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SOCIAL INSURANCE WITH VARIABLE RETIREMENT
AND PRIVATE SAVING

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1. Introduction

Much analysis of retirement income uses the perfect certainty life-cycle model. Yet, when individuals are asked about the ages at which they expect to retire, the answers reveal considerable individual uncertainty in two ways. First, many of the answers change significantly when the question is repeated only a year later. Second, the answers have some, but highly limited, predictive value when one examines actual retirements.[1] Individuals face considerable uncertainty about future job opportunities, wealth accumulation, financial needs, disability, and labor disutility. To help with these uncertainties (as well as to provide retirement income) advanced countries generally have both disabilty and retirement programs. In the United States, these programs overlap. Retirement benefits become available at age 62 while disability benefits can be claimed up to 65. In light of the expense of determining disability and the inevitable presence of errors in determination, it seems to us appropriate to have both kinds of programs. In this paper, we examine the optimal design of a retirement program to insure

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[1]See, e.g., Diamond and Hausman (1981).

ex ante identical individuals against the risk of losing their earnings ability. We do not attempt to examine this question in the explicit presence of a disability program.

In a previous paper (Diamond and Mirrlees 1978) we studied the same question assuming the government controls savings. We analyzed a series of models in which a consumer may or may not be able to work in the latter part of his life, and may or may not choose to do so if he is able. The government pays a pension to those who are not working, financing the pension in part from contributions taken from workers. Whether or not a consumer who is capable of work chooses to work is affected by this social insurance system. The government's budget constraint is, in turn, affected by the work decision. The government seeks to maximize ex ante expected utility, allowing for the effect of incentives on its budget. We found that, with identical consumers and plausible conditions on preferences, the insurance system should be so chosen that, until the socially desired retirement age, all who can work do so; and that this moral hazard constraint is binding in the sense that the optimal system leaves consumers indifferent whether or not to work (and requires them to work). After the socially desired retirement age no one would choose to work if offered compensation equal to the marginal product of labor. The optimal system has an implicit tax on earnings which decreases with age, reaching zero at the socially desired retirement age. We also found that, under the optimal system, there is plausibly an incentive for consumers to save, an incentive which it is socially optimal to frustrate.

It is not particularly easy to render private saving impossible. In the present paper, we study models in which the government takes account of incentives to save as well as incentives to work, and examine the extent to which the findings of our previous paper are modified when private saving is allowed. This change in the problem introduces quite new technical difficulties. The analysis in the present paper does not depend on that of the previous one. Two models are studied: a two period model is used to show how savings affect the moral hazard constraint, and to demonstrate that, under the same circumstances as in the previous models, the moral hazard constraint binds. Under the same plausible assumptions as led to a desire to save in our previous paper, wealth taxation is desired, if possible. We also solve a continuous-time model, so as to obtain properties of contributions and pension payments over time, and to provide numerical solutions for comparison with the results of our previous paper. As in our previous paper, the optimal net return to work is found to increase with age, equalling the marginal product of labor at the socially desired retirement age for those who have remained able to work.

2. A Two-Period Model

We begin by considering a two-period model where everyone works in the first period and has the probability of being able to work in the second period. We assume that the utility effect of an inability to work is additive, and so can be ignored in designing the optimal program. Achieved lifetime utility, u, is then a concave function of first period consumption, second period consumption, and number of periods of work. Since everyone works in the first period, we cannot distinguish the first period wage from lump sum income. We denote their sum, measured in units of second period income, by W. The second period wage is written as w. We write one plus the interest rate as r. r is, for the present, a fixed parameter.

An individual can have one of two different plans. If he plans on working only one period, he has no uncertainty and optimized lifetime expected utility can be written as

$$V_1(W, r) \equiv \text{Max}_{c_1} \; u(c_1, W - rc_1, 1). \tag{1}$$

Alternatively, an individual can plan on two periods of work if able. Expected lifetime utility is then

$$V_2(W, w, r) \equiv \text{Max}_{c_2} \left[\theta u(c_2, W + w - rc_2, 2) + (1-\theta)u(c_2, W - rc_2, 1)\right] \tag{2}$$

We assume that the economy is sufficiently poor for second period work to be socially desirable. This is equivalent to requiring that, when individuals receive their marginal products in the second period, and incomes W' in the first that balance the government's budget, they want to work:

$$V_1(W', r) < V_2(W', m, r) \tag{3}$$

where m, the marginal product of labor, is assumed to be independent of other economic variables and $W' = mr + A$, where A is the level of revenues made available to the retirement program.

The government seeks to maximize expected utility subject to its budget constraint (which is assumed to be in expected value terms) and the need to induce able individuals to work in the second period.

$$\begin{aligned} &\text{Max}_{W,w} \; V_2(W, w, r) \\ &\text{s.t. } W + \theta w - mr - \theta m \leqq A \\ &\qquad V_2(W, w, r) \geq V_1(W, r) \end{aligned} \tag{4}$$

Assuming nonsatiation, the resource constraint is binding, and the Lagrange multiplier, λ, is positive. There are two cases to examine, depending on whether the moral hazard constraint is binding or not.

If the moral hazard constraint is not binding, we have an optimal insurance problem without incentive problems. The solution would call for equating the marginal utility of second period consumption for one and two period workers (and would satisfy the budget constraint)

$$u_2(c_2, W - rc_2, 1) = u_2(c_2, W + w - rc_2, 2) \quad (5)$$

For this solution to hold, it must be the case that compensating the disabled enough to give workers and disabled the same marginal utility of consumption, it does not pay an individual to plan on claiming a retirement benefit whether able to work or not.

While this is a possible solution, it seems to us to be empirically implausible today. A sufficient condition to rule out this solution is that equating utilities of workers and nonworkers leaves the disabled with a higher marginal utility of consumption:

Moral Hazard Assumption

$$\begin{aligned} &\text{Max}_{c_2}\ u(c_2, W + w - rc_2, 2) = \text{Max}_{c_1}\ u(c_1, W - rc_1, 1) \\ &\text{implies } u_2(c_2, W + w - rc_2, 2) < u_2(c_1, W - rc_1, 1) \end{aligned} \quad (6)$$

Assuming (6), the moral hazard constraint is binding and its Lagrangian, μ, is positive.

