Physics 212

Lecture 3

Today's Concepts:

Electric Flux and Field Lines

⇓

Gauss's Law

Introduce a new constant: ε_0

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

$$k \equiv \frac{1}{4\pi\varepsilon_o}$$

$k = 9 \times 10^9$ N m^2 / C^2

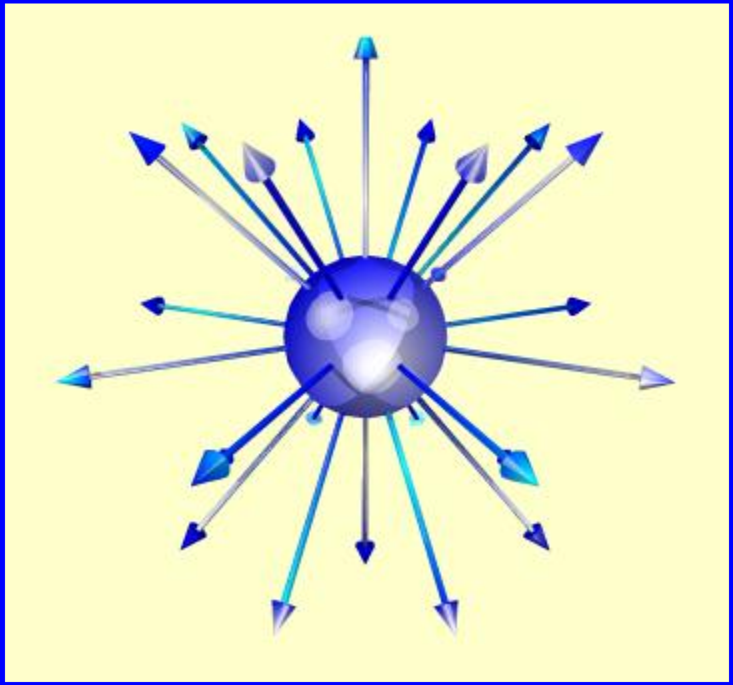
$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 / N m^2

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$$

Plan for Today

- A little more about electric field lines
- Electric field lines and flux
  - *An analogy*
- Introduction to Gauss's Law
  - *Gauss' Law will make it easy to calculate electric fields for some geometries.*

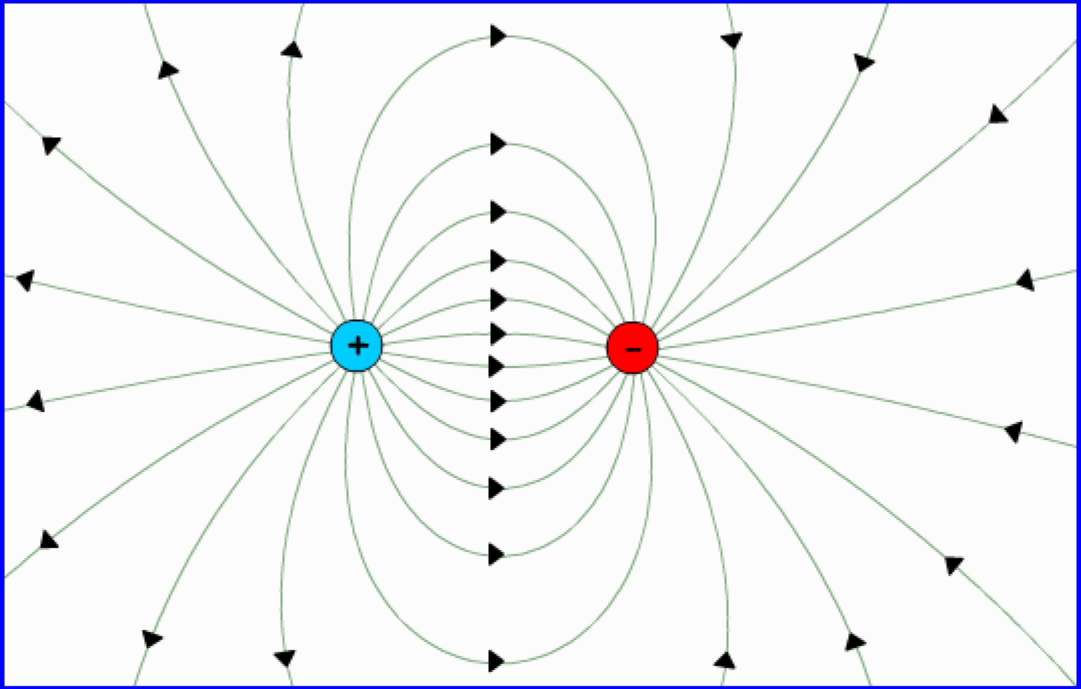
Electric Field Lines



Direction & Density of Lines
represent
Direction & Magnitude of *E*

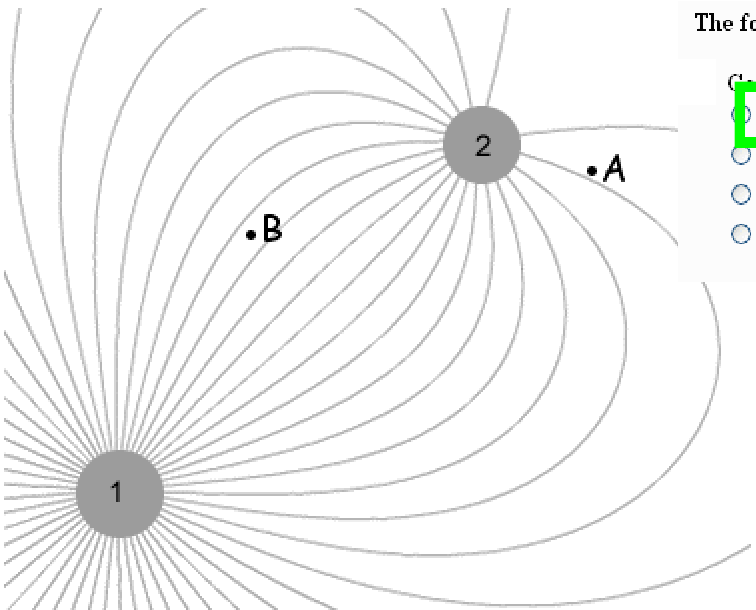
Point Charge:
Direction is radial
Density α $1/R^2$

