



Lecture 08

Gravity Load Analysis of RC Structures

By:

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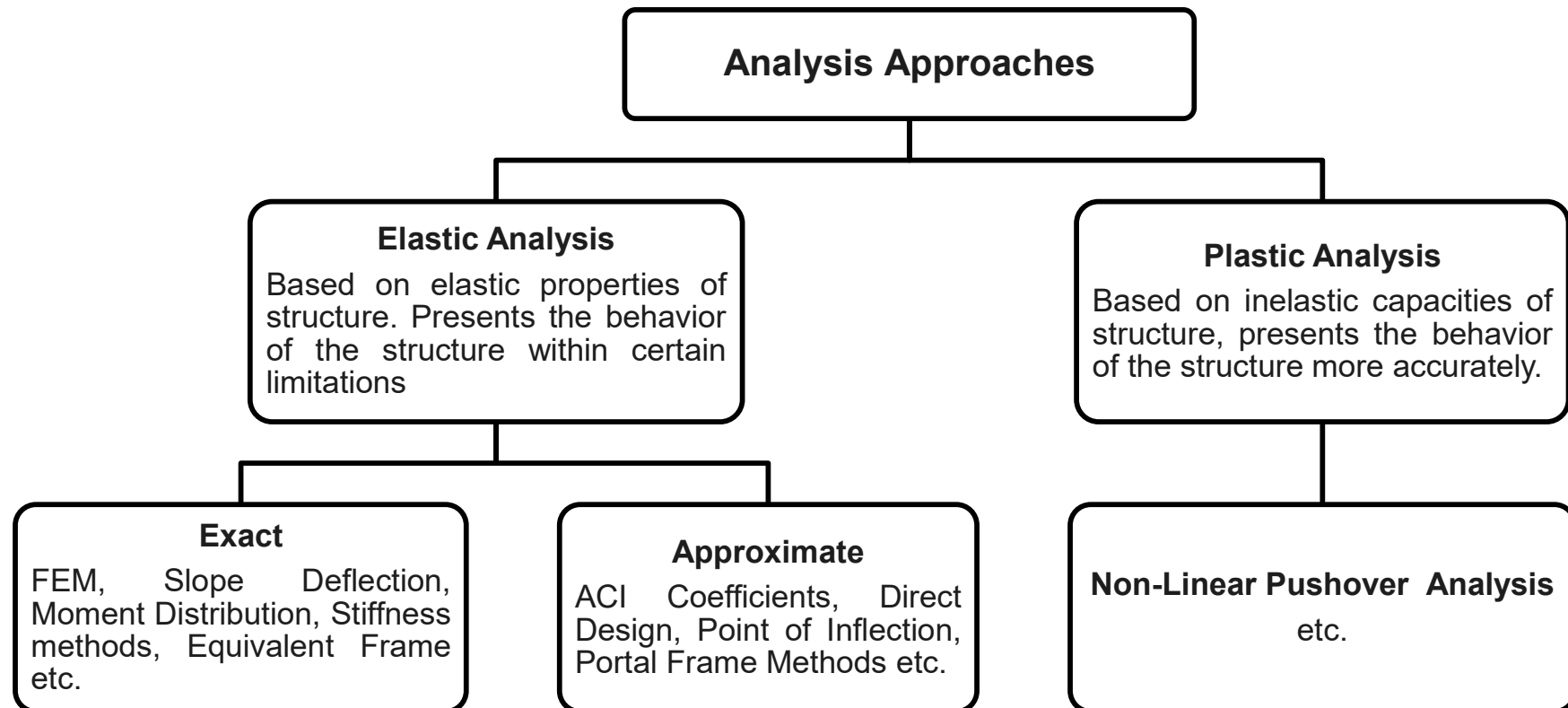
Lecture Contents

- Analysis Approaches
- Point of Inflection Method
- Equivalent Frame Method
- Case Study
- Limit Analysis
- Plastic Analysis
- References



General

□ Analysis Approaches





General

□ Analysis Approaches

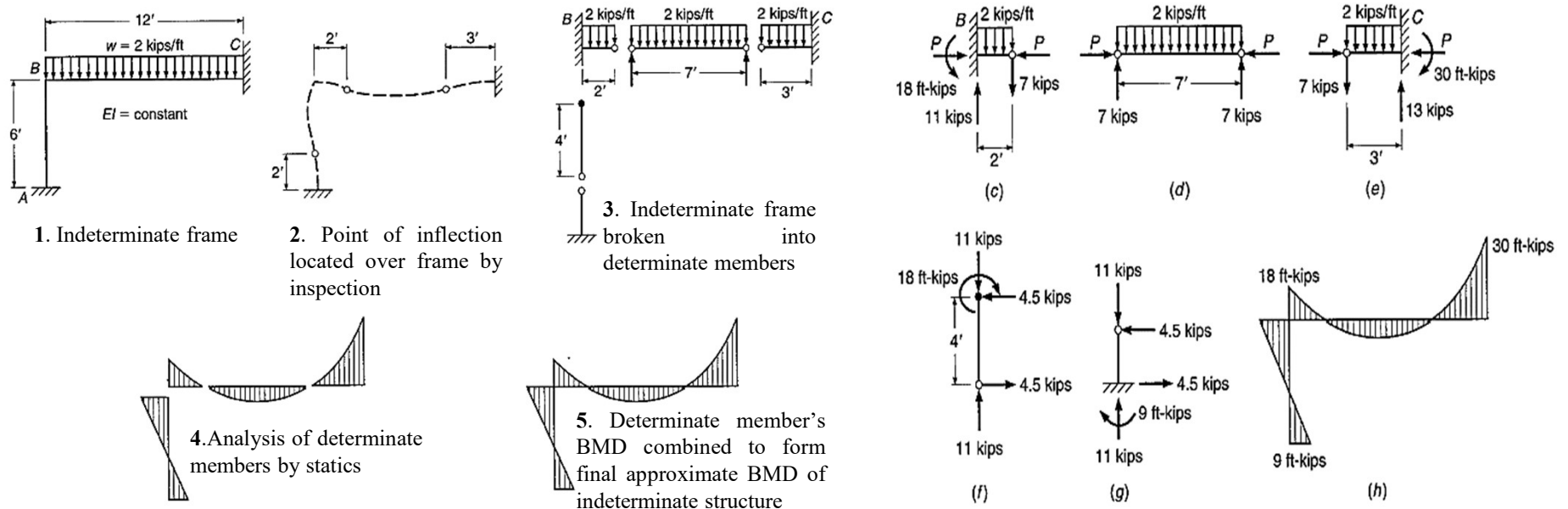
- The approximate analysis methods such as ACI Coefficients and Direct Design Method have been discussed in detail in earlier lectures.
- In this lecture, another approximate method known as Point of Inflection Method will be briefly discussed.
- The exact analysis methods such as Slope Deflection, Moment Distribution and Stiffness method etc. have already been studied. The Equivalent Frame Analysis method will be discussed in detail in this lecture.



General

□ Point of Inflection Method

- In this method, points of inflection are located on the frame and the members are assumed separate determinate members at point of inflection.
- The individual members can be analyzed by statics as shown below.

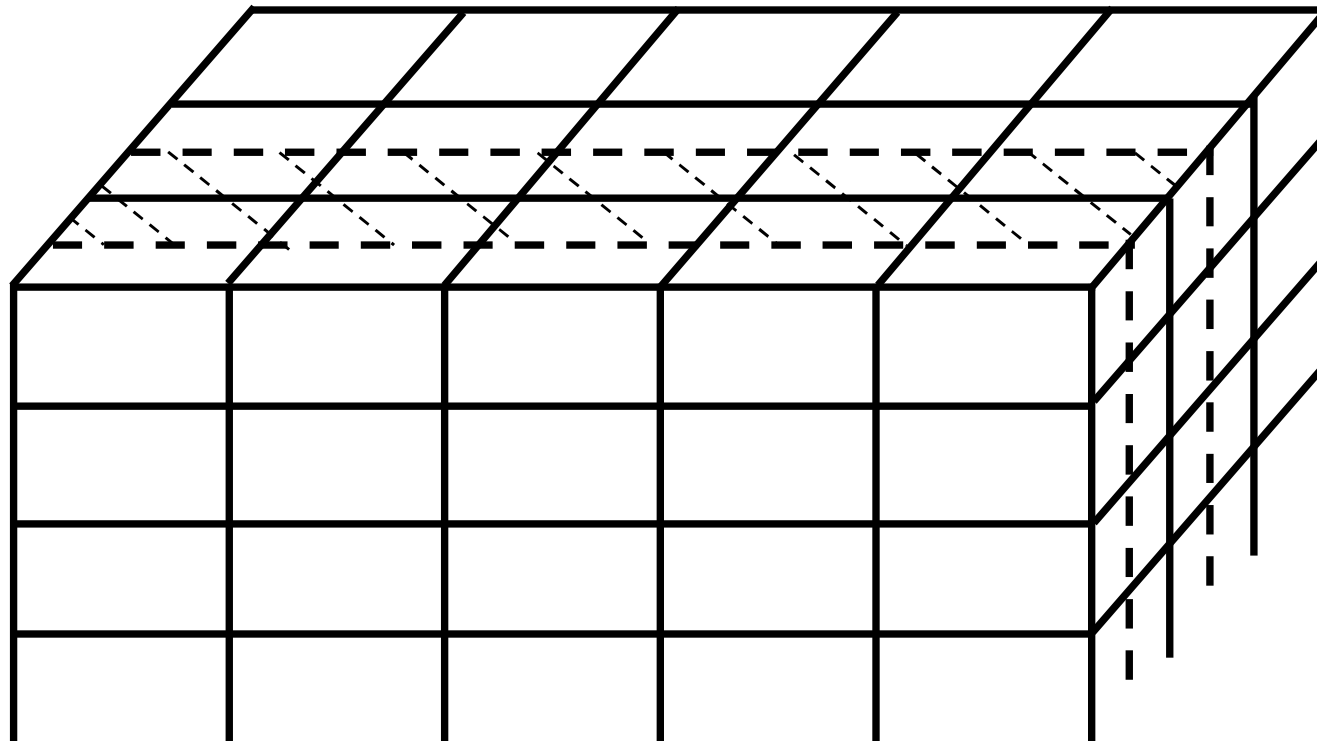




General

□ Equivalent Frame Method (ACI 8.11)

- Consider a 3D structure shown in figure. It is intended to transform this 3D system to 2D system for facilitating analysis. This is done using the transformation technique of Equivalent Frame Method (ACI 318-14 8.11).

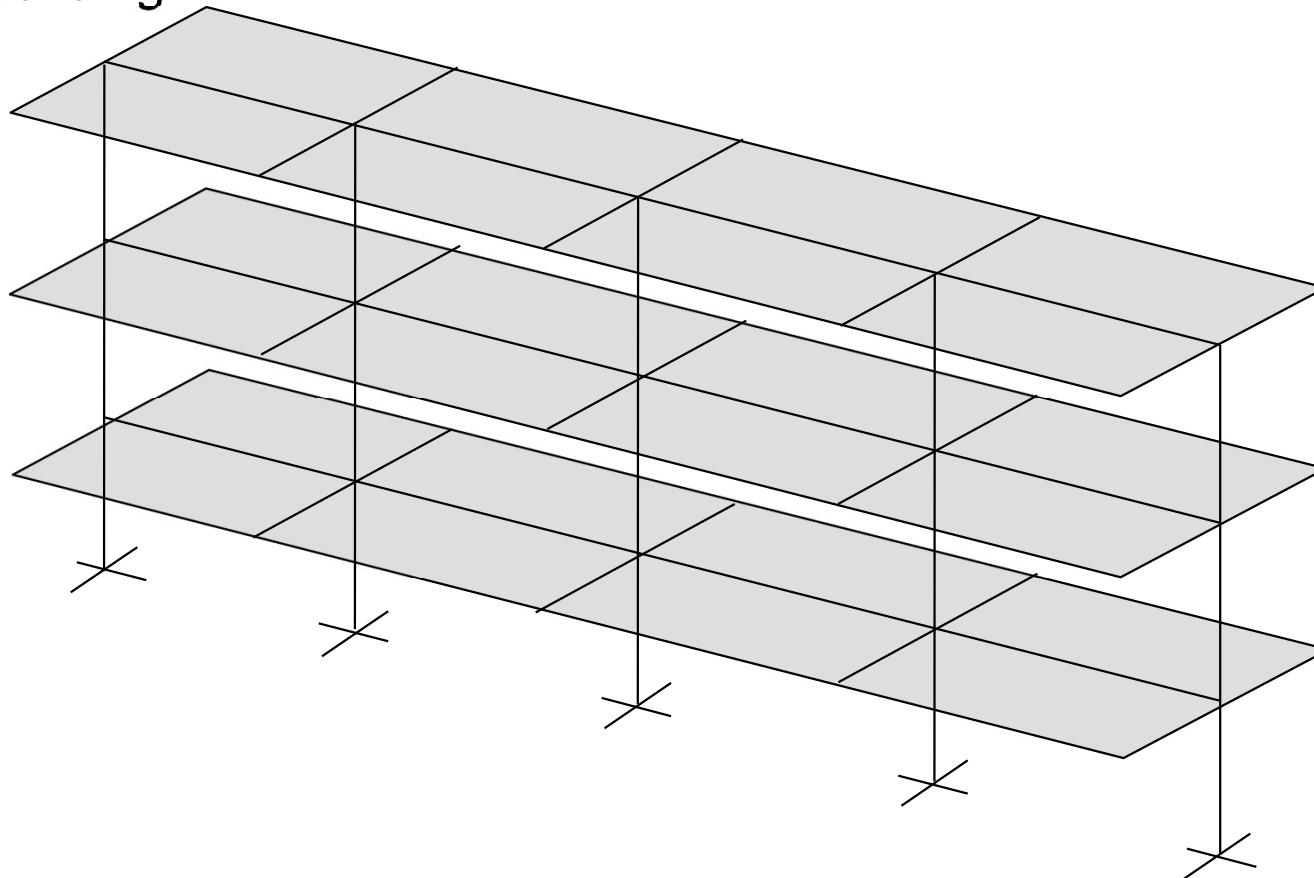




General

□ Equivalent Frame Method (ACI 8.11)

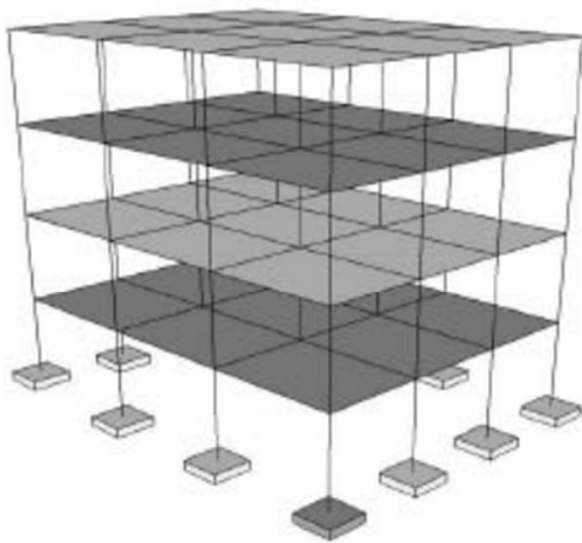
- The width of the frame is same as in DDM. The length of the frame extends up to full length of 3D system and the full height of the building.



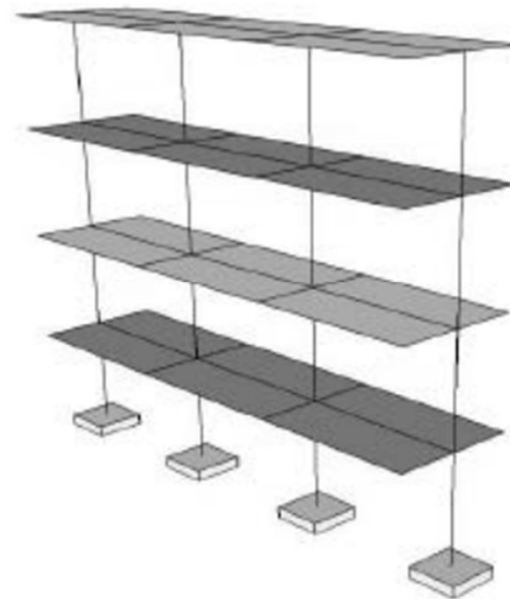


General

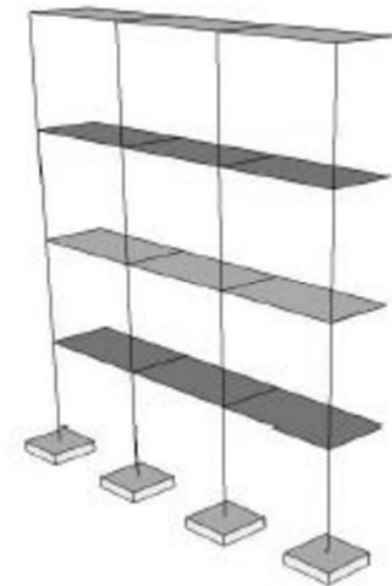
□ Equivalent Frame Method (ACI 8.11)



3D Building



2D Interior Frame



2D Exterior Frame



General

□ Equivalent Frame Method (ACI 8.11)

- This 3D frame is converted to a 2D frame by taking the effect of stiffnesses of laterally present members (slabs and beams).

In EFA method, the horizontal members of the converted 2D frame are called slab-beam members and the vertical members are called equivalent columns.

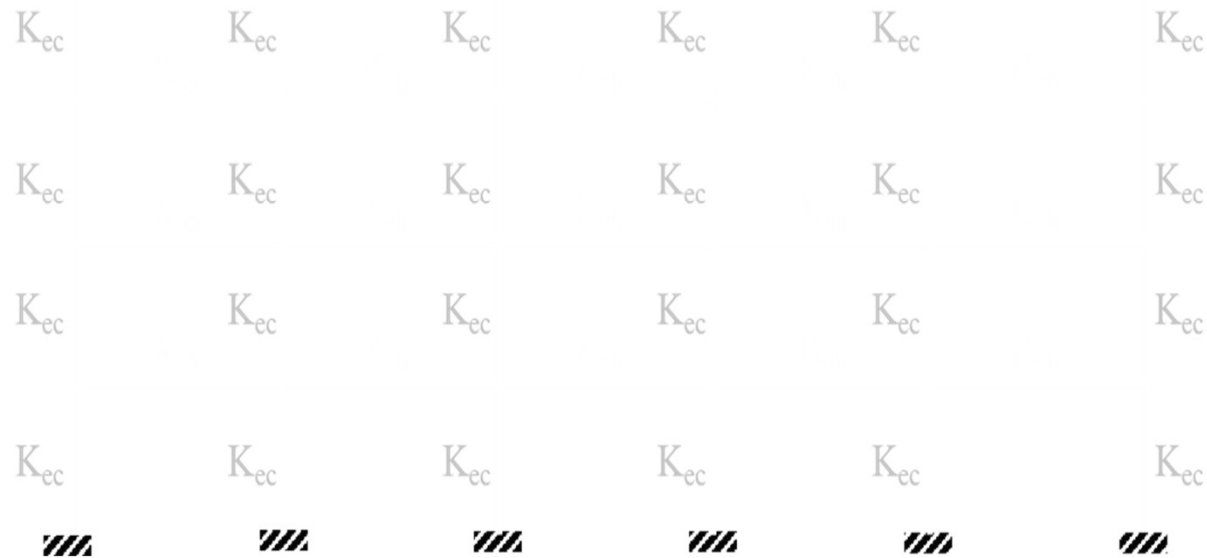




General

□ Equivalent Frame Method (ACI 8.11)

- This 3D frame is converted to a 2D frame by taking the effect of stiffnesses of laterally present members (slabs and beams).



