Two Port Network & Network Functions

A pair of terminals through which a current may enter or leave a network is known as a port. A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero. For example, most circuits have two ports. We may apply an input signal in one port and obtain an output signal from the other port. The parameters of a two-port network completely describes its behaviour in terms of the voltage and current at each port. Thus, knowing the parameters of a two port network permits us to describe its operation when it is connected into a larger network. Two-port networks are also important in modeling electronic devices and system components. For example, in electronics, two-port networks are employed to model transistors and Op-amps. Other examples of electrical components modeled by two-ports are transformers and transmission lines.

Four popular types of two-port parameters are examined here: impedance, admittance, hybrid, and transmission. We show the usefulness of each set of parameters, demonstrate how they are related to each other.

A Typical one port or two terminal network is shown in figure 1.1. For example resistor, capacitor and inductor are one port network.

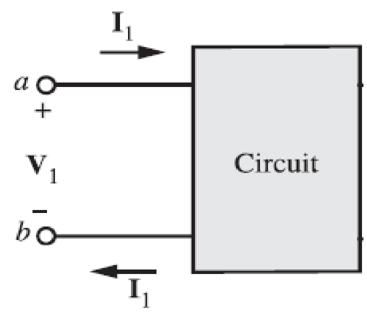


Fig.1.1

Fig. 1.2 represents a two-port network.A four terminal network is called a two-port network when the current entering one terminal of a pair exits the other terminal in the pair. For example, I1 enters terminal 'a' and exit terminal 'b' of the input terminal pair 'a-b'. Example for four-terminal or two-port circuits are op amps, transistors, and transformers.

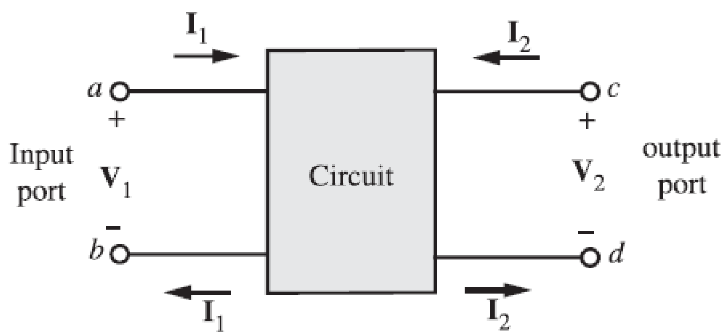


Fig.1.2

To characterize a two-port network requires that we relate the terminal quantities V_1, V_2, I_1 *and* I_2.The various terms that relate these voltages and currents are called parameters. Our goal is to derive four sets of these parameters.

1.3 Open circuit Impedance Parameter (z Parameter):

Let us assume the two port network shown in figure is a linear network then using superposition theorem, we can write the input and output voltages as the sum of two components, one due to I_1 and other due to I_2:

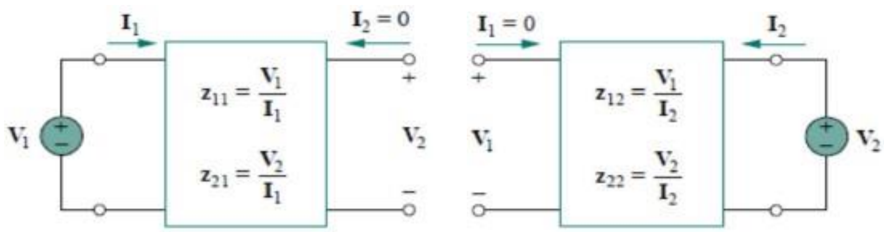
$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

the z terms are called the z parameters, and have units of ohms. The values of the parameters can be evaluated by setting $I_1 = 0$ or $I_2 = 0$.
The **z** parameters are defined as follows:
Thus