Electric Field Lines



Dipole Charge Distribution:
Direction & Density

Checkpoint



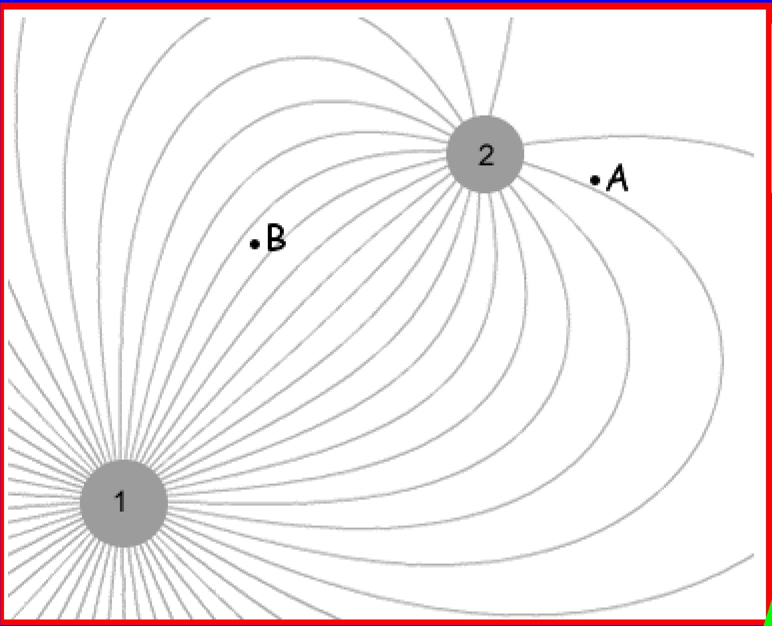
The following three questions pertain to the electric field lines due to two charges are shown above.

Compare the magnitude of the two charges

- ◯ $|Q_1| > |Q_2|$
- ◯ $|Q_1| = |Q_2|$
- ◯ $|Q_1| < |Q_2|$
- ◯ There isn't enough information to determine the relative magnitude of the charges.

Field lines are are denser near Q_1 so $|Q_1| > |Q_2|$

Checkpoint

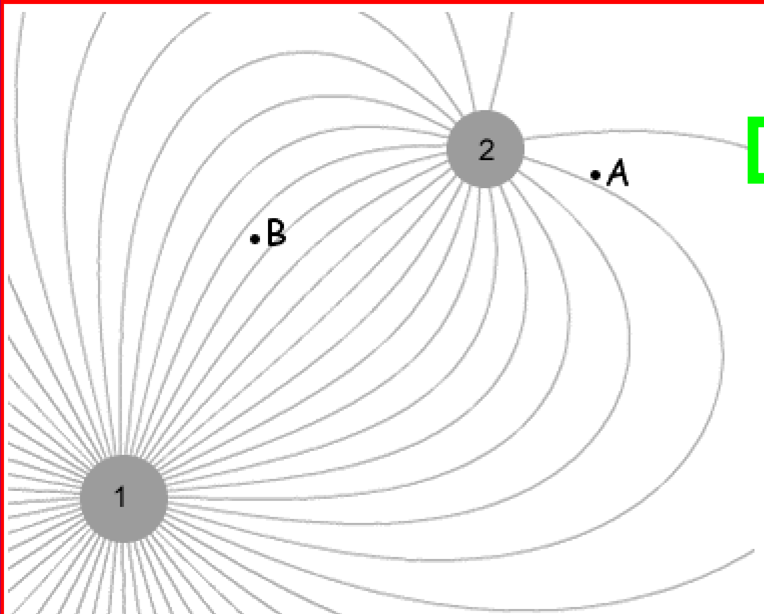


What do we know about the signs of the charges from looking at the picture?

- Q_1 and Q_2 have the same sign
- **Q_1 and Q_2 have opposite signs**
- There is not enough information in the picture to determine the relative signs of the charges

The electric field lines connect the charges. A test charge will move towards one charge and away from the other. So charges 1 and 2 have opposite signs.

Checkpoint

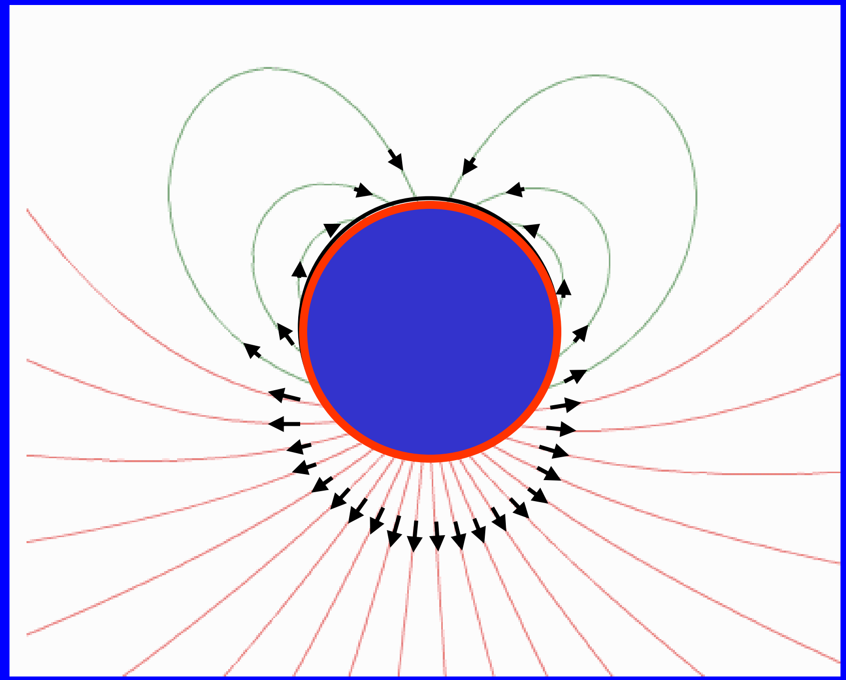


Compare the magnitude of the electric field at points A and B.