The optimum is then defined by the two constraints to the government's choice problem, (4). If there are multiple solutions to these equations, the optimum is the one with the largest W. This can be seen from the fact that V_1 increases with W and V_2 equals V_1 at any solution. That is, it is optimal to provide the greatest amount of insurance consistent with the moral hazard constraint. A possible case is shown in Figure 1, where the two constraints are shown.[2] Since V_2 is monotonic in w, there is a unique w for any W, although the relationship need not be monotonic. Assuming that labor is disliked, the optimal wage is positive. As can be seen from the diagram, assumption (3) on available resources implies that the optimal wage, w*, is less than the marginal product.

Having done this analysis, it is straight forward to consider the case where the utility function of those unable to work in the second period takes the general form v. The expected utility of someone with a plan of working two periods if able, V_2, is $\theta u(c_2, W + w - rc_2, 2) + (1 - \theta)v(c_2, W - rc_2, 1)$. Expected utility of someone with a plan of only one period of work, V_1, is $\theta u(c_1, W - rc_1, 1) + (1 - \theta)v(c_1, W - rc_1, 1)$. There are two types of optima. Either marginal utilities of workers and disabled are equated

$$u_2(c_2, W + w - rc_2, 2) = v_2(c_2, W - rc_2, 1)$$

or expected utilities of those with different plans are equated $V_2 = V_1$. To see which solution holds, one calculates the full optimum, equating marginal utilities, and checks whether the moral hazard constraint is binding.

3. Comparative Statics

When the moral hazard constraint is binding, the optimum is defined by the moral hazard constraint and the resource constraint. To do comparative statics, we need to examine the change in these two equations as parameters change. An increase in resources available to the program is a move to the

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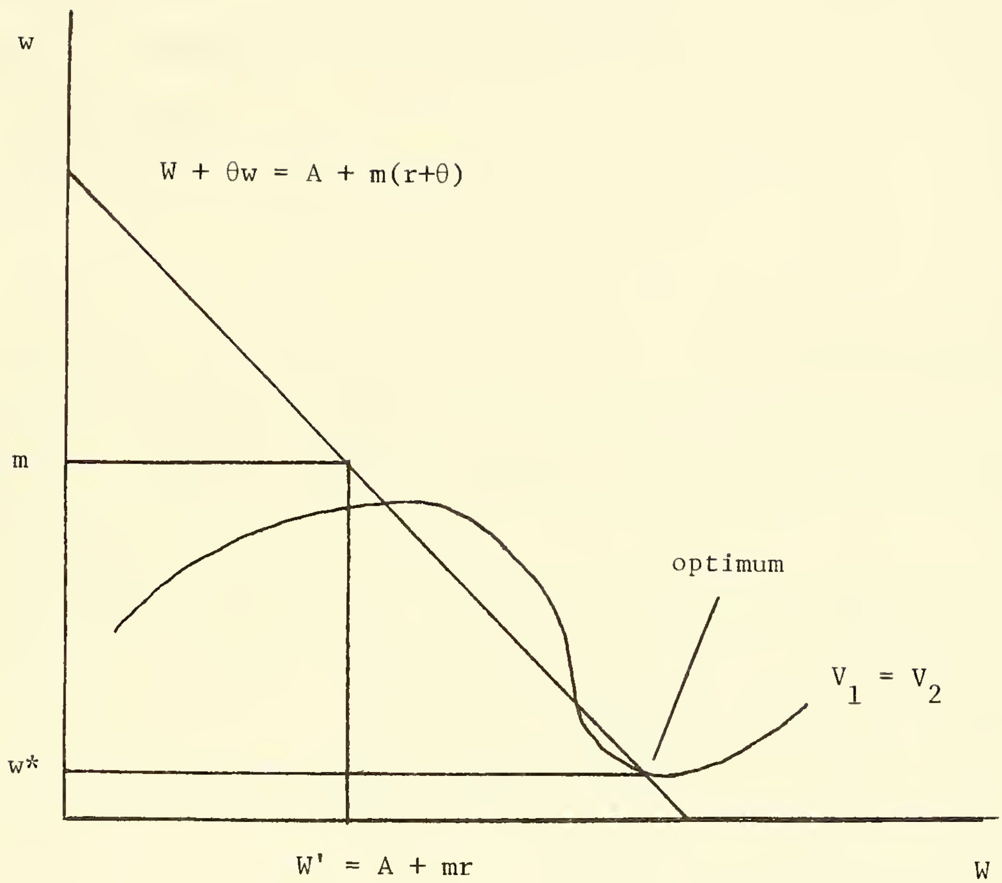
[2]We are indebted to John Burbidge for correcting a previous version of this diagram.

Figure 1



right along the curve $V_2 = V_1$. This raises W_1 but not necessarily w.

The probability of being able to work affects both equations. A rise in the fraction able to work rotates the resource constraint around the point (m, W'), with the line becoming flatter. A rise in θ does not affect V_1, for it affects only the additive disutility of disability. It increases V_2, for $u(c_2, W + w - rc_2, 2) > u(c_2, W - c_2, 1)$ with a binding moral hazard constraint. Thus the curve $V_2 = V_1$ is shifted down. W is raised. w may not be.

4. Wealth Taxation

Suppose now that the government can set an interest rate for consumers that can be different from the marginal product of capital. We denote the latter by $\bar{r}$. Since there would be no taxation of savings in the first best, the additional policy tool does not affect our conclusion that the moral hazard constraint is binding when assumption (6) holds. Assuming it is binding, we form the Lagrangian expression

$$L = V_2(W, w, r) - \lambda[W - m\bar{r} + \theta(w-m) + c_2^*(W, w, r)(r - \bar{r}) - A] - \mu(V_2 - V_1) \quad (7)$$

where c_2^* is individually optimized consumption.

To sign $(r - \bar{r})$ we examine the expression $\frac{\partial L}{\partial r} + c_2 \frac{\partial L}{\partial W}$.

$$\frac{\partial L}{\partial r} = \frac{\partial V_2}{\partial r} - \lambda\left[-c_2^* - (r - \bar{r})\frac{\partial c_2^*}{\partial r}\right] - \mu\left(\frac{\partial V_2}{\partial r} - \frac{\partial V_1}{\partial r}\right) = 0$$

$$\frac{\partial L}{\partial W} = \frac{\partial V_2}{\partial W} - \lambda\left[1 - (r - \bar{r})\frac{\partial c_2^*}{\partial W}\right] - \mu\left(\frac{\partial V_2}{\partial W} - \frac{\partial V_1}{\partial W}\right) = 0 \quad (8)$$

By Roy's identity, $\frac{\partial V_i}{\partial r} = - c_i \frac{\partial V_i}{\partial W}$. This identity also implies that

$(\frac{\partial c_2^*}{\partial r} + c_2 \frac{\partial c_2^*}{\partial W})$ is a compensated change and is negative.

