# ELECTROMAGNETIC THEORY

For
ELECTRICAL ENGINEERING
ELECTRONICS & COMMUNICATION ENGINEERING

# ELECTROMAGNETIC THEORY

## SYLLABUS

Elements of vector calculus: divergence and curl; Gauss' and Stokes' theorems, Maxwell's equations: differential and integral forms. Wave equation, Poynting vector. Plane waves: propagation through various media; reflection and refraction; phase and group velocity; skin depth. Transmission lines: characteristic impedance; impedance transformation; Smith chart; impedance matching; S parameters, pulse excitation. Waveguides: modes in rectangular waveguides; boundary conditions; cut-off frequencies; dispersion relations. Basics of propagation in dielectric waveguide and optical fibers. Basics of Antennas: Dipole antennas; radiation pattern; antenna gain.

## ANALYSIS OF GATE PAPERS

| Exam Year | 1 Mark Ques. | 2 Mark Ques. | Total |
|---|---|---|---|
| 2003 | 2 | 7 | 16 |
| 2004 | 2 | 6 | 14 |
| 2005 | 2 | 6 | 14 |
| 2006 | 2 | 8 | 18 |
| 2007 | 2 | 7 | 16 |
| 2008 | 2 | 5 | 12 |
| 2009 | 2 | 3 | 8 |
| 2010 | 3 | 2 | 7 |
| 2011 | 4 | 3 | 10 |
| 2012 | 4 | 5 | 14 |
| 2013 | 1 | 2 | 5 |
| 2014 Set-1 | 2 | 3 | 8 |
| 2014 Set-2 | 2 | 4 | 10 |
| 2014 Set-3 | 2 | 3 | 8 |
| 2014 Set-4 | 4 | 3 | 10 |
| 2015 Set-1 | 2 | 4 | 10 |
| 2015 Set-2 | 2 | 3 | 8 |
| 2015 Set-3 | 2 | 4 | 10 |
| 2016 Set-1 | 2 | 4 | 10 |
| 2016 Set-2 | 3 | 4 | 11 |
| 2016 Set-3 | 2 | 4 | 10 |
| 2017 Set-1 | 1 | 3 | 7 |
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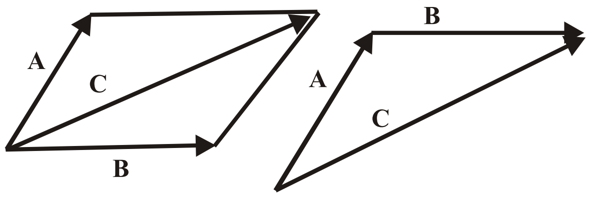
# 1 VECTORS & CO-ORDINATE SYSTEMS

## 1.1 VECTORS

A vector $\overline{A}$ has both magnitude and direction. The magnitude of $\overline{A}$ is a scalar written as A or |A|. A unit vector $a_A$ along $\overline{A}$ is defined as a vector whose magnitude is unity and its direction is along A, that is,

$$a_A = \frac{\overline{A}}{|A|} \qquad a_A = \frac{A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$




### 1.1.1 VECTOR ADDITION & SUBTRACTION

Two vectors $\overline{A}$ and $\overline{B}$ can be added together to give another vector $\overline{C}$; that is,

$\overline{C}=\overline{A}+\overline{B}$

| Law | Addition | Multiplication |
|---|---|---|
| Commutative | $\overline{A}+\overline{B}=\overline{B}+\overline{A}$ | $k\overline{A}=\overline{A}k$ |
| Associative | $\overline{A}+(\overline{B}+\overline{C}) = (\overline{A}+\overline{B})+\overline{C}$ | $k(\ell \overline{A}) = (k\ell)\overline{A}$ |
| Distributive | $k(\overline{A}+\overline{B})=k\overline{A}+k\overline{B}$ | |

The position vector $r_P$ (or **radius vector)** of point P is as the directed from the origin O to P; i.e.,

$R_P = \overline{OP} = X\hat{a}_x + Y\hat{a}_y + Z\hat{a}_z$

The **distance vector** is the displacement from one point to another.

$\overline{r}_{PQ} = \overline{r}_Q - \overline{r}_P$

$= (x_Q - x_P)\hat{a}_x + (y_Q - y_P)\hat{a}_y + (z_Q - z_P)\hat{a}_z$

### 1.1.2 VECTOR MULTIPLICATION

- Scalar (or dot) product: $\overline{\boldsymbol{A}}\,.\,\overline{\boldsymbol{B}}$
- Vector (or cross) product: $\overline{\boldsymbol{A}}\times\overline{\boldsymbol{B}}$
- Scalar triple product: $\overline{\boldsymbol{A}}\,.\,(\overline{\boldsymbol{B}}\times\overline{\boldsymbol{C}})$
- Vector triple product: $\overline{\boldsymbol{A}}\times(\overline{\boldsymbol{B}}\times\overline{\boldsymbol{C}})$

1. **Dot Product**

   The **dot product** of two vector $\overline{\boldsymbol{A}}$ and $\overline{\boldsymbol{B}}$ written as $\overline{\boldsymbol{A}}.\overline{\boldsymbol{B}}$, is defined geometrically as the product of the magnitudes of **A** and **B** and the cosine of the angle between them.

   $\overline{\boldsymbol{A}}\,.\,\overline{\boldsymbol{B}} = AB\cos\theta_{AB}$

   $\overline{\boldsymbol{A}}\,.\,\overline{\boldsymbol{B}} = A_xB_x + A_yB_y + A_zB_z$

2. **Cross Product**

   $$\overline{A}\times\overline{B} = AB\sin\theta_{AB}\,\hat{a}_n,\quad \overline{A}\times\overline{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

   Note that the cross product has the following basic properties:

   i) It is not commutative:

   $\overline{A}\times\overline{B} \neq \overline{B}\times\overline{A}$

   It is anti-commutative:

   $\overline{A}\times\overline{B} = -\overline{B}\times\overline{A}$

   ii) It is not associative:

   $\overline{A}\times(\overline{B}\times\overline{C}) \neq (\overline{A}\times\overline{B})\times\overline{C}$

   iii) It is distributive:

   $\overline{A}\times(\overline{B}+\overline{C}) = \overline{A}\times\overline{B} + \overline{A}\times\overline{C}$

3. **Scalar Triple Product**

   Given three vectors $\overline{\boldsymbol{A}}$, $\overline{\boldsymbol{B}}$, and $\overline{\boldsymbol{C}}$, we define the scalar triple product as

   $\overline{A}.(\overline{B}\times\overline{C}) = \overline{B}.(\overline{C}\times\overline{A}) = \overline{C}.(\overline{A}\times\overline{B})$

   $$\overline{A}.(\overline{B}\times\overline{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

4. **Vector Triple Product.**

   $\overline{A}\times(\overline{B}\times\overline{C}) = \overline{B}(\overline{C}\,.\,\overline{A}) - \overline{C}(\overline{A}\,.\,\overline{B})$

### 1.1.3 COMPONENTS OF A VECTOR

The vector component $A_B$ of A along B is simply the scalar component

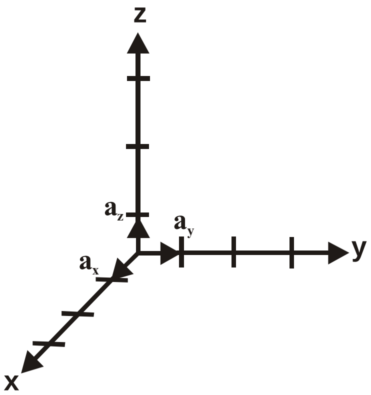
$$A_B = A\cos\theta = \frac{\overline{A}.\overline{B}}{|\overline{B}|}$$

## 1.2 COORDINATE SYSTEMS AND TRANSFORMATION

An **orthogonal system** is one in which the coordinates are mutually perpendicular.

### 1.2.1 CARTESIAN COORDINATES (X, Y, Z)

A point P can be represented as (x, y, z) as in Figure




The ranges of the coordinate variable x, y, and z are

$-\infty < x < \infty$ x=distance from yz plane

$-\infty < y < \infty$ y=distance from xz plane

$-\infty < z < \infty$ z=distance from xy plane

A vector A in **Cartesian** (otherwise known as **rectangular**) coordinates can be written as

$(A_x, A_y, A_z)$ or $\overline{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

Where, $a_x$, $a_y$, and $a_z$ are unit vectors along the x, y and z directions respectively,as shown in Figure.

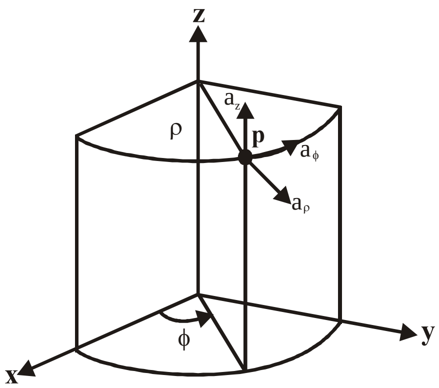
### 1.2.2 CYLINDRICAL COORDINATES $(\rho, \varphi, z)$

Azimuthal angle, is measured from the positive x-axis in the xy-plane; and z is the same as in the Cartesian system.

$\rho$ =radius

$\varphi$ =azimuthal angle

z =height



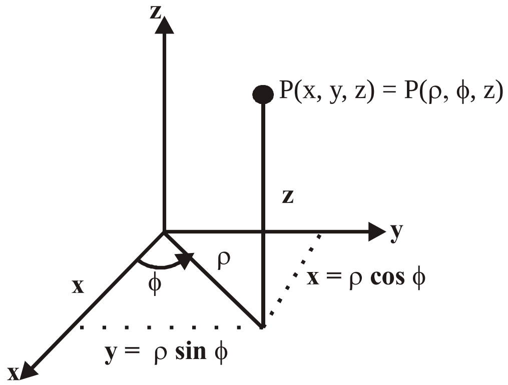

The ranges of the variables are

$0 \le \rho < \infty$

$0 \le \varphi < 2\pi$

$-\infty < z < \infty$

A vector $\overline{A}$ in cylindrical coordinates can be written as




$(A_\rho, A_\varphi, A_z)$ or $A_\rho\hat{a}_\rho + A_\varphi\hat{a}_\varphi + A_z\hat{a}_z$

$\rho = \sqrt{x^2 + y^2}$, $\varphi = \tan^{-1}\frac{y}{x}, z = z$ or

$x = \rho\cos\varphi$, $y = \rho\sin\varphi, z = z$

$$\begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \text{ or}$$

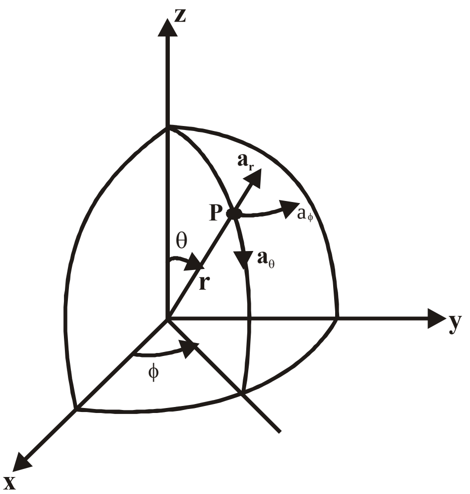
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\varphi \\ A_z \end{bmatrix}$$

### 1.2.3 SPHERICAL COORDINATES $(r, \theta, \varphi)$

R=radius

$\theta$ =elevation angle

$\varphi$ =azimuthal angle




**Range:**

$0 \le r < \infty, \; 0 \le \theta \le \pi \;\; \& \, 0 \le \varphi < 2\pi$

$(A_r, A_\theta, A_\varphi)$ or $A_r\hat{a}_r + A_\theta\hat{a}_\theta + A_\varphi\hat{a}_\varphi$

$|A| = (A_r^2 + A_\theta^2 + A_\varphi^2)^{1/2}$

$r = \sqrt{x^2+y^2+z^2}, \;\; \theta = \tan^{-1}\dfrac{\sqrt{x^2+y^2}}{z},$

$\varphi = \tan^{-1}\dfrac{y}{x}$ or

$x = r\sin\theta\cos\varphi, \; y = r\sin\theta\sin\varphi, z = r\cos\theta$

### 1.2.4 CONSTANT-COORDINATE SURFACES

X = Constant
Y = Constant
Z = Constant
ρ = Constant
φ = Constant
Z = Constant
r = Constant
θ= Constant

## 1.3 DIFFERENTIAL LENGTH, AREA, & VOLUME

### A. Cartesian Coordinates

1) Differential displacement is given by
   $\vec{dl}$= dx $\hat{a}_x$ + dy $\hat{a}_y$ + dz $\hat{a}_z$
2) Different normal area is given by
   $\overline{dS}$ = dy dz $\hat{a}_x$ or
   = dx dz $\hat{a}_y$ or
   = dz dy $\hat{a}_z$
3) Different volume is given by
   V = dx dy dz

### B. Cylindrical Coordinates

1) Different displacement is given by
   $\overline{dl} = d\rho\hat{a}_\rho + \rho\, d\varphi\,\hat{a}_\varphi + dz\,\hat{a}_z$
2) Different normal area is given by
   $\overline{dS} = \rho\, d\varphi\, dz\,\hat{a}_\rho$ or
   $= d\rho\, dz\,\hat{a}_\varphi$ or
   $= \rho\, d\varphi\, d\rho\,\hat{a}_z$
3) Differential volume is given by
   dV = $\rho\, d\rho\, d\varphi\, dz$

### C. Spherical Coordinates

1) The different displacement is
   $\overline{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\, d\varphi\,\hat{a}_\varphi$
2) The differential normal area is
   $\overline{dS} = r^2\sin\theta\, d\theta\, d\varphi\,\hat{a}_r$ or
   $= r\sin\theta\, dr\, d\varphi\,\hat{a}_\theta$ or
   $= r\, dr\, d\theta\,\hat{a}_\varphi$
3) The differential volume is
   $dV = r^2\sin\theta\, dr\, d\theta\, d\varphi$

   The **line integral** $\int_L \overline{A}.\overline{dl}$ is the integral of the tangential component of A along curve L.
   Given a vector field A, continuous in a region containing the smooth surface S, we define the surface Integral or the flux of A through S as
   $\psi = \int_S \overline{A}.\overline{dS}$

## 1.4 DEL OPERATOR

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

$$\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \varphi}\hat{a}_\varphi + \frac{\partial}{\partial z}\hat{a}_z$$

$$\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\hat{a}_\varphi$$

### 1.4.1 APPLICATION OF DEL OPERATOR

- The gradient of a scalar V, written as ∇ V

- The divergence of a vector $\overline{A}$, written as $\nabla.\overline{A}$
- The curl of a vector A, written as $\nabla \times \overline{A}$
- The Laplacian of a scalar V, written $\nabla^2 V = \nabla.\nabla V$

### 1.4.2 GRADIENT OF A SCALAR

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \varphi}\hat{a}_\varphi + \frac{\partial V}{\partial z}\hat{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\hat{a}_\varphi$$

### 1.4.3 DIVERGENCE OF A VECTOR

The divergence of $\overline{A}$ at a given point P is the outward flux per unit volume as the volume shrinks about P.

$$\text{div}\overline{A} = \nabla.\overline{A} = \lim_{\Delta v \to 0}\frac{\oint_S \overline{A}.\overline{dS}}{\Delta v}$$

$$\nabla.\overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla.\overline{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla.\overline{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

### 1.4.4 GAUSS DIVERGENCE THEOREM

The **divergence theorem** states that the total outward flux of a vector field $\overline{A}$ through the closed surface S is the same as the volume integral of the divergence of $\overline{A}$.

$$\oint_S \overline{A}.\overline{dS} = \int_v \nabla.\overline{A}\, dv$$

### 1.4.5 CURL OF A VECTOR

The **curl** of $\overline{A}$ is an axial (or rotational) vector whose magnitude is the maximum circulation of $\overline{A}$ per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented so as to make the circulation maximum. i.e.

$$\text{Curl } \overline{A} = \nabla \times \overline{A} = \left(\lim_{\Delta s \to 0}\frac{\oint_L \overline{A}.\overline{dl}}{\Delta S}\right)_{max} a_n$$

$$\nabla \times \overline{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}; \quad \overline{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

$$\nabla \times \overline{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\varphi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\varphi & A_z \end{vmatrix}; \quad \overline{A} = A_\rho\hat{a}_\rho + A_\varphi\hat{a}_\varphi + A_z\hat{a}_z$$

$$\nabla \times \overline{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_r & rA_\theta & r\sin\theta\, A_\varphi \end{vmatrix}; \quad \overline{A} = A_r\hat{a}_r + A_\theta\hat{a}_\theta + A_\varphi\hat{a}_\varphi$$

### 1.4.6 STOKES'S THEOREM

**Stokes's theorem** states that the circulation of a vector field A around a (closed) path L is equal to the surface integral of the curl of $\overline{A}$ over the open surface S bounded by L provided that A and $\nabla \times \overline{A}$ are continuous on S.

$$\oint_L \overline{A}.\overline{dl} = \int_S (\nabla \times \overline{A}).\overline{dS}$$

### 1.4.7 LAPLACIAN OF A SCALAR

The Laplacian **of a scalar field V, written as**

$\nabla^2 V = \nabla.\nabla V$, **is the divergence of the gradient of V.**

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}$$

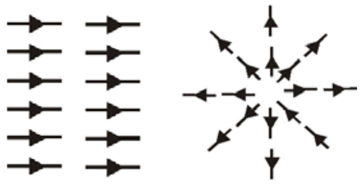
$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right)$$

$$+\frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \varphi^2}$$

## 1.5 CLASSIFICATION OF VECTOR FIELDS

a) $\nabla.\overline{A} = 0, \nabla \times \overline{A} = 0$

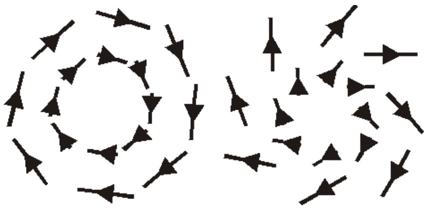
b) $\nabla.\overline{A} \neq 0, \nabla \times \overline{A} = 0$



(a) (b)

c) $\nabla.\overline{A} = 0, \nabla \times \overline{A} \neq 0$

d) $\nabla.\overline{A} \neq 0, \nabla \times \overline{A} \neq 0$



(c) (d)

# 2 ELECTROSTATICS

## 2.1 COULOMB'S LAW

It states that the force F between two point charges $Q_1$ and $Q_2$ is:

1. Along the line joining them
2. Directly proportional to the product $Q_1$, $Q_2$ of the charges
3. Inversely proportional to the square of the distance R between them.

$$|\bar{F}| = \frac{kQ_1Q_2}{R^2}$$

$$\varepsilon_0 = 8.854\times10^{-12} \approx \frac{10^{-9}}{36\pi} F/m \quad \text{or}$$

$$k = \frac{1}{4\pi\varepsilon_0} \approx 9\times10^9 m/F$$

$$\bar{F}_{12} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$

$$\overline{\ }_{12} = \overline{\ }_2 - \overline{\ }_1$$

$$\hat{a}_{R_{12}} = \frac{R_{12}}{|R|}$$

$$\overline{\ }_{21} = -\overline{\ }_{12}$$

## 2.2 ELECTRIC FIELD INTENSITY

The **electric field intensity (** or **electric filed strength) E** is the force per unit positive charge when placed in the electric field.

$$E = \lim_{Q\to0}\frac{F}{Q} \qquad \text{or Simply}$$

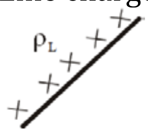
$$\bar{E} = \frac{\bar{F}}{Q}$$

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R = \frac{Q(\bar{r}-\bar{s}')}{4\pi\varepsilon_0|r-r'|^3}$$

$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\bar{r}-\bar{r}_k)}{|r-r_k|^3}$$

### 2.2.1 ELECTRIC FIELDS DUE TO CONTINOUS CHARGE DISTRIBUTIONS

1) Line charge:

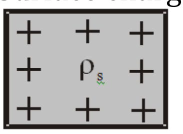



$$dQ = \rho_L\, dl \to Q = \int_L \rho_L dl$$

$$\bar{E} = \int_L \frac{\rho_L dl}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

Unit: $\rho_L$ :c/m

2) Surface charge:

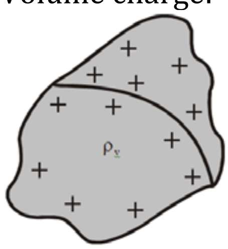



$$dQ = \rho_S dS \to Q = \int_S \rho_S dS$$

$$\bar{E} = \int_s \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

Unit: $\rho_s$ =c/m²

3) Volume charge:

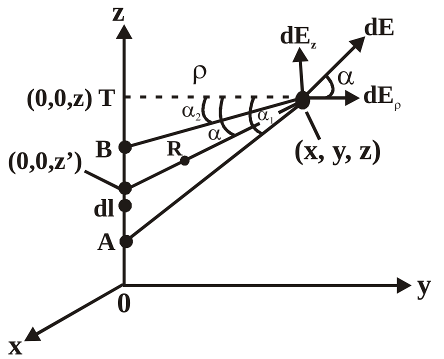


$$dQ = \rho_v dv \to Q = \int \rho_v dv$$

$$\bar{E} = \int_V \frac{\rho_v dV}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

Unit: volts/meter

### 2.2.2 ELECTRIC FIELD DUE TO A LINE CHARGE




$$E=\frac{\rho_L}{4\pi\varepsilon_0}\int\frac{\rho a_\rho+(z-z')a_z}{\left[\rho^2+(z-z')^2\right]^{3/2}}dz'$$

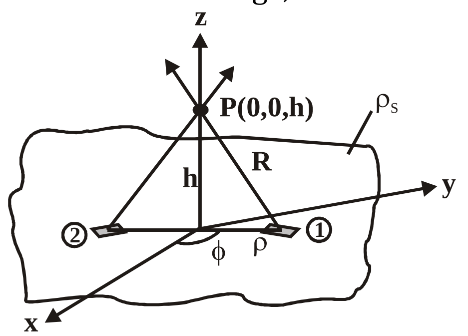
$$R=\left[\rho^2+(z-z')^2\right]^{1/2}=\rho\sec\alpha$$

$$z'=OT-\rho\tan\alpha,\ dz'=-\rho\sec^2\alpha\, d\alpha$$

$$E=\frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{\rho\sec^2\alpha\left[\cos\alpha\hat{a}_\rho+\sin\alpha\hat{a}_z\right]d\alpha}{\rho^2\sec^2\alpha}$$

$$=-\frac{\rho_L}{4\pi\varepsilon_0\rho}\int_{\alpha_1}^{\alpha_2}\left[\cos\alpha\hat{a}_\rho+\sin\alpha\,\hat{a}_z\right]d\alpha$$

Thus for a finite line charge,

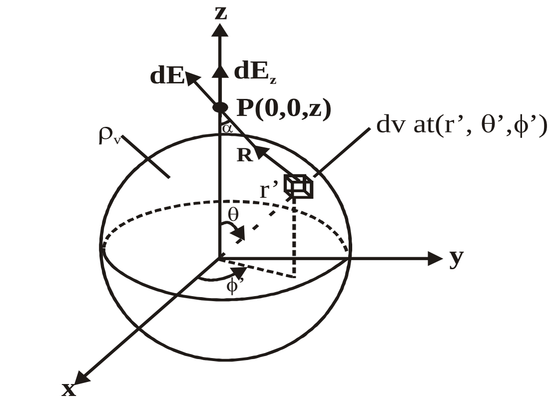



$$E=\frac{\rho_L}{4\pi\varepsilon_0\rho}\left[-(\sin\alpha_2-\sin\alpha_1)\hat{a}_\rho+(\cos\alpha_2-\cos\alpha_1)\hat{a}_z\right]$$

As a special case, for an finite line charge, point B is at (0,0,∞) and A at (0,0,- ∞) so that $\alpha_1=\pi/2$, $\alpha_2=-\pi/2$; the z-component vanishes and eq. Becomes

$$\bar{E}=\frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho \text{ volts/meter}$$

### 2.2.3 ELECTRIC FIELD DUE TO A SURFACE CHARGE




$$dQ=\rho_S dS$$

$$Q=\int_S\rho_S dS$$

$$\overline{dE}=\frac{dQ}{4\pi\varepsilon_0R^2}\hat{a}_R$$

$$\bar{R}=\rho(-\hat{a}_\rho)+h\hat{a}_z$$

$$|R|=\left[\rho^2+h^2\right]^{1/2}$$

$$\hat{a}_R=\frac{\bar{R}}{|R|}\quad dQ=\rho_s dS=\rho_s\rho d\varphi d\rho$$

$$\overline{dE}=\frac{\rho_s\rho d\varphi d\rho\left[-\rho\hat{a}_\rho+h\hat{a}_z\right]}{4\pi\varepsilon_0\left[\rho^2+h^2\right]^{3/2}}$$

$$\bar{E}=\int dE_z=\frac{\rho_S}{4\pi\varepsilon_0}\int_{\varphi=0}^{2\pi}\int_{\rho=0}^{\infty}\frac{h\rho\, d\rho\, d\varphi}{\left[\rho^2+h^2\right]^{3/2}}\hat{a}_z$$

$$=\frac{\rho_s h}{4\pi\varepsilon_0}2\pi\int_0^\infty\left[\rho^2+h^2\right]^{-3/2}\frac{1}{2}d(\rho^2)\hat{a}_z$$

$$=\frac{\rho_s h}{2\varepsilon_0}\left\{-\left[\rho^2+h^2\right]^{-1/2}\right\}_0^\infty\hat{a}_z$$

$$\bar{E}=\frac{\rho_S}{2\varepsilon_0}\hat{a}_z \text{ or } \bar{E}=\frac{\rho_S}{2\varepsilon_0}\hat{a}_n$$

For conducting charged plate

$$\bar{E}=\frac{\rho_S}{2\varepsilon_0}\hat{a}_n+\frac{-\rho_S}{2\varepsilon_0}(-\hat{a}_n)=\frac{\rho_S}{\varepsilon_0}\hat{a}_n \text{ volts/meter}$$

### 2.2.4 ELECTRIC FIELD DUE TO A VOLUME CHARGE

$$dQ=\rho_v dv$$

$$Q = \int_V \rho_v dv = \rho_v \int_V dV$$

$$= \rho_v \frac{4\pi a^3}{3}$$

$$\overline{dE} = \frac{\rho_v dv}{4\pi\varepsilon_0 R^2} \hat{a}_R$$

$$dv = r^2 \sin\theta dr\, d\theta\, d\varphi$$

$$\overline{E}_z = E.\hat{a}_z = \int dE \cos\alpha = \frac{\rho_v}{4\pi\varepsilon_0} \int \frac{dv \cos\alpha}{R^2}$$

$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 z^2} \hat{a}_z$$

$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{a}_r$$

### 2.2.5 ELECTRIC FLUX DENSITY

$\overline{\ } = \varepsilon_0 \overline{\ }$ : units:c/m$^2$

$\Psi = \int_S D.dS$ (Total Flux)

$\overline{D} = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho$ (Line charge)

$\overline{D} = \frac{\rho_S}{2} \hat{a}_n$ (Sheet charge)

$\overline{D} = \int \frac{\rho_v dv}{4\pi R^2} \hat{a}_R$ (Volume charge)

### 2.3 GAUSS'S LAW – MAXEWELL'S EQUATION

**Gauss's law** states that the total electric flux $\psi$ through any closed surface is equal to the total charge enclosed by that surface.

$$\Psi = Q_{enc}$$

$$\Psi = \oint d\Psi = \oint_S D.dS$$

$$= \text{Total charge enclosed } Q = \int \rho_v dv$$

$$Q = \oint_S \overline{D}.\overline{dS} = \int_V \rho_v dv$$

$$\oint_S \overline{D}.\overline{dS} = \int_V \nabla.D\, dv$$

$\rho_v = \nabla.D$ ; $\rho_v$ =volume charge density

Gauss's law provides an easy means of finding $\overline{\ }$ or $\overline{\ }$ for symmetrical charge distributions such as a point charge, an infinite line charge, an infinite cylindrical surface charge, and a spherical distribution of charge. A continues charge distribution.

### 2.3.1 APPLICATIONS OF GAUSS'S LAW (Find out electric flux density)

**A. Point Charge**

$$Q = \oint D.dS = D_r \oint dS = D_r 4\pi r^2$$

$$\overline{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

**B. Infinite Line Charge**

$$\rho_L l = Q = \oint \overline{D}.\overline{dS} = D_\rho \oint dS = D_\rho 2\pi\rho l$$

$$\overline{D} = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho$$

**C. Infinite Sheet of Charge**

$$\rho_S \int_S dS = Q = \oint D.dS = D_z \left[ \int_{top} dS + \int_{bottom} dS \right]$$

$$\rho_S A = D_z (A + A)$$

$$\overline{D} = \frac{\rho_S}{2} \hat{a}_z$$

$$\overline{E} = \frac{\overline{D}}{\varepsilon_0} = \frac{\rho_S}{2\varepsilon_0} \hat{a}_z$$

**D. Uniformly Charged Sphere**

$$Q_{enc} = \int_V \rho_v dv = \rho_v \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{r} r^2 \sin\theta dr\, d\theta\, d\varphi$$

$$= \rho_v \frac{4}{3} \pi r^3$$

$$\Psi = \oint D.dS = D_r \oint dS = D_r \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta\, d\theta\, d\varphi$$

$$D_r 4\pi r^2 = \frac{4\pi r^3}{3} \rho_v$$

$$D = \frac{r}{3} \rho_v a_r, 0 < r \le a$$

$$Q_{enc} = \int \rho_v dv = \rho_v \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{a} r^2 \sin\theta dr\, d\theta\, d\varphi$$

$$= \rho_v \frac{4}{3} \pi a^3$$

$$\Psi = \oint D.dS = D_r 4\pi r^2$$

$$\overline{D} = \begin{cases} \frac{r}{3}\rho_v \hat{a}_r & 0 < r \le a \\ \frac{a^3}{3r^2}\rho_v \hat{a}_r & r \ge a \end{cases}$$

## 2.4 ELECTRIC POTENTIAL

Potential of a point in electrical field is a work done to carry a unit change from infinite to that point.

$$dW = -\overline{F}.\overline{dI} = -Q\overline{E}.\overline{dI}$$

$$W = -Q\int_{\infty}^{r} \overline{E}.\overline{dI}$$

$$V_{AB} = \frac{W}{Q} = -\int_{\infty}^{r} \overline{E}.\overline{dI}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r$$

$$V_{AB} = -\int_{rA}^{rB} \frac{Q}{4\pi\varepsilon_0 r^2} a_r.dr\,\hat{a}_r$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_{AB} = V_B - V_A;\ V_{final} - V_{initial}$$

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

The **potential** at any point is the potential difference between that point and a chosen point at which the potential is zero. $(r = \infty)$

$$V = -\int_{\infty}^{r} \overline{E}.\overline{dI}$$

$$V(r) = \frac{Q}{4\pi\varepsilon_0 |r - r'|}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{n} \frac{Q_k}{|r - r_k|} \quad \text{(point charges)}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\rho_L(r')dl'}{|r - r'|} \quad \text{(line charge)}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_S(r')dS'}{|r - r'|} \quad \text{(surface charge)}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho_v(r')dv'}{|r - r'|} \quad \text{(volume charge)}$$

### 2.4.1 RELATIONSHIP BETWEEN E AND V AND MAXWELL'S EQUATION

$$V_{BA} = -V_{AB}$$

i.e. $V_{BA} + V_{AB} = \oint \overline{E}.\overline{dI} = 0$

$$\oint \overline{E}.\overline{dI} = 0$$

$$\oint \overline{E}.\overline{dI} = \int \left(\overline{\nabla \times E}\right).\overline{dS} = 0$$

$$\nabla \times \overline{E} = 0$$

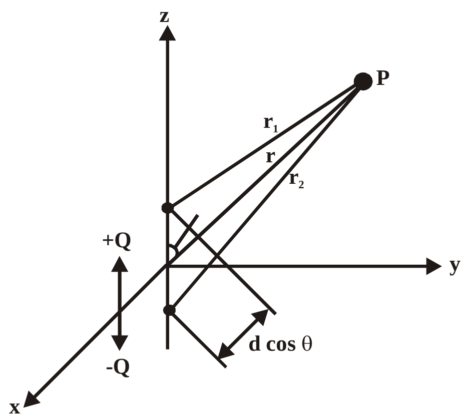
$$dV = -E.dI = -E_x dx - E_y dy - E_z dz$$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}$$

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

$$E = -\nabla V$$

## 2.5 ELECTIC DIPOLE




An electrical dipole is formed when a small distance separates two point charges of equal magnitude but opposite sign.

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2}$$

$(\because r_1 > r \text{ and } r_2 > r \Rightarrow r_1 r_2 \approx r^2)$

$\overline{\ } = Q\overline{\ }$

dipole moment may be written as

$$V = \frac{\bar{p}.\hat{a}_r}{4\pi\varepsilon_0 r^2}$$

$$E = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta\right]$$

$$= \frac{Qd\cos\theta}{2\pi\varepsilon_0 r^3}\hat{a}_r + \frac{Qd\sin\theta}{4\pi\varepsilon_0 r^3}\hat{a}_\theta$$

$$\bar{E} = \frac{\bar{p}}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right)$$

An **electric flux line** is an imaginary path or line drawn in such a way that its direction at any point is the direction of the electric field at that point.

### 2.6 ENERGY DENSITY IN ELECTROSTATIC FIELDS

Work done to carry charges $Q_1$, $Q_2$ and $Q_3$ in a free space

$W_E = W_1 + W_2 + W_3$

$= 0 + Q_2 V_{21} + Q_3 (V_{31} + V_{32})$

If the charges were positional in reverse order,

$W_E = W_3 + W_2 + W_1$

$= 0 + Q_2 V_{23} + Q_1 (V_{12} + V_{13})$

$2W_E = Q_1(V_{12}+V_{13})+Q_2(V_{21}+V_{23})+Q_3 (V_{31}+V_{32})$

$= Q_1 V_1 + Q_2 V_2 + Q_3 V_3$

$$W_E = \frac{1}{2}(Q_1 V_1 + Q_2 V_2 + Q_3 V_3)$$

$$W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k \qquad \text{(in joules)}$$

$$W_E = \frac{1}{2}\int \rho_L V dl \qquad \text{(line charge)}$$

$$W_E = \frac{1}{2}\int \rho_S V dS \qquad \text{(surface charge)}$$

$$W_E = \frac{1}{2}\int \rho_V dV \qquad \text{(volume charge)}$$

$$W_E = \frac{1}{2}\int_V (\nabla.D) V.dv$$

from Maxwell's eq.

$\nabla.VA = A.\nabla V + V(\nabla.A)$ by vector identities

$$(\nabla.A)V = \nabla.VA - A.\nabla V$$

$$W_E = \frac{1}{2}\int_V (\nabla.VD)\,dv - \frac{1}{2}\int_V (D.\nabla V)dv$$

$$W_E = \frac{1}{2}\oint_V (VD).dS - \frac{1}{2}\int_V (D.\nabla V)dv$$

$$W_E = -\frac{1}{2}\int_V (D.\nabla V)\,dv = \frac{1}{2}\int_V (D.E)\,dv$$

$$W_E = \frac{1}{2}\int D.E\,dv = \frac{1}{2}\int \varepsilon_0 E^2\,dv$$

The **current density** at a given point is the current through a unit normal area at that point.

= - e

$$\frac{mu}{\tau} = -e\bar{E}$$

$$u = -\frac{e\tau}{m}\bar{E}$$

$$\rho_V = -ne$$

Thus the conduction current density is

$$= \rho_V u = \frac{ne^2\tau}{m}E = \sigma\bar{E}$$

### 2.6.1 CONDUCTORS

A **perfect conductor** cannot contain an electrostatic filed within it.

$= 0, \quad \rho_v = 0,$

$V_{ab} = 0$ inside a conductor

$$E = \frac{V}{l}$$

$$J = \frac{I}{S} = \sigma E = \frac{\sigma V}{l}$$

$$R = \frac{V}{I} = \frac{l}{\sigma S} = \frac{\rho_c l}{S}$$

$$R = \frac{V}{I} = \frac{\int E.dI}{\int \sigma E.dS}$$

$\int \rho_v dv E.u = \int E.\rho_v u\,dv$ Rate of change of energy

$P = \int E.Jdv$

Which is known as Joule's law. The power density $W_p$ (in watts/$m^3$) is given by the integrand in eq. That is,

$$W_P = \frac{dP}{dv} = \bar{E}.\bar{J} = \sigma|E|^2 \quad (\because \bar{J}=\sigma\bar{E})$$

## 2.7 CONTINUITY EQUATION

$$I_{out} = \oint \bar{J}.\overline{dS} = \frac{-dQ_{in}}{dt}$$

$$\oint_S \bar{J}.dS = \int_V \nabla.\bar{J}dv$$

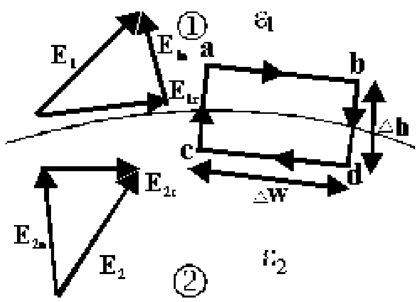
$$\frac{-dQ_{in}}{dt} = -\frac{d}{dt}\int_V \rho_v dv = -\int_V \frac{\partial \rho_v}{\partial t} dv$$

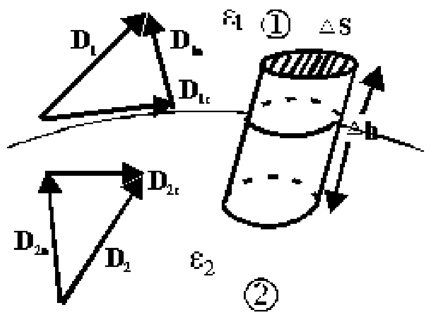
$$\int_V \nabla.\bar{J}dv = -\int_V \frac{\partial \rho_v}{\partial t} dv$$

$$\nabla.\bar{J} = -\frac{\partial \rho_V}{\partial t}$$

Current continuity Eq.

## 2.8 BOUNDRAY CONDITIONS




- Dielectric ($\varepsilon_{r1}$) and dielectric ($\varepsilon_{r2}$)
- Conductor and dielectric
- Conductor and free space

$$\oint \bar{E}.\overline{dl} = 0$$

$$\oint \bar{D}.\overline{dS} = Q_{enc} \quad \text{(Gauss Law)}$$

$$E = E_t + E_n$$

$$0 = E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w$$

$$+E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2}$$

$E_{1t} = E_{2t}$

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

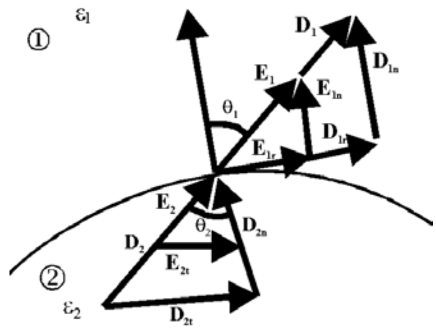
$$\frac{D_{it}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2} \quad .......(1)$$

$$\Delta Q = \rho_S \Delta S = D_{1n}\Delta S - D_{2n}\Delta S \quad D_{1n} - D_{2n} = \rho_S$$

if $\rho_s = 0$ $\quad D_{1n} = D_{2n}$ $\quad$ ........(2)

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

## 2.9 POISSON'S & LAPLACE'S EQUATIONS




Poisson's and Laplace's equations are easily derived from Gauss's law (for a linear material medium)

$\nabla . D = \nabla . \varepsilon E = \rho_v$

where $E = -\nabla V$

$\nabla . (\varepsilon \nabla V) = \rho_v$

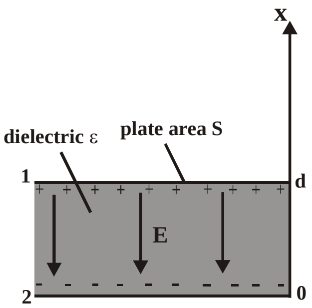
$\nabla^2 V = -\frac{\rho_V}{\varepsilon}$ (Poisson's Equation in electrostatics)

for homogeneous and linear medium.

$\nabla^2 V = 0$ Which is known as Laplace's equation.

## 2.10 CAPACITANCES

### A. Parallel- Plate Capacitor




$\sigma_S = \frac{Q}{S}$; S=area of cross section

$E = \frac{\rho S}{\varepsilon}(-\hat{a}_x)$ Electric filed between plates

$= -\frac{Q}{\varepsilon S}\hat{a}_x$ **V**

$= \int_2^1 E.dl = -\int_0^d \left[-\frac{Q}{\varepsilon S}a_x\right].dxa_x = \frac{Qd}{\varepsilon S}$

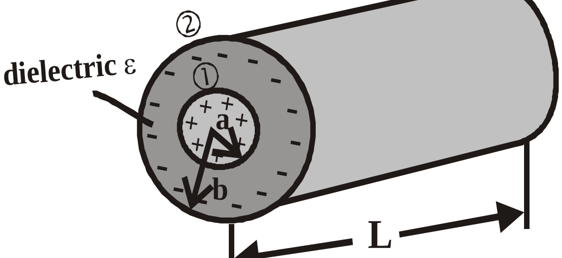
$C = \frac{Q}{V} = \frac{\varepsilon S}{d}$

$W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$

$W_E = \frac{1}{2}\int_V \varepsilon \frac{Q^2}{\varepsilon^2 S^2}dv = \frac{\varepsilon Q^2 Sd}{2\varepsilon^2 S^2}$

$= \frac{Q^2}{2}\left(\frac{d}{\varepsilon S}\right) = \frac{Q^2}{2C} = \frac{1}{2}QV$

**B. Coaxial Capacitor**




$Q = \varepsilon\oint \overline{E}.\overline{dS} = \varepsilon E_\rho 2\pi\rho L$

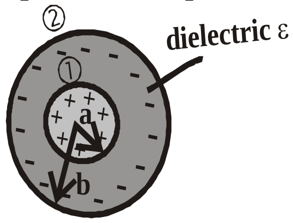
$E = \frac{Q}{2\pi\varepsilon\rho L}$

$V = -\int_2^1 \overline{E}.\overline{dl} = -\int_b^a \left[\frac{Q}{2\pi\rho L}\hat{a}_\rho\right].d_\rho\hat{a}_\rho$

$= \frac{Q}{2\pi\rho L}1n\frac{b}{a}$

$C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{1n\frac{b}{a}}$

**C. Spherical Capacitor**




$Q = \varepsilon\oint \overline{E}.\overline{dS} = \varepsilon E_r 4\pi r^2$

$\overline{E} = \frac{Q}{4\pi\varepsilon r^2}\hat{a}_r$

$V = -\int_2^1 \overline{E}.\overline{dl} = -\int_b^a \left[\frac{Q}{4\pi\varepsilon r^2}\hat{a}_r\right].dr\hat{a}_r$

$= \frac{Q}{4\pi\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$

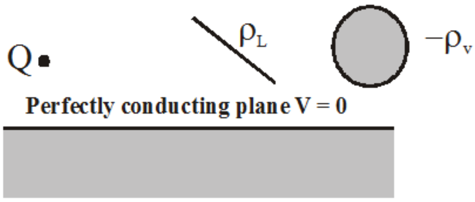
$C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$

$R = \frac{V}{I} = \frac{\int \overline{E}.\overline{dl}}{\oint \sigma\overline{E}.\overline{dS}}$

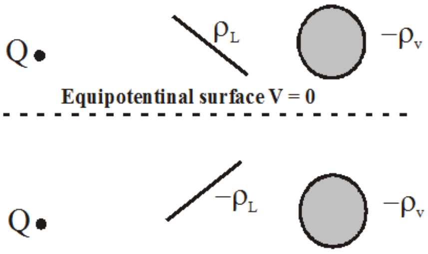
$C = \frac{Q}{V} = \frac{\varepsilon\oint \overline{E}.\overline{dS}}{\int \overline{E}.\overline{dl}}$

$RC = \frac{\varepsilon}{\sigma}$ =Relaxation Time

## 2.11 METHOD OF IMAGES



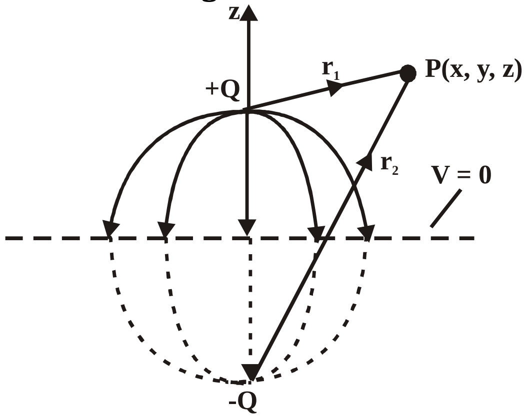

(a)




(b)

The **image theory** states that a given charge configuration above an infinite grounded perfect conducting plane may be replaced by the charge configuration itself, its image, and equipotential surface in place of the conducting plane.

**A. A Point Charge Above Grounded Conducting Plane**




$$\overline{E} = \overline{E}_{+} + \overline{E}_{-}$$

$$= \frac{Qr_1}{4\pi\varepsilon_0 r_1^3} + \frac{-Qr_2}{4\pi\varepsilon_0 r_2^3}$$

$$\overline{E} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{x\hat{a}_x + y\hat{a}_y + (z-h)\hat{a}_z}{\left[x^2+y^2+(z-h)^2\right]^{3/2}} - \frac{x\hat{a}_x + y\hat{a}_y + (z+h)\hat{a}_z}{\left[x^2+y^2+(z+h)^2\right]^{3/2}}\right]$$

$$V = V_{+} + V_{-}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left\{\frac{1}{\left[x^2+y^2+(z-h)^2\right]^{1/2}} - \frac{1}{\left[x^2+y^2+(z+h)^2\right]^{1/2}}\right\}$$

$$\rho_S = D_n = \varepsilon_0 E_n \big|_{z=0}$$

$$= \frac{-Qh}{2\pi\left[x^2+y^2+h^2\right]^{3/2}}$$

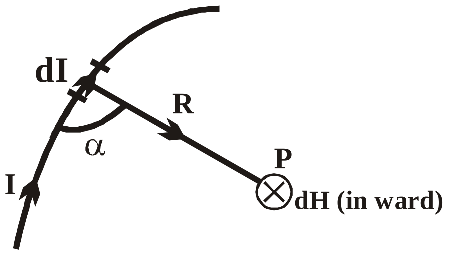
# 3 MAGNETOSTATICS

## 3.1 INTRODUCTION

Magnetostatics is the study of magnetic fields in systems where the currents are steady (not changing with time). It is the magnetic analogue of electrostatics, where the charges are stationary. The magnetization need not be static; the equations of magnetostatics can be used to predict fast magnetic switching events that occur on time scales of nanoseconds or less.

| Term | Electric | Magnetic |
|---|---|---|
| Basic law | $F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}\hat{a}_r$ (Coulomb's Law)<br>$\oint \overline{D}.\overline{dS} = Q_{enc}$ (Gauss Law) | $\overline{dB} = \frac{\mu_0 I \overline{dI} \times \hat{a}_R}{4\pi R^2}$ (Biot-Savart's Law)<br>$\oint \overline{H}.\overline{dI} = I_{enc}$ (Ampere's Law) |
| Force law | $\overline{F} = Q\overline{E}$ | $\overline{F} = Q\overline{u} \times \overline{B}$ |
| Source element | dQ | $Q\overline{u} = I\,\overline{dI}$ |
| Field intensity | $E = \frac{V}{l}(V/m)$ | $H = \frac{I}{l}(A/m)$ |
| Flux density | $D = \frac{\Psi}{S}(C/m^2)$ | $B = \frac{\Psi}{S}(Wb/m^2)$ |
| Relationship between fields | $D = \varepsilon E$ | $B = \mu H$ |
| Potentials | $E = -\nabla V$<br>$V = \int \frac{\rho_L dl}{4\pi\varepsilon r}$ | $H = -\nabla V_m$ $(J = 0)$<br>$A = \int \frac{\mu I dI}{4\pi R}$ |
| Flux | $\Psi = \int \overline{D}.\overline{dS}$<br>$\Psi = Q = CV$<br>$I = C\frac{dV}{dt}$ | $\Psi = \int \overline{B}.\overline{dS}$<br>$\Psi = LI$<br>$V = L\frac{dI}{dt}$ |
| Energy density | $W_E = \frac{1}{2}\overline{D}.\overline{E}$ | $W_m = \frac{1}{2}\overline{B}.\overline{H}$ |
| Poisson's equation | $\nabla^2 V = -\frac{\rho_v}{\varepsilon}$ | $\nabla^2 A = -\mu J$ |

### 3.2 BIOT – SAVATRT'S LAW




**Biot – Savart's law** states that the magnetic filed intensity dH produced at a point P, as shown in Figure by the differential current element I d*l*

$$dH\alpha\frac{I\,dl\sin\alpha}{R^2}$$

$$dH=\frac{kI\,dl\sin\alpha}{R^2}$$

$$dH=\frac{I\,dl\sin\alpha}{4\pi R^2}$$

$$\overline{dH}=\frac{I\,\overline{dl}\times\hat{a}_R}{4\pi R^2}=\frac{I\,\overline{dI}\times\overline{R}}{4\pi R^3}$$

Current elements

I dI = K dS = J dv

$$\overline{H}=\int_L\frac{I\,\overline{dl}\times\hat{a}_R}{4\pi R^2}\qquad\text{(line current)}$$

$$\overline{H}=\int_S\frac{K\,\overline{dS}\times\hat{a}_R}{4\pi R^2}\qquad\text{(surface current)}$$

$$\overline{H}=\int_V\frac{\overline{J}\,dv\times\hat{a}_R}{4\pi R^2}\qquad\text{(volume current)}$$

### 3.3 MAGNETIC FILED INTENSITY

#### 1) Due to current carrying conductor

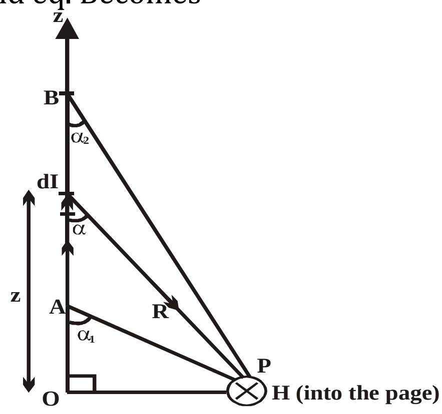
$$\overline{dH}=\frac{I\,\overline{dl}\times\overline{R}}{4\pi R^3}$$

But $\overline{dl}$ = dz $\hat{a}_z$ and $\overline{R}$ = $\rho\hat{a}_\rho-z\hat{a}_z$, so

$$\overline{dI}\times\overline{R}=\rho\,dz\,\hat{a}_\varphi$$

$$\overline{H}=\int\frac{I\rho\,dz}{4\pi\left[\rho^2+z^2\right]^{3/2}}\hat{a}_\varphi$$

Letting z = ρ cot α, dz = - ρ cosec² α dα, and eq. Becomes




$$\overline{H}=-\frac{I}{4\pi}\int_{\alpha_2}^{\alpha_1}\frac{\rho^2\operatorname{cosec}^2\alpha\,d\alpha}{\rho^3\operatorname{cosec}^3\alpha}\hat{a}_\varphi$$

$$=-\frac{I}{4\pi\rho}\hat{a}_\varphi\int_{\alpha_1}^{\alpha_2}\sin\alpha\,d\alpha$$

$$\overline{H}=\frac{I}{4\pi\rho}(\cos\alpha_2-\cos\alpha_1)\hat{a}_\varphi$$

General eq.

$$\overline{H}=\frac{I}{2\pi\rho}\hat{a}_\phi$$

due to a infinite length conductor

$$\hat{a}_\varphi=\hat{a}_l\times\hat{a}_\rho$$

#### 2) Due to a ring current conductor

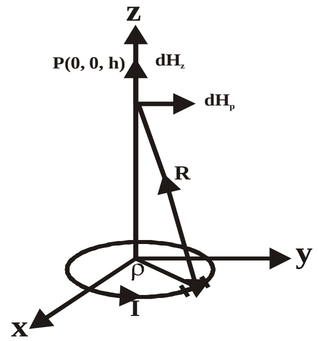
$$\overline{dH}=\frac{I\,\overline{dI}\times\overline{R}}{4\pi R^3}$$

where $\overline{dI}=\rho\,d\varphi\,\hat{a}_\varphi, R=(0,0,h)-(x,y,0)$

$=-\rho\hat{a}_\rho+h\hat{a}_z$ and

$$\overline{dI}\times\overline{R}=\begin{vmatrix}\hat{a}_\rho & \hat{a}_\varphi & \hat{a}_z\\ 0 & \rho\,d\varphi & 0\\ -\rho & 0 & h\end{vmatrix}$$

$$=\rho h\,d\varphi\,\hat{a}_\rho+\rho^2\,d\varphi\,\hat{a}_z$$



$$\overline{dH} = \frac{I}{4\pi\left[\rho^2 + h^2\right]^{3/2}}\left(\rho h\, d\varphi\, \hat{a}_\rho + \rho^2 d\varphi\, \hat{a}_z\right)$$

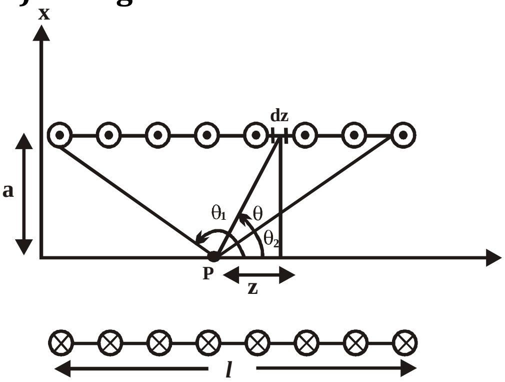
$$= dH_\rho \hat{a}_\rho + dH_z \hat{a}_z$$

$$\overline{H} = \int dH_z\, \hat{a}_z = \int_0^{2\pi} \frac{I\rho^2 d\varphi\, \hat{a}_z}{4\pi\left[\rho^2 + h^2\right]^{3/2}}$$

$$= \frac{I\rho^2\, 2\pi\, \hat{a}_z}{4\pi\left[\rho^2 + h^2\right]^{3/2}}$$

$$\overline{H} = \frac{I\rho^2 \hat{a}_z}{2[\rho^2 + h^2]^{3/2}}$$

**3) Along the axis of solenoid**




$$\overline{H} = \frac{NI}{2l}\left(\cos\theta_2 - \cos\theta_1\right)\hat{a}_z$$

$$dH_z = \frac{I\, dl\, a^2}{2\left[a^2 + z^2\right]^{3/2}} = \frac{I a^2\, n\, dz}{2\left[a^2 + z^2\right]^{3/2}}$$

where dL = ndz = (N/*l*) dz. tan θ = a /z; that is,

$$dz = -\, a\, \text{cosec}^2 q dq$$

$$= -\frac{\left[z^2 + a^2\right]^{3/2}}{a^2} \sin\theta\, d\theta$$

$$dH_z = -\frac{nI}{2}\sin\theta\, d\theta$$

$$H_z = -\frac{nI}{2}\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$H = \frac{nI}{2}\left(\cos\theta_2 - \cos\theta_1\right)\hat{a}_z$$

as required. Substituting n = N / *l* gives

At the center of the solenoid,

$$\cos\theta_2 = \frac{l/2}{\left[a^2 + l^2/4\right]^{1/4}} = -\cos\theta_1$$

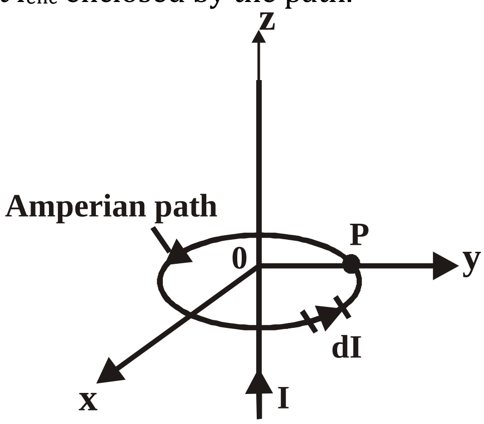
$$H = \frac{Inl}{2\left[a^2 + l^2/4\right]^{1/2}}\hat{a}_z$$

If l >> a or $Q_2 = 0^0$, $Q_1 = 80^0$

$$= n\, I\, \hat{a}_z = \frac{NI}{l}\hat{a}_z$$

### 3.4 AMPERE'S CIRCUIT LAW – MAXEWLL'S EQUATION

**Ampere's circuit law** states that the line integral of the tangential component of **H** around a closed **path** is the same as the net current $I_{enc}$ enclosed by the path.




$$\oint \overline{H}.\overline{dI} = I_{enc}$$

$$I_{enc} = \oint_L \overline{H}.\overline{dI} = \int_S \left(\nabla \times \overline{H}\right).\, dS$$

$$I_{enc} = \int_S \overline{J}.\overline{dS}$$

$\nabla \times \overline{H} = \overline{J}$ =Volume current density (A/m$^2$)

Maxwell's third Eq.