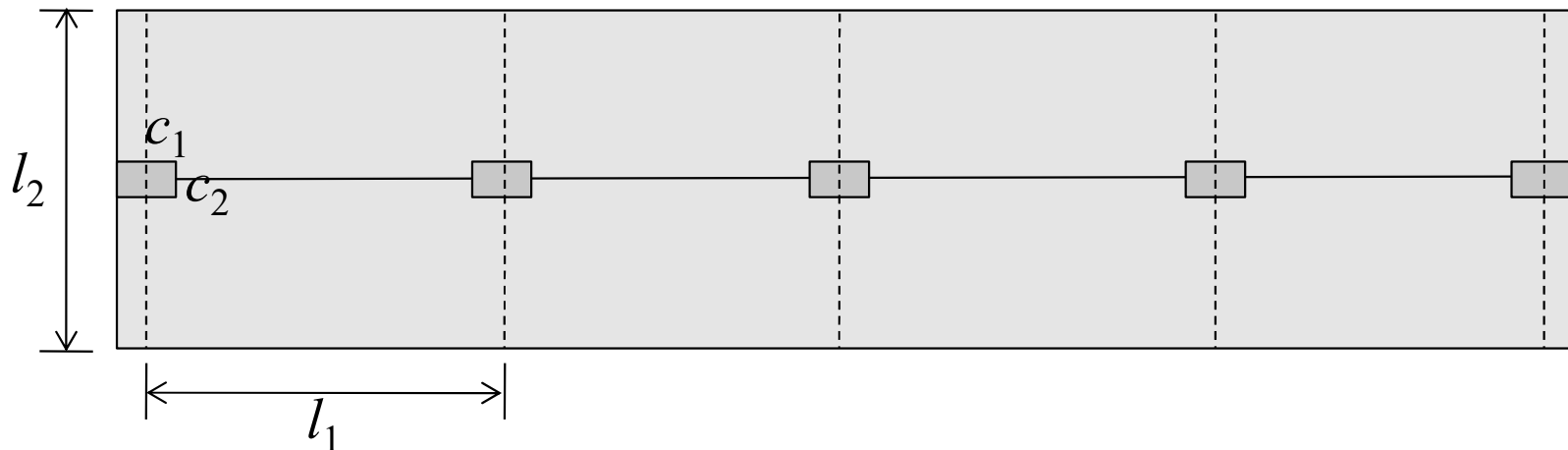
K_{sb} represents the combined stiffness of slab and longitudinal beam and K_{ec} represents the modified column stiffness



General

□ Stiffness of Slab Beam Member K_{sb}

- The stiffness of slab beam ($K_{sb} = kEI_{sb}/l$) consists of combined stiffness of slab and any longitudinal beam present within.
- For a span, the k factor is a direct function of ratios c_1/l_1 and c_2/l_2
- Tables are available for determination of k for various conditions of slab systems.



Plan view of a floor of width l_2 which is to be converted to slab beam line element



General

□ Stiffness of Slab Beam Member K_{sb}

❖ Determination of k

Moment-Distribution Factors for Slabs without Drop Panels ^a							
FEM (uniform load w) = $Mwl_2\ell_1^2$				K (stiffness) = $kE\ell_2t^3/12\ell_1$			
Carryover factor = COF							
		c_2/ℓ_2					
c_1/ℓ_1		0.00	0.05	0.10	0.15	0.20	0.25
0.00	M	0.083	0.083	0.083	0.083	0.083	0.083
	k	4.000	4.000	4.000	4.000	4.000	4.000
	COF	0.500	0.500	0.500	0.500	0.500	0.500
0.05	M	0.083	0.084	0.084	0.084	0.085	0.085
	k	4.000	4.047	4.093	4.138	4.181	4.222
	COF	0.500	0.503	0.507	0.510	0.513	0.516
0.10	M	0.083	0.084	0.085	0.085	0.086	0.087
	k	4.000	4.091	4.182	4.272	4.362	4.449
	COF	0.500	0.506	0.513	0.519	0.524	0.530
0.15	M	0.083	0.084	0.085	0.086	0.087	0.088
	k	4.000	4.132	4.267	4.403	4.541	4.680
	COF	0.500	0.509	0.517	0.526	0.534	0.543
0.20	M	0.083	0.085	0.086	0.087	0.088	0.089
	k	4.000	4.170	4.346	4.529	4.717	4.910
	COF	0.500	0.511	0.522	0.532	0.543	0.554
0.25	M	0.083	0.085	0.086	0.087	0.089	0.090
	k	4.000	4.204	4.420	4.648	4.887	5.138
	COF	0.500	0.512	0.525	0.538	0.550	0.563
$x = (1 - c_2/\ell_2^3)$		1.000	0.856	0.729	0.613	0.512	0.421

^a c_1 and c_2 are the widths of the column measured parallel to ℓ_1 and ℓ_2 .

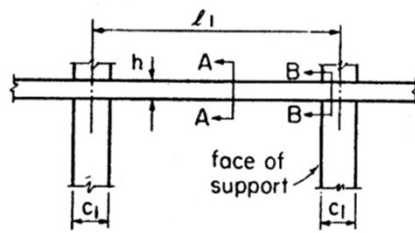
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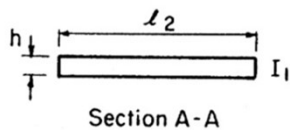
General

□ Stiffness of Slab Beam Member K_{sb}

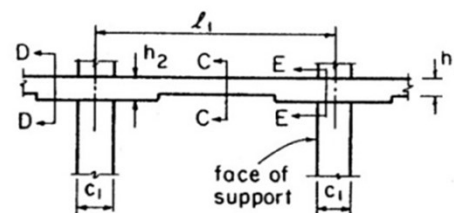
❖ Determination of I_{sb}



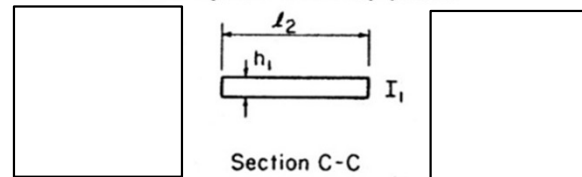
Slab system without beams



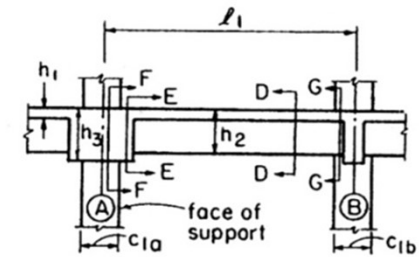
Section A-A



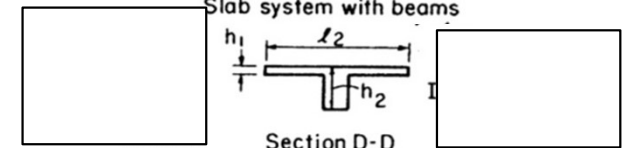
Slab system with drop panels



Section C-C



Slab system with beams



Section D-D



General

□ Stiffness of Equivalent Column K_{ec}

- Stiffness of equivalent column consists of stiffness of actual columns (above and below the slab) plus stiffness of torsional members.
- Mathematically,

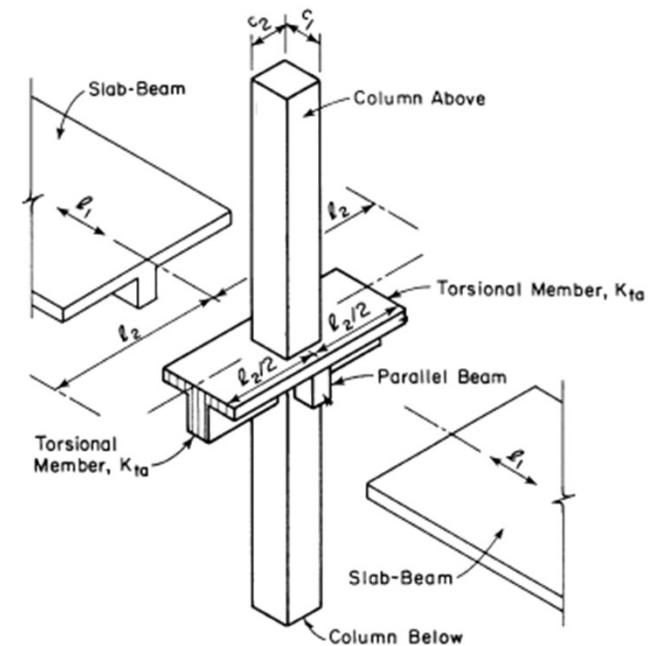
$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t}$$

$$K_{ec} = \frac{\sum K_c \times \sum K_t}{\sum K_c + \sum K_t}$$

where;

$\sum K_c$ = sum of flexural stiffnesses of columns above and below the slab.

$\sum K_t$ = Torsional stiffness of attached torsional members

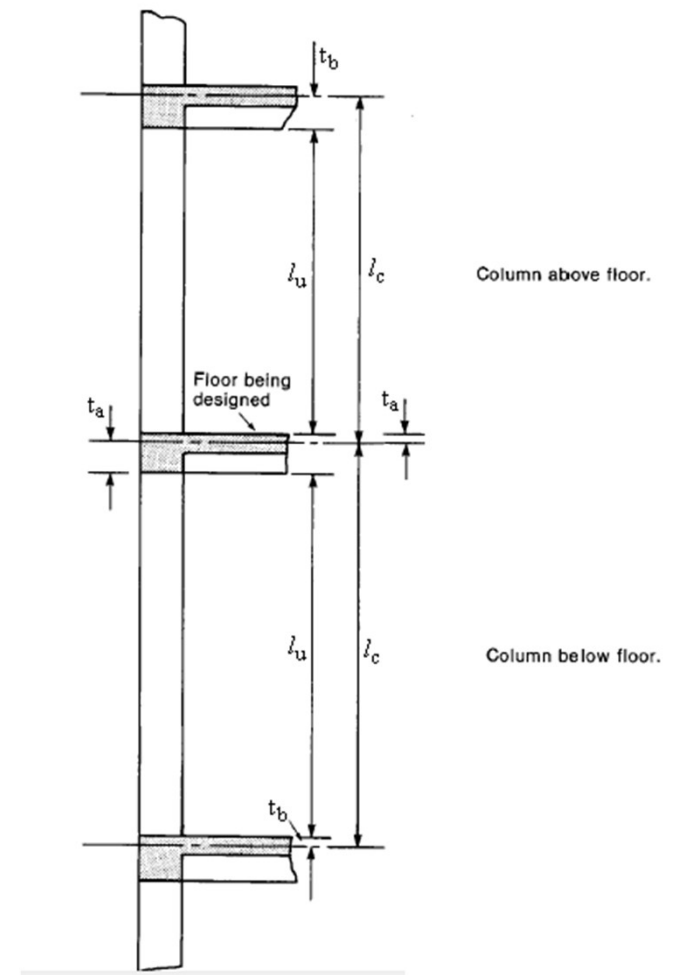




General

□ Stiffness of Equivalent Column K_{ec}

- General formula of flexural stiffness is given by $K = kEI/I$
- Design aids are available from which value of k can be readily obtained for different values of (t_a/t_b) and (l_u/l_c) .
- These design aids can be used if moment distribution method is used as method of analysis.





General

□ Stiffness of Equivalent Column K_{ec}

❖ Determination of k

TABLE A-17 Stiffness and Carryover Factors for Columns

$$K_c = \frac{EL_c}{l_c}$$

		l_c/l_u									
		1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	
0.00	k_{AB}	4.20	4.40	4.60	4.80	5.00	5.20	5.40	5.60	5.80	
	C_{AB}	0.57	0.65	0.73	0.80	0.87	0.95	1.03	1.10	1.17	
0.2	k_{AB}	4.31	4.62	4.95	5.30	5.65	6.02	6.40	6.79	7.20	
	C_{AB}	0.56	0.62	0.68	0.74	0.80	0.85	0.91	0.96	1.01	
0.4	k_{AB}	4.38	4.79	5.22	5.67	6.15	6.65	7.18	7.74	8.32	
	C_{AB}	0.55	0.60	0.65	0.70	0.74	0.79	0.83	0.87	0.91	
0.6	k_{AB}	4.44	4.91	5.42	5.96	6.54	7.15	7.81	8.50	9.23	
	C_{AB}	0.55	0.59	0.63	0.67	0.70	0.74	0.77	0.80	0.83	
0.8	k_{AB}	4.49	5.01	5.58	6.19	6.85	7.56	8.31	9.12	9.98	
	C_{AB}	0.54	0.58	0.61	0.64	0.67	0.70	0.72	0.75	0.77	
1.0	k_{AB}	4.52	5.09	5.71	6.38	7.11	7.89	8.73	9.63	10.60	
	C_{AB}	0.54	0.57	0.60	0.62	0.65	0.67	0.69	0.71	0.73	
1.2	k_{AB}	4.55	5.16	5.82	6.54	7.32	8.17	9.08	10.07	11.12	
	C_{AB}	0.53	0.56	0.59	0.61	0.63	0.65	0.66	0.68	0.69	
1.4	k_{AB}	4.58	5.21	5.91	6.68	7.51	8.41	9.38	10.43	11.57	
	C_{AB}	0.53	0.55	0.58	0.60	0.61	0.63	0.64	0.65	0.66	
1.6	k_{AB}	4.60	5.26	5.99	6.79	7.66	8.61	9.64	10.75	11.95	
	C_{AB}	0.53	0.55	0.57	0.59	0.60	0.61	0.62	0.63	0.64	
1.8	k_{AB}	4.62	5.30	6.06	6.89	7.80	8.79	9.87	11.03	12.29	
	C_{AB}	0.52	0.55	0.56	0.58	0.59	0.60	0.61	0.61	0.62	



General

□ Stiffness of Equivalent Column K_{ec}

❖ Determination of k

t_a/t_b		l_c/l_u								
		1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45
2.0	k_{AB}	4.63	5.34	6.12	6.98	7.92	8.94	10.06	11.27	12.59
	C_{AB}	0.52	0.54	0.56	0.57	0.58	0.59	0.59	0.60	0.60
2.2	k_{AB}	4.65	5.37	6.17	7.05	8.02	9.08	10.24	11.49	12.85
	C_{AB}	0.52	0.54	0.55	0.56	0.57	0.58	0.58	0.59	0.59
2.4	k_{AB}	4.66	5.40	6.22	7.12	8.11	9.20	10.39	11.68	13.08
	C_{AB}	0.52	0.53	0.55	0.56	0.56	0.57	0.57	0.58	0.58
2.6	k_{AB}	4.67	5.42	6.26	7.18	8.20	9.31	10.53	11.86	13.29
	C_{AB}	0.52	0.53	0.54	0.55	0.56	0.56	0.56	0.57	0.57
2.8	k_{AB}	4.68	5.44	6.29	7.23	8.27	9.41	10.66	12.01	13.48
	C_{AB}	0.52	0.53	0.54	0.55	0.55	0.55	0.56	0.56	0.56
3.0	k_{AB}	4.69	5.46	6.33	7.28	8.34	9.50	10.77	12.15	13.65
	C_{AB}	0.52	0.53	0.54	0.54	0.55	0.55	0.55	0.55	0.55
3.5	k_{AB}	4.71	5.50	6.40	7.39	8.48	9.69	11.01	12.46	14.02
	C_{AB}	0.51	0.52	0.53	0.53	0.54	0.54	0.54	0.53	0.53
4.0	k_{AB}	4.72	5.54	6.45	7.47	8.60	9.84	11.21	12.70	14.32
	C_{AB}	0.51	0.52	0.52	0.53	0.53	0.52	0.52	0.52	0.52
4.5	k_{AB}	4.73	5.56	6.50	7.54	8.69	9.97	11.37	12.89	14.57
	C_{AB}	0.51	0.52	0.52	0.52	0.52	0.52	0.51	0.51	0.51
5.0	k_{AB}	4.75	5.59	6.54	7.60	8.78	10.07	11.50	13.07	14.77
	C_{AB}	0.51	0.51	0.52	0.52	0.51	0.51	0.51	0.50	0.49
6.0	k_{AB}	4.76	5.63	6.60	7.69	8.90	10.24	11.72	13.33	15.10
	C_{AB}	0.51	0.51	0.51	0.51	0.50	0.50	0.49	0.49	0.48
7.0	k_{AB}	4.78	5.66	6.65	7.76	9.00	10.37	11.88	13.54	15.34
	C_{AB}	0.51	0.51	0.51	0.50	0.50	0.49	0.48	0.48	0.47
8.0	k_{AB}	4.78	5.68	6.69	7.82	9.07	10.47	12.01	13.70	15.54
	C_{AB}	0.51	0.51	0.50	0.50	0.49	0.49	0.48	0.47	0.46
9.0	k_{AB}	4.80	5.71	6.74	7.89	9.18	10.61	12.19	13.93	15.83
	C_{AB}	0.50	0.50	0.50	0.49	0.48	0.48	0.47	0.46	0.45

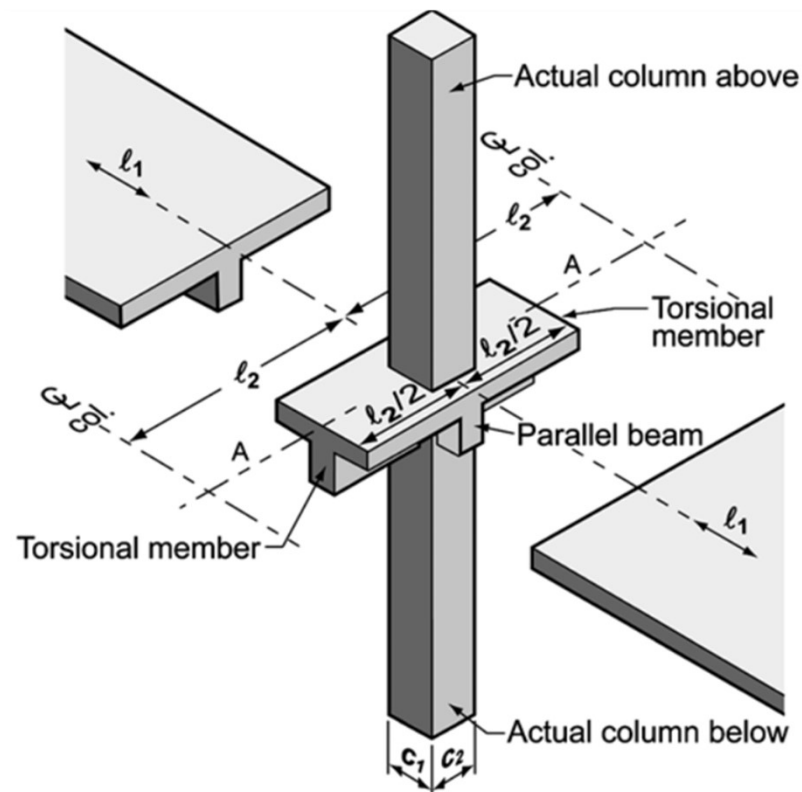
Source: Reinforced Concrete Mechanics and Design 6th Ed. Page #1100



General

□ Stiffness of Torsional Member K_t

- Torsional members (transverse members) provide moment transfer between the slab-beams and the columns.
- Assumed to have constant cross-section throughout their length.





General

□ Stiffness of Torsional Member K_t

- The torsional stiffness K_t of the torsional member is given as:

$$K_t = \sum \left[\frac{9E_{cs}C}{l_2 \left(1 - \frac{c_2}{l_2}\right)^3} \right]$$

- If beams frame into the support in the direction of analysis, the torsional stiffness K_t needs to be increased.

