

Sergio Bittanti ed.

COUNT RICCATI
AND
THE EARLY DAYS
OF THE
RICCATI
EQUATION



Pitagora Editrice Bologna

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Milano

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Alexander Longhi del.

Innocent Alexandri Sculp. Venetis

PREFACE

For many years, I studied the Riccati Equation knowing only a little about its origin. The same probably applies to other scientists who analyzed the equation, here and there in the world.

During the organization activity of the workshop on "The Riccati Equation in Control, Systems, and Signals" (Como, Italy, June 26-28, 1989), I came across some historical papers on Count Riccati and his equation(s). This raised my interest, so that I read more and more papers (many of which in Italian or Latin) on the subject.

In the attempt of serving all those who are too much involved in analytic research to spend time on historical issues, I eventually decided to report the results of my reading in this booklet.

Here, the reader will first find the original paper by Jacopo Francesco Riccati published in 1724. In this paper, Riccati reduces a second order differential equation to a first order one, an equation which we would now call a Riccati Equation. The known term of this equation is a power of the independent variable. At the end of the paper, the question was posed of

finding all exponents of the known term for which the separation of variables was possible. From various documents, it is apparent that Riccati already knew infinitely many cases for which the separation of variables was actually possible. His main concern was looking for the most general solution of the problem.

Riccati's problem was considered by many of the most famous mathematicians. Here, two papers are reprinted; the first one is an Euler's paper of 1764, while the second one is due to Liouville and was published in 1841.

Finally, a paper summarizing my views on the history and prehistory of the Riccati equation can be found.

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Sergio Bittanti
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