1}(E) \cap M_{n_0}^{-1}(E) \cap E$ is residual in $U(t,\eta)$. Thus B is nonempty.

Take $t_0 \in B$. By (11) we have: $t_0 \in E \cap U(\frac{s+t}{2}, \delta)$ and $M_{n_0}(s,t_0)$, $N_{n_0}(s,t_0) \in E \cap U(t,\eta) \subseteq E \cap U(t,\delta)$.

Now, according to (1), (9) and (10), we get

$$f\Big(\frac{s+t}{2}\Big) - \frac{2}{3}\epsilon + f\Big(\frac{s+t}{2}\Big) - \frac{2}{3}\epsilon \leq f(M_{n_0}(s,t_0)) + f(N_{n_0}(s,t_0)) \leq$$
$$f(s) + f(t_0) \leq f(s) + f(t) + \frac{2}{3}\epsilon.$$

Hence,

$$f\Big(\frac{s+t}{2}\Big) \leq \frac{f(s)+f(t)}{2} + \epsilon,$$

Now, letting $\epsilon \longrightarrow 0+$ we get assertion (8).

In virtue of Theorem 2 (see M. Kuczma [1], p. 459) jointly with Example III (M. Kuczma [1], p. 438) and comments (M. Kuczma [1], p. 439, line 24-26), there exists J-convex function $g : I \longrightarrow \mathbb{R}$ such that the set $R := \{x \in I : g(x) = f(x)\}$ is residual in I.

Now, notice that $R \cap E$ is a residual set in I. Fix $x_0 \in R \cap E$. As the function $f|_E$ is continuous at the point x_0, there exists $\gamma > 0$ such that $g|_{R\cap E} = f|_{R\cap E}$ is bounded on $U(x_0,\gamma) \cap R \cap E$. In view of Theorem 2.7 g is continuous function and, by J-convexity, g is convex. Consequently, by Lemma 3.2 the function f is convex.$\blacksquare$

J. Matkowski and M. Wróbel in [9] generalized a result of Zs. Pales [12] to the following

Theorem 3.4 *Let $M, N : I^2 \longrightarrow I$ be continuous, strict means on I, and suppose that $\varphi : I \longrightarrow \mathbb{R}$ is a non-constant and continuous solution of equation*

$$\varphi(M(x,y)) + \varphi(N(x,y)) = \varphi(x) + \varphi(y), \quad x, y \in I. \tag{12}$$

Then φ is one-to-one, and for every lower semicontinuous function $f : I \longrightarrow \mathbb{R}$ satisfying inequality (1), *the function $f \circ \varphi^{-1}$ is convex on $\varphi(I)$.*

Our next aim is to generalize the above result to the case of strictly increasing in each variable mean on I. Our result simultaneously improves the main theorem of M. Lewicki [5].

Definition 3.5 Let $I \subseteq \mathbb{R}$ be an open interval and $M : I^2 \longrightarrow I$ be a mean on I. Assume that $\varphi : I \longrightarrow \mathbb{R}$ is a strictly monotone function. By $M_\varphi : \varphi(I)^2 \longrightarrow \varphi(I)$ we denote a mean on $\varphi(I)$ defined by

$$M_\varphi(x,y) := \varphi(M(\varphi^{-1}(x), \varphi^{-1}(y))), \quad x, y \in I.$$

Corollary 3.6 *Let $I \subseteq \mathbb{R}$ be an open interval and $M, N : I^2 \longrightarrow I$ be means on I. Furthermore, suppose that M, N are continuous and strictly increasing in each variable. Assume that there exists a nonconstatnt, continuous solution $\varphi : I \longrightarrow \mathbb{R}$ of equation* (12). *If $f : I \longrightarrow \mathbb{R}$ is a Baire measurable function satisfying inequality* (1)*, then the function $f \circ \varphi^{-1} : \varphi(I) \longrightarrow \mathbb{R}$ is convex on $\varphi(I)$.*

Our proof is based on the papers of J. Matkowski and M. Wróbel [9], J. Matkowski [6] and M. Lewicki [4].

Proof First we show that the function φ is strictly monotone. With the notation of Lemma 2.1, there exists a continuous mean $K : I^2 \longrightarrow I$ on I such that (3) holds. By (12) and obvious induction we get

$$\varphi(M_k(x, y)) + \varphi(N_k(x, y)) = \varphi(x) + \varphi(y), \quad x, y \in I,\ k \in \mathbb{N}_0.$$

Letting $k \longrightarrow +\infty$ and making use of the continuity of φ, and (3) we hence get

$$2\varphi(K(x, y)) = \varphi(x) + \varphi(y), \quad x, y \in I.$$

The rest of the proof that φ is one-to-one reads as in J. Matkowski and M. Wróbel [9] (see p. 10, lines 19-27).

Now, consider functions M_φ and N_φ. Directly from (12) we get

$$M_\varphi(x, y) + N_\varphi(x, y) = x + y, \quad x, y \in \varphi(I),$$

Hence, by above and assumptions on M, the mean M_φ is strict, continuous and strictly increasing in each variable mean on $\varphi(I)$. Now we show that the function $f \circ \varphi^{-1}$ is (M_φ, N_φ)-Wright convex.

Fix $x, y \in \varphi(I)$ and put $s := \varphi^{-1}(x)$ and $t := \varphi^{-1}(y)$

$$\begin{aligned} &(f \circ \varphi^{-1})(M_\varphi(x, y)) + (f \circ \varphi^{-1})(N_\varphi(x, y)) = \\ &(f \circ \varphi^{-1})(\varphi(M(\varphi^{-1}(x), \varphi^{-1}(y)))) + (f \circ \varphi^{-1})(N(\varphi^{-1}(x), \varphi^{-1}(y)))) = \\ &f(M(s, t)) + f(N(s, t)) \le f(s) + f(t) \le (f \circ \varphi^{-1})(x) + (f \circ \varphi^{-1})(y). \end{aligned}$$

Applying Theorem 3.3 we get the thesis. ∎

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