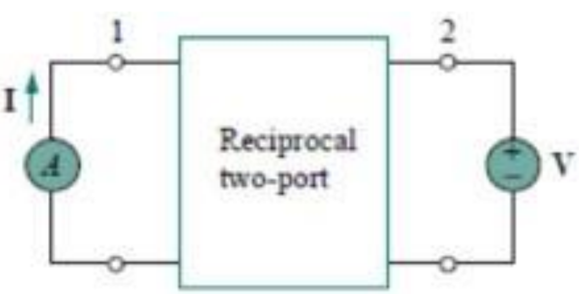
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

In the preceding equations, letting $I_1 = 0$ or $I_2 = 0$ is equivalent to open-circuiting the input or output port. Hence, the **z** parameters are called open-circuit impedance parameters.
Here z_{11} is defined as the open-circuit input impedance, z_{22} is called the open-circuit output impedance, and z_{12} and z_{21} are called the open-circuit transfer impedances.
If $z_{12} = z_{21}$, the network is said to be **reciprocal network**. Also, if $z_{11} = z_{22}$ then the network is called a **symmetrical network.**

We obtain z_{11} and z_{21} by connecting a voltage V_1 (or a current source I_1) to port 1 with port 2 open-circuited as in fig.



Similarly z_{12} and z_{22} by connecting a voltage V_2 (or a current source I_2) to port 2 with port 1 open-circuited as
in fig.

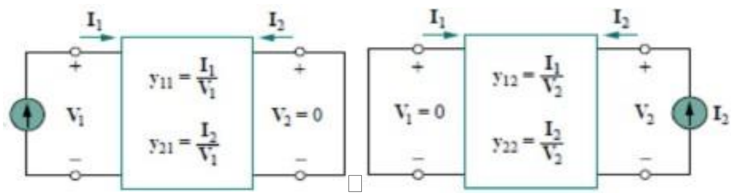


1.4 Admittance Parameter(y Parameter):

The terminal currents can be expressed in terms of the terminal voltages: The y terms are known as the admittance parameters (or, simply, y parameters) and have units of siemens.

$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

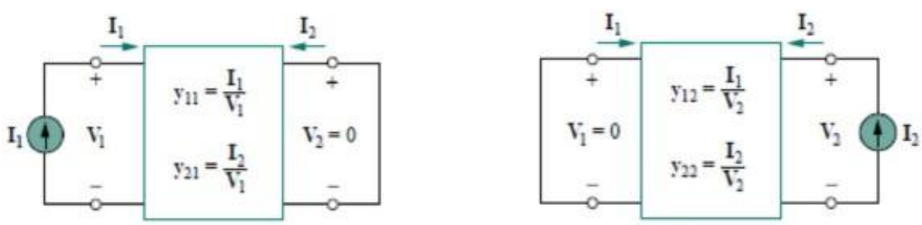


The y terms are called the y parameters, and have units of seimens. The values of the parameters can be evaluated by setting $V_1 = 0$ or $V_2 = 0$.

The y parameters are defined as follows:

Thus

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



In the preceding equations, letting $V_1 = 0$ or $V_2 = 0$ is equivalent to short-circuiting the input or output port. Hence, the y parameters are called short-circuit admittance parameters.

If $y_{12} = y_{21}$, the network is said to be **reciprocal network**. Also, if $y_{11} = y_{22}$ then it is called a **symmetrical network.**

A reciprocal network ($y_{12} = y_{21}$) can be modeled by the equivalent circuit in Fig.8.3

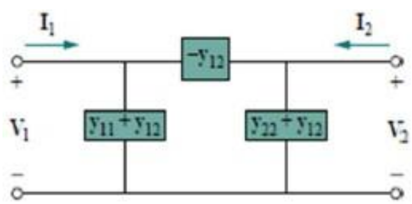


Fig.8.3

1.6 Transmission Parameters:

The transmission parameters are defined by the equations:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Putting the above equations in matrix form, we get

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

A, **B**, **C** and **D** parameters represent the open-circuit voltage ratio, the negative short-circuit transfer impedance, the open-circuit transfer admittance, and the negative short-circuit current ratio, respectively.