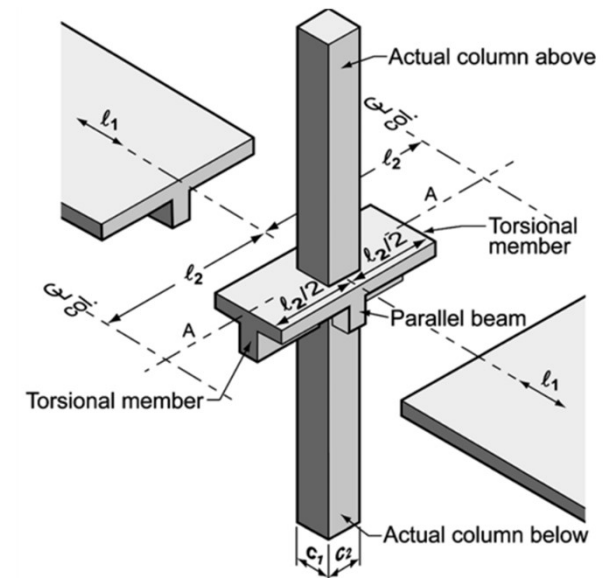
$$K_{ta} = \frac{K_t I_{sb}}{I_s}$$

where;

E_{cs} = modulus of elasticity of slab concrete;

I_{sb} = I of slab with beam;

I_s = I of slab without beam = $l_2 h^3 / 12$

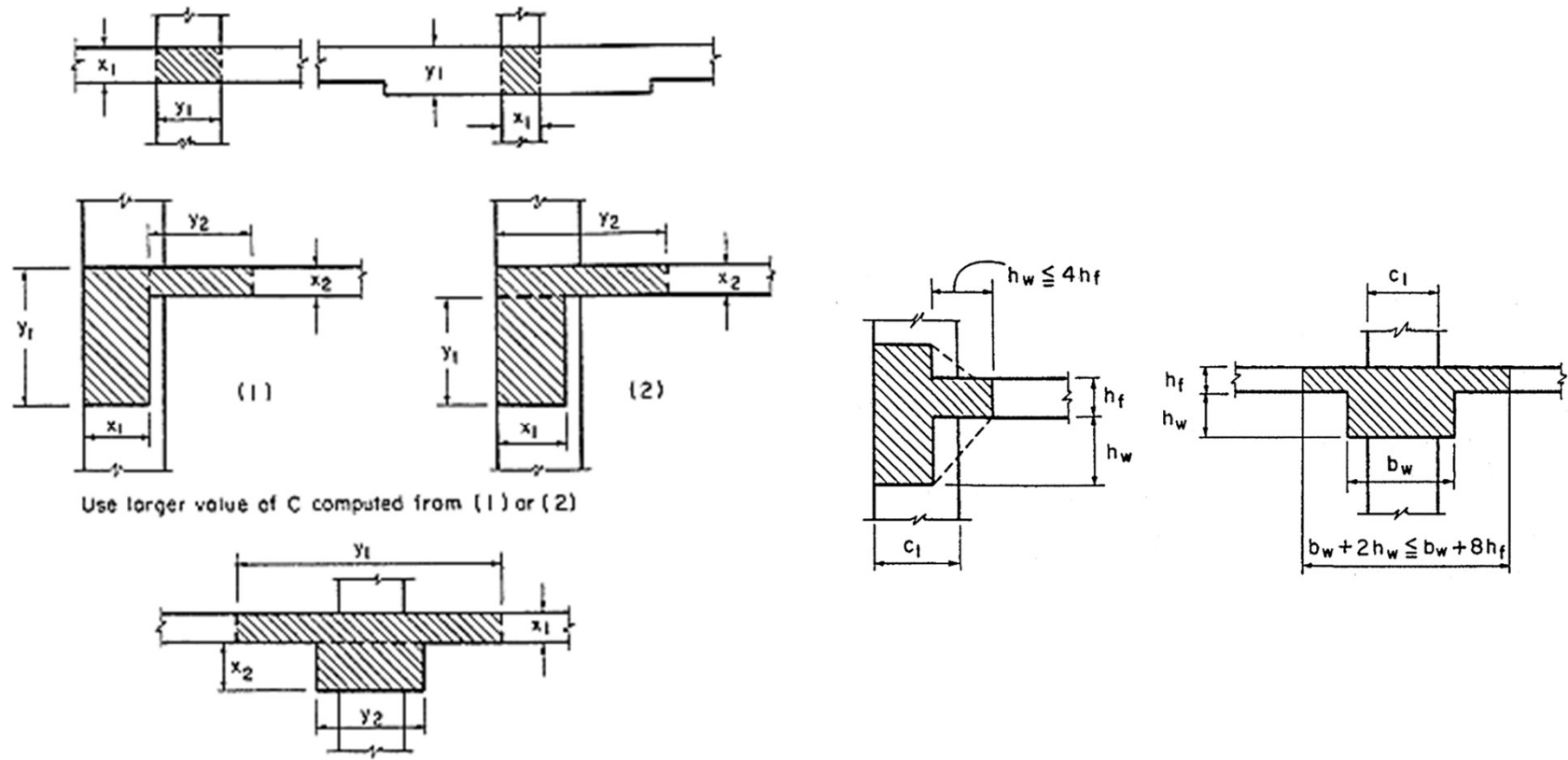




General

□ Stiffness of Torsional Member K_t

❖ Cross Sectional Constant C



$$C = \sum \left[\left(1 - 0.63 \frac{x_1}{y_1} \right) \frac{x_1^3 y_1}{3} \right] + \left[\left(1 - 0.63 \frac{x_2}{y_2} \right) \frac{x_2^3 y_2}{3} \right]$$



General

□ Moment Distribution Method

- The original derivation of EFM assumed that moment distribution would be the procedure used to analyze the slabs.
- In lieu of computer software, moment distribution is a convenient hand calculation method for analyzing partial frames in the Equivalent Frame Method.
- Next slides discuss the application of Moment Distribution Method to complete the analysis using EFM.



General

□ Moment Distribution Method

❖ Distribution Factors

i. Slab Beam Distribution Factors

$$DF_{2 \rightarrow 1} = \frac{K_{sb1}}{K_{sb1} + K_{sb} + K_{ec}}$$

K_{sb1}

$$DF_{2 \rightarrow 3} = \frac{K_{sb}}{K_{sb1} + K_{sb} + K_{ec}}$$

K_{ec}

K_{sb2}



General

□ Moment Distribution Method

❖ Distribution Factors

ii. Equivalent Column Distribution Factor

$$DF = \frac{K_{ec}}{K_{sb1} + K_{sb2} + K_{ec}}$$

K_{sb1}

K_{sb2}

K_{ec}



General

□ Moment Distribution Method

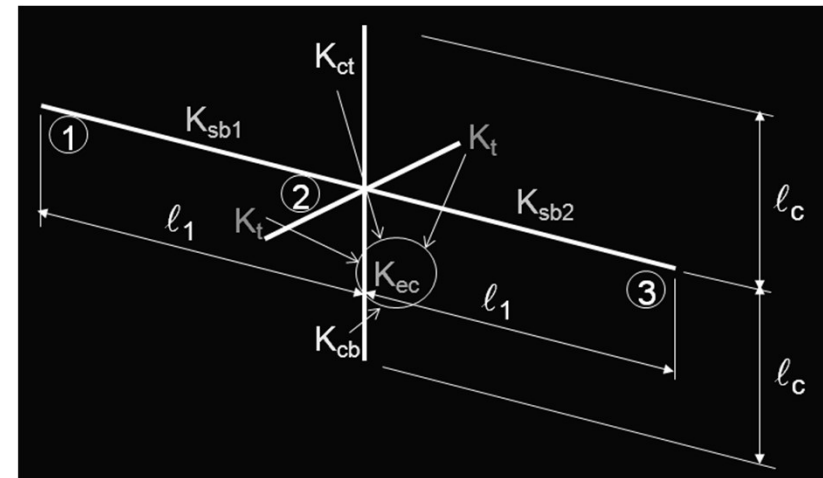
❖ Distribution of Unbalanced Moments to Columns

- Portion of Unbalanced Moment from Beam to Upper Column

$$DF_{uc} = \frac{K_{ct}}{K_{cb} + K_{ct}}$$

- Portion of Unbalanced Moment from Beam to Lower Column

$$DF_{lc} = \frac{K_{cb}}{K_{cb} + K_{ct}}$$





General

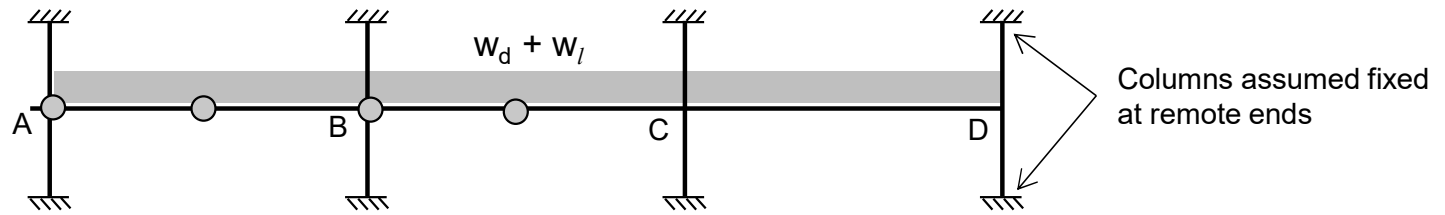
□ Arrangement of Live Loads

- ACI 8.11.1 states that when the loading pattern is known, the equivalent frame shall be analyzed for that load.
- When $LL \leq 0.75DL$
 - Maximum factored moment when Full factored LL on all spans
- Other cases
 - Pattern live loading using 0.75 (Factored LL) to determine maximum factored moment.

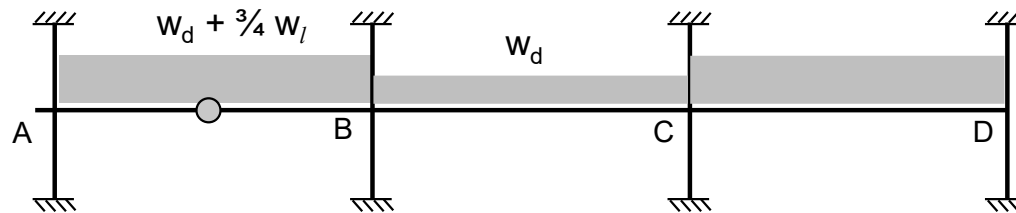


General

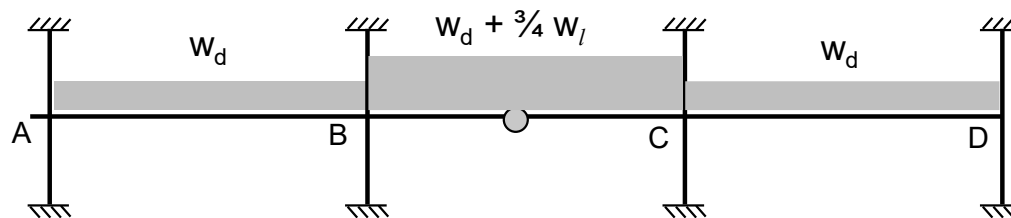
□ Arrangement of Live Loads



(1) Loading pattern for design moments in all spans with $L \leq 3/4 D$



(2) Loading pattern for positive design moment in span AB'

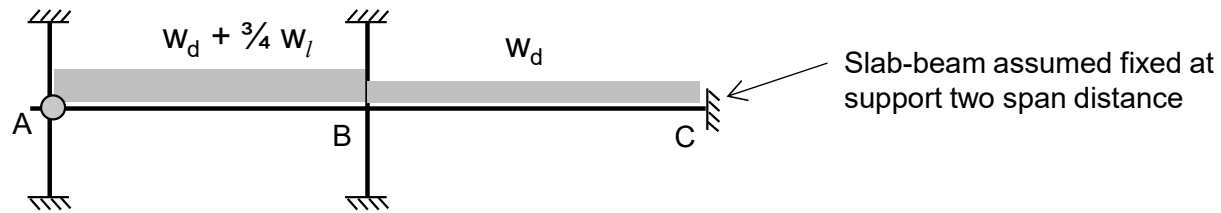


(3) Loading pattern for positive design moment in span BC'



General

□ Arrangement of Live Loads



(4) Loading pattern for negative design moment at support A'



(5) Loading pattern for negative design moment at support B'



General

□ Summary of Analysis Steps in EFM

- Extract the 3D frame from the 3D structure.
- Extract a story from 3D frame for gravity load analysis.
- Identify EF members i.e., slab beam, torsional member and columns.
- Find stiffness (kEI/I) of each EF member using tables.
- Assign stiffnesses of each EF member to its corresponding 2D frame member.



General

□ Summary of Analysis Steps in EFM

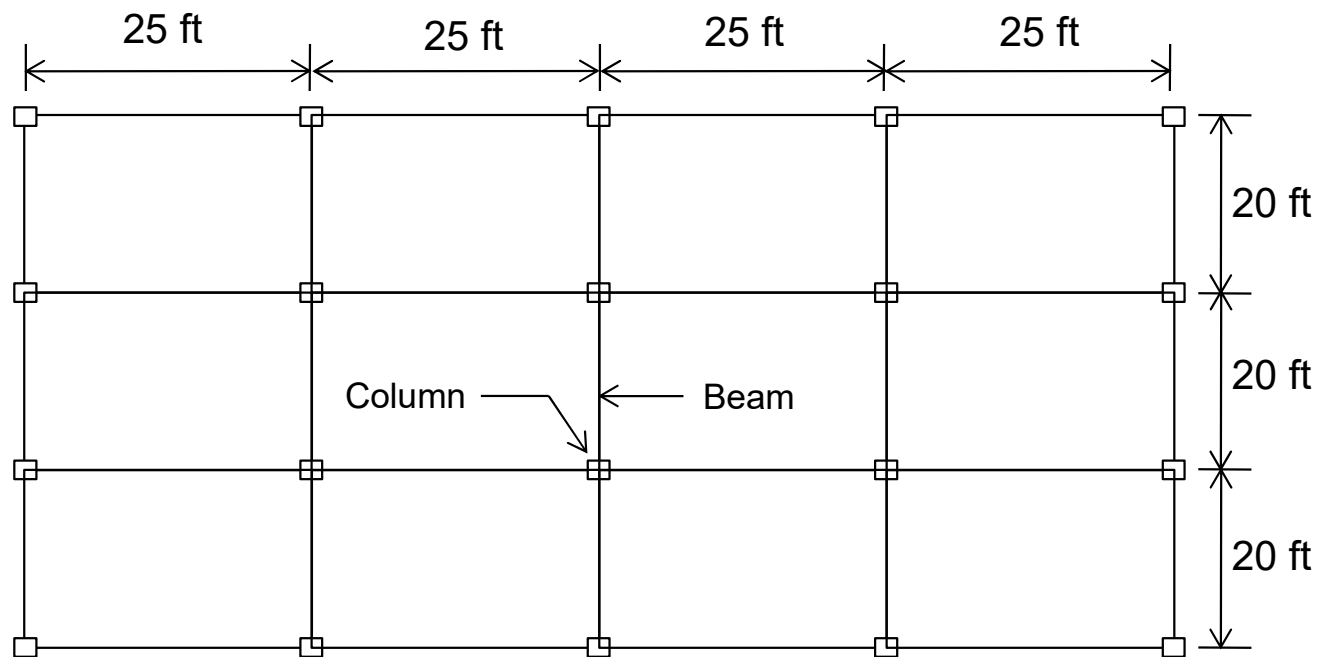
- Analyze the obtained 2D frame using Moment Distribution method of analysis to get longitudinal moments based on center-to-center span.
- Distribute slab-beam longitudinal moment laterally using lateral distribution procedures of DDM.
- Slab analysis can be done using DDM.



Example 8.1

□ Problem Statement

Analyze the building whose typical floor is shown below, using Equivalent Frame Method.



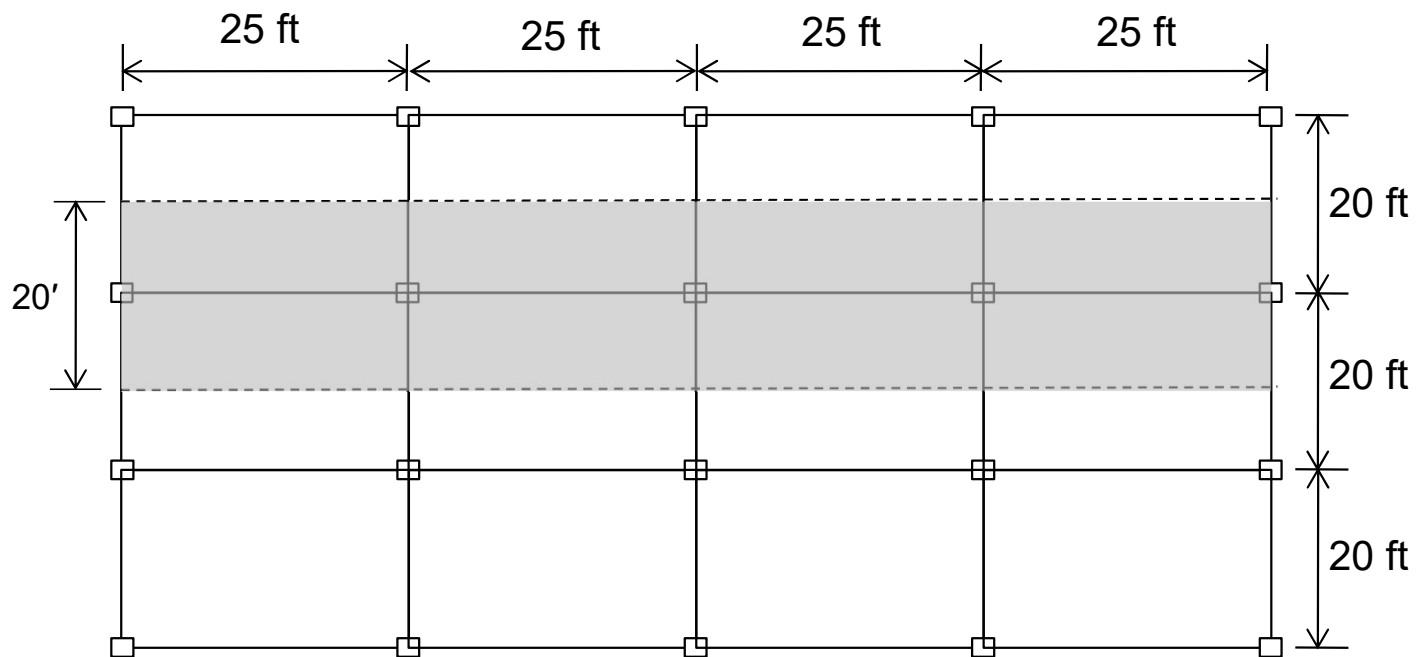
Slab thickness: 7", Columns: 14" × 14" and Beams: 14" × 20"



Example 8.1

□ Solution

➤ Step 1: 3D Frame Selection



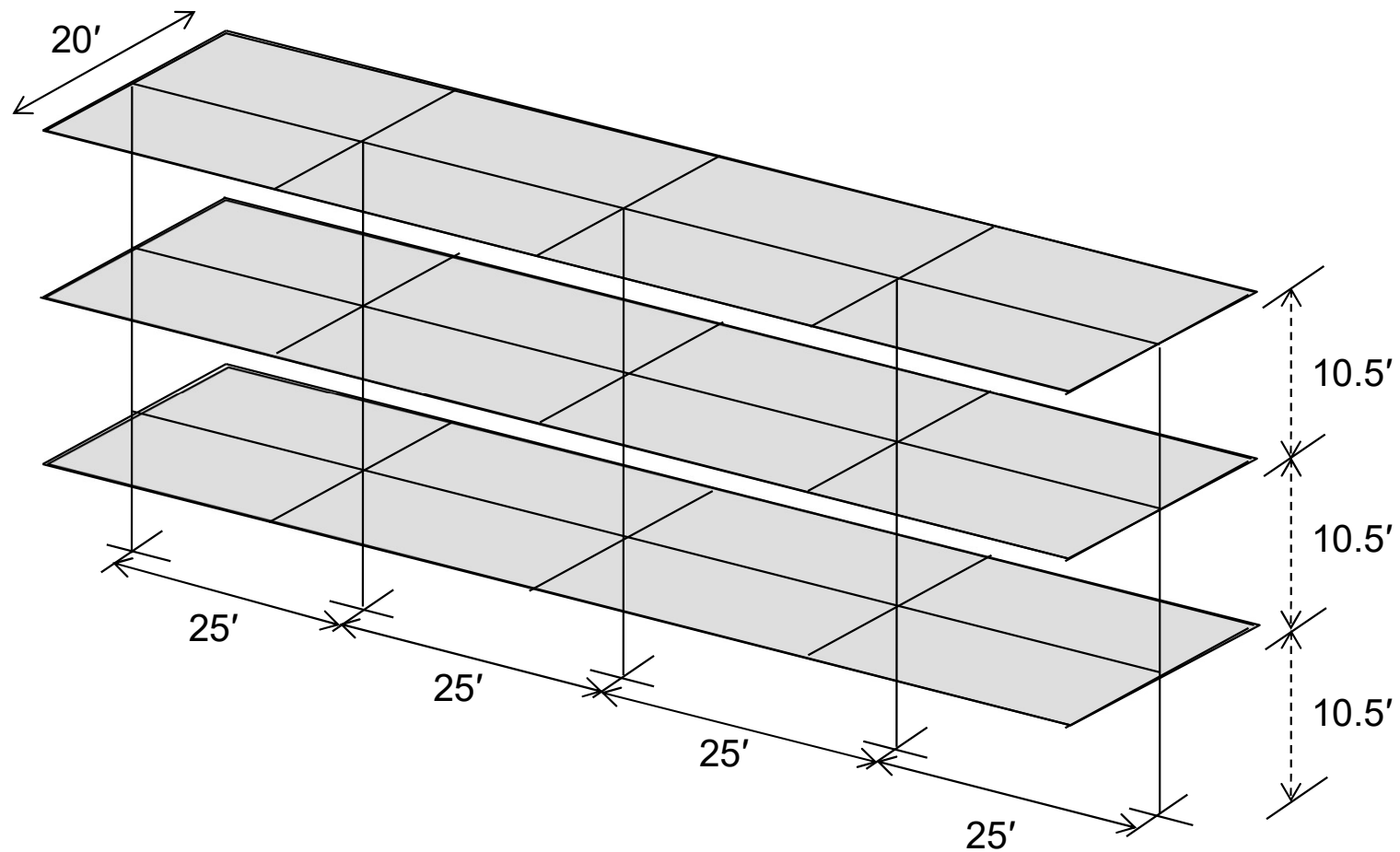
Slab thickness: 7", Columns: 14" × 14" and Beams: 14" x 20"