### 3.4.1 APPLICATIONS OF AMPERE'S LAW

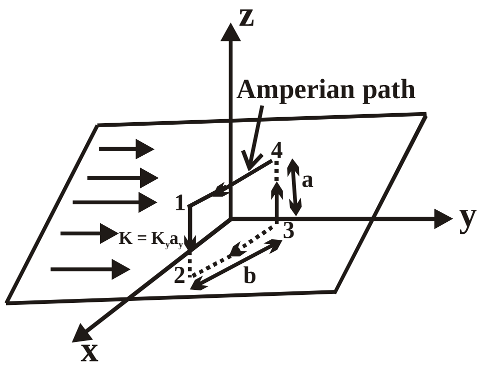
To determine $\overline{H}$ for some symmetrical current distribution (as in Gauss's Law)

**A. Infinite Line Current**

$$I = \int H_\varphi \hat{a}_\varphi . \rho\, d\varphi\, \hat{a}_\varphi = H_\varphi \int \rho\, d\varphi = H_\varphi . 2\pi\rho$$

$$\overline{H} = \frac{I}{2\pi\rho}\hat{a}_\varphi$$

**B. Infinite Sheet of Current**




Uniform current density $\overline{K} = k_y\,\hat{a}_y$

$$\oint H.dI = I_{enc} = K_y b$$

$$\overline{H} = \begin{cases} H_0\hat{a}_x & z>0 \\ -H_0\hat{a}_x & z<0 \end{cases}$$

$$\oint H.dI = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1\right) H.dI$$

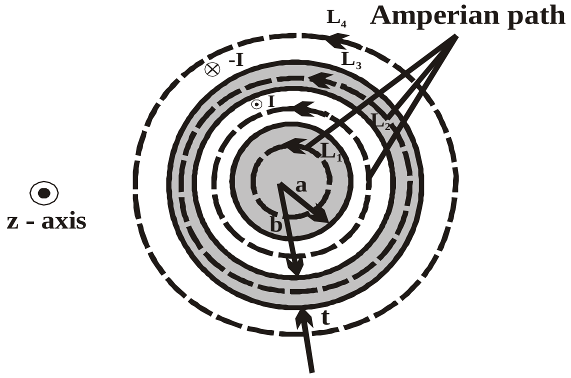
$$= 0(-a) + (-H_0)(-b) + 0(a) + H_0(b)$$

$$= 2H_0 b$$

$$\overline{H} = \begin{cases} \frac{1}{2}K_y\hat{a}_x, & z>0 \\ -\frac{1}{2}K_y\hat{a}_x, & z<0 \end{cases}$$

$$\overline{H} = \frac{1}{2}K \times \hat{a}_n$$

## C. Infinity Long Coaxial Transmission Line




For region $0 \le \rho \le a$, we apply Ampere's law to path $L_1$, giving

$$\oint_{L_1} \overline{H}.\overline{dI} = I_{enc} = \int \overline{J}.\overline{dS}$$

$$\overline{J} = \frac{I}{\pi a^2}\hat{a}_z, \overline{dS} = \rho\, d\varphi\, d\rho\, \hat{a}_z$$

$$I_{enc} = \int \overline{J}.\overline{dS} = \frac{I}{\pi a^2}\int\int \rho\, d\varphi\, d\rho$$

$$= \frac{I}{\pi a^2}\pi\rho^2 = \frac{I\rho^2}{a^2}$$

$$H_\varphi \oint dl = H_\varphi 2\pi\rho = \frac{I\rho^2}{a^2}$$

$$H_\phi = \frac{I\rho}{2\pi a^2}$$

For region $a \le \rho \le b$, we use path $L_2$ as the Amperian path.

$$\oint_{L_1} H.dI = I_{enc} = I$$

$$H_\phi 2\pi\rho = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

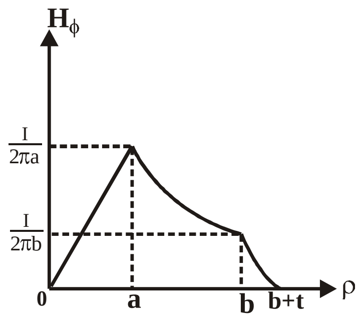
For region $b \le \rho \le (b + t)$, we use path $L_3$, getting

$$\oint H.dI = H_\phi.2\pi\phi = I_{enc}$$

$$I_{enc} = I\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

$$H_\varphi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

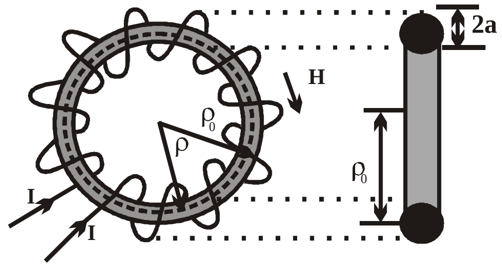
$$\oint_{L_4} H.dI = I - I = 0$$




$$\overline{H} = \begin{cases} \frac{I\rho}{2\pi a^2}\hat{a}_\varphi, & 0 \le \rho \le a \\ \frac{I}{2\pi\rho}\hat{a}_\varphi, & a \le \rho \le b \\ \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2 - 2bt}\right]\hat{a}_\varphi, & b \le \rho \le b+t \\ 0, & \rho \ge b+t \end{cases}$$

## D. TOROID

It has N turns and carries current I. Determine H inside and outside the toroid




$$\oint H.dI = I_{enc} \rightarrow H.2\pi\rho = NI$$

$$H = \frac{NI}{2\pi\rho}, \text{for } (\rho_0 - a) < \rho < (\rho_0 + a)$$

$$H_{apporax} = \frac{NI}{2\pi\rho_0} = \frac{NI}{l}$$

### 3.5 MAGNETIC FLUX DENSITY – MAXWELL'S EQUATION

$\overline{B} = \mu_0 \mu_R \overline{H}$

Where $\mu_0$ is a constant known as the permeability of free space. The constant is in henrys/meter (H/m) and has the value of

$\mu_0 = 4\pi\times10^{-7}$ H/m

Magnetic flux through a surface S

$$\psi = \int_S \overline{B}.\overline{dS}$$

An **isolated magnetic** charge does not exist.

$$\oint B.dS = 0$$

$$\oint_S B.dS = \int_V \nabla.B\,dv = 0$$

$\nabla.B = 0$ fourth Maxwell's eq.

**MAXWELL'S EQUATIONS FOR STATIC EM FILEDS**

| **Differential (or point) Form** | **Integral Form** | **Remarks** |
|---|---|---|
| $\nabla.D = \rho_v$ | $\oint_S D.dS = \int_v \rho_v dv$ | Gauss's law |
| $\nabla.B = 0$ | $\oint_S B.dS = 0$ | Nonexistence of magnetic monopole |
| $\nabla\times E = 0$ | $\oint_L E.dI = 0$ | Conservations of Electrostatic filed |
| $\nabla\times H = J$ | $\oint_L H.dI = \int_S J.dS$ | Ampere's law |

**MAGNETIC SCLAR & VECTOR POTENTIALS**

$E = -\nabla V$, Similarly magnetic scalar potential $V_m$ related to H

$H = -\nabla V_m$ if J = 0

$J = \nabla\times H = \nabla\times(-\nabla V_m) = 0$

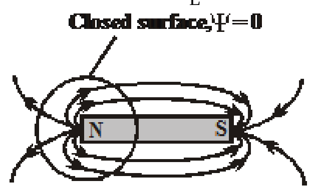
$\nabla^2 V_m = 0$, if (J = 0)

Magnetic vector potential A related as

$\overline{B} = \nabla\times\overline{A}$

$$V = \int_L \frac{dQ}{4\pi\varepsilon_0 r}$$

Similarly $A = \int_L \frac{\mu_0 I\,dI}{4\pi R}$ for line current




$$A = \int_S \frac{\mu_0 K\,dS}{4\pi R}$$

for surface current

$$A = \int_V \frac{\mu_0 J dv}{4\pi R}$$

for volume current

From Stokes's Theorem

$$\psi = \int_S \overline{B}.\overline{dS} = \int_S \left(\overline{\nabla x A}\right).\overline{dS} = \oint_L \overline{A}.\overline{dI}$$

$\Psi = \oint_L A.dI$ flux through a given area

### 3.6 FORCE DUE TO MAGNETIC FILED

**A. Force on a Charged Particle**

$\overline{F} = Q\left(\overline{E} + \left(\overline{u}\times\overline{B}\right)\right)$ Lorentz force for magnetic fields and electric fields

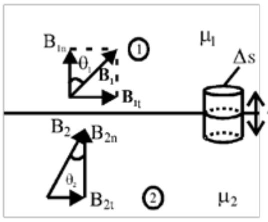
**B. Force on a Current Element**

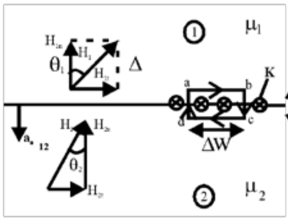
$$F = \oint_L I\,dI\times B$$

**C. Force between Two Current Elements**

$$\overline{F_1} = \frac{\mu_0 I_1 I_2}{4\pi}\oint_{L_1}\oint_{L_2}\frac{\overline{dI_1}\times\left(\overline{dI_2}\times\hat{a}_{R_{21}}\right)}{R_{21}}$$

## 3.7 MAGNETIC TORQUE AND MOMENT





The **torque T** (or mechanical moment of force) on the loop is the vector product of the force **F** and the moment arm **r.**

T = r × F

T = | F0 | ω sin α

T = BI *l* w sin α

m = I S an.

The **magnetic dipole moment** is the product of current and area of the loop; its direction is normal to the loop.

T = m × B

### A MAGNETIC DIPOLE

$$A = \frac{\mu_0 I}{4\pi}\oint \frac{dI}{r}$$

$$A = \frac{\mu_0 m \times a_r}{4\pi r^2}$$

$$\overline{B} = \frac{\mu_0 m}{4\pi \hat{r3}}\left(2\cos\theta \hat{a}_r + \sin\theta \hat{a}_\theta\right)$$

## 3.8 MAGNETIC BOUNDRAY CONDITIONS

$$\oint B.dS = 0$$

$$\oint \overline{H}.\overline{dI} = I$$

$B_{1n}\Delta S - B_{2n}\Delta S = 0$ for cylindrical close surface on boundary

$B_{1n} = B_{2n}$ or $\mu_1 H_{1n} = \mu_2 H_{2n}$

first Boundary conditions

$$K.\Delta w = H_{1t}.\Delta w + H_{1n}.\frac{\Delta h}{2} + H_{2n}.\frac{\Delta h}{2} - H_{2t}.$$

$$\Delta w - H_{2n}.\frac{\Delta h}{2} - H_{1n}.\frac{\Delta h}{2}$$

$$H_{1t} - H_{2t} = K$$

Second Boundary conditions

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$$

$$(H_1 - H_2) \times a_{n12} = K$$

if k = 0 $H_{1t} = H_{2t}$ or $\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$

$$B_1\cos\theta_1 = B_{1n} = B_{2n} = B_2\cos\theta_2$$

$$\frac{B_1}{\mu_1}\sin\theta_1 = H_{1t} = H_{2t} = \frac{B_2}{\mu_2}\sin\theta_2$$

$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$ Law of refraction for magnetic flux lines at a boundary with no surface current

## 3.9 INDUCTOR AND INDUCTANCES

$\lambda = N\Psi$ flux linkage $\lambda$

$\lambda \propto I$ or $\lambda = LI$

$L = \frac{\lambda}{I} = \frac{N\Psi}{I}$ inductance of circuit.

$$W_m = \frac{1}{2}LI^2$$

$$L = \frac{2W_m}{I^2}$$

$$\Psi_{12} = \int_{S_1} B_2.dS$$

flux linkage from S, due to I2

$$M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1\Psi_{12}}{I_2}$$

mutual inductance

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2\Psi_{21}}{I_1}$$

M12 = M21

$$L_1 = \frac{\lambda_{11}}{I_1} = \frac{N_1\Psi_1}{I_1}$$

$$L_2 = \frac{\lambda_{22}}{I_2} = \frac{N_2\Psi_2}{I_2}$$

Total magnetic energy in magnetic filed

$$W_m = W_1 + W_2 + W_3$$

$$= \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 \pm M_{12} I_1 I_2$$

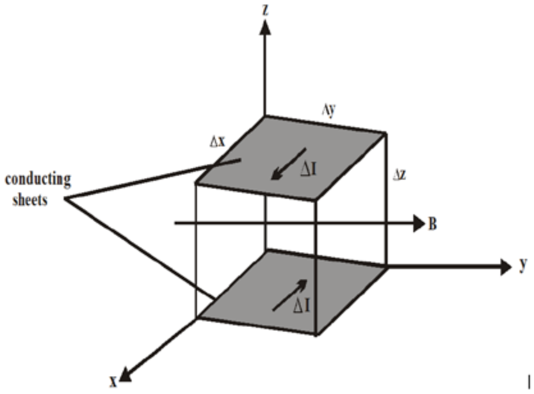
Relationship bet parameters

RC = $\frac{\varepsilon}{\sigma}$ and $L_{ext}$ C = με

## 3.10 MAGNETIC ENERGY

Sibilantly to the energy in an electrostatic filed.

$$W_E = \frac{1}{2}\int \overline{\mathbf{D}}.\overline{\mathbf{E}}\, dv = \frac{1}{2}\int \varepsilon E^2\, dv$$




$$W_m = \frac{1}{2} L I^2$$

Consider a differential volume in a magnetic field as shown in Figure. Let the volume be covered with conducting sheets at the top and bottom surface with current ΔI.

$$\Delta L = \frac{\Delta \Psi}{\Delta I} = \frac{\mu H\, \Delta x\, \Delta z}{\Delta I}$$

where $\Delta I = H\, \Delta y$. Substituting eq. into eq. we have

$$\Delta W_m = \frac{1}{2}\Delta L\, \Delta I^2 = \frac{1}{2}\mu H^2\, \Delta x\, \Delta y\, \Delta z$$

$$\Delta W_m = \frac{1}{2}\mu H^2\, \Delta v$$

The magnetostatic energy density $w_m$ ( in J /m$^3$) is defined as

$$w_m = \lim_{\Delta v \to 0} \frac{\Delta W_m}{\Delta v} = \frac{1}{2}\mu H^2$$

$$w_m = \frac{1}{2}\mu H^2 = \frac{1}{2} B\,.\,H = \frac{B^2}{2\mu}$$

Thus the energy in a magnetostatic field in a linear medium is

$$W_m = \int w_m\, dv$$

$$W_m = \frac{1}{2}\int B.H\, dv = \frac{1}{2}\int \mu H^2\, dv$$

# GATE QUESTIONS

**Q.1** The electric field on the surface of a perfect conductor is 2 V/m. The conductor is immersed in water with $\in = 80 \in_0$. The surface charge density on the conductor is

a) $0 C/m^2$

b) $2 C/m^2$

c) $1.8 \times 10-^{11} C/m^2$

d) $1.41 \times 10^{-9} C/m^2$

$(\in = 10^9)/(36\pi) F/m)$

**[GATE – 2002]**

**Q.2** if the electric field intensity is given by $E = (xu_x + yu_y + zu_z)$ volt/m the potential difference between X (2, 0, 0) and Y (1, 2, 3) is

a) + 1 volt b) – 1 volt

c) + 5 volt d) + 6 volt

**[GATE – 2003]**

**Q.3** The unit $\nabla \times H$ is

a) Ampere b) Ampere/meter

c) Ampere/meter² d) Ampere-meter

**[GATE – 2003]**

**Q.4** A parallel plate air – filled capacitor has plate area of $10^{-4}$ $m^2$ and plate separation of $10^{-3}$ $m$ .It is connected to a 0.5 V, 3.6 GHz source. The magnitude of the displacement current is

$(\varepsilon_0 = 1/36\pi \times 10^{-9} F/m)$

a) 10 mA b) 100 mA

c) 10 A d) 1.59 mA

**[GATE – 2004]**

**Q.5** If C is a closed curve enclosing a surface S, then the magnetic field intensity $\vec{H}$, the current density $\vec{J}$ and the electric flux density $\vec{D}$ are related by

a) $\iint_S \vec{H}.\vec{dS} = \oint_C \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right).\vec{dl}$

b) $\int_C \vec{H}.\vec{dl} = \oiint_S \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right).\vec{dS}$

c) $\oiint_S \vec{H}.\vec{dS} = \int_C \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right).\vec{dl}$

d) $\oint_C \vec{H}.\vec{dl} = \iint_S \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right).\vec{dS}$

**[GATE – 2007]**

**Q.6** For static electric and magnetic fields in an inhomogeneous source-free medium, which of the following represents the correct form of two of Maxwell's equations?

a) $\nabla.E = 0$

b) $\nabla.E = 0 \quad \nabla \times B = 0 \quad \nabla.B = 0$

c) $\nabla \times E = 0$

d) $\nabla \times E = 0 \quad \nabla \times B = 0 \quad \nabla.B = 0$

**[GATE – 2008]**

**Q.7** A magnetic field in air is measured to be $\vec{B} = B_0 \left( \frac{x}{x2+y2} \hat{y} - \frac{y}{x2+y2} \hat{x} \right)$ what current distribution leads to this field?

[Hint : the algebra is trivial in cylindrical coordinates.]

a) $\vec{j} = -\frac{B_0 \hat{Z}}{\mu_0} \left( \frac{1}{x^2+y^2} \right), r \neq 0$

b) $\vec{j} = -\frac{B_0 \hat{Z}}{\mu_0} \left( \frac{2}{x^2+y^2} \right), r \neq 0$

c) $\vec{j} = 0, r \neq 0$

d) $\vec{j} = \frac{B_0 \hat{Z}}{\mu_0} \left( \frac{1}{x^2+y^2} \right), r \neq 0$

**[GATE – 2009]**

**Q.8** If a vector field $\vec{V}$ is related to another vector field $\vec{A}$ through $\vec{V} = \nabla x \vec{A}$ which of the following is true? Note: C and $S_C$ refer to any closed contour and any surface whose boundary is C.

a) $\oint_C \vec{V}.\vec{dl} = \iint_{SC} \vec{A}.\vec{dS}$

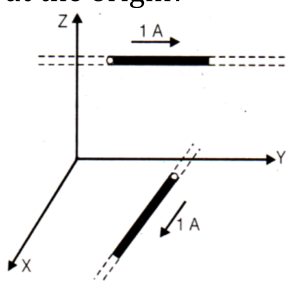
b) $\oint_C \vec{A}.\vec{dl} = \iint_{SC} \vec{V}.\vec{dS}$

c) $\oint_C \nabla \times V.\vec{dl} = \iint_{SC} \nabla \times \vec{A}.\vec{dS}$

d) $\oint_C \nabla \times \vec{A}.\vec{dl} = \iint_{SC} \vec{V} \times \vec{dS}$

**[GATE – 2009]**

**Q.9** Two infinitely long wires carrying current are as shown in the figure below. One wire is in the y-z plane and parallel to the y-axis. The other wire is in the x-y plane and parallel to the x-axis. Which components of the resulting magnetic field are non-zero at the origin?




a) x, y, z components
b) x, y components
c) y, z components
d) x, z components

**[GATE – 2009]**

**Q.10** Consider a closed surface S surrounding a volume V. If $\vec{r}$ is the position vector of a points inside S, with $\hat{n}$ the unit normal on S, the value of the integral $\oiint 5\vec{r}.\hat{n}$ dS S

a) 3 V b) 5 V
c) 10 V d) 15 V

**[GATE – 2011]**

**Q.11** The electric and magnetic fields for a TEM wave of frequency 14 GHz in a homogeneous medium of relative permittivity $\varepsilon_r$ and relative permeability $\mu_r = 1$ are given by

$\vec{E} = E_p e^{j(\omega t - 280\pi y)} \hat{u}_z V/m$

$\vec{H} = 3e^{j(\omega t - 280\pi y)} \hat{u}_x A/m$

Assuming the speed of light in free space to be $3\times10^8$ m/s, intrinsic impedance of free space to be $120\pi$, the relative permittivity $\varepsilon_r$ of the medium and the electric field amplitude $E_P$ are

a) $\varepsilon_r = 3,\ E_\rho = 120\pi$

b) $\varepsilon_r = 3,\ E_\rho = 360\pi$

c) $\varepsilon_r = 9,\ E_\rho = 360\pi$

d) $\varepsilon_r = 9,\ E_\rho = 120\pi$

**[GATE – 2011]**

**Statement for linked answer questions 12 and 13**

An infinitely long uniform solid wire of radius a carries a uniform dc current of density $\bar{j}$.

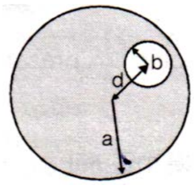
**Q.12** The magnetic field at a distance r from the centre of the wire is proportional to

a) r for r < a and $1/r^2$ for r > a
b) 0 for r < a and 1/r for r > a
c) r for r < a and 1/r for r > a
d) 0 for r < a and $1/r^2$ for r > a

**[GATE – 2012]**

**Q.13** A hole of radius b(b<a) is now drilled along the length of the wire at a distance d from the centre of the wire as shown below.

a) uniform and depends only on
b) uniform and depends only on b
c) uniform and depends on both b and d
d) non uniform




**[GATE – 2012]**

**Q.14** The direction of vector A is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which $\nabla . A = 0$ is

a) – 2 b) 2
c) 1 d) 0

**[GATE – 2012]**

**Q.15** The force on a point charge +q kept at a distance d from the surface of an infinite grounded metal plate in a medium of permittivity ∈ is

a) 0

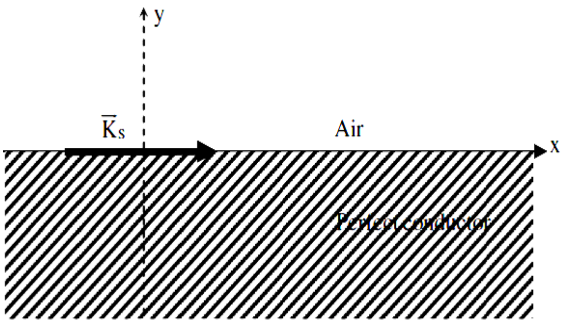
b) $\frac{q^2}{16\pi\epsilon d^2}$ way from the plate

c) $\frac{q^2}{16\pi\epsilon d^2}$ towards the plate

d) $\frac{q^2}{4\pi\epsilon d^2}$ towards the plate

**[GATE – 2014-1]**

**Q.16** A region shown below contains a perfect conducting half-space and air. The surface current $\vec{K}_s$ on the surface of the perfect conductor is $\vec{K}_s = \hat{x}2$ amperes per meter. The tangential $\vec{H}$ field in the air just above the perfect conductor is




a) $(\hat{x} + \hat{z})2$ amperes per meter

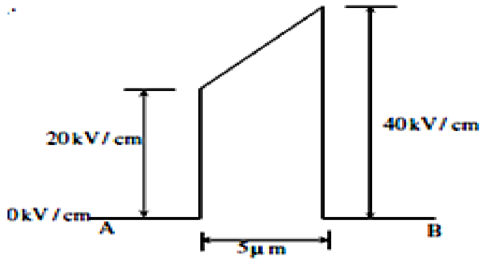
b) $\hat{x}2$ amperes per meter

c) $-\hat{z}2$ amperes per meter

d) $\hat{z}2$ amperes per meter

**[GATE – 2014-3]**

**Q.17** The electric field (assumed to be one-dimensional) between two points A and B is shown. Let $\Psi_A$ and $\Psi_B$ be the electrostatic potentials at A and B, respectively. The value of $\Psi_B . - \Psi_A$ in Volts is ----------




**[GATE – 2014-4]**

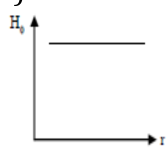
**Q.18** If $E = -(2y^3 - 3yz^2)\hat{x} - (6xy^2 - 3xz^2)\hat{y} + (6xyz)\hat{z}$ is the electric field in a source free region, a valid expression for the electrostatic potential is

a) $xy^3 - yz^2$ b) $2xy^3 - xyz^2$
c) $y^3 - xyz^2$ d) $2xy^3 - 3xyz^2$

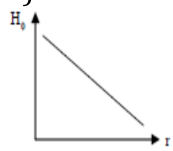
**[GATE – 2014-4]**

**Q.19** Consider a straight, infinitely long, current carrying conductor lying on the z-axis. Which one of the following plots (in linear scale) qualitatively represents the dependence of $H_\phi$ on r, where $H_\phi$ is the magnitude of the azimuthal component of magnetic field outside the conductor and r is the radial distance from the conductor?

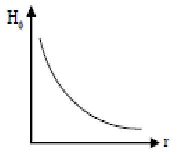
a)



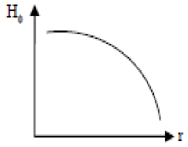

b)



c)




d)




**[GATE – 2015-1]**

**Q.20** A vector $\vec{P}$ is given by $\vec{P}=x^3y\vec{a}_x - x^2y^2\overrightarrow{a_y} - x^2yz\vec{a}_z$. Which one of the following statements is TRUE?

**a)** $\vec{P}$ is solenoidal, but not irrotational

b) $\vec{P}$ is irrotational, but not solenoidal,

c) $\vec{P}$ is neither solenoidal nor irrotational

d) $\vec{P}$ is both solenoidal and irrotational

**[GATE – 2015-1]**

**Q.21** In a source free region in vacuum, if the electrostatic potential $\varphi = 2x^2 + y^2 + cz^2$, the value of constant c must be

**[GATE – 2015-2]**

**Q.22** A vector field $D = 2\rho^2 a_\rho + z a_z$ exists inside a cylindrical region enclosed by the surfaces $\rho = 1$, $z = 0$ and z = 5. Let S be the surface bounding this cylindrical region. The surface integral of this field on S $\left(\oiint_s D.ds\right)$ is ______.

**[GATE – 2015-3]**

**Q.23** Concentric spherical shells of radii 2 m, 4 m, and 8 m carry uniform surface charge densities of 20 nC/m², -4nC/m² and $\rho_s$, respectively. The value of $\rho_s$ (nC/m²) required to ensure that the electric flux density $\vec{D} = \vec{0}$ at radius 10 m is _________

**[GATE – 2016-1]**

**Q.24** The current density in a medium is given by

$$\vec{J} = \frac{400\sin\theta}{2\pi(r^2+4)}\hat{a}_r \text{Am}^{-2}$$

The total current and the average current density flowing through the portion of a spherical surface r = 0.8 m, $\frac{\pi}{12} \le \theta \le \frac{\pi}{4}, 0 \le \varnothing \le 2\pi$ are given, respectively, by

a) 15.09 A, 12.86 $Am^{-2}$

b) 18.73 A, 13.65 $Am^{-2}$

c) 12.86 A, 9.23 $Am^{-2}$

d) 10.28 A, 7.56 $Am^{-2}$

**[GATE – 2016-1]**

**Q.25** A uniform and constant magnetic field B = $\hat{z}$ B exists in the direction in vacuum. A particle of mass m with a small charge q is introduced into this region with an initial velocity $v = \hat{x}v_x + \hat{z}v_z$.. Given that B, m, q, $v_x$ and $v_z$ are all non-zero, which one of the following describes the eventual trajectory of the particle?

a) Helical motion in the $\hat{z}$ direction.

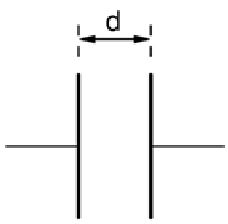
b) Circular motion in the xy plane.

c) Linear motion in the $\hat{z}$ direction.

d) Linear motion in the $\hat{x}$ direction

**[GATE – 2016-2]**

**Q.26** The parallel-plate capacitor shown in the figure has movable plate. The capacitor is charged so that the energy stored in it is E when the plate separation is d. The capacitor is then isolated electrically and the plates are moved such that the plate separation becomes 2d.



At this new plate separation, what is the energy stored in the capacitor, neglecting fringing effects?

a) 2E  b) $\sqrt{2}$ E

c) E  d) E/2

**[GATE – 2016-2]**

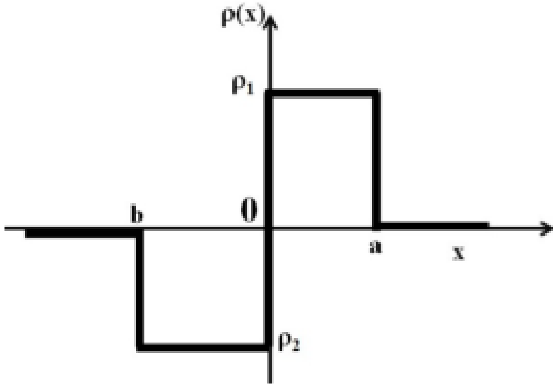
**Q.27** Faraday's law of electromagnetic induction is mathematically described by which one of the following equations?

a) $\nabla.\vec{B} = 0$  b) $\nabla.\vec{D} = \rho v$

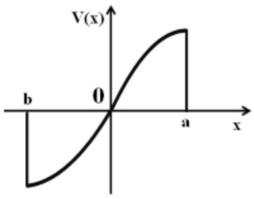
c) $\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$  d) $\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$

**[GATE – 2016-3]**

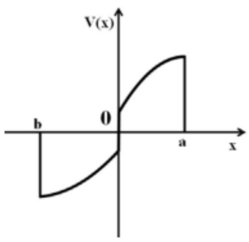
**Q.28** Consider the charge profile shown in the figure. The resultant potential distribution is best described by



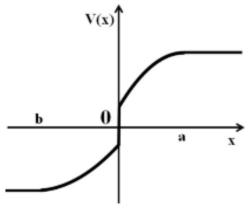

a)



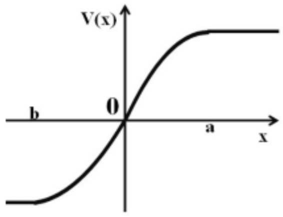

b)




c)

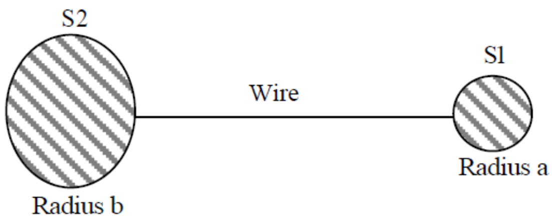



d)




**[GATE – 2016-3]**

**Q.29** Two conducting spheres S1 and S2 of radii a and b (b>a) respectively, are placed far apart and connected by a long, thin conducting wire, as shown in the figure.




For some charge placed on this structure, the potential and surface electric field on S1 are Va and Ea, and that on S2 are Vb and Eb, respectively, which of the following is CORRECT?

a) $V_a = V_b$ and $E_a < E_b$

b) $V_a > V_b$ and $E_a > E_b$

c) $V_a = V_b$ and $E_a > E_b$

d) $V_a > V_b$ and $E_a = E_b$

**[Gate-2017, Set-2]**

**Q.30** An electron ($q_1$) is moving in free space with velocity $10^5$ m/s towards a stationary electron ($q_2$) far away.

The closest distance that this moving electron gets to the stationary electron before the repulsive force diverts its path is __________ $\times 10^{-8}$m.
[Given, mass of electron m = $9.11\times10^{31}$ kg, charge of electron e = -1.6 x $10^{-19}$C, and permittivity $\varepsilon_0 = (1/36\pi)X10^{-9}F/m$]

**[Gate-2017, Set-2]**

**Q.31** If the vector function $\vec{F} = \hat{a}_x(3y - k_1 z) + \hat{a}_y(k_2 x - 2z) - \hat{a}_z(k_3 y + z)$ is irrotational, then the values of the constants k1, k2 and k3 respectively, are

a)0.3, -2.5,0.5

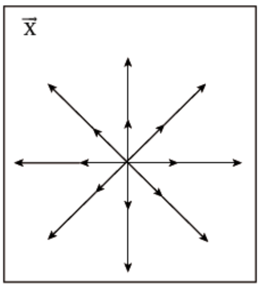
b) 0.0, 3.0, 2.0

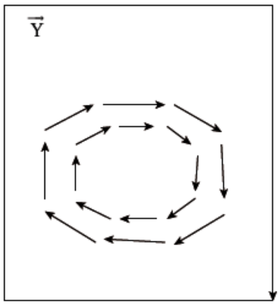
c) 0.3, 0.33, 0.5

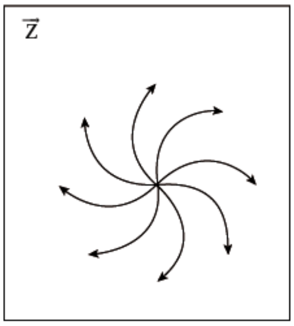
d) 4.0, 3.0, 2.0

**[Gate-2017, Set-2]**

**Q.32** The figures show diagrammatic representations of vector fields $\vec{X}$, $\vec{Y}$ and $\vec{Z}$ respectively of Which one of the following choices is true?




(A) $\bar{\nabla}.\vec{X} = 0,\ \bar{\nabla}\times\vec{Y} \neq 0,\ \bar{\nabla}\times\vec{Z} = 0$

(B) $\bar{\nabla}.\vec{X} \neq 0,\ \bar{\nabla}\times\vec{Y} = 0,\ \bar{\nabla}\times\vec{Z} \neq 0$

(C) $\bar{\nabla}.\vec{X} \neq 0,\ \bar{\nabla}\times\vec{Y} \neq 0,\ \bar{\nabla}\times\vec{Z} \neq 0$

(D) $\bar{\nabla}.\vec{X} = 0,\ \bar{\nabla}\times\vec{Y} = 0,\ \bar{\nabla}\times\vec{Z} = 0$

**[Gate-2017(EE), Set-2]**

**Q.33** Consider an electron, a neutron and a proton initially at rest and placed along a straight line such that the neutron is exactly at the center of the line joining the electron and proton. At t = 0, the particles are released but are constrained to move along the same straight ling. Which of these will collide first?

a) The particles will never collide
b) All will collide together
c) Proton and neutron
d) Electron and neutron

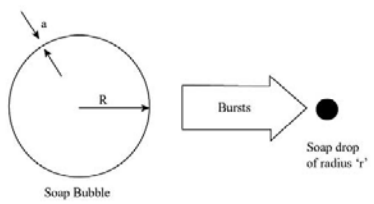
**[Gate-2017(EE), Set-1]**

**Q.34** Consider a solid sphere of radius 5 cm made of a perfect electric conductor. If one million electrons are added to this sphere, these electrons will be distributed.

a) Uniformly over the entire volume of the sphere
b) Uniformly over the outer surface of the sphere
c) Concentrated around the center of the sphere
d) Along a straight line passing through the center of the sphere.

**[Gate-2017(EE), Set-2]**

**Q.35** A thin soap bubble of radius R = 1 cm, and thickness a = 3.3 μm (a << R). is at a potential of 1 V with respect to a reference point at infinity. The bubble bursts and becomes a single spherical drop of soap (assuming all the soap is contained in the drop) of radius r. The volume of the soap in the thin bubble is $4\pi R^2 a$ and that of the drop is $-\frac{4}{3}\pi r^3$. The potential hi volts, of the resulting single spherical drop with respect to the same reference point at infinity is _____. (Given the answer up to two decimal places)



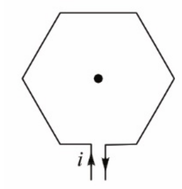

**[Gate-2017(EE), Set-2]**

**Q.36** A solid iron cylinder is paced in a region containing a uniform magnetic field such that the cylinder ax's is parallel to the magnetic field direction, the magnetic field lines inside the cylinder will

a) Bend closer to the cylinder axis
b) Bend farther away from the axis
c) remain uniform as before
d) Cease to exist inside the cylinder

**[Gate-2017(EE), Set-1]**

**Q.37** The magnitude of magnetic flux density (B) in micro Teslas ($\mu T$). At the center of a loop of were wound as a regular hexagon of side length 1 m carrying a Current ($I$ = 1 A) and placed in vacumm as shown in the figure ____ (Give the answer up to two decimal places.)




**[Gate-2017(EE), Set-1]**

**Q.38** A positive charge of 1nC is placed at (0, 0, 0.2) where all dimensions are in meters. Consider the *x-y* plane to be a conducting ground plane. Take $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m. The $Z$ component of the $E$ field at (0, 0, 0.1) is closest to

a) 899.18 V/m
b) – 899.18 V/m
c) 999.09 V/m
d) – 999.09 V/m

**[Gate-2018(EE)]**

**Q.39** The capacitance of an air-filled parallel-plate capacitor is 60 pF. When a dielectric slab whose thickness is half the distance between the plates, is placed on one of the plates covering it entirely, the capacitance becomes 86 pF. Neglecting the fringing effects, the relative permittivity of the dielectric is _______ (up to 2 decimal places).

**[Gate-2018(EE)]**

# ANSWER KEY:

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** |
| (d) | (c) | (c) | (a) | (d) | (d) | (c) | (b) | (d) | (d) | (d) | (c) | (c) | (a) |
| **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** |
| (c) | (d) | * | (d) | (c) | (a) | * | * | * | (a) | (a) | (a) | (c) | (d) |
| **29** | **30** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | | | |
| (c) | $5.058 \times 10^{-8}$ | (b) | (c) | (b) | (b) | 10.03 | (a) | 0.69 | (d) | 2.53 | | | |

# EXPLANATIONS

**Q.1 (d)**

$D = \rho_s = \varepsilon E = 80.\varepsilon.2$

$= 80x2x8.854x10^{-12}\ c/m^2$

$= 1.41x10^{-9}\ c/m^2$

**Q.2 (c)**

$V = -\int Edl$

$= -\left[\int_1^2 xdxu_x + \int_2^0 ydyu_y + \int_3^0 zdzu_z\right]$

$= -\left[\left.\frac{X^2}{2}\right|_1^2 + \left.\frac{y^2}{2}\right|_2^0 + \left.\frac{z^2}{2}\right|_3^0\right]$

$= -\frac{1}{2}[2^2 - 1^2 + 0^2 - 2^2 + 0^2 - 3^2]$

$= -\frac{1}{2} \times -10 = 5V$

**Q.3 (c)**

$\nabla x\vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$

J is current density = $A/m^2$

**Q. 4 (a)**

Displacement current

$I_d = AJ_d$

$= A\frac{\partial D}{\partial t} = A \in \frac{\partial E}{\partial t}$

$|I_d| = |A \in \omega E|$

$= A\omega \in \frac{V}{d}$

After putting values we get

$I_d$= 10 mA

**Q.5 (d)**

**Q.6 (d)**

**Q.7 (c)**

$\vec{B} = B_0\left(\frac{x}{x2+y2}\hat{a}_y - \frac{y}{x2+y2}\hat{a}_x\right)$

Convert to cylindrical coordinates and put

x = r cos ϕ

$\hat{a}_x = \cos\phi \hat{a}_r - \sin\phi \hat{a}_\phi$

$y = r\sin\phi$

$\hat{a}_y = \sin\phi\hat{a}_r + \cos\phi\hat{a}_r$

Putting the values

$\overline{B} = B_0\hat{a}_\phi$

$\overline{H} = \frac{B_0}{\mu_0}\hat{a}_\phi$ = constant

$J = \nabla xH = \nabla x$ [constant]=0

**Q.8 (b)**

This represents stoke's theorem

$\oint_C \overline{A}.\overline{dt} = \int\int_S (\nabla x\overline{A}).\overline{dS} = \int\int_S \overline{V}.\overline{dS}$

**Q.9 (d)**

**Q.10 (d)**

$\oint\oint_S 5\vec{r}.\hat{n}ds = \iiint_V \nabla.5\vec{r}dv$ (using divergence theorem)

$5\iiint_V \nabla.\vec{r}dv = 5x3 = 15volt$

**Q.11 (d)**

Given

$\vec{E} = E_p e^{j(\omega t - 280\pi y)}\hat{u}_z V/m$

$\vec{H} = 3e^{j(\omega t - 280\pi y)}\hat{u}_x A/m$

From given expression we conclude that

$\beta = 280\pi = \frac{2\pi}{\lambda}$

$\lambda = \frac{1}{140}$ meter

$v = f\lambda$

$= 14 \times 10^9 \times \frac{1}{140} m/\sec$

$v = 1 \times 10^8 m/\sec$

but, $v = \frac{c}{\sqrt{\epsilon_r \mu_r}}$

$$1\times10^8 = \frac{3\times10^8}{\sqrt{1\times\epsilon_r}}$$

$\therefore \quad \epsilon_r = 8$

$$\frac{E_P}{H_P} = \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\times1}{\epsilon_0\times9}}$$

$$= \frac{1}{3}\sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\Rightarrow \quad \frac{E_P}{3} = \frac{1}{3}\times120\pi$$

$$\therefore E_\rho = 120\pi$$

**Q.12 (c)**

We know that magnetic flux density at a distance r from the wire is

$$|B| = \frac{\mu_0 I}{2\pi r}$$

For $r < a \rightarrow \quad I = J.\pi r^2$

$|B| \propto r$ for r < a (inside the wire)

for r > a , $I = J\times\pi a^2$

so, $|B| = \frac{\mu_0 J\pi a^2}{2\pi r}$

$|B| \propto= \frac{1}{r}$ for r > a. (outside the wire)

**Q.13 (c)**

It is uniform and depends on both b and d.

**Q.14 (a)**

$$|A| = kr^n$$

$\overline{A} = kr^n\hat{a}_r$ (since it is radially outward)

$\nabla.\overline{A}$ in spherical coordinate is

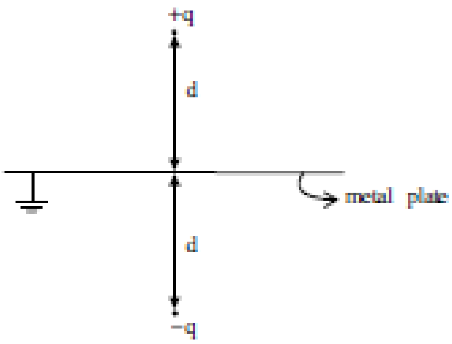
$$\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial r}\left(A_\theta\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial r}A_\theta$$

$$\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 kr^n\right) + 0 + 0$$

$$\nabla.\vec{A} = \frac{k}{r^2}\frac{\partial}{\partial r}\left(r^{n+2}\right)$$

So $\nabla.\vec{A}$ will be zero if $\frac{\partial}{\partial r}(r^{n+2})$ will be zero, and $\frac{\partial}{\partial r}(r^{n+2})$ will be zero if $r^{n+2}$ will be constant and this is possible if $n+2=0 \Rightarrow n=-2$

**Q.15 (C)**




$$F = \frac{1}{4\pi\epsilon}\frac{Q_1Q_2}{R^2}$$

$$F = \frac{1}{4\pi\epsilon}\frac{9^2}{(2d)^2}\frac{9^2}{16\pi\epsilon d^2}$$

Since the charges are opposite polarity the force between them is attractive.

**Q.16 (D)**

Given medium (1) is perfect conductor

Medium (2) is air

$\therefore H_1 = 0$

From boundary conditions

$(H_1 - H_2)\times a_n = K_s$

$$\left.\begin{aligned} H_1 &= 0 \\ a_n &= a_y \end{aligned}\right| K_s = 2\hat{a}_x$$

$-H_2 \times a_y = 2\hat{a}_x$

$-(H_x a_x + H_y a_y + H_z a_z)\times a_y = 2a_x$

$-H_x a_z + H_z a_x = 2a_x$

$\therefore H_z = 2$

$H = 2a_z$

**Q.17 (-15)**

A

(0kV/cm, 20kV/cm)
B
$(5\times10^{-4}\text{kV / cm}, 40\text{kV / cm})$

$$E-20=\frac{40-20}{5\times10^{-4}}(x-0)\Rightarrow E=4\times10^{4}x+20$$

$$V_{AB}=-\int_A^B E.dI=-\int_0^{5\times10^{-4}/cm}\left(4\times10^{4}x+20\right)dx$$

$$=-\left(4\times10^{4}\frac{x^2}{2}+20x\right)\Bigg|_0^{5\times10^{-4}}=$$

$$-(2\times10^{4}\times25\times10^{-8}+20\times5\times10^{-4})$$

$$=-(50\times10^{-4}+100\times10^{-4})=$$

$-150\times10^{-4}$kV $V_{AB}$ $V_{AB}$ = -15V

**Q.18 (d)**

Given

$$E=-(2y^3-3yz^2)\hat{a}_x-$$
$$(6xy^2-3xz^2)\hat{a}_y+6xyz\hat{a}_z$$

By verification option (D) satisfy
E= -∇V

**Q.19 (c)**

$$H_\Phi=\frac{I}{2\pi r}$$

r is the distance from current element.

$$H_\Phi\infty\frac{I}{r}$$

**Q.20 (a)**

$$P=x^3y\hat{a}_x-x^2y^2\hat{a}_y-x^2yz\hat{a}_z$$

$$\nabla P=3x^2y-2x^2y-x^2y=0$$

It is silenoidal

$$\nabla\times P=\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x^3y & -x^2y^2 & -x^2yz\end{vmatrix}$$

$$=\hat{a}_x(-x^2y)-\hat{a}_y(-2xyz)+\hat{a}_z(-2xy^2-x^3)\neq0$$

So, P is solenoidal but not irrotational.

**Q.21 (-3)**

$$2x^2+y^2+cz^2$$

$$E=-\nabla\varphi=-4x\hat{a}_x-2y\hat{a}_y-2cz\hat{a}_z$$

$\nabla E = 0$
-4-2-2c=0
C=-3

**Q.22 (78.53)**

$$D=2\rho^2\hat{a}_\rho+z\hat{a}_z$$

$$\oint_s D.ds=\int_V(\nabla.D)dv$$

$$\nabla.D=\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho)+\frac{1}{\rho}\frac{\partial D_\varnothing}{\partial\varnothing}+\frac{\partial D_z}{\partial z}$$

$$=\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho2\rho^2)+0+1$$

$$=\frac{1}{\rho}2(3)\rho^2+1$$

$$=6\rho+1$$

$$\int_V(\nabla.D)dv=\int_{\rho=0}^{1}\int_{\varnothing=0}^{2\pi}\int_{z=0}^{5}(6\rho+1)\rho d\rho d\varnothing dz$$

$$=\left(\frac{6\rho^3}{3}+\frac{\rho^2}{2}\right)\Bigg|_0^1(2\pi)(5)$$

$$=\left(2+\frac{1}{2}\right)10\pi\int_V(\nabla.D)dv=78.53$$

**Q.23 (-0.25)**

Consider a Gaussian surface a sphere of radius 10m
To ensure $\vec{D}=\vec{0}$ at radius 10m, the total charge enclosed by Gaussian surface is zero
$Q_{enc}=0$

$$\Rightarrow20\times2^2-4\times4^2+P_s\times8^2$$

$$=0\Rightarrow P_s=-0.25nc/m^2$$

**Q.24 (a)**

**Q.25 (a)**

Force due to B on q is

$$F=q(V\times B)=q[V_xV_B](-\hat{y})$$

⇒Helical motion in z-direction.

**Q.26 (a)**

If capacitor is electrically isolated then charge is same

We know $C_1d_1 = C_2d_2 \& \frac{C_1}{V_1} = \frac{C_2}{V_2}$

If 'd' is doubled then C will be C/2 and V will be 2V

Given $E = \frac{1}{2}CV^2 \Rightarrow E_{new} = \frac{1}{2}.\frac{C}{2}$

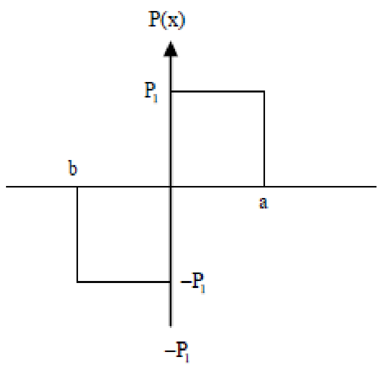
$\times(2V)^2 = 2\times\frac{1}{2}CV^2 = 2E$

**Q.27 (c)**

**Q.28 (d)**

Electrical $\varepsilon_0 = -\frac{q}{\varepsilon}N_dX_{no} = -\frac{q}{\varepsilon}N_dX_{po}$

Potential $\psi(x) = |\varepsilon_0|\left(x - \frac{x^2}{2w_0}\right)$




**Q.29 (c)**

Two spheres are joined with a conducting wire, the voltage on two spheres is same.

$V_a = V_b$ The capacitance of sphere $\propto$ radius

$\frac{C_a}{C_b} = \frac{a}{b}$

We know Q = CV

$\frac{Q_a}{Q_b} = \frac{C_a}{C_b} = \frac{a}{b}$

$$\frac{E_a}{E_b} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{a}{a^2}}{\frac{1}{4\pi\varepsilon_0}\frac{b}{b^2}} = \frac{b}{a} > 1$$

$E_a > E_b$

**Q.30 5.058 (4.55-5.55)**

Work done due to field and external agent must be zero

$qV = \frac{1}{2}MV^2$

$-1.6x10^{-19}x\frac{1.6x10^{-19}}{4\pi\varepsilon_0\gamma} = \frac{1}{2}mx\left(10^5\right)^2$

$\gamma = 5.058x10^{-8}m$

**Q.31 (b)**

curl $\vec{F} = 0$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \\ 3y - k_1z & k_2x\text{-}2z & \text{-}k_3y\text{-}z \end{vmatrix} = 0$$

$\bar{i}(-k_3 + 2) - \bar{j}(0 + k_1) + \bar{k}(k_2 - 3) = 0$

$k_1 = 0, k_2 = 3, k_3 = 2$

**Q.32 (c)**

$\vec{X}$ is radial and irrational and hence

$\bar{V}.\vec{X} \neq 0$ While y is rotational and its curl is non

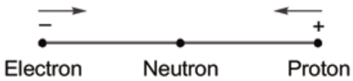
zero $\bar{V}\times\vec{Y} \neq 0$

The field z is also rotational and hence

$\bar{V} \times \bar{Z} \neq 0$

**Q.33 (b)**

Given that electron, neutron and proton are in a straight line




The electron will move towards proton and proton will more towards electron and force will be same $F = \frac{q_1 q_2}{4\pi \in_0 R^2}$. But acceleration of electron will be more than proton as 4n C mass of electron < mass of proton. Since neutron are neutral they will not move. Thus electron will hit neutron first.

**Q.34 (b)**

Static charge resides only on the surface of a conductor. Therefore the electrons will be uniformly over the entire surface of the sphere.

**Q.35 10.00-10.50**

Charge must be same

$$\left(4\pi R^2\right)a = \left(\frac{4}{3}\pi r^3\right)p_v$$

$$r = 3\sqrt{3R^2 a}$$

$$= 0.996 \times 10^{-3}$$

The potential of thin bubble is 1 V.

$$I = \frac{Q}{4\pi \in_0 \times 1 \times 10^{-2}}$$

$$Q = 4\pi \in_0 \times 1 \times 10^{-2} C$$

Potential at soap drop,

$$V = \frac{Q}{4\pi \in_0 r}$$

$$= \frac{4\pi \in_0 \times 10^{-2}}{4\pi \in_0 \times 0.996610^{-3}}$$

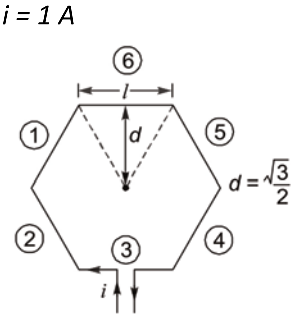
$$= 10.03 V$$

Ans : 10.00 to 10.50

**Q.36 (a)**

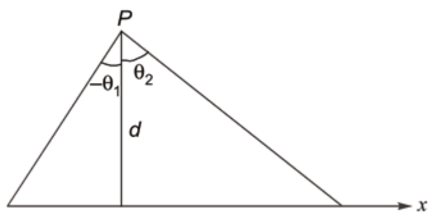
Iron being a ferromagnetic material, magnetic lines of force bend closer to cylindrical axis.