TWO-PORT PARAMETER CONVERSION TABLE

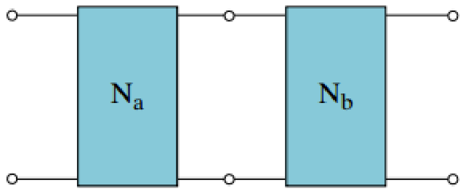
Desired Parameters	Given Parameters [z]	[y]	[h]	[t]
[z]	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix}$	$\begin{bmatrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{A}{C} & \frac{\Delta_t}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$
[y]	$\begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{bmatrix}$	$\begin{bmatrix} \frac{D}{B} & \frac{-\Delta_t}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$
[h]	$\begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$	$\begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{bmatrix} \frac{B}{D} & \frac{\Delta_t}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$
[t]	$\begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta_y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$	$\begin{bmatrix} \frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_t = AD - BC$

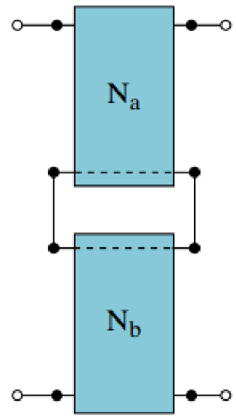
Table 1.11

Interconnection of Two port network:

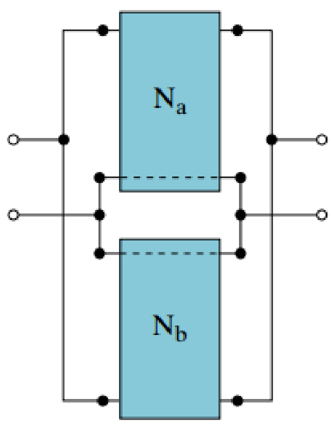
A large, complex network may be divided into subnetworks for the purposes of analysis and design. The subnetworks are modeled as two-port networks, interconnected to form the original network. The two-port networks may therefore be regarded as building blocks that can be interconnected to form a complex network. The interconnection can be in series, in parallel, or in cascade. Although the interconnected network can be described by any of the six parameter sets, a certain set of parameters may have a definite advantage. For example, when the networks are in series, their individual z parameters add up to give the z parameters of the larger network. When they are in parallel, their individual y parameters add up to give the y parameters of the larger network. When they are cascaded, their individual transmission parameters can be multiplied together to get the transmission parameters of the larger network.