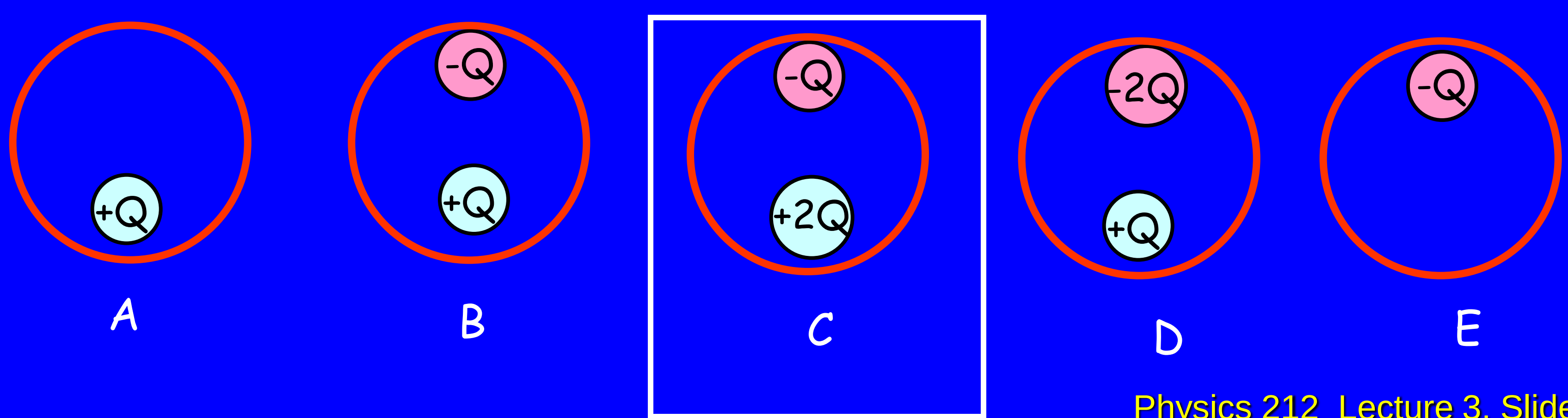
- $|E_A| > |E_B|$
- $|E_A| = |E_B|$
- $|E_A| < |E_B|$
- There isn't enough information to determine the relative magnitude of the electric field at points A and B..

Density of lines is greater at B than at A.
Therefore, magnitude of field at B is greater than at A.

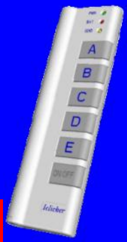
Point Charges

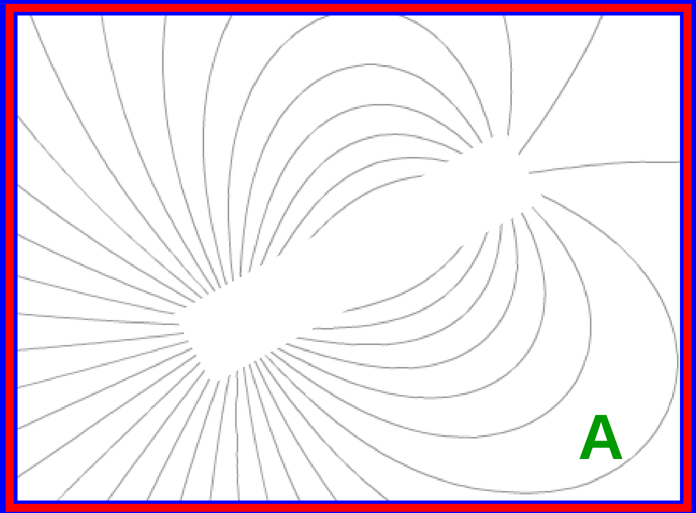


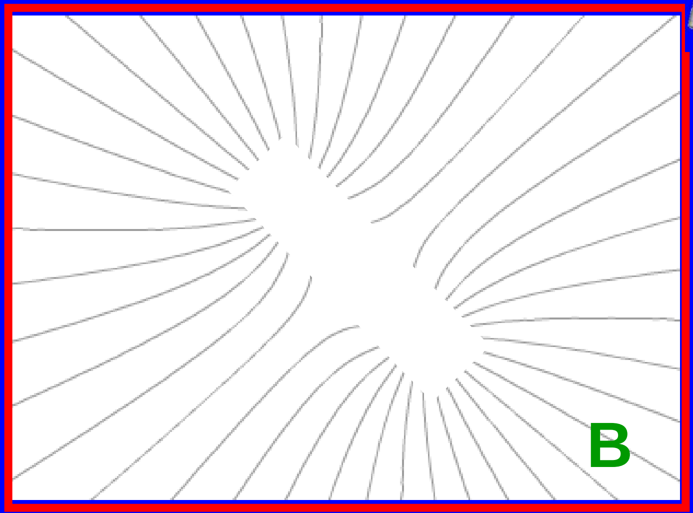
What charges are inside the red circle?

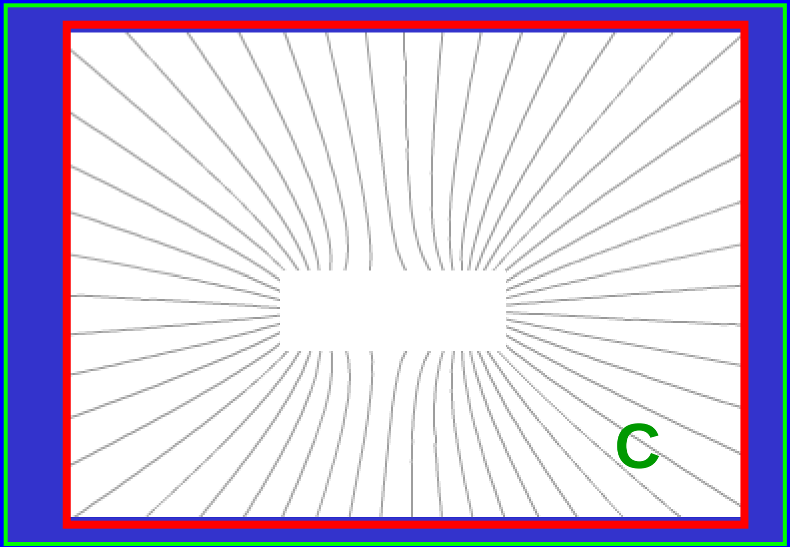


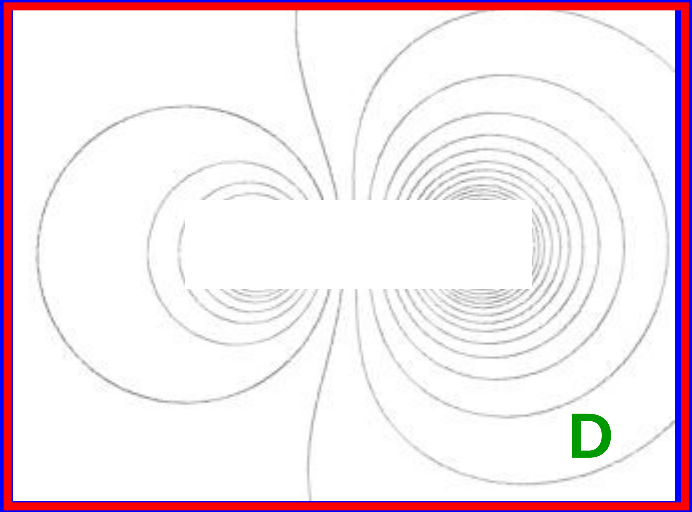
Which of the following field line pictures best represents the electric field from two charges that have the same sign but different magnitudes?









