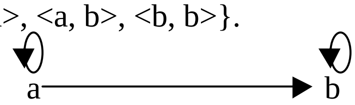
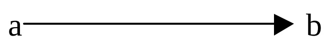
LING 501, Fall 2004: Binary relations

Reflexivity

A binary relation R on a domain A is **reflexive** if for all x in A, R(x, x). For example, let A = {a, b}; R = {<a, a>, <a, b>, <b, b>}.



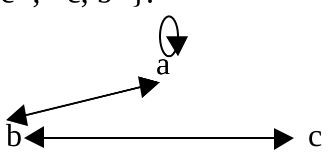
R is **irreflexive** if for all x in A, not R(x, x). Example: A = {a, b}; R = {<a, b>}.



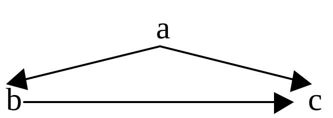
R is **nonreflexive** if it is neither reflexive nor irreflexive. Example: A = {a, b}; R = {<a, a>, <a, b>}.

Symmetry

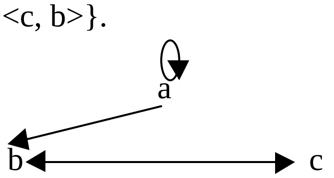
R is **symmetric** if for all x, y in A, if R(x, y) then R(y, x). Example: A = {a, b, c}; R = {<a, a>, <a, b>, <b, a>, <b, c>, <c, b>}.



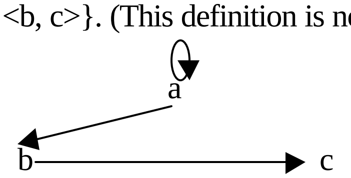
R is **asymmetric** if for all x, y in A, if R(x, y) then not R(y, x). Example: A = {a, b, c}; R = {<a, b>, <b, c>, <a, c>}.



R is **nonsymmetric** if it is neither symmetric nor asymmetric. Example: A = {a, b, c}; R= {<a, a>, <a, b>, <b, c>, <c, b>}.



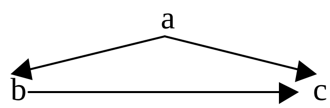
R is **antisymmetric** if all x, y in A, if R(x, y) and R(y, x), then x = y. Example: A = {a, b, c}; R = {<a, a>, <a, b>, <b, c>}. (This definition is not in Hodges!)



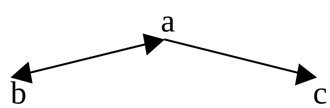
Transitivity

R is **transitive** if for all distinct x, y, z in A, if R(x, y) and R(y, z), then R(x, z). Example: A = {a, b, c};

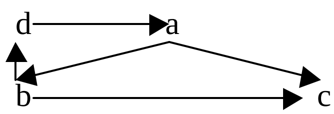
R = {<a, b>, <b, c>, <a, c>}.



R is **intransitive** if for all distinct x, y, z in A, if R(x, y) and R(y, z), then ~R(x, z). Example: A = {a, b, c}; R = {<a, b>, <b, a>, <a, c>}.

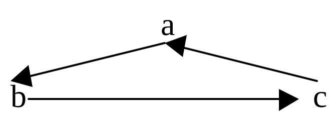


R is **nontransitive** if it is neither transitive nor intransitive. Example: A = {a, b, c, d}; R = {<a, b>, <b, c>, <a, c>, <b, d>, <d, a>}.

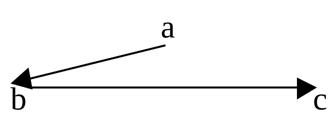


Connectedness

R is **connected** if for all distinct x, y in A, either R(x, y) or R(y, x). Example: A = {a, b, c}; R = {<a, b>, <b, c>, <c, a,>}.



R is **nonconnected** iff there are distinct x, y in A such that neither R(x, y) nor R(y, x). Example: A = {a, b, c}; R= {<a, b>, <b, c>}.



R is **disconnected** iff A can be partitioned into subdomains B, C such that for all x in B and y in C, neither R(x, y) nor R(y, x). Example: A = {a, b, c, d}; R = {<a, b>, <c, d>}; in this example, B = {a, b}, C = {c, d}.