Using these facts we have

$$\frac{\partial L}{\partial r} + c_2 \frac{\partial L}{\partial W} = \lambda(r - \bar{r})(\frac{\partial c_2^*}{\partial r} + c_2 \frac{\partial c_2^*}{\partial W}) + \mu \frac{\partial V_1}{\partial W} (c_2^* - c_1^*) = 0 \qquad (9)$$

where c_1^* is optimized consumption for someone planning to work for one period only. Thus we have

$$\text{sign } (r - \bar{r}) = - \text{ sign } (c_2^* - c_1^*) \qquad (10)$$

While in general $c_2^* - c_1^*$ can be of either sign, it is plausible that it is positive. That is, if the wage for second period work is just high enough for workers to be indifferent between working one period and working two periods when able, then a worker planning to work for two periods would consume more in the first period.[3]

The intuition for this result can be seen from the following considerations. The government is constrained in the desirable size of the insurance program by the constraint that workers be willing to plan on two periods of work. Since someone planning only one period of work would plan to save more, a tax on savings falls more heavily on such a person than on someone planning two periods of work. This permits a larger social insurance program while still satisfying the moral hazard constraint.

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[3]A sufficient condition for this conclusion is the assumption in our earlier paper that was seen to imply a desire to save at the optimum when the government controlled savings.

5. Another Two-Period Model

In the model considered above, it was certain that workers were able to work in the first period. In that model we can not examine the issue whether the moral hazard constraint is effective in all periods. If some individuals may be unable to work in both periods, there are two moral hazard constraints--depending on plans to work zero or one period rather than the desired two periods. To analyze this problem, we restrict ourselves to the case of an intertemporally additive utility function in which disability has an additive effect on utility. For notational simplicity we assume that the marginal produce of labor is one and the discount rates for both utility and output are zero. We continue to omit the disutility of disability from the expressions for expected utility. We find that, at the optimum, both constraints are binding (given the moral hazard condition). Also, the wage is monotonically increasing. These results carry over to the continuous time model.

Denote the sum paid to someone who does no work by W_0. Let w_1 and w_2 be the wages in the two periods. The lifetime budget constraints of those who have worked one and two periods are

$$W_1 = W_0 + w_1 \tag{11}$$

$$W_2 = W_1 + w_2$$

We assume that at the social optimum it is desirable that all who are able to work, do so. Let θ_i be the fraction able to work in period i. Provided that the moral hazard constraints are satisfied, the resource constraint is

$$R \equiv (1-\theta_1)W_0 + (\theta_1 - \theta_2)(W_1 - 1) + \theta_2(W_2 - 2) = A \tag{12}$$

where A is the value of resources available for this program.

There are three plans an individual can have--to work for 0, 1, or 2 periods. For those able to work in period one, these plans give expected utilities

$$V_0(W_0) = \text{Max}_{c_0}\left[u_2(c_0) + u_2(W_0 - c_0)\right]$$

$$V_1(W_1) = \text{Max}_{c_1}\left[u_1(c_1) + u_2(W_1 - c_1)\right] \tag{13}$$

$$V_2(W_1, W_2) = \text{Max}_{c_2}\left[u_1(c_2) + \theta_3 u_1(W_2 - c_2) + (1 - \theta_3)u_2(W_1 - c_2)\right]$$

where u_1 is the utility of a worker and u_2 that of a nonworker able to work (with $u_2(c) > u_1(c)$ for all c). $\theta_3 = \theta_2/\theta_1$ is the conditional probability of further working ability. We assume that the consumption decision is made after the ability to work in that period is known. With this assumption, we have $c_0 = 1/2\ W_0$.

Given the additive structure, we state the moral hazard assumption in one period form:

$$u_1'(c) = u_2'(b) \text{ implies } u_1(c) < u_2(b). \tag{14}$$

We can write the sum of expected utilities as the utility of those never able to work plus the expected utility of those who are able to work in the first period. Thus the social welfare problem is

$$\left.\begin{array}{l} \underset{W_0, W_1, W_2}{\text{Maximize}}\ (1-\theta_1)V_0(W_0) + \theta_1 V_2(W_1, W_2) \\ \\ \text{Subject to } R = A,\ V_2 \geq V_1,\ V_2 \geq V_0. \end{array}\right\} \tag{15}$$

For any feasible value of W_0, consider the optimal setting of W_1 and W_2. This problem is identical to that considered in Section 2 and is equivalent to the problem

$$\underset{W_1, W_2}{\text{Maximize}}\ \theta_1 V_2$$

$$\text{Subject to } V_2 \geq V_1$$

$$(\theta_1 - \theta_2)(W_1 - 1) + \theta_2(W_2 - 2) = A - (1 - \theta_1)W_0. \tag{16}$$

When the moral hazard assumption (14) is satisfied, the solution of this problem is the solution of the two constraints. Before considering the choice of W_0, we note that at the optimum wages we have $c_1 < c_2$. That is, those planning to work in the future would consume more in the present. To see this, assume $c_1 \geq c_2$. Then $u_1'(c_1) \leq u_1'(c_2)$ and $u_2'(W_1 - c_1) \geq u_2'(W_1 - c_2)$. From the first order condition for individual savings, we have $u_2'(W_1 - c_1) \leq u_1'(W_2 - c_2)$. From the moral hazard assumption (14) we have $u_2(W_1 - c_1) > u_1(W_2 - c_2)$. This would contradict the moral hazard constraint since

$$\begin{aligned} V_2(W_1, W_2) &< u_1(c_2) + \theta_3 u_2(W_1 - c_1) + (1-\theta_3)u_2(W_1 - c_2) \\ &\leq u_1(c_2) + \theta_3 u_2(W_1 - c_2) + (1-\theta_3)u_2(W_1 - c_2) \\ &\leq V_1(W_1). \end{aligned}$$

Now consider the optimal choice of W_0. There are two possibilities. If the moral hazard constraint $V_2 \geq V_0$ is binding, W_0 is a solution to $V_2 = V_0$, evaluated at the W_1 and W_2 values which are optimal for that W_0. A second possibility is that this constraint is not binding. Then, W_0 is determined by the first order condition

$$\lambda = V_0'(W_0) = u_2'(W_0/2) \tag{18}$$

where λ is the Lagrangian on the resource constraint in the suboptimisation (16).