Example 8.1

□ Solution

➤ Step 2: 3D Frame Extraction



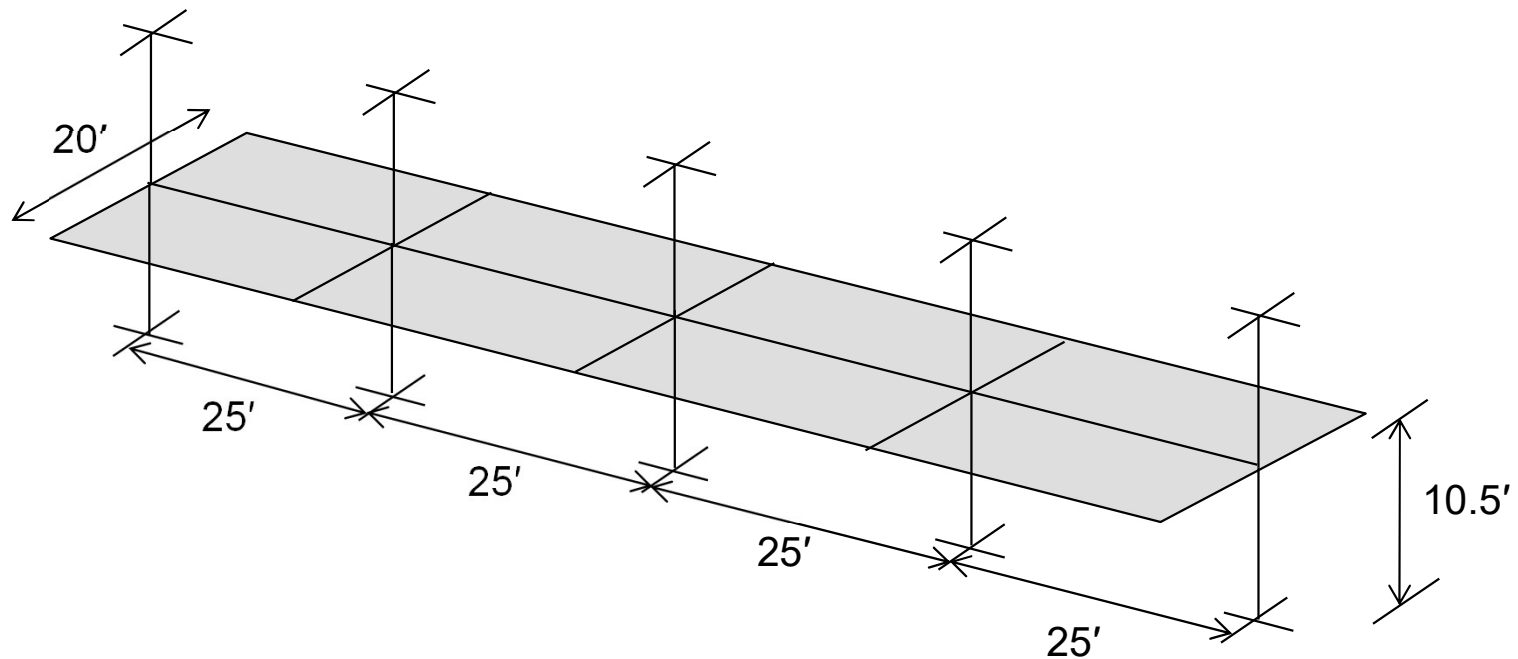


Example 8.1

□ Solution

➤ Step 3: Extraction of Single Story from 3D frame

- According to ACI 8.11.2.5, it shall be permitted to assume that the far ends of columns built integrally with the structure are considered to be fixed for gravity load analysis.



Frame Extracted from at Intermediate Story



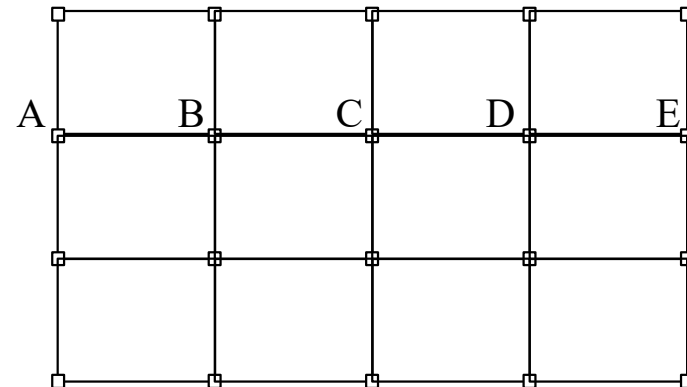
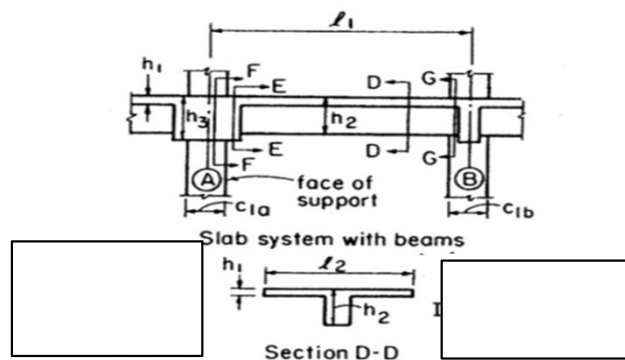
Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

i. Slab-beam Stiffness Calculation

Slab beam stiffness (K_{sb})							
Span	l_1 and c_1	l_2 and c_2	c_1/l_1	c_2/l_2	k	I_{sb}	$K_{sb}=kEI_{sb}/l_1$
AB	25' & 14"	20' and 14"	0.0467	0.058	4.051	25844	349E
The remaining spans will have the same values as the geometry is same. Table A-20 (Reinforced concrete: Mechanics and Design, 3 rd Ed)							





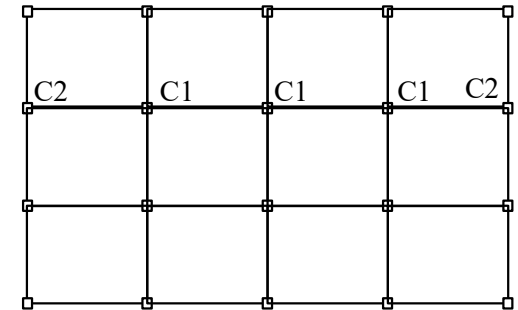
Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t}$$



Calculation of K_t				
Column location	l_2	c_2	$C = \sum (1 - 0.63x/y)x^3y/3$ (in ⁴)	$K_t = \sum 9E_{cs} C / \{l_2(1 - c_2/l_2)^3\}$
C2	20'	14"	11208	$3792.63E_{cs}$
C1	20'	14"	12190	$4295.98E_{cs}$

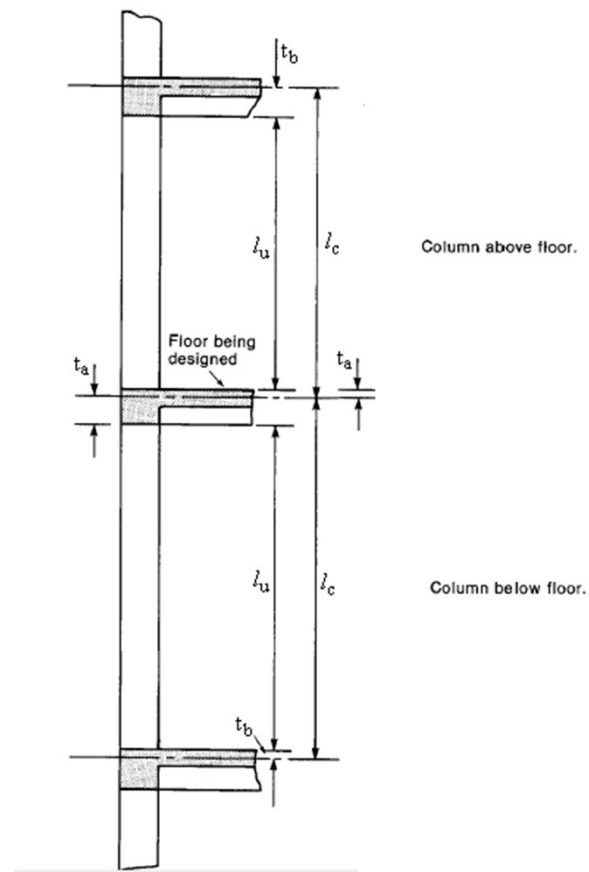


Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation





Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation

$K_c = k \frac{EL_c}{\ell_c}$

t_a/t_b	ℓ_c/ℓ_u									
	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	
2.4	k_{AB}	4.66	5.40	6.22	7.12	8.11	9.20	10.39	11.68	13.08
	C_{AB}	0.52	0.53	0.55	0.56	0.56	0.57	0.57	0.58	0.58
2.6	k_{AB}	4.67	5.42	6.26	7.18	8.20	9.31	10.53	11.86	13.29
	C_{AB}	0.52	0.53	0.54	0.55	0.56	0.56	0.56	0.57	0.57
2.8	k_{AB}	4.68	5.44	6.29	7.23	8.27	9.41	10.66	12.01	13.48
	C_{AB}	0.52	0.53	0.54	0.55	0.55	0.55	0.56	0.56	0.56
3.0	k_{AB}	4.69	5.46	6.33	7.28	8.34	9.50	10.77	12.15	13.65
	C_{AB}	0.52	0.53	0.54	0.54	0.55	0.55	0.55	0.55	0.55
3.5	k_{AB}	4.71	5.50	6.40	7.39	8.48	9.69	11.01	12.46	14.02
	C_{AB}	0.51	0.52	0.53	0.53	0.54	0.54	0.54	0.53	0.53
4.0	k_{AB}	4.72	5.54	6.45	7.47	8.60	9.84	11.21	12.70	14.32
	C_{AB}	0.51	0.52	0.52	0.53	0.53	0.52	0.52	0.52	0.52
4.5	k_{AB}	4.73	5.56	6.50	7.54	8.69	9.97	11.37	12.89	14.57
	C_{AB}	0.51	0.52	0.52	0.52	0.52	0.52	0.51	0.51	0.51
5.0	k_{AB}	4.75	5.59	6.54	7.60	8.78	10.07	11.50	13.07	14.77
	C_{AB}	0.51	0.51	0.52	0.52	0.51	0.51	0.51	0.50	0.49
6.0	k_{AB}	4.76	5.63	6.60	7.69	8.90	10.24	11.72	13.33	15.10
	C_{AB}	0.51	0.51	0.51	0.51	0.50	0.50	0.49	0.49	0.48
7.0	k_{AB}	4.78	5.66	6.65	7.76	9.00	10.37	11.88	13.54	15.34
	C_{AB}	0.51	0.51	0.51	0.50	0.50	0.49	0.48	0.48	0.47
8.0	k_{AB}	4.78	5.68	6.69	7.82	9.07	10.47	12.01	13.70	15.54
	C_{AB}	0.51	0.51	0.50	0.50	0.49	0.49	0.48	0.47	0.46
9.0	k_{AB}	4.80	5.71	6.74	7.89	9.18	10.61	12.19	13.93	15.83
	C_{AB}	0.50	0.50	0.50	0.49	0.48	0.48	0.47	0.46	0.45



Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t}$$

Calculation of $\sum K_c$ for Column C2							
Column location	l_c	l_u	l_c/l_u	I_c (in ⁴) 14" × 14" column	t_a/t_b	k_{AB}	$K_c = k(EI_c/l_c)$
C2 (bottom)	10.5' (126")	106"	1.20	$14 \times 14^3/12 = 3201$	$16.5/3.5 = 4.71$	7.57	$192E_{cc}$
C2 (top)	10.5' (126")	106"	1.20	$14 \times 14^3/12 = 3201$	$3.5/16.5 = 0.21$	5.3	$135E_{cc}$
$\sum K_c = 192E_{cc} + 135E_{cc} = 327E_{cc}$							



Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t}$$

Calculation of $\sum K_c$ for Column C1							
Column location	l_c	l_u	l_c/l_u	I_c (in ⁴) 14" × 14" column	t_a/t_b	k_{AB}	$K_c = k(EI_c/l_c)$
C1 (bottom)	10.5' (126")	106"	126/106 ≈ 1.20	14 × 14 ³ /12 = 3201	16.5/3.5 = 4.71	7.57	192E _{cc}
C1 (top)	10.5' (126")	106"	126/106 ≈ 1.20	14 × 14 ³ /12 = 3201	3.5/16.5 = 0.21	5.3	135E _{cc}
$\sum K_c = 192E_{cc} + 135E_{cc} = 327E_{cc}$							



Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation

For column C2 (exterior column), we have

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t} = \frac{1}{327E_{cc}} + \frac{1}{3792.63E_{cs}}$$

Because the slab and the columns have the same strength concrete, $E_{cc} = E_{cs} = E_c$. Therefore,

$$K_{ec} = 301E_c$$

Similarly, for column C1 (interior column), we get

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t} = \frac{1}{327E_{cc}} + \frac{1}{4295.98E_{cs}}; \quad K_{ec} = 303E_c$$

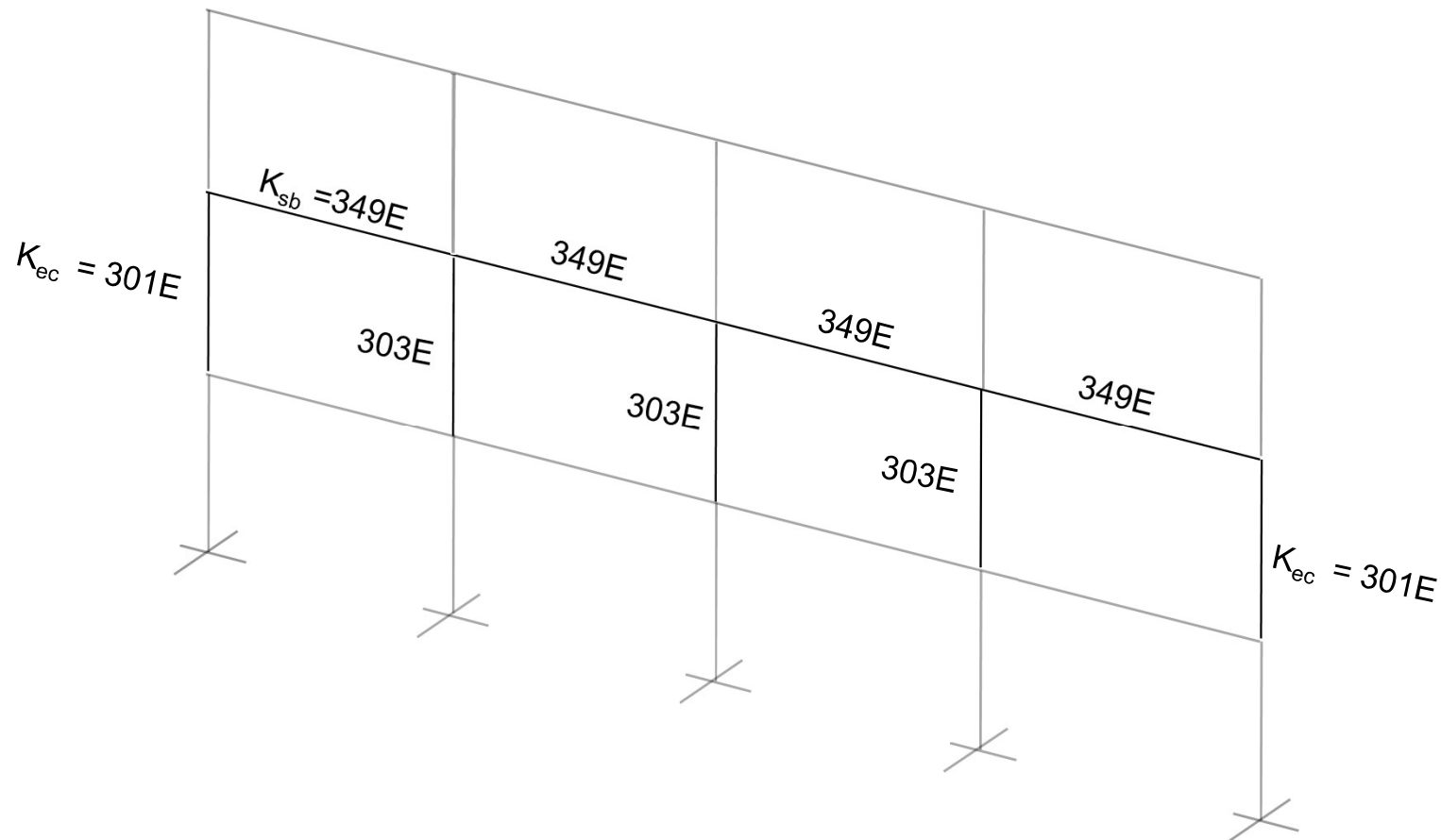


Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation





Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation

- As the ground story is same as 1st one, therefore the stiffness calculated shall also be assigned to ground story.
- For the top story, the slab beam stiffness will be same as lower stories. However, the equivalent stiffness of the top story column is computed next.



Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation

Calculation of $\sum K_c$ for Column C2 (Top Story)							
Column location	I_c	I_u	I_c / I_u	I_c (in ⁴) 14" × 14" column	t_a / t_b	k_{AB}	$K_c = k(EI_c / I_c)$
C2 (bottom)	10.5' (126")	100"	126/106 ≈ 1.20	$14 \times 14^3 / 12 = 3201$	16.5/3.5 = 4.71	7.57	192E _{cc}
$\sum K_c = 192E_{cc}$ Similarly for interior column, $\sum K_c = 192E_{cc}$							



Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffnesses

ii. Equivalent Column Stiffness Calculation

For column C2, we have

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t} = \frac{1}{192E_{cc}} + \frac{1}{3792.63E_{cs}}$$

Because the slab and the columns have the same strength concrete, $E_{cc} = E_{cs} = E_c$. Therefore,

$$K_{ec} = 182E_c$$

Similarly, for column C1, we get

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t} = \frac{1}{92E_{cc}} + \frac{1}{4295.98E_{cs}}; \quad K_{ec} = 183E_c$$

