

① Optimal basis diff from text

② Kruskal's algo, MST of G , cycle detection, unique MST

③ Task scheduling

$\rightarrow (s_i, t_i)$, non overlapping
 $t_i \leq s_j$ or $t_j \leq s_i$

\rightarrow Minimize # of machines

\rightarrow Order by s_i

④ Activity selection

$\rightarrow (s_i, t_i)$, compatible

\rightarrow Maximum size subset of mutually compatible activities

$t_1 \leq t_2 \leq t_3 \leq \dots \leq t_n$

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
t_i	4	5	6	7	9	9	10	11	12	14	16

$\{a_1, a_4, a_8, a_{11}\}$ or $\{a_2, a_4, a_9, a_{11}\}$

are both ~~largest~~ optimal

\rightarrow optimal substructure

$S_{i,j} = \{ \text{activities that start after } a_i \text{ finishes and end$

by time } a_j \text{ starts } \}

we want to find a maximum set of mutually compatible activities in $S_{i,j}$. Let $a_{i_2} \in S_{i,j}$.

$$A_{i_2, k} = A_{i,j} \cap S_{i_2, k}$$

$$A_{i, j_2} = A_{i,j} \cap S_{i, j_2}$$

$$|A_{i_2, k}| + |A_{i, j_2}| = |A_{i,j}| + 1$$

Let $C(i, j)$ = optimal soln for S_{ij}

$$C(i, j) = \begin{cases} 0, & \text{if } S_{ij} = \emptyset \\ \max_{a_k \in S_{ij}} \{ C(i, k) + C(k, j) + 1, & \text{if } S_{ij} \neq \emptyset \end{cases}$$

Greedy choice:

→ choose activity with earliest finish time

Let $S_k = \{a_i \in S : s_i \leq f_k\}$

S_k is the only subproblem to solve if a_i is to be included.

But is a_i necessary in the optimal solution?

Thm: a_i is included in any maximum-size subset of mutually compatible activities.

Proof: Exchange argument

Elements of greedy strategy

→ Optimal substructure

→ Greedy choice

→ safe to make greedy choice

→ Greedy vs dynamic programming