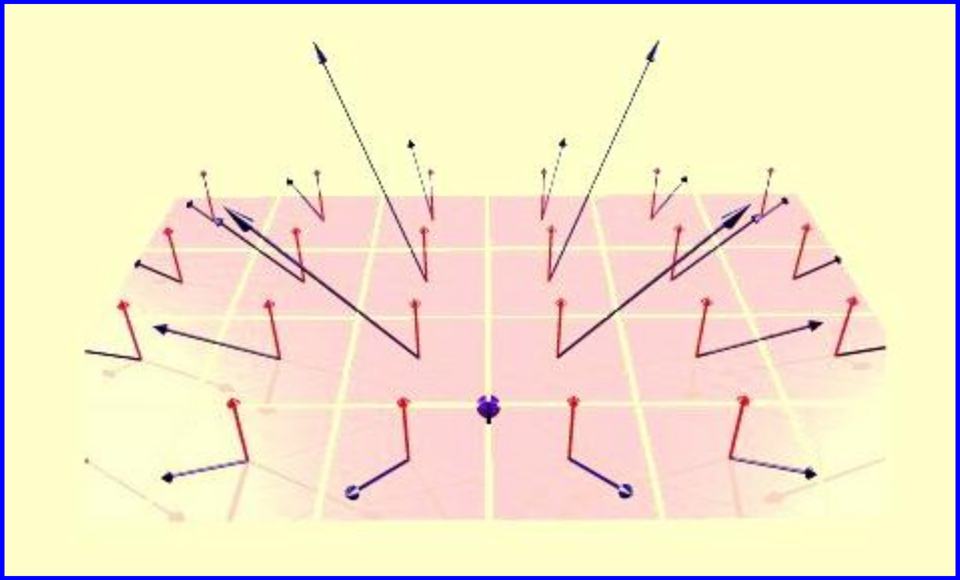


Electric Flux "Counts Field Lines"

