

2- Pulse Digital Modulation

a) Pulse Code Modulation (PCM)

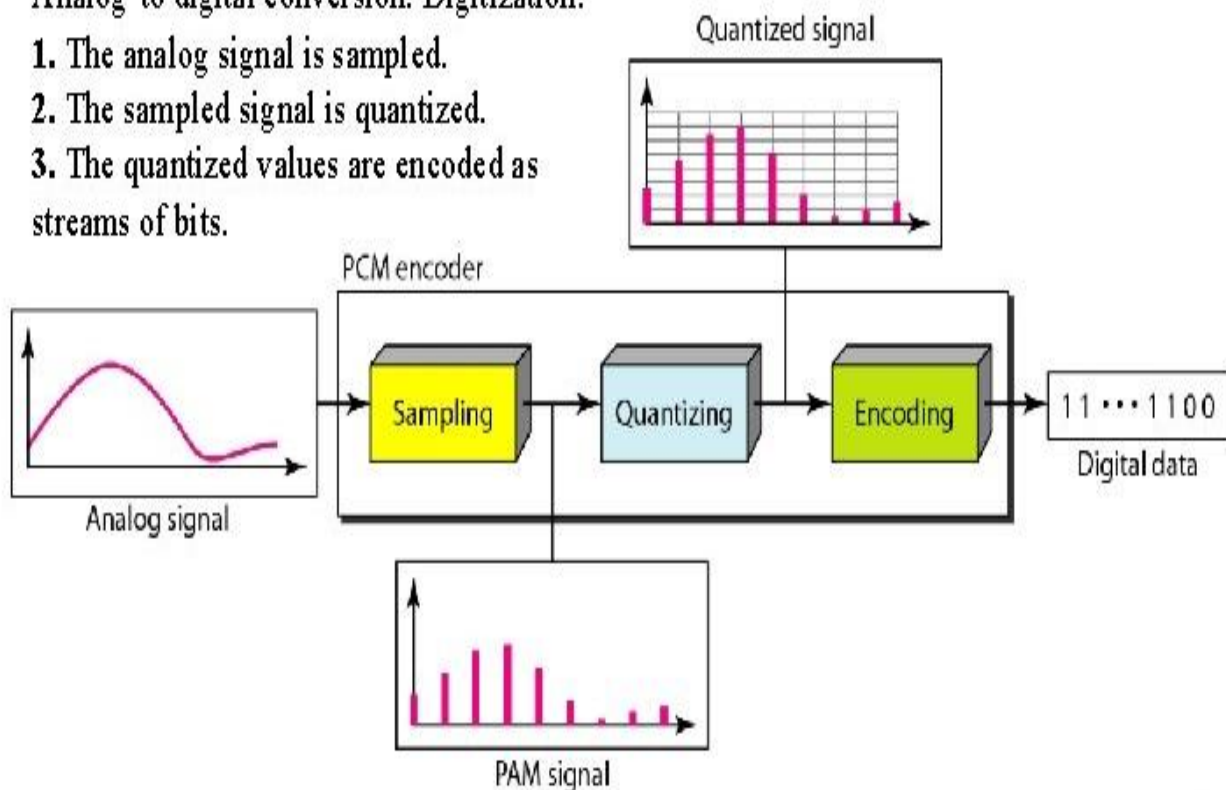
Pulse-code modulation (PCM) is used to digitally represent sampled analog signals. It is the standard form of digital audio in computers, CDs, digital telephony and other digital audio applications. The amplitude of the analog signal is sampled at uniform intervals and each sample is quantized to its nearest value within a predetermined range of digital levels

PCM is the most useful and widely used of all the pulse modulations mentioned. As shown in figure below, PCM basically is a tool for converting an analog signal into a digital signal (A/D conversion). An **analog** signal is characterized by an amplitude that can take on any value over a continuous range. This means that it can take on an infinite number of values. On the other hand, **digital** signal amplitude can take on only a finite number of values.

Pulse Code Modulation.

Analog to digital conversion. Digitization.

1. The analog signal is sampled.
2. The sampled signal is quantized.
3. The quantized values are encoded as streams of bits.