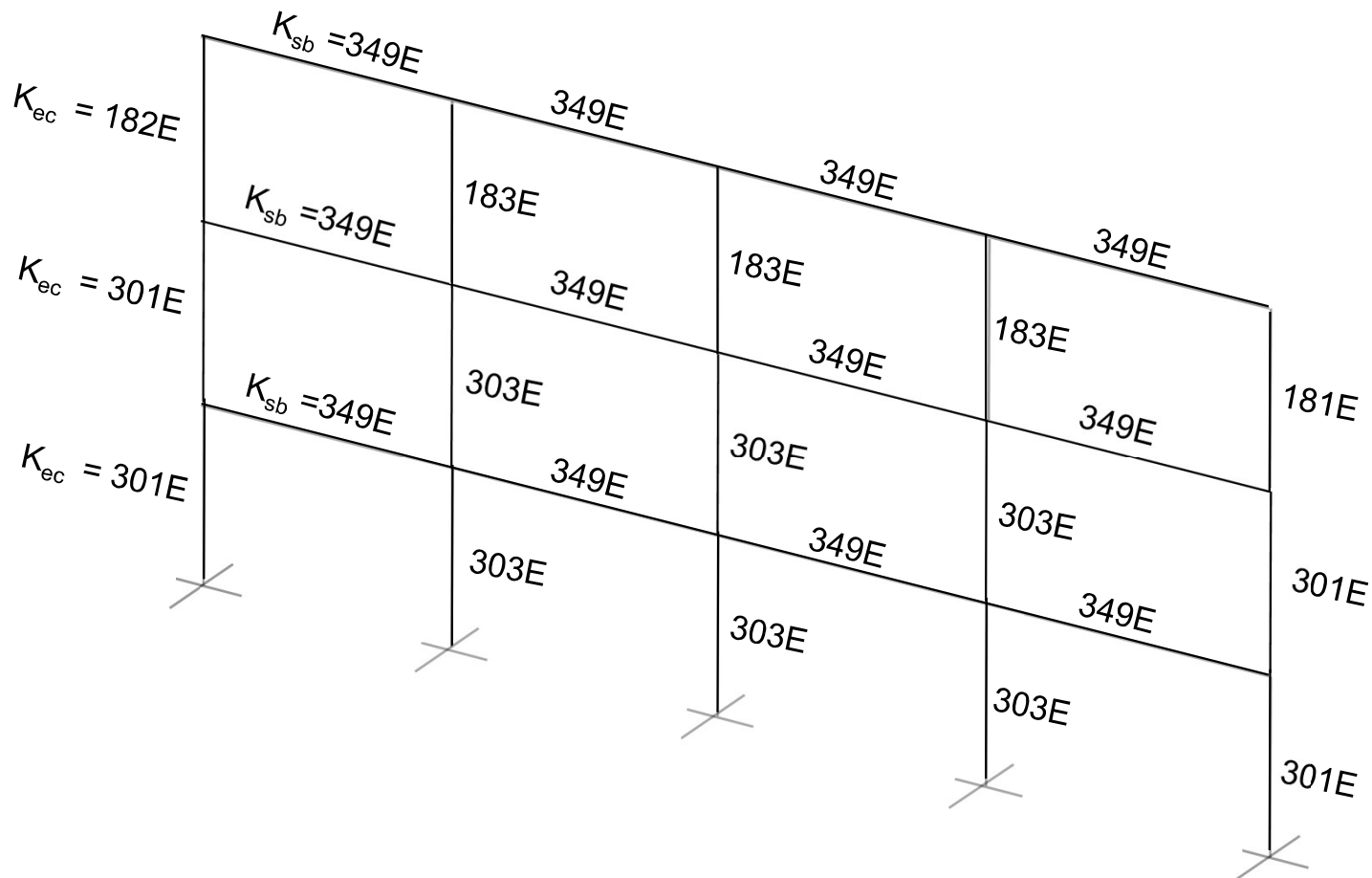


Example 8.1

□ Solution

➤ Step 4: Calculation of Stiffness

❖ Equivalent Frame

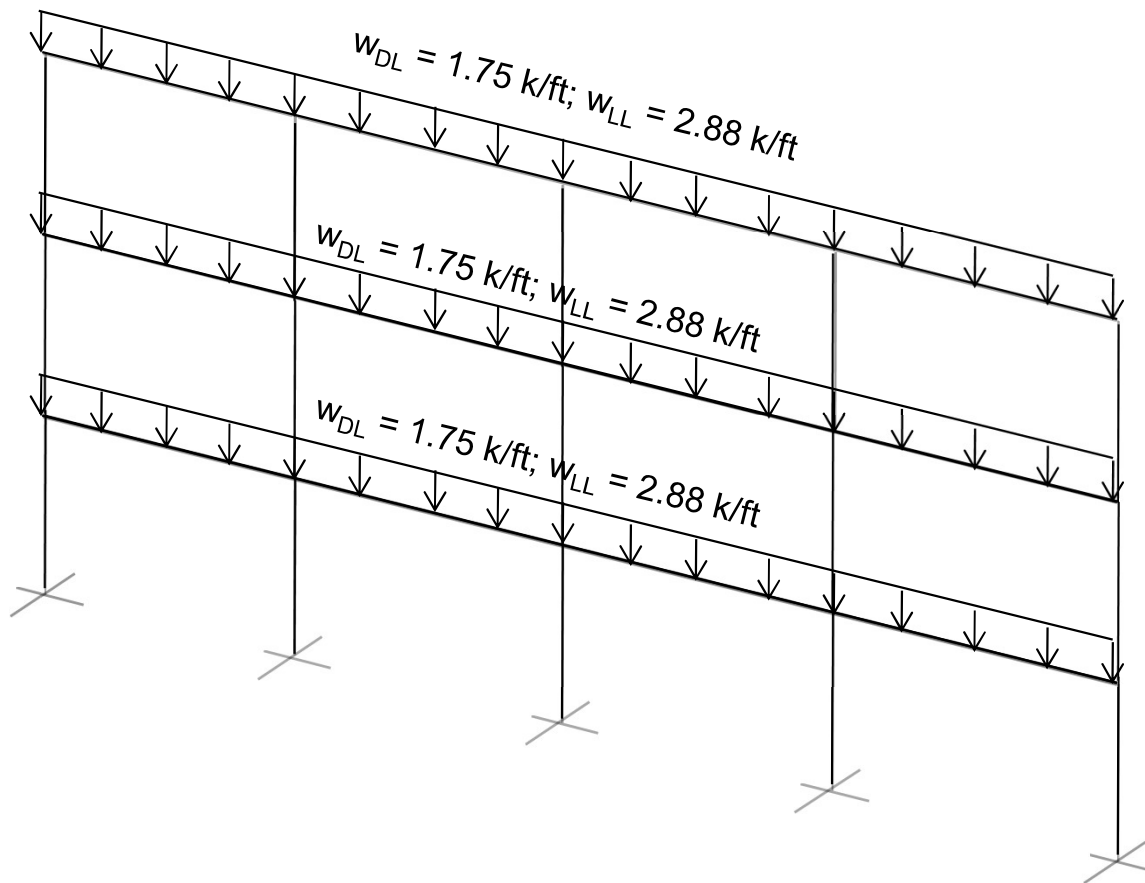




Example 8.1

□ Solution

➤ Step 5: Frame Analysis Using Moment Distribution Method



Load on frame for Bending Analysis:

As the horizontal frame element represents slab beam, load is computed by multiplying slab load with width of frame

$$w_{DL} = 0.0875 \times 20 = 1.75 \text{ kip/ft}$$

$$w_{LL} = 0.144 \times 20 = 2.88 \text{ kip/ft}$$



Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Dead Loads (interior story).**

Joint	A		B		C		D		E						
CarryOver	0.5034		0.5034		0.5034		0.5034		0.5034						
DF	0.000	0.463	0.537	0.348	0.303	0.348	0.348	0.303	0.348	0.348	0.303	0.348	0.537	0.463	0.000
	Slab	Column	Slab	Slab	Column	Slab	Slab	Column	Slab	Slab	Column	Slab	Slab	Column	Slab
FEM	0.000	0.000	91.802	-91.802	0.000	91.802	-91.802	0.000	91.802	-91.802	0.000	91.802	-91.802	0.000	0.000
Bal	0.000	-42.532	-49.270	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	49.270	42.532	0.000
Carry over			0.000	-24.804		0.000	0.000		0.000	0.000		24.804	0.000		
Bal	0.000	0.000	0.000	8.638	7.527	8.638	0.000	0.000	0.000	-8.638	-7.527	-8.638	0.000	0.000	0.000
Carry over			4.349	0.000		0.000	4.349		-4.349	0.000		0.000	-4.349		
Bal	0.000	-2.015	-2.334	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.334	2.015	0.000
Carry over			0.000	-1.175		0.000	0.000		0.000	0.000		1.175	0.000		
Bal	0.000	0.000	0.000	0.409	0.357	0.409	0.000	0.000	0.000	-0.409	-0.357	-0.409	0.000	0.000	0.000
Carry over			0.206	0.000		0.000	0.206		-0.206	0.000		0.000	-0.206		
Bal	0.000	-0.095	-0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.111	0.095	0.000
Carry over			0.000	-0.056		0.000	0.000		0.000	0.000		0.056	0.000		
Bal	0.000	0.000	0.000	0.019	0.017	0.019	0.000	0.000	0.000	-0.019	-0.017	-0.019	0.000	0.000	0.000
Carry over			0.010	0.000		0.000	0.010		-0.010	0.000		0.000	-0.010		
Bal	0.000	-0.005	-0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.005	0.000
Carry over			0.000	-0.003		0.000	0.000		0.000	0.000		0.003	0.000		
Bal	0.000	0.000	0.000	0.001	0.001	0.001	0.000	0.000	0.000	-0.001	-0.001	-0.001	0.000	0.000	0.000
Carry over			0.000	0.000		0.000	0.000		0.000	0.000		0.000	0.000		
Bal	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Carry over			0.000	0.000		0.000	0.000		0.000	0.000		0.000	0.000		
Bal	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Carry over			0.000	0.000		0.000	0.000		0.000	0.000		0.000	0.000		
Bal	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Carry over			0.000	0.000		0.000	0.000		0.000	0.000		0.000	0.000		
Total	0.000	-44.647	44.647	-108.771	7.901	100.870	-87.237	0.000	87.237	-100.870	-7.901	108.771	-44.647	44.647	0.000



Example 8.1

□ Solution

- Step 5: Frame Analysis Using Moment Distribution Method
 - ❖ Analysis Results for Dead Loads (top storey).

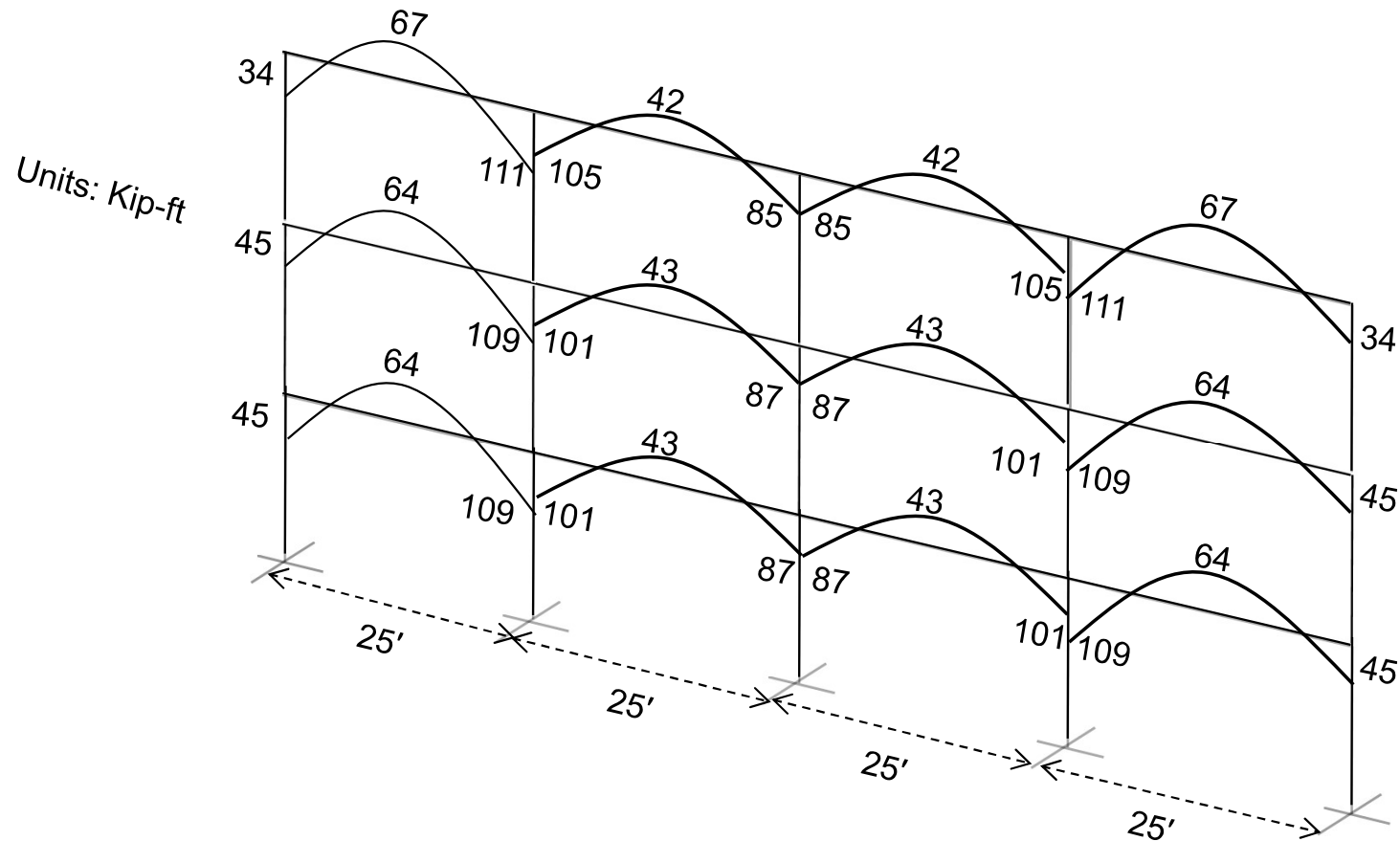
Joint	A			B			C			D			E		
CarryOver	0.5034			0.5034			0.5034			0.5034			0.5034		
DF	0.000	0.344	0.656	0.396	0.209	0.396	0.396	0.209	0.396	0.396	0.209	0.396	0.656	0.344	0.000
	Slab	Column	Slab	Slab	Column	Slab	Slab	Column	Slab	Slab	Column	Slab	Slab	Column	Slab
FEM	0.000	0.000	91.802	-91.802	0.000	91.802	-91.802	0.000	91.802	-91.802	0.000	91.802	-91.802	0.000	0.000
Bal	0.000	-31.611	-60.191	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	60.191	31.611	0.000
Carry over			0.000	-30.301		0.000	0.000		0.000	0.000		30.301	0.000		
Bal	0.000	0.000	0.000	11.985	6.330	11.985	0.000	0.000	0.000	-11.985	-6.330	-11.985	0.000	0.000	0.000
Carry over			6.034	0.000		0.000	6.034		-6.034	0.000		0.000	-6.034		
Bal	0.000	-2.078	-3.956	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3.956	2.078	0.000
Carry over			0.000	-1.992		0.000	0.000		0.000	0.000		1.992	0.000		
Bal	0.000	0.000	0.000	0.788	0.416	0.788	0.000	0.000	0.000	-0.788	-0.416	-0.788	0.000	0.000	0.000
Carry over			0.397	0.000		0.000	0.397		-0.397	0.000		0.000	-0.397		
Bal	0.000	-0.137	-0.260	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.260	0.137	0.000
Carry over			0.000	-0.131		0.000	0.000		0.000	0.000		0.131	0.000		
Bal	0.000	0.000	0.000	0.052	0.027	0.052	0.000	0.000	0.000	-0.052	-0.027	-0.052	0.000	0.000	0.000
Carry over			0.026	0.000		0.000	0.026		-0.026	0.000		0.000	-0.026		
Bal	0.000	-0.009	-0.017	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.017	0.009	0.000
Carry over			0.000	-0.009		0.000	0.000		0.000	0.000		0.009	0.000		
Bal	0.000	0.000	0.000	0.003	0.002	0.003	0.000	0.000	0.000	-0.003	-0.002	-0.003	0.000	0.000	0.000
Carry over			0.002	0.000		0.000	0.002		-0.002	0.000		0.000	-0.002		
Bal	0.000	-0.001	-0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000
Carry over			0.000	-0.001		0.000	0.000		0.000	0.000		0.001	0.000		
Bal	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Carry over			0.000	0.000		0.000	0.000		0.000	0.000		0.000	0.000		
Bal	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Total	0.000	-33.835	33.835	-111.406	6.776	104.631	-85.344	0.000	85.344	-104.631	-6.776	111.406	-33.835	33.835	0.000



Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Dead Loads (values at centerline).**





Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Dead Loads**

E-W Interior Frame Analysis (Top Story)						
Length	Longitudinal moment section	Longitudinal moments (LM)	Column Strip Moment %age factor (Graph A4)	Column Strip slab Moment (CSSM) = 0.15CSM	Column strip Beam Moment (BM)= 0.85CSM	Middle Strip slab Moment
25'-0" (Exterior)	Ext -	34	0.93	5	27	2.38
	+	67	0.8	8	46	13.4
	Int -	111	0.8	13	75	22.2
25'-0" (Interior)	-	105	0.8	13	71	21
	+	42	0.8	5	29	8.4
	-	85	0.8	10	58	17



Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Dead Loads**

E-W Interior Frame Analysis (Intermediate Story)						
Length	Longitudinal moment section	Longitudinal moments (LM)	Column Strip Moment %age factor (Graph A4)	Column Strip slab Moment (CSSM) = 0.15CSM	Column strip Beam Moment (BM)= 0.85CSM	Middle Strip slab Moment
25'-0" (Exterior)	Ext -	45	0.93	6	36	3.15
	+	64	0.8	8	44	12.8
	Int-	109	0.8	13	74	21.8
25'-0" (Interior)	-	101	0.8	12	69	20.2
	+	43	0.8	5	29	8.6
	-	87	0.8	10	59	17.4



Example 8.1

□ Solution

➤ Step 5: Frame Analysis Using Moment Distribution Method

❖ Analysis Results for Dead Loads

- Analysis of columns for DL (factors for moment distribution)

The computed unbalanced longitudinal moments shall be transferred to columns and shall be distributed to top and bottom columns as follows:

$$DF_{uc} = \frac{K_{ct}}{K_{cb} + K_{ct}} \quad (\text{Portion of unbalanced moment to upper column})$$

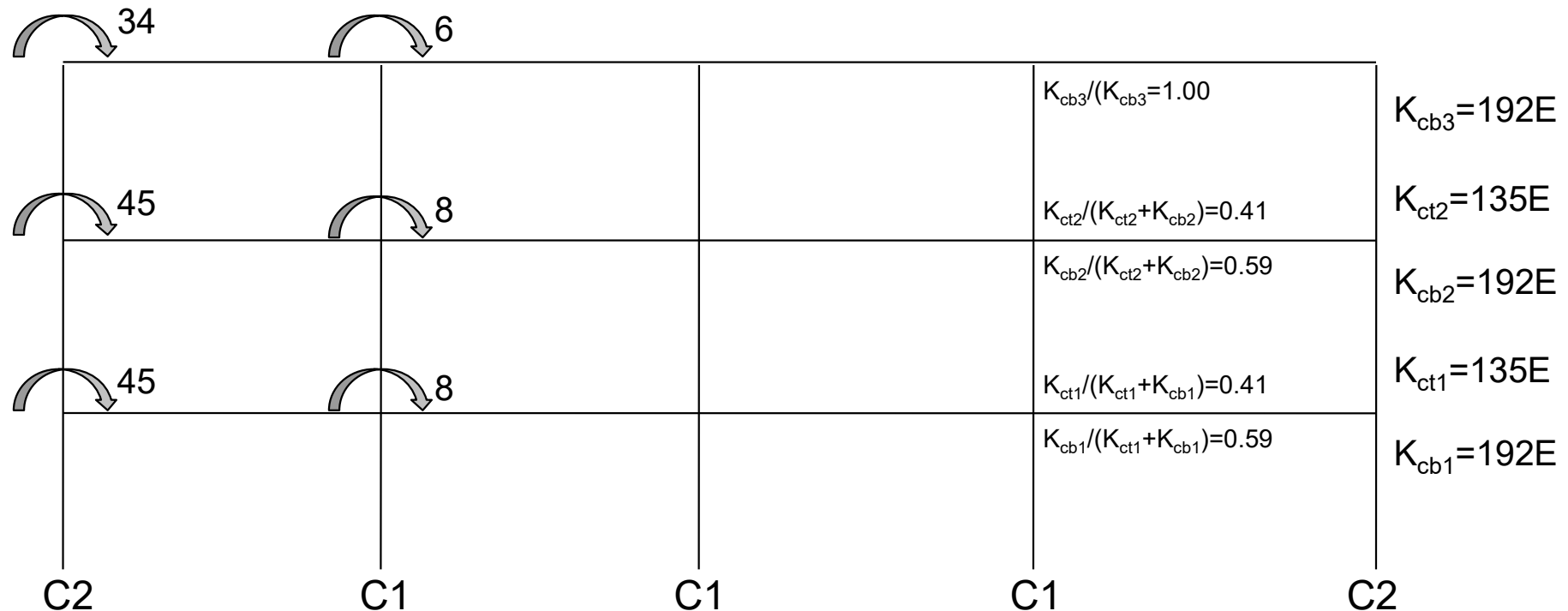
$$DF_{uc} = \frac{K_{cb}}{K_{cb} + K_{ct}} \quad (\text{Portion of unbalanced moment to lower column})$$



Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Dead Loads**
 - Analysis of columns for DL (factors for moment distribution)

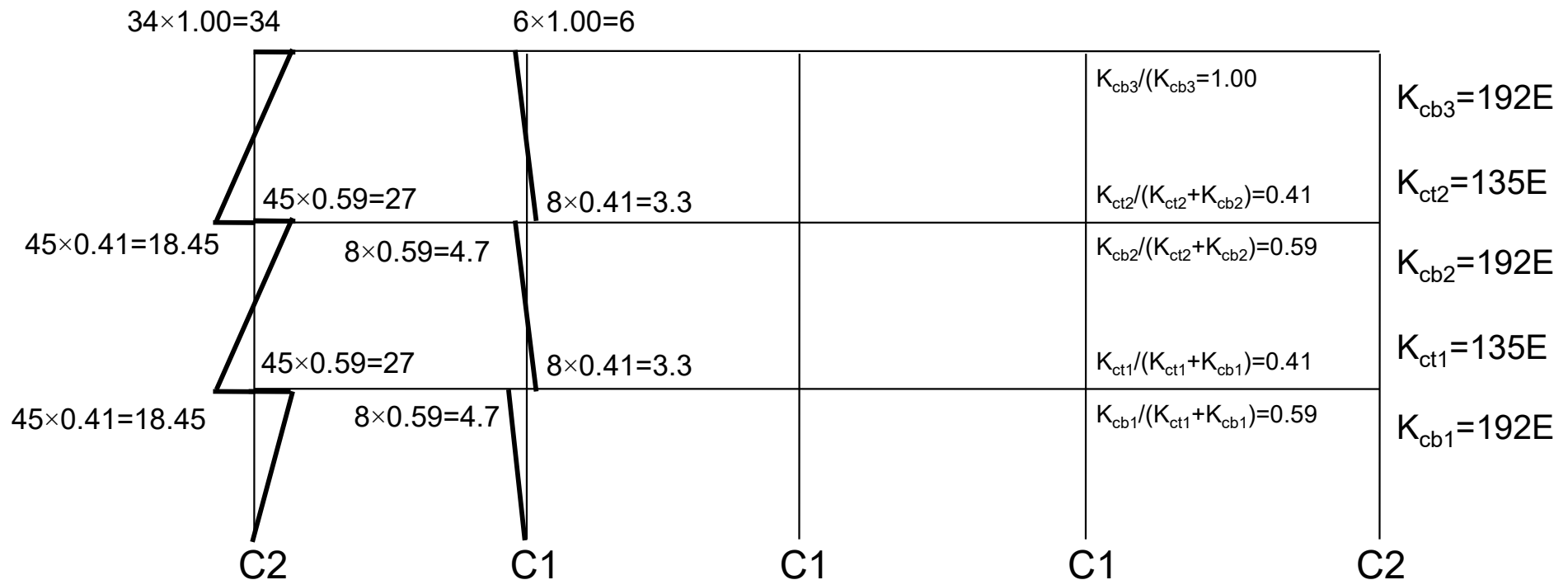




Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Dead Loads**
 - Analysis of columns for DL