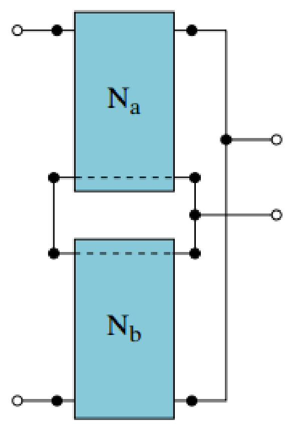
(a) Cascade



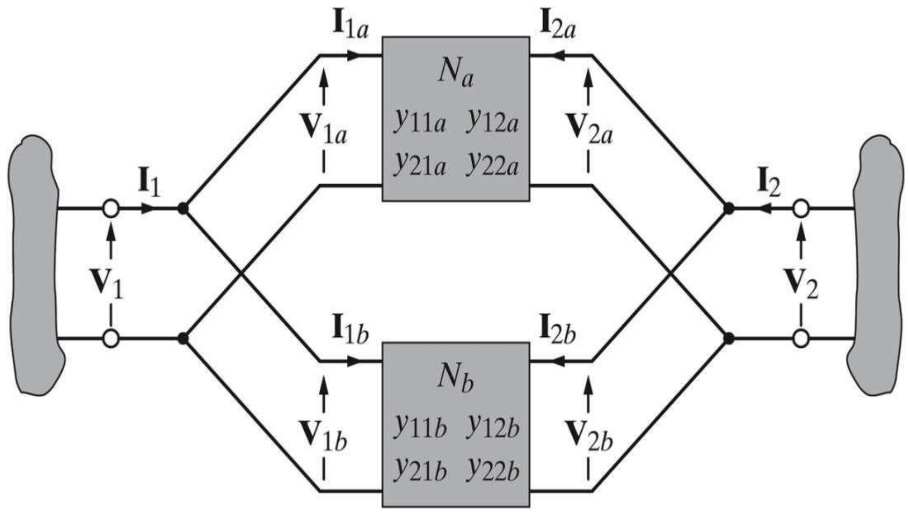
(b) Series



(c) Parallel



(d) Series-Parallel



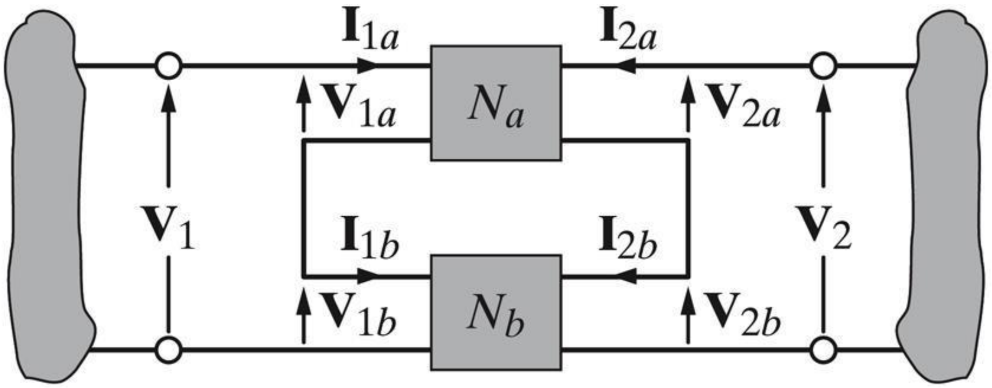


Fig.8.21

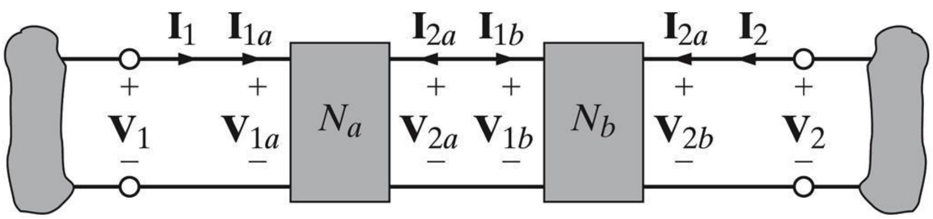


Fig.8.22

1.12 Network Functions for One Port and Two Port Network

1.13 Driving Point Functions:
The impedance or admittance found at a given port is called a driving point impedance(or admittance).

1.Driving Point Impedance:
$Z_{11}(s) = V_1(s)/I_1(s)$

2. Driving Point Admittance:
$Y_{11}(s) = I_1(s)/V_1(s) = 1/Z_{11}(s)$

1.14 Transfer Function:

The transfer function relates the transform of a quantity at one port to the transform of another quantity at another port.Thus transfer functions which relate voltages and currents have following possible forms:
The ratio of one voltage to another voltage, or the **voltage transfer ratio.**
The ratio of one current to another current ,or the **current transfer ratio**.
The ratio of one current to another voltage or one voltage to another voltage.

Transfer function for the two port network:

Denominator	Numerator	
	$V_2(s)$	$I_2(s)$
$V_1(s)$	$G_{12}(s)$	$Y_{12}(s)$
$I_1(s)$	$Z_{12}(s)$	$\alpha_{12}(s)$