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A}$$

Flux through surface S

Integral of $\vec{E} \cdot d\vec{A}$ on surface S

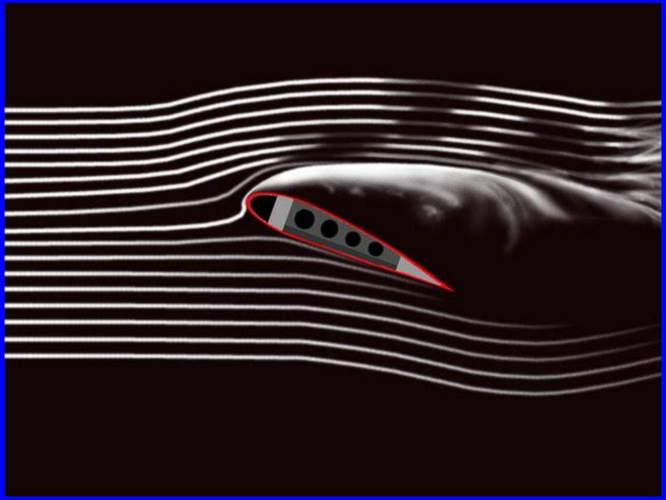


Electric Field/Flux Analogy: Velocity Field/Flux

$$\Phi_S = \int_S \vec{v} \cdot d\vec{A} = flowrate$$

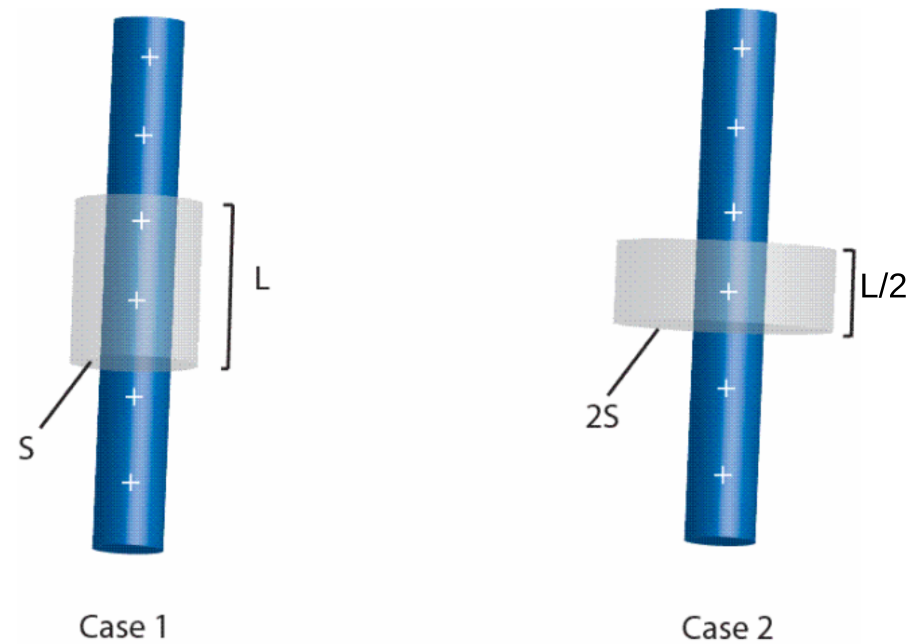
Flux through surface S

Integral of $\vec{v} \cdot d\vec{A}$ on surface S



Checkpoint

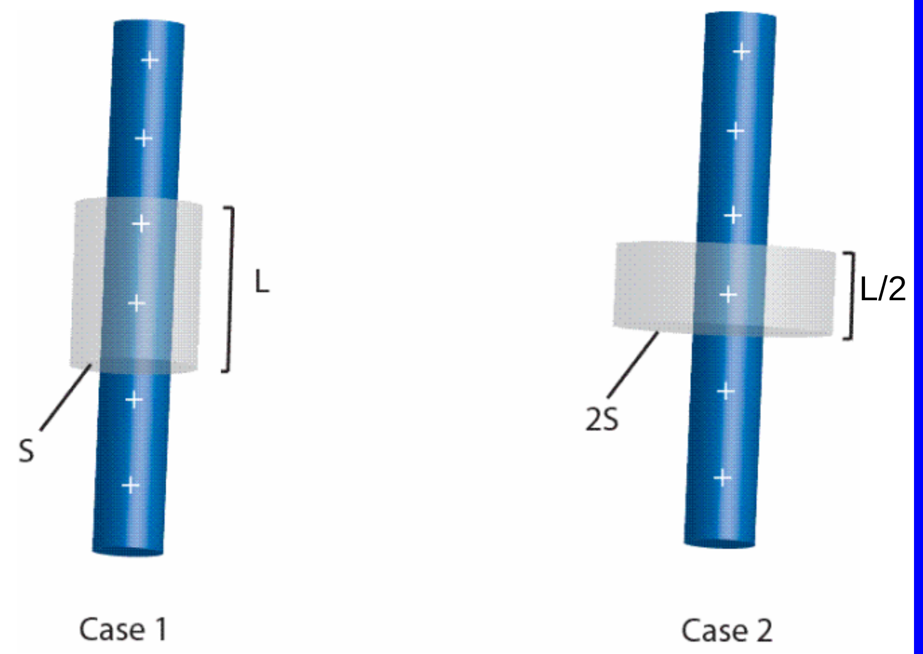
An infinitely long charged rod has uniform charge density λ and passes through a cylinder (gray). The cylinder in Case 2 has twice the radius and half the length compared with the cylinder in Case 1.



(A)	(B)	(C)	(D)
$\Phi_1=2\Phi_2$	$\Phi_1=\Phi_2$	$\Phi_1=1/2\Phi_2$	none

Checkpoint

An infinitely long charged rod has uniform charge density l and passes through a cylinder (gray). The cylinder in Case 2 has twice the radius and half the length compared with the cylinder in Case 1.



Definition of Flux:

$$\Phi \equiv \int_{surface} \vec{E} \cdot d\vec{A}$$

E constant on barrel of cylinder
E perpendicular to barrel surface
(E parallel to dA)

$$\Phi = E \int_{barrel} d\vec{A} = EA_{barrel}$$

$\Phi_1 = 2\Phi_2$
(A)

RESULT: GAUSS' LAW
Φ proportional to charge enclosed !

Case 1

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 s}$$

$$A_1 = (2\pi s)L$$

$$\Phi_1 = \frac{\lambda L}{\varepsilon_0}$$

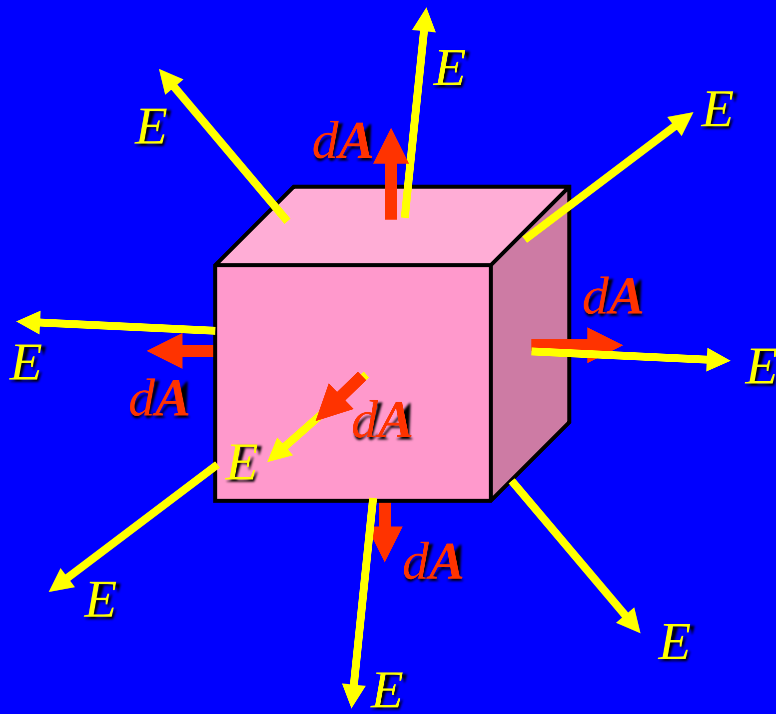
Case 2

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 (2s)}$$

$$A_2 = (2\pi(2s))L/2 = 2\pi s L$$

$$\Phi_2 = \frac{\lambda(L/2)}{\varepsilon_0}$$

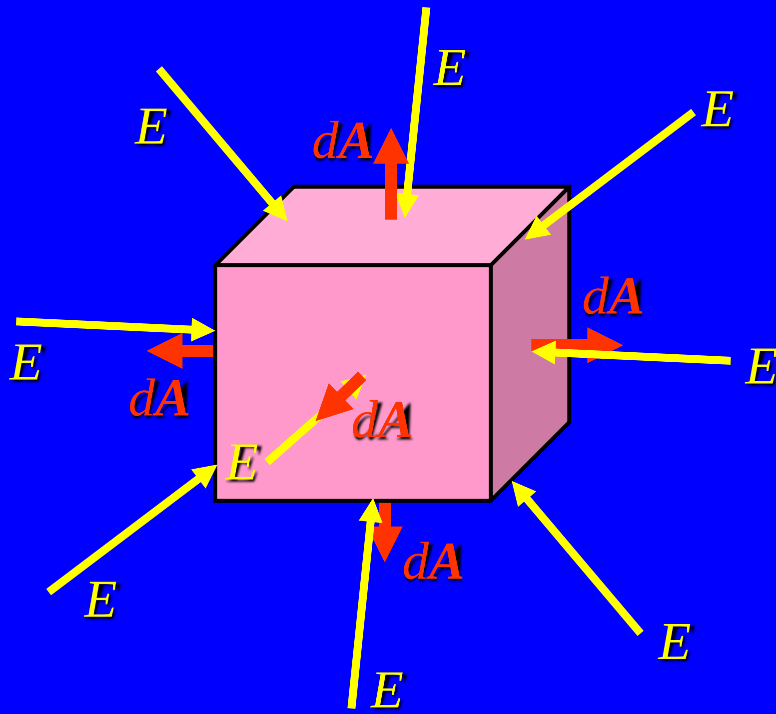
Direction Matters:



For a closed surface,
A points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} > 0$$