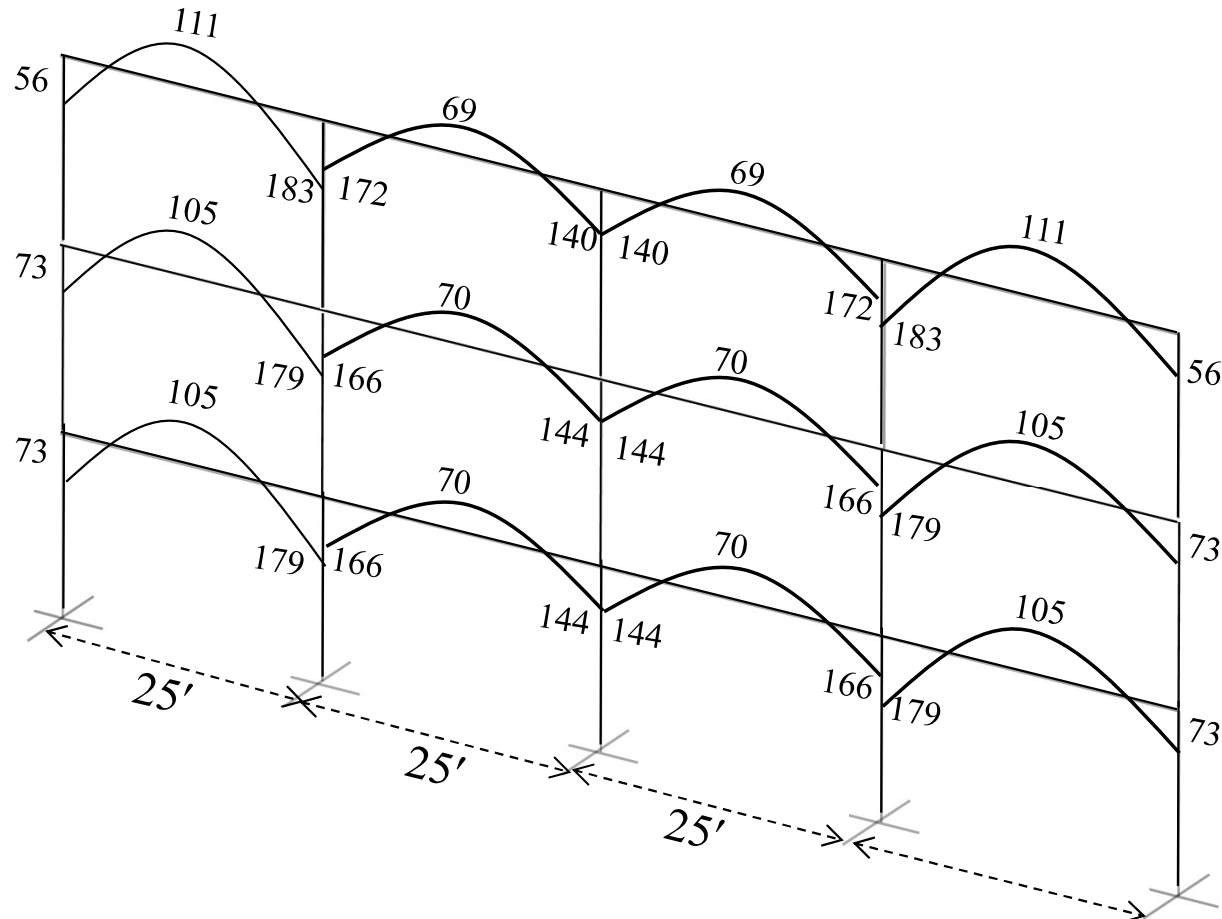


Example 8.1

□ Solution

➤ Step 5: Frame Analysis Using Moment Distribution Method

❖ Analysis Results for Live Loads





Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Live Loads**
 - Distribution of Moments to Slab and Beam

E-W Interior Frame Analysis (Top Story)						
Length	Longitudinal moment section	Longitudinal moments (LM)	Column Strip Moment %age factor (Graph A4)	Column Strip slab Moment (CSSM) = 0.15CSM	Column strip Beam Moment (BM)= 0.85CSM	Middle Strip slab Moment
25'-0" (Exterior)	Ext -	56	0.93	7.8	44	3.92
	+	111	0.8	13.3	75	22.2
	Int-	183	0.8	22.0	124	36.6
25'-0" (Interior)	-	172	0.8	20.6	117	34.4
	+	69	0.8	8.3	47	13.8
	-	140	0.8	16.8	95	28



Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Live Loads**
 - Distribution of Moments to Slab and Beam

E-W Interior Frame Analysis (Interior Story)						
Length	Longitudinal moment section	Longitudinal moments (LM)	Column Strip Moment %age factor (Graph A4)	Column Strip slab Moment (CSSM) = 0.15CSM	Column strip Beam Moment (BM)= 0.85CSM	Middle Strip slab Moment
25'-0" (Exterior)	Ext -	73	0.93	10.2	58	5.11
	+	105	0.8	12.6	71	21
	Int -	179	0.8	21.5	122	35.8
25'-0" (Interior)	-	166	0.8	19.9	113	33.2
	+	70	0.8	8.4	48	14
	-	144	0.8	17.3	98	28.8



Example 8.1

□ Solution

➤ Step 5: Frame Analysis Using Moment Distribution Method

❖ Analysis Results for Live Loads

- Analysis of columns for LL (factors for moment distribution)

The computed unbalanced longitudinal moments shall be transferred to columns and shall be distributed to top and bottom columns as follows:

$$DF_{uc} = \frac{K_{ct}}{K_{cb} + K_{ct}} \quad (\text{Portion of unbalanced moment to upper column})$$

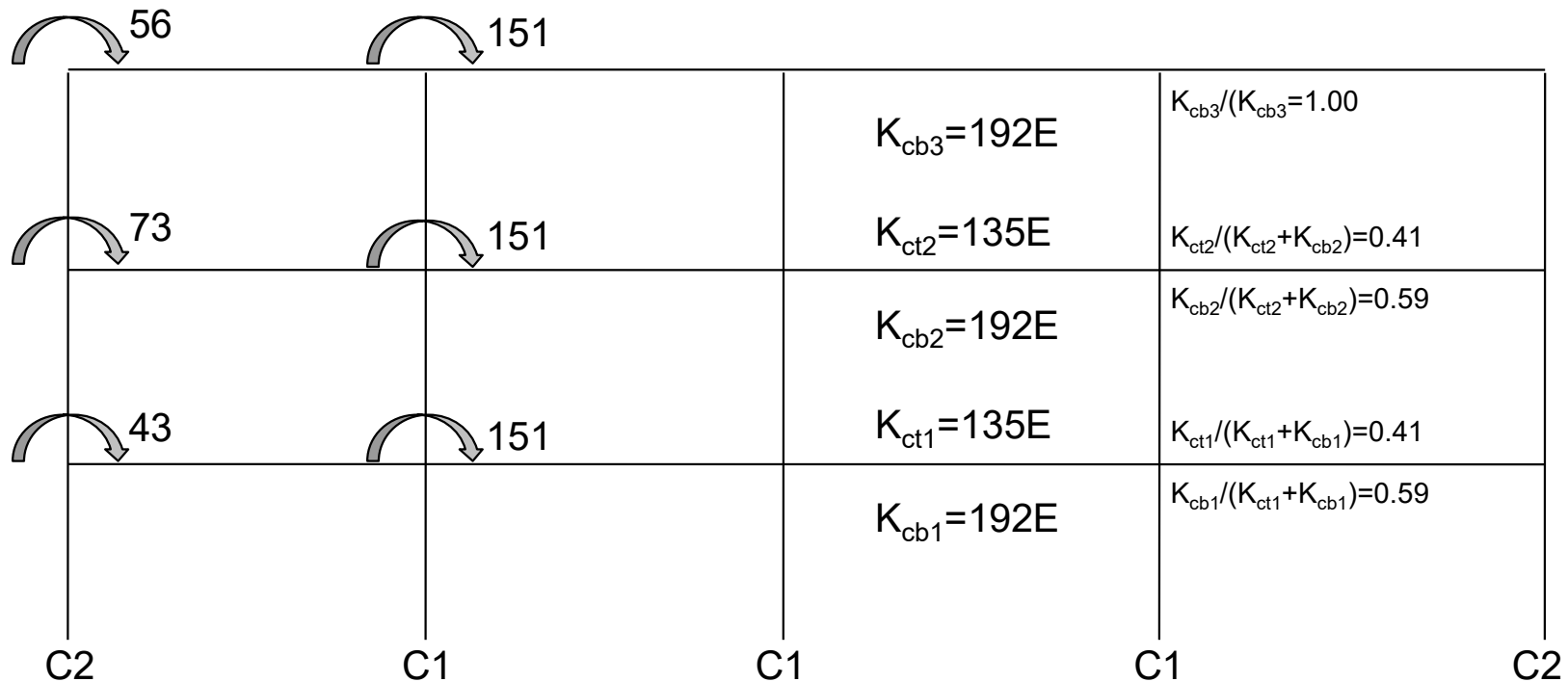
$$DF_{uc} = \frac{K_{cb}}{K_{cb} + K_{ct}} \quad (\text{Portion of unbalanced moment to lower column})$$



Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Live Loads**
 - Analysis of columns for LL (factors for moment distribution)

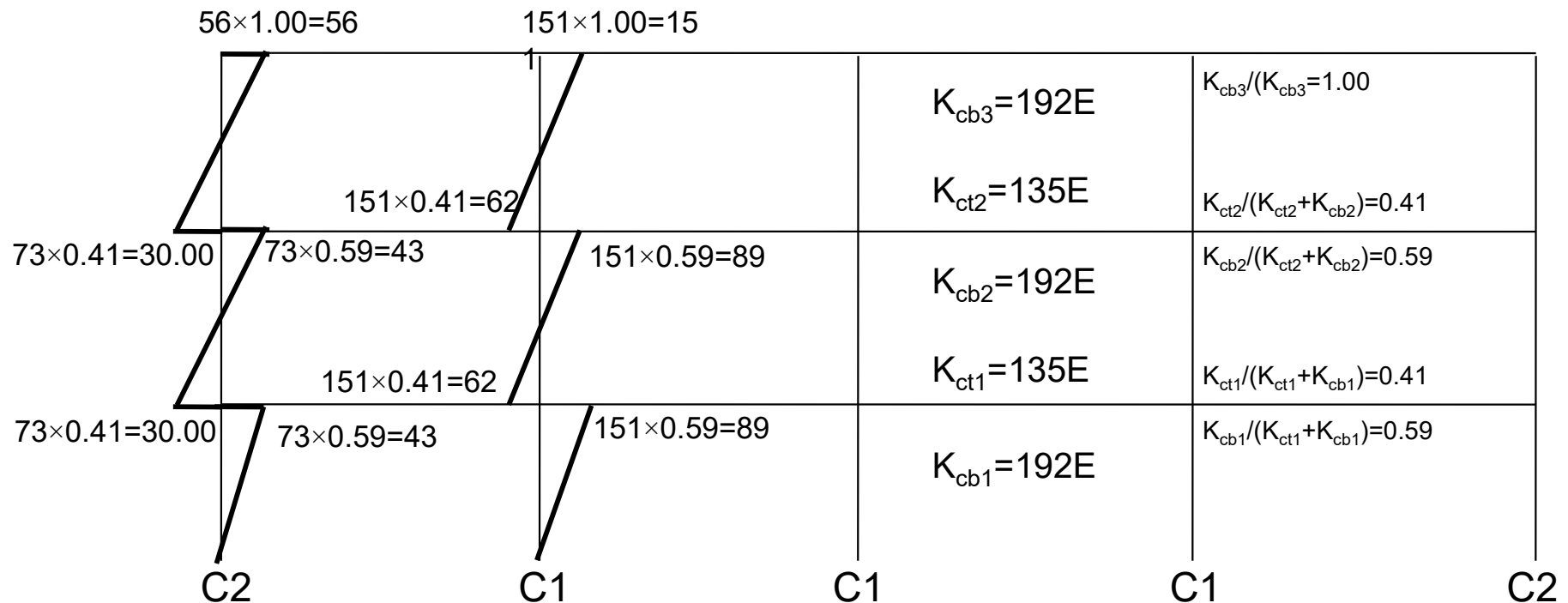




Example 8.1

□ Solution

- **Step 5: Frame Analysis Using Moment Distribution Method**
 - ❖ **Analysis Results for Live Loads**
 - Analysis of columns for LL (factors for moment distribution)





Case Study

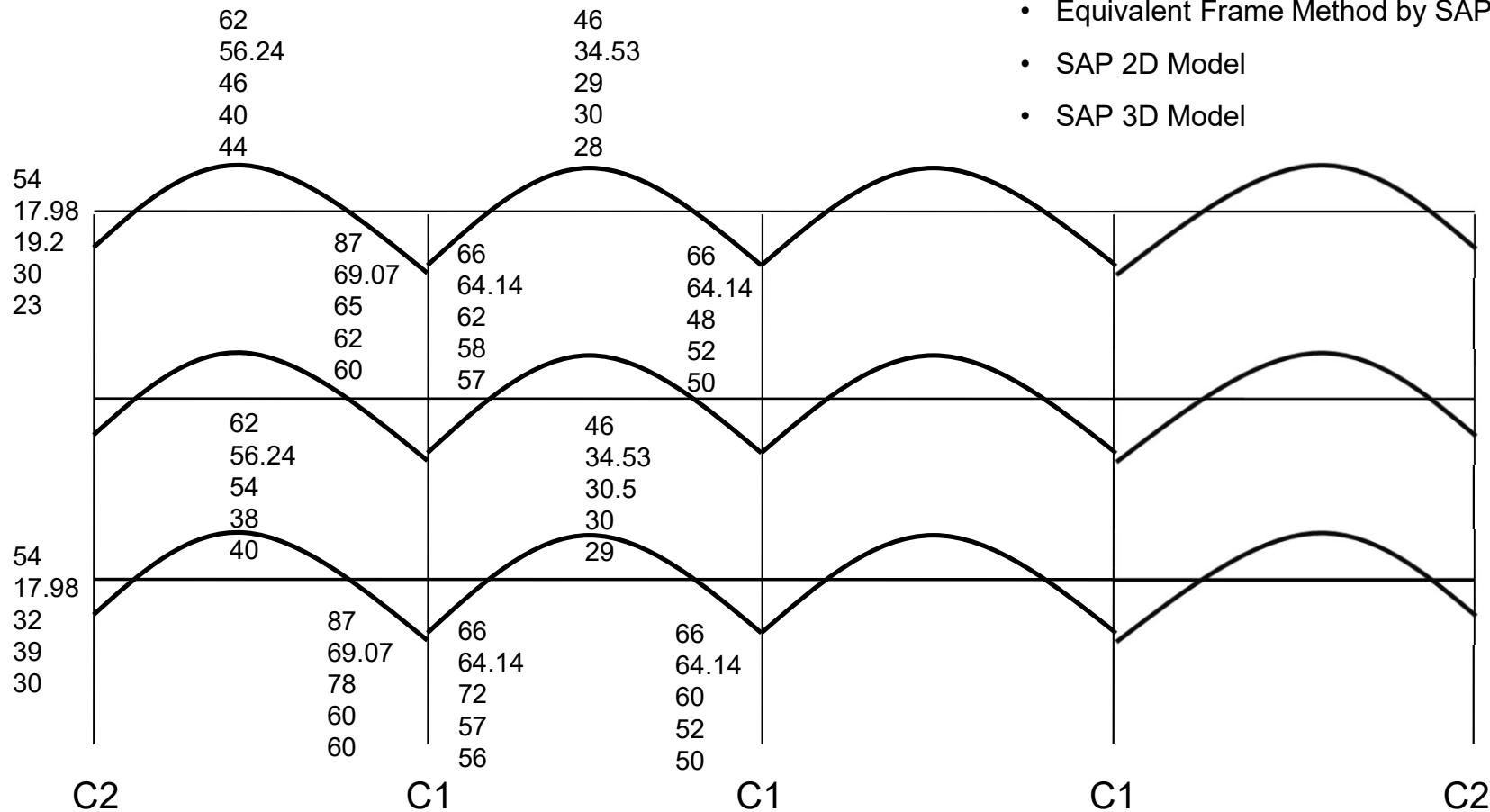
Comparison of the Results of
EFM, ACI Coefficient Method, DDM & SAP 2D Model
with respect to SAP2000 3D Line Model



Case Study

Dead Load Bending Moment in Beams

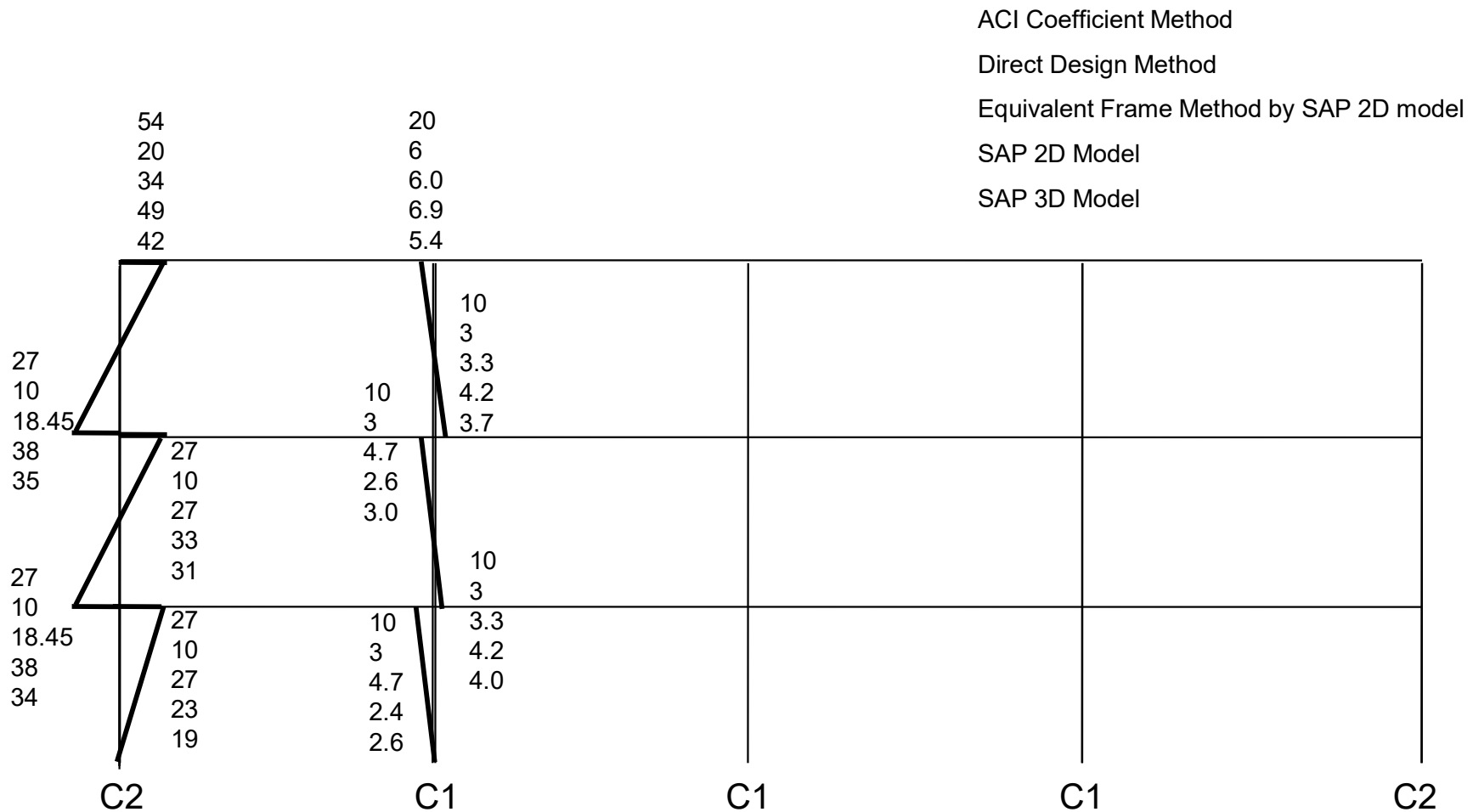
- ACI Coefficient Method
- Direct Design Method
- Equivalent Frame Method by SAP 2D model
- SAP 2D Model
- SAP 3D Model