We now argue that the moral hazard assumption (14) rules out this latter possibility. To obtain λ we solve the suboptimisation (16), using Lagrangian techniques. Forming the Lagrangian expression

$$L = \theta_1 V_2(W_1, W_2) - \lambda\{(1-\theta_1)W_0 + (\theta_1-\theta_2)(W_1-1) + \theta_2(W_2 - 2)-A\} - \mu\{V_1(W_1) - V_2(W_1,W_2)\}, \tag{19}$$

we set the derivatives with respect to W_1 and W_2 equal to zero, and solve for λ. This gives

$$\lambda = \frac{V_1'\frac{\partial V_2}{\partial W_2}}{\frac{\partial V_2}{\partial W_2} + \theta_3\left(V_1' - \frac{\partial V_2}{\partial W_1} - \frac{\partial V_2}{\partial W_2}\right)} \tag{20}$$

To check which type of solution occurs, we need to inquire whether at the value of W_0 which equates λ with $u_2'(1/2\ W_0)$, we have V_0 larger or smaller than V_1 or V_2 (which are equal). To evaluate λ we notice that

$$V_1' - \frac{\partial V_2}{\partial W_1} - \frac{\partial V_2}{\partial W_2} = u_2'(W_1 - c_1) - \left[\theta_3 u_1'(W_2 - c_2) + (1 - \theta_3)u_2'(W_1 - c_2)\right] \tag{21}$$

$$= u_1'(c_1) - u_1'(c_2)$$

$$> 0, \text{ since } c_1 < c_2.$$

Therefore $\lambda < V_1'$. Finally we show that $V_1' < V_0'$, contradicting (18). Assume the contrary. Then $W_1 - c_1 \leq W_0 - c_0$. Also $u_1'(c_1) \geq u_2'(c_0)$, implying that $u_1(c_1) < u_2(c_0)$. Consequently

$$\begin{aligned} V_0 &= u_2(c_0) + u_2(W_0 - c_0) \\ &> u_1(c_1) + u_2(W_1 - c_1) \\ &= V_1 = V_2 \end{aligned} \tag{22}$$

This contradicts the constraint $V_2 \geq V_0$. Therefore $\lambda < V_0'(W_0)$, and (18) cannot hold.

This argument shows that, when the moral hazard assumption (14) holds, it is optimal to make the consumer indifferent among his three possible plans. The method of proof might seem difficult to extend to many periods. Nevertheless we are able to prove the same result for a continuous-time model (Proposition 4 in Section 7 below).

Before turning to the continuous time model, we show that the optimum has $0 < w_1 < w_2 \leq 1$. (This property of the optimum also carries over to the continuous time model.) The optimum is a solution to the resource constraint and the equalities $V_0 = V_1 = V_2$. Comparing V_1 with V_0, we note that $W_0 \geq W_1$ would imply $V_1 < V_0$ since $u_2(c) > u_1(c)$ for all c. Thus $w_1 > 0$.

Next assume that $w_2 > 1$ for the optimal program. Decreasing w_2 to 1 ends labor supply in the second period. This has no effect on utility (since workers are indifferent to continued labor supply) and saves revenue, since workers were paid more than their marginal products. The extra revenue can be used to raise W_0 and W_1 (keeping $V_0(W_0) = V_1(W_1)$), thereby raising welfare.

To obtain the remaining inequality, $w_1 < w_2$, or, equivalently,

$$2W_1 < W_0 + W_2 \tag{23}$$

we use the fact that $u_1'(c_1) = u_2'(W_1 - c_1) = \mu$, say, and the basic inequality for concave functions. From (13) we have

$$\begin{aligned} V_2(W_1,W_2) &= u_1(c_2) + \theta_3 u_1(W_2 - c_2) + (1 - \theta_3)u_2(W_1 - c_2) \\ &< u_1(c_1) + \mu(c_2 - c_1) \\ &+ \theta_3[u_1(c_1) + \mu(W_2 - c_2 - c_1)] \\ &+ (1 - \theta_3)[u_2(W_1 - c_1) + \mu(c_1 - c_2)] \\ &= (1 + \theta_3)V_1(W_1) - 2\theta_3 u_2(W_1 - c_1) + \mu\theta_3(W_2 - 2c_1) \end{aligned} \tag{24}$$

Similarly, since $c_0 = W_0/2$,

$$V_0(W_0) = 2u_2(W_0/2) < 2u_2(W_1 - c_1) + 2\mu(W_0/2 - W_1 + c_1) \tag{25}$$

Combining (24) and (25),

$$V_2 < (1 + \theta_3)V_1 - \theta_3 V_0 + \mu\theta_3(W_2 - 2W_1 + W_0)$$

Since $V_0 = V_1 = V_2$, (23) follows.

We shall prove that the optimal net wage increases with age also in a continuous time model (Propositon 6 of section 7).

6. The Continuous-Time Model

We next study a population of identical consumers, each of whom lives for T periods, and has utility (u_i concave)

$$\int_o^T u_i(c(t))dt \tag{26}$$

where i = 1 when he is working, i = 2 when he is not working. We continue to ignore the additive utility effect of disability. Marginal utilities are assumed to range over the entire positive half-line for both utility functions. When working, the rate of productivity is unity. The probability that an individual is unable to work at age t is F(t).

$$f(t) = F'(t) > 0. \quad F(0) = 0. \quad F(t) < 1 \text{ for } t < T. \tag{27}$$

The interest rate is zero, and there is no taxation of savings or the return from savings. If the consumer is still working at age t, he receives pay, net of insurance contribution, at rate w(t). In addition everyone receives a once-and-for-all lump-sum payment of W(0). We also use the notation

$$W(s) = \int_0^s w(t)dt + W(0) \tag{28}$$

for the present value of life-time earnings and benefits, net of contributions, received by a worker who, by choice or necessity, retires at age s. Once a worker has retired, he cannot return to work (e.g. because the contribution level is related to years of work rather than age).

A consumer who retires at age s has, at that date, wealth equal to $W(s) - \int_0^s c(t)dt$, which is allocated evenly among the remaining years of life. Defining

b(s) = consumption rate after retirement of a worker who retires at age s

c(t) = consumption at age t of a worker who retires after t

we therefore have

$$b(s)(T - s) = W(s) - \int_0^s c(t)dt \tag{29}$$

Expected utility for a worker who plans to retire at s, if still able to work at that age, is

$$\int_o^s u_1\{c(t)\}[1-F(t)]dt + \int_o^s (T - t)u_2\{b(t)\}f(t)dt + (T - s)u_2\{b(s)\}[1 - F(s)] \tag{30}$$

That is, for $t < s$, the probability of being able to work, and so have utility $u_1(c(t))$ is $(1 - F(t))$; for $t \geq s$ there is zero probability of working; for $t < s$, the probability of retiring at t is $f(t)$, giving $(T-t)$ years of retirement each with utility $u_2(b(t))$; for $t = s$, the probability of retiring as planned is $1 - F(s)$.