Direction Matters:

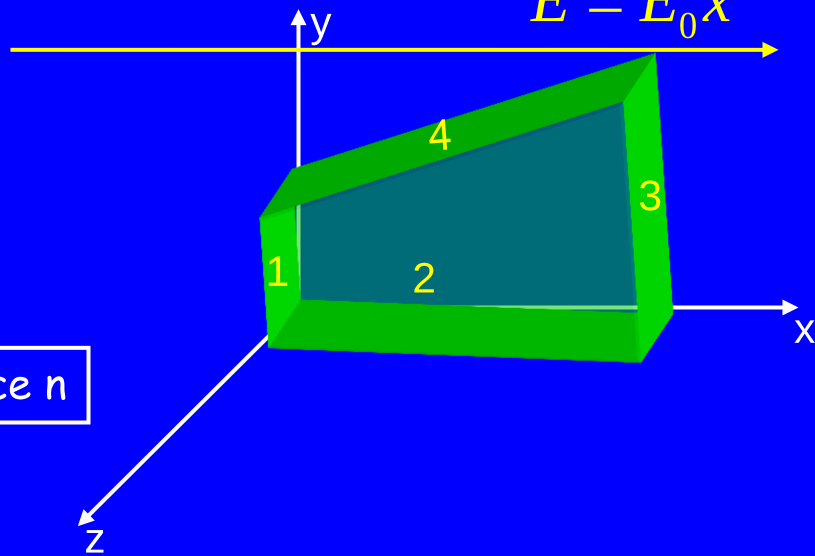


For a closed surface, A points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} < 0$$

Trapezoid in Constant Field

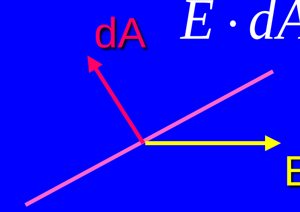
$\vec{E} = E_0 \hat{x}$



Label faces:
1: x = 0
2: z = +a
3: x = +a
4: slanted

Define Φ_n = Flux through Face n

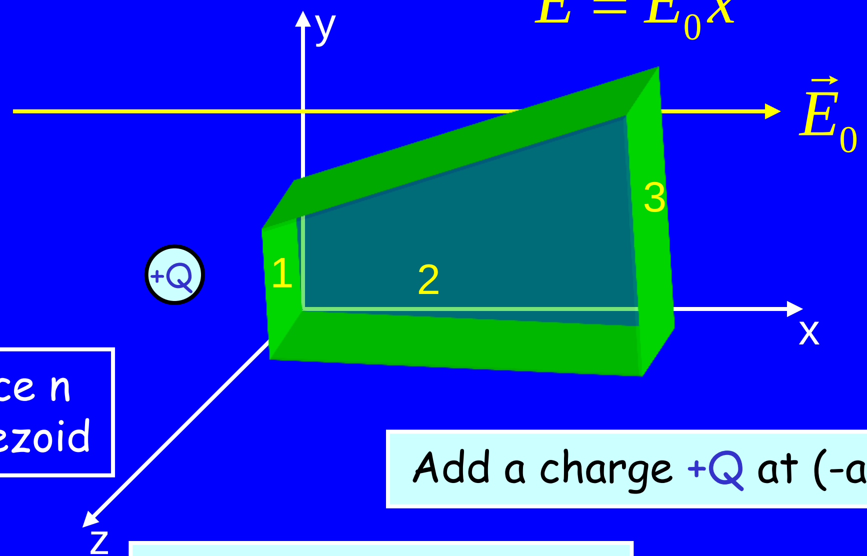
$\vec{E} \cdot d\vec{A} < 0$



A ☐ $\Phi_1 < 0$	A ☐ $\Phi_2 < 0$	A ☐ $\Phi_3 < 0$	A ☐ $\Phi_4 < 0$
B ☐ $\Phi_1 = 0$	B ☐ $\Phi_2 = 0$	B ☐ $\Phi_3 = 0$	B ☐ $\Phi_4 = 0$
C ☐ $\Phi_1 > 0$	C ☐ $\Phi_2 > 0$	C ☐ $\Phi_3 > 0$	C ☐ $\Phi_4 > 0$

Trapezoid in Constant Field + Q

$\vec{E} = E_0 \hat{x}$



Label faces:
1: x = 0
2: z = +a
3: x = +a

Define Φ_n = Flux through Face n
Φ = Flux through Trapezoid

Add a charge +Q at (-a,a/2,a/2)

How does Flux change?