**Q.37** 0.69 μT




Here B at point P is

$$B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_1 + \sin\theta_2\right)$$



For each segment of hexagon (1, 2, 3, 4, 5, 6)

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_1 + \sin\theta_2\right)$$

*Here*, $d = \frac{\sqrt{3}}{2} l$ Where $l = 1$

and $\theta_1 = \theta_2 = 30^\circ$

$$\Rightarrow \quad B = \frac{6 \times \mu_0 I}{\frac{4\pi\sqrt{3}}{2}}\left(\sin 30^\circ + \sin 30^\circ\right)$$

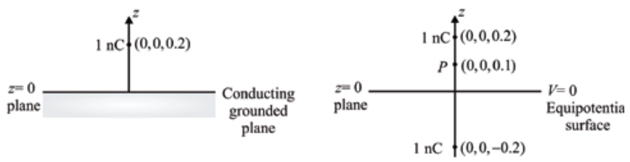
$$= \frac{4\pi \times 10^{-7} \times 6 \times 1}{4\pi \times \frac{\sqrt{3}}{2}}$$

$$= \frac{12}{\sqrt{3}} \times 10^{-7} = \frac{1.20}{\sqrt{3}} \times 10^{-6} T$$

$B = 0.69\ \mu T$

**Q.38 (d)**

**Given :** Charge = 1nC at (0,0,0.2). Since the charge is placed above conducting grounded plane there will be an image charge below the grounded conducting plane as per method of image concept




Electric field at any point is given by,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} = \frac{Q\vec{R}}{\left|\vec{R}\right|^3}$$

$\vec{R}$ = displacement vector between charge to point of interest.

Electric field at point *P* due to point charge +1nC,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{1\times 10^{-9}\left[(0-0)\vec{a}_x + (0-0)\vec{a}_y + (0.1-0.2)\vec{a}_z\right]}{(0.1)^3}$$

$\vec{E}_1 = 898.755$ V/m

Electric field point *P* due to point charge -1nC.

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{1\times 10^{-9}\left[(0-0)\vec{a}_x + (0-0)\vec{a}_y + (0.1-0.2)\vec{a}_z\right]}{(0.3)^3}$$

= 99.86 V/m

Total electric field at point *P* due to both charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E} = 898.755 - 99.86 = -998.61 \text{ V/m}$$

**Q.39 2.53**

**Given :** Capacitance of an air-filled parallel plate capacitor,

$$C_0 = \frac{\varepsilon_0 A}{d} = 60\,\text{pF} \qquad ...(i)$$

Overall capacitance,

$$C_{eq} = 86\,\text{pF}$$

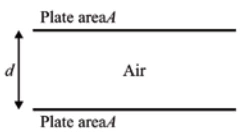



Fig. 1

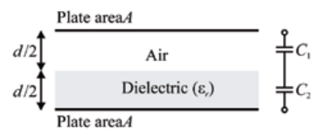



Fig. 2

From fig. 2, after filling with dielectric it has become series combination of two capacitances overall capacitance for series combination is given by,

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 86 \text{ pF} \qquad ...(ii)$$

$$C_1 = \frac{\varepsilon_0 A}{d/2} \text{ and } C_2 = \frac{\varepsilon_0 \varepsilon_r A}{d/2}$$

From equation (ii),

$$C_{eq} = \frac{\frac{\varepsilon_0 A}{d/2} \times \frac{\varepsilon_0 \varepsilon_r A}{d/2}}{\frac{\varepsilon_0 A}{d/2} + \frac{\varepsilon_0 \varepsilon_r A}{d/2}} = \frac{\varepsilon_0 A}{d/2} \frac{(\varepsilon_r)}{(1+\varepsilon_r)}$$

$$86 = 120 \frac{(\varepsilon_r)}{(1+\varepsilon_r)}$$

$$86(1+\varepsilon_r) = 120\,\varepsilon_r$$

$$86 = 34\,\varepsilon_r$$

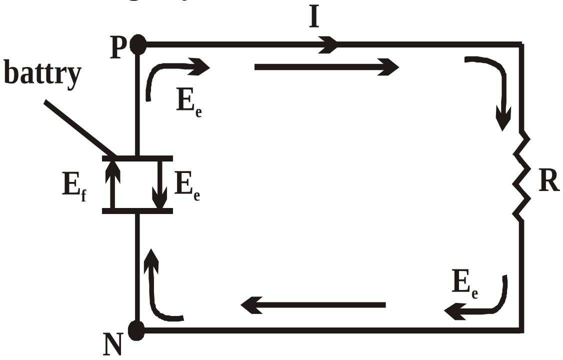
$$\varepsilon_r = 2.529$$

Hence, the relative permittivity of the dielectric is **2.53.**

# 4 MAXWELL'S EQUATIONS

## 4.1 FARADAY'S LAW

Faraday discovered that the **induced emf,** $V_{emf}$ (in volts), in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit.




$$V_{emf} = -\frac{d\lambda}{dt} - N\frac{d\Psi}{dt}$$

$E = E_f + E_e$

$\oint_L \bar{E}.\bar{dl} = \oint_L \bar{E}_f.\bar{dl} + 0 = \int_N^P \bar{E}_f.\bar{dl}$ (through battery)

$$V_{emf} = \int_N^P E_f.dl = -\int_N^P E_e.dl = IR$$

### 4.1.1 TRANSFORMER & MOTIONAL EMFS

$$V_{emf} = -\frac{d\Psi}{dt}$$

$$Vemf = \oint_L \bar{E}.\bar{dl} = -\frac{d}{dt}\int_S \bar{B}.\bar{dS}$$

$$V_{emf} = \oint_L \bar{E}.\bar{dl} = -\int_S \frac{\partial \bar{B}}{\partial t}.\bar{dS}$$

$$\int_s (\nabla x \bar{E}).\bar{dS} = -\int_S \frac{\partial \bar{B}}{\partial t}.dS$$

$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$ Faraday's Law

Modified Maxwell's equation

$V_{emf} = \oint_L E_m.dl = \oint_L (u \times B).dl$ e.m.f. Of moving object of L length

**Moving Loop in Time –Varying Field**

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} + \nabla \times (\bar{u} \times \bar{B})$$

## 4.2 DISPLACEMENT CURRENT

$\nabla x \bar{H} = \bar{J}$

$\nabla.(\nabla x \bar{H}) = 0 = \nabla.\bar{J}$

where $\nabla.\bar{J} = -\frac{\partial \rho_v}{\partial t}$ current continuity equation

$\nabla.\bar{J} = -\frac{\partial \rho_v}{\partial t} \neq 0$

it means ∇ × H = J is not right equation.

$\nabla \times \bar{H} = \bar{J} + \bar{J}_d$

$\nabla.(\nabla x \bar{H}) = 0 = \nabla.\bar{J} + \nabla.\bar{J}_d$

$$\nabla.\bar{J}_d = -\nabla.\bar{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla.\bar{D}) = \nabla.\frac{\partial \bar{D}}{\partial t}$$

$\bar{J}_d = \frac{\partial \bar{D}}{dt}$ Displacement current density

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \bar{J} + \varepsilon\frac{\partial \bar{E}}{\partial t}$$

Modified Maxwell's equation

Based on the displacement current density, we define the displacement current as

$$I_d = \int J_d.dS = \int \frac{\partial D}{\partial t}.dS$$

**MAXWELL'S EQUATIONS IN FINAL FORMS**

| Differential Forms | Integral Forms | Remarks |
|---|---|---|
| $\nabla.\bar{D} = \rho_v$ | $\oint_S \bar{D}.\bar{dS} = \int_V \rho_v dV$ | Gauss's law |
| $\nabla.\bar{B} = 0$ | $\oint_S \bar{B}.\bar{dS} = 0$ | Nonexistene of isolated magnetic |

| | | charge* |
|---|---|---|
| $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$ | $\oint_L \bar{E}.d\bar{l} = -\frac{\partial}{\partial t}\int_S \bar{B}.d\bar{S}$ | Faraday's day |
| $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$ | $\oint_L \bar{H}.d\bar{l} = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right).dS$ | Ampere's circuit law |

## 4.3 TIME – VARYING POTENTIALS

$$V = \int_v \frac{\rho_v dv}{4\pi\varepsilon R}$$

$$A = \int_V \frac{\mu J\, dv}{4\pi R}$$

$$V = \int_v \frac{[\rho_v] dv}{4\pi\varepsilon R}$$

Retarded Electric potential

$$A = \int_V \frac{\mu [J] dv}{4\pi R}$$ ; Magnetic vector potential

Retarded Magnetic potential

$$\bar{B} = \nabla x \bar{A}$$

$$\nabla x \bar{E} = -\frac{\partial}{\partial t}(\nabla x \bar{A})$$

$$\nabla x \left(E + \frac{\partial \bar{A}}{\partial t}\right) = 0$$

$$E + \frac{\partial \bar{A}}{\partial t} = -\nabla V$$

$$\bar{E} = -\nabla V - \frac{\partial \bar{A}}{\partial t}$$

Modified electric filed equation

Electromagnetic wave equation

$$\nabla.E = \frac{\rho_v}{\varepsilon} = -\nabla^2 V - \frac{\partial}{\partial t}(\nabla.A)$$

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla.A) = -\frac{\rho_v}{\varepsilon} \qquad .......(1)$$

$$\nabla \times B = J + \frac{\partial D}{\partial t}$$

Maxwell's equation

$$\nabla \times \nabla \times A = \mu J + \varepsilon\mu \frac{\partial}{\partial t}\left(-\nabla V - \frac{\partial A}{\partial t}\right)$$

$$= \mu J - \mu\varepsilon \nabla\left(\frac{\partial V}{\partial t}\right) - \mu\varepsilon \frac{\partial^2 A}{\partial t^2}$$

$$\nabla x \nabla x \bar{A} = \nabla(\nabla.\bar{A}) - \nabla^2 A$$

by vector identity

$$\nabla^2 A - \nabla(\nabla.A) = -\mu J + \mu\varepsilon \nabla\left(\frac{\partial V}{\partial t}\right) + \mu\varepsilon \frac{\partial^2 A}{\partial t^2}$$

by compare

$$\nabla.A = -\mu\varepsilon \frac{\partial V}{\partial t} \qquad .........(2)$$

Lorentz condition for potentials by (1) and (2)

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_V}{\varepsilon}$$

Those are wave equations

$$\nabla^2 A - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\mu J$$

$$t' = t - \frac{R}{\mu}$$

$$u = \frac{1}{\sqrt{\mu\varepsilon}}$$

## 4.4 TIME – HARMONIC FIELD

A **time – harmonic field** is one carries periodically or sinusoidally with time.

$z = x + jy = r\angle\varphi$

$z = re^{j\emptyset} = r\,(\cos + j\sin\emptyset)$

$r = |z| = \sqrt{x^2 + y^2}$

$\emptyset = \tan^{-1}\frac{y}{x}$

$z^* = x - jy = r\,\angle{-\emptyset} = re^{-j\emptyset}$

$\emptyset = \omega t + \theta$

Time varying $re^{j\emptyset} = re^{j\theta}e^{j\omega t}$

$Re\,(re^{j\emptyset}) = r\cos(\omega t + \theta)$

$Im\,(re^{j\emptyset}) = r\sin(\omega t + \theta)$

The complex term $I_0e^{j\theta}$, which result from dropping the time factor $e^{j\omega t}$ in I(t), is called the phasor current, denoted by $I_s$, that is,

$I_s = I_0e^{j\theta} = I_0\,\angle\theta$

Where the subscript s denotes the pharos form of I(f). Thus $I(t) = I_0\cos(\omega t + \theta)$, the instantaneous form, can be expressed as

$I(t) = Re\,(I_se^{j\omega t})$

In general, a phasor could be scalar or vector. If a vector A(x, y, z, t) is a time – harmonic field, the phasor from of A is $A_s$(d, y, z); the two quantities are related as

$A = Re\,(A_se^{j\omega t})$

For example, if $A = A_o\cos(\omega t - \beta x)\,a_y$, we can write A as

$A = Re\,(A_o e^{-j\beta x} a_y e^{j\omega t})$

$\frac{\partial A}{\partial t} = \frac{\partial}{\partial t} Re\,(A_s e^{j\omega t}) \quad = Re\,(j\omega A_s e^{j\omega t})$

$\frac{\partial A}{\partial t} \rightarrow j\omega A_s$ $\frac{\partial}{\partial t}$ replaces with $j\omega$

$\int A \partial t \rightarrow \frac{A_s}{j\omega}$

| **Point From** | **Integral Form** |
|---|---|
| $\nabla . \overline{D}_s = \rho_{vs}$ | $\oint \overline{D}_s . \overline{dS} = \int \rho_{vs} dv$ |
| $\nabla . \overline{B}_s = 0$ | $\oint \overline{B}_s . \overline{dS} = 0$ |
| $\nabla . \overline{E}_s = -j\omega \overline{B}_s$ | $\oint \overline{E}_s . \overline{dI} = -j\omega \int \overline{B}_s . \overline{dS}$ |
| $\nabla . \overline{H}_s = \overline{J}_s +$ $j\omega \overline{D}_s$ | $\oint \overline{H}_s . \overline{dI} = \int \left( j_s + j\omega \overline{D}_s \right) . \overline{dS}$ |

# 5 UNIFORM PLANE WAVES

## 5.1 UNIFORM PLANE WAVES

In general, **waves** are means of transporting energy or information.

1. Free space $(\sigma = 0,\ \varepsilon = \varepsilon_0,\ \mu = \mu_0)$
2. Lossless dielectrcs
   $(\sigma = 0,\ \varepsilon = \varepsilon_r\varepsilon_0,\ \mu = \mu_r\mu_0, \text{or}\ \sigma << \omega\varepsilon)$
3. Lossy dielectrics
   $(\sigma \neq 0,\ \varepsilon = \varepsilon_r\varepsilon_0, \mu = \mu_r\mu_0)$
4. Good conductor
   $(\sigma \approx \infty, \varepsilon = \varepsilon_0, \mu = \mu_r\,\mu_0, \text{or}\ \sigma >> \omega\varepsilon)$

### 5.1.1 WAVES IN GENERALMEDIUM

A **wave** is a function of both space and time.

$$\frac{\partial^2 E}{\partial t^2} - u^2\frac{\partial^2 E}{\partial z^2} = 0$$

Harmonic time dependence $e^{i\omega t}$ eq.

$$\frac{d^2E}{dz^2} + \beta^2 E_S = 0$$

$$E = Ae^{j(\omega t - \beta z)} + Be^{j(\omega t + \beta z)}$$

$$E = A\sin(\omega t - \beta z)$$

Imaginary part of eq. should be zero for real solution

$u = f\lambda \qquad \omega = 2\pi f$

$\beta = \dfrac{\omega}{u} \qquad T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$

$\beta = \dfrac{2\pi}{\lambda} \qquad \omega t - \beta z = \text{const}$

$$\frac{dz}{dt} = \frac{\omega}{\beta} = u$$

### 5.1.2 WAVE PROPAGATION IN LOSSY DIELECTRICS

A linear, isotopic, homogenous, lossy dielectric medium that is change free ($\rho_v$ = 0)

$$\nabla . \overline{E}s = 0$$

$$\nabla . \overline{H}s = 0$$

$$\nabla \times \overline{E}s = -j\omega\mu\overline{H}s$$

$$\nabla \times \overline{H}s = (\sigma + j\omega\varepsilon)\overline{E}s$$

$$\nabla \times \nabla \times \overline{E}s = -j\omega\mu\ \nabla \times \overline{H}s$$

$$\nabla \times \nabla \times \overline{A} = \nabla(\nabla . A) - \nabla^2 A$$

$$\nabla\left(\nabla . \overline{E}_s\right) - \nabla^2\overline{E}_s = -j\omega\mu(\sigma + j\omega\mu)\overline{E}_s$$

$$\nabla^2 E_S - \gamma^2 E_S = 0$$

where

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

and ϒ is called the propagation constant (in per meter) of the medium. By a similar procedure, it can be shown that for the H field,

$$\nabla^2\overline{H}s - \gamma^2\overline{H}s = 0$$

$$\gamma = \alpha + j\beta$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

The wave propagation along + $a_z$ Electric filed

$$E_s = E_{xs}(z)\hat{a}_x$$

Electric wave eq. $\left(\nabla^2 - \gamma^2\right)E_{xs}(z) = 0$

Solution of eq. $E_{xs}(z) = E_0 e^{-\gamma z} + E_0^{'} e^{\gamma z}$

Wave propagation along $a_z$ axis ($E_0$ = 0) wave does not trading along - $a_z$.

Real part

$$E(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\hat{a}_x$$

$$H_0 = \frac{E_0}{\eta}$$

where $\eta \to$ interstice impedance of medium

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta|\angle\theta_n = |\eta|e^{j\theta_\eta}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}, \tan 2\theta_n = \frac{\sigma}{\omega\varepsilon}$$

$$\overline{H} = \text{Re}\left[\frac{E_0}{|\eta e^{j\theta_n}|}, e^{-\alpha z} e^{j(\omega t-\beta z)} \hat{a}_y\right]$$

$$\overline{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_n) \hat{a}_y$$

1Np = 20 log $_{10}$ e = 8.686 dB

Loss tangent (determine how lossy a medium)

$$\frac{|J_s|}{|J_{ds}|} = \frac{|\sigma E_s|}{|j\omega\varepsilon E_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta$$

$$\tan\theta = \frac{\sigma}{\omega\varepsilon}$$

$$\nabla \times \overline{H}_S = (\sigma + j\omega\varepsilon)\overline{E}_S = j\omega\varepsilon\left[1 - \frac{j\sigma}{\omega\varepsilon}\right]\overline{E}_S$$

$$= j\omega\varepsilon_c \overline{E}_S$$

$$\varepsilon_c = \varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]$$

complex permittivity of medium

$$\tan\theta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon}$$

### 5.1.3 PLANE WAVES IN LOSSLESS DIELECTRICS

In a lossless dielectric, σ << ωε. It is a special case of that in Section expect that

$\sigma \approx 0, \varepsilon = \varepsilon_0\varepsilon_r, \ \mu = \mu_0\mu_r$

Substituting these into eqs. Gives

$$\alpha = 0, \beta = \omega\sqrt{\mu\varepsilon} u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}, \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \angle 0^0$$

Also and thus E and H are in time phase with each other.

### 5.1.4 PLANE WAVES IN FREE SPACE

$$\sigma = 0, \varepsilon = \varepsilon_0, \mu = \mu_0$$

$$\alpha = 0, \beta = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c}$$

$$u = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c, \lambda = \frac{2\pi}{\beta}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \approx 377\Omega$$

$$E = E_0 \cos(\omega t - \beta z) a_x$$

$$H = H_0 \cos(\omega t - \beta z) a_y = \frac{E_0}{\eta_0} \cos(\omega t - \beta z) a_y$$

Direction of wave propagation $a_z$

Em wave that has no electric or magnetic filed components along the direction of propagation, such a wave is called a transverse electromagnetic (TEM) wave. Each of E and H is called uniform plane wave

### 5.1.5 PLANE WAVES IN GOOD CONDUCTORS

$$\sigma \approx \infty, \varepsilon = \varepsilon_0, \mu = \mu_0\mu_r$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}, \ \lambda = \frac{2\pi}{\beta}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^0$$

$$\overline{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_x$$

$$\overline{H} = \frac{E_0}{\sqrt{\frac{\omega\mu}{\sigma}}} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ) \hat{a}_y$$

Skin depth which an EM wave can penetrate the medium

$$E_0 e^{-\alpha\delta} = E_0 e^{-1}$$

$$\delta = \frac{1}{\alpha}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

for a good conductor

$$\eta = \frac{1}{\sigma\delta}\sqrt{2}\, e^{j\pi/4} = \frac{1+j}{\sigma\delta}$$

$$\overline{E} = E_0 e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right) \hat{a}_x$$

The phenomenon whereby filed intensity in a conductor rapidly decrease is known as skin effect. The skin depth is useful depth in calculating the ac resistance due to skin effect. The dc resistance

$$R_{dc} = \frac{l}{\sigma s}$$

Skin depth at high freq.

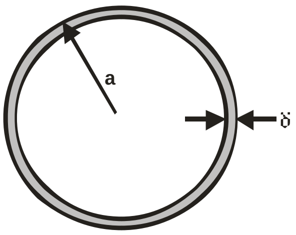
$\delta << a$

The surface a skin resistance Rs ($\Omega$ /m$^2$) (the real part of the $\eta$ a good conductor)

$$R_S = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f \mu}{\sigma}}$$

$$R_{ac} = \frac{l}{\sigma \delta w} = \frac{R_S l}{w}$$

$$\frac{R_{ac}}{R_{dc}} = \frac{\dfrac{l}{\sigma 2\pi a\delta}}{\dfrac{l}{\sigma\pi a^2}} = \frac{a}{2\delta}$$




$\delta << a$ at high frequencies

## 5.2 THE POYNTING VECTOR

Energy transported from one point to another point by means of EM waves. The rate of such energy can be obtained.

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\nabla x \bar{H} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t}$$

$$\bar{E}.\left(\nabla \times \bar{H}\right) = \sigma \bar{E}^2 + E.\varepsilon \frac{\partial \bar{E}}{\partial t}$$

$\nabla.\left(\bar{A} \times \bar{B}\right) = \bar{B}.\left(\nabla \times \bar{A}\right) - \bar{A}.\left(\nabla \times \bar{B}\right)$ vector identity

$$\bar{H}.\left(\nabla x \bar{E}\right) + \nabla.\left(\bar{H} x \bar{E}\right) = \sigma E^2 + \bar{E}.\sigma \frac{\partial \bar{E}}{\partial t}$$

$$\bar{H}.\left(\nabla x \bar{E}\right) = \bar{H}\left(-\mu \frac{\partial H}{\partial t}\right) = -\frac{\mu}{2}\frac{\partial}{\partial t}\left(\bar{H}.\bar{H}\right)$$

$$-\frac{\mu}{2}\frac{\partial H^2}{\partial t} - \nabla.\left(\bar{E} x \bar{H}\right) = \sigma E^2 + \frac{1}{2}\varepsilon \frac{\partial E^2}{\partial t}$$

$$\int_V \nabla.\left(\bar{E} \times \bar{H}\right) dv = -\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right] dv - \int_V \sigma E^2\, dv$$

$$\oint_S \left(\bar{E} \times \bar{H}\right).dS = -\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right] dv - \int_V \sigma E^2\, dv$$

| ↓ | ↓ | ↓ |
|---|---|---|
| Total power leaving the volume | = Rate of decrease in energy stored in electric and magnetic fields | Ohmic power dissipated |

Equation is referred to as Poynting's theorem. The various terms in the equation are identified using energy – conservation arguments for EM fields. The quantity E × H is known as the Poynting vector – is watts per square meter (W/m$^2$); that is, $- = E \times H$

**Poynting's theorem** states that the net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within volume minus the conduction losses.

$\hat{a}_k = \hat{a}_E \times \hat{a}_H$ – points along ak

$$E\left(z,t\right) = E_0 e^{-\alpha z}\cos\left(\omega t - \beta z\right)\hat{a}_x$$

$$H\left(z,t\right) = \frac{E_0}{|\eta|} e^{-\alpha z}\cos\left(\omega t - \beta z - \theta_\eta\right)\hat{a}_y$$

$$P_{ave}(z) = \frac{1}{T}\int_0^T P(z,t)dt$$

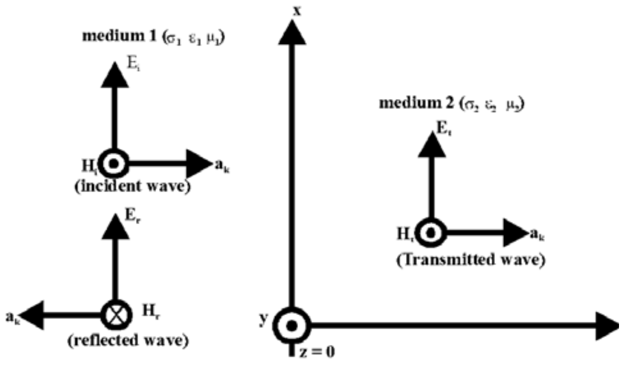
$$P_{ave}(z) = \frac{1}{2}\text{Re}\left(\bar{E}_s x \bar{H}_s^*\right)$$

$$P_{ave}(z) = \frac{E_0^2}{2|\eta|} e^{-2\alpha z}\cos\theta_\eta \hat{a}_z$$

$$P_{ave} = \int_S P_{ave}.dS$$

## 5.3 REFLECTION OF PLANE WAVE

### 5.3.1 AT NORMAL INCIDENCE




**Incident wave**

($E_i$, $H_i$) is traveling along + $a_z$ in medium 1. If we suppress the time factor $e^{jwt}$ and assume that

$$\bar{E}_{is}(z) = E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x$$

$$\bar{H}_{is}(z) = H_{io}\, e^{-\gamma_1 z}\, \hat{a}_y = \frac{E_{io}}{\eta_1} e^{-\gamma z}\, \hat{a}_y$$

**Reflected wave**

$$E_{rs}(z) = E_{ro}\, e^{\gamma_1 z}\, a_x$$

$$\bar{H}_{rs}(z) = H_{ro}\, e^{\gamma_1 z}(-\hat{a}_y) = -\frac{E_{ro}}{\eta_1} e^{-\gamma_1 z}\, \hat{a}_y$$

**Transmitted wave**

$$\bar{E}_{ts}(z) = E_{to}\, e^{-\gamma_1 z}\, \hat{a}_x$$

$$\bar{H}_{ts}(z) = H_{to}\, e^{\gamma_1 z}(-\hat{a}_y) = -\frac{E_{to}}{\eta_1} e^{-\gamma_1 z}\, \hat{a}_y$$

The interface at z = 0, 2 tan the boundary conditions

$E_{11} = E_{at}$ , $H_{11} = H_{2t}$

$$E_1 = E_i + E_r, H_1 = H_i + H_r$$

$$E_2 = E_t, H_2 = H_t$$

$$E_i(0) + E_r(0) = E_t(0) \to E_{io} + E_{ro} = E_{to}$$

$$H_i(0) + H_r(0) = H_t(0) \to \frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2}$$

$$E_{ro} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{io}$$

$$E_{to} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{io}$$

$$\Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Reflection coefficient

$$\tau = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Transmission coefficient

1. $1 + \Gamma = \tau$
2. Both $\Gamma$ and $\tau$ are dimensionless and may be complex.
3. $0 \le |\Gamma| \le 1$

**Case A**

If $E_z$ = 0 The totally reflected wave combines with the incident wave to form a standing wave.

$$\Gamma = \frac{E_{ro}}{E_{io}} = -1, \sigma_1 = 0, \alpha_1 = 0, \gamma_1 = j\beta_1$$

$$E_1 = 2E_{io} \sin\beta_{1z} \sin\omega t\, \hat{a}_x$$

$$H_1 = \frac{2E_{io}}{\eta_1} \cos\beta_{1z} \cos\omega t\hat{a}_y$$

**Case B**

If $\eta_1$, $\Gamma$ < 0. For this case, the locations of $|E_1|$ maximum are given by eq. whereas those of $|E_1|$ minimum are given by eq. All theses are illustrated in Figure.

1. $|H_1|$ minimum occurs whenever there is $|E_1|$ maximum which versa.
2. The transmitted wave ( not shown in Figure) in medium 2 is a purely traveling wave and consequently there are no maxima or minima in this region. The ratio of $|E_1|_{max}$ (or $|E_1|_{min}$ (or $|H_1|_{max}$ to $|H_1|_{min}$ ) is called the standing wave ratio s; that is,

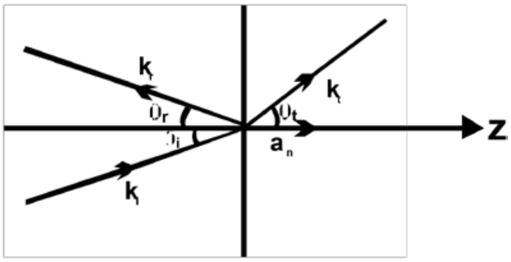
$$s = \frac{|E_1|_{max}}{|E_1|_{min}} = \frac{|H_1|_{max}}{|H_1|_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = \frac{s-1}{s+1}$$

Since $|\Gamma| \le 1$, it follows that $1 \le s \le \infty$. The standing – wave ratio is dimensionless and it is customarily expressed in decibels (dB) as

S in dB = 20 $\log_{10}$ s

**5.3.2 AT OBLIQUE INCIDENCE**



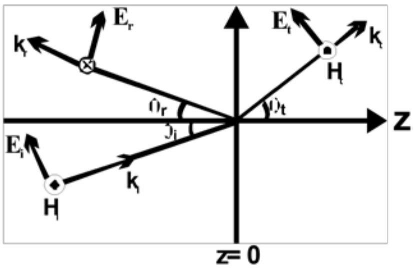

$$H = \frac{1}{\omega\mu} k \times E = \frac{a_k \times E}{\eta} t$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_i}{k_t} = \frac{u_2}{u_1} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

where $n_1 = c\sqrt{\mu_1\varepsilon_1} = c/u_1$ and $n_2 = c\sqrt{\mu_2\varepsilon_2} = c/u_2$ are the refractive indices of the media.

**A. Parallel Polarization**




$$\Gamma_{\|} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\tau_{\|} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$E_{to} = \tau_{\|} = E_{io}$$

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t}$$

$$= \sqrt{1-(u_2-u_1)^2\sin^2\theta_t}$$

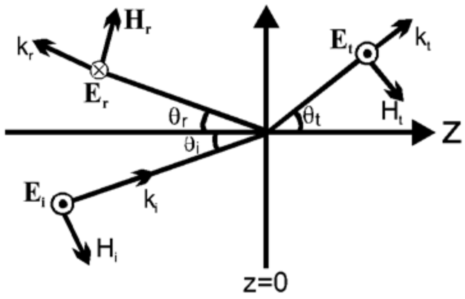
$$1+\Gamma_{\|} = \tau_{\|}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

$$\sin^2\theta_{B\|} = \frac{1-\mu_2\varepsilon_1/\mu_1\varepsilon_2}{1-(\varepsilon_1/\varepsilon_2)^2}$$

$$\sin^2\theta_{B\|} = \frac{1}{1+\varepsilon_1\varepsilon_2} \rightarrow \sin\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}}$$

$$\tan\theta_{B\|} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{\frac{n_2}{n_1}}$$

**B. Perpendicular Polarization**




$$\Gamma_{\perp} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$E_{ro} = \Gamma_{\perp}E_{io}$$

$$\tau_{\perp} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$E_{to} = \tau_{\perp}E_{io}$$

$$1+\Gamma_{\perp} = \tau_{\perp}$$

$$\sin^2\theta_{B\perp} = \frac{1-\mu_1\varepsilon_2/\mu_2\varepsilon_1}{1-(\mu_1/\mu_2)^2}$$

$$\sin\theta_{B\|} = \sqrt{\frac{\mu_2}{\mu_1+\mu_2}}$$

$$\tan\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1}}$$

# GATE QUESTIONS

**Q.1** Two coaxial cables 1 and 2 are filled with different dielectric constants $\varepsilon_{r1}$ and $\varepsilon_{r2}$ respectively. The ratio of the wavelengths in the two cables, $(\lambda_1/\lambda_2)$s

a) $\sqrt{\varepsilon_{r1}/\varepsilon_{r2}}$ b) $\sqrt{\varepsilon_{r2}/\varepsilon_{r1}}$

c) $\varepsilon_{r1}/\varepsilon_{r2}$ d) $\varepsilon_{r2}/\varepsilon_{r1}$

**[GATE – 2000]**

**Q.2** A uniform plane wave in air impinges at $45^0$ angle on a lossless dielectric material with dielectric constant$\epsilon_r$. The transmitted wave propagates in a $30^\circ$ direction with respect to the normal. The value of $\epsilon_r$ is

a) 1.5 b) $\sqrt{1.5}$

c) 2 d) $\sqrt{2}$

**[GATE – 2000]**

**Q.3** A material has conductivity of $10^{-2}$ mho/m and a relative permittivity of 4. The frequency at which the conduction current in the medium is equal to the displacement current is

a) 45 MHz b) 90 MHz

c) 450 MHz d) 9000 MHz

**[GATE – 2001]**

**Q.4** If a plane electromagnetic wave satisfies the equation $\frac{\partial^2 E_x}{\partial z^2} = c^2 \frac{\partial^2 E_x}{\partial t^2}$, the wave propagates in the

a) x-direction

b) z-direction

c) y- direction

d) xy plane at an angle of $45^\circ$ between the x and z directions

**[GATE – 2001]**

**Q.5** Distilled water at $25^0$C is characterized by $\sigma = 1.7\times10^{-4}$ mho/m and $\epsilon = 78\epsilon_0$ at a frequency of 3 GHz. Its loss tangent tan $\delta$ is

a) $1.3\times10^{-5}$ b) $1.3\times10^{-3}$

c) $1.7\times10^{-4}/78$ d) $1.7\times10^{-4}/(78\epsilon_0)$

$(\epsilon = 10^{-9}/(36\pi) F/m)$

**[GATE – 2002]**

**Q.6** A plane wave is characterized by $\vec{E} = \left(0.5\hat{x} + \hat{y}e^{j\pi/2}\right)e^{j\omega t - jkz}$ This wave is

a) linearly polarized

b) circularly polarized

c) elliptically polarized

d) unpolarized

**[GATE – 2002]**

**Q.7** If the electric field intensity associated with uniform plane electromagnetic wave travelling in a perfect dielectric medium is given by $E(z,t) = 10\cos(2\pi\times10^7 t - 0.1\pi/z)$ volt/m, the velocity of the travelling wave is

a) $3.00\times10^8$ m/sec b) $2.00\times10^8$ m/sec

c) $6.28\times10^7$ m/sec d) $2.00\times10^7$ m/sec

**[GATE – 2003]**

**Q.8** A uniform plane wave travelling in air is incident on the plane boundary between air and another dielectric medium with $\varepsilon_r = 4$. The reflection coefficient for the normal incidence, is

a) zero b) $0.5\angle180^0$

c) $0.333\angle0^0$ d) $0.333\angle180^0$

**[GATE – 2003]**

**Q.9** Medium 1 has the electrical permittivity $\varepsilon_1 = 1.5\varepsilon_0$ farad/m and occupies the region to the left of $x = 0$ plane. Medium 2 has the electrical permittivity $\varepsilon_2 = 2.5\varepsilon_0$ farad/m and occupies the region to the right of $x = 0$ plane. If $E_1$ in medium 1 is $E_1 = (2u_x - 3u_y + 1u_z)$ volt/m, then $E_2$ in medium 2 is

a) $(2.0u_x - 7.5u_y + 2.5u_z)$volt / m

b) $(2.0u_x - 7.5u_y + 2.5u_z)$volt / m

c) $(1.2u_x - 3.0u_y + 1.0u_z)$ volt/m

d) $(1.2u_x - 2.0u_y + 0.6u_z)$ volt/m

**[GATE – 2003]**

**Q.10** The depth of penetration of electromagnetic wave in a medium having conductivity $\sigma$ at a frequency of 1 MHz will be penetration at a frequency of 4 MHz will be

a) 6.25 cm b) 12.50 cm

c) 50.00 cm d) 100.00cm

**[GATE – 2003]**

**Q.11** If $\vec{E} = \left(\hat{a}_x + j\hat{a}_y\right)e^{jkz - j\omega t}$ and $\vec{H} = \left(\frac{k}{\omega\mu}\right)\left(\hat{a}_x + j\hat{a}_y\right)e^{jkz - j\omega t}$, the time averaged pointing vector is

a) null vector b) $\left(\frac{k}{\omega\mu}\right)\hat{a}_z$

c) $\left(\frac{2k}{\omega\mu}\right)\hat{a}_z$ d) $\left(\frac{k}{2\omega\mu}\right)\hat{a}_z$

**[GATE – 2004 ]**

**Q.12** The magnetic field intensity vector of plane wave is given by.

$H(x,t) = 10\sin(50000t + 0.004x + 30)\hat{a}_y$

where $\hat{a}_y$ denotes the unit vector in y direction. The wave is propagating with a phase velocity.

a) $5\times10^4$ m / s b) $3\times10^8$ m / s

c) $1.25\times10^7$ m / s d) $3\times10^6$ m / s

**[GATE – 2005]**

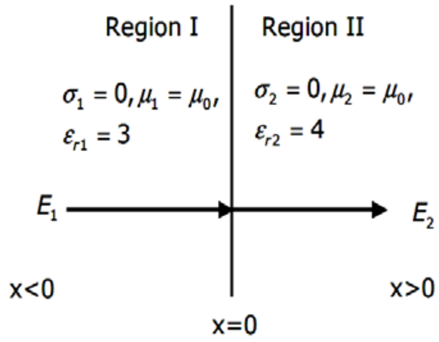
**Q.13** When a plane wave travelling in free-space is incident normally on a medium having $\varepsilon_r = 4.0$, the fraction of power transmitted into the medium is given by

a) 8/9 b) 1/2

c) 1/3 d) 5/6

**[GATE – 2006]**

**Q.14** A medium is divided into regions I and II abour X = 0 plane, as shown in the figure below, an electromagnetic wave with electric field $E = 4\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$ is incident normally on the interface from region-I. The electric field $E_2$ in region – II at the interface is




a) $E_2 = E_1$

b) $4\hat{a}_x + 0.75\hat{a}_y - 1.25\hat{a}_z$

c) $3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$

d) $-3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$

**[GATE – 2006]**

**Q.15** A medium of relative permittivity $\varepsilon_{r2} = 2$ forms an interface with free-space. A point source of electromagnetic energy is located in the medium at a depth of 1 meter from the interface. Due to the total internal reflection, the transmitted beam has a circular cross-section over the interface. The area of the beam cross – section at the interface is given by

a) $2\pi m^2$ b) $\pi^2 m^2$

c) $\pi / 2m^2$ d) $\pi m^2$

**[GATE – 2006]**

**Q.16** The electric field of an electromagnetic wave propagating in the positive z-direction is given by.

$$E = \hat{a}_x \sin(\omega t - \beta z) + \hat{a}_y \sin\left(\omega t - \beta z + \frac{\pi}{2}\right)$$

The wave is

a) linearly polarized in the z-direction
b) elliptically polarized
c) left-hand circularly polarized
d) right-hand circularly polarized

**[GATE – 2006]**

**Q.17** A plane wave of wavelength $\lambda$ is travelling in a direction making an angle $30^0$ with positive x-axis and $90^0$ with positive y-axis The $\vec{E}$ field of the plane wave can be represented as ($E_0$ is constant)

a) $\vec{E} = \hat{y}E_0 e^{j\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x - \frac{\pi}{\lambda}z\right)}$

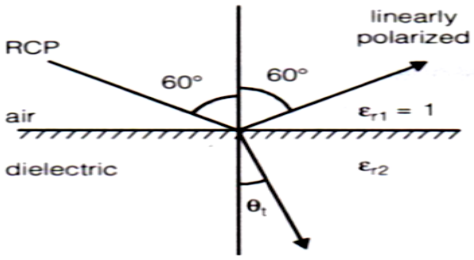
b) $\vec{E} = \hat{y}E_0 e^{j\left(\omega t - \frac{\pi}{\lambda}x - \frac{\sqrt{3}\pi}{\lambda}z\right)}$

c) $\vec{E} = \hat{y}E_0 e^{j\left(\omega t + \frac{\sqrt{3}\pi}{\lambda}x + \frac{\pi}{\lambda}z\right)}$

d) $\vec{E} = \hat{y}E_0 e^{j\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x + \frac{\pi}{\lambda}z\right)}$

**[GATE – 2007]**

**Q.18** A Right Circularly Polarized (RCP) plane wave is incident at an angle of $60^0$ to the normal, on an air-dielectric interface. If the reflected wave is linearly polarized, the relative dielectric constant $\varepsilon_{r2}$ is




a) $\sqrt{2}$ b) $\sqrt{3}$
c) 2 d) 3

**[GATE – 2007]**

**Q.19** The $\vec{H}$ field (in A/m) of a plane wave propagating in free space is given by

$$\vec{H} = \hat{x}\frac{5\sqrt{3}}{\eta_0}\cos(\omega t - \beta z) + \hat{y}\frac{5}{\eta_0}\sin\left(\omega t - \beta z + \frac{\pi}{2}\right)$$

The time average power flow density in watts is

a) $\frac{\eta_0}{100}$ b) $\frac{100}{\eta_0}$
c) $50\eta_0^2$ d) $\frac{50}{\eta_0}$

**[GATE – 2007]**

**Q.20** A uniform plane wave in the free space is normally incident on an infinitely thick dielectric slab (dielectric constant $\varepsilon_r = 9$).The magnitude of the reflection coefficient is.

a) 0 b) 0.3
c) 0.5 d) 0.8

**[GATE – 2008]**

**Q.21** A plane wave having the electric field component $\vec{E}_i = 24\cos(3x10^8 t + \beta y)\hat{a}_x V/m$ and travelling in free space is incident normally on a lossless medium with $\mu = \mu_0$ and $\varepsilon = 9\varepsilon_0$ which occupies the region $y \ge 0$. The reflected magnetic field component is given by.

a) $\frac{1}{10\pi}\cos(3x10^8 t + y)\hat{a}_x$ A/m

b) $\frac{1}{20\pi}\cos(3x10^8 t + y)\hat{a}_x$ A/m

c) $-\frac{1}{20\pi}\cos(3x10^8 t + y)\hat{a}_x$ A/m

d) $-\frac{1}{10\pi}\cos(3x10^8 t + y)\hat{a}_x$ A/m

**[GATE – 2010]**

**Q.22** The electric field component of a time harmonic plane EM wave travelling in a nonmagnetic lossless dielectric medium has an amplitude of 1 V/m. If the relative permittivity of the medium is 4, the magnitude of the time average power density vector (in W/m²) is

a) $\frac{1}{30\pi}$  b) $\frac{1}{60\pi}$

c) $\frac{1}{120\pi}$  d) $\frac{1}{240\pi}$

**[GATE – 2010]**

**Q.23** Consider the following statement regarding the complex Poynting vector $\vec{P}$ for the power radiated by a point source in an infinite homogenous and lossless medium. Re $\vec{P}$ denotes the real part of $\vec{P}$, S denotes a spherical surface whose centre is at the point source, and $\vec{n}$ denotes the unit surface normal on S. Which of the following statement is TRUE?

a) Re $\vec{P}$ remains constant at any radial distance from the source

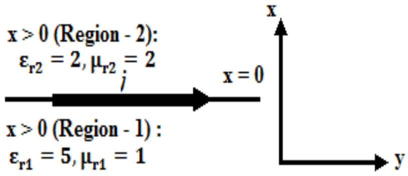
b) Re $\vec{P}$ increases with increasing radial distance from the source

c) $\oiint_S \text{Re}(\vec{P})\hat{n}\ dS$ remains constant at any radial constant at any radial distance from the source

d) $\oiint_S \text{Re}(\vec{P})\hat{n}\ dS$ decreases with increasing radial distance from the source

**[GATE – 2011]**

**Q.24** A current sheet $\vec{J} = 10\hat{u}_y$ A/m lies on the dielectric interface x = 0 between two dielectric media with $\varepsilon_{r1} = 1, \mu_{r1} = 1$ in Region-1 (x < 0) and $\varepsilon_{r2} = 2, \mu_{r2} = 2$ in Region-1 at x=0 is $\vec{H}_1 = 3\hat{u}_x + 30\hat{u}_y$ A/m, the magnetic field in Region – 2 at x = 0⁺ is




a) $\vec{H}_2 = 1.5\hat{u}_x + 30\hat{u}_y - 10\hat{u}_z$ A/m

b) $\vec{H}_2 = 3\hat{u}_x + 30\hat{u}_y - 10\hat{u}_z$ A/m

c) $\vec{H}_2 = 1.5\hat{u}_x + 40\hat{u}_y$ A/m

d) $\vec{H}_2 = 3\hat{u}_x + 30\hat{u}_y + 10\hat{u}_z$ A/m

**[GATE – 2011]**

**Q.25** A plane wave propagating in air with $\vec{E} = \left(8\hat{a}_x - 6\hat{a}_y + 5\hat{a}_z\right)e^{|\omega t + 3x + 4y}$ is incident on a perfectly conducting slab positioned at $x \le 0$. the $\vec{E}$ field of the reflected waves is

a) $\left(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z\right)e^{|\omega t + 3x + 4y}V/m$

b) $\left(-8\hat{a}_x + 6\hat{a}_y - 5\hat{a}_z\right)e^{|\omega t + 3x + 4y}V/m$

c) $\left(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z\right)e^{|\omega t - 3x - 4y}V/m$

**[GATE – 2012]**

**Q.26** The electric field of a uniform plane electromagnetic wave in free space, along the positive x direction, is given by $\vec{E} = 10(\hat{a}_y + j\hat{a}_z)e^{-j25x}$. The frequency and polarization of the wave. Respectively, are

a) 1.2 GHz and left circular

b) 4 Hz and left circular

c) 1.2 GHz and right circular

d) 4 GHz and right circular

**[GATE – 2012]**

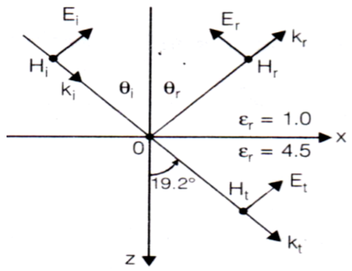
**Q.27** A coaxial-cable with an inner diameter of 1 mm and outer diameter of 2.4 mm is filled with a dielectric of relative permittivity 10.89. Given $\mu_0 = 4\pi \times \frac{10^{-7}H}{m}, \varepsilon_0 = \frac{\frac{10^{-9}}{36\pi}F}{m}$, the characteristic impedance of the cable as

a) 330Ω b) 100Ω
c) 143.3Ω d) 43.4Ω

**[GATE – 2012]**

**Statement for Linked Answer Questions 28 and 29**

A monochromatic plane wave of wavelength $\lambda = 600\mu m$ is propagating in the direction as shown in the figure below. $\vec{E}_i$, $\vec{E}_r$ and $\vec{E}_t$ denote incident, reflected, and transmitted electric field vectors associated with the wave




**Q.28** The expression for $\vec{E}_r$ is

a) $0.23\frac{E_0}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)e^{-j\frac{\pi\times10^4(x-z)}{3\sqrt{2}}}$ V/m

b) $-\frac{E_0}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)e^{j\frac{\pi\times10^4 z}{3}}$ V/m

c) $0.44\frac{E_0}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)e^{-j\frac{\pi\times10^4(x-z)}{3}}$ V/m

d) $\frac{E_0}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)e^{-j\frac{\pi\times10^4(x-z)}{3}}$ V/m

**[GATE – 2013]**

**Q.29** The angle of incidence $\theta_i$ and the expression for $\bar{E}_i$ are

a) $60^\circ$ and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4(x+z)}{3\sqrt{2}}}$ V/m

b) $45^\circ$ and $\frac{E_0}{\sqrt{2}}(\hat{a}_x + \hat{a}_z)e^{-j\frac{\pi\times10^4 z}{3}}$ V/m

c) $45^\circ$ and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4(x+z)}{3\sqrt{2}}}$ V/m

d) $60^\circ$ and $\frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-j\frac{\pi\times10^4 z}{3}}$ V/m

**[GATE – 2013]**

**Q.30** If the electric field of a plane wave is $\vec{E}(Z,t) = \hat{x}3\cos(\omega t - kz + 3o^\circ) - s$ $(-kz + 45^\circ)$ $(mV/m)$, the polarization state of the plane wave is

a) left elliptical b) left circular
c) right elliptical d) right circular

**[GATE – 2014-2]**

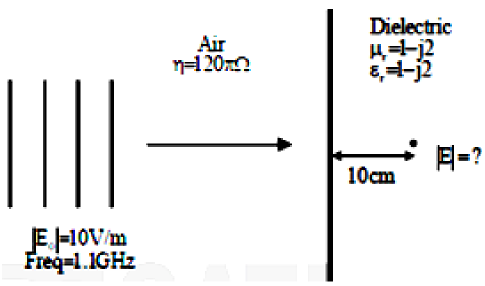
**Q.31** Assume that a plane wave in air with an electric field $\vec{E} = 10\cos(\omega t - 3x - \sqrt{3}z)\hat{a}_y$ V/m is incident on a non-magnetic dielectric slab of relative permittivity 3 which covers the region. Z > 0 The angle of transmission in the dielectric slab is ________.degrees.

**[GATE – 2014-3]**

**Q.32** The electric field component of a plane wave travelling in a lossless dielectric medium is given by $\vec{E}(z,t) = \hat{a}_y 2\cos\cos\left(10^8 t - \frac{z}{\sqrt{2}}\right)$ V/m. The wavelength (in m) for the wave is ________.

**[GATE – 2015-1]**

**Q.33** Consider a uniform plane wave with amplitude $(E_0)$ of 10V / m and 1.1GHz frequency travelling in air, and incident normally on a dielectric medium with complex relative permittivity $(\varepsilon_r)$ and permeability $(\mu_r)$ as shown in the figure.



The magnitude of the transmitted electric field component (in V/m) after it has travelled a distance of 10 cm inside the dielectric region is ____________.

**[GATE – 2015-1]**

**Q.34** The electric field intensity of a plane wave traveling in free space is give by the following expression

$E(x,t) = a_y 24\pi\cos(\omega t - k_0 x)(V/m)$

In this field, consider a square area 10cm x 10 cm on a plane x + y = 1. The total time-averaged power (in mW) passing through the square area is_____

**[GATE – 2015-1]**

**Q.35** The electric field of a uniform plane electromagnetic wave is

$\vec{E} = (a_x + j4\vec{a}_y)\exp[j(2\pi\times10^7 t - 0.2z)]$

The polarization of the wave is

a) Right handed circular
b) Right handed elliptical
c) Left handed circular
d) Left handed elliptical

**[GATE – 2015-2]**

**Q.36** The electric field of a plane wave propagating in a lossless non-magnetic medium is given by the following expression

$\vec{E}(z,t) = \hat{a}_x 5\cos(2\pi\times10^9 t + \beta z)$

$+\hat{a}_y 3\cos(2\pi\times10^9 t + \beta z - \frac{\pi}{2})$

The type of the polarization is

a) Right Hand Circular.
b) Left Hand Elliptical
c) Right Hand Elliptical
d) Linear

**[GATE – 2015-2]**

**Q.37** The electric field of a uniform plane wave travelling along the negative z direction is given by the following equation: $\vec{E}_w^i = (\hat{a}_x + j\hat{a}_y)E_0 e^{jkz}$

This wave is incident upon a receiving antenna placed at the origin and whose radiated electric field towards the incident wave is given by the following equation:

$\vec{E}_a = (\hat{a}_x + 2\hat{a}_y)E_1 \frac{1}{r} e^{jkr}$

The polarization of the incident wave, the polarization of the antenna and losses due to the polarization mismatch are, respectively,

a) Linear, Circular (clockwise),-5dB
b) Circular (clockwise), Linear,-5dB
c) Circular (clockwise), Linear,-3dB
d) Circular (anti clockwise), Linear,-3dB

**[GATE – 2016-1]**

**Q.38** Let the electric field vector of a plane electromagnetic wave propagating in a homogenous medium be expressed as $E = \hat{x}E_x e^{-j(\omega t - \beta z)}$, where the propagation constant $\beta$ is a function of the angular frequency $\omega$. Assume that $\beta(\omega)$ and $E_x$ are known and are real. From the information available, which one of the following CANNOT be determined?

a) The type of polarization of the wave.
b) The group velocity of the wave.
c) The phase velocity of the wave.
d) The power flux through the z = 0 plane.

**[GATE – 2016-2]**

**Q.39** A positive charge q is placed at x= 0 between two infinite metal plates placed at x=-d and at x=+d respectively. The metal plates lie in the yz plane.


The charge is at rest at t= 0, when a voltage +V is applied to the plate at –d and voltage –V is applied to the plate at x= +d. Assume that the quantity of the charge q is small enough that it does not perturb the field set up by the metal plates. The time that the charge q takes to reach the right plate is proportional to

a) d/V  b) $\sqrt{d}$ /V
c) d/ $\sqrt{V}$  d) $\sqrt{d/V}$

**[GATE – 2016-2]**

**Q.40** If a right-handed circularly polarized wave is incident normally on a plane perfect conductor, then the reflected wave will be

a) right-handed circularly polarized
b) left-handed circularly polarized
c) elliptically polarized with tilt angle of 45°
d) horizontally polarized

**[GATE – 2016-3]**

**Q.41** The expression for an electric field in free space is

$E = E_0 = \left(\hat{x} + \hat{y} + j2\hat{z}\right)e^{-j(\omega t - kx + ky)}$

where x, y, z represent the spatial coordinates, t represents time, and ω,k are constants. This electric field,

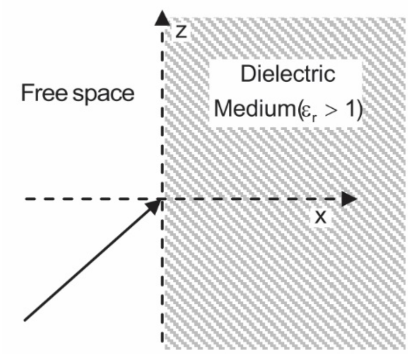
a) does not represent a plane wave
b) represents a circular polarized plane wave propagating normal to the z-axis
c) represents an elliptically polarized plane wave propagating along x-y plane.
d) represents a linearly polarized plane wave

**[GATE – 2017-1]**

**Q.42** A uniform plane wave traveling in free space and having the electric field

$$\vec{E} = \left(\sqrt{2}\hat{a}_x - \hat{a}_z\right)\cos\left[6\sqrt{3}x10^8 t - 2\pi\left(x + \sqrt{2}z\right)\right] V/mE$$

is incident on a dielectric medium (relative permittivity > 1, relative permeability = 1) as shown in the figure and there is no reflected wave



# ANSWER KEY:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (b) | (c) | (a) | (b) | (a) | (c) | (b) | (d) | (c) | (b) | (a) | (c) | (a) | (c) |
| **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** |
| (d) | (c) | (a) | (d) | (d) | (c) | (a) | (c) | (c) | (a) | (c) | (a) | * | (a) |
| **29** | **30** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** | **41** | **42** |
| (c) | (a) | 30 | 8.8 | 0.1 | 53.3 | (d) | (b) | (c) | (d) | (c) | (b) | (c) | 2 |

# EXPLANATIONS

**Q.1 (b)**

$$v_1 = \frac{1}{\sqrt{\mu_1 \in_1}}$$

$$v_2 = \frac{1}{\sqrt{\mu_2 \in_2}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{\in^2}{\in^1}}$$

$$v = \eta\lambda$$

n = frequency of operation

$$v \propto \lambda$$

$$\therefore \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{\in^2}{\in^1}}$$

**Q.2 (c)**

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{1}{\in^r}}$$

$$\frac{\sin 30^0}{\sin 45^0} = \frac{1}{\sqrt{\in^r}}$$

$$\Rightarrow \quad \frac{1/2}{1/\sqrt{2}} = \frac{1}{\sqrt{\in_r}}$$

$$\Rightarrow \quad \varepsilon_r = 2$$

**Q.3 (a)**

$$|\sigma E| = |j\omega \in E|$$

$$\sigma = 2\pi f \in_0 \in_r$$

$$\Rightarrow \quad f = \frac{\sigma}{2\pi \times \in_0 \in_r}$$

$$= \frac{9\times10^9 \times 2\times10^{-2}}{4}$$

$$F = 45\times10^6 = 45\text{MHz}$$

**Q.4 (b)**

**Q.5 (a)**

$$\tan\delta = \frac{\sigma}{\omega \in}$$

$$= \frac{1.7\times10^{-4}}{2\pi\times3\times10^9\times78\in_0}$$

$$= 0.13\times10^{-4}$$

$$= 1.3\times10^{-5}$$

**Q.6 (c)**

$$\vec{E}_x = 0.5e^{j(\omega t - kz)}\hat{x}$$

$$\vec{E}_y = 1e^{j(\omega t - kz)}\hat{y}$$

$$\frac{\vec{E}_x}{\vec{E}_y} = 0.5e^{-\frac{\pi}{2}} = \frac{1}{2}\angle 90^\circ$$

Since $\left|\frac{\vec{E}_x}{\vec{E}_y}\right| \neq 1$, so elliptically polarised.