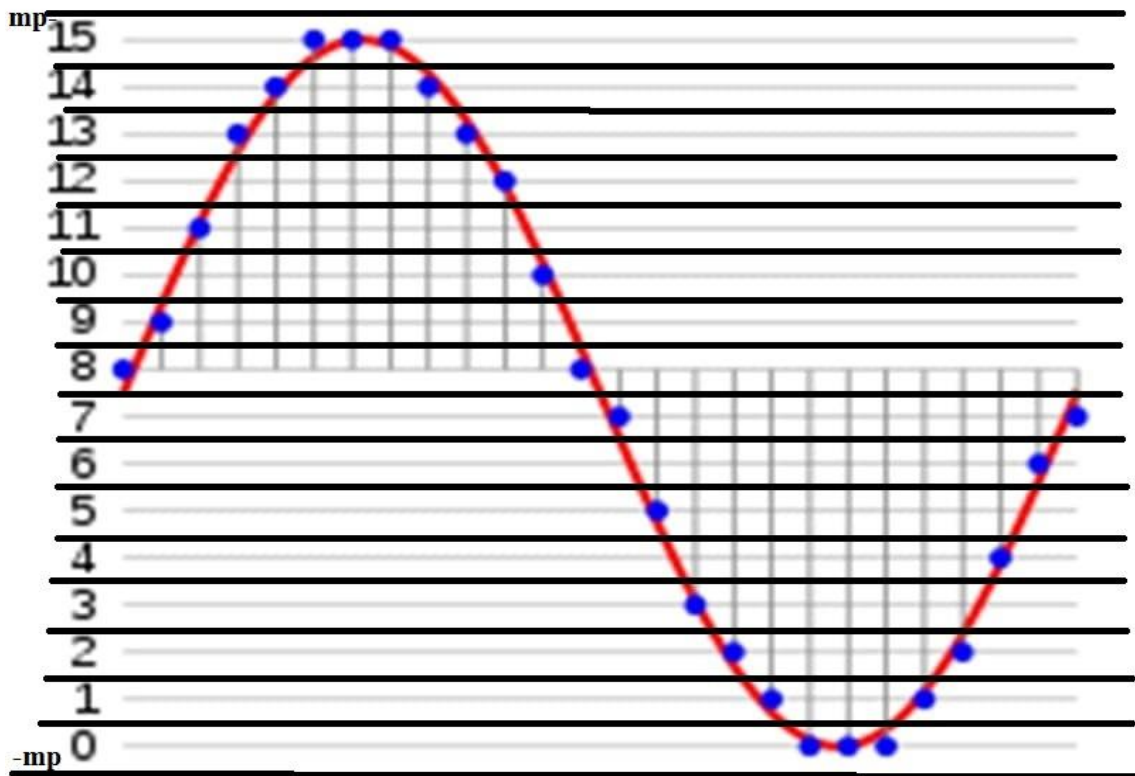


Quantization

Quantization is the process of mapping continuous infinite values to a smaller set of discrete finite values. When an information signal is pulse amplitude modulated, it becomes discrete in time only. It remains analogue in amplitudes since all the values within the specified range are allowed. PAM signal is said to be quantized when each pulse of the PAM signal is adjusted in amplitude to coincide with the nearest level within a finite set.

This is achieved by dividing the distance between min and max into ***L zones*** (***Quantized zone***), each of height Δv .

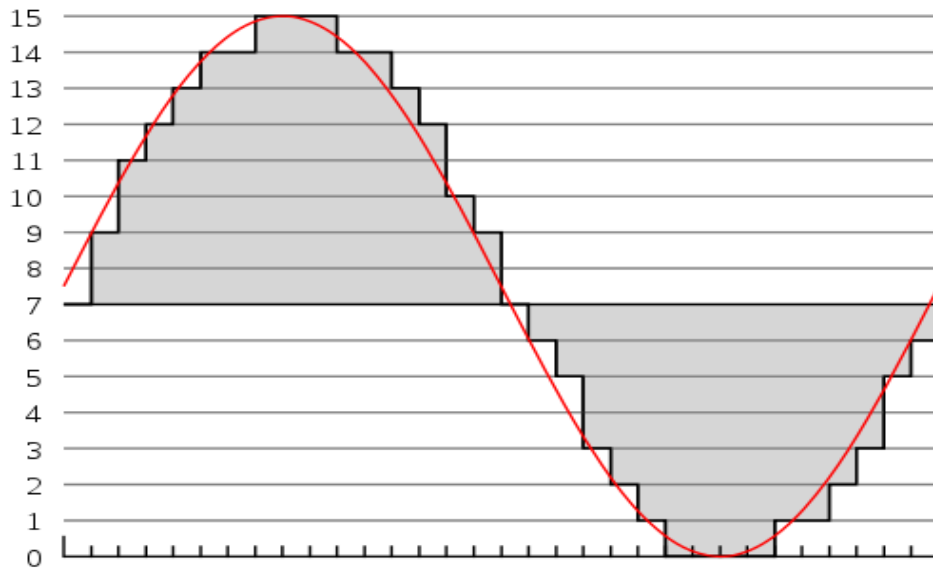
$$\Delta v = \frac{m_{\max} - m_{\min}}{L} = \frac{m_p - (-m_p)}{L} = \frac{2m_p}{L}$$