A ☐ Φ_1 increases

B ☐ Φ_1 decreases

C ☐ Φ_1 remains same

A ☐ Φ_3 increases

B ☐ Φ_3 decreases

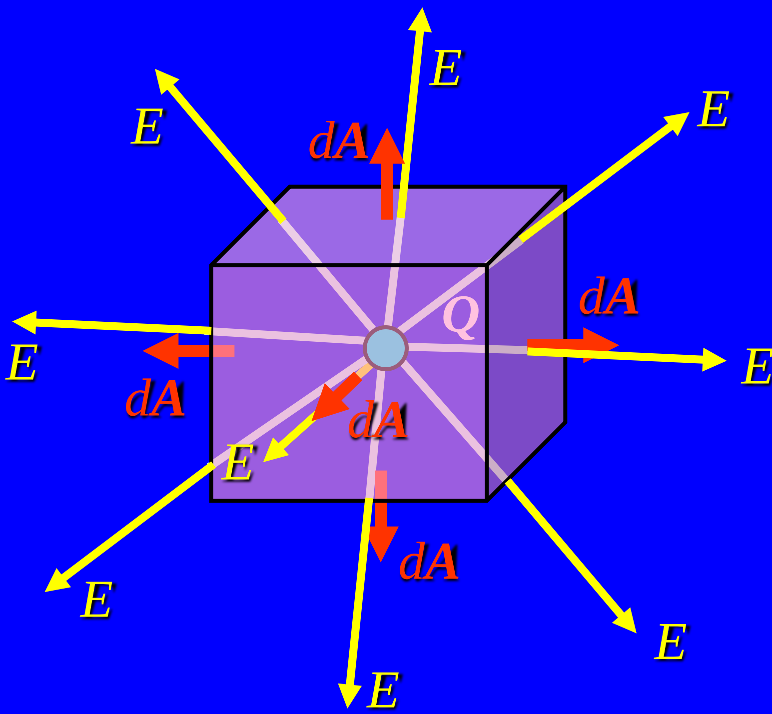
C ☐ Φ_3 remains same

A ☐ Φ increases

B ☐ Φ decreases

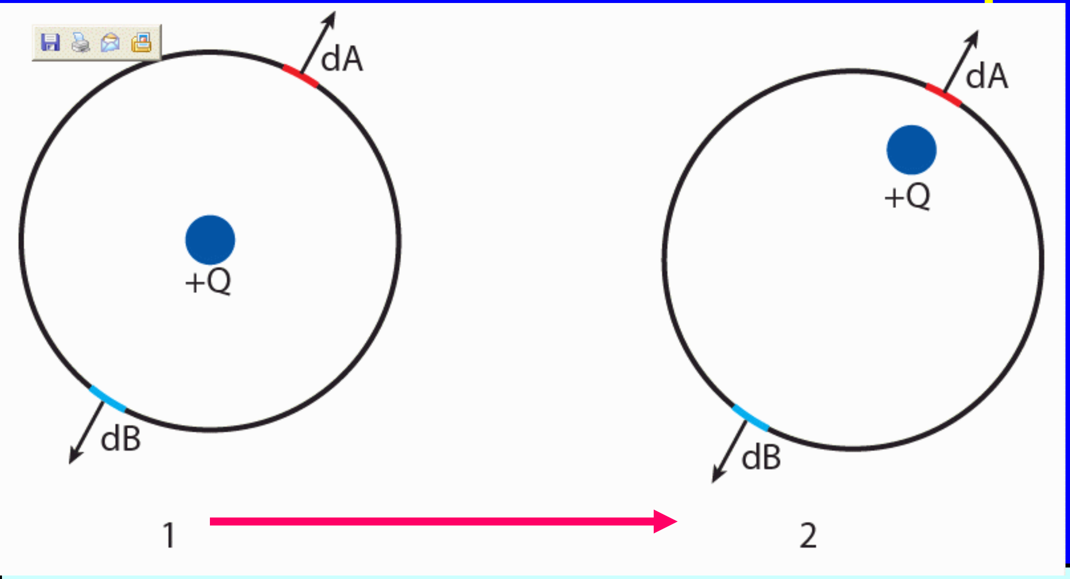
C ☐ Φ remains same

Gauss Law



$$\Phi_S \equiv \oint_{\substack{closed\\surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

Checkpoint



(A)
Φ increases

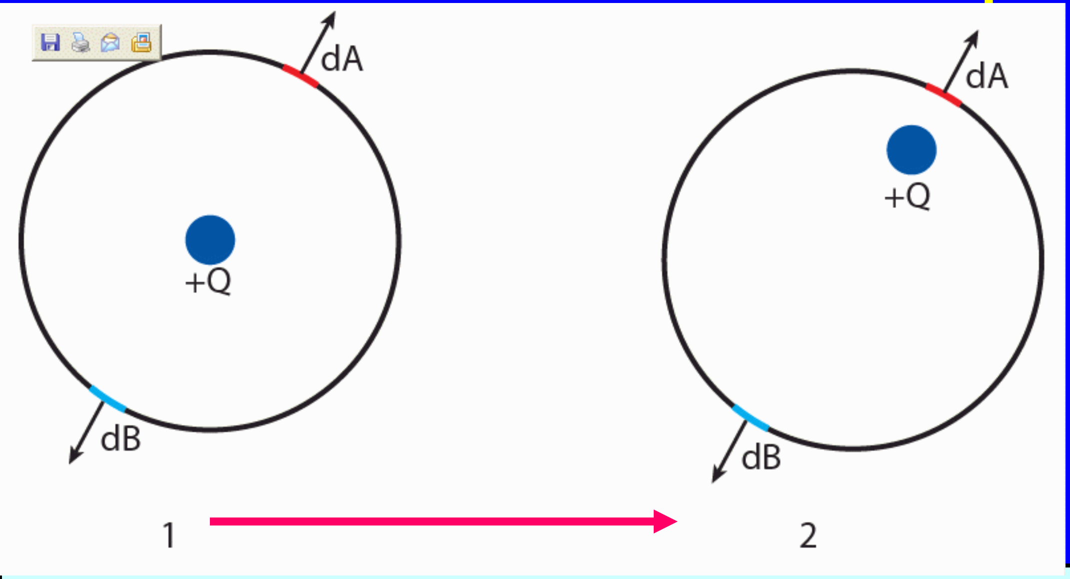
(B)
Φ decreases

(C)
Φ stays same

What happens to total flux through the sphere as we move Q ?

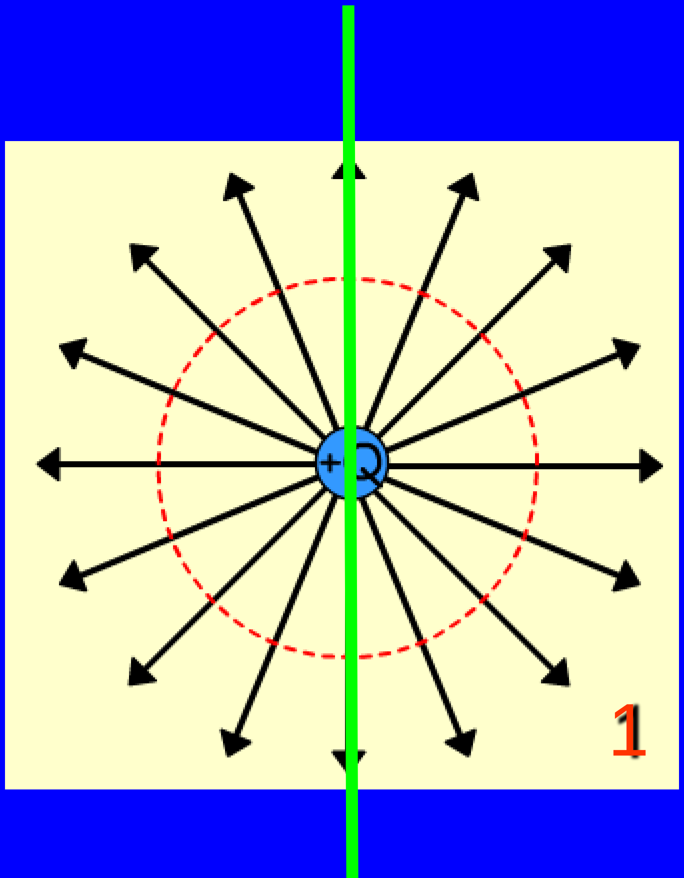
The same amount of charge is still enclosed by the sphere, so flux will not change.