Case Study

❑ Dead Load Bending Moment in Columns

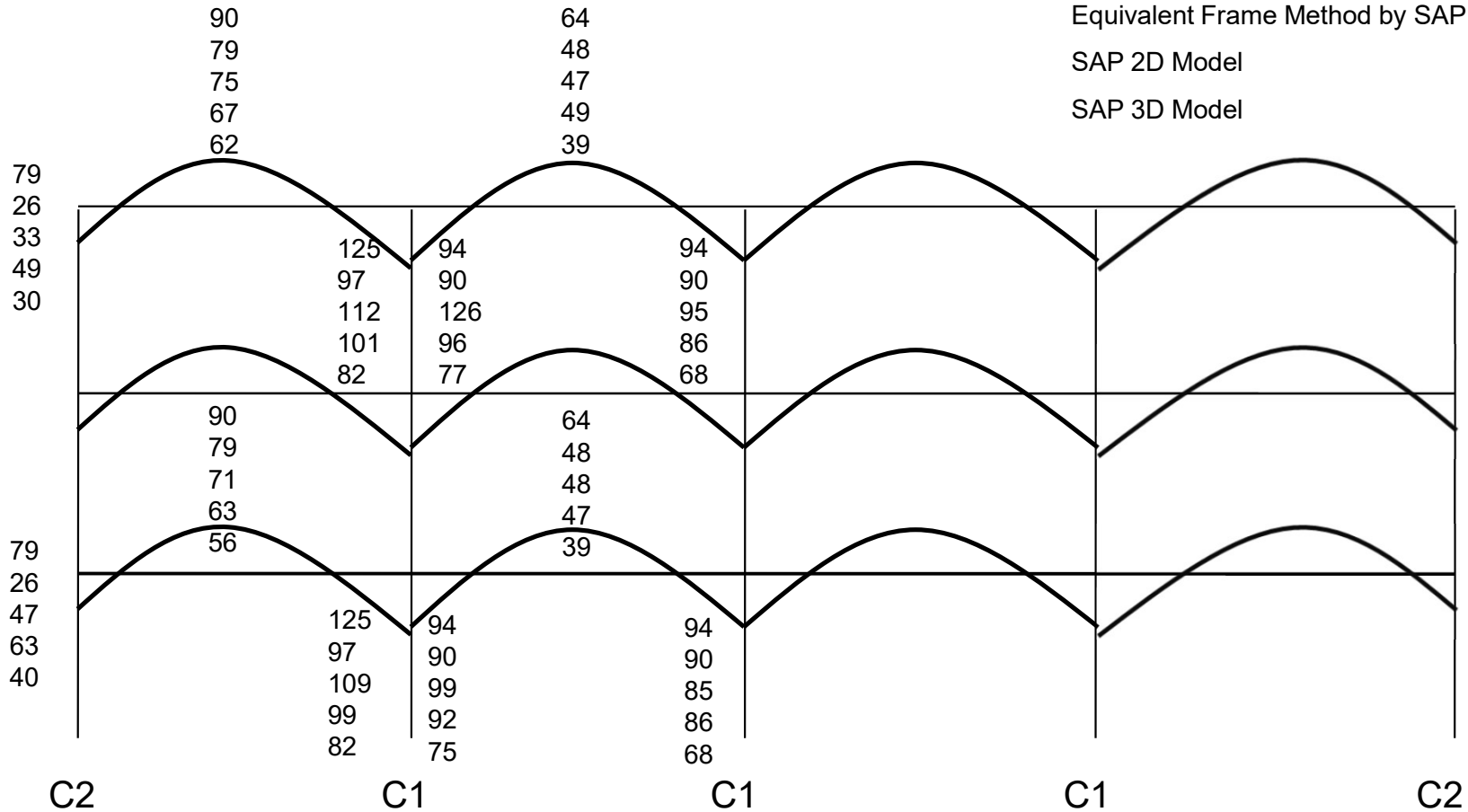




Case Study

□ Live Load Bending Moment in Beams

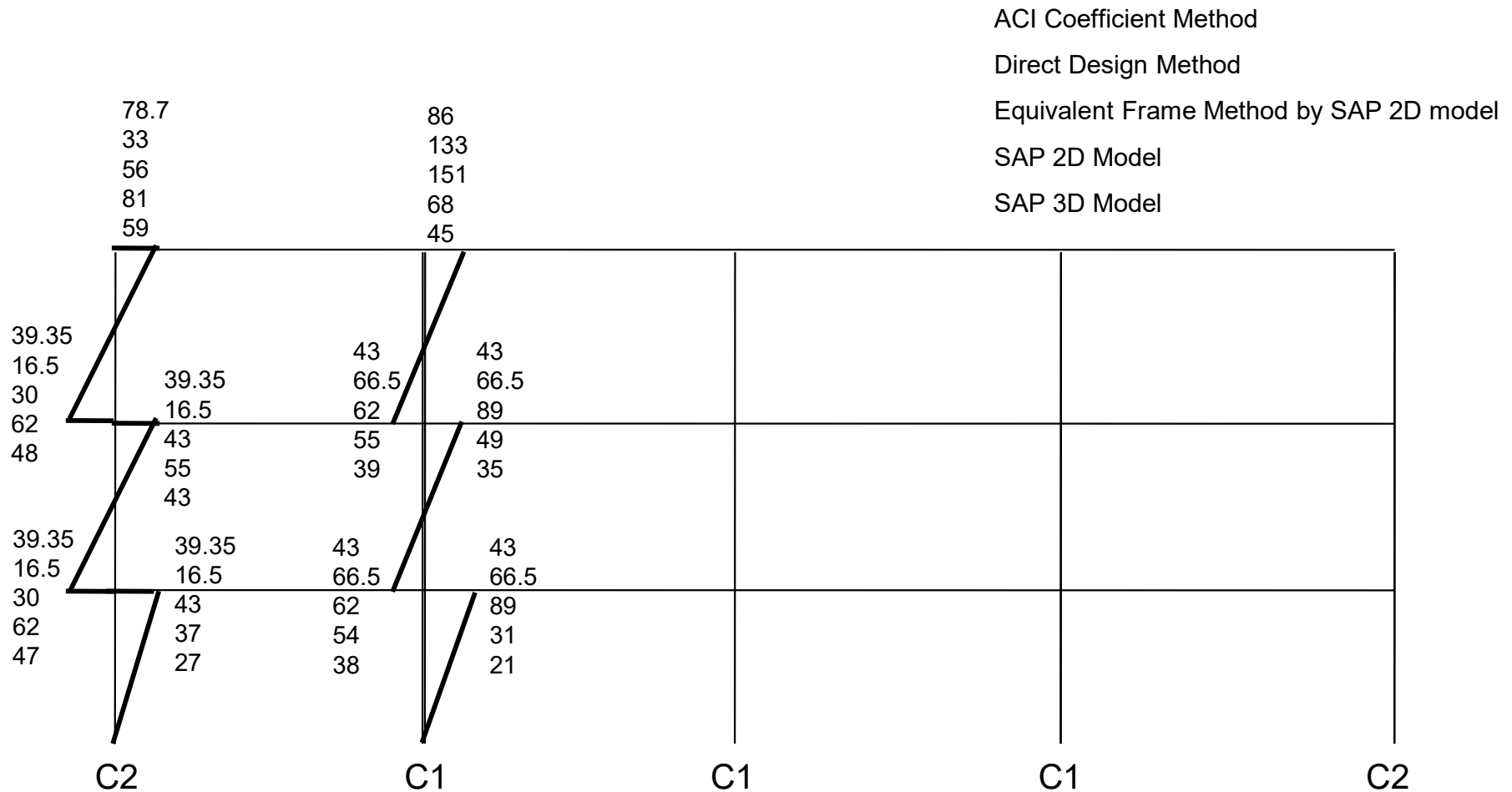
- ACI Coefficient Method
- Direct Design Method
- Equivalent Frame Method by SAP 2D model
- SAP 2D Model
- SAP 3D Model





Case Study

□ Live Load Bending Moment in Columns





Limit Analysis

□ Introduction

- Most RC structures are designed using following approach:
 - Moments, shears, and axial forces in RC structures are found by elastic theory.
 - The actual proportioning of members is done by strength methods, in which inelastic section and member response is considered.
- Although this design approach is safe and conservative but is inconsistent to total analysis-design process.



Limit Analysis

□ Redistribution

- A frame normally will not fail when the nominal moment capacity of just one critical section is reached:
 - A plastic hinge will form at that section.
 - Large rotation at constant resisting moment will occur.
 - Load transfer to other locations (having more capacity) along the span will occur.
 - On further increase in load, additional plastic hinges may form at other locations along the span.
 - As a result, structure will collapse, but only after a significant redistribution of moments.



Limit Analysis

□ Redistribution

- Full use of the plastic capacity of reinforced concrete beams and frames requires an extensive analysis of all possible mechanisms and an investigation of rotation requirements and capacities at all proposed hinge locations.
- On the other hand, a restricted amount of redistribution of elastic moments can safely be made without complete analysis yet may be sufficient to obtain most of the advantages of limit analysis.



Limit Analysis

□ Redistribution of Moments in Continuous Flexural Members

- A limited amount of redistribution is permitted by ACI Code 6.6.5. depending upon a rough measure of available ductility, without explicit calculation of rotation requirements and capacities.



Limit Analysis

□ Redistribution of Negative Moments in Continuous Flexural Members

- The net tensile strain in the extreme tension steel at nominal strength ϵ_t given in eq. below, is used as an indicator of rotation capacity.
- The ACI Code Section 6.6.5 states “except where approximate values for moments are used, it shall be permitted to increase or decrease negative moments calculated by elastic theory at supports of continuous flexural members for any assumed loading arrangement by not more than $1000\epsilon_t$ percent, with a maximum of 20 percent”.

$$\epsilon_t = \frac{\epsilon_u(d - c)}{c}$$



Limit Analysis

□ Redistribution of Negative Moments in Continuous Flexural Members

$$\epsilon_t = \epsilon_u(d - c)/c$$

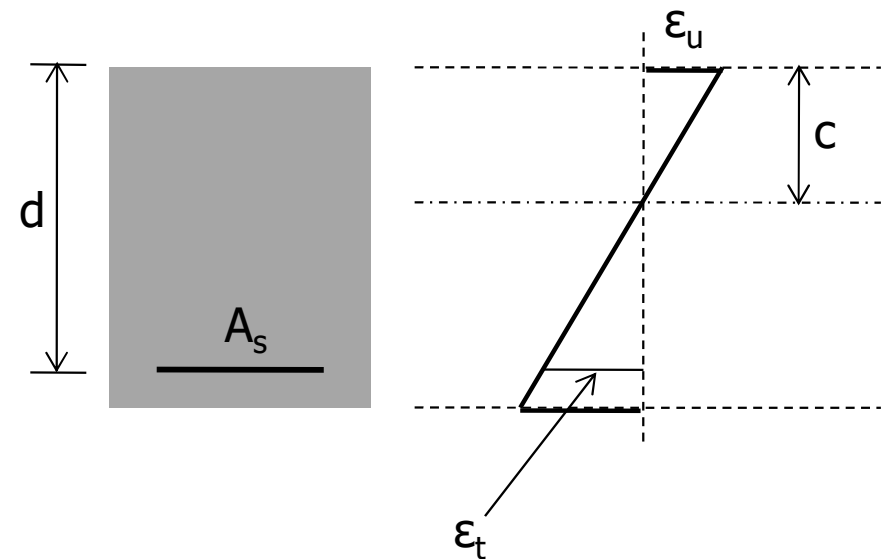
$$\epsilon_u = 0.003$$

As example, for given A_s if:

$$d = 16.5'' ; c = 4''$$

$$\epsilon_t = 0.009$$

$$1000\epsilon_t = 9 \% < 20 \%$$





Limit Analysis

□ Redistribution of Negative Moments in Continuous Flexural Members

- Graphical representation of ACI code provision

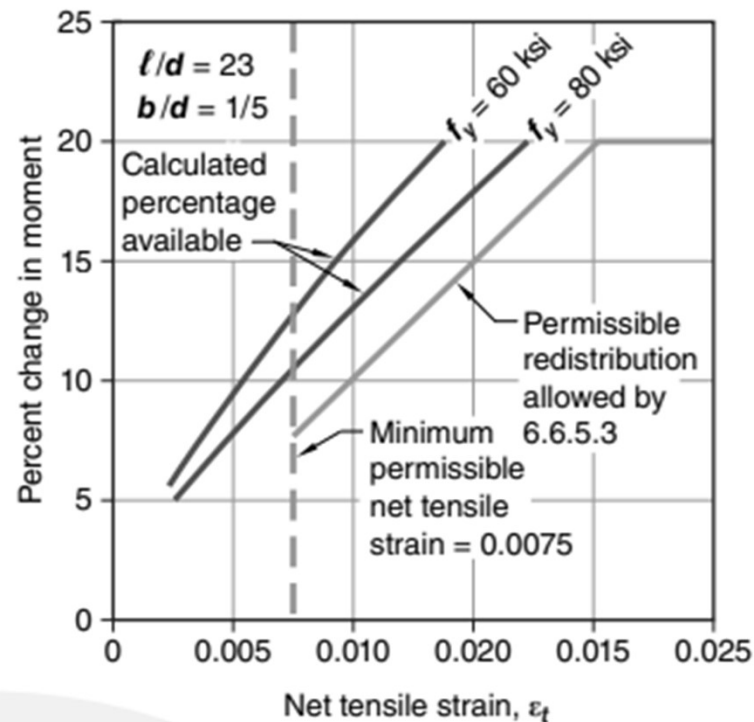


Fig. R6.6.5—Permissible redistribution of moments for minimum rotation capacity.



Limit Analysis

□ Redistribution of Negative Moments in Continuous Flexural Members

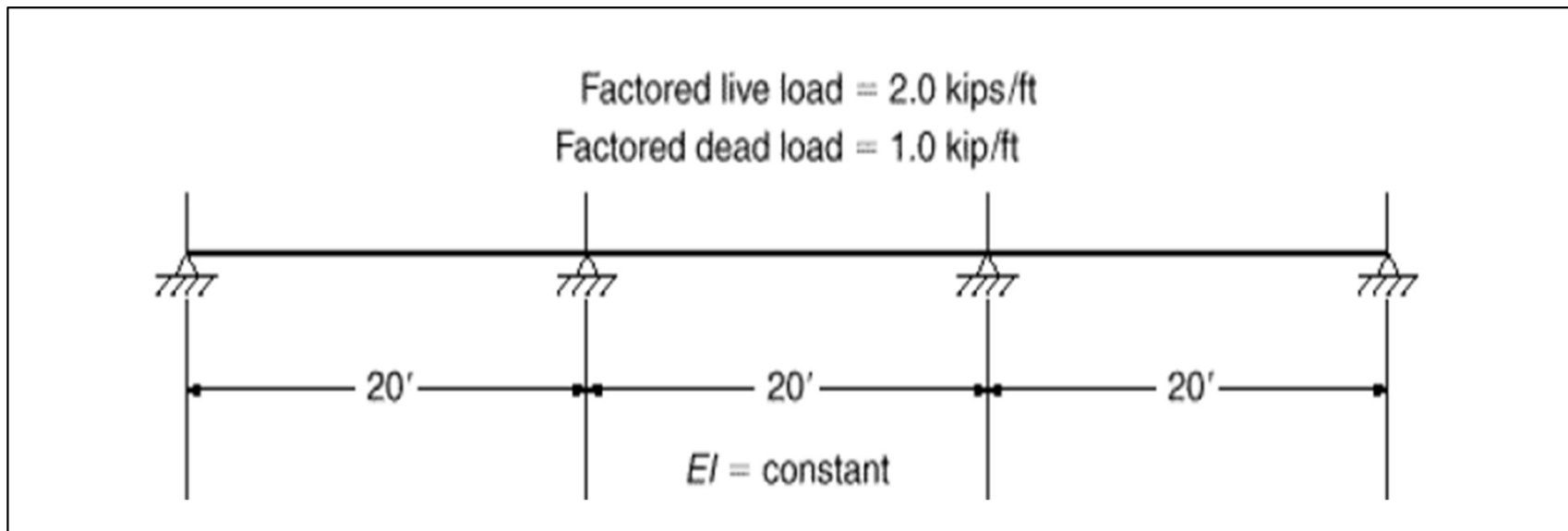
- The modified negative moments shall be used for calculating moments at sections within the spans.
- Redistribution of negative moments shall be made only when ϵ_t is equal to or greater than 0.0075 at the section at which moment is reduced (ACI 6.6.5.1).



Example 8.2

□ Problem Statement

- For the beam shown, find moment redistribution.

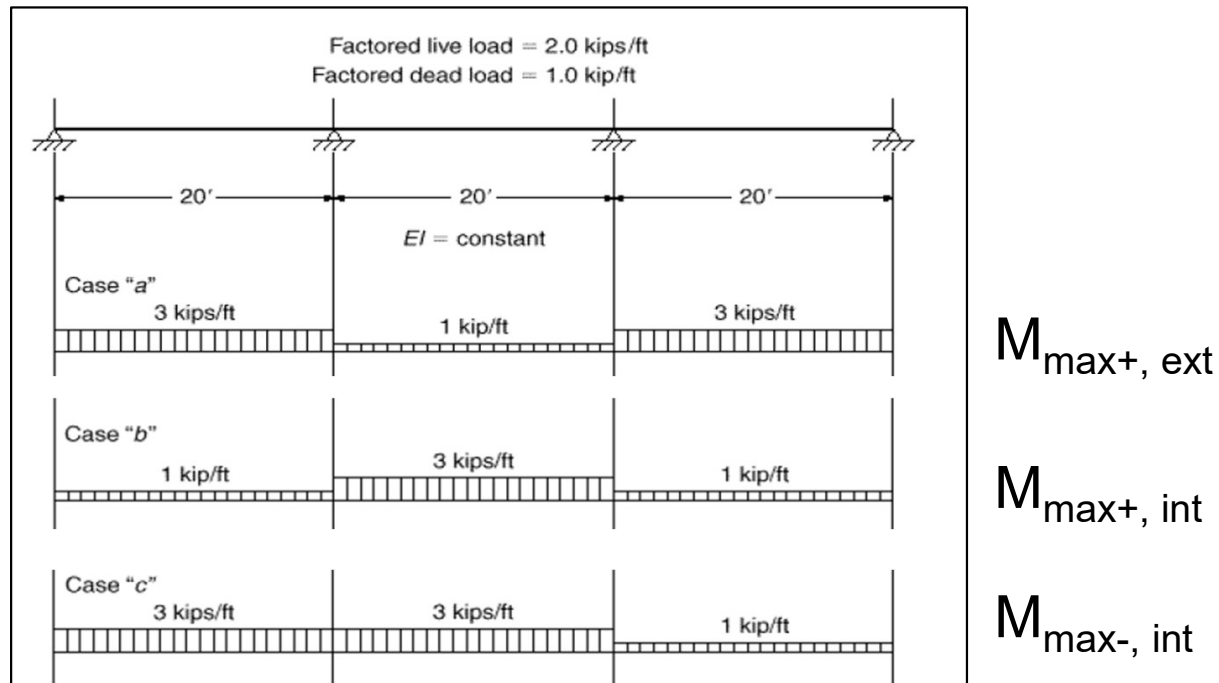




Example 8.2

□ Solution

- To obtain maximum moments at all critical design sections, it is necessary to consider three alternative loadings.
- It will be assumed that 20 % adjustment of support moment is permitted throughout.





