

## thy.1 Computably Enumerable Sets

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sec

**Definition thy.1.** A set is *computably enumerable* if it is empty or the range of a computable function.

**Historical Remarks** Computably enumerable sets are also called *recursively enumerable* instead. This is the original terminology, and today both are commonly used, as well as the abbreviations “c.e.” and “r.e.”

You should think about what the definition means, and why the terminology is appropriate. The idea is that if  $S$  is the range of the computable function  $f$ , then [explanation](#)

$$S = \{f(0), f(1), f(2), \dots\},$$

and so  $f$  can be seen as “enumerating” the elements of  $S$ . Note that according to the definition,  $f$  need not be an increasing function, i.e., the enumeration need not be in increasing order. In fact,  $f$  need not even be injective, so that the constant function  $f(x) = 0$  enumerates the set  $\{0\}$ .

Any computable set is computably enumerable. To see this, suppose  $S$  is computable. If  $S$  is empty, then by definition it is computably enumerable. Otherwise, let  $a$  be any element of  $S$ . Define  $f$  by

$$f(x) = \begin{cases} x & \text{if } \chi_S(x) = 1 \\ a & \text{otherwise.} \end{cases}$$

Then  $f$  is a computable function, and  $S$  is the range of  $f$ .

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## Bibliography