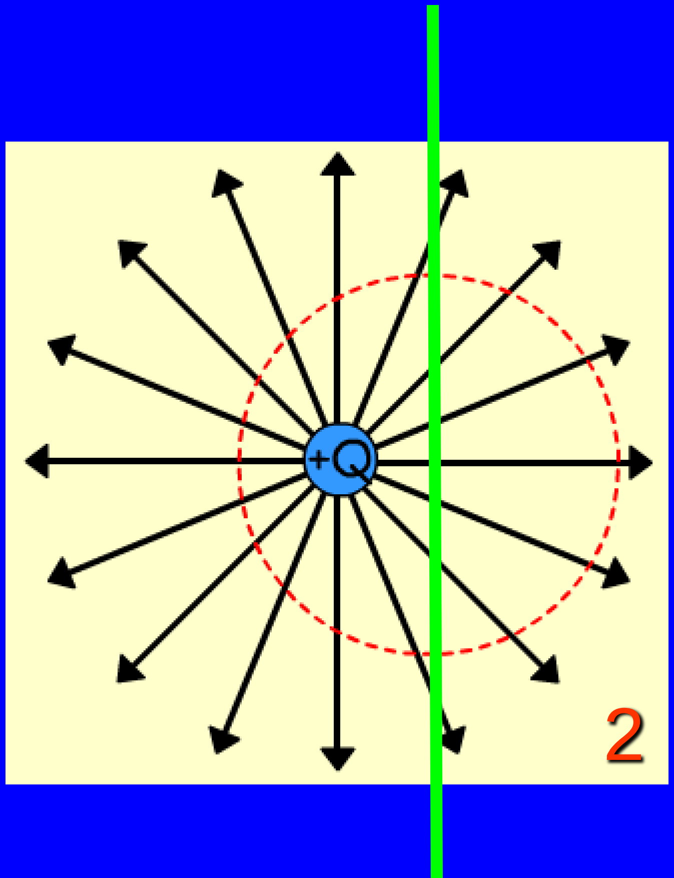
Checkpoint



(A)	(B)	(C)
$d\Phi_A$ increases $d\Phi_B$ decreases	$d\Phi_A$ decreases $d\Phi_B$ increases	$d\Phi_A$ stays same $d\Phi_B$ stays same

Think of it this way:

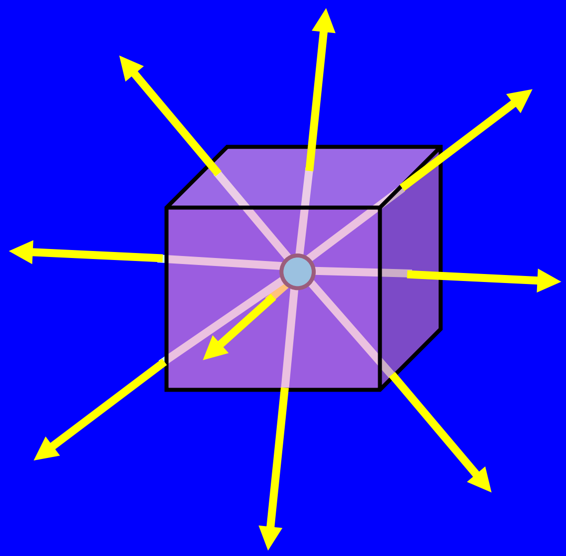




The total flux is the same in both cases (just the total number of lines)

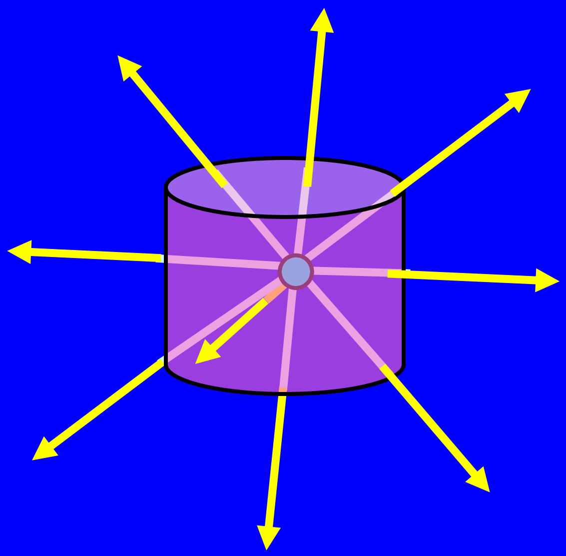
The flux through the right (left) hemisphere is smaller (bigger) for case 2.

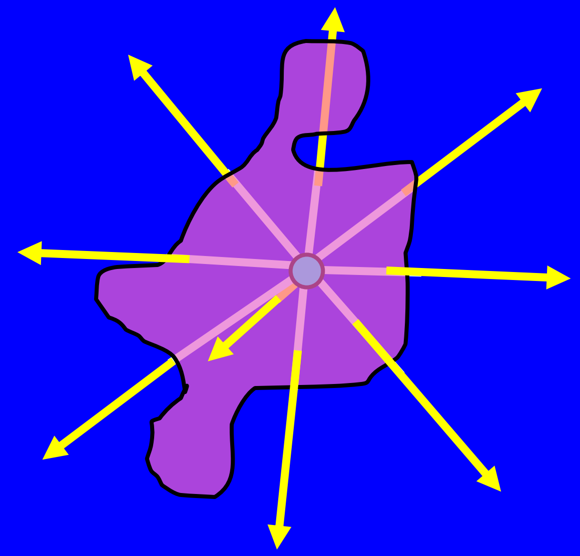
Things to notice about Gauss's Law



$$\Phi_S \equiv \oint_{\substack{closed \\ surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

If $Q_{enclosed}$ is the same, the flux has to be the same, which means that the integral must yield the same result for any surface.





Things to notice about Gauss's Law

$$\oint_{\substack{closed\\surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

In cases of high symmetry it may be possible to bring E outside the integral. In these cases we can solve Gauss Law for E

$$\oint_{\substack{closed\\surface}} \vec{E} \cdot d\vec{A} = EA = \frac{Q_{enclosed}}{\varepsilon_o} \Rightarrow E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

So - if we can figure out $Q_{enclosed}$ and the area of the surface A, then we know E !

This is the topic of the next lecture…

- Prelecture 4 and Checkpoint 4 due Thursday
- Homework 2 due next Monday