Quantized Levels

The ***midpoint*** of each zone is assigned a value from ***0 to L-1*** (resulting in ***L values***)

Each sample falling in a zone is then ***approximated*** to the value of the midpoint $\frac{\Delta v}{2}$.



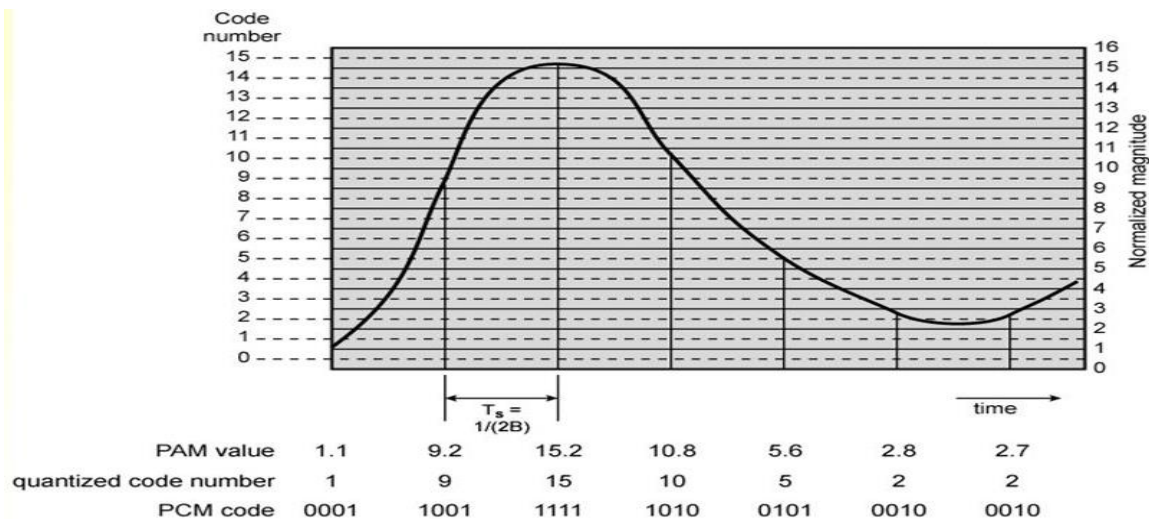
Encoding

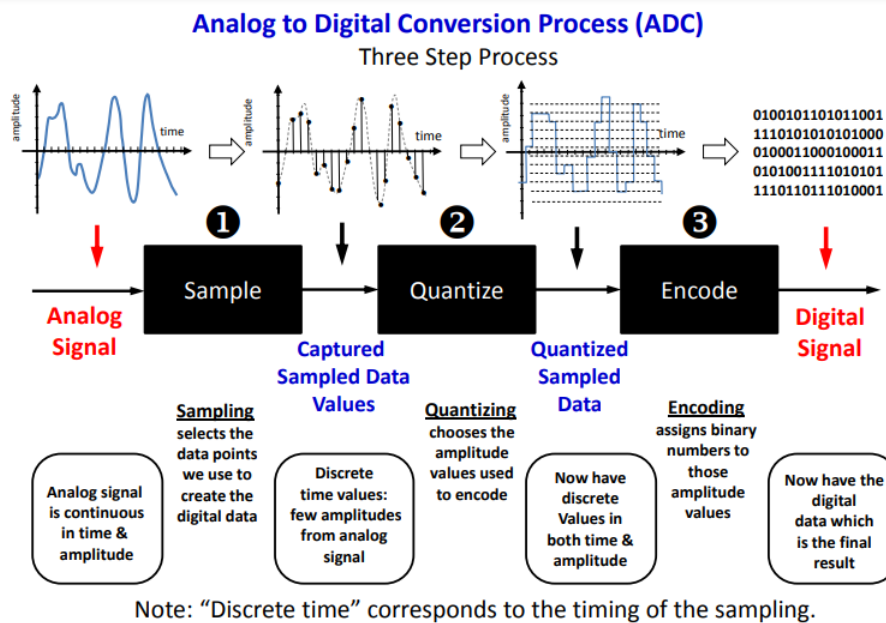
Each zone is then assigned a binary code. The number of bits required to encode the zones, or the number of bits per sample as it is commonly referred to, is obtained as follows:

$$L = 2^n \quad \text{or} \quad n = \log_2 L$$

The 8 zone (or level) codes are therefore: 000, 001, 010, 011, 100, 101, 110, and 111. This code, formed by binary representation of the 8 decimal digits from 0 to 7, is known as the **natural binary code (NBC)**.

Also coding for the case of $L = 16$ was shown in figure below. This code, formed by binary representation of the 16 decimal digits from 0 to 15: 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110 and 1111





Bit rate (f_b) and bandwidth (B_{PCM}) requirements of PCM

- **Bit rate** of a PCM signal can be calculated from the number of bits per sample \times the sampling rate

$$f_b = n f_s$$

- **Bandwidth** of binary encoded PCM wave form for no aliasing $f_s \geq 2B$

$$B_{PCM} \geq nB$$

Or

$$B_{PCM} \geq \frac{f_b}{2}$$

Minimum bandwidth

$$B_{min,PCM} = nB = \frac{f_b}{2}$$

Quantization Error

- When a signal is quantized, we introduce an error the coded signal is an approximation of the actual amplitude value.
- The difference between actual $m(t)$ and coded value (midpoint) $\overline{m}(t)$ is referred to as the quantization error $q(t)$.

$$q(t) = \overline{m}(t) - m(t)$$

- **The more zones (more quantization level), the smaller Δ which results in smaller errors.**



- BUT, the more zones the more bits required to encode the samples higher bit rate f_b

$$f_b = n f_s$$

Because $\overline{q(t)^2}$ is the mean square value or **power of the quantization noise**, we shall denote it by N_q ,

$$N_q = \overline{q(t)^2} = \frac{(\Delta v)^2}{12}$$

$$N_q = \overline{q(t)^2} = \frac{m_p^2}{3L}$$

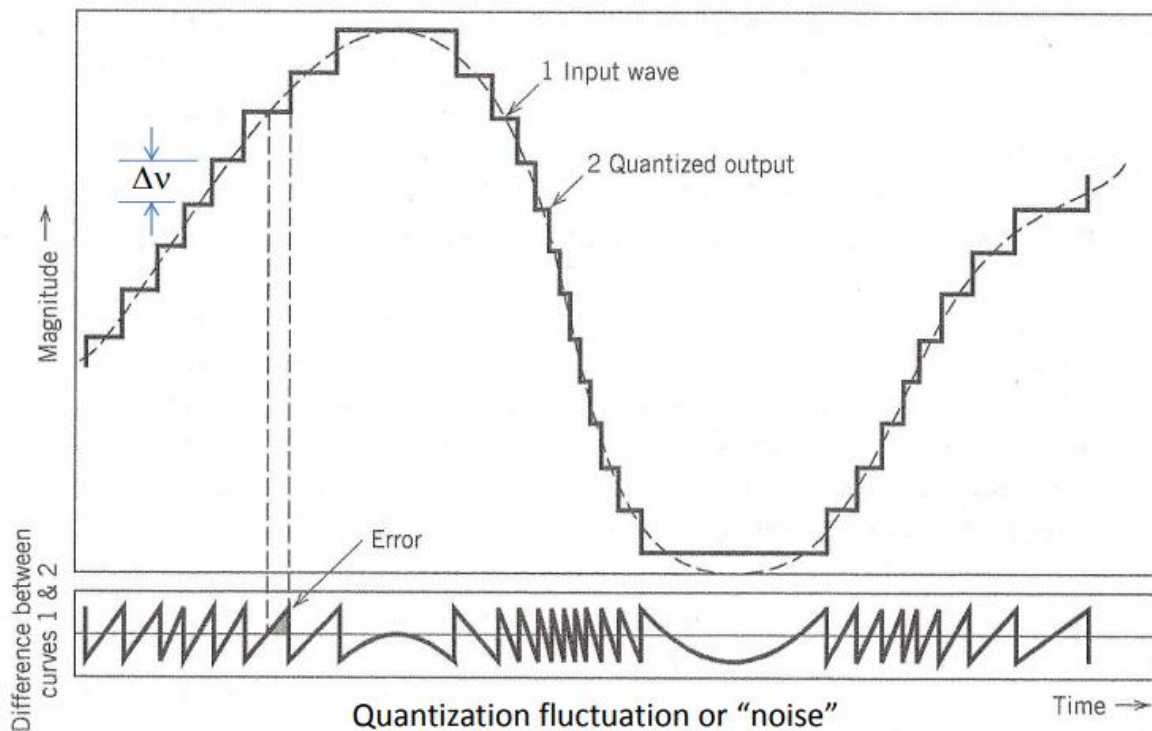
Since the power of the message signal $m(t)$ is $\overline{m(t)^2}$ then SN_{QR}