**Q.7 (b)**

$$\omega = 2\pi\times10^7$$

$$\beta = 0.1\pi$$

Comparing with Acos $(\omega t - \beta z)$

$$v = \frac{\omega}{\beta} = \frac{2\pi\times10^7}{0.1\pi}$$

$$= 2\times10^8 m/s$$

**Q.8 (d)**

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_z + \eta_1}$$

$$= \frac{\sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} - \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} + \sqrt{\frac{\mu_0}{\epsilon_0}}}$$

$$= \frac{\frac{1}{\sqrt{\epsilon_r}} - 1}{\frac{1}{\sqrt{\epsilon_r}} + 1} = \epsilon_r = 4$$

$$= \frac{-1/2}{3/2} = \frac{-1}{3} \Rightarrow 0.333\angle 180^0$$

**Q.9 (c)**

$$E_1 = 2u_x - 3u_y + 1u_z$$

$$E_{1t} = -3u_y + u_z = E_{2t}\ (x = 0\ plane)$$
$$E_{1n} = 2u_x$$
$$D_{1n} = D_{2n} = \in E$$
$$\Rightarrow \quad \in_1 E_{1n} = \in_2 E_{2n}$$
$$= 1.5 \in_0 .2u_x = 2.5 \in_0 .E_{2n}$$
$$E_{2n} = \frac{3}{2.5}u_x = 1.2u_x$$
$$E_2 = E_{2t} + E_{2n}$$
$$E_2 = -3u_y + u_z + 1.2u_x$$

**Q.10 (b)**

$$\delta \propto \frac{1}{\sqrt{f}}$$
$$\frac{\delta_2}{\delta_1} = \sqrt{\frac{f_1}{f_2}}$$
$$\delta_2 = \sqrt{\frac{1MHz}{4MHz}} \times 25cm$$
$$\delta_2 = 12.5cm$$

**Q.11 (a)**

$$P_{avg} = \frac{1}{2}Re\left[\vec{E} x \vec{H}\right]$$
$$\frac{1}{2}Re\left[\left(\hat{a}_x + j\hat{a}_y\right)\right]e^{jkz-ej\omega t}$$
$$x\frac{k}{\omega\mu}\left(\hat{a}_x + j\hat{a}_y\right)e^{-jkz-j\omega t}$$
$$P_{avg} = \frac{k}{2\omega\mu}[0] = 0$$
$$\begin{bmatrix} 1 & 1 \\ -j & 1 \end{bmatrix} = 1 + j^2 = 1 - 1 = 0$$

**Q.12 (c)**

$$\omega = 50,000$$
$$\beta = -0.004$$
$$v\rho = \frac{\omega}{\beta}$$
$$= \frac{5\times10^4}{-4\times10^{-3}}$$
$$= -1.25\times10^7 m/s$$

**Q.13 (a)**

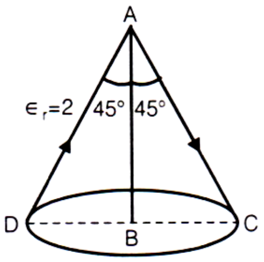
$$P_1 = (1-\Gamma^2)P_i$$
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
$$= \frac{\sqrt{\frac{\mu_0}{\in_0\in_r}} - \sqrt{\frac{\mu_0}{\in_0}}}{\sqrt{\frac{\mu_0}{\in_0\in_r}} + \sqrt{\frac{\mu_0}{\in_0}}}$$
$$\Gamma = \frac{\frac{1}{2}-1}{\frac{1}{2}+1} = \frac{-1}{3}$$
$$T = 1 - \Gamma^2 = 1 - \frac{1}{9} = \frac{8}{9}$$
$$\therefore P_t = T.P_i$$
$$\frac{P_t}{P_i} = \frac{8}{9}$$

**Q.14 (c)**

$$E_{1t} = 3\hat{a}_y + 5\hat{a}_z = E_{2t}$$
$$D_{n1} = D_{n2}$$
$$dH = \frac{kl\,dl \sin\alpha}{R^2}$$
$$E_{2n} = \frac{3x4\hat{a}_x}{4} = 3\hat{a}_x$$
$$E_2 = E_{2t} + E_{2n}$$
$$= 3\hat{a}_y + 5\hat{a}_z + 3\hat{a}_x$$

**Q.15 (d)**

$$\sin\theta = \frac{1}{\sqrt{\in_r}} = \frac{1}{\sqrt{2}}$$
$$\theta = 45^0 = \frac{\pi}{4}$$




BD = AB = 1m

$$Area = \pi r^2 = \pi \times BD^2 = \pi m^2$$

**Q.16 (c)**

**Q.17 (a)**

**Q.18 (d)**

$$\tan\theta_B = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}$$

$$\tan 60^0 = \sqrt{\frac{\varepsilon_{r2}}{1}}$$

$$\sqrt{3} = \sqrt{\varepsilon_{r2}}$$

$$\varepsilon_{r2} = 3$$

**Q.19 (d)**

For free space,

$$P = \frac{E^2}{2\eta_0}, E = \eta_0 H$$

$$P = \frac{\eta_0 H^2}{2\eta_0} = \frac{\eta_0 H^2}{2}$$

$$H = \frac{1}{\eta_0}\sqrt{(5\sqrt{3})^2 + (5)^2} = \frac{10}{\eta_0}$$

$$P = \frac{\eta_0}{2} \times \frac{100}{\eta_0^{\,2}}$$

$$P = \frac{50}{\eta_0} \text{ Watts}$$

**Q.20 (c)**

Dielectric constant of free space, $\in_{r1} = 1$ dielectric constant of dielectric slab, $\in_{r2} = 9$ Magnitude of reflection coefficient,

$$\rho = \left|\frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}}\right| = \left|\frac{1-3}{1+3}\right| = \frac{2}{4} = 0.5$$

**Q.21 (a)**

$$\vec{E_1} = 24\cos\left(3\times10^8 t - \beta y\right)\hat{a}_z$$

The wave is travelling in + y direction

$$\vec{H_i} = \frac{1}{\eta}\left(\hat{a}_y x \vec{E_1}\right)$$

$$= \frac{24\cos}{120\pi}\left(3x10^8 t - \beta y\right)\hat{a}_x$$

$$= \frac{1}{5\pi}\cos\left(3x10^8 t - \beta y\right)\hat{a}_x$$

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\sqrt{\frac{\mu_0}{\in_1}} - \sqrt{\frac{\mu_0}{\in_2}}}{\sqrt{\frac{\mu_0}{\in_1}} + \sqrt{\frac{\mu_0}{\in_2}}}$$

$$= \frac{\sqrt{\in_2} - \sqrt{\in_1}}{\sqrt{\in_2} + \sqrt{\in_1}} = \frac{\sqrt{9} - \sqrt{1}}{\sqrt{9} + \sqrt{1}}$$

$$= \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

Therefore, reflected magnetic field component,

$$\vec{H_r} = \frac{1}{5\pi}\cos\left(3x10^8 t - \beta y\right)\hat{a}_x$$

Which indicates that the reflected wave is travelling in –y direction.

**Q.22 (c)**

Magnitude of the time-average power density vector,

$$P_{av} = \frac{E^2}{2\eta}$$

Where, $\eta = \sqrt{\mu / \in}$

$$\eta = \sqrt{\frac{\mu_0}{4\in_0}} = \frac{\eta_0}{2} = \frac{120\pi}{2}$$

$$P_{av} = \frac{1^2}{2\left(\frac{120\pi}{2}\right)} = \frac{1}{120\pi} W/m^2$$

**Q.23 (c)**

**Q.24 (a)**

For magnetic field boundary relations are

$$B_{\eta 1} = B_{\eta 2}$$

and $H_{t1} - H_{t2} = -\vec{J_s} x \vec{a_n}$

$\Rightarrow \quad B_{x1} = B_{x2}$

$\therefore$ (x is normal component)

$$\mu_1 H_{x1} = \mu_2 H_{x2}$$

$\Rightarrow \quad 1 \times 3 = 2 \times H_{x_2}$

$$H_{x_2} = 1.5$$

$$\Rightarrow H_{t_1} - H_{t_2} = -10\hat{u}_y x \hat{u}_x$$

$$= +10\hat{u}_z$$

$$H_{t2} = H_{t1} - 10\hat{u}_z$$

$H_{t2} = 30\hat{u}_z - 100\hat{u}_z$

**Q.25 (c)**

$\vec{E} = (8\hat{a}_x + 6\hat{a}_y + 5\hat{a}_z)e^{j(\omega t+3x-4y)}$ v/m

Since perfect conductor will reflect wave totally. Let reflected wave is $\vec{E}_R$. Tangential component of incident wave is

$\vec{E}_t = 6\hat{a}_y + 5\hat{a}_z$

Since at the boundary net tangential field will be zero.
For this tangential component of reflected wave $(\vec{E}_{tr})$ and tangential component of incident wave must cancel out each other, for this

$\vec{E}_{tr} = -6\hat{a}_y - 5\hat{a}_z$

Reflected wave will have normal component such that it will cancel out the normal component of incident wave so it will be $-8\hat{a}_x$

Also the direction of propagation will be in –x direction. So $\vec{E} = (-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z)e^{j(\omega t-3x-4y)}$ v/m.

**Q.26 (a)**

Out of phase by 900 and equal amplitude the wave is left circularly polarized check the trace of E with the time

$a_y = \sin \omega t$

$a_z = \cos \omega t$

**Q.27 (*)**

Characteristic impedance of the co-axial cable is given by

$$Z_0 = \frac{138}{\sqrt{\epsilon_r}}\log\frac{D}{d}$$

$$= \frac{138}{\sqrt{10.89}}\log\left(\frac{2.4}{1}\right) = 15.89\Omega$$

**Q.28 (a)**

**Q.29 (c)**

Electric field boundary condition Electric lies in the x – z plane the plane of incidence in the case of parallel poarization.

$\sqrt{\xi_{r1}}\sin\theta_i = \sqrt{\xi r_2}\sin\theta t$

$1\sin\theta_i = \sqrt{4.5}\sin(19.2^0)$

$\sin\theta_i = 2.12\times 0.3289$

$\sin\theta_i = 0.697$

$\theta_i = 0.697$

$\theta_i = \sin^{-1}(0.697)$

$\theta_i = 44.2$

$\theta_i = 45^0$

Incidence electric field

$\vec{E}_i = \vec{E}_{io}(\cos\theta_i\hat{a}_x - \sin\theta_i\hat{a}_z)e^{j\beta}$
$(x\sin\theta_i + z\cos\theta_i)$

$Ei = E0(\cos 45\circ)$

$$E_i = E_o\frac{1}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{\frac{-j\pi\times10^4}{3}\frac{1}{\sqrt{2}}(x+z)}$$

$$E_i = \frac{E_0}{\sqrt{2}}(\hat{a}_x - \hat{a}_z)e^{-\frac{j\pi\times10^4(x+z)}{3\sqrt{2}}}\ V/m.$$

**Q.30 (a)**

$E(z,t) = 3\cos(\text{cot-kz} + 30°)$

$a_x$ -4 -sin$(\omega t - kz + 45°)a_y$

$E_x$ =3cos $(\omega t - kz + 30°)$

$E_y = -\ 4\cos(\omega t - kz + 45°)$

At $z = 0\ E_x = 3\cos(\omega t + 30°)$.

$E_y = -\ 4\sin(\omega t + 45°)$

$|E_x| \neq |E_y| \rightarrow$ so

Elliptical polarization
Q=30° —135°=-105°
∴left hand elliptical (LEP)

**Q.31 (30)**

Given $E = 10\cos(\omega t - 3x - \sqrt{3}z)a_y$

$E = E_{0}e^{-J\beta(x\cos\theta_x + y\cos\theta_y + z\cos\theta_z)}$

So, $\beta_x = \beta\cos\theta_x = 3$

$\beta_y = \beta\cos\theta_y = 0$

$\beta_z = \beta\cos\theta_z = \sqrt{3}$

$\beta_x^2 + \beta_y^2 + \beta_z^2 = \beta^2$

$9 + 3 = \beta^2 \Rightarrow \beta = \sqrt{13}$

$$\beta\cos\theta_z = \sqrt{3} \Rightarrow \cos\theta_z$$
$$= \sqrt{\frac{3}{13}} \Rightarrow \theta_z = 61.28 = \theta_i$$
$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{E_2}{E_1}}$$
$$\Rightarrow \frac{\sin 61.28}{\sin\theta_t} = \sqrt{\frac{3}{1}} \Rightarrow \frac{0.8769}{\sqrt{3}} = \sin\theta_t$$
$$\theta_t = 30.4 \Rightarrow \theta_t \approx 30^o$$

**Q.32 (8.885)**

$$E(z,t) = 2\cos(10^8 t - \frac{z}{\sqrt{2}})a_y$$
$$\lambda = \frac{2\pi}{\beta} = 2\pi\sqrt{2}$$
$$\lambda = 8.885\text{m}$$

**Q.33 (0.1)**

| (1) air | (2) Dieletric($\sigma = 0$) |
|---|---|
| $\eta_1 = 120\pi\Omega$ | $\mu_r = 1 - j2$ |
| $E_1 = 10V/m$ | $\epsilon_r = 1 - j2$ |
| | $\eta_2 = 120\pi\Omega$ |

$$\eta_1 = \eta_2$$
So, $E_2 = E_1 = 10V/m$

$E_3 \to$ Electric field in the dieletric after travelling10cm

$$E_3 = E_2 e^{-\gamma z} \to$$
$$r = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\in)}$$
$$\alpha + j\beta = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r}$$
$$\alpha + j\beta = j\omega\sqrt{\mu_0\epsilon_0}(1 - j2)$$
$$= j\omega\sqrt{\mu_0\epsilon_0} + 2\omega\sqrt{\mu_0\epsilon_0}$$
$$\alpha = 2\omega\sqrt{\mu_0\epsilon_0} = \frac{2\times 2\pi\times 1.1\times 10^9}{3\times 10^8} = 46.07$$
z = 10 cm
$$E_3 = 10e^{-10\times 10^{-2}\times 46.07} = 10e^{-4.6} = 0.1$$

**Q.34 (53.3)**

$$E(x,t) = a_y 24\pi\cos(\omega t - k_0 x)(V/m)$$
$$P_{avg} = \frac{1}{2}\frac{E_0^2}{\eta}a_x$$
$$= \frac{1}{2}\frac{(24\pi)^2}{120\pi}a_x$$
$$= 7.53a_x$$
Power (P) = $\int_s P_{avg}.ds$

x+ y= 1

$\frac{a_x + a_y}{\sqrt{2}} \to$ unit vector normal To the surface

$$P = \int_s 7.53a_x.\frac{a_x}{\sqrt{2}}dydz$$
$$= \frac{7.53}{\sqrt{2}}\times 10\times 10\times 10^{-2}$$
$$P = 53.3mW$$
$$P = 53.3\times 10^{-3}W$$

**Q.35 (d)**

$$E = (a_x + 4ja_y)e^{j(2\pi\times?10^7 t - 0.2z)}$$
$$\omega = 2\pi\times 10^7$$
$$E_z = \cos\omega t$$
$$\beta = 0.2$$
$$E_y = 4\cos(\omega + \frac{\pi}{2}) = -\sin\omega t$$
So, it left hand elliptical polarization

**Q.36 (b)**

$$E_x = 5\cos(\omega t + \beta z)$$
$$E_y = 3\cos\left(\omega t + \beta z - \frac{\pi}{2}\right)$$
$$\phi = -\frac{\pi}{2}$$
But the wave is propagating along negative z-direction

So, Left hand elliptical (LED).

**Q.37 (c)**

**Q.38 (d)**

Option (a): The polarization is linear

Option (b): Vg = $\frac{C}{V_p}$

Option (c): Vp = $\frac{\omega}{\beta}$

Option (D): It is not possible to find the intrinsic impedance of the medium. So it is not possible to find power flux.

**Q.39 (c)**

**Q.40 (b)**

If incident wave is right handed polarized then the reflected wave is left handed polarized.

**Q.41 (c)**

Given the direction of propagation is $\hat{a}_x - \hat{a}_y$

The orientation of $\overline{E}$ field is $\hat{a}_x + \hat{a}_y + j2\hat{a}_z$

The dot product between above two is 1 – 1 + 0 = 1

It is a plane wave

We observed that

$\overline{P} = \hat{a}_x - \hat{a}_y, \hat{a}_x + \hat{a}_y$ and $j2\hat{a}_z$ are normal to each other

So electric field can be resolved into two normal component along $\hat{a}_x + \hat{a}_y$ and $j2\hat{a}_z$

The magnitude are $\sqrt{2}$ and 2 and $\theta = \frac{\pi}{2}$

So elliptical polarization

**Q.42 2**

$$\vec{E} = \left(\sqrt{2}\hat{a}_{11} - \hat{a}_z\right)\cos\left(\sigma\sqrt{3}\pi x10^8 t - 2\pi\left(x + \sqrt{2}z\right)\right) V/m$$

The wave is parallel to the x, z plane

Since, there is no reflection,

Angle of incidence in Brewster angle is

$$\theta_i = \theta_B = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \tan^{-1}\sqrt{\varepsilon_2}$$

$$\tan\theta_i = \sqrt{2}$$

$$\sqrt{\varepsilon_2} = \sqrt{2}$$

$$\varepsilon_2 = 2$$

# 6 TRANSMISSION LINE

## 6.1 INTRODUCTION

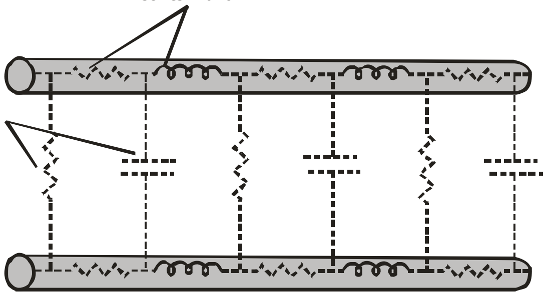
Transmission lines are commonly used in power distribution (at low frequencies) and in communications (at high frequencies). There are various kinds of transmission lines such as the twisted–pair and coaxial cables.

1) The line parameters R, L, G and not discrete or lumped but distributed as shown in figure. By this we mean that the parameters are uniformly distributed along the entire length of the line.

| Parameters | Coaxial Line | Two – Wire Line | Planer Line |
|---|---|---|---|
| R ( Ω /m) | $\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}-\frac{1}{b}\right]$ $(\delta << a, c-b)$ | $\frac{1}{\pi a\delta\sigma_c}$ $\delta << a$ | $\frac{2}{w\delta\sigma_c}$ $(\delta << t)$ |
| L (H/m) | $\frac{\mu}{2\pi}\ln\frac{b}{a}$ | $\frac{\mu}{\pi}\cosh^{-1}\frac{d}{2a}$ | $\frac{\mu d}{w}$ |
| G(S/m) | $\frac{2\pi a}{\ln\frac{b}{a}}$ | $\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$ | $\frac{\sigma w}{d}$ |
| C (F/m) | $\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$ | $\frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$ | $\frac{\varepsilon w}{d}$ $(w >> d)$ |

2) For each line, the conductor are characterized by $\sigma_c, \mu_c, \varepsilon_c = \varepsilon_0$, and the homogeneous dielectric separating the conductor is characterized by $\sigma, \mu, \varepsilon$.

3) G ≠ 1/R; R is the resistance per unit length of the conductor comprising the line and G is the conductance per unit length of the conductor comprising the line and G is the conductance per unit length due to the dielectric medium separating the conductors.




4) The value of L shown in Table is the external inductance per unit length; that is, $L = L_{ext}$. The effects of internal inductance $L_{in}$ ( $= R/\omega$) are negliable as high frequencies at which most communication systems operate.

5) For each line,

$$LC = \mu\varepsilon \text{ and } \frac{G}{C} = \frac{\sigma}{\varepsilon}$$

## 6.2 TRANSMISSION LINE EQUATIONS

$$V = -\int E.dl, \quad I = \oint H.dl$$

By KVL to the circuit

$$V(z,t) = R\Delta z\, I(z,t) + L\,\Delta z\frac{\partial I(z,t)}{\partial t} + V(z+\Delta z, t)$$

$$\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

By KCL to the node of circuit

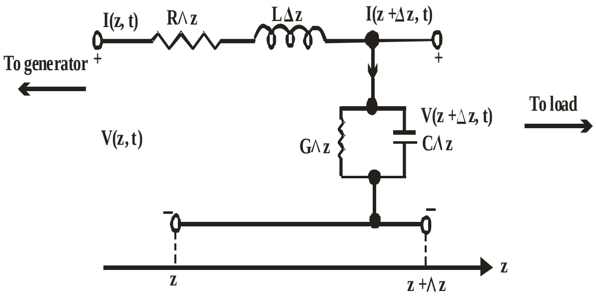
$$I(z,t) = I(z+\Delta z,t) + \Delta I$$

$$= I(z+\Delta z,t) + G\,\Delta z\, V(z+\Delta z,t) + C\,\Delta z\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z}$$

$$= GV(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$$




$$V(z,t) = \text{Re}\left[V_s(z)e^{j\omega t}\right]$$

$$I(z,t) = \text{Re}\left[I_s(z)e^{j\omega t}\right]$$

$$-\frac{dV_s}{dz} = (R + j\omega L)I_s$$

$$-\frac{dI_s}{dz} = (G + j\omega C)V_s$$

$$\frac{d^2V_s}{dz^2} = (R + j\omega L)(G + j\omega C)V_s$$

$$\frac{d^2V_s}{dz^2} - \gamma^2 V_s = 0$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\frac{d^2I_s}{dz^2} - \gamma^2 I_s = 0$$

$$\lambda = \frac{2\pi}{\beta}$$

$$u = \frac{\omega}{\beta} = f\lambda$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{\omega}{\mu}$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{\mu}$$

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\rightarrow +z \quad -z \leftarrow$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\rightarrow +z \quad -z \leftarrow$$

The **characteristic impedance** $Z_0$ of the line is the ratio of positively travelling voltage wave to current wave at any point on the line.

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0$$

**A. Lossless Line (R = 0 = G)**

A **transmission line** is said to be **lossless** if the conductors of the line are perfect ( $\sigma_c \approx \infty$) and the dielectric medium separating them is lossless ( $\sigma \approx 0$).

$R = 0 = G$

$\alpha = 0,$

$$\gamma = j\beta = j\omega\sqrt{LC}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

$$X_0 = 0, Z_0 = R_0 = \sqrt{\frac{L}{C}}$$

**B. Distortionless Line (R /L = G/C)**

A **distortionless line** is one which the attenuation constant $\alpha$ is frequency independent while the phase constant $\beta$ is linearly dependent on frequency.

$$\frac{R}{L} = \frac{G}{C}$$

$$\gamma = \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta$$

$$\alpha = \sqrt{RG} \quad \beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{R(1 + j\omega L/R)}{G(1 + j\omega C/G)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, \quad = R_0 + jX_0$$

$$R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \quad X_0 = 0$$

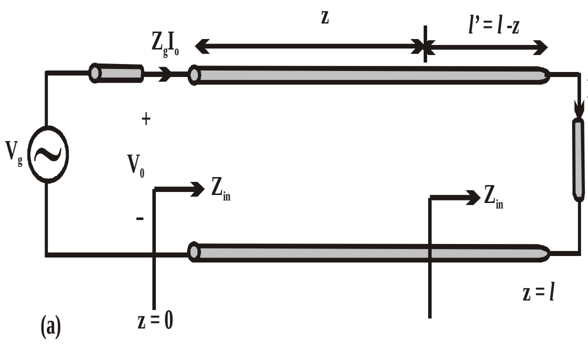
$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

A losses line is also a distortionless line, but a distortionless line is not necessarily lossless. Lines are desirable in power transmission, telephone lines are required to be distortionless.

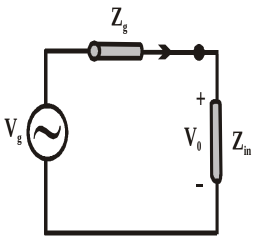
| **Case** | **Propagation Constant** $\gamma = \alpha + j\beta$ | **Characteristic Impedance** $Z_0 = R_0 + jX_0$ |
|---|---|---|
| General | $\sqrt{(R + j\omega L)(G + j\omega C)}$ | $\sqrt{\frac{R + j\omega L}{G + j\omega C}}$ |

| | | |
|---|---|---|
| Lossless | $0 + j\omega\sqrt{LC}$ | $\sqrt{\frac{L}{C}} + j0$ |
| Distortionless | $\sqrt{RG} + j\omega\sqrt{LC}$ | $\sqrt{\frac{L}{C}} + j0$ |

## 6.3 INPUT IMPEDANCE, SWR, & POWER



(a)



(b)

$$V_s(z) = V_o^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

$V_0 = V(z = 0)$, $I_0 = I(z = 0)$

$$V_0^+ = \frac{1}{2}(V_0 + Z_0 I_0)$$

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_0)$$

$$V_0 = \frac{Z_{in}}{Z_{in} + Z_g} V_g, I_0 = \frac{V_g}{Z_{in} + Z_g}$$

$$V_L = V(z = l),\ I_L = I(z = l)$$

$$V_0^+ = \frac{1}{2}(V_L + Z_0 I_L) e^{\gamma l}$$

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_0) e^{-\gamma l}$$

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_0(V_0^+ + V_0^-)}{V_0^+ - V_0^-}$$

$$\frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l, \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sin \gamma l$$

$$\tan \gamma l = \frac{\sinh \gamma l}{\cosh \gamma l} = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right] \quad \text{(lossy)}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \beta l}{Z_0 + jZ_L \tan \beta l}\right] \quad \text{(lossless)}$$

The **voltage reflection coefficient** at any point on the line is the ratio of the magnitude of the reflected voltage wave to that of the incident wave.

$$\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

The **current reflection coefficient** at any point on the line is negative of the voltage reflection coefficient at that point.

Voltage standing wave Ratio s (VSWR)

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$1 < S \le \infty$

$-1 \le T \le +1$

max for min (zin) input impedance

$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = sZ_0$$

$$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{s}$$

Power Transmission at the *l* distance

$$P_{ave} = \frac{1}{2} Re\left[V_s(l) I_s^*(l)\right]$$

$$P_{ave} = \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right)$$

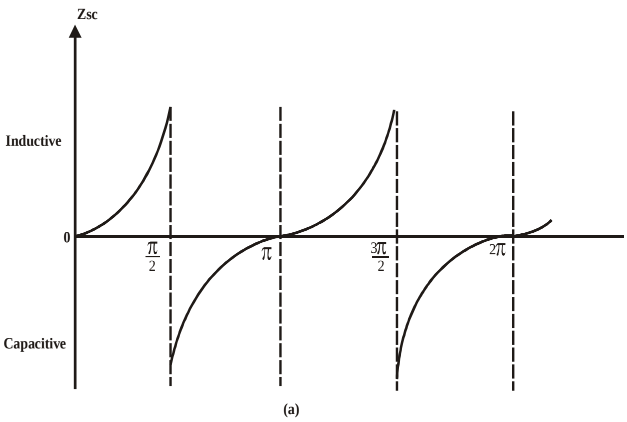
$P_t = P_i - P_r$

Special care of Transmission Line

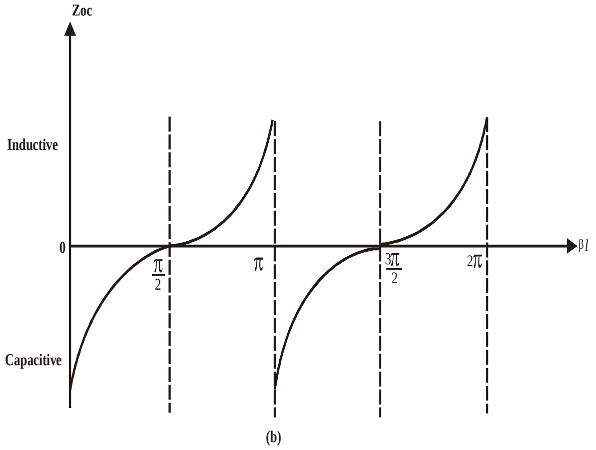
### A. Shorted Line ($Z_L = 0$)

$$Z_{sc} = Z_{in}\Big|_{Z_L = 0} = jZ_0 \tan \beta l$$

$$\Gamma_L = -1, s = \infty$$



(a)



(b)

**B. Open – Circuited Line ($Z_L = \infty$)**

$$Z_{oc} = \lim_{Z_L \to \infty} Z_{in} = \frac{Z_0}{j\tan\beta l} = -jZ_0 \cot\beta l$$

$$\Gamma_L = 1, \quad s = \infty$$

$$Z_{sc}Z_{oc} = Z_0^2$$

**C. Matched Line ($Z_L = Z_0$)**

$Z_{in} = Z_0$

$\Gamma_L = 0, s = 1$

The incident power is fully absorbed by the load. Thus maximum power transfer is possible when a transmission line is matched to the load.

The Smith chart is a graphical mean of obtaining line characteristics such as Γ, s, and $Z_{in}$, It is constructed within a circle of unit radius and based on the formula for $\Gamma_L$ given above. For each r and x, it has two explicit circle (the constant s – circle). It is conveniently used in determining the location of a stub tuner and its length. It is also used with the slotted line to determine the value of the unknown load impedance.

## 6.4 APPLICATIONS OF TRANSMISSION LINES

**A. Quarter – Wave Transformer (Matching)**

$$l = \frac{\lambda}{4} \text{ or } \beta l = \frac{2\pi}{\lambda}\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$$

When $Z_0 \neq Z_L$ the load is mismatched and a reflected wave exist on the line. However, for minimum power transfer, it is desired that the load be matched to the transmission line ($Z_0 = Z_L$) so there is no reflection the mateling is achieved by using shorted sections of transmission lines.

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\pi/2}{Z_0 + jZ_L \tan\pi/2}\right] = \frac{Z_0^2}{Z_L}$$

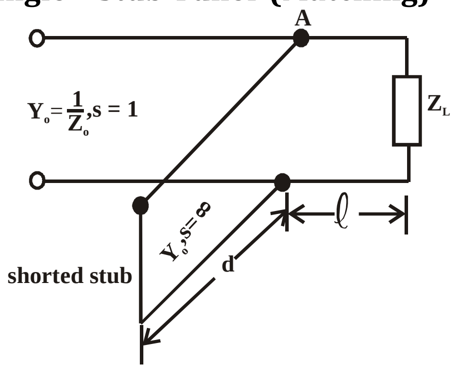
$$\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}$$

$$Z_0' = \sqrt{Z_0 Z_L}$$

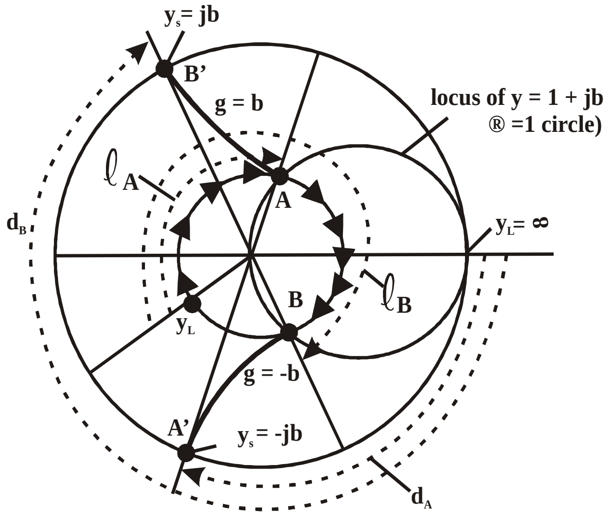
If for example, a 120 - Ω load is to be matched to a 75 -Ω line, the quarter – wave transformer must have a characteristic impedance of $\sqrt{(75)(120)}$ ≈ 95 Ω.

Thus, the main disadvantage of the quarter
– wave transformer is that it is a narrow
– band or frequency
– sensitive device

**B. Single – Stub Tuner (Matching)**




The major drawback of using a quarter – wave transformer as a line – matching device is eliminated by using a single – stub tuner. The tuner consists of an open or shorted section of transmission line of length d connected in parallel with the main line at some distance $l$ from the load as in Figure. Notice that the stub has the same characteristic impedance as the main line. It is difficult to use a series stub although it is theoretically feasible. An open – circuited stub radiates some energy are preferred.




As we intend that $Z_{in} = Z_0$, that is, $Z_{in} = 1$ or $y_{in} = 1$ at point A on the line, we first draw the locus y = 1 + jb ( r = 1 circle) on the Smith chart as shown in Figure 11.20. If a shunt stub of admittance $y_s$ = - jb is introduced at A, then

$$y_{in} = 1 + jb + y_s = 1 + jb - jb = 1 + j0$$

We determine the length d of the stub by finding the distance from $P_{sc}$ (at which $z'_L = 0 + j0$) to the required stub admittance $y_s$. For the stub at A, we obtain $d = d_A$ as the distance from P to A', where A' corresponds to $y_s$ = -jb located on the periphery of the chart as in Figure 11.20. Similarly, we obtain $d = d_B$ as the distance from $P_{sc}$ to B' ($y_s$ = jb). Thus we obtain $d = d_A$ and $d = d_B$, corresponding to A and B, respectively, as shown in Figure 11.20. Note that $d_A + d_B = \lambda/2$ always. Since we have two possible shunted stubs, we normally choose to match the shorter stub or one at a position closer to the load. Instead of having a single stub shunted across the line, we may have two stubs. This is called double – stub matching and allows for the adjustment of the load impedance.

# GATE QUESTIONS

**Q.1** The magnitude of the open-circuit and short – circuit input impedances of a transmission line are $100\Omega$ and $25\Omega$ respectively. The characteristic impedance of the line is.

a) $25\Omega$ b) $50\Omega$
c) $75\Omega$ d) $100\Omega$

**[GATE – 2000]**

**Q.2** A uniform plane electromagnetic wave incident normally on a plane surface of a dielectric material is reflected with a VSWR of 3. What is the percentage of incident power that is reflected?

a) 10% b) 25%
c) 50% d) 75%

**[GATE – 2001]**

**Q.3** A transmission line is distortion less if

a) $RL = \frac{1}{GC}$ b) RL = GC
c) LG = RC d) RG = LC

**[GATE – 2001]**

**Q.4** In an impedance Smith chart, a clockwise movement along a constant resistance circle gives rise to

a) a decrease in the value of reactance
b) an increase in the value of reactance
c) no change in the reactance value
d) no change in the impedance value

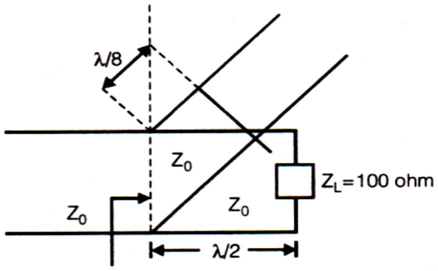
**[GATE – 2002]**

**Q.5** The VSWR can have any value between

a) 0 and 1 b) – 1 and + 1
c) 0 and $\infty$ d) 1 and $\infty$

**[GATE – 2002]**

**Q.6** A short-circuited stub is shunt is shunt connected to a transmission line as shown in the figure. If $Z_0 = 50\ \Omega$, the admittance Y seen at the junction of the stud and the transmission line is




a) (0.01 – j0.02) mho
b) (0.02-j0.01) mho
c) (0.04-j0.02) mho
d) (0.02 + j0) mho

**[GATE – 2003]**

**Q.7** A lossless transmission line is terminated in a load which reflects a part of the incident power. The measured VSWR is 2. The percentage of the power that is reflected back is

a) 57.73 b) 33.33
c) 0.11 d) 11.11

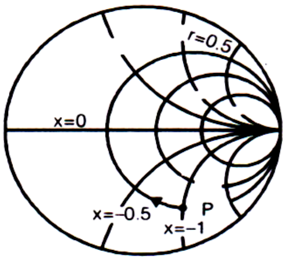
**[GATE – 2004]**

**Q.8** A plane electromagnetic wave propagating in free space is incident normally on a large slab of loss-less, non-magnetic, dielectric material with $\varepsilon > \varepsilon_0$. Maxima and minima are observed when the electric field is measured in front of the slab. The maximum electric field is found to be 5 times the minimum field. The intrinsic impedance of the medium should be

a) $120\pi\Omega$ b) $60\pi\Omega$
c) $600\pi\Omega$ d) $24\pi\Omega$

**[GATE – 2004]**

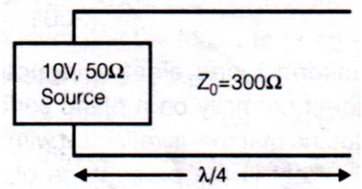
**Q.9** Consider an impedance Z=R+jX marked with port P in an impedance Smith chart as shown in the figure. The movement from point P along a constant resistance circle in the clockwise direction by an angle $45^0$ is equivalent to




a) Adding an inductance in series with Z
b) Adding a capacitance in series with Z
c) Adding an inductance in shunt across Z
d) Adding a capacitance in shunt across Z

**[GATE – 2004]**

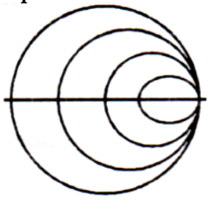
**Q.10** Consider a $300\Omega$, quarter-wave long (at 1 GHz) transmission line as show in the figure. It is connected to a 10V. $50\Omega$ source at one end and is left open circuited at the other end. The magnitude of the voltage at the open circuit end of the line is




a) 10 V
b) 5 V
c) 60 V
d) 60/7 V

**[GATE – 2004]**

**Q.11** Many circles are drawn in a smith chart used for transmission line calculation. The circles shown in the figure represent

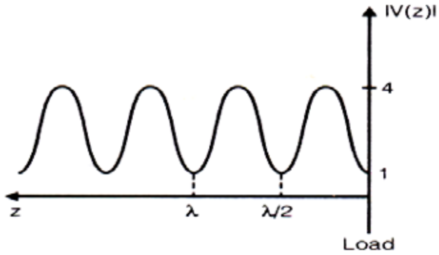


a) Unit circles
b) Constant resistance circles
c) constant reactance circles
d) constant reflection coefficient circles

**[GATE – 2005]**

**Common data for questions 12 and 13**

Voltage standing wave pattern in a lossless transmission line with characteristic impedance $50\Omega$ and a resistive load is shown in the figure.




**Q.12** The value of the load resistance is
a) $50\Omega$
b) $200\Omega$
c) $12.5\Omega$
d) $0\Omega$

**[GATE – 2005]**

**Q.13** The reflection coefficient is given by
a) -0.6
b) -1
c) 0.6
d) 0

**[GATE – 2005]**

**Q.14** Characteristic impedance of transmission line is $50\Omega$. Input impedance of the open circuited line is $Z_{OC} = 100 + j150\Omega$. When the transmission line is short-circuited, then value of the input impedance will be
a) $50\Omega$
b) $100 + j150\Omega$
c) $7.69 + j11.54\Omega$
d) $7.69 - j11.54\Omega$

**[GATE – 2005]**

**Common data for questions 15 & 16:**

A 30-volts battery with zero source resistance is connected to a coaxial line of characteristic impedance of 50 ohms at t=0 second and terminated in an unknown resistive load, the line length is such that it takes $400\mu s$ for an electromagnetic wave to travel from source end to load end and vice – versa. At t = $400\mu s$, the voltage at the load end is found to be 40 volts.

**Q.15** The load resistance is

a) 25 ohms b) 50 ohms
c) 75 ohms d) 100 Ohms

**[GATE – 2006]**

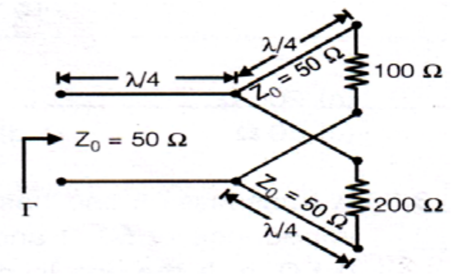
**Q.16** The steady – state current through the load resistance is

a) 25 Amps b) 0.3 Amps
c) 0.6 Amps d) 0.4 Amps

**[GATE – 2006]**

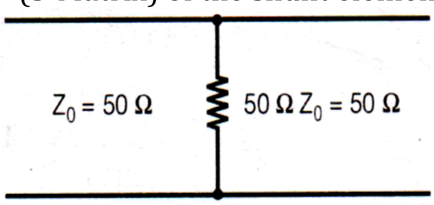
**Q.17** The parallel branches of a 2-wire transmission line are terminated in $100\Omega$ and $200\Omega$ resistors as shown in the figure. The characteristic impedance of the line is $Z_0 = 50\Omega$ and each section has length of $\lambda/4$. The voltage reflection coefficient $\Gamma$ at the input is

a) $-j\frac{7}{5}$ b) $\frac{-5}{7}$
c) $j\frac{5}{7}$ d) $\frac{5}{7}$




**[GATE – 2007]**

**Q.18** A load of $50\Omega$ is connected in shunt in a 2-wire transmission line of $Z_0 = 50\Omega$ as shown in the figure. The 2-port scattering parameter matrix (S-Matrix) of the shunt element is




a) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ d) $\begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}$

**[GATE – 2007 ]**

**Q.19** In the design of a single mode step index optical fiber closer upper cut – off, the single – mode operation is not preserved if

a) radius as well as operating wavelength are halved
b) radius as well as operating wavelength are doubled
c) radius is halved and operating wavelength is doubled
d) radius is doubled and operating wavelength is halved

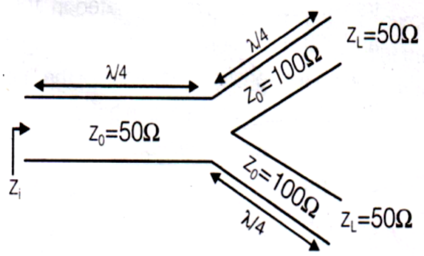
**[GATE –2008]**

**Q.20** One end of a loss-less transmission line having the characteristic impedance of $75\Omega$ and length of 1 cm is short-circuited. At 3 GHz, the input impedance at the other end of the transmission line is

a) 0 b) Resistive
c) Capacitive d) Inductive

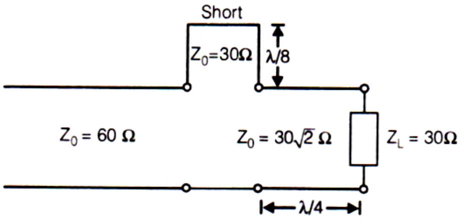
**[GATE –2008]**

**Q.21** A transmission line terminates in two branches, each of length $\lambda/4$, as shown. The branches are terminated by $50\Omega$ loads. The lines are lossless and have the characteristic impedance shown. Determine the impedance $Z_i$ as seen by the source.



a) $200\Omega$ b) $100\Omega$
c) $50\Omega$ d) $25\Omega$
**[GATE – 2009]**

**Q.22** In the circuit shown, all the transmission line sections are lossless. The voltage standing wave ratio (VSWR) on the $60\Omega$ line is




a) 1.00 b) 1.64
c) 2.50 d) 3.00
**[GATE–2010]**

**Q.23** A transmission line has a characteristic impedance of $50\Omega$ and a resistance of $0.1\Omega/\text{m}$. If the line is distortion less, the attenuation constant (in Np/m) is
a) 500 b) 5
c) 0.014 d) 0.002
**[GATE – 2010]**

**Q.24** A transmission line of characteristic impedance $50\Omega$ is terminated in a load impedance as $Z_L$. The VSWR of the line is measured as 5 and the first of the voltage maxima in the line is observed at a distance of $\lambda/4$ from the load. The value of $Z_L$ is
a) $10\Omega$
b) $250\Omega$
c) $(19.23+j46.15)\Omega$
d) $(19.23-j46.15)\Omega$
**[GATE – 2011]**

**Q.25** A transmission line of characteristic impendence $50\Omega$ is terminated by a $50\Omega$ load. When excited by a sinusoidal voltage source at 10GHz, the phase difference between two points spaced 2mm apart on the line is found to be $\pi/4$ radians. The phase velocity of the wave along the line is
a) $0.8\times10^8\,\text{m/s}$ b) $1.2\times10^8\,\text{m/s}$
c) $1.6\times10^8\,\text{m/s}$ d) $3\times10^8\,\text{m/s}$
**[GATE – 2011]**

**Q.26** A transmission line with a characteristic impedance of $100\Omega$ is used to match a $50\Omega$ section to a $200\Omega$ section. If the matching is to be done both at 429 MHz and 1GHz, the length of the transmission line can be approximately
a) 82.5 cm b) 1.05 m
c) 1.58 m d) 1.75 m
**[GATE – 2012]**

**Q.27** The return loss of a device is found to be 20dB. The voltage standing wave ratio (VSWR) and magnitude of reflection coefficient are respectively
a) 1.22 and 0.1 b) 0.81 and 0.1
c) -1.22 and 0.1 d) 2.44 and 0.2
**[GATE – 2013]**

**Q,28** For a parallel plate transmission line, let v be the speed of propagation and Z be the characteristic impedance. Neglecting fringe effects, a reduction of the spacing between the plates by a factor of two results in

(A) halving of v and no change in Z
(B) no change in v and halving of Z

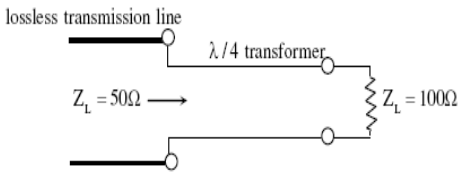
(C) no change in both v and Z
D) halving of both v and Z

**[GATE – 2014-1]**

**Q.29** The input impedance of a $\frac{\lambda}{8}$ section of a lossless transmission line of characteristic impedance $50\ \Omega$ is found to be real when the other end is terminated by a load $Z_L(R + jX)\ \Omega$ if X is 30 Ω, the value of R (in Ω) is______.

**[GATE – 2014-1]**

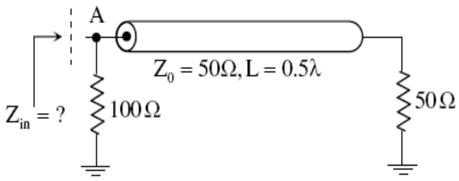
**Q.30** To maximize power transfer, a lossless transmission line is to be matched to a resistive load impedance via a $\lambda/4$ transformer as shown.




The characteristic impedance (in Ω) of the λ/a transformer is _____.

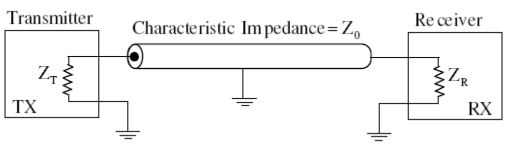
**[GATE – 2014-2]**

**Q.31** In the transmission line shown, the impedance $Z_{in}$ (in ohms) between node A and the ground is _________.




**[GATE–2014-2]**

**Q.32** In the following figure, the transmitter Tx sends a wideband modulated RF signal via a coaxial cable to the receiver Rx. The output impedance $Z_T$ of Tx, the characteristic impedance Zo of the cable and the input impedance $Z_R$ of Rx are all real.




Which one of the following statements is TRUE about the distortion of the received signal due to impedance mismatch?

a) The signal gets distorted if $Z_R$ # $Z_0$, irrespective of the value of $Z_T$

b) The signal gets distorted if $Z_T$ # $Z_0$, irrespective of the value of $Z_R$

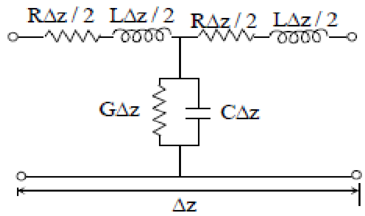
c) Signal distortion implies impedance mismatch at both ends: $Z_T$ # $Z_0$ and $Z_R$ # $Z_0$

d) Impedance mismatches do NOT result in signal distortion but reduce power transfer efficiency

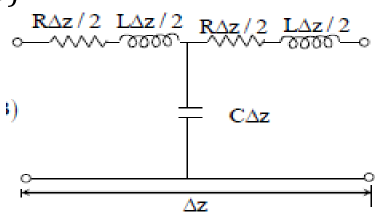
**[GATE–2014-3]**

**Q.33** A coaxial cable is made of two brass conductors. The spacing between the conductors is filled with Teflon ($\varepsilon_r$ = 2. 1, $\tan\delta$ = 0). Which one of the following circuits can represent the lumped element model of a small piece of this cable having length Δz ?

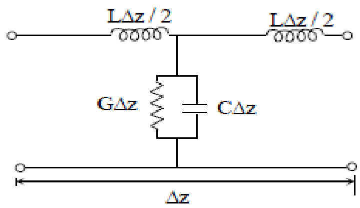
a)



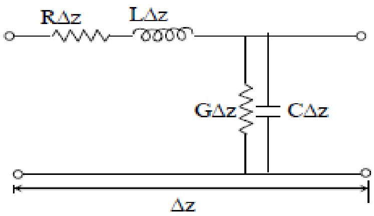

b)




c)

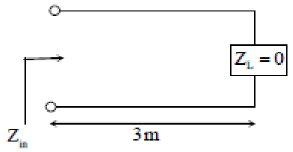



d)




**[GATE–2015-3]**

**Q.34** Consider the 3 m long lossless air filled transmission line shown in the figure. It has a characteristics impedance of $120\pi\ \Omega$, is terminated by a short circuit , and is excited with a frequency of 37.5 MHz. What is the nature of the input impedance ($Z_{in}$)?



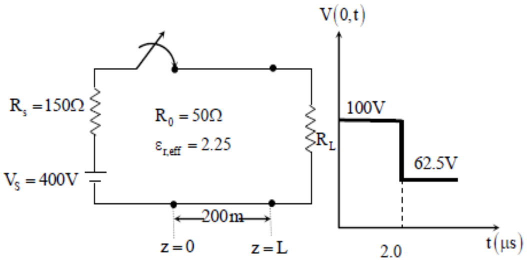

a) Open b) Short
c) Inductive d) Capacitive

**[GATE–2015-3]**

**Q.35** A coaxial capacitor of inner radius 1 mm and outer radius 5 mm has a capacitance per unit length of 172 pF/m. If the ratio of outer radius to inner is doubled, the capacitance per unit length (in pF/m) is ________ .

**[GATE–2015-3]**

**Q.36** A 200 m long transmission line having parameters shown in the figure is terminated into a load $R_L$. The line is connected to a 400 V source having source resistance $R_S$ through a switch which is closed at t = 0. The transient response of the circuit at the input of the line (z = 0) is also drawn in the figure. The value of $R_L$ (in $\Omega$) is ________



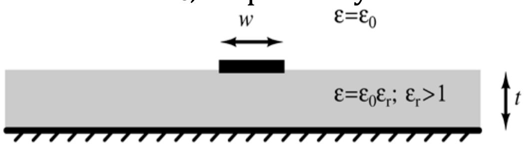

**[GATE–2015-3]**

**Q.37** The propagation constant of a lossy transmission line is (2 + j5) $m^{-1}$ and its characteristic impedance is (50+ j0) ω at ω = $10^6$ rad $s^{-1}$. The values of the line constants L, C, R, G are, respectively,

a) L = 200 μH/m, C= 0.1 μH/m, R= 50 Ω/m, G= 0.02 S/m
b) L = 250 μH/m, C= 0.1 μH/m, R= 100 Ω/m, G= 0.04 S/m
c) L = 200 μH/m, C= 0.2 μH/m, R= 100 Ω/m, G= 0.02 S/m
d) L = 250 μH/m, C= 0.2 μH/m, R= 50 Ω/m, G= 0.04 S/m

**[GATE – 2016-1]**

**Q.38** A lossless micro strip transmission line consists of a trace of width w. It is drawn over a practically infinite ground plane and is separated by a dielectric slab of thickness t and relative permittivity $\varepsilon_r$ >1. The inductance per unit length and the characteristic impedance of this line are L and $Z_0$, respectively.