Example 8.2

□ Solution

Decrease in exterior positive moment:

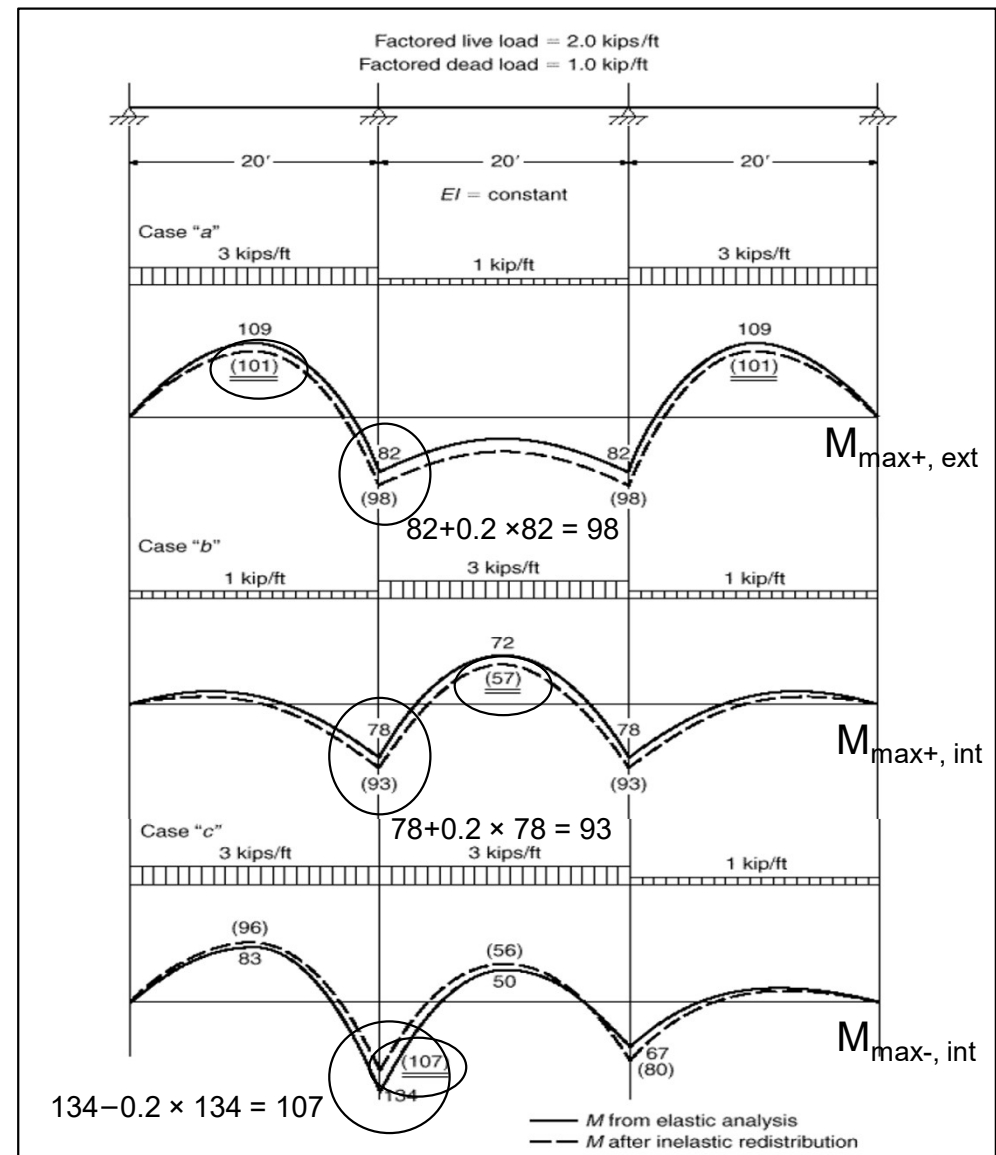
If negative moment is increased 20 %, the result is decrease in $M_{\max+, \text{ext}}$ from 109 to 101

Decrease in interior positive moment:

If negative moment is increased 20 %, the result is decrease in $M_{\max+, \text{int}}$ from 72 to 57

Decrease in interior negative moment:

If negative moment is decreased 20 %, positive moments increase in both spans.





Example 8.2

□ Conclusion on Redistribution of Moments

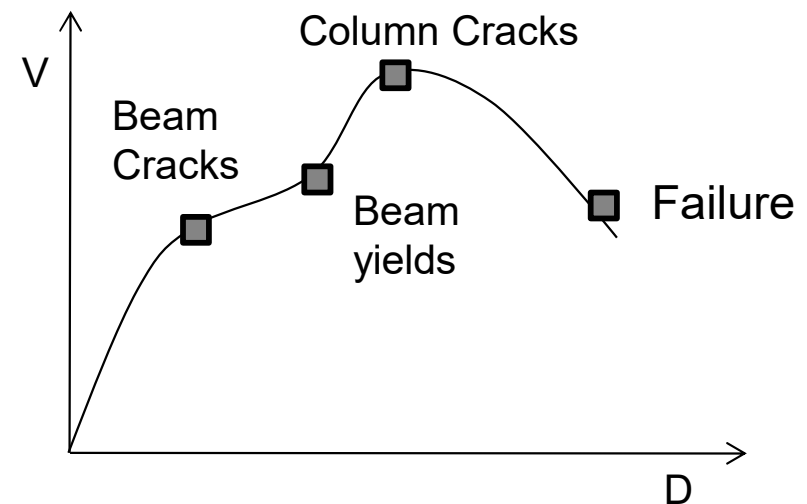
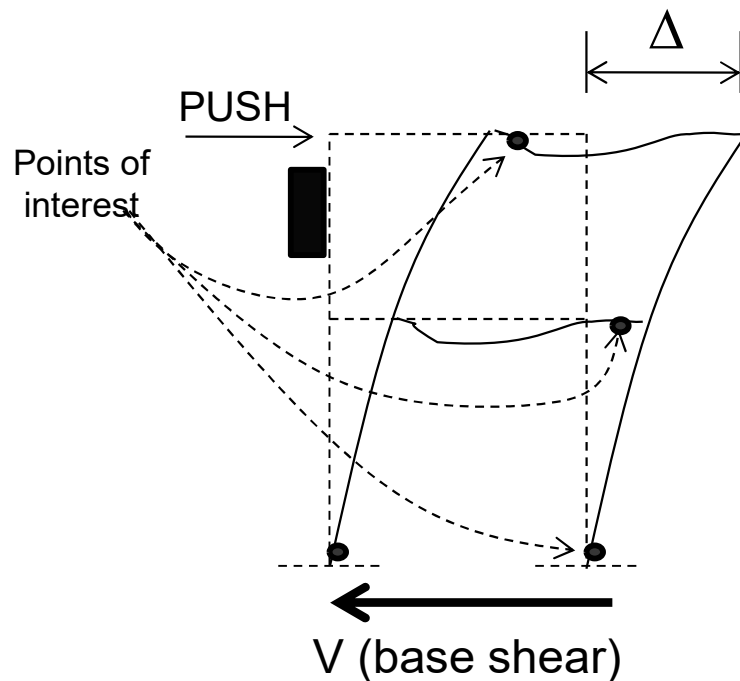
- The net result is a reduction in design moments over the entire beam.
- This modification of moments does not mean a reduction in safety factor below that implied in code safety provisions; rather, it means a reduction of the excess strength that would otherwise be present in the structure because of the actual redistribution of moments that would occur before failure.
- It reflects the fact that the maximum design moments are obtained from alternative load patterns, which could not exist concurrently.
- The result is a more realistic appraisal of the actual collapse load of the indeterminate structure.



Plastic Analysis

□ Non-Linear Static (Pushover) Analysis

- Points on the structure whose performance (when it yields, cracks or fails) is required to be monitored are selected.
- The structure is pushed at the top.

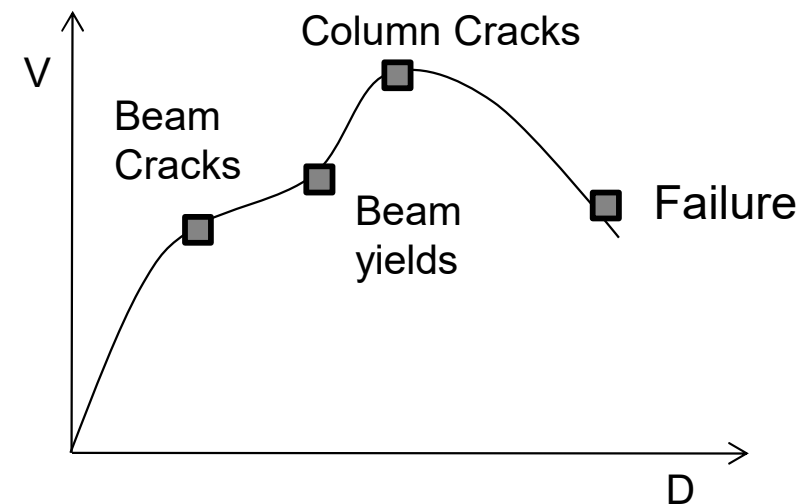
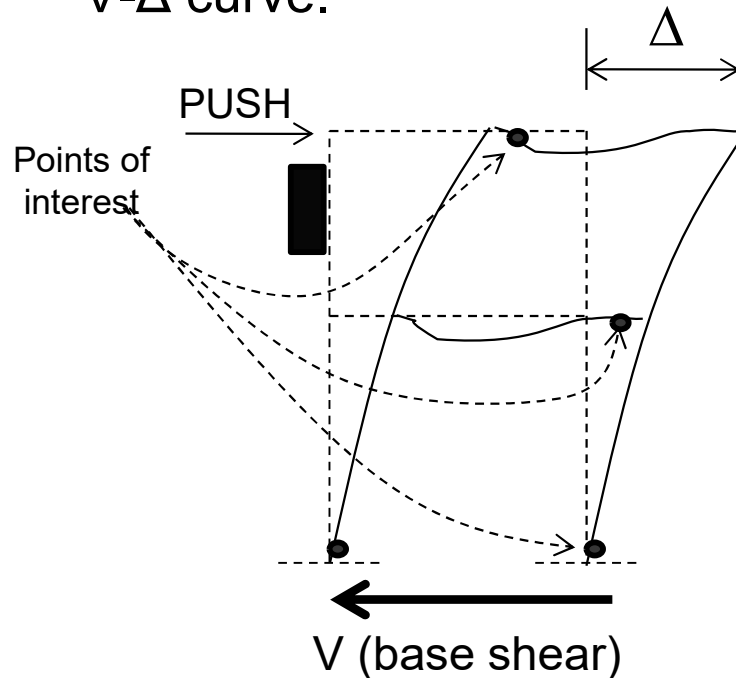




Plastic Analysis

□ Non-Linear Static (Pushover) Analysis

- Top Drift (D) and corresponding base-shear (V) is calculated and plotted on V - Δ curve.
- Structure is further pushed in steps and V - Δ curve is plotted. Also, performance of the selected points is monitored and marked on the V - Δ curve.

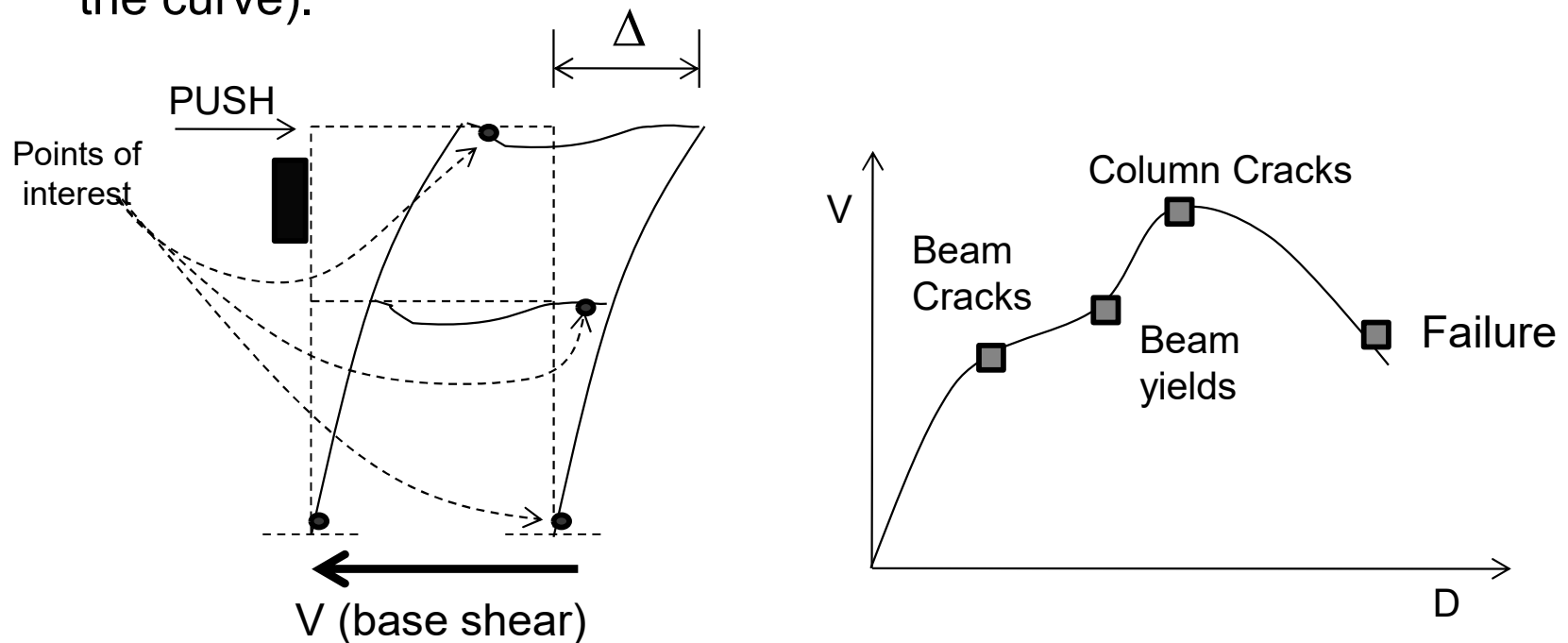




Plastic Analysis

□ Non-Linear Static (Pushover) Analysis

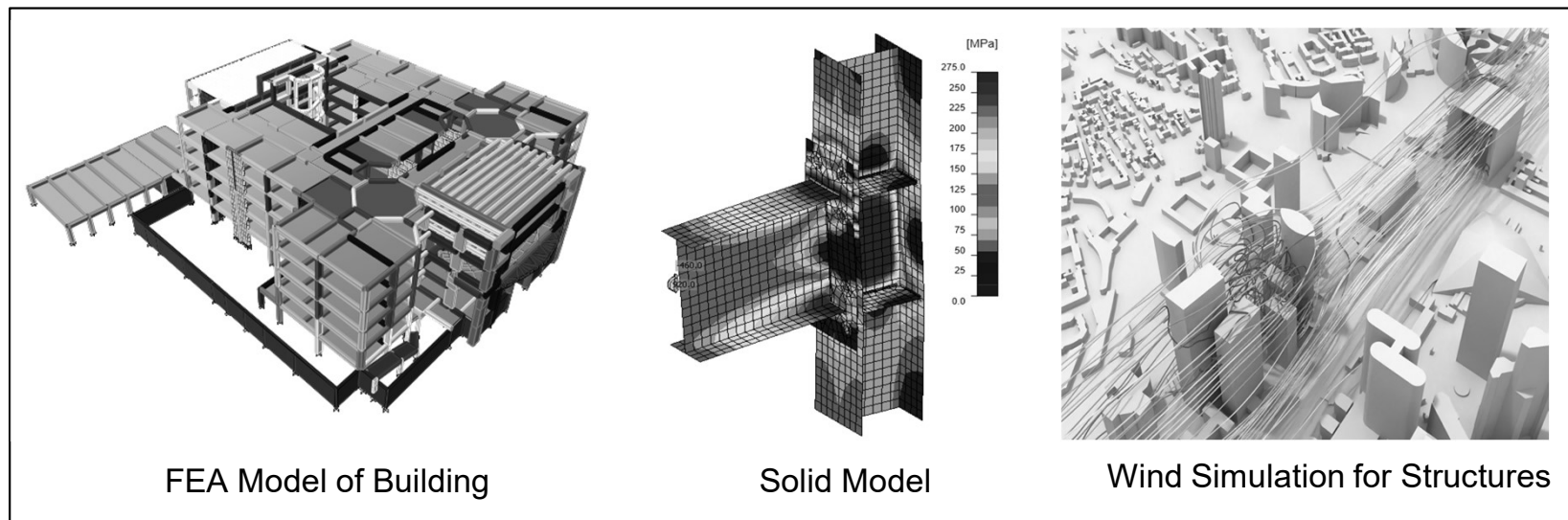
- Therefore, a single chart that shows the performance of the whole structure (or separate charts for all points of interests) is obtained.
- These charts can be used to identify points where the strengthening of structure is required (i.e points that fail or start to fail in the start of the curve).





Modern Analysis Tools

- In earlier times, structural analysis was largely limited to simplified models and manual calculations, focusing primarily on static loads and linear elastic behavior.
- Today, modern tools can perform analysis with full nonlinear material and geometric behavior from the elastic to plastic stages including cracking, buckling, post-buckling, P-Delta effects, contact, complete element separation, collision, and effects of falling debris.





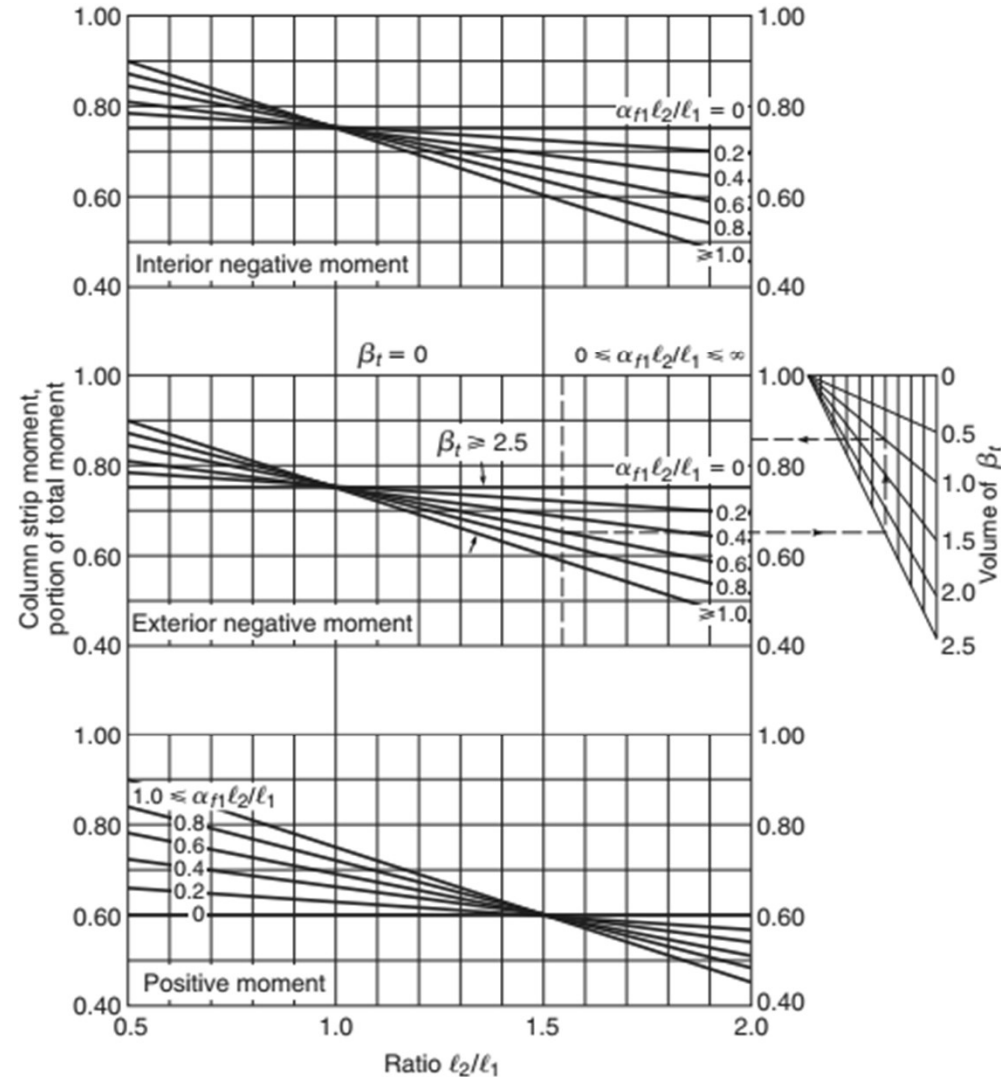
References

- Reinforced Concrete - Mechanics and Design (7th Ed.) by James MacGregor.
- Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.
- Building Code Requirements for Structural Concrete (ACI 318-19)
- Portland Cement Association (PCA 2002)



Appendix

GRAPH A.4
Interpolation charts for
lateral distribution of slab
moments.



Source: Design of Concrete Structures 14th / 15th edition by Nilson, Darwin and Dolan.