Given the function W, which is set by the government, and the budget constraint (29), the consumer chooses the function c, and the age s, to maximize (30). We define

$$V(s) = \text{the maximum of (30) for given } s, \tag{31}$$

and write $c_s(t)$, $b_s(t)$ for the corresponding consumption rates of a worker who plans to retire at s. From (29) and (30) it is easily seen that expected utility is maximized (by the consumer) for given s if and only if

$$u_1'\{c_s(t)\}[1 - F(t)] = \int_t^s u_2'\{b_s(z)\}f(z)z + u_2'\{b_s(s)\}[1 - F(s)] \tag{32}$$

for all $t \leq s$. That is, at each date, the marginal utility of consumption when working is set equal to the expected marginal utility of consumption at retirement, with the expectation of retirement date taken conditional on having worked up to that date.

When $s < T$, (32) implies, setting $t = s$, that

$$u_1'\{c_s(s)\} = u_2'\{b_s(s)\} \tag{33}$$

(When $s = T$, $b_T(T)$ does not matter; but, dividing (32) by $1 - F(t)$, and letting $t \to T$, we see that (33) holds of the limits $\text{Lim}\ c_T(s)$ and $\text{Lim}\ b_T(s)$ as $s \to T$.) Notice that (29) implies

$$b_s(0) = W(0)/T \tag{34}$$

for all s. Thus, given the wage function, $b_s(0)$ is independent of the planned retirement date. We shall denote it by b_o.

Differentiating (32) with respect to t, we obtain the differential equation

$$[1 - F(t)]\dot{u}_1'\{c_s(t)\} = [u_1'\{c_s(t)\} - u_2'\{b_s(t)\}]f(t) \tag{35}$$

where the dot, here and later, means differentiation with respect to the t in $c_s(t)$. This shows that c_s is an increasing function of t so long as

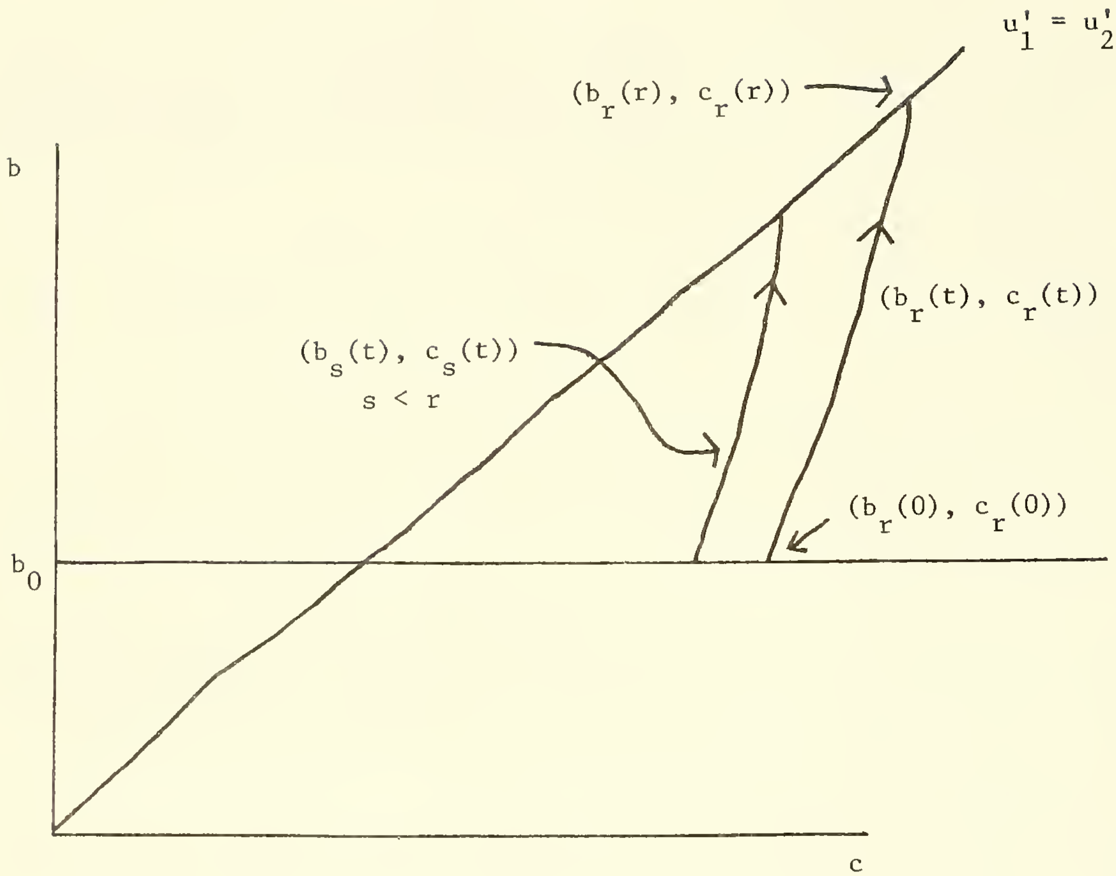
$$u_1'\{c_s(t)\} < u_2'\{b_s(t)\}. \tag{36}$$

This inequality defines the region below the equal marginal utility curve (EMU) in the (c, b)-plane, which is shown in Figure 2.

We shall also want to know how V varies with s. Differentiating the expression (30) for V(s) with respect to s, and using the first order conditions (32) to simplify in the usual way, we get

$$\frac{dV}{ds} = [u_1\{c_s(s)\} - u_2\{b_s(s)\} + (T - s)u_2'\{b_s(s)\}\dot{b}_s(s)][1 - F(s)] \tag{37}$$

Figure 2
Optimal trajectory



(Recollect that $\dot{b}_s(s)$ means $\frac{\partial}{\partial t} b_s(t)$ evaluated at $t = s$.) That is, planning to work longer has no direct effect on expected utility if the worker is not working at s, an event that happens with probability F(s). If the worker is working at s (probability 1-F(s)), instantaneous utility is higher by $u_1 - u_2$ during the period of additional work. This additional work also changes benefits for the rest of the retirement period.