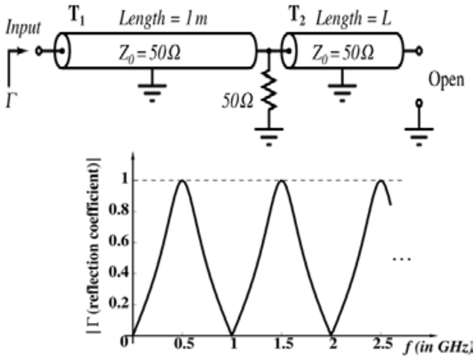

Which one of the following inequalities is always satisfied?

a) $Z_o > \sqrt{\dfrac{Lt}{\varepsilon_0\varepsilon_r w}}$ b) $Z_o < \sqrt{\dfrac{Lt}{\varepsilon_0\varepsilon_r w}}$

c) $Z_o > \sqrt{\dfrac{Lw}{\varepsilon_0\varepsilon_r t}}$ d) $Z_o < \sqrt{\dfrac{Lw}{\varepsilon_0\varepsilon_r t}}$

**[GATE – 2016-2]**

**Q.39** A microwave circuit consisting of lossless transmission lines $T_1$ and $T_2$ is shown in the figure. The plot shows the magnitude of the input reflection coefficient Γ as a function of frequency f. The phase velocity of the signal in the transmission lines is 2× $10^8$ m/s.




The length L ( in meters) of $T_2$ is ___

**[GATE – 2016-2]**

**Q.40** The voltage of an electromagnetic wave propagating in a coaxial cable with uniform characteristic impedance $V(l) = e^{-\gamma l + j\omega t}$ volts, Where l is the distance along the length of the cable in meters. $\gamma = (0.1 + j40)\,m^{-1}$ is the complex propagation constant, and $\omega = 2\pi x10^9\, rad/s$ is the angular frequency. The absolute value of the attenuation in the cable in dB/meter is ________.

**[GATE – 2017-1]**

**Q.41** A two – wire transmission line terminates in a television set. The VSWR measured on the line is 5.8. The percentage of power that is reflected from the television set is ____________

**[GATE – 2017-1]**

**Q.42** A lossy transmission line has resistance per unit length R = 0.05Ω/m The line is distortionless and characteristic impedance of 50Ω The attenuation constant (in Np/m, correct to three decimal places) of the line is_____. / m.

**[GATE – 2017-1]**

# ANSWER KEY:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (b) | (b) | (c) | (b) | (d) | (a) | (d) | (d) | (a) | (c) | (b) | (c) | (a) | (d) |
| **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** |
| (d) | (b) | (d) | (b) | (d) | (d) | (d) | (b) | (d) | (a) | (c) | (b) | (a) | (b) |
| **29** | **30** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** | **41** | **42** |
| 40 | 70.7 | 33.3 | (c) | (b) | (d) | 120.2 | 30 | (b) | (b) | 0.1 | 0.868 | 49.82 | 0.001 |

# EXPLANATIONS

**Q.1 (b)**

$Z_0 = \sqrt{Z_{OC}.Z_{SC}}$

$= \sqrt{100 \times 25}$

$Z_0 = 10 \times 5 = 50\Omega$

**Q.2 (b)**

$S = \frac{1+\Gamma}{1-\Gamma}$

$\Rightarrow \quad 3 = \frac{1+\Gamma}{1-\Gamma}$

$\Gamma = 0.5$

$\frac{P_r}{P_i} = \Gamma^2 = 0.25$

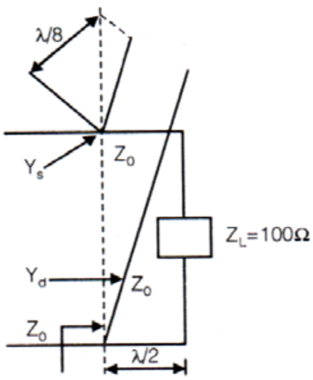
$\therefore$ 25% of incident power is reflected

**Q.3 (c)**

**Q.4 (b)**

**Q.5 (d)**

**Q.6 (a)**




For $Y_d . \beta d = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$

$Z_d = \frac{Z_o[Z_L + jZ_o \tan \beta d]}{Z_o + jZ_L \tan \beta d}$

$Z_d = \frac{50[100 + j50 \tan \pi]}{(50 + j100 \tan \pi)}$

$= 100$

$Y_d = \frac{1}{Z_d} = \frac{1}{100} = 0.01$

For $Y_s$' $\beta d = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}, Z_L \to 0$

$Z_S = \frac{Z_o\left[Z_L + jZ_0 \tan \frac{\pi}{4}\right]}{\left[Z_0 + jZ_L \tan \frac{\pi}{4}\right]} = jZ_0$

$Y_s = \frac{1}{Z_s} = \frac{1}{jZ_0} = -0.02j$

$\therefore \quad Y = Y_d + Y_s$

= 0.01 – 0.02j

**Q.7 (d)**

$\rho = \frac{1+\Gamma}{1-\Gamma}$

$\Rightarrow \quad 2 = \frac{1+\Gamma}{1-\Gamma}$

$\Rightarrow \quad \Gamma = \frac{1}{3}$

$\therefore \quad \frac{P_{ref}}{P_{inc}} = \Gamma^2 = \frac{1}{3}$

$\Rightarrow$ 11.11% of $P_i$ is reflected

**Q.8 (d)**

$$SWR = \frac{E_{max}}{E_{min}}$$

$$\Rightarrow \quad S = \frac{5E_{min}}{E_{min}} = 5$$

$$5 = \frac{1-|\Gamma|}{1-|\Gamma|} \quad |\Gamma| = \frac{2}{3} \quad \therefore \quad \Gamma = -\frac{2}{3}$$

As $\eta_2 < \eta_1$

$$\Rightarrow -\frac{2}{3} = \frac{\eta_2 - 120\pi}{\eta + 120\pi}$$

$$\Rightarrow \quad \eta_2 = 24\pi$$

**Q.9 (a)**

Movement on const. r-circle by an $\angle 45^0$ in C.W. direction. R is same and reactance increases i.e. addition of induction in series with Z.

**Q.10 (c)**

$$\frac{V_L}{V_{in}} = \frac{Z_o}{Z_{in}}$$

$$\Rightarrow \quad V_L = \frac{10 \times 300}{50} = 60V$$

**Q.11 (b)**

**Q.12 (c)**

$$S = \frac{V_{max}}{V_{min}}$$

$$= \frac{4}{1} = 4$$

$$S = \frac{1+\Gamma}{1-\Gamma}$$

$$Z_{max} = Z_0.S$$

$$Z_{min} = Z_0 / S$$

As minima is at load,

$$\therefore Z_L = Z_{min} = \frac{50}{4} = 12.5\Omega$$

**Q.13 (a)**

$$G = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{12.5-50}{12.5+50}$$

$$\Gamma = -0.6$$

**Q.14 (d)**

$$Z_0^2 = Z_{OC}.Z_{SC}$$

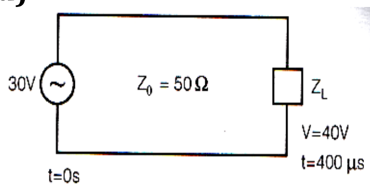
$$Z_{SC} = \frac{50 \times 50}{100 + j150}$$

$$= \frac{50}{2+3j}$$

$$= \frac{50(2-3j)}{13}$$

$$Z_{SC} = 7.69 - 11.54j$$

**Q.15 (d)**




$$\Gamma = \frac{V_r}{V_i} = \frac{10}{30} = \frac{1}{3}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

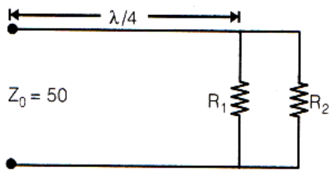
$$\Rightarrow \frac{1}{3} = \frac{Z_L - 50}{Z_L + 50}$$

$$\Rightarrow Z_L = 100\Omega$$

**Q.16 (b)**

$$I_L = \frac{30}{100} = 0.3A$$

**Q.17 (d)**




$$Z_{in} = \frac{Z_0^2}{Z_L}; ifl = \frac{\lambda}{4}$$

$$R_1 \text{ due to } 100\Omega = \frac{50^2}{100} = 25$$

$$R_2 \text{ due to } 200\Omega = \frac{50^2}{200} = \frac{25}{2}$$

$$\Rightarrow R_1 \| R_2 = 25 \| \frac{25}{2} = \frac{25}{3}$$

$$Z_S = \frac{(50)^2}{25/3} = 300\Omega$$

$$\Rightarrow \Gamma = \frac{Z_S - Z_0}{Z_S + Z_0}$$

$$= \frac{300-50}{300+50} = \frac{5}{7}$$

**Q.18 (b)**

The line is terminated with *50Ω* at the center and so matched on both the sides.

**Q.19 (d)**

**Q.20 (d)**

$f = 3GHz$

$$\lambda = \frac{C}{f}, \beta = \frac{2\pi}{\lambda}$$

$$\beta l = \frac{2\pi f l}{c}$$

$$= \frac{2\pi \times 3\times 10^9}{3\times 10^8} \times 0.01$$

$$= \frac{\pi}{5} = 36°$$

Input impedance

$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

Given that $Z_L = 0, Z_0 = 75\Omega$

$$Z_{in} = \frac{Z_0 . jZ_0 \tan\beta l}{Z_0}$$

$$= jZ_0 \tan\beta l$$

$$= j\times 75 \times \tan 36°$$

$$= j54.49\Omega$$

Hence , input impedance is inductive.

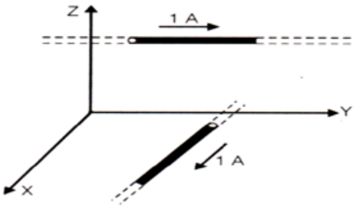
**Q.21 (d)**

$$Z_1 = \frac{Z_0^2}{Z_{L_1}} = \frac{(100)^2}{50} = 200\Omega$$

$$Z_2 = \frac{Z_0^2}{Z_2} = \frac{(100)^2}{50} = 200\Omega$$

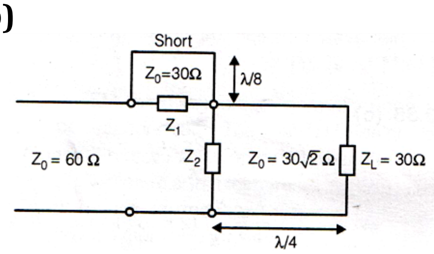
$$Z_L = Z_1 \| Z_2 = 100\Omega$$

$$Z_i = \frac{Z_0^2}{Z_L} = \frac{(50)^2}{100} = 25\Omega$$




a) x, y, z components
b) x, y components
c) y, z components
d) x, z components

**Q.22 (b)**




$$Z_{in} = Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

Input impedance

$$Z_1 = 30\left[\frac{0 + j30\tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right)}{30+0}\right]$$

$$\Rightarrow Z_1 = j30$$

$$Z_2 = 30\sqrt{2}\left[\frac{30 + j30\sqrt{2}\tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right)}{30\sqrt{2} + j30\tan\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{8}\right)}\right]$$

$$= 30\sqrt{2}\left[\frac{\frac{30}{\tan\pi/2} + j30\sqrt{2}}{\frac{30\sqrt{2}}{\tan\pi/2} + j30}\right]$$

$$= 60\Omega$$

Or , $Z_2 = \frac{(30\sqrt{2})^2}{30} = 60\Omega$

Load impedance,

$$Z_L = Z_1 + Z_2 = j30 + 60$$

Magnitude of reflection coefficient,

$$|\rho| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \left|\frac{60 + j30 - 60}{60 + j30 + 60}\right|$$

$$= \left|\frac{j30}{120 + j30}\right| = \left|\frac{j1}{4 + j}\right|$$

$$= \frac{1}{\sqrt{16+1}} = \frac{1}{\sqrt{17}}$$

VSWR on 60Ω line,

$$VSWR == \frac{1+|\rho|}{1-|\rho|} = \frac{1+\frac{1}{\sqrt{17}}}{1-\frac{1}{\sqrt{17}}} = 1.6$$

**Q.23 (d)**

For distortion less transmission line,

$$LG = RC \Rightarrow \frac{L}{C} = \frac{R}{G}$$

Characteristic impedance,

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$$

Attenuation constant,

$$\alpha = \sqrt{RG} = \sqrt{R}.\sqrt{\frac{R}{Z_0}}$$

$$= \frac{R}{Z_0} = \frac{0.1}{50}$$

$$= 0.002 Np/m$$

**Q.24 (a)**

$$VSWR = 5 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\therefore |\Gamma| = \frac{2}{3} \Rightarrow |\Gamma| = \pm\frac{2}{3}$$

If maximum is at $\lambda/4$ from load then minimum will be at load itself

Reflection coefficient $\Gamma = |\Gamma| e^{j\theta}$

Where , $\theta = 720° \left(\frac{x_{vm}}{\lambda} - \frac{1}{4}\right)$

$x_{vm}$ = distance of minima from load

Here $x_{vm} = 0$

So, $\theta = -180°$

Therefore , $\Gamma = |\Gamma| e^{-j180°}$

$$\Rightarrow \Gamma = -\frac{2}{3}$$

Now $-\frac{2}{3} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L - 50}{Z_0 + 50}$

$$\Rightarrow Z_L = 10\Omega$$

**Q.25 (c)**

Phase difference $= \frac{2\pi}{\lambda}$ (path difference)

$$\Rightarrow \frac{\pi}{4} = \frac{2\pi}{\lambda}(2\times10^{-3})$$

$$\therefore \lambda = 8\times2\times10^{-3}$$

$$= 16\times10^{-3} m$$

$$f = 10GHz = 10\times10^{9} Hz$$

Hence the phase velocity of wave along the line is

$$v = f\lambda = 10\times10^{9}\times16\times10^{-3}$$

$$\therefore v = 1.6\times10^{8} m/s$$

**Q.26 (b)**

**Q.27 (a)**

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Only options (a) satisfy this

$$\frac{1+.01}{1-0.1} = \frac{1.1}{0.9} = 1.22$$

**Q.28 (b)**

$$Z_o = \frac{276}{\sqrt{\epsilon_r}}\log\left(\frac{d}{r}\right)$$

d → distance between the two plates

so, $Z_o$ — changes, if the spacing between the plates changes.

V= $\frac{1}{\sqrt{LC}}$ → independent of spacing between the plates

**Q.29 (40)**

Given, $\ell = \lambda/s$

$Z_o$ =50 Ω

$$Z_{in}\left(l = \lambda/8\right) = Z_o\left[\frac{Z_L + JZ_o}{Z_o + KZ_L}\right]$$

$$Z_{in} = 50\left[\frac{Z_L + J50}{50 + JZ_L}\right]$$

$$= 50\left[\frac{Z_L + J50}{50 + JZ_L} \times \frac{50 - JZ_L}{50 - JZ_L}\right]$$

$$Z_{in} = 50\left[\frac{50Z_L + 50Z_L + j(50^2 - Z_L^2)}{50^2 + Z_L^2}\right]$$

Given, $Z_{in} \rightarrow$ Real

So, $I_{mg}\ (Z_{in}) = 0$

$50^2 - Z_L^2 = 0$

$Z_L^2 = 50^2$

$R^2 = 50^2 - X^2 = 50^2 - 30^2$

$R = 40\Omega$

**Q.30 (70.7)**

Here impedance is matched by using QWT $(\lambda/4)$

$\therefore Z_0' = \sqrt{Z_L Z_{in}}$

$= \sqrt{100 \times 50} = 50\sqrt{2}$

$= Z_0' = 70.7\Omega$

**Q.31 (33.33)**

Here $l = \frac{\lambda}{2}$

$Z_{in}\left(l = \lambda/2\right) = Z_L = 50\Omega$

$\therefore Z_{in} = (100 \| 50)$

$= \frac{100}{3} = 33.33\Omega$

**Q.32 (c)**

Signal distortion implies impedance mismatch at both ends. i.e.,

$Z_T \# Z_0$

$Z_R \# Z_0$

**Q.33 (b)**

Loss tangent $\tan\delta = 0 = \frac{\sigma}{\omega\epsilon}$

$\sigma = 0$

G $\rightarrow$ Conductivity of the dielectric material

So, $\sigma = 0 = G$

**Q.34 (d)**

$z_{in} = JZ_0 \tan\beta l$

$$\beta l = \frac{2\pi}{\lambda}.l = \frac{2\pi}{8}(3) = \frac{3\pi}{4}$$

$$\lambda = \frac{3\times10^8}{37.5\times10^6} = 8$$

Short circuited line

$0 < \beta l \frac{\pi}{2} \rightarrow$ inductor

$\frac{\pi}{2} < \beta l < \pi \rightarrow$ Capacitor

**Q.35 (120.22)**

$$C = \frac{2\pi\epsilon l}{In\left(\frac{b}{a}\right)}$$

$$\frac{C}{l} = \frac{2\pi\epsilon}{In\left(\frac{b}{a}\right)} = C_1$$

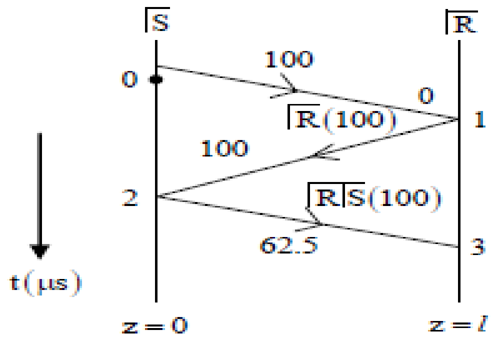
$$\frac{C_1}{C_2} = \frac{In(b_2/a_2)}{In(b_1/a_1)}$$

$$\frac{172pF}{C_2} = \frac{In(10/1)}{In(5)}$$

$$C_2 = \frac{In(5)}{In(10)}172pF$$

$C_2 = 120.22pF$

**Q.36 (30)**




Given V(t = 2 μs, Z=0)=62.5

$62.5 = V(t=0, z=0) + V(t=1, z=0) + V(t=2, z=0)$

$62.5 = 100 +$

**Q.37 (b)**

We know

$$\delta = \sqrt{(R + j\omega L)(G + j\omega L)}.....(1)$$

$$z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega L}}............(2)$$

From (1) and (2)

$$R + j\omega L = z_0 \times \delta = 50 \times (2 + j5)$$

$\Rightarrow R = 100\Omega/m$ & $L = 250\mu H/m$

From (1) and (2)

$$G + j\omega c = \frac{\delta}{z_0} = \frac{2 + j5}{50}$$

$$\Rightarrow G = \frac{0.04s}{m} \;\&\; c = 0.1\mu F/m$$

**Q.38 (b)**

**Q.39 (0.1)**

**Q.40 0.8686 (0.85-0.88)**

Given $\gamma = (0.1 + j40)m^{-1}$

Here $\alpha = 0.1\omega_p / m$

We know that $1\omega_p / m = 8.686dB / m$

$0.1\omega_p / m = 0.8686dB / m$

**Q.41 49.82 %(48.0-51.0)**

Percentage of power reflected is

$|\Gamma|^2$ x100

$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1}$$

$$= \frac{5.8 - 1}{5.8 + 1} = \frac{4.8}{6.8} = 0.7058$$

% Power reflected

$|\Gamma|^2$ x100 $= 49.82\%$

**Q.42 0.001**

Given $R = 0.05\Omega/m$

Condition for distortion less transmission line

$$RC = LG$$

$$\frac{R}{G} = \frac{L}{C}$$

$$\frac{R}{G} = 50x50 \Rightarrow R = G$$

$$\sqrt{\frac{L}{C}} = z_0 = 50$$

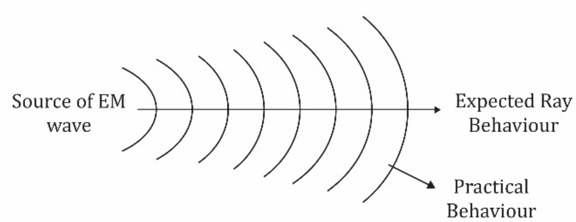
$$\Rightarrow \frac{L}{C} = 50^2$$

Alternation constant= $\sqrt{RG}$

$$= \sqrt{\frac{0.05x0.05}{50x50}} = \frac{0.05}{50} = 0.001$$

# 7 WAVEGUIDES

EM waves cannot travel like a single ray or a uniform beam as discussed in the previous sections. EM wave tends to open out by forming spherical wavefronts and hence reducing the power density of the wave as its travel forward.




A waveguide can be used to confine the wave within the stipulated limits and can therefore give directionality for a wave. This helps the EM wave to have same power density without having to get weakened.

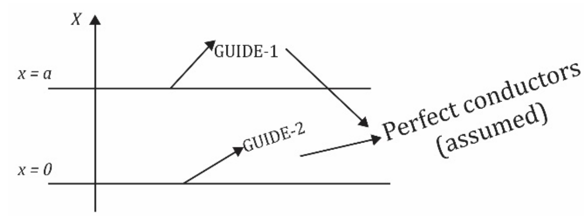
The most popular waveguides are:

Parallel Plane waveguides – for 1-dimensional restriction of the wave.

Rectangular waveguides – for 2-dimensional restriction of the wave.

## 7.1 PARALLEL PLANE WAVEGUIDES:

Consider a wave having E(x, z, t)/H(x, z, t) between two infinite conducting sheets at x = 0 and x = a




The wave is assumed to expand in x-direction only and not in y-direction and wave is expected to propagate in the z-direction. The boundary conditions for 'a', the conductor can be defined as $E_{tan} = 0$, i.e. no E-field can exist parallel to the conductor of surface.

## 7.2 WAVE BEHAVIOUR BETWEEN GUIDES:

When $\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu_0\varepsilon_0}$

There are only two possibilities for $\gamma$

i.e. $\bar{\gamma} = \text{real when } \left(\frac{m\pi}{a}\right)^2 > \omega^2\mu_0\varepsilon_0$

$= \alpha + j0$

= only attenuation of E field but no wave or phase changes exist.

$\bar{\gamma} = \text{imaginary when } \omega^2\mu_0\varepsilon_0 > \left(\frac{m\pi}{a}\right)^2$

$= 0 + j\bar{\beta}$

= An un-attenuated wave exists.

## 7.3 CUT-OFF FREQUENCY

For wave propagation to exist between the guides, the frequency 'ω' should be sufficiently large enough, else there cannot be wave propagation possible. Every guide has a cut-off frequency below which waves cannot propagate in the guide.

$$\omega^2\mu_0\varepsilon_0 > \left(\frac{m\pi}{a}\right)^2$$

$$\omega > \frac{m\pi}{a\sqrt{\mu_0\varepsilon_0}}$$

$$\omega_c = \text{cut} - \text{off frequency}$$

$$\omega_c = \frac{m\pi c}{a}$$

$$f_c = \frac{mc}{2a}$$

$$c = \text{free space velocity} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\lambda_c = \frac{2a}{m}$$ m = integer 0, 1, 2, 3 ...

## 7.4 PHASE VELOCITY & GUIDE WAVELENGTH:

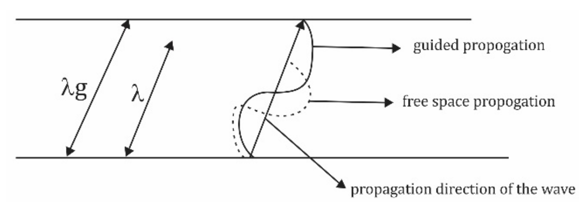
The velocity of the wave when represented as phase velocity

$$V_p = \frac{\omega}{\beta}$$

$$\overline{\beta} = \sqrt{\omega^2\mu_0\varepsilon_0 - \left(\frac{m\pi}{a}\right)^2}$$

$$V_P = \frac{\omega}{\sqrt{\omega^2\mu_0\varepsilon_0 - \left(\frac{m\pi}{a}\right)^2}}$$

$$= \frac{1}{\sqrt{\mu_0\varepsilon_0 - \left(\frac{m\pi}{\omega a}\right)^2}}$$




The $V_P$ expression can be rewritten as,

$$\lambda_g f = \frac{\lambda c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

The wavelength of the wave in the guide is therefore larger than that in free-space due to the increased phase change, such that exactly at the guide walls E field has zero value.

$$V_P = \frac{c}{\sqrt{1 - \left(\frac{m\pi}{a\omega\sqrt{\mu_0\varepsilon_0}}\right)^2}}$$

$$V_P = \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$= \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

As seen in the above equation, $V_p > V_c$ implies that the phase changes of the wave are faster than in free space. The wave has a dynamic change in rate of phase change with distance $(\beta)$ that is essential to satisfy the boundary conditions at the conducting walls.

### 7.4.1 GROUP VELOCITY

The velocity of a wave can be expressed as

$$V_p = \frac{\omega}{\beta}$$

When the phase changes with time and space are linear to each other i.e. $\beta \propto \omega$. Otherwise, in all other circumstances term group velocity is used to represent the propagation rate.

$$V_g = \frac{d\omega}{d\beta}$$

In guided waves as $\lambda_g$ is increased due to changes in phase with space being changed, $v_g$ represents the energy propagation rate such that

$$V_p V_g = c^2$$

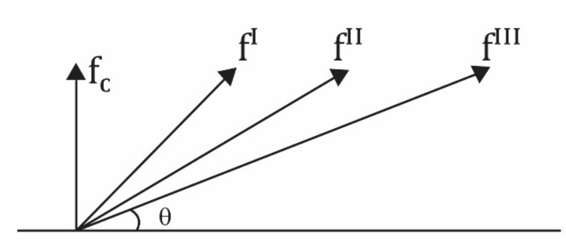
Hence

$$V_g = c\cos\theta$$

Hence, no propagation exists exactly at cut-off frequencies.

### 7.5 WAVE ANGLE

Every wave above $f_c$ travels along the guide axis in a pre-determined angle that depends on how larger is f over $f_c$. At exact $f_c$ there is no propagation along the axis but oscillates.




When

$$f_c = \frac{mc}{2a} \qquad f''' > f'' > f' > f_c$$

The angle made with the guide walls is called as wave angle, such that

$$\sin\theta = \frac{f_c}{f}$$

$$\Rightarrow V_p = \frac{c}{\cos\theta}$$

$$\lambda_g = \frac{\lambda}{\cos\theta}$$

Note that $f = f_c$, $\theta = 90^\circ$ and as the frequency increase angle made with the walls decreases.

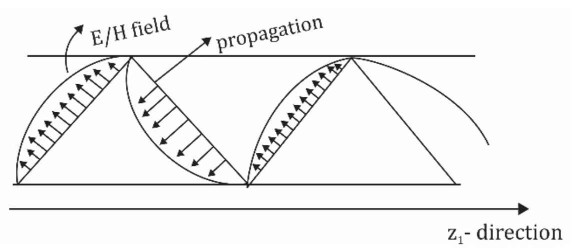
### 7.6 MODES OF OPERATION

$$f_c = \frac{mc}{2a} \qquad \text{when } m = 1$$

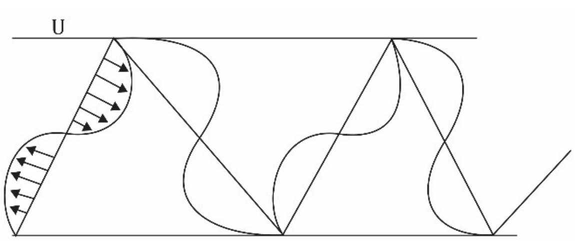
$$f_c = \frac{c}{2a}$$

is the least cut-off frequency of the guide and all the frequencies above $f_c$ and below

$2f_c$ propagate in the guide as shown below.



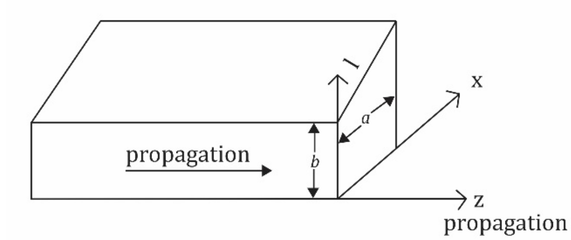

Similarly all frequencies from $2f_c$ to $3f_c$ propagation as shown. They have their own wave angles depending on their frequency.




## 7.7 RECTANGULAR WAVEGUIDES

A parallel plane waveguide confines waves in one dimension only. A more practical waveguide needs to confine the wave in both the dimensions and allow propagation in one dimensions.

A rectangular waveguide has two sets of finite conducting sheets as shown below.




Both x and y dimensions are restricted by the sheets and hence $\gamma_x = \frac{m\pi}{a}$ and $\gamma_y = \frac{n\pi}{b}$

Propagation constant

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu_0\varepsilon_0}$$

Cut-off frequency of the guide is

$$\omega_c = \left[\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right]c$$

$$f_c = \frac{\left[\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right]c}{2\pi}$$

Phase velocity of the guide is

$$V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Wave angle

$$\sin\theta = \frac{f_c}{f}$$

TE/ TM/ TEM waves in guided conditions

$$E_x(x,y,z), E_y(x,y,z), E_z(x,y,z)$$

$$H_x(x,y,z), H_y(x,y,z), H_z(x,y,z)$$

## 7.8 TE WAVES IN RECTANGULAR WAVEGUIDES

$$E_x = E_{x0}\left[\frac{n\pi}{b}\right]\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$E_y = E_{y0}\left[\frac{m\pi}{a}\right]\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$H_x = H_{x0}\left[\frac{m\pi}{a}\right]\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$H_y = H_{y0}\left[\frac{n\pi}{b}\right]\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$H_z = H_{z0}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

## 7.9 DOMINATE MODE OF TE WAVES

When m = 0 and n = 1

$$E_x = E_{x0}\left[\frac{n\pi}{b}\right]\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$H_y = H_{y0}\left[\frac{n\pi}{b}\right]\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$H_z = H_{z0}\cos\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

When n = 0 and m = 1

$$H_x = H_{x0}\left[\frac{m\pi}{a}\right]\sin\left(\frac{m\pi}{a}x\right)e^{-\gamma z}$$

$$E_y = E_{y0}\left[\frac{m\pi}{a}\right]\sin\left(\frac{m\pi}{a}x\right)e^{-\gamma z}$$

$$H_z = H_{z0}\cos\left(\frac{m\pi}{a}x\right)e^{-\gamma z}$$

The dominant mode is $TE_{10}$ or $TE_{01}$

The existing modes are $TE_{mo}$ or $TE_{on}$ and $TE_{mn}$

## 7.10 TM WAVES IN RECTANGULAR WAVEGUIDE

$$E_x = E_{x0}\left[\frac{m\pi}{a}\right]\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$E_y = E_{y0}\left[\frac{n\pi}{b}\right]\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$E_z = E_{z0}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$H_x = H_{x0}\left[\frac{n\pi}{b}\right]\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

$$H_y = H_{y0}\left[\frac{m\pi}{a}\right]\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\gamma z}$$

## 7.11 DOMINATE MODE IN TM WAVES

When m = 0 or n = 0, all components of E and H vanish. Dominant mode is $TM_{11}$. The existing modes are $TM_{mn}$ only.

## 7.12 NUMERICAL APERTURE

The most important parameter of an optical fiber is its numerical aperture (NA). The value of NA is decided by the refracted indices of the core and the cladding.

By definition, the refractive index n of a medium is defined as

$$n = \frac{\text{speed of light in vaccum}}{\text{speed of light in medium}}$$

$$= \frac{c}{\mu_m} = \frac{\frac{1}{\sqrt{\mu_0\varepsilon_0}}}{\frac{1}{\sqrt{\mu_m\varepsilon_m}}} \qquad (1)$$

Since $\mu_m = \mu_0$ in most practical cases,

$$n = \sqrt{\frac{\varepsilon_m}{\varepsilon_0}} = \sqrt{\varepsilon_r} \qquad (2)$$

indicating that the refractive index is essentially the square-root of the dielectric constant.

Keep in mind that $\varepsilon_r$ can be complex. For common materials, n = 1 for air, n = 1.33 for water, and n = 1.5 for glass.

As a light ray propagates from medium 1 to medium 2, Snell's law must be satisfied.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (3)$$

Where $\theta_1$ is the incident angle in medium 1 and $\theta_2$ is the transmission angle in medium 2.

The total reflection occurs when $\theta_2 = 90^\circ$, resulting in

$$\theta_1 = \theta_c = \sin^{-1}\frac{n_2}{n_1} \qquad (4)$$

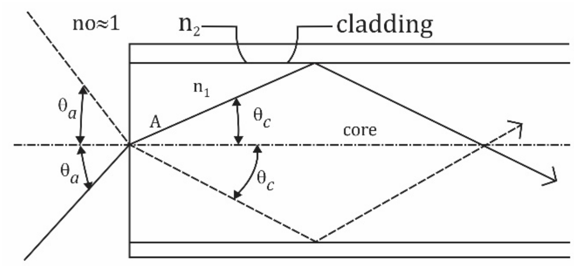
Another way of looking at the light-guiding capability of fiber is to measure the acceptance angle $\theta_a$, which is the maximum angle over which light rays entering the fiber will be trapped in its core. We know that the maximum angle occurs when $\theta_c$ is the critical angle, there by satisfying the condition for total internal reflection. Thus, for a step-index fiber.

$$NA = \sin\theta_a = n_1 \sin\theta_c = \sqrt{n_1^2 - n_2^2} \qquad (5)$$

Where $n_1$ is the refractive index of the core and $n_2$ is the refractive index of the cladding as shown in figure.

Since most fiber cores are made of silica $n_1 = 1.48$. Typical values of NA range between 0.19 and 0.25. The larger the value of NA, the more optical power the fiber can capture from a source. Because such optical fibers may support the numerous modes, they are called multi-mode step-index fibers. The mode volume V is given by

$$v = \frac{\pi d}{\lambda}\sqrt{n_1^2 - n_2^2} \qquad (6)$$




Where d is the fiber core diameter and $\lambda$ is the wavelength of the optical source. From equation (6), the number N of modes propagating in a step-index fiber can be estimated as

$$N = \frac{v^2}{2} \qquad (7)$$

### 7.13 ATTENUATION

Attenuation is the reduction in the power of the optical signal. Power attenuation (or fiber loss) in an optical fiber is governed by

$$\frac{dp}{dz} = -\alpha p \qquad (8)$$

Where $\alpha$ the attenuation and p is the optical power. In equation (8), it is assumed that a wave propagates along z. By solving equation (8), the power p (0) at the input of the fiber and the power p (l) of the light after l are related as

$$p(l) = p(0)e^{-\alpha z} \qquad (9)$$

It is customary to express attenuation $\alpha$ in decibels per kilometer and length l of the fiber in kilometers. In this case equation (9) becomes

$$\alpha_l = 10\log_{10}\frac{p(0)}{p(1)}$$

Thus, the power of the light reduces by $\alpha$ decibels per kilometer as it propagates, through the fiber. Equation (10) may be written as

$$p(p) = p(0) = 10^{-\alpha l/10}$$

For $l = 100\text{km}$

$$\frac{p(0)}{p(l)} \sim \begin{cases} 10^{-100} & \text{For coaxial cable} \\ 10^{-2} & \text{For Fiber} \end{cases}$$

Deciding that much more power is lost in the coaxial cable than in fiber.

## 7.14 DISPERSION

The spreading of pulses of light as they propagate down a fiber is called dispersion. As the pulses representing 0s spread, they overlap epochs that represent 1s. Dispersion is beyond a certain limit. It may confuse the receiver. The dispersive effects in single mode fibers are much smaller than in multimode fibers.

# GATE QUESTIONS

**Q.1** A rectangular waveguide has dimensions $1\,cm \times 0.5\,cm$, Its cut-off frequency is
a) 5 GHz b) 10 GHz
c) 15 GHz d) 12 GHz
**[GATE – 2000]**

**Q.2** A TEM wave is incident normally upon a perfect conductor. The E and H fields at the boundary will be. Respectively,
a) minimum and minimum
b) maximum and maximum
c) minimum and maximum
d) maximum and minimum
**[GATE – 2000]**

**Q.3** The dominant mode in a rectangular waveguide is $TE_{10}$ because this mode has
a) no attenuation
b) no cut-off
c) no magnetic field component
d) the highest cut-off wavelength
**[GATE – 2001]**

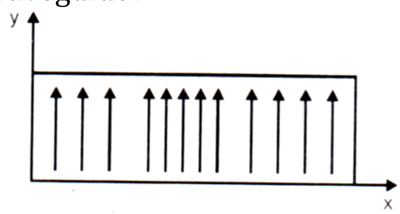
**Q.4** The phase velocity of waves propagating in a hollow metal waveguide is
a) greater than the velocity of light in free space
b) less than the velocity of light in free space
c) equal to the velocity of light in free space
d) equal to the group velocity
**[GATE – 2001]**

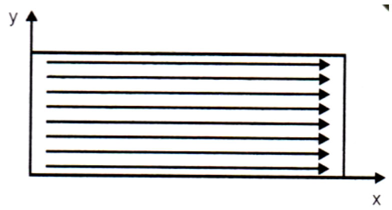
**Q.5** The phase velocity for the $TE_{10}$ mode in an air-filled rectangular waveguide is
a) Less than c b) equal to c
c) Greater to c d) none of the above
**Note**: (c is the velocity of plane waves in free space)
**[GATE – 2002]**

**Q.6** A rectangular metal wave guide filled with a dielectric material of relative permittivity $\varepsilon_r = 4$ has the inside dimensions $3.0\,cm \times 1.2\,cm$. The cut-off frequency for the dominant mode is
a) 2.5 GHz b) 5.0 GHz
c) 10.0 GHz d) 12.5 GHz
**[GATE – 2003]**

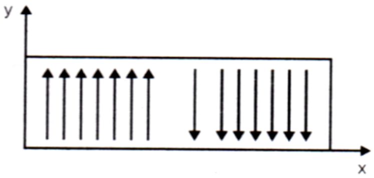
**Q.7** The phase velocity of an electromagnetic wave propagating in a hallow metallic rectangular waveguide in the $TE_{10}$ mode is
a) equal to its groups velocity
b) less than the velocity of light in free space
c) equal to the velocity of light in free space
d) greater than the velocity of light in free space
**[GATE – 2004]**

**Q.8** Which one of the following does represent the electric field lines for the $TE_{02}$ mode in the cross-section of a hollow rectangular metallic waveguide?

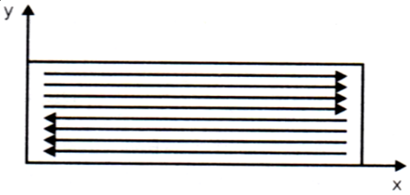
a) 


b) 

c)




d)




**[GATE – 2005]**

**Q.9** In a microwave test bench, why is the microwave signal amplitude modulation at 1 KHz?

a) To increase the sensitivity of measurement

b) To transmit the signal to a far – off place

c) To study amplitude modulation

d) Because crystal detector fails at microwave frequencies

**[GATE – 2006]**

**Q.10** A rectangular waveguide having $TE_{10}$ mode as dominant mode is having a cutoff frequency of 18-GHz for the $TE_{30}$ mode. The inner broad-wall dimension of the rectangular waveguide is

a) 5/3 cms  b) 5 cms

d) 5/2 cms  d) 10 cms

**[GATE – 2006]**

**Q.11** The $\vec{E}$ field in a rectangular waveguide of inner dimensions $a \times b$ is given by

$$\vec{E} = \frac{\omega\mu}{h^2}\left(\frac{\pi}{a}\right)H_0 \sin\left(\frac{2\pi x}{a}\right)\sin(\omega t - \beta z)\,\hat{y}$$

Where H0 is a constant, and a and b are the dimensions along the x-axis and the y-axis respectively. The mode of propagation in the waveguide is.

a) $TE_{20}$  b) $TM_{11}$

c) $TM_{20}$  d) $TE_{10}$

**[GATE – 2007]**

**Q.12** An air-filled rectangular waveguide has inner dimensions of 3cm x 2 cm. The wave impedance of the $TE_{20}$ mode of propagation in the waveguide at a frequency of 30 GHz is (free space impedance $\eta_0 = 377\Omega$ )

a) $308\Omega$  b) $355\Omega$

c) $400\Omega$  d) $461\Omega$

**[GATE – 2007]**

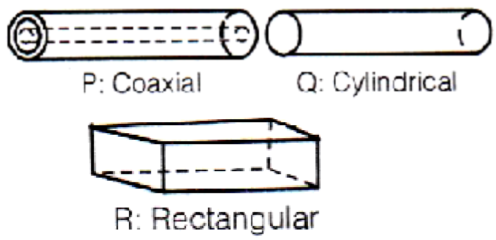
**Q.13** A rectangular waveguide of internal dimensions (a = 4cm and b = 3 cm) is to be operated in $TE_{11}$ mode. The minimum operating frequency is

a) 6.25 GHz  b) 6.0 GHz

c) 5. 0 GHz  d) 3.75 GHz

**[GATE – 2008]**

**Q.14** Which of the following statements is true regarding the fundamental mode of the metallic waveguides shown?




a) Only P has no cutoff-frequency

b) Only Q has no cutoff-frequency

c) Only R has no cutoff-frequency

d) All these have cutoff-frequency

**[GATE – 2009]**

**Q.15** The modes in a rectangular waveguide are denoted by $TE_{mn}/TM_{mn}$ where m and n are the eigen numbers along the larger and smaller dimensions of the waveguide respectively. Which one of the following statement is TRUE?

a) The $TM_{10}$ mode of the waveguide does not exist

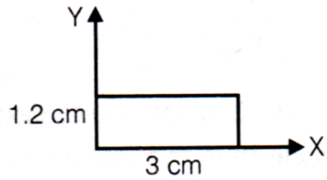
b) The$TE_{10}$ mode of the waveguide does not exist

c) The $TM_{10}$ and the $TE_{10}$ modes both exist and have the same cut-off frequencies

d) The $TM_{10}$ and the $TM_{01}$ modes both exist and have the same cut-off frequencies

**[GATE – 2011]**

**Q.16** The magnetic field along the propagation direction inside a rectangular waveguide with the cross – section shown in figure is $H_Z = 3\cos(2.094\times10^{2}x)\cos(2.618\times10^{2}y)$ $\cos(6.283\times10^{10}t-\beta z)$




a) $v_p > c$ b) $v_p = c$

c) $0 < v_p < c$ d) $v_p = 0$

**[GATE – 2012]**

**Q.17** A two-port network has sattering parameters given by $[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, If the port -2 of the two-port is short circuited, the $S_{11}$ parameter for the resultant one-port network is

a) $\dfrac{S_{11}-S_{11}S_{22}+S_{12}S_{21}}{1+S_{22}}$

b) $\dfrac{S_{11}-S_{11}S_{22}-S_{12}S_{21}}{1+S_{22}}$

c) $\dfrac{S_{11}-S_{11}S_{22}+S_{12}S_{21}}{1-S_{22}}$

d) $\dfrac{S_{11}-S_{11}S_{22}+S_{12}S_{21}}{1-S_{22}}$

**[GATE – 2014-1]**

**Q.18** Which one of the following field patterns represents a TEM wave travelling in the positive x direction?

a) $E=+8\hat{y}, H=-4\hat{z}$

b) $E=-2\hat{y}, H=-3\hat{z}$

c) $E+2\hat{z}, H=+2\hat{y}$

d) $E=-3\hat{y}, H=+4\hat{z}$

**[GATE – 2014-2]**

**Q.19** For a rectangular waveguide of internal dimensions $a\times b(a>b)$ the cut-off frequency for the $TE_{11}$ mode is the arithmetic mean of the cut-off frequencies for $TE_{10}$, mode and $TE_{20}$ mode. If a = $\sqrt{5}$ cm, the value of b (in cm) is_______.

**[GATE – 2014-2]**

**Q.20** Consider an air filled rectangular waveguide with a cross-section of 5 cm x 3 cm. For this waveguide, the cut-off frequency (in MHz) of $TE_{21}$ mode is ________.

**[GATE – 2014-3]**

**Q.21** The longitudinal component of the magnetic field inside an air-filled rectangular waveguide made of a perfect electric conductor is given by the following expression

$H_z(x,y,z,t)=0.1\cos(25\pi x)\cos(30.3\pi y)$ $\cos(12\pi\times10^{9}t-\beta z)(A/m)$

The cross-sectional dimensions of the waveguide are given as a = 0.08m and b= 0.033m. The mode of propagation inside the waveguide is

a) $TM_{12}$ b) $TM_{21}$

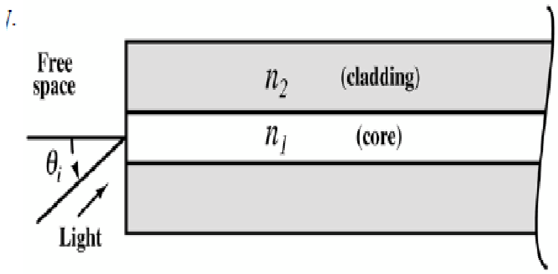
c) $TE_{21}$ d) $TE_{12}$

**[GATE – 2015-1]**

**Q.22** An air - filled rectangular waveguide of internal dimensions $a cm\times b cm(a>b)$ has a cutoff frequency of 6 GHz for the dominant $TE_{10}$, mode. For the same waveguide, if the cutoff frequency of the $TM_{11}$ mode is 15 GHz, the cutoff frequency of the $TE_{01}$ mode in GHz is _____.

**[GATE – 2015-2]**

**Q.23** Light from free space is incident at an angle $\theta_i$ to the normal of the facet of a step-index large core optical fibre. The core and cladding refractive indices are $n_1 = 1.5$ and $n_2 = 1.4$, respectively.




The maximum value of $\theta_i$ (in degrees) for which the incident light will be guided in the core of the fibre is _________.

**[GATE – 2016-2]**

**Q.24** An optical fiber is kept along the $\hat{z}$ direction. The refractive indices for the electric fields along $\hat{x}$ and $\hat{y}$ directions in the fiber are $n_x$ = 1.5000 and $n_y$ = 1.5001 respectively ($n_x \neq n_y$ due to the imperfection in the fiber cross-section). The free space wavelength of a light wave propagating in the fiber is 1.5 μm. If the light wave is circularly polarized at the input of the fiber, the minimum propagation distance after which it becomes linearly polarized, in centimeter, is _________.

**[GATE – 2017-1]**

**Q.25** Standard air – filled rectangular waveguides of dimensions a = 2.29 cm and b= 1.02 cm are designed for radar applications. It is desired that these waveguides operate only in the dominant TE10 mode but not higher than 95% of the next higher cutoff frequency. The range of the allowable operating frequency f is.

a) $8.19\text{GHz} \le f \le 13.1\text{GHz}$

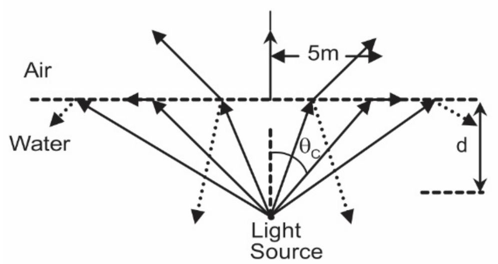
b) $8.19\text{GHz} \le f \le 12.45\text{GHz}$

c) $6.55\text{GHz} \le f \le 13.1\text{GHz}$

d) $1.64\text{GHz} \le f \le 10.24\text{GHz}$

**[GATE – 2017-2]**

**Q.26** The permittivity of water at optical frequencies is 1.75 $\varepsilon_0$ It is found that an isotropic light source at a distance d under water forms an illuminated circular area of radius 5m, as shown in the figure. The critical angle is $\theta_c$




The value of d (in meter) is ___________ .

**[GATE – 2017-2]**

**Q.27** The cutoff frequency of TE01 mode of an air filled rectangular waveguide having inner dimensions a cm × b cm (a > b) is twice that of the dominant TE10 mode. When the waveguide is operated at frequency which is 25% higher than the cutoff frequency of the dominant mode, the guide wavelength is found to be 4 cm. The value of b (in cm, correct to two decimal places) is ____.

**[GATE – 2018]**

**Q.28** The distance (in meters) a wave has to propagate in a medium having a skin depth of 0.1m so that the amplitude of the wave attenuates by 20 dB, is

a) 0.12 b) 0.23
c) 0.46 d) 2.3

**[GATE – 2018]**

# ANSWER KEY:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (c) | (a) | (d) | (a) | (c) | (a) | (d) | (d) | (d) | (c) | (a) | (c) | (a) | (a) |
| **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** |
| (a) | (a) | (b) | (b) | 2 | 7810 | (c) | 13.7 | 32.5 | 0.375 | (b) | 4.33 | 0.75 | (b) |

# EXPLANATIONS

**Q.1 (c)**

$\lambda_c = 2a$

(For $TE_{10}$ domain mode)

$= 2\times 1cm$

$= 2\times 10^{-2} m$

$f_c = \frac{c}{\lambda_c}$

$= \frac{3\times 10^8}{2\times 10^{-2}} = 15GHz$

**Q.2 (a)**

For good conductors

$E = \epsilon_0 e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right) a_x$

Where, $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

Showing that $\delta$ measures the exponential damping of wave as it travels through the conductor

∴ E & H i.e. electric and magnetic fields can hardly propagated through good conductors.

**Q.3 (d)**

Domain mode in WG has lowest cut-off frequency and hence the highes cut-off frequency and highest cut off wavelength.

**Q.4 (a)**

**Q.5 (c)**

$v_p = \frac{v_c}{\sqrt{1+\left(\frac{f_c}{f}\right)^2}}$

For $TE_{10} f_c < f$

$\Rightarrow v_p > v_c$

**Q.6 (a)**

$v = \frac{C}{\sqrt{\epsilon_o}}$

$= \frac{3\times 10^8}{2} = 1.5\times 10^8$

$\lambda_c = \frac{2a}{m} = 2\times 3 = 6cm$

(for $TE_{10}$)

$f_c = \frac{v}{\lambda_c} = \frac{1.5\times 10^8}{6\times 10^{-2}} = 2.5GHz$

**Q.7 (d)**

**Q.8 (d)**

**Q.9 (d)**

**Q.10 (c)**

$TE_{30} \lambda_c = \frac{2a}{m}$

$= \frac{2a}{3}$

$\lambda_c = \frac{3\times 10^8}{18\times 10^9} = \frac{1}{60}$

$\frac{1}{60} = \frac{2a}{3}$

$\Rightarrow a = \frac{1}{40}$

$= 2.5cm = \frac{5}{2} cm$

**Q.11 (a)**

**Q.12 (c)**

For $TE_{20}, \lambda_c = a = 3\times 10^{-2} m$

$f_c = \frac{c}{\lambda_c}$

$= \frac{3\times 10^8}{3\times 10^{-2}}$

$= 10^{10} Hz$

$$n^1 = \frac{n}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{377}{\sqrt{1-\left(\frac{10^{10}}{3\times10^{10}}\right)^2}}$$

$$n^1 = 400\Omega$$

**Q.13 (a)**

Minimum operating frequency

$$f_0 = \frac{C}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

For $TE_{11}$ mode ,

$$f_0 = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{0.04}\right)^2+\left(\frac{1}{0.03}\right)^2}$$

$$= \frac{3\times10^8}{2}\times\frac{0.05}{0.04\times0.03}$$

$$= 6.25GHz$$

**Q.14 (a)**

P is coaxial line and support TEM wave $\therefore$ P has no cut-off frequency Q and R waveguides and cut-off frequency of each depends upon their dimensions.

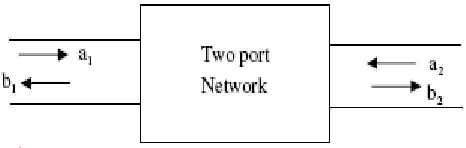
**Q.15 (a)**

In case of rectangular wave guide $TE_{mn}$ exist for all values of m and except m=0 and n=0 For $TE_{mn}$ to exist both values of m ad n must be non zero.

**Q.16 (a)**

Phase velocity is always greater than C inside a waveguide while group velocity is always less than c is inside a waveguide.

**Q.17 (b)**




$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}; \; S_1 = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$

By verification Answer B satisfies.

**Q.18 (b)**

For TEM wave

Electric field (E), Magnetic field (H) and Direction of propagation (P) are orthogonal to each other.

Here P = + $a_x$

By verification

$$E = -2\alpha_{y,} H = -3\alpha_{z,}$$

$$E\times H = -\alpha_{y,} \times -\alpha_{z,} = +\alpha_x \;\; \to P$$

**Q.19 (2)**

$$t_{c10} = \frac{C}{2}\sqrt{\left(\frac{1}{a}\right)^2}$$

$$t_{c10} = K\left(\frac{1}{a}\right); \; t_{c20} = K\left(\frac{2}{a}\right);$$

$$t_{c11} = K\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

$$\text{Given } t_{cll} = \frac{f_{c10}+f_{c20}}{2}$$

$$K\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{K}{2}\left[\frac{1}{a}+\frac{2}{a}\right]$$

$$\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{3}{2a}$$

$$\frac{1}{5}+\frac{1}{b^2} = \frac{9}{4(5)} \Rightarrow \frac{1}{5}+\frac{9}{20} = \frac{1}{b^2}$$

$$-0.2+0.45 = \frac{1}{b^2}$$

$$\therefore \frac{1}{b^2} = \frac{1}{2^2} \Rightarrow b = 2cm$$

**Q.20 (7810MHz)**

$$f_c(TE_{21}) = \frac{C}{2}\sqrt{\left(\frac{2}{9}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$$\frac{3\times10^{10}}{2}\sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{1}{3}\right)^2}$$

$$= 1.5\times10^{10}\sqrt{0.16+0.111}$$

$$= 0.52\times1.5\times10^{10}$$

=7.81GHz

=7810MHz.