$$S_o = \overline{m(t)^2}$$

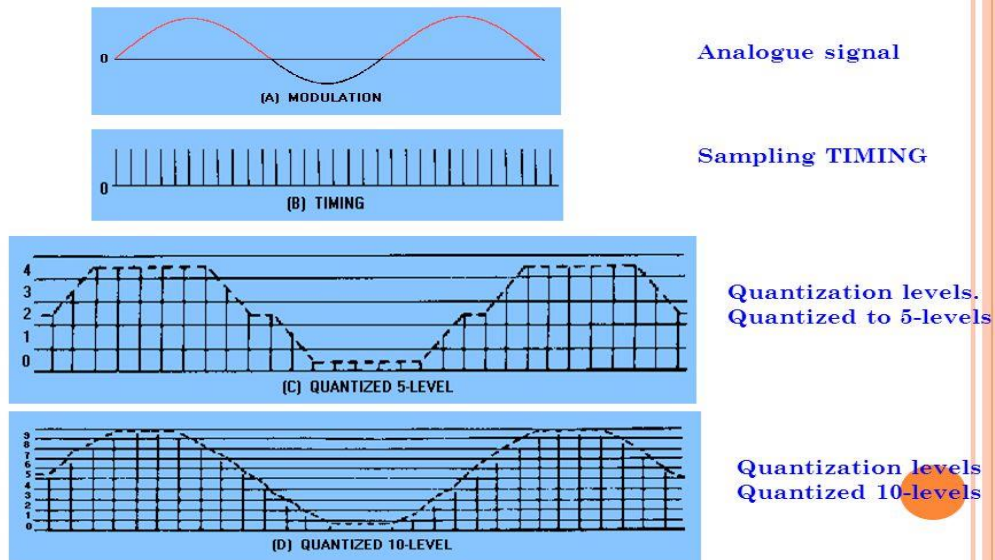
$$N_o = N_q = \frac{m_p^2}{3L^2}$$

$$\frac{S_o}{N_o} = 3L^2 \frac{\overline{m(t)^2}}{m_p^2}$$

$$\left(\frac{S_o}{N_o}\right)_{dB} = 10 \log_{10} \left(\frac{S_o}{N_o}\right)$$



QUANTIZATION EXAMPLE



Two approaches keep SN_QR fixed for all sample values

1. The quantization levels follow a logarithmic curve. Smaller Δ 's at lower amplitudes and larger Δ 's at higher amplitudes. (**using non-uniform quantizing**).
2. **Companding**: The sample values are **compressed** at the sender into logarithmic zones, and then **expanded** at the receiver. The zones are fixed in height.

Quantization Types

- 1- Uniform Quantization: the representation level are equally spaced (uniformly spaced)
- 2- Non- Uniform Quantization: the representation level have variable spacing from one another