The following propositions tell us something about the behavior of $c_s(t)$ and $b_s(t)$ as s varies, W remaining unchanged:

<u>Proposition 1</u>. If $c_s(0) < c_{s'}(0)$, then $c_s(t) < c_{s'}(t)$ $(t \geq 0)$.

<u>Proposition 2</u>. If $c_s(0) < c_{s'}(0)$, then $b_s(t) > b_{s'}(t)$ $(t > 0)$.

Proposition 2 follows from Proposition 1 by equation (29). To prove the first result, suppose to the contrary that there is a smallest date t' such that $c_s(t') = c_{s'}(t')$. Then by (29), $b_s(t) > b_{s'}(t)$ when $t \leq t'$. Evaluating (32) at t and 0, we have

$$u_1'\{c_s(t)\}[1 - F(t)] = u_1'\{c_s(0)\} - \int_0^t u_2'\{b_s(z)\}f(z)dz \tag{38}$$

for any t, and in particular for $t = t'$. But then $b_s(t) > b_{s'}(t)$ for $t \leq t'$ and $c_s(0) < c_{s'}(0)$ together imply that $c_s(t') < c_{s'}(t')$, contradicting the assumption that there exists t' for which $c_s(t') = c_{s'}(t')$. This proves Proposition 1.

Whether or not lower initial consumption is associated with an earlier planned retirement date may depend on the net-wage function. We shall see later that lower s implies lower c (and higher b) at fixed t if the net-wage function is optimal.

7. Optimization of the Continuous-Time Model

When the government has set the wage profile, the socially chosen retirement date r must satisfy

$$V(s) \leq V(r) \qquad (s \leq r) \tag{39}$$

The government is constrained by these inequalities, and by the budget constraint

$$\int_0^r [W(t) - t]f(t)dt + [W(r) - r][1 - F(r)] = A \tag{40}$$

where A equals net resources available for the retirement program. Subject to these constraints, it wishes to maximize V(r). Notice that we need not, in our mathematical formulation, concern ourselves with putative retirement dates greater than r, since W(t) can be set equal to zero for $t > r$. The moral hazard problem that has to be guarded against is that consumers may retire too early.

We make the same assumption as in Section 5, that

$$u_1'(c) = u_2'(b) \text{ implies } u_1(c) < u_2(b) \tag{41}$$

Under this assumption, we can show that a moral hazard problem exists.

In the first best, the retirement date r can be chosen by the government. Then the marginal utility of consumption should be the same at all dates, both for workers and retired. Thus c_r and b_r are constant, and

$$u_1'(c_r) = u_2'(b_r) \tag{42}$$

For this to be the outcome of a net wage function, we must have, from (29)

$$W(s) = sc_r + (T - s)b_r$$

It is readily verified that (29) and (32) are satisfied by setting

$$c_s(t) = c_r, \quad b_s(t) = b_r$$

for all s and t. This means that the same choice of c and b is optimal for the consumer, whatever s is chosen. Now, since $\dot{b}_s = 0$, (37) implies that

$$\dot{V}(s) = [u_1(c_r) - u_2(b_r)][1 - F(s)] \tag{43}$$
$$< 0$$

for s < T by (41) and (42). Thus V(0) > V(r), and, free to choose, consumers would not work. Assumption (41) implies the presence of moral hazard in a most extreme form. (The opposite assumption implies, by (43), that the first best is attainable.) It follows similarly that the moral hazard constraints, (39), can not be satisfied with a strict inequality over any interval ending at r.

In the rest of this section, we show that a wage schedule such that V(s) is constant is optimal. That is, we show that all of the constraints in (39) are binding. In the next section we consider the socially optimal planned retirement date.

If it were possible to express the government's maximization problem as the maximization of a concave function subject to constraints described by the nonnegativity of concave functions, we should only have to derive first-order conditions for the problem, and show that they are satisfied by the constant-utility solution. But in this problem, unlike that studied in our earlier paper, it is not possible to express the budget constraint as the nonnegativity of a concave functional of V(•). Since we are bound by constraints $V(s) \leq V(r)$, which are concave only in the function V(•), the problem seems to be definitely not a concave program. We must therefore proceed by showing that any well behaved wage function that implies $V(s) \leq V(r)$ for all $s < r$, and $V(s') < V(r)$ for some $s' < r$, can be improved upon. (A wage function is well behaved if the implied $V(s)$ is differentiable and the set of intervals on which $V(s) < V(r)$ is finite.) We do this by constructing an infinitesimal variation which raises welfare without violating the constraints. Assuming that a well behaved optimum exists - and in this context one can not believe otherwise - it will follow that the constant-utility wage schedule is optimal. For completeness, and reassurance, we shall also show that the constant-utility wage schedule can not be improved upon by small changes in the wage schedule.

We describe the wage schedule by $W(0)$ and $w(t) = \dot{W}(t)$, and contemplate an infinitesimal variation $\delta W(0)$ and δw. r is also a control variable, but we shall first suppose r fixed. We start with a well-behaved wage schedule such that

$$V(s) \leq V(r) \qquad (s \leq r)$$

and such that the budget constraint (40) is satisfied. In terms of $W(0)$ and w, (40) can be written (using integration by parts)

$$W(0) + \int_{o}^{r} \left[w(t) - 1\right]\left[1 - F(t)\right]dt = A \tag{45}$$

To preserve this budget equality, our variation must satisfy

$$\delta W(0) + \int_{o}^{r} \delta w(t)\left[1 - F(t)\right]dt = 0. \tag{46}$$

To find the effect of variation in wages on $V(s)$, we use the expression for expected utility (30), which is maximized subject to

$$c(t) = w(t) + b(t) - (T - t)\dot{b}(t), \quad b(0) = W(0)/T,$$

which comes from (29). Variation of w induces changes in b(t) for t > 0 which, by the usual envelope argument, can be ignored. Therefore we have

$$\delta V(s) = \int_0^s u_1'\{c_s(t)\}\delta w(t)[1 - F(t)]dt + u_1'\{c_s(0)\}\delta W(0). \tag{47}$$

Let t_2 be the least upper bound of those t for which $V(t) < V(r)$, and suppose that $t_2 > 0$. By the argument above and the assumption that the wage path is well behaved we have $t_2 < r$.

$$V(t) = V(r) \qquad (t_2 \leq t \leq r) \tag{48}$$

and there exists $t_0 < t_2$ such that

$$V(t) < V(r) \qquad (t_0 \leq t < t_2) \tag{49}$$

We use the following result:

<u>Proposition 3</u>. <u>If</u> (48) <u>holds</u>,

$$u_1'\{c_s(t)\} < u_2'\{b_s(t)\} \tag{50}$$

<u>for</u> $t_2 \leq t < s \leq r$.