**Q.21 (C)**

$$\frac{m\pi x}{a} = 25\pi x \Rightarrow m = 25a = 2$$

$$\frac{n\pi y}{b} = 30.3\pi y \Rightarrow n = 30.3b = 1$$

Given is Hz means TE mode

$\therefore$ mode $= TE_{21}$

**Q.22 (13.7)**

$TE_{10}$

$$f_c = \frac{1}{2\sqrt{\mu\in}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$a = \frac{1}{40}$$

$TE_{11}$

$$15\times10^9 = \frac{3\times10^8}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

$$= \frac{1}{b} = 91.65$$

$TE_{01}$

$$f_c = \frac{3\times10^8}{2}.\frac{1}{b}$$

$f_c = 13.7$GHz

**Q.23 (32.58)**

$$\theta_i = \sin^{-1}\sqrt{n_2{}^2 - n_1{}^2} =$$

$$\sin^{-1}\sqrt{(1.5)^2-(1.4)^2} = \sin^{-1}\sqrt{0.29} = 32.58$$

**Q.24 0.375 (0.36-0.38)**

The phase difference for linear polarization should be $\pi$

So the wave must travel a minimum distance such that the extra phase difference of must occur $\pi/2$

$$\beta_y l_{min} - \beta_x l_{min} = \frac{\pi}{2}$$

$$l_{min}\frac{\omega}{c}\left[n_y - n_x\right] = \frac{\pi}{2}$$

$$\frac{2\pi l_{min}}{\lambda_0}\left[n_y - n_x\right] = \frac{\pi}{2}$$

$$l_{min} = \frac{\lambda_0}{4\left[n_y - n_x\right]}$$

$$= \frac{1.5x10^{-6}}{4[0.0001]} = \frac{1.5}{4}x10^{-2}$$

$$= 0.375x10^{-2}\,m = 0.375cm$$

**Q.25 (b)**

Cut off frequency of $TE_{10}$ is

$$f_c = \frac{c}{2}\frac{1}{a}$$

$$= \frac{3x10^8}{2}x\frac{1}{2.29x10^{-2}}$$

$$= 65.5x10^8\,Hz$$

Since, $b < \frac{a}{2}$

Next higher mode is $TE_{20}$

$$f_c\big|_{TE_{20}} = \frac{c}{2}\frac{2}{a} = 13.1GHz$$

$$f \le 0.95x13.1 = 12.45GHz$$

**Q.26 4.33 (4.2-4.4)**

$$\theta_c = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{\gamma2}}{\varepsilon_{\gamma1}}}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{1.75}}\right) = 49.106$$

$$\tan\theta_c = \frac{5}{d} = \frac{5}{\tan\theta_c} = 4.33m$$

**Q.27 0.75**

Cut off frequency for $TE_{01} = \frac{C}{2b}$

Cut off frequency for $TE_{10} = \frac{C}{2a}$

It is given that

$$\frac{C}{2b} = 2\frac{C}{2a}$$
$$\frac{a}{b} = 2$$

Given waveguide frequency

$$f = \frac{5}{4}\left(\frac{C}{2a}\right)$$

(25% higher than the cut off frequency of dominant mode)

$$\lambda_{guide} = \frac{\lambda_{01}}{\sqrt{1-\left(\frac{4}{5}\right)^2}}$$
$$= \frac{5\lambda_{01}}{3} = 4cm$$
$$\lambda_{01} = \frac{3x4}{5} = 2.4cm$$
$$\lambda_{10} = 2.4x1.25 = 3$$
$$a = \lambda_{10} / 2 = 1.5cm$$
$$b = 0.75cm$$

**Q.28 (b)**

$$\text{Skin Depth} = \frac{1}{\sqrt{\pi f\mu\sigma}} = 0.1m$$

Attenuation constant

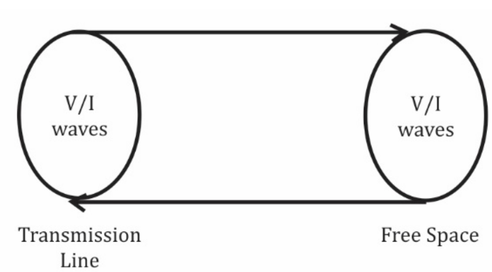
$$\sqrt{\pi f\mu\sigma} = \frac{1}{0.1} = 10$$

Amplitude of wave varies as $e^{-\alpha z}$

$$e^{-\alpha z} = \frac{1}{10}$$
$$-\alpha z = \ln\left(\frac{1}{10}\right)$$
$$10z = 2.3$$
$$z = \frac{2.3}{10} = 0.23m$$

# 8 ANTENNAS

An interface between free-space and transmission line that convert V/I waves to E/H waves and vice-versa is called an Antenna




The process of converting V/I waves into E/H waves is called as Radiation.

The process of converting E/H waves into V/I waves is called as Induction.

## 8.1 RADIATED POWER OF A HERTZIAN DIPOLE:

From $E_\theta$ and $H_\phi$ in the above equation,

Time averaged pointing vector= $\frac{1}{2}E_\theta H_\phi$

Power Radiated

$$= \int \frac{1}{2}E_\theta H_\phi = W_r = 80\pi^2 \left(\frac{dI}{\lambda}\right)^2 I_{rms}^2$$

The term $80\pi^2\left(\frac{dl}{\lambda}\right)^2$ is called Radiation Resistance.

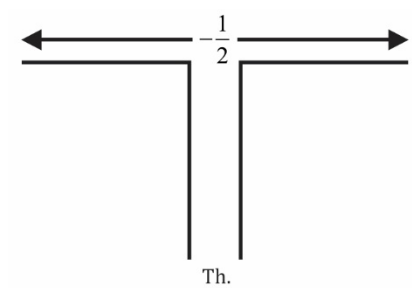
$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$ when applied for a more practical centre fed-dipole of very short length dI.

(short dipole) $R_r = 20\pi^2\left(\frac{dl}{\lambda}\right)^2$

(short monopole) $R_r = 10\pi^2\left(\frac{dl}{\lambda}\right)^2$

## 8.2 RADIATION OF HALF-WAVE DIPOLE:

A more practical radiating device is a half wave dipole or opened out transmission line of length $\lambda/2$ corresponding to a frequency 'f'.




All practical antennas are some or other modifications of the $\lambda/2$ dipole.

Fields of a dipole are expressed as

$$E_\theta = \frac{60I_m}{r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$H_\phi = \frac{I_m}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

Power radiated $W_r = \int \frac{1}{2}\left(E_\theta H_\phi\right) ds = I_{rms}^2$

Radiation Resistance of a dipole $= 73\Omega$

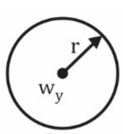
Radiation Resistance of n dipoles in parallel $l = \left(n^2 \times 73\right)\Omega$

## 8.3 BASIC TERMS AND DEFINITIONS IN ANTENNAS

### 1. Isotropic Radiator

A hypothetical point source antenna that radiates in all directions equally and uniformly.

Power Density $= \frac{W_r}{4\pi r^2}$




### 2. Radiation Power Density

Power per unit area $V(r,\theta,\phi) = \frac{dW_r}{ds} = \frac{1}{2}\frac{E^2(r,\theta,\phi)}{n}$

It is a function of distance and direction from the antenna.

$$W_r = \int_{sphere} V(r,\theta,\phi).r^2 \sin\theta d\theta d\phi$$

Where, $ds = r^2 \sin\theta d\theta d\phi$ (incremental surface of the sphere)

### 3. Radiation Intensity

Power per solid angle

$$\psi(\theta,\phi) = \frac{dW_r}{d\Omega} = \frac{1}{2}\frac{E^2(\theta,\phi)}{n}$$

$$ds = r^2 \sin\theta d\theta d\phi = r^2 d\Omega$$

Where $d\Omega = \sin\theta d\theta d\phi$

$\psi(\theta,\phi) = \frac{dW_r}{ds} r^2 =$ independent of 'r' and depends on 'θ' and 'φ' only

$$W_r = \int_{sphere} \psi(\theta,\phi)\sin\theta d\theta d\phi$$

### 4. Gain-(Directivity or power) Gains-Directivity ($G_D$):

$$G_D = \frac{\text{Radiation intensity in a direction from antenna}}{\text{Radiation intensity on an average}}$$

$$\frac{\psi(\theta,\phi)}{\frac{W_r}{4\pi}} = \frac{4\pi\psi(\theta,\phi)}{W_r} = \frac{4\pi\psi(\theta,\phi)}{\int \psi(\theta,\phi)d\Omega}$$

Power Gain

$$G_P = \frac{\text{Radiation intensity in a direction from antenna}}{\text{Input power fed to the antenna on an average}}$$

$$\frac{\psi(\theta,\phi)}{\frac{W_{in}}{4\pi}} = \frac{4\pi\psi(\theta,\phi)}{W_{in}}$$

$$= \frac{4\pi\psi(\theta,\phi)}{W_r} = \frac{W_r}{W_{in}}$$

$$= \frac{4\pi\psi(\theta,\phi)}{W_r} \text{(Efficiency)}$$

Where Effeciency $= \frac{W_r}{W_{in}}$

$= \frac{W_r}{W_r + W_L} = \frac{R_r}{R_r + R_L}$

$W_L$ = power losses

$G_P = (\text{Efficiency}) G_D$

Directivity $D = G_D (\text{maximum})$

**5. Radiation Resistance**

**6. Radiation pattern-Beam Angle**

**8. Effective Length**

**8. Effective Area**

## 8.4 STUDY OF VARIOUS ANTENNA

### 8.4.1 YAGI-UDA ANTENNA:

- Active element is a folded dipole.
- Reflector (5% greater in length than folded dipole) and directors (5% decrease in length than folded dipole) are the parasitic elements.
- Single reflector exists but multiple directors are possible.
- A unidirectional or single beam radiation pattern is formed.

### 8.4.2 HELICAL ANTENNA:

- A simplest antenna to produce circularly polarized waves.
- It has two modes of operation: a) normal mode b) Axial mode
- When $h << \lambda$ it is normal mode as radiation is normal to the axis of the helix.
- Narrow band width and loss radiation efficiency limit the usage of this mode.
- When C is comparable to $\lambda$ it is axial mode with spacing of $\lambda/4$ between the turns of the helix.

### 8.4.3 SLOT ANTENNA:

- A sheet of metal with length $\lambda/2$ and width d when removed from it is called a slot antenna.
- The removed portion from the metal is called the complementary slot antenna
- $Z_c . Z_s = \frac{\eta^2}{4}$

Where $Z_c$ : impedance of complimentary slot

$Z_s$ : impedance of slot

- $Z_c = (73 + j43)\Omega$,

  $Z_s = (363 - j211)\Omega$

### 8.4.4 LOG-PERIODIC ANTENNA:

- An array of dipoles whose lengths & spacings are varied with periodicity that changes or repeats logarithmically.
- $\frac{S_{nH}}{S_n} = \frac{L_{nH}}{L_n} = \frac{R_{nH}}{R_n} = \frac{1}{\tau}$
- The value of $\tau$ lies between 0 and 1.

### 8.4.5 LONG-WIRE ANTENNA:

- Any long wire of more than $\lambda/2$ length is called as harmonic or long-wire antenna.
- It may be Resonant or non-resonant.
- Resonant: - Terminated with short and open circuit.
- Non-Resonant: - Terminated with characteristic impedance.
- E field relations for long wire antennas:
- Resonant: -

$$E(r,\theta,\phi) = \frac{60I_{rms}}{r}\frac{\cos\left(\frac{n\pi}{2}\cos\theta\right)}{\sin\theta}$$

n=odd

$$= \frac{60I_{rms}}{r}\frac{\sin\left(\frac{n\pi}{2}\cos\theta\right)}{\sin\theta}$$

n=even

## 8.4.6 HORN ANTENNA

Used for microwave frequencies and more an extended or flared out waveguide.

$$G_d = \frac{7.5A_P}{\lambda^2} \quad \text{Where } A_P = a_E.a_H$$

## 8.4.8 LOOP ANTENNA

Works on the principle of induced voltage due to EM waves

(Following Faraday's Law)

Used for direction and sense finding.

Directivity D = 3/2 small loop $[c << \lambda]$

$= \frac{0.68C}{\lambda}$ large loop $[c >> \lambda]$

Radiation Resistance

$$R_r = 31200\left[\frac{NA}{\lambda^2}\right]^2 \quad \text{small loop}$$

$$R_r = 6000\left[\frac{C}{\lambda}\right] \quad \text{large loop}$$

# GATE QUESTIONS

**Q.1** For an 8 feet (2.4m) parabolic dish antenna operating at 4 GHz, the minimum distance required for far field measurement is closed to

a) 7.5 cm b) 15 cm
b) 15 cm d) 150 cm

**[GATE – 2000]**

**Q.2** If the diameter of a $\lambda/2$ dipole antenna is increased from $\frac{\lambda}{100}$ to $\frac{\lambda}{50}$. then its

a) bandwidth increases
b) bandwidth decreases
c) gain increases
d) gain decreases

**[GATE – 2000]**

**Q.3** The frequency range for satellite communication is

a) 1 kHz to 100 kHz
b) 100 kHz to 10 kHz
c) 10 mHz to 30 mHz
d) 1 GHz to 30 GHz

**[GATE – 2000]**

**Q.4** In a uniform linear array, four isotropic radiating elements are spaced $\lambda/4$ apart. The progressive phase shift between the elements required for forming the main beam at $60^0$ off the end – fire is:

a) $-\pi$ radians b) $-\pi/2$ radians
c) $\frac{-\pi}{4}$ radians d) $\frac{-\pi}{8}$ radians

**[GATE – 2001]**

**Q.5** A medium wave radio transmitter operating at a wavelength of 492 m has a tower antenna of height 124 m. what is the radiation resistance of the antenna?

a) $25\Omega$ b) $36.5\Omega$
c) 50 $\Omega$ d) 73 $\Omega$

**[GATE – 2001]**

**Q.6** A person with a receiver is 5 km away from the transmitter. What is the distance that this person must move further to detect a 3 dB decrease in signal strength?

a) 942 m b) 2070 m
b) 4978 m d) 5320 m

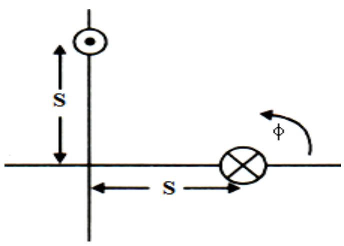
**[GATE – 2002]**

**Q.7** The line-of-signal communication requires the transmit and receive antennas to face each other. If the transmit antenna is vertically polarized, for best reception the receiver antenna should be

a) Horizontally Polarized
b) Vertically polarized
c) at $45°$ with respect to horizontal polarization
d) at $45^0$ with respect to vertical polarization

**[GATE – 2002]**

**Q.8** Two identical antennas are placed in the $\phi=\pi/2$ plane as shown in the figure. The elements have equal amplitude excitation with $180°$ polarity difference, Operating at wavelength $\lambda$. The correct value of the magnitude of the far-zone resultant electric field strength normalized with that of a signal element, both computed for $\phi=0$, is



a) $2\cos\left(\frac{2\pi s}{\lambda}\right)$ b) $2\sin\left(\frac{2\pi s}{\lambda}\right)$

c) $2\cos\left(\frac{\pi s}{\lambda}\right)$ d) $2\sin\left(\frac{\pi s}{\lambda}\right)$

**[GATE – 2003]**

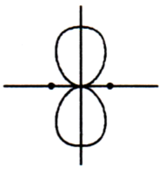
**Q.9** Consider a lossless antenna with a directive gain of + 6dB. If 1 mW of power is fed to it the total power radiated by the antenna will be

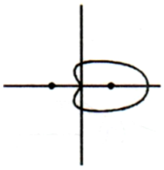
a) 4 mW b) 1 mW

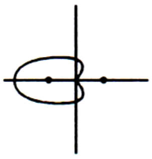
d) 7 mW d) 1/4 mW

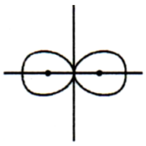
**[GATE – 2004]**

**Q.10** Two identical and parallel dipole antennas are kept apart by a distance of $\lambda/4$ in the H-Plane. They are fed with equal currents but the right most antenna has a phase shift of + $90^0$. The radiation pattern is given as

a) 

b) 

c) 

d) 

**[GATE – 2005]**

**Q.11** A mast antenna consisting of a 50 meter long vertical conductor operators over a perfectly conducting ground plane. It is base-fed at a frequency of 600 KHz. The radiation resistance of the antenna in Ohms is

a) $\frac{2\pi^2}{5}$ b) $\frac{\pi^2}{5}$

c) $\frac{4\pi^2}{5}$ d) $20\pi^2$

**[GATE – 2006]**

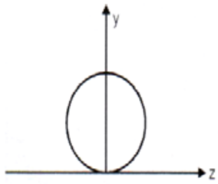
**Q.12** A transmission line is feeding 1 watt of power to a horn antenna having a gain of 10 dB. The antenna is matched to the transmission line. The total power radiated

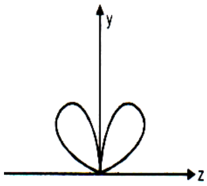
a) 10 Watts b) 1 Watt

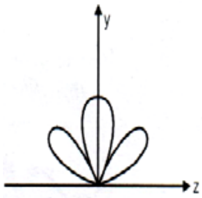
c) 0.1 Watt d) 0.01 Watt

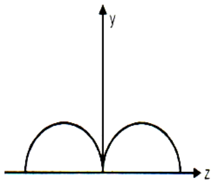
**[GATE – 2006]**

**Q.13** A $\frac{\lambda}{2}$ dipole is kept horizontal at a height of $\frac{\lambda_0}{2}$ above a perfectly conducting infinite ground plane. The radiation pattern in the plane of the dipole (E Plane) Looks ( approximately as

a) 


b) 


c) 


d) 


**[GATE – 2007]**

**Q.14** At 20 GHz, the gain of a parabolic dish antenna of 1 meter diameter and 70% efficiency is

a) 15 dB b) 25 dB

b) 35 dB d) 45 dB

**[GATE – 2008]**

**Q.15** For a Hertz dipole antenna, the half power Beam width (HPBW) in the E-plane is

a) $360^0$ b) $180^0$

c) $90^0$ d) $45^0$

**[GATE – 2008]**

**Q.16** The radiation pattern of an antenna in spherical co-ordinates is given by

$F(\theta) = \cos^4\theta, 0 \le \pi/2$

The directivity of the antenna is

a)10 dB b) 12.6 dB

c) 11.5 dB d) 18 dB

**[GATE – 2012]**

Q.17 In spherical coordinates, let $\hat{a}_\theta . \hat{a}_\phi$ denote until vectors along the θ, $\phi$ directions.

$$E = \frac{100}{r} \sin\theta \cos(\omega t - \beta r) \hat{a}_\theta \ V/m$$

and

$$H = \frac{0.265}{r} \sin\theta \cos(\omega t - \beta r) \hat{a}_\phi \ A/m$$

represent the electric and magnetic field components of the EM wave of large distances r from a dipole antenna, in free space. The average power (W) crossing the hemispherical shell located at r = 1km, $0 \le / \theta \le \pi/2$ is______.

**[GATE – 2014-1]**

Q.18 For an antenna radiating in free space, the electric field at a distance of 1 km is found 12 to be 12 mV/m. Given that intrinsic impedance of the free space is 120πΩ, the magnitude of average power density due to this antenna at a distance of 2 km from the antenna (in $nW/m^2$) is _________.

**[GATE–2014-4]**

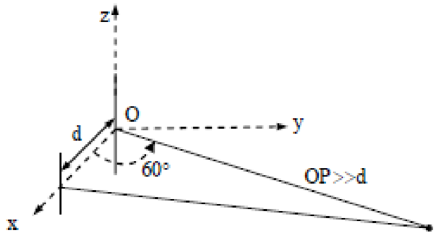
Q.19 Match column A with column B.

| column A | column B. |
|---|---|
| (1)Point electromagnetic source | (P)Highly directional |
| (2)Dish antenna | (Q)End free |
| (3)Yagi-Uda antenna | (R)Isotropic |

a) 1→P, 2→Q, 3→R
b) 1→R, 2→P, 3→Q
c) 1→Q, 2→P, 3→R
d) 1→R, 2→Q, 3→P

**[GATE–2014-4]**

Q.20 Two half±wave dipole antennas placed as shown in the figure are excited with sinusoidally varying currents of frequency 3 MHz and phase shift of π/ 2 between them (the element at the origin leads in phase). If the maximum radiated E-field at the point P in the x-y plane occurs at an azimuthal angle of 60° the distance d (in meters) between the antennas is ________.




**[GATE–2015-2]**

Q.21 The directivity of an antenna array can be increased by adding more antenna elements, as a larger number of elements

a) improves the radiation efficiency
b) increases the effective area of the antenna
c) results in a better impedance matching
d) allows more power to be transmitted by the antenna

**[GATE–2015-3]**

Q.22 Two loss1ess X-band horn antennas are separated by a distance of 200λ. The amplitude reflection coefficients at the terminals of the transmitting and receiving antennas are 0.15 and 0.18, respectively. The maximum directivities of the transmitting and receiving antennas (over the isotropic antenna) are 18 dB and 22 dB, respectively. Assuming that the input power in the lossless transmission line connected to the antenna is 2 W, and that the antennas are perfectly aligned and polarization matched, the power (in mW) delivered to the load at the receiver is____.

**[GATE–2016-1]**

Q.23 The far-zone power density radiated by a helical antenna is approximated as:

$$\vec{W}_{rad} = \vec{W}_{average} \approx \hat{a}_r C_0 \frac{1}{r^2} \cos^4 \theta$$

The radiated power density is symmetrical with respect to ∅ and exists only in the upper hemisphere:

$0 \le \theta \le \frac{\pi}{2}; 0 \le \varnothing \le 2\pi$; $C_o$ is a constant. The power radiated by the antenna (in watts) and the maximum directivity of the antenna, respectively, are

a) $1.5C_o$, 10dB b) $1.256C_o$, 10Db
c) $1.256C_o$, 12dB d) $1.5C_o$, 12dB

**[GATE–2016-1]**

**Q.24** A radar operating at 5 GHz uses a common antenna for transmission and reception. The antenna has a gain of 150 and is aligned for maximum directional radiation and reception to a target 1 km away having radar cross-section of 3 $m^2$. If it transmits 100 kW, then the received power (in μW) is ______.

**[GATE–2016-3]**

**Q.25** Consider a wireless communication link between a transmitter and a receiver located in free space, with finite and strictly positive capacity. If the effective areas of the transmitter and the receiver antennas, and the distance between them are all doubled, and everything else remains unchanged, the maximum capacity of the wireless link

a) increases by a factor of 2
b) decrease by a factor 2
c) remains unchanged
d) decreases by a factor of $\sqrt{2}$

**[GATE–2017-1]**

# ANSWER KEY:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (d) | (b) | (d) | (c) | (b) | (b) | (b) | (d) | (b) | (a) | (a) | (b) | (b) | (d) |
| **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | | | |
| (c) | (a) | 55.5 | 47.5 | (b) | 50 | (b) | 3 | (b) | 2 | (c) | | | |

## EXPLANATIONS

**Q.1 (d)**

$$d=\frac{2l^2}{\lambda}$$

$$\lambda=\frac{3\times10^8}{4\times10^9}=7.5$$

$$d=\frac{2\times2.4\times2.4}{7.5}=1.536m$$

**Q.2 (b)**

**Q.3 (d)**

The frequency range for satellite communication is 1GHz to 30GHz.

**Q.4 (c)**

$\Psi=\beta d\cos\theta+\alpha$

For an array to be end fire $\Psi = 0$

$0=\beta d\cos\theta+\alpha$

$\alpha=-\beta d$

$$\Rightarrow\Psi=\beta d(\cos\theta-1)$$

$$\Rightarrow\Psi=\frac{2\pi}{\lambda}\times\frac{\lambda}{4}(\cos60^\circ-1)$$

$$=\frac{\pi}{\lambda}\times\left(-\frac{1}{2}\right)=-\frac{\pi}{4}\quad\text{radians}$$

**Q.5 (b)**

$$\frac{\lambda}{4}=\frac{492}{4}=123\,;\ L$$

∴ It is a quarter wave monopole antenna

So, $R_a=36.5\Omega$

**Q.6 (b)**

Signal strength $=\dfrac{p}{4\pi R^2}$

$$p_1=\frac{p}{4\pi(5000)^2}$$

$$p_2=\frac{p}{4\pi(x)^2}$$

Given $\dfrac{p_1}{p_2}=\dfrac{1}{2}$ (3dB decrease)

$$\Rightarrow\left(\frac{5000}{x}\right)^2=\frac{1}{2}$$

$\Rightarrow x=5000\sqrt{2}$

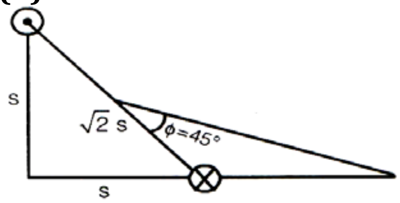
∴ Required distance

$=5000\sqrt{2}-5000$

$=2071m$

**Q.7 (b)**

**Q.8 (d)**




Normalized array factor

$$=2\left|\cos\frac{\Psi}{2}\right|$$

$\Psi=\beta d\sin\theta\cos\Phi+\delta$

$\theta=90^\circ, d=\sqrt{2}s$

$\Phi=45^\circ,\delta=180^\circ$

$$\Rightarrow2\left|\cos\frac{\Psi}{2}\right|$$

$$=2\cos\left[\frac{\beta d\sin\theta\cos\Phi+\delta}{2}\right]$$

$$=2\cos\left[\frac{2\pi}{\lambda.2}\sqrt{2}s\cos45^\circ+\frac{180^\circ}{2}\right]$$

$$=2\cos\left[\frac{\pi s}{\lambda}+90^\circ\right]$$

$$=2\sin\left(\frac{\pi s}{\lambda}\right)$$

**Q.9 (b)**

When an antenna is lossless then the antenna efficiency is 100% .In such case whole of the power fed to antenna is radiated in the form of radiation. Therefore if power fed to antenna is 1 m W.

**Q.10 (a)**

**Q.11 (a)**

Since antenna is installed at conducting ground,

$$R_{rad} = 40\pi^2\left(\frac{dl}{\lambda}\right)^2 \Omega$$

$$\lambda = \frac{3\times10^8}{600\times10^3} = 0.5\times10^3$$

$$R_a = 40\pi^2\left(\frac{50}{0.5\times10^3}\right)^2$$

$$= \frac{40\pi^2}{100} = \frac{2\pi^2}{5}$$

**Q.12 (b)**

Gain of antenna is directive gain i.e. different from that of amplifier. It radiates same power. So1 W.

**Q.13 (b)**

The radiation pattern of the dipole $(\lambda/2)$ kept horizontally above perfectly conducting ground at different heights.

**Q.14 (d)**

Gain of parabolic dish antenna

$$G = n\pi^2\left(\frac{D}{\lambda}\right)^2$$

$$= 0.7\pi^2\left(\frac{1\times20\times10^9}{3\times10^8}\right)^2$$

$$= 30705.4359$$

$$G(dB) = 10\log_{10} G$$

$$= 10\log_{10}(30705.4359)$$

$$= 45dB$$

**Q.15 (c)**

**Q.16 (a)**

$$F(\theta) = \cos^4\theta, 0 \le \theta \le \pi/2$$

We know that directivity D is

$$D = \frac{4\pi U_{max}}{\pi_{rad}} \qquad ....(1)$$

$F(\theta)$ is nothing but radiation intensity $u(\theta,\phi)$ and $\pi_{rad}$ is radiated power.

$$\pi_{rad} = \int_0^{2\pi}\int_0^{\pi/2} \cos^4\theta\sin\theta d\phi \qquad .....(2)$$

Above equation is written from the formula

$$\pi_{rad}\int_\theta\int_\phi u(\theta,\phi)\,d\Omega$$

Where $d\Omega$ is solid angle and $d\Omega = \sin\theta d\theta\ d\phi$

So from (2)

$$\pi_{rad} = \int_0^{2\pi}\int_0^{\pi/2} \cos^4\theta\sin d\theta d\phi$$

$$\pi_{rad} = 2\pi\int_0^{\pi/2} \cos^4\theta\sin\theta d\theta$$

$$\pi_{rad} = -2\pi\left[\frac{\cos^5\theta}{5}\right]_0^{\pi/2}$$

$$\pi_{rad} = \frac{2\pi}{5} \quad .....(3)$$

So $D = \dfrac{4\pi U_{max}}{\left(\frac{2\pi}{5}\right)} = 10U_{max}$

$$= 10[F(\theta)]_{max} = 10$$

In dB directivity

$$= 10\log_{10} D = 10dB$$

**Q.17 (55.5)**

$$E_\theta = \frac{100}{r}\sin\theta e^{-J\beta r}$$

$$HQ = \frac{0.265}{r}\sin\theta e^{-J\beta r}$$

$$P_{avg} = \frac{1}{2}\int_s E_\theta H_Q^*.?ds$$

$$= \frac{1}{2}\int_s \frac{100(0.265)}{r^2}\sin^2\theta r^2\sin\theta\ d\theta\ d\phi$$

$$P_{avg} = \frac{1}{2}\int_s (26.5)\sin^2 d\theta\ d\phi$$

$$= 13.25\int_{\theta=0}^{\pi/2} \sin^3 d\theta\ d\phi$$

Channel capacity remains same.

P= 55.5w

**Q.18 (47.7)**

Electric field of an antenna is

$$E_\theta = \frac{\eta I_0 dI}{4\pi}\sin\theta \left[ \underset{\substack{\downarrow \\ \text{Radiation field}}}{\frac{J\beta}{r}} \quad \underset{\substack{\downarrow \\ \text{inductive field}}}{\frac{1}{r^2}} \quad \underset{\substack{\downarrow \\ \text{Electrostanic field}}}{\frac{J}{\beta r^3}} \right]$$

$$\therefore E\alpha\frac{1}{r}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1} \Rightarrow E_2 = 6mv/m$$

$$P = \frac{1}{2}\frac{E^2}{\eta} = \frac{1}{2}\frac{36\times10^{-8}}{120\pi} = 47.7nw/m^2$$

**Q.19 (b)**

1. Point electromagnetic source, can radiate fields in all directions equally, so isotropic.
2. Dish antenna →highly directional
3. Yagi — uda antenna →End fire

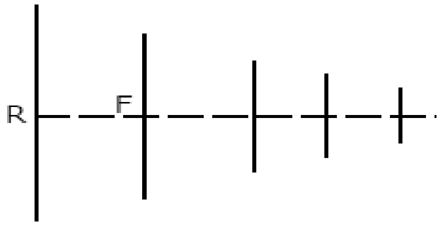



Figure: Yagi — uda antenna

**Q.20 (50)**

$$\psi = \delta + \beta d\cos\theta$$

For maximum field,

$$\psi = 0 \,|\, \lambda = \frac{3\times10^8}{3\times10^6} = 100m$$

$$\delta + \beta d\cos\theta = 0$$

$$-\frac{\pi}{2} + \frac{2\pi}{\lambda}d\cos 60 = 0$$

$$\frac{\pi}{2} = \frac{2\pi}{100}(d)\frac{1}{2}$$

d = 50 m

**Q.21 (b)**

$$D = \frac{4\pi}{\lambda^2}Ae$$

$$D\uparrow \rightarrow Ae\uparrow$$

**Q.22 (3)**

**Q.23 (b)**

$$P_{rad} = \int_{\theta=0}^{\pi/2}\int_{\varnothing=0}^{2\pi} W_{rad} r^2 \sin\theta d\theta d\varnothing$$

$$= \int_{\beta=0}^{\pi/2}\int_{\varnothing=0}^{2\pi} \frac{C_0}{r^2}\cos^4 \omega r^2 \sin\omega d\theta d\varnothing$$

$$= C_0 \times \frac{-\cos^{20}}{5}\Big|_0^{\pi/2} .2\pi = \frac{2\pi}{5}C_0 = 1.256C_0$$

$$\text{Max. directivity} = \frac{\max(W_{rad})}{P_{rad}} \times 4\pi r^2$$

$$= \frac{\frac{C_0}{r^2}}{1.256C_0} \times 4\pi r^2 = \frac{4\pi}{1.256} = 10$$

Max. directivity in dB=10log10=10dB

**Q.24 (2)**

**Q.25 (c)**

$$C = B\log_2\left[1 + \frac{S}{N_0 B}\right]$$

Where,

$$S = \frac{P_t G_t A_{er}}{4\pi r^2}$$

$$S^1 = \frac{P_t A_{er}}{4\pi r^2}.\frac{4\pi}{\lambda^2}A_e t$$

$$= \frac{P_t A_{er} A_e t}{\lambda^2 (r^2)} = P_t \frac{4A_{er}A_e t}{4\lambda^2 r^2}$$

$$= \frac{P_t A_{er} A_e t}{A^2 (r^2)} = S$$

Channel capacity remain same.

## 9 ASSIGNMENT QUESTIONS

**Q.1** Which of the following statement regarding electric flux is true?

1. Electric flux begins on positive charges and terminates on negative charges.
2. Flux is in the same direction as the electric field $\overline{E}$
3. Flux density is proportional to the magnitude of $\overline{E}$
4. In the SI system of units, total flux emanating from a charge of Q(C) is Q(C).

A single line will emanate from 1 C of charge.

a) 1 only b) 1 & 2 only
c) 1, 2 & 3 only d) 1, 2, 3 & 4

**Q.2** Two concentric spherical shells carry equal and opposite uniformly distributed charges over their surfaces as shown in the figure. The electric field on the surface of the inner shell will be

a) zero b) $\frac{Q}{4\pi\epsilon_0 R^2}$
c) $\frac{Q}{8\pi\epsilon_0 R^2}$ d) $\frac{Q}{16\pi\epsilon_0 R^2}$

**Q.3** Joule/Coulomb is the unit of
a) Electric field potential
b) Electric flux density
c) Charge
d) None of the above

**Q.4** The electric potential due to an electric dipole of length L at a point distance r away from it will be doubled if the
a) length L of the dipole is doubled
b) r is doubled
c) r is halved
d) L is halved

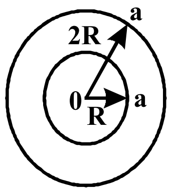
**Q.5** The force between two charged particles is given by,

$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{r^2}$, where the symbols have their usual meanings. The dimensions of $\epsilon_0$ in free space in SI system are:

a) $M^{-1} L^{-3} T^6 A^4$ b) $M^{-1} L^{-3} T^4 A^2$
c) $ML^{-3} T^4 A^3$ d) $M^{-1} L^{-3} T^2$

**Q.6** The energy spent in moving a charge of 10 coulomb from one point 'a' to another point 'b' is 50 joules. The potential difference between points 'a' and 'b' is
a) 2 volts b) 5 volts
c) 10 volts d) 100 volts

**Q.7** P is a point at a large distance from the centre O of a short dipole formed by two point charges all lying on a horizontal plane. If θ is the angle between OP and the dipole axis, then θ, component of the E-field at P is




a) given by sin θ b) given by cos θ
c) given by tan θ d) independent of θ

**Q.8** Two equal positive point charges are placed along X-axis at + $X_1$ and $-X_1$ respectively. The electric field vector at a point P on the positive Y axis will be directed
a) in the +X direction
b) in the –X direction
c) in the +Y direction
d) in the –Y direction

**Q.9)** Two concentric spherical conducting shells are held at two different potentials. Their centre coincides with the origin of a spherical (r, θ) coordinate system. Which one of the following gives the correct nature of the field in the angular gap region?

a) It is purely radial and independent of the coordinates $(r,\theta,\phi)$

b) It is purely radial, independent of $(\theta, \phi)$ and varies as $r^{-2}$

c) The field lines are the like the latitude of the Earth and depend on $(r, \theta)$ but not on $\phi$

d) The field lines are radial and independent of $(\theta, \phi)$ but varies as $r^{-1}$

**Q.10** Which one of the following pairs is NOT correctly matched?

a) Gauss's Theorem: $\oint_s \bar{D}\cdot d\bar{s} = \oint_v \nabla\cdot\bar{D}\,dv$

b) Gauss's Law: $\oint \bar{D}\cdot d\bar{s} = \int_v \rho dv$

c) Coulomb's Law: $V = -\dfrac{d\varphi_m}{dt}$

d) Stoke's Theorem: $\oint_l \bar{\xi}\cdot d\bar{l} = \int_s \left(\nabla\times\bar{\xi}\right)\cdot d\bar{s}$

**Q.11** When a dielectric is placed in an electric field, the electric flux density D, electric field intensity E and polarization P are related as

a) $\bar{D} = \varepsilon_0 \bar{E}\times\bar{P}$ b) $\bar{D} = \varepsilon_0 \bar{E} - \bar{P}$

c) $\bar{D} = \varepsilon_0\left(\bar{E}+\bar{P}\right)$ d) $\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$

**Q.12** A, B and C are three parallel plate condensers with similar surface area of plates and similar separation. A contains air as dielectric, B contains glass as dielectric and C contains polythene. If the electric field intensities in air, glass and polythene are respectively $E_1$, $E_2$ and $E_3$ then (assume that the plates are filled with the given dielectrics)

a) $E_1 = E_2 = E_3$ b) $E_1 = E_2 > E_3$

c) $E_1 > E_2 < E_3$ d) $E_1 < E_2 > E_3$

**Q.13** Maxwell's divergence equation for the electric field is

a) $\nabla\cdot E = \dfrac{\rho}{\varepsilon_0}$ b) $\nabla\cdot E = \dfrac{\rho}{2\pi\varepsilon_0}$

c) $\nabla\times E = \dfrac{\rho}{\varepsilon_0}$ d) $\nabla\times E = \dfrac{\rho}{2\pi\varepsilon_0}$

**Q.14** The dipole moment per unit volume as a function of E in the case of an insulator is given by (symbols have the usual meanings)

a) $P = \varepsilon_0 E(\varepsilon_r - 1)$ b) $P = \varepsilon_0 E$

c) $P = \dfrac{E}{\varepsilon_0\varepsilon_r}$ d) $P = \dfrac{E}{\varepsilon_0(\varepsilon_r - 1)}$

**Q.15** A parallel plate capacitor of plate area A and plate separation 't' has a capacity C. If a metallic plate ρ of area A and of negligible thickness is introduced in the capacitor at a distance $\frac{t}{2}$ from either of the two plates, as shown in the given figure, then the capacity of the capacitor will become

a) $\dfrac{C}{2}$ b) C

c) 2C d) 4C

**Q.16** An infinite plane Z = 10 m carries a uniformly distributed charge of density 2nC/m². The electric field intensity at the origin is

a) 0.2 $a_z$n V/m b) 2 $a_z$n V/m

c) -2 $a_z$n V/m d) -36 $\pi a_z$ V/m

**Q.17** If an isolated conducting sphere in air has radius = $\dfrac{1}{4\pi \epsilon_0}$ its capacitance will be

a) zero b) 1F

c) 4 πF d ) 0F

**Q.18** Which one of the following is the Poisson's equation for a linear and isotropic but inhomogeneous medium?

a) $\nabla^2 V = -\rho/\epsilon$ b) $\bar{\nabla}\left(\epsilon\bar{\nabla}V\right) = -\rho$

c) $\bar{\nabla}\cdot\bar{\nabla}(\in V)=-\rho$ d) $\nabla^2 V=-\rho/\in$

**Q.19** The displacement flux density at a point on the surface of a perfect conductor is $\bar{D}=2(\hat{a}_x-\sqrt{3}\hat{a}_z)\,C/m^2$ and is pointing away from the surface. The surface charge density at that point ($C/m^2$) will be
a) 2 b) -2
c) 4 d) -4

**Q.20** Charge needed within a unit sphere centered at the origin for producing a potential field. $V=-\frac{6r^5}{\epsilon_0}$, for $r\le 1$ is
a) $12\pi C$ b) $60\pi C$
c) $120\pi C$ d) $180\pi C$

**Q.21** The equation $\nabla\cdot J=0$ is known as
a) Poisson's equation
b) Laplace equation
c) Continuity equation
d) Maxwell equation

**Q.22** The dimension of flux density is
a) $MT^{-1}Q^{-1}$ b) $MT^{-2}Q^{-2}$
c) $MT^{1}Q^{1}$ d) $MT^{-1}Q^{-2}$

**Q.23** The energy stored per unit volume in an electric field (with usual notations) is given by
a) $\frac{1}{2}\varepsilon H^2$ b) $\frac{1}{2}\varepsilon H$
c) $\frac{1}{2}\varepsilon E^2$ d) $\varepsilon E^2$

**Q.24** A positive charge of Q coulomb is located at point A (0,0,3) and a negative charge of magnitude Q coulomb is located at point B (0,0,-3). The electric field intensity at point C (4,0,0) is in the
a) negative x-direction
b) negative z-direction
c) positive x-direction
d) positive z-direction

**Q.25** The force between two point charges of 1 nC each with a 1 mm separation in air is
a) $9\times10^{-3}N$ b) $9\times10^{-6}N$
c) $9\times10^{-9}N$ d) $9\times10^{-12}N$

**Q.26** Consider the following statements:
A parallel plane capacitor is filled with a dielectric of relative permittivity $\varepsilon_{r_1}$ and connected to a d.c. voltage of V volts. If the dielectric is changed to another with relative permittivity $\varepsilon_{r_2}=2\varepsilon_{r_1}$, keeping the voltage constant, then
1. the electric field intensity within the capacitor doubles.
2. the displacement flux density D doubles.
3. the charge Q on the plates is reduced to half.
4. the energy stored in the capacitor is doubled.

Select the correct answer using the codes given below:
a) 1 and 2 b) 2 and 3
c) 2 and 4 d) 3 and 4

**Q.27** The resistivity of the wire material can be expressed in terms of LMTI system of dimensional parameter as
a) $ML^2T^{-2}I^{-2}$ b) $ML^2T^{-3}I^{-2}$
c) $ML^3T^{-3}I^{-2}$ d) $ML^3T^{-2}I^{-2}$

**Q.28** An electronic charge Q is placed in a dielectric medium. Which of the following quantities are independent of the dielectric constant $\in$ of the medium?
a) Electric potential V and Electric field intensity E
b) Displacement density D and Displacement $\Psi$
c) Electric field intensity E and Displacement density D
d) Electric potential V and Displacement $\Psi$

**Q.29)** Two coaxial cylindrical sheets of charge are present in free space $\rho_s = 5C/m^2$ at r=2m & $\rho_s = -2C/m^2$ at r=4m. The displacement flux density $\overline{D}$ at r = 3 m is

a) $\overline{D} = 5\overline{a}_r C/m^2$ b) $\overline{D} = \frac{2}{3}\overline{a}_r C/m^2$

c) $\overline{D} = \frac{10}{3}\overline{a}_r C/m^2$ d) $\overline{D} = \frac{18}{3}\overline{a}_r C/m^2$

**Q.30** An electric potential field is produced in air by point charges 1 μC and 4 μC located at (-2,1,5) and (1,3,-1) respectively. The energy stored in the field is

a) 2.57 mJ b) 5.14 mJ
c) 10.28 mJ d) 12.50 mJ

**Q.31** Which one of the following potentials does NOT satisfy Laplace's Equation?

a) $V = 10xy$ b) $V = r\cos\varphi$

c) $V = \frac{10}{r}$ d) $V = \rho\cos\varphi + 10$

**Q.32** Laplacian of a scalar function V is

a) Gradient of V
b) Divergence of V
c) Gradient of the gradient of V
d) Divergence of the gradient of V

**Q.33** An infinitely long uniform charge of density 30 nC/m is located at y = 3, z = 5. The field intensity at (0, 6, 1) is E = 64. 7$a_y$ – 86.3$a_z$ V/m. What is the field intensity at (5,6,1)?

a) E b) $\left(\frac{6^2+}{5^2+6^2+1^2}\right)E$

c) $\left(\frac{6^2+}{5^2+6^2+1^2}\right)^{1/2}E$ d) $\left(\frac{5^2+6^2+1^2}{6^2+}\right)^{1/2}E$

**Q.34** An infinitely long line charge of uniform charge density $\rho_0$ C/m is situated parallel to and at a distance from the grounded infinite plane conductor. This field problem can be solved by which one of the following?

a) By conformal transformation
b) By method of images
c) By Laplace's equation
d) By Poisson's equation

**Q.35** The electric field lines and equipotential lines

a) are parallel to each other
b) are one and the same
c) cut each other orthogonally
d) can be inclined to each other at any angle

**Q.36** Curl curl $\overline{A}$ is given by

a) $\nabla(\nabla\cdot\overline{A}) - \nabla^2\overline{\phantom{A}}$ b) $\nabla(\nabla\cdot A) - \nabla^2\overline{A}$

c) $\nabla(\nabla\cdot\overline{A}) + \nabla^2\overline{\phantom{A}}$ d) $\nabla(\nabla\cdot A) - \nabla\overline{A}$

**Q.37** If a charge of one coulomb is to be brought to a distance of one meter from a charge of 2 coulombs, then the work required is

a) $\frac{1}{2\pi\ {}_0}$ N – m b) $\frac{1}{4\pi\ {}_0}$ N – m

c) $2\pi\in_0$ N – m d) $4\pi\in_0$ N – m

**Q.38** Ohm's law in point form in field theory can be expressed as

a) $V = RI$ b) $\vec{j} = \frac{\vec{E}}{\sigma}$

c) $\bar{J} = \sigma\vec{E}$ d) $R = \frac{\rho l}{A}$

**Q.39** The force between a charge q and a grounded infinite conducting plane kept at a distance d from it is given by

a) $\frac{q}{4\pi\varepsilon_0 d^2}$ b) $\frac{q^2}{4\pi\varepsilon_0 d^2}$

c) $\frac{q}{16\pi\varepsilon_0 d^2}$ d) $\frac{q^2}{16\pi\varepsilon_0 d^2}$

**Q.40** The magnetic field intensity (in amperes/meter) at the centre of a circular coil of diameter 1 meter and carrying a current of 2 amperes is

a) 8  b) 4
c) 3  d) 2

**Q.41** A straight conductor of length 1m moving with a velocity v in the presence of a magnetic field of flux density B directed at an angle θ with the direction of v experience a force. Which of the following statement (s) are true for the magnitude of the force?

1. It is independent of θ
2. It is proportional to $l^2$
3. It is proportional to B
4. It is independent of v

Select the correct answer from the codes given below:

a) 1, 2 and 3  b) 4 alone
c) 3 alone  d) 2 and 4

**Q.42** The Lorentz force law is given by

a) $\bar{f} = Q\bar{E}$  b) $\bar{f} = \bar{V} \times \bar{B}$
c) $\bar{f} = Q(\bar{E} + \bar{V} \times \bar{B})$  d) $\bar{f} = Q(\bar{V} \times \bar{B})$

**Q.43** A solid cylindrical conductor of radius 'R' carrying a current 'I' has a uniform current density. The magnetic field intensity '$\bar{H}$' inside the conductor at the radial distance 'r' (r < R) is

a) Zero  b) $\frac{1}{2\pi r}$
c) $\frac{I_r}{2\pi R^2}$  d) $\frac{IR^2}{2\pi R^2}$

**Q.44** If the vector field $\vec{A} = x\vec{a_x} + y\vec{a_y} + kz\vec{a_z}$ represents solenoidal field, then the value of k must be

a) 1  b) 2
c) -1  d) -2

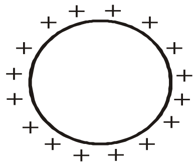
**Q.45** The value of $\oint_C \vec{dl}$ along a circle of radius 2 units is

a) zero  b) $2\pi$
c) $4\pi$  d) $8\pi$

**Q.46** The field at any point on the axis of a current carrying circular coil will be

a) perpendicular to the axis
b) parallel to the axis
c) at an angle of 45° with the axis
d) zero

**Q.47** The given figure shows the surface charge distribution of q coulomb/m². What is the force on a unit charge placed at the centre of circle?

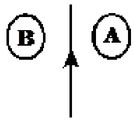


a) Zero  b) $\frac{q}{4\pi R^2}N$
c) $\frac{q^2}{4\pi R}N$  d) $\frac{q^2}{4\pi R}N$

**Q.48** Which of the following equations is not correct?

a) $a_x \times a_x = |a_x|^2$
b) $(a_x \times a_y) + (a_y \times a_x) = 0$
c) $a_x \times (a_y \times a_z) = a_x \times (a_z \times a_y)$
d) $a_r \cdot a_\theta + a_\theta \cdot a_r = 0$

**Q.49** A straight current carrying conductor and two conducting loops A and B are shown in the given figure. If the current in the straight wire is decreasing, then the induced currents in the two loops A and B will be




a) clockwise in both A and B
b) anticlockwise in both A and B
c) anticlockwise in A and clockwise in B
d) clockwise in A and anticlockwise in B

**Q.50** The region $z \le 0$ is a perfect conductor. On its surface at the origin, the surface current density $\vec{K}$ is $(5\vec{a_x} - 6\vec{a_y})\text{A/m}$. If the region z > 0 were free space, then the magnetic field intensity $\vec{H}$ in A/m, at the origin would be

a) $\vec{H} = 0$

b) $\vec{H} = 5\vec{a_x} - 6\vec{a_y}$

c) $\vec{H} = 6\vec{a_x} - 5\vec{a_y}$

d) $\vec{H} = -\left(6\vec{a_x} + 5\vec{a_y}\right)$

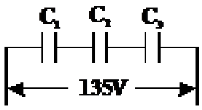
**Q.51** $\in = 2 \uparrow \vec{E} = \vec{a_x}$
$\in = 3 \uparrow \vec{E} = 2\vec{a_x}$
The electric field across an interface is shown in the figure. The surface charge density (in coulomb/m$^2$) on the interface is

a) $-4 \in_0$
b) $-3 \in_0$
c) $+3 \in_0$
d) $+4 \in_0$

**Q.52** The polarization of a dielectric material is given by

a) $\overline{P} = \varepsilon_r \overline{E}$
b) $\overline{P} = (\varepsilon_r - 1)\overline{E}$
c) $\overline{E} = \overline{E}\varepsilon_o(\varepsilon_r - 1)$
d) $\overline{P} = (\varepsilon_r - 1)\varepsilon_o$

**Q.53** If the charge on each of the capacitors in the given figure is 4500 μC, what is the total capacitance (in μF), assuming that the voltage distribution across $C_1$, $C_2$, $C_3$ is in the ratio of 2:3:4?




a) 325
b) 11.1
c) 22.2
d) 33.3

**Q.54** Consider the following statements regarding field boundary conditions:

1. The tangential component of electric field is continuous across the boundary between two dielectrics.
2. The tangential component of electric field at a dielectric conductor boundary is non-zero.
3. The discontinuity in the normal component of flux density at a dielectric conductor boundary is equal to the surface charge density on the conductor.
4. The normal component of the flux-density is continuous across the charge-free boundary between two dielectrics.

Which of these statements are currect

a) 1, 2 and 3 are correct
b) 2, 3 and 4 are correct
c) 1, 2 and 4 are correct
d) 1, 3 and 4 are correct

**Q.55** The infinite parallel metal plates are charged with equal surface charge density of the same polarity. The electric field in the gap between the plates is

a) the same as that produced by one plate
b) double of the field produced by one plate.
c) dependent on coordinates of field point
d) zero

**Q.56** Two positive charges, Q Coulombs each, are placed at points (0, 0, 0) and (2, 2, 0) while two negative charges, Q coulombs each in magnitude, are placed at points (0, 2, 0) and (2, 0, 0). The electric field intensity at the point (1,1,0) is

a) zero
b) $\frac{Q}{8\pi\varepsilon_0}$
c) $\frac{Q}{4\pi\varepsilon_0}$
d) $\frac{Q}{16\pi\varepsilon_0}$

**Q.57** Consider the following statements associate with the basic electrostatic properties of ideal conductors:

1. The resultant field inside is zero.

2. The net charge density in the interior is zero.
3. Any net charge resides on the surface
4. The surface is always equipotential.
5. The field just outside is zero.

Which of these statements are currect

a) 1, 2, 3 and 4 are correct
b) 3, 4 and 5 are correct
c) 1, 2 and 3 are correct
d) 2 and 3 are correct

**Q.58)** A capacitor is made up of two concentric spherical shells. The radii of the inner and outer shells are $R_1$ and $R_2$ respectively and $\in$ is the permittivity of the medium between the shells. The capacitance of the capacitor is given by

a) $\frac{1}{4\pi\in}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

b) $\frac{1}{4\pi\in}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

c) $4\pi\in\frac{R_1R_2}{(R_1+R_2)}$

d) $4\pi\in\frac{R_1R_2}{R_2-R_1}$

**Q.59** In a hundred-turn coil, if the flux through each turn is $(t^3-2t)$m Wb, the magnitude of the induced emf in the coil at a time of 4s is

a) 46 mV
b) 56 mV
c) 4.6 V
d) 5.6 V

**Q.60** A coil of resistance of 5Ω and inductance 0.4 H is connected to a 50 V d.c. supply. The energy stored in the field is

a) 10 joules
b) 20 joules
c) 40 joules
d) 80 joules

**Q.61** For an elliptically polarzed wave incident on the interface of a dielectric at the Brewster angle, the reflected wave will be

a) elliptically polarized
b) linearly polarized
c) right circularly polarized
d) left circularly polarized

**Q.62** If the current element represented by $4\times10^{-4}\vec{a}_y$ Am is placed in a magnetic field of $\vec{H}=\frac{5}{\mu}\hat{a}_x A/m$ the force on the current element is

a) $-2\vec{a}_z$mN
b) $2\vec{a}_z$mN
c) $-2\vec{a}_z$N
d) $2\vec{a}_z$N

**Q.63** In an electrodynamometer, a moving coil has an area A, turn N and carries a current I producing a magnetic flux B. The torque on the moving coil is proportional to

a) I
b) $I^2$
c) $B.I^2$
d) $A.N.B.I^2$

**Q.64** The three values of a one-dimensional potential function $\phi$ shown in the given figure and satisfying Laplace's equation are related as


a) $\varphi_2=\frac{2\varphi_3+\varphi_1}{3}$
b) $\varphi_2=\frac{2\varphi_1+\varphi_3}{3}$
c) $\varphi_2=\frac{2\varphi_1+2\varphi_3}{3}$
d) $\varphi_2=\frac{\varphi_1+3\varphi_3}{2}$

**Q.65** The skin depth (δ) due to RF current is equal to (ω = angular frequency, μ = permeability and σ = conductivity) .

a) $\frac{2}{\omega\mu\sigma}$
b) $\frac{2}{\sqrt{\omega\mu\sigma}}$
c) $\left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$
d) $\frac{\sqrt{2}}{\omega\mu\sigma}$

**Q.66** The electric field of an electromagnetic wave at a point in free space is in the positive Y direction and the magnetic field is in the negative X direction. The direction of power flow will be in the

a) + X direction
b) +Y direction
c) +Z direction
d) –Z direction

**Q.67** The electric field vector E of a wave in free space ($\varepsilon_o$, $\mu_o$) is given by $E = y\left[A\cos\omega\left(t-\frac{z}{c}\right)\right]$ Its magnetic vector $\overline{H}$ will be given by

a) $\overline{H} = y\left[A\sin\omega\left(t-\frac{z}{c}\right)\right]$

b) $\overline{H} = z\left[A\cos\omega\left(t-\frac{z}{c}\right)\right]$

c) $\overline{H} = x\left[-j\sqrt{\frac{\epsilon_o}{\mu_o}}A\cos\omega\left(t-\frac{z}{c}\right)\right]$

d) $\overline{H} = x\left[-j\sqrt{\frac{\epsilon_o}{\mu_o}}A\sin\omega\left(t-\frac{z}{c}\right)\right]$

**Q.68** What is the phase velocity of plane wave in a good conductor?

a) $\sqrt{\pi f\mu\sigma}$
b) $\sqrt{\frac{\pi f\sigma}{\mu}}$
c) $\sqrt{\frac{\pi f}{\mu\sigma}}$
d) $2\sqrt{\frac{\pi f}{\mu\sigma}}$

**Q.69** An electromagnetic plane wave propagates in a dielectric medium characterized by $\varepsilon_r$ = 4 and $\mu_r$ = 1. The phase velocity of the wave will be

a) $0.75\times10^8 m/s$
b) $1.5\times10^8 m/s$
c) $3\times10^8 m/s$
d) $6\times10^8 m/s$

**Q.70** If H = 0.2cos ($\omega t$ - $\beta x$)$a_z$ A/m is the magnetic field of a wave in free space, then the average power passing through a circle of radius 5 cm in the x= 1 plane will be=approximately

a) 30mW
b) 60mW
c) 120mW
d) 150mW

**Q.71** The depth of penetration or skin depth for an electromagnetic field at a frequency 'f' in a conductor of resistivity $\rho$ and permeability $\mu$ is

a) inversely proportional to $\rho$ and f & directly proportional to $\mu$
b) directly proportional to $\rho$ and inversely proportional to f & $\mu$
c) directly proportional to f and inversely proportional to $\rho$ & $\mu$
d) inversely proportional to $\rho$ and $\mu$ and directly proportional to f.

**Q.72** When a plane wave is incident normally from dielectric '1' $(\mu_o, \epsilon_1)$ onto dielectric '2' $(\mu_o, \epsilon_2)$, the electric field of the transmitted wave is -2 times the electric field of the reflected wave. The ratio $\frac{\epsilon_2}{\epsilon_1}$ is

a) 0.5
b) 1
c) 2
d) 4

**Q.73** If, for the transmission of a parallel polarized wave from a dielectric medium of permittivity $\epsilon_1$ into a dielectric medium of permittivity $\epsilon_2$ there exists a value of the angle of incidence $\theta_p$ for which the reflection coefficient is zero, then

a) $\tanh\theta_p = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$
b) $\tan\theta_p = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$
c) $\tan\theta_p = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$
d) $\tanh\theta_p = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

**Q.74** For electromagnetic wave propagation in free space, the free space is defined as

a) $\sigma = 0, \epsilon = 1, \mu \neq 1, \vec{P} \neq 0, \vec{J} = 0$
b) $\sigma = 0, \epsilon = 1, \mu = 1, \vec{P} = 0, \vec{J} = 0$
c) $\sigma \neq 0, \epsilon > 1, \mu < 1, \vec{P} \neq 0, \vec{J} = 0$
d) $\sigma = 0, \epsilon = 1, \mu = 1, \vec{P} \neq 0, \vec{J} \neq 0$

**Q.75** The Poynting vector $P = \vec{E} \times \vec{H}$ has the dimensions of

a) Power/unit area
b) Volts
c) Power
d) Volt/unit length

**Q.76** Consider the following statements regarding a plane wave propagating through free space:
The direction of field
1) 'E' is perpendicular to the direction of propagation
2) 'H' is perpendicular to the direction of propagation
3) 'E' is perpendicular to the direction of field 'H'

Which of these statements are correct ?

a) 1 and 2 b) 2 and 3
c) 1 and 3 d) 1, 2 and 3

**Q.77** If the velocity of electromagnetic wave in free space is $3\times10^8$ m/s, the velocity in a medium with $\varepsilon_r$ of 4.5 and $\mu_r$ of 2 would be

a) $1\times10^8 m/s$ b) $3\times10^8 m/s$
c) $9\times10^8 m/s$ d) $27\times10^8 m/s$

**Q.78** If $H = 0.1\sin(10^8\pi t + \beta y)a_x$ A/m for a plane wave propagating in free space, then the time average Poynting vector is

a) $(0.6\pi\sin^2\beta y)\hat{a}_y W/m^2$
b) $-0.6\pi \hat{a}_y W/m^2$
c) $1.2\pi\hat{a}_x W/m^2$
d) $-1.2\pi\hat{a}_y W/m^2$

**Q.79** In a uniform plane wave, the value of $\frac{|E|}{|H|}$ is

a) $\sqrt{\frac{\mu}{\varepsilon}}$ b) $\sqrt{\frac{\varepsilon}{\mu}}$
c) 1 d) $\sqrt{\mu\varepsilon}$

**Q.80** Which one of the following statements is NOT correct for a plane wave with

$\overline{H} = 0.5e^{-0.1x}\cos(10^6 t - 2x)\hat{a}_z \frac{A}{m}$

a) The wave frequency is $10^6$ r.p.s.
b) The wavelength is 3.14 m
c) The wave travels along + x-direction
d) The wave is polarized in the z-direction

**Q.81** What is the value of standing wave Ratio (SWR) in free space for transmission coefficient $\Gamma = -\frac{1}{3}$

a) $\frac{2}{3}$ b) 0.5
c) 4.0 d) 2.0

**Q.82** A ray is incident on the boundary of a dielectric medium of dielectric constant $\in_1$ with another perfect dielectric medium of dielectric constant $\in_2$. The incident ray makes an angle of $\theta$ with the normal to the boundary surface. The ray transmitted into the other medium makes an angle of $\theta_2$ with the normal. If $\in_1 = 2 \in_2$ and $\theta_1 = 60^o$, which one of the following is correct?

a) $\theta_2 = 45^o$
b) $\theta_2 = \sin^{-1}0.433$
c) $\theta_2 = \sin^{-1} 0.612$
d) There will be no transmitted wave

**Q.83** The electric field of a wave propagation through a lossless medium ($\mu_0$, $81\varepsilon_0$) is $\overline{E} = 10\cos(6\pi\times10^8 t - \beta x)\overline{a}_y$ What is the phase constant $\beta$ of the wave ?

a) 2 π rad/m b) 9π rad/m
c) 18 πrad/m d) 81 π rad/m

**Q.84** Which one of the followig gives the values of the attenuation factor α and phase shift factor $\beta$ for a wave propagated in a good conductor having $\sigma/(\omega\varepsilon) >> 1$ ?

a) $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$;

b) $\alpha = \sqrt{\mu/\varepsilon};\beta = \sqrt{\omega\mu/\varepsilon}$

c) $\alpha = \frac{\lfloor \alpha\sqrt{\mu/\varepsilon} \rfloor}{2};\beta = \sqrt{\omega\mu\varepsilon}$

d) $\alpha = 0;\beta = \omega^2\mu\varepsilon$

**Q.85** Consider the two fields,
$E = 120\pi\cos(10^6\,\pi t - \beta x)\,a_y$ V/m and
$H = A\ \cos(10^6\,\pi t - \beta x)\,a_z$ A/m
The values of A and $\beta$ which will satisfy the Maxwell's equation in a linear isotropic, homogenous, loss-less medium with $\varepsilon_r = 8$ and $\mu_r = 2$ will be

| | A (in A/m) | β(in rad/m) |
|---|---|---|
| a) | 1 | 0.0105 |
| b) | 1 | 0.042 |
| c) | 2 | 0.0105 |
| d) | 2 | 0.042 |

**Q.86** The velocity of a traveling electromagnetic wave in free space is given by

a) $\mu_0\varepsilon_0$ b) $\sqrt{\mu_0\varepsilon_0}$

c) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ d) $\frac{1}{\mu_0\varepsilon_0}$

**Q.87** The equation of a plane wave may by written as (with usual symbols)

a) $\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\mu\varepsilon}\frac{\partial^2 E_y}{\partial x^2}$ b) $\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{\mu\varepsilon}\frac{\partial^2 E}{\partial t^2}$

c) $\frac{\partial^2 E_y}{\partial t^2} = \frac{\mu}{\varepsilon}\frac{\partial^2 E}{\partial x^2}$ d) $\frac{\partial^2 E_y}{\partial t^2} = \frac{\varepsilon}{\mu}\frac{\partial^2 E}{\partial x^2}$

**Q.88** For time-varying electromagnetic fields with electric and magnetic field given by |E| & |H| respectively the rate of energy flow per unit area has a magnitude given by

a) $|E|\ |H| \cos\theta$ b) $|E|\ |H| \sin\theta$

c) $(|E|+|H|)\sin\theta$ d) $(|E|-|H|)\cos\theta$

**Q.89** The E field of a plane electromagnetic wave traveling in a non- magnetic non conducting medium is given by $\vec{E} = \hat{a}_x 5\cos(10^9 t + 30Z)$. What is the dielectric constant of the medium ?

a) 30 b) 10
c) 9 d) 81

**Q.90** The expression for $\overline{B}$, given that in the free space
$\overline{E} = 15\cos\left(6\pi\times10^8 t - 2\pi z\right)i_x$ V/m is

a) $5\times10^{-8}\cos\left(6\pi\times10^8 t - 2\pi z\right)i_y$

b) $-90\pi\times10^8\sin\left(6\pi\times10^8 t - 2\pi z\right)\ i_x$

c) $-45\times10^8\sin\left(6\pi\times10^8 t - 2\pi z\right)i_y$

d) $5\times10^{-8}\lfloor\cos\left(6\pi\times10^8 t - 2\pi z\right)i_y - \sin\left(6\pi\times10^8 t - 2\pi z\right)\ i_z\rfloor$

**Q.91** A uniform plane electromagnetic wave with electric component $E = E_m \cos(\omega t - px)$ propagates in vacuum along the positive x-direction. The mean Poynting Vector is given by

a) $\frac{1}{2}\frac{\beta\epsilon_0 c^2 E_m^2}{\omega}$ b) $\frac{1}{2}\frac{\beta\mu_0 c^2 E_m^2}{\omega}$

c) $\frac{1}{2}\frac{\beta\mu_0 c^2 E_m}{\omega}$ d) $\frac{1}{2}\frac{\beta\epsilon_0 c^2 E_m}{\omega}$

**Q.92** In a lossless medium the intrinsic impedance $\eta = 60\pi$ and $\mu_r = 1$. What is the value of the dielectric constant $\varepsilon_r$?

a) 2 b) 1
c) 4 d) 8

**Q.93** If the $\vec{E}$ field in a plane electromagnetic wave traveling along the z-axis is given by
$\vec{E} = \left(\vec{a}_x + \vec{a}_y\right)f\left(\omega t - \beta z\right) + \left(\vec{a}_x + \vec{a}_y\right)\ f\left(\omega t + \beta z\right)$
The H field associated with the wave is ($z_0$ is the characteristic impedance)

a) $\left(\overrightarrow{a_x}+\overrightarrow{a_y}\right)\ f\left(\omega t-\beta z\right)/z_0$
$+\left(\overrightarrow{a_x}+\overrightarrow{a_y}\right)f\left(\omega t-\beta z\right)/z_0$

b) $\left(\overrightarrow{a_x}-\overrightarrow{a_y}\right)\ f\left(\omega t-\beta z\right)/z_0$
$+\left(\overrightarrow{a_x}-\overrightarrow{a_y}\right)f\left(\omega t+\beta z\right)/z_0$

c) $\left(\overrightarrow{a_y}-\overrightarrow{a_x}\right)\ f\left(\omega t-\beta z\right)/z_0$
$+\left(-\overrightarrow{a_y}+\overrightarrow{a_x}\right)f\left(\omega t+\beta z\right)/z$

d) $\left(\overrightarrow{a_x}-\overrightarrow{a_y}\right)\ f\left(\omega t-\beta z\right)/z_0$
$+\left(\overrightarrow{a_y}-\overrightarrow{a_x}\right)f\left(\omega t+\beta z\right)/z$

**Q.94** If the electric field $\overline{E}=0.1\ te^{-t}\overline{a_x}$ and $\in=4\in_0$, then the displacement current crossing an area of 0.1 m² at t = 0 will be

a) zero  b) $0.04\in_0$
c) $0.4\in_0$  d) $4\in_0$

**Q.95** Match List I with List II and select the correct answer using the codes given below the lists:

**List I**

A. $\overline{\nabla}\cdot\overline{D}=\rho$

B. $\overline{\nabla}\cdot\overline{D}=\frac{\partial\rho}{\partial t}$

C. $\overline{\nabla}\times\overline{H}=\overline{J}_c$

D. $\overline{\nabla}\times\overline{E}=-\frac{\partial\overline{B}}{\partial t}$

**List II**

1. Ampere's Law
2. Dauss's Law
3. Fraday's Law
4. Continuity equation

**Codes:**

| | A | B | C | D |
|---|---|---|---|---|
| a) | 4 | 2 | 1 | 3 |
| b) | 2 | 4 | 1 | 3 |
| c) | 4 | 2 | 3 | 1 |
| d) | 2 | 4 | 3 | 1 |

**Q.96** If the static magnetic flux density is $\overline{B}$, then

a) $\nabla\times\vec{B}=0$  b) $\nabla\cdot\vec{B}=0$
c) $\nabla\cdot\vec{B}=\overline{J}$  d) $\nabla\times\vec{B}=\overline{J}$

**Q.97** Match List I with List II and select the correct answer using the codes given below the lists:

**List I (Law/Quantity)**

A. Dauss's Law Ampere's Law
B. Ampere's Law
C. Fraday's Law
D. Poynting vector

**List I (Mathematical expression)**

1. $\overline{\nabla}\cdot\overline{D}=\rho$

2. $\overline{\nabla}\times\overline{E}=-\frac{\partial\overline{B}}{\partial t}$

3. $S=\overline{E}\times\overline{H}$

4. $\overline{F}=q\left(\overline{E}+\overline{v}\times\overline{B}\right)$

5. $\overline{\nabla}\times\overline{H}=\overline{J}_c+\frac{\partial\overline{D}}{\partial t}$

**Codes:**

| | A | B | C | D |
|---|---|---|---|---|
| a) | 1 | 2 | 4 | 3 |
| b) | 3 | 5 | 2 | 1 |
| c) | 1 | 5 | 2 | 3 |
| d) | 3 | 2 | 4 | 1 |

**Q.98** If a vector field $\overline{B}$ is solenoidal, which of these is true?

a) $\oint_s \overline{B}\cdot d\overline{l}=0$  b) $\oint_s \overline{B}\cdot d\overline{s}=0$
c) $\nabla\times\overline{B}=0$  d) $\nabla\cdot\overline{B}\neq 0$

**Q.99** A medium behaves like dielectric when the

a) Displacement current is just equal to the conduction current.
b) Displacement current is less than the conduction current.
c) Displacement current is much greater than the conduction current.
d) Displacement current is almost negligible.

**Q.100** For linear isotropic materials, both $\vec{E}$ and $\vec{H}$ have the time dependence $e^{j\omega t}$ and regions of interest are free of charges. The value of $\vec{\nabla}\times\vec{H}$ is given by

a) $\sigma\vec{E}$ b) $j\omega\varepsilon\vec{E}$

c) $\sigma\vec{E}+j\omega\varepsilon\vec{E}$ d) $\sigma\vec{E}-j\omega\varepsilon\vec{E}$

**Q.101** For electrostatic fields in charge free atmosphere, which one of the following is correct?

a) $\nabla\times\vec{E}=0\, and\, \nabla\cdot\vec{E}=0$

b) $\nabla\times\vec{E}\neq 0\, but\, \nabla\cdot\vec{E}=0$

c) $\nabla\times\vec{E}=0\, \text{but}\, \nabla\cdot\vec{E}\neq 0$

d) $\nabla\times\vec{E}\neq 0\, but\, \nabla\cdot\vec{E}\neq 0$

**Q.102** Maxwell equation $\vec{\nabla}\times\overline{E}=-\frac{\partial\overline{B}}{\partial t}$ is represented in integral form as

a) $\oint\overline{E}\cdot d\overline{l}=-\frac{\partial}{\partial t}\int\overline{B}.\overline{dl}$

b) $\oint\overline{E}\cdot d\overline{l}=-\frac{\partial}{\partial t}\int\overline{B}.\overline{ds}$

c) $\oint\overline{E}\times d\overline{l}=-\frac{\partial}{\partial t}\oint\overline{B}\cdot d\overline{l}$

d) $\oint\overline{E}\times d\overline{l}=-\frac{\partial}{\partial t}\oint\overline{B}\cdot d\overline{s}$

**Q.103** In a source-free imperfect dielectric medium (specified by loss tangent tan δ), Maxwell's curl equation can be written as

a) $\nabla\times\overline{H}=j\omega\varepsilon\overline{E}(1+j\tan\delta)$

b) $\nabla\times\overline{H}=j\omega\varepsilon\overline{E}(1-j\tan\delta)$

c) $\nabla\times\overline{H}=-j\omega\varepsilon\overline{E}(1+j\tan\delta)$

d) $\nabla\times\overline{H}=-j\omega\varepsilon\overline{E}(1-j\tan\delta)$

**Q.104** $\overline{E}=\overrightarrow{a_x}[20\cos(\omega t-\beta z)+5\cos(\omega t+\beta z)]V/m$. The associated magnetic field is

a) $\frac{\overrightarrow{a_y}}{120\pi}[20\cos(\omega t-\beta z)+5\cos(\omega t+\beta z)]A/m$

b) $\frac{\overrightarrow{a_y}}{120\pi}[20\cos(\omega t-\beta z)-5\cos(\omega t+\beta z)]A/m$

c) $\frac{\overrightarrow{a_x}}{120\pi}[20\cos(\omega t-\beta z)+5\cos(\omega t+\beta z)]A/m$

d) $\frac{\overrightarrow{a_x}}{120\pi}[20\cos(\omega t-\beta z)-5\cos(\omega t+\beta z)]A/m$

**Q.105** The frequency of the power wave associated with an electromagnetic wave having an E field as $E=e^{-z/\delta}\cos\left(\omega t-\frac{z}{\delta}\right)$, is given by

a) $\frac{\omega}{8\pi}$ b) $\frac{\omega}{4\pi}$

c) $\frac{\omega}{2\pi}$ d) $\frac{\omega}{\pi}$

**Q.106** The velocity of the plane wave $\sin^2(\omega t - \beta x)$ is

a) $\frac{2\omega}{\beta}$ b) $\frac{\omega}{2\beta}$

c) $\frac{\omega^2}{\beta^2}$ d) $\frac{\omega}{\beta}$

**Q.107** Which one of the following statements is correct ?
For a lossless dielectric medium, the phase constant for a traveling wave, β is proportional to

a) $\varepsilon_r$ b) $\sqrt{\varepsilon_r}$

c) $\frac{1}{\varepsilon_r}$ d) $\frac{1}{\sqrt{\varepsilon_r}}$

**Q.108** If $\vec{n}$ is the polarization vector and $\vec{k}$ is the direction of propagation of a plane electromagnetic wave, then

a) $\vec{n}=\vec{k}$ b) $\vec{n}=-\vec{k}$

c) $\vec{n}\cdot\vec{k}=0$ d) $\vec{n}\times\vec{k}=0$

**Q.109** The ratio of conduction current density to the displacement current density is (symbols have the usual meaning)

a) $\frac{j\sigma}{\omega\varepsilon}$ b) $\frac{\sigma}{j\varpi\varepsilon}$

c) $\frac{\sigma\omega}{j\varepsilon}$ d) $\frac{\sigma\varepsilon}{j\omega}$

**Q.110** The electric field of a uniform plane wave is given by

$\vec{E} = 10\cos(3\pi\times10^8 t - \pi z)\vec{a}_x$

Match List I with List II pertaining to the above wave and select the correct answer using the codes given below the lists:

**List I (Parameter)**

A) Phase velocity
B) Wavelength
C) Frequency
D) Phase constant

**List II (Values in MKS units)**

1.2
2.3.14
3.377
4.1.5 × 10⁸
5.3.0 × 10⁸

**Codes :**

| | A | B | C | D |
|---|---|---|---|---|
| a) | 5 | 4 | 3 | 2 |
| b) | 3 | 4 | 2 | 1 |
| c) | 4 | 3 | 1 | 2 |
| d) | 5 | 1 | 4 | 2 |

**Q.111** List II gives mathematical expressions for the variables given in List I. Match List I with List II and select the correct answer using the codes given below the lists:

**List I**

A. Intrinsic impedance
B. Velocity of wave propagation
C. Skin depth
D. Attenuation constant

**List II**

1. $\frac{1}{\sqrt{\mu\varepsilon}}$

2. $\sqrt{\frac{\mu}{\varepsilon}}$

3. $\frac{1}{\sqrt{\pi f\,\mu\sigma}}$

4. $\frac{1}{f\sqrt{\mu\omega}}$

5. $\sqrt{\frac{\omega\mu\sigma}{2}}$

**Codes:**

| | A | B | C | D |
|---|---|---|---|---|
| a) | 1 | 2 | 3 | 4 |
| b) | 2 | 1 | 4 | 5 |
| c) | 2 | 1 | 3 | 5 |
| d) | 1 | 2 | 5 | 3 |

**Q.112** A plane wave propagates in z-direction with field components $E_y$ and $H_x$ with a time & z-dependence of the form exp (jωt - jβz). $E_y$ / $H_x$ is given by

a) $\frac{\omega\mu}{\beta}$ b) $\frac{\omega\varepsilon}{\beta}$

b) $\frac{-\beta}{\omega\mu}$ d) $\frac{-\beta}{\omega\varepsilon}$

**Q.113 List I (Medium)**

A. Loss-less dielectric
B. Good conductor
C. Poor conductor
D. Lossy

**List II (Expression for intrinsic impedance for plane wave propagation)**

1. $\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

2. $\sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right)$

3. $\sqrt{\frac{\mu}{\varepsilon}}$

4. $(1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$

**Codes :**

| | A | B | C | D |
|---|---|---|---|---|
| a) | 4 | 3 | 2 | 1 |
| b) | 4 | 3 | 1 | 2 |
| c) | 3 | 4 | 2 | 1 |
| d) | 3 | 4 | 1 | 2 |

**Q.114** Match List I with List II and select the correct answer using the codes given below the lists: (Symbols have the usual meanings)

**List I**
A. Poisson's equation
B. Laplace's equation2.
C. Joule's equation
D. Helmholtz's equation
**List II**
1. $\nabla^2\varphi = 0$
2. $\nabla^2 E + K_0{}^2 E = 0$
3. $\nabla^2\varphi = \frac{-P}{\varepsilon_0}$
4. $\frac{dP}{dV} = U_j \bar{E}\cdot\bar{J}$

**Codes:**

| | A | B | C | D |
|---|---|---|---|---|
| a) | 2 | 1 | 4 | 3 |
| b) | 3 | 4 | 1 | 2 |
| c) | 3 | 1 | 4 | 2 |
| d) | 2 | 4 | 1 | 3 |

**Q.115** Tangential component of the electric field on a perfect conductor will be
a) infinite
b) zero
c) same as the normals field component and 90º out of phase]
d) same as the normal component but 180º out of phase.

**Q.116** Consider the following statements
For a uniform plane electromagnetic wave
1. The direction of energy flow is the same as the direction of propagation of the wave.
2. lectric and magnetic fields are in time quadrature.
3. Electric and magnetic fields are in space quadrature.
Of these statements
a) 2 alone is correct
b) 1 and 2 are correct
c) 1 and 3 are correct
d) 3 alone is correct

**Q.117** In a dielectric medium of relative permittivity 4, the electric field intensity is $20\sin(10^8 t - \beta z)\bar{a}_y$ V/m and z being in m. The phase shift constant β is
a) $\frac{1}{3} rad/m$ b) $\frac{2}{3} rad/m$
b) c) $\frac{2\pi}{3} rad/m$ d) $\frac{1}{6\pi} rad/m$

**Q.118** The unit of the Poynting vector is
a) power
b) power density
c) energy
d) energy density

**Q.119** The intrinsic impedance of free space is
a) 377Ω b) $\sqrt{\mu_0 \varepsilon_o}$
c) $j\sqrt{\frac{\varepsilon_o}{\mu_o}}$ d) $\sqrt{\frac{\varepsilon_o}{\mu_o}}$

**Q.120** If a plane electromagnetic wave traveling in the direction $\vec{\beta} = \vec{a}_x\beta_x + \vec{a}_y\beta_y + \vec{a}_z\beta_z$ has electric field $E = A\cos(\omega t - \vec{\beta}\cdot\bar{r})$, then the phase velocities $v_x : v_y : v_z$ is equal to
a) $\frac{1}{\beta_x^2}:\frac{1}{\beta_y^2}:\frac{1}{\beta_z^2}$ b) $\beta_x^2 : \beta_y^2 : \beta_z^2$
c) $\frac{1}{\beta_x}:\frac{1}{\beta_y}:\frac{1}{\beta_z}$ d) $\beta_x : \beta_y : \beta_z$

**Q.121** If
$\bar{B} = B_0 z\cos\omega t\,\hat{a}_y$ and $\bar{E} = \hat{a}_x E_x$, then
a) $E_x = 0$
b) $E_x = +B_0 z\,\omega\sin\omega t$
c) $E_x = +B_0\cos\omega t$
d) $E_x = \frac{1}{2}B_0 z^2\,\omega\sin\omega t$

**Q.122** In the relation $S = \frac{1+|\Gamma|}{1-|\Gamma|}$; the values of S and Γ (where S stands for wave ratio and Γ is reflection coefficient), respectively vary as

a) 0 to 1 and -1 to 0
b) 1 to ∞ and 0 to +1
c) -1 to +1 and 1 to ∝
d) -1 to 0 and 0 to 1

**Q.123** Three media are characterized by
$1. \in_f = 8, \mu_f = 2, \sigma = 0$
$2. \in_f = 1, \mu_f = 9, \sigma = 0$
$3. \in_f = 4, \mu_f = 4, \sigma = 0$
$\in_r$ is relative permittivity, $\mu_r$ is relative permeability and σ is conductivity. The values of the intrinsic impedances of the media 1, 2 and 3 respectively are
a) 188 Ω, 377 Ω and 1131 Ω
b) 377 Ω, 1131 Ω and 188 Ω
c) 188 Ω, 1131Ω and 377 Ω
d) 1131Ω, 188 Ω and 377 Ω

**Q.124** For a perfect conductor, the field strength at a distance equal to the skin depth is X% of the field strength at its surface. The value of 'X%' is
a) Zero
b) 50%
c) 36%
d) 26%

**Q.125** If $\vec{E} = (\hat{x} + j\hat{y})e^{-j\beta z}$ ,then wave is said to be which one of the following
a) Right circularly polarized
b) Right elliptically polarized
c) Left circularly polarized
d) Left elliptically polarized

**Q.126** A right circularly polarized wave is incident from air onto a polystyrene $(\in_r = 2.7)$
The reflected wave is
a)right circularly polarized
b)left circularly polarized
c)right elliptically polarized
d)left elliptically polarized

**Q.127** An elliptically polarized wave travelling in the positive z-direction in air has x and y components
$E_x = 3\sin(\omega t - \beta z) \, V/m$
$E_y = 6\sin(\omega t - \beta z + 75°) \, V/m$
Determine the power density?
a) $8 \, W/m^2$
b) $4 \, W/m^2$
c) $62.5m \, W/m^2$
d) $125m \, W/m^2$

**Q.128** We have the following six sets of electro-magnetic waves with the frequency ranges as follows:
30 – 300 KHz
10 – 30 KHz
3 – 30 MHz
300 – 3000 KHz
30 – 300 MHz
> 300 MHz
These are designated in the following orders :
a) VLF, LF, MF, HF, VHF and UHF
b)LF, VLF, MF, HF, VHF and UHF
c) LF, VLF, HF,MF, VHF and UHF
d) VHF, VLF, HF, MF, LF and UHF

**Q.129** At a certain frequency f, a uniform plane wave is found to have established a wavelength λ in a good conductor. If the source frequency is doubled, then the wavelength would change to
a) $\frac{\lambda}{\sqrt{2}}$
b) $\sqrt{2}\lambda$
c) $2\lambda$
d) $4\lambda$

**Q.130** In free space $\overline{H}(z,t) = 0.10\cos(4\times10^7 t - \beta z)T_x A/m$.
The expression for $\overline{E}(z,t)$ is
a) $\overline{E}(z,t) = 37.7\cos(4\times10^7 t - \beta z)T_y$
b) $\overline{E}(z,t) = 2.65\times10\cos(4\times10^7 t - \beta z)T_z$
c) $\overline{E}(z,t) = 37.7\cos(4\times10^7 t - \beta z)T_x$
d) $\overline{E}(z,t) = -37.7\cos(4\times10^7 t - \beta z)T_y$

**Q.131** For a lossless transmission line
1. series resistance is zero.
2. shunt conductance is zero

3. shunt conductance is infinite
4. series resistance is infinite.

a) 1 & 2 b) 2 & 3
c) 2 & 4 d) 3 & 4

**Q.132** A $\frac{\lambda}{4}$ transformer is used for

a) light loads
b) high frequency load
c) connecting high impedance and low frequency loads.
d) reducing distortion in transmission lines.

**Q.133** A generator of 50 ohm internal impedance and operating at 1 GHz feeds a 75 Ω load via a coaxial line of characteristic impedance 50 ohm. The Voltage Standing Wave Ratio on the feed line is

a) 0.5 b) 1.5
c) 2.5 d) 1.75

**Q.134** A loss-less 50 ohm transmission line is terminated in (A) 25 ohm and (B) 100 ohm loads. Which one of the following statements would be correct if the voltage standing wave patterns measured in the two cases are compared ?

a) The two patterns will be identical in all respects and cannot be distinguished
b) The two patterns will have identical locations of maxima /minima but the VSWR will be higher in the case of A.
c) The two patterns will have identical locations of maxima/minima but the VSWR will be higher in the case of B.
d) The two patterns will be identical except for a relative spatial shift of quarter wavelength in the two cases.

**Q.135** A quarter wave transformers, made of air- filled coaxial line, matches two transmission lines of characteristics impedance 50 ohms and 72 ohms respectively. If the inner conductor of the coaxial line is made 10 mm in diameter, what should be the diameter of the outer conductor (approx.)?

a) 16 mm b) 20 mm
c) 27 mm d) 32 mm

**Q.136** The input impedance of short-circuited loss-less line of length less than a quarter wavelength is

a) purely resistive
b) purely inductive
c) purely capacitive
d) complex.

**Q.137** For a transmission line, the open-circuit and short-circuit impedance are 20Ω and 5 Ω respectively. Then the characteristics impedance of the line is

a) 100 ohms b) 50 ohms
c) 25 ohms d) 10 ohms

**Q.138** The characteristic impedance of a transmission line with inductance 0.294μH/m & capacitance 60pF/m, is

a) 49Ω b) 60Ω
c) 70Ω d) 140Ω

**Q.139** The magnitude of the reflection coefficient of a transmission line in terms of the load and characteristic impedance is given by

a) $\rho = \left|\frac{Z_L + Z_0}{Z_0 - Z_L}\right|$ b) $\rho = \left|\frac{Z_0 Z_L}{Z_0 - Z_L}\right|$

c) $\rho = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|$ d) $\rho = \left|\frac{Z_L - Z_0}{Z_0 Z_L}\right|$

**Q.140** What are the directions of $\bar{E}$ and $\bar{H}$ in TEM mode transmission lines with respect to **the** direction of propagation?

a) Both $\bar{E}$ and $\bar{H}$ are entirely transverse to the direction of propagation.

b) $\bar{E}$ is entirely transverse to and h has a component in the direction of propagation.
c) $\bar{H}$ is entirely transverse to and $\bar{E}$ has a component in the direction of propagation.
d) Both $\bar{E}$ and $\bar{H}$ have components in the direction of propagation.

**Q.141** In an ideal transmission line with matched load, the voltage standing wave ratio & reflection coefficient are respectively
a) 1 and 1
b) infinity and one
c) infinity and zero
d) one and zero

**Q.142** A 75 ohm transmission line is to be terminated in two resistive loads $R_1$ and $R_2$ such that the standing patterns in the two cases have the same SWR. To obtain the desired result, the values $R_1$ & $R_2$ (in ohms) should be
a) 250 and 200 respectively
b) 225 and 25 respectively
c) 100 and 150 respectively
d) 50 and 125 respectively

**Q.143** One end of a loss-less transmission line of length $\frac{3}{8}\lambda$ and characteristic impedance $R_0$ is short-circuited, and the other end is terminated in $R_0$. The impedance measured at $\frac{\lambda}{8}$ away from the end terminated in $R_0$ is :
a) zero  b) $R_0$
c) $\frac{R_0}{2}$  d) *Infinite*

**Q.144** For a quarter wavelength ideal transmission line of characteristic impedance 50 ohms and load impedance 100 ohms, the input impedance will be
a) 25 Ω  b) 50 Ω
c) 100 Ω  d) 150 Ω

**145** A transmission line of characteristic impedance $Z_o$ = 50 ohms, phase velocity $v_p = 2 \times 10^8$ m/s, and length $l$ = 1m is terminated by a load $Z_L$ = (30 – j40) ohms. The input impedance of the line for a frequency of 100 MHz will be
a) (30 + j40) ohms
b) (30 – j40) ohms
c) (50 + j40) ohms
d) (50 – j40) ohms

**Q.146** A line of characteristic impedance $z_o$ ohms, phase velocity $v_p = 2 \times 10^8$ m/s and length $l$ = 2m is terminated by a load impedance $z_L$ ohms. The reflection coefficients at the input end and load end are respectively $|1|$and$|R|$. The ratio $\frac{|1|}{|R|}$ for a frequency of 50 MHz will be
a) 1  b) −1
c) $\sqrt{-1}$  d) 2

**Q.147** For transmission of wave form a dielectric medium of permittivity $\in_1$ into a dielectric medium of lower permittivity $\in_2, (\in_1 > \in_2)$ the critical angle of incidence Q (relative to the interface) is given by
a) $\sin^{-1}\sqrt{\frac{\in_2}{\in_1}}$  b) $\cos^{-1}\sqrt{\frac{\in_2}{\in_1}}$
b) $\tan^{-1}\sqrt{\frac{\in_2}{\in_1}}$  d) $\sin^{-1}\left(\frac{\in_2}{\in_1}\right)$

**Q.148** A 50Ω characteristic impedance line is connected to a load which shows a reflection coefficient of 0.268. If $V_{in}$ = 15V, then the net power delivered to the load will be
a) 0.139W  b) 1.39W
c) 0.278W  d) 2.78W

**Q.149** For a transmission line with homogenous dielectric, the capacitance per unit length is 'C', the relative permittivity of the dielectric is '$\in_r$' and velocity of light in free space is 'v'. The characteristic impedance $Z_0$ is equal to

a) $\frac{\in_r}{vC}$ b) $\frac{\in_r}{\sqrt{vC}}$

c) $\frac{\sqrt{\in_r}}{vC}$ d) $\sqrt{\frac{\in_r}{vC}}$

**Q.150** A transmission line has primary constants R, L, G and C, and secondary constants $Z_0$ and $\gamma(=\alpha+j\beta)$. If the line is lossless, then

a) R=0, G≠0 and α=0

b) R=0, G=α,0 and β=|γ|

c) G=0 and α=β

d) R=0, G=0, α=0 and β=|γ|

**Q.151** A transmission line having 50 Ω impedance is terminated in a load of (40+j30)Ω. The VSWR is

a) j0.033 b) 0.8+j0.6

c) 1 d) 2

**Q.152** A (75-j40) Ω load is connected to a coaxial line of $Z_0$ = 75Ω at 6MHz. The load matching on the line can be accomplished by connecting

a) a short-circuited stub at the load

b) an inductance at the load

c) a short-circuited stub at a specific distance from the load

d) a capacitance at a specific distance from the load.

**Q.153** If R,L,C and G are the resistance, inductance, capacitance & conductance of a transmission line respectively, then the condition for distortionless transmission line is

a) $\frac{R}{C}=\frac{G}{L}$ b) $RC=LG$

c) $R^2C=G^2L$ d) $RC^2=GL^2$

**Q.154** The input impedance of a $\frac{\lambda}{8}$ long short- circuited section of a lossless transmission line is

a) zero b) inductive

c) capacitive d) infinite

**Q.155** Match List I (Parameters) with List II (Values) for a transmission line with a series impedance $Z=R+j\omega L\frac{\Omega}{m}$ and a shunt admittance $Y=G+j\omega C\frac{mho}{m}$, and select the correct answer using the codes given below the lists

**List I**

A. Characteristic impedance $Z_0$

B. Propagation constant $\gamma$

C. The sending-end input impedance $Z_s$

**List II**

1. $\sqrt{zY}$

2. $\sqrt{\frac{Z}{Y}}$

3. $\sqrt{\frac{Y}{Z}}$

when the line is terminated in its characteristic impedance $Z_0$

**Codes:**

| | A | B | C |
|---|---|---|---|
| a) | 3 | 1 | 1 |
| b) | 2 | 3 | 3 |
| c) | 2 | 1 | 2 |
| d) | 1 | 2 | 2 |

**Q.156** In a line the VSWR of a load is 6dB. The reflection coefficient will be

a) 0.033 b) 0.33

c) 0.66 d) 3.3

**Q.157** $Z_L$=200Ω & it is desired that $Z_i$=50Ω. The quarter wave transformer should have a characteristic impedance of

a) 100 Ω b) 40Ω

c) 10,000Ω d) 4Ω

**Q.158** Consider the following statements regarding transmission lines transmitting high frequency (HF) signals:
HF transmission lines
1. have appreciable ohmic and dielectric losses.
2. have small physical length
3. are loss-free lines

Of these statements:
a) 1 and 2 are correct
b) 2 and 3 are correct
c) 3 alone is correct
d) 1, 2 and 3 are correct

**Q.159** Values of VSWR measured by 1 terminating the load end of transmission line first with horn, then with a short and finally with a matched load, will be respectively
a) 20, 1.9 and 1.01
b) 20, 1.01 and 1.9
c) 1.9, 1.01 and 20
d) 1.9, 20 and 1.01

**Q.160** A lossless line of length $\frac{\lambda}{4}$ and characteristic impedance $Z_0$ transforms a resistive load R into an impedance $\left(\frac{Z_0^2}{R}\right)$. When the line is $\frac{\lambda}{2}$ long, the transformed impedance will be
a) $\left(\frac{Z_0^2}{R}\right)$ b) $2\left(\frac{Z_0^2}{R}\right)$
c) $Z_o$ d) $R$

**Q.161** A 50 Ω loss-less transmission line is terminated in 100 Ω load and is excited by a 30 MHz source of internal resistance of 50 Ω. What should be the length of the transmission line for maximum power transfer?
a) 5.0 m b) 1.25 m
c) 2.5 m d) 10.0 m

**Q.162** A 75Ω transmission line is first short terminated and the minima locations are noted. When the short is replaced by a resistive load $R_L$, the minima locations are not altered and the VSWR is measured to be 3. The value of $R_L$ is
a) 25Ω b) 50Ω
c) 225Ω d) 250Ω

**Q.163** A UHF short circuited lossless transmission lines can be used to provide appropriate values of impedance., Match List I with List II and select the correct answer using the code given below the list

**List-I**
A. $l < \lambda/4$
B. $\lambda/4 < \lambda/2$
C. $l = \lambda/4$
D. $l = \lambda/2$

**List-II**
1.Capacitive
2.Inductive
3.0
4. ∞

**Codes:**

| | A | B | C | D |
|---|---|---|---|---|
| a) | 2 | 1 | 4 | 3 |
| b) | 3 | 1 | 4 | 2 |
| c) | 2 | 4 | 1 | 3 |
| d) | 3 | 4 | 1 | 2 |

**Q.164** A loss-less transmission line of length λ/8 is short circuited at one end. The impedance measured at the other end is jZ at a frequency of 13.5 MHz. If the frequency is raised to 27 MHz, the impedance measured would be

a) j2Z b) –jZ
c) zero d) infinity

**Q.165** A 50Ω distortionless trnsmission line has a capacitance of $10^{-10}$f/m. What is its inductance per meter?
a) 0.25μH b) 500μ H
c) 5000 μH d) 50μH

**Q.166** What is the attenuation constant α for distortionless transmission line?
a) $\alpha = 0$ b) $\alpha = R\sqrt{\frac{C}{L}}$
c) $\alpha = R\sqrt{\frac{L}{C}}$ d) $\alpha = \sqrt{\frac{RL}{C}}$

**Q.167** A transmitter in free space radiates a mean power of 'P' Watts uniformly in all directions. At a distance 'd', sufficiently far from the source, in plane, the electric field 'E' should be related to 'P' and 'd' as
a) $E\alpha pd$ b) $E\alpha \frac{p}{d}$
c) $E\alpha\sqrt{pd}$ d) $E\alpha \frac{\sqrt{p}}{d}$

**Q.168** With the symbols having their standard meanings, cut off frequency for a rectangular wave guide is
a) $\frac{1}{\sqrt{\mu\in}}\sqrt{\frac{m\pi}{a}+\frac{n\pi}{b}}$
b) $\frac{1}{2\pi\sqrt{\mu\in}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$
c) $\frac{1}{2\pi\sqrt{\mu\in}}\left(\frac{m\pi}{a}+\frac{n\pi}{b}\right)$
d) $\frac{1}{\sqrt{\mu\in}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$

**Q.169)** $\eta = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-\omega^2\mu\in}$
represents the propagation constant in rectangular wave guide for
a)TE waves only
b)TM waves only
c)TEM waves
d)TE and TM waved.

**Q.170** Consider a rectangular wave guide of internal dimensions $8cm \times 4cm$ Assuming critical wavelength would be.
a)8cm b)16cm
c)4cm d)32cm

**Q.171** The ratio of transverse electric field to the transverse magnetic field is called as
a)wave guide impedance
b)waveguide wave length
c)phase velocity
d)Poynting vector

**Q.172** The mode with lowest cut off frequency for on electromagnetic propagating between two perfectly conduction parallel planes of infinitive extent is
a) $TE_{10}$ b) $TM_{10}$
c) $TM_{01}$ d)TEM

**Q.173** The dominant mode in a circular waveguide is a
a)TEM b) $TM_{01}$
c) $TE_{21}$ d) $TE_{11}$

**Q.174** The cut off frequency of the dominant mode of rectangular wave guide having aspect ratio more than 2 is 10GHz. The inner broad wall dimension is given by
a) 3 cm b)2 cm
c)1.5 cm d)2.5 cm

**Q.175** Match List-I (Type of transmission structure) will List-II (Dominant Mode) and select the correct answer

**List-I**
A. Coaxial line
B. Rectangular wave guide
C. Micro strip line
D Coplanar wave guide

**List-II**
1. TE
2. Quasi TEM
3. Hybrid
4. TEM

**Codes:**

| | A | B | C | D |
|---|---|---|---|---|
| a) | 1 | 4 | 2 | 3 |
| b) | 4 | 1 | 3 | 2 |
| c) | 1 | 4 | 3 | 2 |
| d) | 4 | 1 | 2 | 3 |

**Q.176** When a particular mode is excited in a wave guide there appears an extra electric component in the direction of propagation. The resulting mode is
a)TE
b)TM
c)longitudinal
d)TEM

**Q.177** For a wave propagating in air filled rectangular wave guide.
a) guide wave length is never less than free space wave length
b) wave impedance is never less than the free space impedance .
c) TEM mode is possible if the dimensions of the wave guide are properly chosen
d) Propagation constant is always a real quantity

**Q.178** A cylindrical cavity resonates at 9 GHz in $TE_{111}$ mode. The bandwidth 3db is measured to be 2.4MHz. The Q of the cavity at 9GHz is
a) $9000/2.4\sqrt{3}$
b) $9000/2.4\sqrt{2}$
c) $9000/{}^{2.4\sqrt{3}}\!/\!_{\sqrt{2}}$
d) $9000/2.4$

**Q.179** For a hollow wave guide, the axial current must necessarily be
a) a combination of conduction and displacement currents
b) conduction current only
c) time varying conduction current and displacement current
d) displacement current only

**Q.180** Phase velocity $'V_p'$ & group velocity $'V_g'$ wave guide ('C is velocity of light) are related as
a) $V_p.V_g = C^2$
b) $V_p/V_g = \text{a constant}$
c) $V_p + V_g = C$
d) $V_p + V_g = \text{a constant}$

**Q.181** A rectangular wave guide $2.29\text{cm}\times1.02\text{cm}$ operates at a frequency of 11GHz in $TE_{10}$ mode. If the maximum potential gradient of the signal is 5kv/cm, then the maximum power handling capacity of the wave guide will be
a)31.11mw
b)31.11w
c)31.11kw
d)3111Mw

**Q.182** In cylindrical wave guides the attenuation will be minimum, at a frequency which is $\sqrt{3}$ times the cut off frequency for the following mode of operations:
1. $TE_{10}$
2. $TM_{11}$
3. $TM_{10}$
4. $TE_{11}$
Which of the above are correct?
a)1,2,3,4
b)2 and 3 only
c)1 and 2 only
d)3 and 4 only

**Q.183** The directivity of a $\lambda/2$ long wire antenna is
a)1.5
b)1.64
c)2
d) $\sqrt{2}$

**Q.184** An antenna located on the surface of a flat earth transmits an average power of 200kW Assuming that all the power is radiated uniformly over the surface of a hemisphere with the antenna at the center, the time average poyting vector at 50km is.

a) zero
b) $\frac{2}{\pi}\vec{a}_r \, W/m^2$
c) $\frac{40}{\pi}\mu W/m^2$
d) $\frac{40}{\pi}\vec{a}_r \, \mu W/m^2$

**Q.185** A dipole with a length of 1.5m operates at 100 MHz while the other has a length of 15m and operates at 10MHz, the dipole are fed with same current. The power radiated by the two antennas will be

a) The longer antenna will radiate 10 times more power than the shorter one.
b) both antenna radiate same power
c) shorter antenna will radiate 10 times more power that the longer antenna
d) longer antenna will radiate $\sqrt{10}$ times more power than the shorter antenna

**Q.186** An antenna can be modeled as an electric dipole of length 5m at 3MHz. Find the radiation resistance of the antenna assuming uniform current over the length.

a) $2\Omega$
b) $1\Omega$
c) $4\Omega$
d) $0.5\Omega$

**Q.187** A half wave dipole working at 100MHz in free space radiates a power of 1000wattss. The field strength at a distance of 10 kms in the direction of maximum radiation is

a) 1.73mv/m
b) 2.12mv/m
c) 2.22mv/m
d) 22/.2mv/m

**Q.188** A short current element has length $l = 0.03\lambda$ .The radiation resistance for uniform current distribution is

a) $0.072\pi^2\Omega$
b) $80\pi^2\Omega$
c) $72\Omega$
d) $80\Omega$

**Q.189** In a three elements antenna

a) All the three elements are equal length
b) The driven elements and the director are.
c) The reflector is longer than the driven element which in turn is longer than the director
d) The director is longer than the driven elements which in turn is longer than the reflector.

**Q.190** Where does the maximum radiation for an end fire array occur?

a) Perpendicular to the line of the array only
b) along the line of the array
c) at $45^\circ$ to the line of the array
d) Both perpendicular to and along the line of the array

**Q.191** Multiple members of antennas are arranged in arrays in order to enhance what property.

a) Both directivity and bandwidth
b) only directivity
c) only bandwidth
d) Neither directivity nor bandwidth

**Q.192** As the aperture area of an antenna increases the gain

a) increases
b) decreases
c) remains steady
d) behaves unpredictably

**Q.193** For a taking antenna for field pattern, what must be the distance R, between transmitting and receiving antennas?

a) $R > \frac{2D^2}{\lambda}$
b) $R > \frac{4D^2\lambda^2}{3}$

c) $R > \frac{D^2}{2\lambda^2}$ d) $R > \frac{2D^2}{2\lambda^2}$

**Q.194** A transmitting antenna has a gain of 10.It is fed with a signal power of 1w. Assuming free space propagation, what power would be captured by a receiving antenna of effective area of $1m^2$ in the bore sight direction at a distance of 1m?

a)10w b)1w

c)2w d)0.8w

**Q.195** A dipole antenna of $\lambda/8$ length has an equivalent loss resistance of $1.5\Omega$ .The efficiency of the antenna is

a) 0.89159% b)8.9159%

c)89.159% d)891.59%

**Q.196** A radio communication link is to be established via the ionosphere The virtual height at the midpoint of the path is 300 km and its critical frequency is 9 MHz. The maximum usable frequency for the link between the stations of distance 800 km assuming flat earth is

a) 11.25MHz b) 12 MHz

c)15MHz d)25.5MHz

**Q.197** **Assertion A** : A wave guide does not support TEM wave.

**Reason R**: For TEM wave to exist on a transmission line, an axial conduction current must be supported

In question below, two statements A (Assertion) & R (Reason) are given.

a) A and R both are correct and R is the correct explanation of A.
b) A and R both are correct and R is not the correct explanation of A.
c) A is correct and R is incorrect.
d) A is incorrect and R is correct.

## ANSWER KEY :

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** |
| (c) | (b) | (a) | (a) | (b) | (b) | (a) | (c) | (b) | (c) | (d) | (c) | (a) | (b) |
| **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** |
| (c) | (d) | (d) | (a) | (c) | (c) | (c) | (a) | (c) | (c) | (a) | (c) | (c) | (b) |
| **29** | **30** | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** | **41** | **42** |
| (a) | (b) | (b) | (d) | (a) | (b) | (c) | (a) | (a) | (c) | (d) | (d) | (c) | (c) |
| **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** | **51** | **52** | **53** | **54** |
| (c) | (d) | (c) | (b) | (a) | (a) | (b) | (d) | (a) | (c) | (d) | (d) | (d) | (a) |
| **55** | **56** | **57** | **58** | **59** | **60** | **61** | **62** | **63** | **64** | **65** | **66** | **67** | **68** |
| (a) | (d) | (c) | (b) | (b) | (a) | (b) | (b) | (c) | (c) | (d) | (d) | (b) | (b) |
| **69** | **70** | **71** | **72** | **73** | **74** | **75** | **76** | **77** | **78** | **79** | **80** | **81** | **82** |
| (b) | (d) | (c) | (b) | (a) | (d) | (a) | (b) | (a) | (d) | (d) | (d) | (c) | (a) |
| **83** | **84** | **85** | **86** | **87** | **88** | **89** | **90** | **91** | **92** | **93** | **94** | **95** | **96** |
| (d) | (c) | (a) | (b) | (d) | (a) | (d) | (a) | (a) | (c) | (d) | (b) | (b) | (b) |
| **97** | **98** | **99** | **100** | **101** | **102** | **103** | **104** | **105** | **106** | **107** | **108** | **109** | **110** |
| (c) | (b) | (c) | (b) | (a) | (b) | (b) | (b) | (c) | (d) | (b) | (c) | (b) | (d) |
| **111** | **112** | **113** | **114** | **115** | **116** | **117** | **118** | **119** | **120** | **121** | **122** | **123** | **124** |
| (c) | (d) | (d) | (c) | (b) | (c) | (b) | (b) | (a) | (c) | (d) | (b) | (c) | (c) |
| **125** | **126** | **127** | **128** | **129** | **130** | **131** | **132** | **133** | **134** | **135** | **136** | **137** | **138** |
| (c) | (d) | (c) | (c) | (a) | (d) | (a) | (b) | (b) | (d) | (c) | (b) | (d) | (c) |
| **139** | **140** | **141** | **142** | **143** | **144** | **145** | **146** | **147** | **148** | **149** | **150** | **151** | **152** |
| (c) | (a) | (d) | (b) | (b) | (a) | (d) | (a) | (a) | (b) | (c) | (d) | (d) | (c) |
| **153** | **154** | **155** | **156** | **157** | **158** | **159** | **160** | **161** | **162** | **163** | **164** | **165** | **166** |
| (b) | (b) | (c) | (b) | (a) | (a) | (d) | (c) | (b) | (a) | (a) | (b) | (a) | (b) |
| **167** | **168** | **169** | **170** | **171** | **172** | **173** | **174** | **175** | **176** | **177** | **178** | **179** | **180** |
| (d) | (b) | (d) | (b) | (a) | (a) | (d) | (c) | (d) | (b) | (a) | (d) | (d) | (a) |
| **181** | **182** | **183** | **184** | **185** | **186** | **187** | **188** | **189** | **190** | **191** | **192** | **193** | **194** |
| (c) | (d) | (b) | (d) | (b) | (a) | (c) | (a) | (c) | (b) | (b) | (a) | (a) | (d) |
| **195** | **196** | **197** | | | | | | | | | | | |
| (c) | (c) | (c) | | | | | | | | | | | |

# EXPLANATIONS

**Q.1 (c)**
*Electric flux flows from +ve charge to negative charge in the form of field lines.
*Direction of flux and filed lines is same
* Flux=charge
* but only one line will not emanate from 1C of charge

**Q.2 (b)**
The electric field on the surface of inner shell is

$$E=\frac{Q}{4\pi\in_0 R^2}$$

as the inner shell and outer shell is separated by a distance R.

**Q.3 (a)**

$$V=\frac{W}{q}$$

Potential is work done per unit charge

**Q.4 (a)**
Potential due to an electric dipole is

$$V=\frac{Q}{4\pi\in_0}\frac{L\cos\theta}{r^2}$$

**Q.5 (b)**

$$F=\frac{1}{4\pi\in_0}\frac{Q_1Q_2}{r^2}$$

$$N=\frac{1}{\in_0}\frac{c^2}{m^2}$$

$$\in_0=N^{-1}m^{-2}C^2$$

$$=(kgm/s^2)^{-1}m^{-2}(As)^2$$

$$=kg^{-1}m^{-3}S^4A^2$$

$$[M^{-1}L^{-3}T^4A^2]$$

**Q.6 (b)**
Potential difference is given by

$$v=\frac{\text{workdone}}{\text{charge}} \text{ or } \frac{\text{Energy}}{\text{charge}}$$

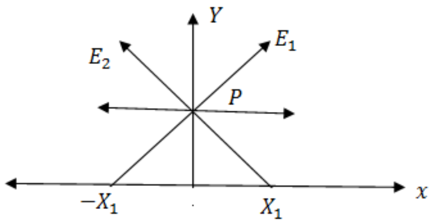
$$=\frac{50}{10}=5\text{volts}$$

**Q.7 (a)**
$\vec{E}$ due to a dipole is given by

$$\vec{E}=\frac{Qd}{4\pi\in_0 r^3}[2\cos\theta\,\hat{a}_r+\sin\theta\,\hat{a}_\theta]V/m$$

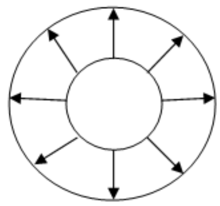
$\theta$ component is given by $\sin\theta$

**Q.8 (c )**




X-component will cancel each other hence the electric field will be only in +y – direction.