From our expression for $\dot{V}(s)$, equation (37), we have

$$u_1\{c_s(s)\} - u_2\{b_s(s)\} + (T - s)u_2'\{b_s(s)\}\dot{b}_s(s) = 0$$

for all s between t_2 and r. Since $u_1'\{c_s(s)\} = u_2'\{b_s(s)\}$, the moral hazard assumption (41) implies that $u_1\{c_s(s)\} < u_2\{b_s(s)\}$. Therefore

$$\dot{b}_s(s) > 0 \qquad (t_2 \leq s \leq r) \tag{51}$$

Now suppose that the conclusion of the proposition is false. Refer to Figure 3. By (51) and the fact that $\dot{c}_s(s) = 0$, the curve $(c_s(t), b_s(t))$ cuts the EMU curve from below at s. For the proposition to be false, it must therefore also cut it also from above at some t' between t_2 and s, i.e., with $\dot{b}_s(t') < 0$. Then the functions c_s and b_s in $[0, t']$ satisfy the conditions (33) and (35), which are sufficient for utility maximization subject to planned retirement date t'. But since $\dot{b}_{t'}(t') = \dot{b}_s(t')$ we have a contradiction of (51), completing the proof of the proposition.

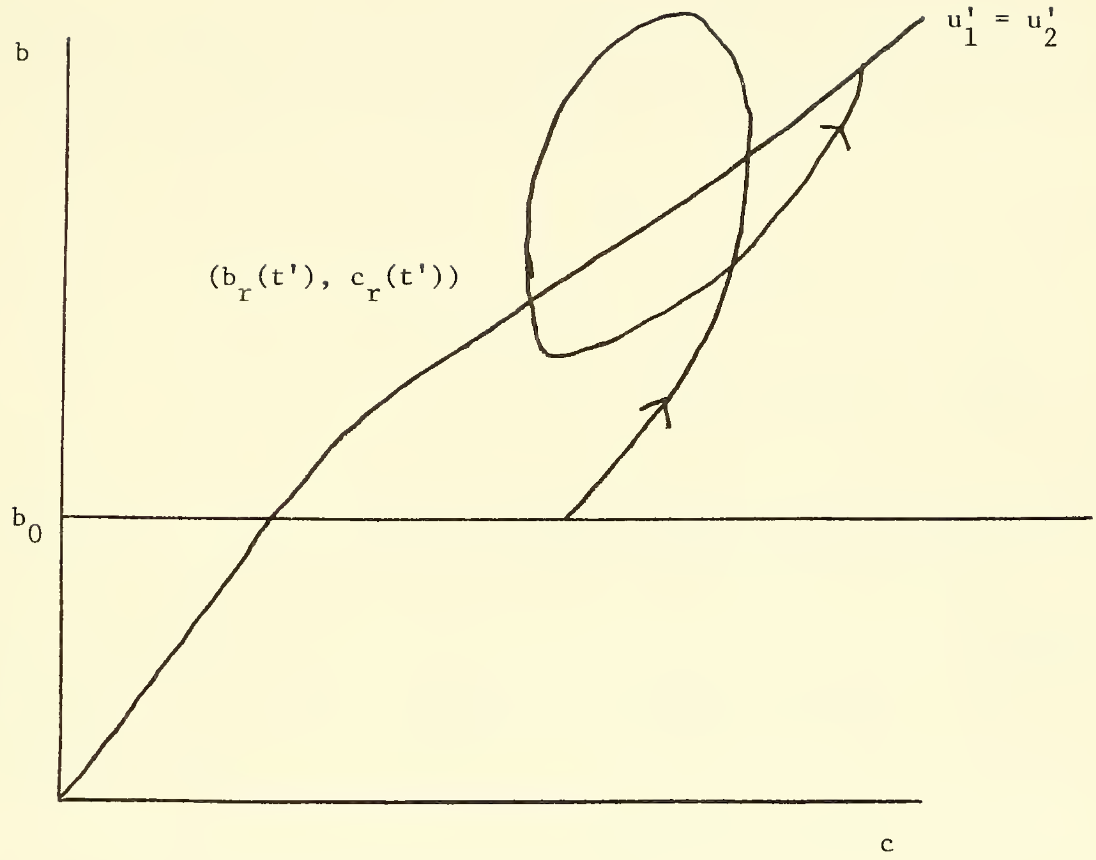
Our next step is to show that

$$\left.\begin{array}{l} c_s(t) < c_r(t) \\ \\ b_s(t) > b_r(t) \end{array}\right\} \quad \text{where } t_2 \leq s < r,\ t < s \qquad (52)$$

We know from Propositions 1 and 2 that (52) holds if $c_s(0) < c_r(0)$. If on the contrary $c_s(0) \geq c_r(0)$, the point $(c_s(s), b_s(s))$ lies below and to the right of $(c_r(s), b_r(s))$, which is below the EMU curve by Proposition 3. This is impossible since $(c_s(s), b_s(s))$ lies on the EMU curve. (52) is proved. Thus the planned consumption paths line up as shown in Figure 2.

We are now ready to construct our variation in the wage schedule which raises expected utility. Let $\delta W(0) = 0$, and let $\delta w(t)$ be nonzero only for $t_1 < t < t_2$, where $t_0 \leq t_1 < t_2$, and

Figure 3



$$u_1'\{c_s(t)\} < u'_2\{b_s(t)\} \text{ for } t_1 \leq t < s,\ t_2 \leq s < r. \quad (53)$$

Since $\dot{b}_{t_2}(t_2) > 0$ by (51), we can find $t_1 < t_2$ such that (53) holds for $s=t_2$.

Then by (52), (53) holds for the full range of s.

Furthermore let δw be positive in the lower half of the interval (t_1, t_2), and negative in the upper half. Since $u_1'(c_r(t))$ is decreasing in (t_1, t_2) by (35), and, by (46)

$$\int_{t_1}^{t_2} \delta w.(1 - F)dt = 0, \quad (54)$$

$$\delta V(r) = \int_{t_1}^{t_2} u_1'(c_r)\delta w.(1 - F)dt > 0. \quad (55)$$

From (35) and (52), we deduce that

$$\dot{u}_1'(c_s(t)) > \dot{u}_1'(c_r(t)) \text{ for } t_2 \leq s \leq r. \quad (56)$$

Therefore

$$\delta V(r) - \delta V(s) = \int_{t_1}^{t_2} (u_1'(c_r) - u_1'(c_s))\delta w(1 - F)dt$$

$$> 0 \qquad (t_2 \leq s \leq r) \quad (57)$$

since $(u_1'(c_r) - u_1'(c_s))$ is a decreasing function of t (by (56)) and $\delta w(1 - F)$ is first positive, then negative and integrates to zero.