**Q.9 (b)**



Assuming inner shell at higher potential

Electric field will directed from higher potential to lower potential and it varies with distance as $r^{-2}$.

**Q.10 (c)**
The correct matching is

$$Faraday's\ Law: V=-\frac{d\phi_m}{dt}$$

**Q.11 (d)**

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

**Q.12 (c)**

$$\vec{E} \propto \frac{1}{\epsilon}$$

$(\epsilon_r)_{air} = 1,\ (\epsilon_r)_{glass} = 3.7 \text{ to } 10,\ (\epsilon_r)_{polythene} = 2.2 \text{ to } 2.4$

$(\epsilon_r)_{glass} = 3.7 \text{ to } 10,\ (\epsilon_r)_{polythene} = 2.2 \text{ to } 2.4$

If $\epsilon$ is max, E is minimum.

**Q.13 (a)**

$$\nabla.\vec{D} = \rho_v$$

$$\nabla.\vec{E} = \frac{\rho_v}{\epsilon_0}$$

**Q.14 (b)**

$$E = \frac{\rho}{4\pi\epsilon_0 r^3}(2\cos\theta \hat{a}_r + \sin\theta \hat{a}_\theta)$$

$$\frac{\rho}{r^3} = \epsilon_0 E$$

Where,

$\frac{\rho}{r^3} =$ dipole moment per unit volume

**Q.15 (c)**

$$C = \frac{\epsilon A}{t}$$

$$C_{new} = \frac{\epsilon A}{t/2} = 2C$$

**Q.16 (d)**

Electric field due to a sheet charge is

$$\vec{E} = \frac{\rho_s}{2\epsilon}\hat{a}_N$$

Where $\hat{a}_N$ → Normal vector to the sheet

$$\vec{E} = \frac{2\times10^{-9}}{2\times\frac{1}{36\pi}\times10^{-9}}(-\hat{a}_z) = -36\pi\ \hat{a}_z$$

**Q.17 (b)**

The capacitance of an isolated sphere is

$$C = 4\pi\epsilon_0 r$$

$$= 1F \quad \left(\because r = \frac{1}{4\pi\epsilon_0}\right)$$

**Q.18 (a)**

From Gauss's law we have

$$\nabla.\vec{D} = \rho_v\ \nabla.(\epsilon\vec{E}) = \rho_v (\vec{D} = \epsilon.\vec{E})$$

$$-\nabla.(\epsilon\nabla V) = \rho_v (\vec{E} = \nabla V)$$

$$\nabla^2 V = \frac{-\rho_v}{\epsilon}$$

**Q.19 (c)**

$$\rho_s = |\vec{D}| = \sqrt{2^2 + \left(2\sqrt{3}\right)^2}$$

$$= 4$$

Since $\vec{D}$ is pointing away from the surface $\rho_s$ must be positive

$$\rho_s = 4\ c/m^2$$

**Q.20 (c)**

$$\nabla^2 v = \frac{-\rho_v}{\epsilon_0} \text{ (using Poisson's eqn) ..(1)}$$

$$\nabla^2 v = \frac{1}{r^2}\frac{\partial}{\partial_r} r^2 \left[\frac{\partial}{\partial_r}\left(\frac{-6r^5}{\epsilon_0}\right)\right]$$

$$= \frac{-6}{\epsilon_0}\times 30r^3 \quad .....(2)$$

from (1) and (2)

$$= \frac{-6}{\epsilon_0}\times 30r^3 = \frac{-\rho_v}{\epsilon_0}$$

$$\rho_v = 180r^3$$

$$Q = \int \rho_v dv = \int_{r=0}^{1}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 180r^3.r^2 \sin\theta dr d\theta d\phi$$

$$= 120\pi\ C$$

**Q.21 (c)**

Continuity equation is given by

$$\nabla.\vec{J} = -\frac{\partial\rho_v}{\partial t}$$

For charge free region $(\rho_v = 0)$

$$\therefore \nabla.\vec{J} = 0$$

**Q.22** **(a)**

Magnetic flux density ,

$B = Wb/m^2$

$1Wb = 1volt - sec$

$\therefore\ B = \dfrac{Volt - sec}{m^2} = \dfrac{ML^2T^{-2}}{Q}TL^{-2}$

$= MT^{-1}Q^{-1}$

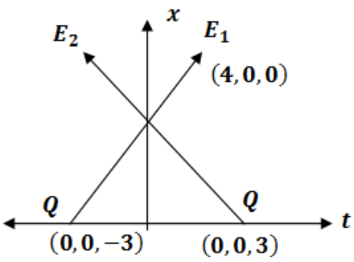
**Q.23** **(c )**

Energy stored per unit volume

$E = \dfrac{1}{2}\in E^2$

**Q.24** **(c )**

Electric fields is in positive x-direction




**Q.25** **(a)**

$\left|\vec{F}\right| = \dfrac{1}{4\pi\in_0}\dfrac{Q_1Q_2}{r^2}$

$= 9\times10^9\times\dfrac{10^{-9}\times10^{-9}}{\left(10^{-3}\right)^2}$

$= 9\times10^{-3}N$

**Q.26** **(c)**

$C_1 = \dfrac{\in_1 A}{d} = \dfrac{\in_0\in_{r_1} A}{d}$

$C_2 = \dfrac{\in_2 A}{d} = 2\,C_1$

$C = QV$

$E = \dfrac{1}{2}CV^2$

**Q.27** **(c)**

Resistivity, $\rho = \dfrac{RA}{L}$

$= \dfrac{v}{I}\ \dfrac{A}{L}$

$= \dfrac{ML^2T^{-2}}{Q}\ \dfrac{1}{I}\ \dfrac{L^2}{L}$

$= \dfrac{ML^2T^{-2}}{IT}\ \dfrac{1}{I}\ L$

$= ML^3T^{-3}I^{-2}$

**Q.28** **(b)**

$D = \in E \qquad \Psi = \int \vec{D}.d\vec{s}$

Since $E \propto \dfrac{1}{\in}$ hence $D$ and $\Psi$ will be independent of $\in$

**Q.29** **(a)**

$\vec{D}$ due to $\rho_s = 5\ c/m^2$ at $r = 3m$ is

$\vec{D}_1 = \dfrac{\rho_s}{2}\hat{a}_r = \dfrac{5}{2}\hat{a}_r$

$\vec{D}$ due to $\rho_s = -5\ c/m^2$ at $r = 3m$ is

$\vec{D}_2 = \dfrac{\rho_s}{2}\left(-\hat{a}_r\right) = \dfrac{5}{2}\hat{a}_r$

Total $\vec{D} = \vec{D}_1 + \vec{D}_2 = 5\hat{a}_r c/m^2$

**Q.30** **(b)**

Energy stored in the field is given by

$E = \dfrac{1}{4\pi\in_0}\dfrac{q_1q_2}{r}$

$= \dfrac{9\times10^9\times10^{-6}\times4\times10^{-6}}{\sqrt{(1+2)^2+(3-1)^2+(-1-5)^2}}$

$= 5.14mJ$

**Q.31** **(b)**

Lapalce's equation is given by

$\nabla^2V = 0$

For V=r cosφ the Laplace's equation will be satisfied.

**Q.32** **(d)**

$\nabla^2V = \nabla.(\nabla V)$

= Divergence of gradient of V.

**Q.33 (a)**

$\rho_L = 30\ nC/m$ at (0,3,5)

$\vec{E} = 64.7\hat{a}_y - 86.3\hat{a}_z V/m$ at (0,6,1)

$\vec{E} = ?$ at (5,6,1)

$$\vec{E} = \frac{\rho_L}{2\pi \in_0 \rho}$$

Where, $\rho$ is the $\perp^r$ distance from infinity long line charge to the point.

Here in both the case $\perp^r$

$$\rho = \sqrt{(3-6)^2 + (5-1)^2} = 5$$

Hence, field remains same

**Q.34 (b)**

Method of images is used for finding the boundary conditions

**Q.35 (c)**

Electric field lines cut the equipotential surface orthogonally.

**Q.36 (a)**

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \vec{A}) - \nabla^2 \vec{A}$$

**Q.37 (a)**

$$V = \frac{1}{4\pi \in_0} \frac{q}{r} = \frac{1}{4\pi \in_0}(v)$$

$$V = \frac{\omega}{q} \Rightarrow \omega = V_q = \frac{1}{4\pi \in_0} \times 2 = \frac{1}{4\pi \in_0} N-m$$

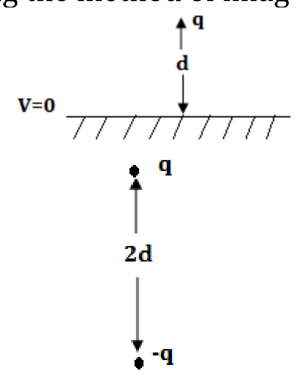
**Q.38 (c)**

ohm's law in field theory is given by

$$\vec{J} = \sigma \vec{E}$$

**Q.39 (d)**

using the method of image charge




Force, $F = \frac{1}{4\pi \in_0} \frac{q^2}{r^2} = \frac{1}{4\pi \in_0} \frac{q^2}{(2d)^2}$

$$= \frac{1}{16\pi \in_0} \frac{q^2}{(d)^2}$$

**Q.40 (d)**

Magnetic field due to a circular coil of radius r at the centre is given by

$$\vec{H} = \frac{I}{2r} \hat{a}_N$$

$$|\vec{H}| = \frac{I}{2r} = \frac{2}{2.\,1/2} = 2\ A/m$$

**Q.41 (c)**

magnetic force is given by

$$\vec{F} = q(\vec{V} \times \vec{B}) = \quad qVB\sin\theta \hat{a}_N$$

**Q.42 (c)**

By Lorentz force law

$$\vec{F} = Q\hat{E} + Q(\vec{V} \times \vec{B})$$

=Electric force +Magnetic force

**Q.43 (c)**

$$I_1 = \oint \vec{H}.d\vec{l}$$

$$\therefore \frac{I}{\pi R^2} \times^{\pi r^2} = H \times 2\pi^2$$

$$\Rightarrow H = \frac{I_r}{2\pi R^2}$$

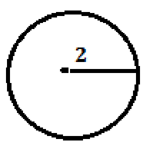
**Q.44 (d)**

A vector field is Solenoidal if

$\nabla.\vec{A} = 0$

$\Rightarrow k = -2$

**Q.45 (c)**



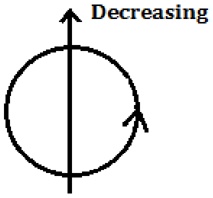

$$\oint dl = 2\pi r = 4\pi$$

**Q.46** **( b)**
The magnetic field due to a current carrying coil will be parallel to the axis

**Q.47** **(a)**
Due to symmetry the force on the unit charge is zero.

**Q.48** **(a)**
$a_x \times a_x = 0$

**Q.49** **(b)**
Anticlockwise in both A and B




**Q.50** **(d)**
By using Boundary condition

$-\vec{H} \times \hat{a}_z = \vec{k}$

Let $\vec{H} = H_x\hat{a}_x + H_y\hat{a}_y + H_z\hat{a}_z$

$\therefore H_x\hat{a}_y - H_y\hat{a}_x = 5\hat{a}_x - 6\hat{a}_y$

$H_y = -5, \quad H_x = -6$

$\therefore \vec{H} = -6\hat{a}_x - 5\hat{a}_y$

**Q.51** **(a)**
boundary condition (Dielectric-dielectric)

$D_{N1} - D_{N2} = \rho_s$

$\in_1 E_{N1} - \in_2 E_{N2} = \rho_s$

$2\in_0 - 6\in_0 = \rho_s$

$\rho_s = -4\in_0$

**Q.52** **(c)**
Polarization is given by

$\vec{P} = \in_0 X_e \vec{E}$

$X_e = \in_r - 1$

$\therefore \vec{P} = \in_0 (\in_r - 1)\vec{E}$

**Q.53** **(d)**

$Q = CV$

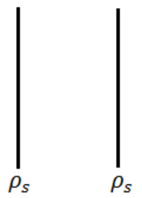
$C = \frac{Q}{V} = \frac{4500\mu}{135}$

$= 33.3\mu F$

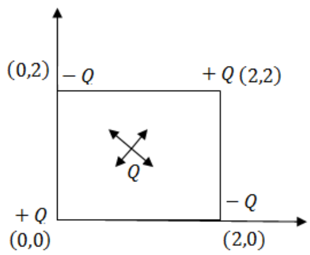
**Q.54** **(d)**

1. $E_{tan_1} = E_{tan2}$
2. $E_{tan} = 0$
3. $D_{N_1} - D_{N_2} = \rho_s$
4. $D_{N_1} = D_{N_2}$ (for charge free boundary)

**Q.55** **(d)**
The electric field between the plates is zero




**Q.56** **(a)**




Net force is zero.

**Q.57** **(a)**
Properties of ideal conductor
* Field inside conductor is zero
* charge density inside conductor zero.
*charge are present on the surface
* conductor surface is equipotential.

**Q.58** **(d)**

$$Q = \frac{4\pi \in V_0}{\frac{1}{R_1} - \frac{1}{R_2}}$$

$$C = \frac{Q}{V} = \frac{\frac{4\pi \in V_0}{\frac{1}{R_1} - \frac{1}{R_2}}}{V_0}$$

$$C=\frac{4\pi\in V_0}{\frac{1}{R_1}-\frac{1}{R_2}}$$
$$=4\pi\in\frac{R_1R_2}{R_2-R_1}$$

**Q.59** **(c)**

Induced emf is given by

$$v=N\frac{d\varphi}{dt}$$

$N\rightarrow$ Number of turns

$\varphi\rightarrow$ flux

$$v=100\left(3t^2-2\right)\Big|_{t=4}\times10^{-3}$$
$$=100(46)\times10^{-3}=4.6v$$

**Q.60** **(b)**

Energy stored in an inductor is

$$E=\frac{1}{2}Li^2=\frac{1}{2}\times0.4\times\left(\frac{50}{5}\right)^2=0.2\times100$$
$$=20J$$

**Q.61** **(b)**

Any wave having arbitrary polarization when incident at Brewster's angle, the reflected wave is always linearly polarized

**Q.62** **(a)**

$$Id\vec{l}=4\times10^4\vec{a}_y$$
$$\vec{B}=\mu\vec{H}=5\vec{a}_x wb$$
$$\vec{F}=\overrightarrow{Idl}\times\vec{B}=4\times10^{-4}\times5\ \left(\vec{a}_y\times\vec{a}_x\right)$$
$$=-2\hat{a}_z mN$$

**Q.63** **(b)**

**Q.64** **(b)**

$\nabla^2V=0$ (Laplace's equation)

If we consider the potential function along x-axis than

$$\frac{\partial^2V}{\partial x^2}=0$$

Here $\phi$ is potential function hence

$$\frac{\partial^2\phi}{\partial x^2}=0\Rightarrow\frac{\partial\phi}{\partial x}=\text{constant}$$

From figure $\frac{\phi_2-\phi_1}{d}=\text{constant}$

$$\frac{\phi_3-\phi_2}{2d}=\text{constant}$$
$$\therefore\frac{\phi_2-\phi_1}{d}=\frac{\phi_3-\phi_2}{2d}\Rightarrow\phi_2=\frac{2\phi_1+\phi_3}{3}$$

**Q.65** **(c)**

Skin depth $(\delta)=\frac{1}{\alpha}=\sqrt{\frac{2}{\omega\mu\sigma}}$

**Q.66** **(c)**

$$\hat{a}_E\times\hat{a}_H=\hat{a}_K$$

Where $\hat{a}_K\rightarrow$ direction of wave

$$\hat{a}_K=\hat{a}_y\times\left(-\hat{a}_x\right)=\left(-\hat{a}_z\right)=\hat{a}_z$$

direction of wave= direction of power $=+z$

**Q.67** **(d)**

$$\hat{a}_H=\hat{a}_k\times\hat{a}_E$$
$$=\hat{a}_z\times\hat{a}_y=-\hat{a}_x$$
$$\frac{E_0}{H_0}=\eta=\sqrt{\frac{\mu_0}{\epsilon_0}}$$
$$\Rightarrow H_0=E_0\sqrt{\frac{\epsilon_0}{\mu_0}}=A\sqrt{\frac{\epsilon_0}{\mu_0}}$$
$$\vec{H}=\sqrt{\frac{\epsilon_0}{\mu_0}}A\cos\omega\left(t-\frac{z}{c}\right)\left(-\hat{a}_x\right)$$
$$=-\sqrt{\frac{\epsilon_0}{\mu_0}}A\cos\omega\left(t-\frac{z}{c}\right)\hat{a}_x$$
$$=-j\sqrt{\frac{\epsilon_0}{\mu_0}}A\sin\omega\left(t-\frac{z}{c}\right)\hat{a}_x$$

**Q.68** **(d)**

Phase velocity, $V\rho=\frac{\omega}{\beta}$

$$=\frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}}=\sqrt{\frac{2\omega}{\mu\sigma}}$$

$$=2\sqrt{\frac{\pi f}{\mu\sigma}}$$

**Q.69 (b)**

Phase velocity in dielectric medium is

$$V_p=\frac{3\times10^8}{\sqrt{\in_r}}=\frac{3\times10^8}{2}=1.5\times10^8\,m/s$$

**Q.70 (b)**

$$\vec{H}=0.2\cos(\omega t-\beta x)$$

$$P_{avg}=\frac{H_0^{\,2}\eta_0}{2}=\frac{0.2\times0.2\times377}{2}$$

$$=7.54\,W/m^2$$

Total avg power is given by

$$\vec{P}=P_{avg}\times area=7.54\times\pi\times5^2\times10^{-4}$$

$$=592.19\times10^{-4}$$

$$=59.21mW$$

$$\approx 60mW$$

**Q.71 (b)**

Skin depth, $\delta=\sqrt{\frac{2}{\omega\mu\sigma}}$

$\rho=\frac{1}{\sigma}\qquad \omega=2\pi f$

$$\therefore \delta=\sqrt{\frac{\rho}{\pi f\mu}}$$

**Q.72 (d)**

$$\frac{E_t}{E_i}=\frac{2\eta_2}{\eta_1+\eta_2}=\frac{2\sqrt{\in_1}}{\sqrt{\in_1}+\sqrt{\in_2}}$$

$$-\frac{2E_r}{E_i}=\frac{2\sqrt{\in_1}}{\sqrt{\in_1}+\sqrt{\in_2}}$$

$$\left(\because \frac{E_r}{E_i}=\frac{\eta_2-\eta_1}{\eta_1+\eta_2}=\frac{\sqrt{\in_1}-\sqrt{\in_2}}{\sqrt{\in_1}+\sqrt{\in_2}}\right)$$

$$\Rightarrow \sqrt{\in_2}=2\sqrt{\in_1}$$

$$\Rightarrow \frac{\in_2}{\in_1}=4$$

**Q.73 (c)**

It is given by Brewster's angle

$$\tan\theta_B=\sqrt{\frac{\in_2}{\in_1}}$$

**Q.74 (b)**

For free space

$$\sigma=0,\in=1,\mu=1,\vec{P}=0,\vec{J}=0$$

**Q.75 (a)**

pointing vector, $\vec{P}=\vec{E}\times\vec{H}$

$$=\frac{v}{m}\times\frac{A}{m}=\frac{Watt}{m^2}$$

**Q.76 (d)**

For plane wave electric field, magnetic field and wave direction are perpendicular to each other.

**Q.77 (a)**

In general

Velocity, $v=\frac{1}{\sqrt{\mu\in}}$

$$=\frac{3\times10^8}{\sqrt{\mu_r\in_r}}$$

$$=\frac{3\times10^8}{\sqrt{4.5\times2}}=1\times10^8\,m/s$$

**Q.78 (b)**

Wave is travelling in –y direction

$$P_{avg}=\frac{H_0^{\,2}\eta_0}{2}=\frac{0.1\times0.1\times12.0\pi}{2}$$

$$=0.6\pi\frac{watt}{m^2}$$

$$\vec{P}_{avg}=-0.6\pi\hat{a}_y\frac{watt}{m^2}$$

**Q.79 (a)**

Intrinsic impedance,

$$\eta = \left|\frac{E}{H}\right| = \sqrt{\frac{\mu}{\epsilon}}$$

**Q.80 (d)**
Polarization is the direction of electric field

$\hat{a}_E = \hat{a}_H \times \hat{a}_k$

$= \hat{a}_z \times \hat{a}_z = -\hat{a}_y$

Since Electric field is oriented in y direction it is polarized in y direction.

**Q.81 (d)**

SWR, $\rho = \frac{1+|\Gamma|}{1-|\Gamma|}$

$= \frac{1+1/3}{1-1/3} = \frac{4/3}{2/3} = 2$

**Q.82 (d)**
By snell's law

$\sqrt{\epsilon_1}\sin\theta_1 = \sqrt{\epsilon_2}\sin\theta_2$

$\sin\theta_2 = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_1$

$= \sqrt{2} \times \frac{\sqrt{3}}{2} = 1.22$

Since $\sin\theta$ cannot be greater than 1 hence there will be no transmitted wave.

**Q.83 (c)**

$v = \frac{\omega}{\beta}$

$\omega = 6\pi \times 10^8 \quad v = \frac{3\times10^8}{\sqrt{\mu_r \epsilon_r}} = \frac{10^8}{3} m/s$

$\beta = \frac{\omega}{v} = \frac{6\pi\times10^8}{10^8/3} = 18\pi \text{ rad/m}$

**Q.84 (a)**
For a good conductor

$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

**Q.85 (d)**

$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r}{\epsilon_r}} \times 120\pi$

$= \sqrt{\frac{2}{8}} \times 120\pi = 60\pi$

$\frac{E_0}{H_0} = \eta$

$\frac{120\pi}{A} = 60\pi \Rightarrow A = 2$

$\beta = \frac{\omega}{v} = \omega\sqrt{\mu\epsilon} = 10^6\pi \times \frac{\sqrt{\mu_r \epsilon_r}}{3\times10^8}$

$= 0.042$

**Q.86 (c)**
Velocity of a wave in free space is given by

$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

**Q.87 (a)**
Plane wave is given by

$\frac{\partial^2 E}{\partial x^2} = \mu\epsilon \frac{\partial^2 E}{\partial t^2}$

**Q.88 (b)**
Rate of Energy flow is given by

$\vec{P} = \vec{E} \times \vec{H} = |\vec{E}||\vec{H}|\sin\theta\,\hat{a}_N$

**Q.89 (d)**

$\vec{E} = 5\cos(10^9 + 30z)\hat{a}_x$

$E_0 = 5, \ \omega = 10^9, \beta = 30$

$\beta = \omega\sqrt{\mu\epsilon_0} = \frac{\omega}{3\times10^8}\sqrt{\epsilon_r}$

$\sqrt{\epsilon_r} = \frac{\beta}{\omega} \times 3\times10^8 = \frac{30}{10^9} \times 3\times10^8$

$= 9$

$\epsilon_r = 81$

**Q.90 (a)**

$$H_0 = \frac{E_0}{\eta_0} = \frac{15}{120\pi} = \frac{1}{8\pi}$$

Direction of magnetic field $= \vec{a}_k \times \vec{a}_E$

$= \vec{a}_z \times \vec{a}_x$

$= \vec{a}_y$

$$\vec{H} = \frac{1}{8\pi}\cos\left(6\pi\times10^8 t - 2\pi z\right)\hat{a}_y$$

$$\vec{B} = \mu_0\vec{H}$$

$$= \frac{4\pi\times10^{-7}}{8\pi}\cos\left(6\pi\times10^8 t - 2\pi z\right)\hat{a}_y$$

$$= 5\times10^{-8}\cos\left(6\pi\times10^8 t - 2\pi z\right)\hat{a}_y$$

**Q.91 (a)**

$$P = \frac{|E_0|^2}{2\eta_0}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0}$$

$$= \sqrt{\mu_0}\times\frac{\omega\sqrt{\mu_0}}{\beta} = \frac{\omega}{\beta}\mu_0$$

$$\sqrt{\epsilon_0} = \frac{\beta}{\omega\sqrt{\mu_0}}$$

$$\therefore P = \frac{E_m^{\ 2}}{2\frac{\omega}{\beta}\mu_0}$$

$$= \frac{1}{2}\frac{\beta\epsilon_0 C^2}{\omega}E_m^{\ 2}$$

**Q.92 (c)**

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = 120\pi\sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$60\pi = \frac{120\pi}{\sqrt{\epsilon_r}} \quad \Rightarrow \epsilon_r = 4$$

**Q.93 (d)**

$$H_0 = \frac{E_0}{\eta_0} = \frac{E_0}{z_0}$$

For the wave traveling in +z direction Direction of $\vec{E} = \hat{a}_H \times \hat{a}_k$

$= \left(\hat{a}_x + \hat{a}_y\right)\times\hat{a}_z$

$= -\hat{a}_y + \hat{a}_x$

For the wave travelling in −z direction Direction of $\vec{E} = \hat{a}_H \times \hat{a}_k$

$= \left(\hat{a}_x + \hat{a}_y\right)\times\left(-\hat{a}_z\right)$

$= \hat{a}_y - \hat{a}_x$

$$\therefore \vec{H} = \frac{\left(\hat{a}_x + \hat{a}_y\right)f(\omega t - \beta z)}{z_0} + \frac{\left(\hat{a}_y - \hat{a}_x\right)f(\omega t - \beta z)}{z_0}$$

**Q.94 (b)**

$\vec{E} = 0.1te^{-t}\vec{a}_x \qquad \epsilon = 4\epsilon_0$

Displacement current density is given by

$$\vec{J}_D = \frac{\partial\vec{D}}{\partial t} = \epsilon\frac{\partial\vec{E}}{\partial t}$$

$$= 4\epsilon_0\left\{0.1e^{-t} - 0.1te^{-t}\right\}$$

$$I_D = J_D A\Big|_{t=0}$$

$$= 0.4\epsilon_0\times0.1 = 0.04\epsilon_0$$

**Q.95 (b)**

$\nabla.\vec{D} = \rho_v \rightarrow$ Gauss's law

$\nabla.\vec{D} = \frac{\partial\rho_v}{\partial t} \rightarrow$ Continuity equation

$\nabla\times\vec{H} = \vec{J}_c \rightarrow$ Ampere law

$\nabla\times\vec{E} = \frac{-\partial\vec{B}}{\partial t} \rightarrow$ Faraday's law

**Q.96 (b)**

For static magnetic flux

$\nabla.\vec{B} = 0$

**Q.97 (c)**

Poynting vector, $S = \vec{E}\times\vec{H}$

**Q.98 (b)**

By divergence theorem

$$\oint \vec{B}.d\vec{S} = \int (\nabla.\vec{B}) dv = 0$$

$\Rightarrow \nabla.\vec{B} = 0 \quad \Rightarrow$ Solenoidal

**Q.99 (c)**

Loss tangent $= \frac{J_C}{J_D} = \frac{\sigma}{\omega \in}$

For perfect dielectric

$$= \frac{J_C}{J_D} = \frac{\sigma}{\omega \in} << 1$$

$$\Rightarrow J_C << J_D$$

**Q.100 (b)**

By time varying Maxwell's eqn

$$\nabla \times \vec{H} = \vec{J}_C + \vec{J}_D$$

$$\vec{J}_C = \sigma \vec{E} \qquad \vec{J}_D = \frac{\partial \vec{D}}{\partial t} = j\omega \in \vec{E}$$

For charge free region , $\sigma = 0$

$$\therefore \nabla \times \vec{H} = j\omega \in \vec{E}$$

**Q.101 (a)**

For electrostatic field

$$\nabla \times \vec{E} = 0, \quad \nabla.\vec{E} = \frac{\rho_v}{\in}$$

For charge free region , $\rho_v = 0$

$$\nabla \times \vec{E} = 0, \quad \nabla.\vec{E} = 0$$

**Q.102 (b)**

$$\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

Taking surface integrate both side , we get

$$\int (\nabla \times \vec{E}).d\vec{S} = \int \left( \frac{-\partial \vec{B}}{\partial t} \right) ds$$

$$\oint \vec{E}.\vec{dl} = \int \frac{-\partial \vec{B}}{\partial t} ds$$

**Q.103 (b)**

Loss tangent $(\tan\delta) = \frac{|J_C|}{|J_D|} = \frac{\sigma}{\omega \in}$

$$\nabla \times \vec{H} = \sigma \vec{E} + j\omega \in \vec{E}$$

$$= j\omega \in \vec{E} \left( 1 + \frac{\sigma}{j\omega \in} \right)$$

$$= j\omega \in \vec{E} (1 - j\tan\delta)$$

**Q.104 (b)**

$$\vec{E} = 20\cos(\omega t - \beta z)\hat{a}_x + 5\cos(\omega t + \beta z)\hat{a}_x$$

First term represents a wave travelling in +z direction, second term represents a wave travelling in –z direction.

$$\vec{H} = \frac{1}{120\pi}\left[ 20\cos(\omega t - \beta t)(\hat{a}_z \times \hat{a}_x) + 5\cos(\omega t - \beta t)(-\hat{a}_z \times \hat{a}_x) \right]$$

$$= \frac{1}{120\pi}\left[ 20\cos(\omega t - \beta t)\hat{a}_y - 5\cos(\omega t - \beta z)\hat{a}_y \right] A/m$$

**Q.105 (c)**

The given wave is travelling in a conducting medium with

$$\alpha = \beta = \frac{1}{\delta}$$

$f = \frac{\omega}{2\pi}$ is the frequency of the wave

**Q.106 (d)**

$$\sin^2(\omega t - \beta x) = \frac{1 - \cos(2\omega t - 2\beta x)}{2}$$

Velocity remain $\frac{\omega}{\beta}$

**Q.107 (b)**

$$\beta = \omega\sqrt{\mu \in} = \omega\sqrt{\mu_0 \mu_r \in_0 \in_r} = \frac{\omega\sqrt{\mu_r \in_r}}{3 \times 10^8}$$

$$\therefore \beta \alpha \sqrt{\in_r}$$

**Q.108 (c)**

Polarization vector gives the direction of electric field.

Direction of Electric field × Direction of wave propagation =Direction of magnetic field

$\vec{n}.\vec{k} = 0$ (since both are perpendicular)

**Q.109 (b)**

$$\frac{\vec{J}_C}{\vec{J}_D}=\frac{\sigma\vec{E}}{j\omega\in\vec{E}}=\frac{\sigma}{j\omega\in}$$

**Q.110 (d)**

$$\vec{E}=10\cos(3\pi\times10^8t-\pi_z)\hat{a}_x$$
$$\omega=3\pi\times10^8\text{rad},\beta=\pi=3.14\,\text{rad}/\text{m}$$
$$v_\rho=\frac{\omega}{\beta}=\frac{3\pi\times10^8}{\pi}=3\times10^8\,\text{m}/\text{s}$$

**Q.111 (c)**

**Q.112 (d)**

$$\frac{E_y}{H_x}=-\eta=-\sqrt{\frac{\mu}{\in}}$$
$$\beta=\omega\sqrt{\mu\in}\Rightarrow\sqrt{\mu}=\frac{\beta}{\omega\sqrt{\in}}$$
$$\therefore\frac{E_y}{H_x}=-\frac{\beta}{\omega\in}$$

**Q.113 (d)**

For lossy dielectric

$$\eta=\sqrt{\frac{\mu}{\in}\left(1+j\frac{\sigma}{\omega\in}\right)}$$

Using Binomial theorem

$$\eta=\sqrt{\frac{\mu}{\in}}\left(1+j\frac{\sigma}{2\omega\in}\right)$$

**Q.114 (c)**

Helmhotz's equation is

$$\nabla^2E=-\gamma^2E$$

Where, $\gamma\rightarrow$ propagation constant

**Q.115 (b)**

Tangential component of electric field

$$E_{tan}=0$$

**Q.116 (c)**

Direction of wave =Direction of energy flow. Electric and magnetic field are in space quadrature i.e. perpendicular in space.

**Q.117 (b)**

$$\in_r=4$$
$$\beta=\omega\sqrt{\mu\in}=\frac{\omega\sqrt{\mu_r\in_r}}{3\times10^8}$$
$$=\frac{10^8\sqrt{4}}{3\times10^8}$$
$$=\frac{2}{3}\text{rad}/\text{m}$$

**Q.118 (b)**

The unit of Poynting vector is power density $\left(\text{watt}/\text{m}^2\right)$

**Q.119 (a)**

Intrinsic impedance of free space

$$\eta_0=\sqrt{\frac{\mu_0}{\in_0}}=\sqrt{\frac{4\pi\times10^7}{\frac{1}{36\pi}\times10^9}}$$
$$=120\pi\text{ or }377\Omega$$

**Q.120 (c)**

$$V_x:V_y:V_z$$
$$\frac{\omega}{\beta_x}:\frac{\omega}{\beta_y}:\frac{\omega}{\beta_z}$$
$$\frac{1}{\beta_x}:\frac{1}{\beta_y}:\frac{1}{\beta_z}$$

**Q.121 (d)**

According to Maxwell's equation

$$\nabla\times\vec{E}=\frac{-\partial\vec{B}}{\partial t}$$

**L.H.S:**

$$\nabla\times\vec{E}=\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ E_x & 0 & 0\end{vmatrix}$$

$$\nabla\times\vec{E}=\frac{\partial E_x}{\partial z}\hat{a}_y-\frac{\partial E_x}{\partial y}\hat{a}_z$$

**R.H.S:**

$$\frac{\partial\vec{B}}{\partial t}=B_0 z\sin(\omega t)\omega\hat{a}_y$$

From (1) we have

$$\frac{\partial E_x}{\partial z}=B_0 z\sin(\omega t)\omega$$

$$E_x=B_0\omega\sin(\omega t)\int zdz$$

$$=\frac{1}{2}B_0 z^2\omega\sin(\omega t)$$

**Q.122 (b)**

$\Gamma\rightarrow 0$ to 1

$S\rightarrow 1$ to $\infty$

**Q.123 (c)**

$$\eta_1=377\sqrt{\frac{\mu_r}{\epsilon_r}}=\frac{377}{2}\approx 188\Omega$$

$$\eta_2=377\sqrt{\frac{\mu_r}{\epsilon_r}}=377\times 3=1131\Omega$$

$$\eta_3=377\sqrt{\frac{\mu_r}{\epsilon_r}}=377\Omega$$

**Q.124 (c)**

In a perfect conductor the amplitude of electric field is given by

$E_0e^{-xz}$

At skin depth , $\delta=\frac{1}{\alpha}$ ,magnitude becomes

$E_0e^{-1}=0.368E_0=36.8\%E_0$

**Q.125 (c)**

$$E_x=e^{-j\beta z}e^{j\omega t}=e^{j(\omega t-\beta z)}$$

$$E_y=je^{-j\beta z}e^{j\omega t}=e^{j(\omega t-\beta z+90^\circ)}$$

As $|E_x|=|E_y|$ and there is a phase shift of $90^\circ$ between $E_x$ and $E_y$ hence it is circularly polarized.

For increasing 't' the electric field moves in left direction hence it is left circularly polarized.

**Q.126 (d)**

It is left elliptically polarized

**Q.127 (c)**

$$\vec{E}=E_x+E_y$$

$$=3\sin(\omega t-\beta z)+6\sin(\omega t-\beta z+75^\circ)$$

$$=3\sin(\omega t-\beta z)+6[\sin(\omega t-\beta z)]\cos 75^\circ+\cos(\omega t-\beta z)\sin 75^\circ$$

$$=3\sin(\omega t-\beta z)+6\sin(\omega t-\beta z)\cos 75^\circ+6\sin 75^\circ\cos(\omega t-\beta z)$$

$$=4.55\sin(\omega t-\beta z)+5.79\cos(\omega t-\beta z)$$

$$E_0=\sqrt{4.55^2+5.79^2}=7.37$$

$$P=\frac{E_0{}^2}{2\eta}=0.072\,{}^{W}/{}_{m^2}$$

$=72m\,{}^{W}/{}_{m^2}$ Hence, option (c) is the nearest answer.

**Q.128 (c)**

**Q.129 (a)**

$$\lambda=\frac{2\pi}{\beta}$$

$$=\frac{2\pi\sqrt{2}}{\sqrt{\omega\mu\sigma}}$$

$$\lambda\alpha\frac{1}{\sqrt{f}}$$

$$\frac{\lambda_1}{\lambda_2}=\frac{\sqrt{f_2}}{\sqrt{f_1}}\Rightarrow\lambda_2=\frac{\lambda}{\sqrt{2}}$$

**Q.130 (d)**

$$\vec{E}=0.10\times 377\cos(4\times 10^7t-\beta z)(\hat{a}_x\times\hat{a}_z)$$

$$=-37.7\cos(4\times 10^7t-\beta z)\hat{a}_y\,{}^{V}/{}_{m}$$

**Q.131 (a)**

For a lossless Tx line

$R=G=0$

**Q.132 (b)**

A $\frac{\lambda}{\mu}$ transfer is used for high frequency load i.e. for impedance matching purpose.

**Q.133 (b)**

$$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\therefore VSWR = \frac{Z_L}{Z_0} \text{ for } Z_L > Z_0$$

$$= \frac{Z_0}{Z_L} \text{ for } Z_0 > Z_L$$

$$VSWR = \frac{75}{50} = 1.5$$

**Q.134 (d)**

$$\text{for} R_L = 100\Omega,\ \bar{R}_L = \frac{R_L}{Z_0} = \frac{100}{50} = 2$$

$$\text{for} R_L = 25\Omega,\ \bar{R}_L = \frac{R_L}{Z_0} = \frac{25}{50} = \frac{1}{2}$$

After $\frac{\lambda}{4}$ distance on transmission line the normalized impedance inverts.

**Q.135 (c)**

The impedance of quarter wave transformer is

$$Z_{0x} = \sqrt{Z_{01}Z_{02}} = \sqrt{50\times 72}$$

$$= 10\times 6 = 60\Omega$$

For coaxial cable , characteristic impedance is given by

$$Z_{0x} = \frac{138}{\sqrt{\epsilon_r}}\log\left(\frac{D}{d}\right)$$

$D \rightarrow$ diameter of outer conductor

$d \rightarrow$ diameter of inner conductor

$$60 = 138\log\left(\frac{D}{10}\right)$$

$$D = 10\times 10^{60/138} = 26.9mm$$

$$\approx 27mm$$

**Q.136 (b)**

$$Z_{SC} = jz_0\tan\beta l$$

$$\text{Since } l < \frac{\lambda}{4}, \text{let } l = \frac{\lambda}{8}$$

$$\beta l = \frac{2\pi}{\lambda}\times\frac{\lambda}{8} = \frac{\pi}{4}$$

$$Z_{SC} = jz_0\tan\frac{\pi}{4} = jz_0$$

Which implies a purely inductive impedance

**Q.137 (d)**

$$Z_0 = \sqrt{Z_{SC}Z_{0C}}$$

$$= \sqrt{20\times 5} = 10\Omega$$

**Q.138 (c)**

For a lossless transmission line

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.294\times 10^{-6}}{60\times 10^{-12}}}$$

$$= 70\Omega$$

**Q.139 (c)**

Reflection coefficient is given by

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} \text{ as } \rho < 1$$

**Q.140 (a)**

For a TEM mode transmission line wave travels in the transverse plane w.r.t. $\vec{E}$ and $\vec{H}$

**Q.141 (d)**

For $Z_L = Z_0$ (Matched Load)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

$$\rho = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = 1$$

**Q.142 (b)**

For $R_1 = 225\Omega$ the VSWR is

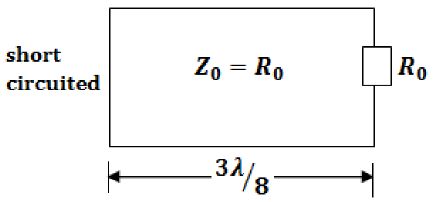
$$\rho = \frac{225}{75} = 3$$

For $R_2 = 75\Omega$ the VSWR is

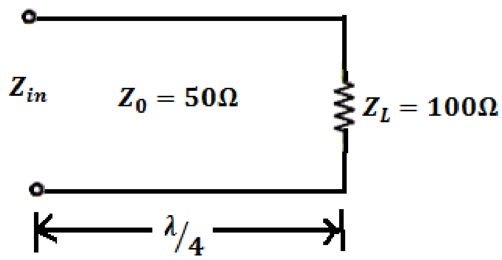
$$\rho = \frac{75}{25} = 3$$

**Q.143 (b)**

This is the case of matched load. In case of matched load the impedance measured at any distances is same as characteristic impedance. Hence, at $\frac{\lambda}{4}$ distance from the load impedance measured is $R_0$.




**Q.144 (a)**




For a distance of $\lambda/4$ the normalized impedance inverts.

Normalized load impedance ,

$$\bar{Z}_L = \frac{Z_L}{Z_0} = \frac{100}{50} = 2$$

Normalized input impedance ,

$$\bar{Z}_{in} = \frac{1}{\bar{Z}_L} = \frac{1}{2}$$

$$\bar{Z}_{in} = \frac{Z_{in}}{Z_0} = \frac{1}{2}$$

$$Z_{in} = \frac{Z_0}{2} = \frac{50}{2} = 25\Omega$$

**Q.145 (b)**

$$Z_{in} = Z_0\left\{\frac{Z_L\cos\beta l + jZ_0\sin\beta l}{Z_0\cos\beta l + jZ_L\sin\beta l}\right\}$$

$$\lambda = \frac{V_p}{f} = \frac{2\times10^8}{100\times10^6} = 2$$

$$\beta l = \frac{2\pi}{\lambda}.l = \frac{2\pi}{2}\times1 = \pi$$

$$Z_{in} = 50\left\{\frac{(30-j40)\cos\pi}{50\cos\pi}\right\}(\sin\pi = 0)$$

$$= (30-j40)\Omega$$

**Q.146 (a)**

$$\lambda = \frac{V_p}{f} = \frac{2\times10^8}{50\times10^6} = 4$$

$$\beta l = \frac{2\pi}{\lambda}\times l = \frac{2\pi}{4}\times2 = \pi$$

$$Z_{in} = Z_0\left\{\frac{Z_L\cos\beta l + jZ_0\sin\beta l}{Z_0\cos\beta l + jZ_L\sin\beta l}\right\}$$

$$= Z_L \quad (\sin\pi = 0)$$

Reflection coefficient at any distance l is

$$\Gamma(l) = \frac{Z(l) - Z_0}{Z(l) + Z_0}$$

Since impedance is same at load and input end hence reflection coefficient is also same

$$\frac{[l]}{[R]} = 1$$

**Q.147 (a)**

By snell's law

$$\beta_1\sin\theta_1 = \beta_2\sin\theta_2$$

$$\sqrt{\epsilon_1}\sin\theta_1 = \sqrt{\epsilon_2}\sin\theta_2$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}$$

For critical angle $\theta_2 = 90°$

$$\therefore \theta_1 = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

**Q.148 (b)**

VSWR, $\rho = \left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) = 1.73$

$$\rho = \frac{Z_L}{Z_0} = 1.73 \Rightarrow Z_L = 86.61$$

Reflected voltage is,

$V_r = \Gamma_L V_{in} = 0.268 \times 15 = 4.02\ V$

Load Voltage is, $V_L = V_i - V_r = 15 - 4.02 = 10.98\ V$

Power delivered to the load is

$P_L = \frac{V_L^2}{Z_L} = \frac{10.98^2}{86.61} = 1.39W$

**Q.149 (c)**

$\frac{v}{\sqrt{\epsilon_r}} = \frac{1}{\sqrt{LC}}$ (Velocity of wave)

Also, characteristics impedance,

$Z_0 = \sqrt{\frac{L}{C}}$

$\Rightarrow \sqrt{L} = Z_0 \sqrt{C}$

$\therefore \frac{V}{\sqrt{\epsilon_r}} = \frac{1}{Z_0 C}$

$\Rightarrow Z_0 = \frac{\sqrt{\epsilon_r}}{VC}$

**Q.150 (d)**

For a lossless line , R=G=0

$\gamma = \sqrt{(R + j\omega L)(G + j\omega c)}$

$\alpha + j\beta = \sqrt{j^2\omega^2 LC} = j\omega\sqrt{LC}$

$\alpha = 0,\ \beta = \omega\sqrt{LC}$

**Q.151 (d)**

Reflection coefficient, $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

$= \frac{40 + j30 - 50}{40 + j30 + 50}$

$= \frac{-10 + j30}{90 + j30} = \frac{-1 + j3}{9 + j3}$

$|\Gamma_L| = \sqrt{\frac{1+9}{81+9}} = \frac{1}{3}$

$VSWR, \rho = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2$

**Q.152 (c)**

Short circuit stub is used for impedance matching. It is connected at a specific distance from the load.

**Q.153 (b)**

A line is said to be distortion less

If $\frac{R}{L} = \frac{G}{C} \Rightarrow RC = LG$

**Q.154 (b)**

$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$

For a short circuit line

$Z_{SC} = jZ_0 \tan\beta l$

$= jZ_0 \Rightarrow$ purely inductive

**Q.155 (c)**

Characteristics impedance, $Z_0 = \sqrt{\frac{Z}{Y}}$

Propagation constant , $V = \sqrt{ZY}$

When $Z_L = Z_0,\ Z_s = Z_0 = \sqrt{\frac{Z}{Y}}$

**Q.156 (b)**

$VSWR_{dB} = 20\log(\rho)$

$6 = 20\log(\rho)$

$\rho = 10^{6/20} = 2$

Reflection coefficient, $\Gamma = \frac{\rho - 1}{\rho + 1} = \frac{1}{3}$

**Q.157 (a)**

$Z_{ox} = \sqrt{Z_i Z_L} = \sqrt{50 \times 200}$

$= 100\Omega$

**Q.158 (a)**

High frequency transmission lines suffers from heavy losses.

**Q.159 (d)**

Low VSWR means less reflections. For short circuit the reflection is highest, then for horn and finally the reflections are minimum for matched load.

**Q.160 (c)**

For $\frac{\lambda}{4}$ distance the normalized impedance inverts whereas for $\frac{\lambda}{2}$ distance the impedance value repeats.

**Q.161 (b)**

For maximum power transfer

$Z_{in} = 50\Omega$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{30\times10^6} = 10m$$

$$Z_{in} = Z_0\left\{\frac{z_L + jz_0\tan\beta l}{z_0 + jz_L\tan\beta l}\right\}$$

$$50 = 50\left\{\frac{100 + j50\tan\beta l}{50 + j100\tan\beta l}\right\}$$

$$50 + j100\tan\beta l = 100 + j50\tan\beta l$$

$$j50\tan\beta l = 50$$

$$j\tan\beta l = 50$$

$$\beta l = \frac{\pi}{4}$$

$$\frac{2\pi}{\lambda}\times l = \frac{\pi}{4} \Rightarrow l = \frac{\lambda}{8} = \frac{10}{8}$$

$$= 1.25m$$

**Q.162 (a)**

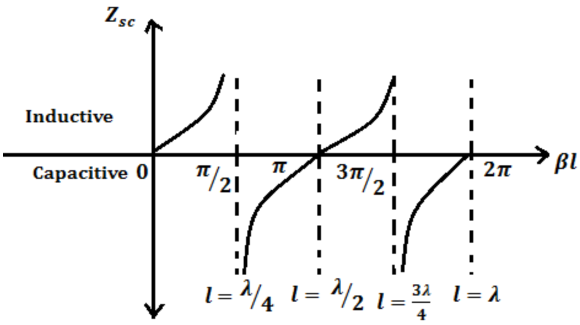
For short circuit the minima occur at the load end. Now, when $R_L$ is connected minima locations are same. This means that $R_L < Z_0$

$$VSWR, \rho = \frac{Z_0}{R_L} = \frac{75}{R_L} \quad (\because Z_0 > R_L)$$

$$R_L = \frac{75}{\rho} = \frac{75}{3} = 25\Omega$$

**Q.163 (a)**

$$Z_{sc} = jz_0\tan\beta l$$




**Q.164 (b)**

$$\lambda = \frac{c}{f}$$

Since, frequency is doubled $\lambda$ is halved

$$\beta l = \frac{2\pi}{\lambda}l$$

Since, $\lambda$ is halved $\beta l$ will be doubled.

Hence the impedance would be $-jz$

**Q.165 (a)**

For a distortion less line

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$L = Z_0^2C = 50\times50\times10^{-10}$$

$$= 0.25\mu H$$

**Q.166 (b)**

For distortion less line

$$\alpha = \sqrt{RG}$$

$$= \sqrt{R.R\frac{C}{L}} = R\sqrt{\frac{C}{L}}$$

**Q.167 (d)**

$$E = \frac{V}{d}$$

$$P = \frac{V^2}{R} \Rightarrow V = \sqrt{PR}$$

$$\therefore E \propto \frac{\sqrt{P}}{d}$$

**Q.168 (b)**

$$\omega_c = \frac{1}{\sqrt{\mu\in}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\in}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

**Q.169 (d)**

For both TE and TM waves the propagation constants is same.

**Q.170 (b)**

For $H_{10}$ mode cut off frequency

$$f_c = \frac{1}{2\pi\sqrt{\mu\in}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$= \frac{1}{2\sqrt{\mu\in}}\frac{1}{a} = \frac{c}{2a}$$

$$\lambda_c = \frac{c}{f_c} = 2a = 16cm$$

**Q.171 (a)**

$$\frac{E}{H} = \eta$$

**Q.172(a)**

$TE_{10}$ mode is called the dominant mode. It has lowest cut off frequency and highest cut off wavelength.

**Q.173 (d)**

$TE_{11}$ is the dominant mode in a circular wave guide

**Q.174 (c)**

For $TE_{10}$ (dominant mode)

$$f_c = \frac{c}{2a}$$

$$a = \frac{c}{2f_c} = \frac{3\times10^8}{2\times10\times10^9}$$

$$= \frac{1.5}{100}m$$

$= 1.5cm$ (broad wall dimension)

Aspect ratio $= \frac{a}{b} = 2$

$$\therefore b = \frac{a}{2} = 0.75cm$$

**Q.175) (d)**

The dominant mode of micro strip line is Quasi TEM.

**Q.176) (b)**

For TM mode the electric field component is non zero in the direction of wave propagation

**Q.177) (a)**

- $\lambda_g = \dfrac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ for wave propagation $f > f_c$
- $\eta_{TM} < \eta_0$ $\eta_{TE} > \eta_0$
- TEM mode is impossible inside a rectangular wave guide
- $V = j\beta$ for wave propagation which is purely imaginary $(\alpha = 0)$

**Q.178) (d)**

$$Q = \frac{\text{Resonantfreq}}{\text{Bandwidth}} = \frac{9\times10^9}{2.4\times10^6}$$

$$= \frac{9000}{2.4}$$

**Q.179) (d)**

only displacement current is the axial current in a hollow wave guide

**Q.180) (a)**

$$V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} > c$$

$$V_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2} < c$$

$$V_p . V_g = c^2$$

**Q.181) (c)**

$$f_c = \frac{3\times10^8}{2\times2.29} = 6.55\text{GHz}$$

Maximum power handling capacity of the wave guide for the TE mode is

$$P = \frac{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{2\eta} E^2_{max} a\, b$$

$$= \frac{\sqrt{1-\left(\frac{6.55}{11}\right)^2}}{2\times120\pi} \times (5\times10)^2 \times 2.29\times10^{-2} \times 1.02\times10^{-2}$$

$$= 31.11\text{kW}$$

**Q.182 (d)**

For cylindrical waveguide $TE_{10}$ and $TM_{11}$ have highest attenuation.

They are called as degenerate mode.

**Q.183 (b)**

Directivity, $D = \frac{4\pi}{\iint F^2(\theta)d\Omega}$

$F(\theta) = \frac{\cos\left(\pi/2 \cos\theta\right)}{\sin\theta}$ for dipole antenna

D=1.64

**Q.184 (d)**

Transmitted powers, $P_T = P_{avg} \times \text{area}$

$$P_{avg} = \frac{P_T}{\text{area}} = \frac{200\text{kw}}{2\pi r^2}$$

$$\vec{P}_{avg} = \frac{40}{\pi} \vec{a}_r \mu \, w/m^2$$

**Q.185 (b)**

For a Hertz dipole

Power radiated, $P_{rad} = I^2 R_{rad}$

$$R_{rad} = 80\pi^2 \left(\frac{l}{\lambda}\right)^2$$

For 1st dipole,

$l = 1.5\text{m}$ $\quad \lambda = \frac{c}{f} = 3\text{m}$

$$R_{rad} = 80\pi\left(\frac{l}{\lambda}\right)^2 = 80\pi\left(\frac{1}{2}\right)^2$$

For 2nd dipole

$$\frac{l}{\lambda} = \frac{15}{30} = \frac{1}{2}$$

**Q.186 (a)**

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^6} = 100\text{m}$$

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2$$

$$= 80\pi^2\left(\frac{5}{100}\right)^2 = 1.97\Omega \approx 2\Omega$$

**Q.187 (c)**

**Q.188 (a)**

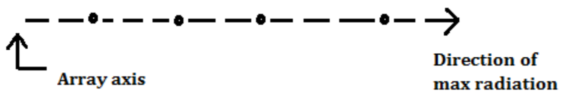
$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2(0.03)^2$$

$$= 0.072\pi^2\Omega$$

**Q.189 (c)**

**Q.190 (b)**

The front view of an end fire array is as shown below.




**Q.191(b)**

Antenna arrays are used to enhance the directivity in a given direction.

**Q.192 (a)**

Gain of the antenna is

$$G = \frac{4\pi A_e}{\lambda^2}$$

$$R > \frac{2D^2}{\lambda^2}$$

**Q.193 (a)**

For taking antenna far field

$$R > \frac{2D^2}{\lambda}$$

**Q.194 (d)**

$$P_r = \frac{P_t G_t G_r}{4\pi r^2}.A$$

$$= \frac{1\times 10}{4\pi} = 0.8w$$

**Q.195 (c)**

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 = 80\pi^2\left(\frac{1}{8}\right)^2 = 12.337\Omega$$

$$R_{loss} = 1.5\Omega$$

$$\eta = \frac{R_{rad}}{R_{input}} = \frac{R_{rad}}{R_{rad} + R_{loss}}$$

$$\eta = \frac{12.337}{12.337 + 1.5} = 0.89159 = 89.159\%$$

**Q.196 (c)**

Maximum usable frequency is given by

$$f_{MUF} = f_c\sqrt{1+\left(\frac{D}{2h}\right)^2}$$

$$= 9\sqrt{1+\left(\frac{800}{2\times 300}\right)^2}$$

$$= 15MHz$$

**Q.197 (c)**

TEM mode can never be supported inside a wave guide