Therefore (57) ensures that the constraints $V(s) \leq V(r)$ are maintained by our variation for $s > t_2$. Since, $\delta V(s) = 0$ for $s \leq t_1$ and $\delta V(s)$ in (t_1, t_2) does not matter (since the moral hazard constraint is not binding), all of the moral hazard constraints continue to hold. (55) implies that the variation brings about an improvement. Thus the supposition that, with an optimal wage path, there is a range of planned retirement ages for which the moral hazard constraint does not bind, is contradicted.

Next, we show that no wage variation can improve on the constant-utility schedule. To do this, we define a function $\mu(s)$ that will play the role of the Lagrange multipliers for the moral hazard constraints (39), by requiring that for all t, $0 \leq t < r$

$$\int_t^r \mu(s) u_1'\{c_s(t)\} ds = u_1'\{c_r(t)\} - u_1'\{c_r(r)\}. \tag{58}$$

Restricting ourselves to cases where $r \neq T$, we show that $\mu > 0$ in $[0,r)$ and that

$$\int_0^r \mu(s) ds < 1. \tag{59}$$

It has already been shown that on the constant-utility path, $c_s(t)$ and $b_s(t)$ are increasing functions of t, that $u_1'\{c_s(s)\} = u_2'\{b_s(s)\}$, and that $c_s(t) < c_r(t)$, $b_s(t) > b_r(t)$ when $s < r$ (cf. (52)). Thus

$$q(s,t) = u_2'\{b_s(t)\} - u_1'\{c_s(t)\}$$

is a positive function, vanishing when $t = s$, and satisfying

$$q(s,t) < q(r,t).$$

From (35) we know that

$$\dot{u}_1'\{c_s(t)\} = - q(s,t)f(t)/[1-F(t)]$$

Differentiation of (58) therefore yields

$$\mu(t) u_1'\{c_t(t)\} = \left[q(r,t) - \int_t^r \mu(s) q(s,t) ds\right] \frac{f(t)}{1 - F(t)} \tag{60}$$

From this we deduce first that $\mu(r) = 0$, since we have assumed $r < T$. Then we see that μ is a continuous function, and that since $q(r,t) > q(s,t)$, $\mu(t) > 0$ so long as $t < r$ and

$$\int_t^r \mu(s)ds < 1. \tag{61}$$

At the same time, since $u_1'\{c_s(t)\} > u_1'\{c_r(t)\}$ when $s < r$, (58) implies that (61) holds if $\mu(s) \geq 0$ for $t \leq s \leq r$.

It follows that, as claimed, $\mu(s) > 0$ for $0 \leq s < r$, and also that (59) holds. The special case $r = T$ would be analytically trickier, since we should have to show that μ is bounded near T. We presume that the result is true for that case as well.

Armed with the $\mu(s)$ function, we calculate, from (47),

$$\begin{aligned}\int_o^r \mu(s)\delta V(s)ds &= \int_o^r \int_o^s \mu(s)u_1'\{c_s(t)\}\delta w(t)[1 - F(t)]dt\, ds \\ &\quad + \int_0^r \mu(s)u_1'\{c_s(0)\}ds\ \delta W(0) \\ &= \int_o^r [\int_t^r \mu(s)u_1'\{c_s(t)\}ds]\delta w(t)[1 - F(t)]dt \\ &\quad + [u_1'\{c_r(0)\} - u_1'\{c_r(r)\}]\delta W(0) \\ &= \int_o^r u_1'\{c_r(t)\}\delta w(t)[1 - F(t)]dt - u_1'\{c_r(r)\}[\int_o^r \delta w[1-F]dt + \delta W(0)] \\ &\quad + u_1'\{c_r(0)\}\delta W(0) \\ &= \delta V(r)\end{aligned} \tag{62}$$

using the budget constraint (46).

From (62) it follows that $\delta V(r) > 0$ can hold only if, for some s, $\delta V(s) > \delta V(r)$. (Otherwise $\int \mu \delta V ds \leq \delta V(r) \int \mu ds < \delta V(r)$.) Since we start with V constant, this involves breaking one of the moral hazard constraints.

We have proved

Proposition 4. At the optimum

$$V(s) = V(r) \quad (s \leq r).$$

We also have the following

Proposition 5. At the optimum, for all $0 < t < s \leq r$,

(i) $u_1'\{c_s(t)\} < u_2'\{b_s(t)\}$

(ii) $\dot{c}_s(t) > 0$

(iii) $b_s(s)$ and $c_s(s)$ are increasing functions of s

(iv) $b_s(t)$ is a decreasing function of s: $c_s(t)$ is an increasing function of s.

(i) follows from Propositions (3) and (4), (ii) from (i), and (iv) from the analysis leading to (52). (iii) is a consequence of the fact that the consumption paths move to the right as s increases.

Using Propositions 4 and 5, we can analyze the optimal wage structure. Setting $\dot{V} = 0$ in (37), and using the fact that

$$(T - t)\dot{b}_s(t) = b_s(t) - c_s(t) + w(t)$$

we have

$$w(s) = \frac{u_2(b_s(s)) - u_1(c_s(s))}{u_2'(b_s(s))} - b_s(s) + c_s(s) \tag{63}$$

Differentiating (63) with respect to s, and using the equality of $u_1'(c_s(s))$ and $u_2'(b_s(s))$, yields

$$\dot{w}(s) = - \frac{u_2(b_s(s)) - u_1(c_s(s))}{[u_2'(b_s(s))]^2} u_2''(b_s(s)) \frac{d}{ds} b_s(s)$$

$$> 0,$$

since $u_2 > u_1$ on the equal marginal utility curve; and, by Proposition 5, $b_s(s)$ is an increasing function of s. We have proved the striking result:

Proposition 6. $w(s)$ is an increasing function.

8. Planned Retirement Date

Since for any given r, V(s) is constant, we can obtain a simple necessary condition for the optimal choice of r. If, keeping V constant, a change in r made more resources available, we would not have an optimum. Therefore r minimizes

$$W(0) + \int_o^r [w(t) - 1][1 - F(t)]dt \tag{64}$$

This implies, in particular,

Proposition 7. If r is optimal

$$w(r) = 1 \quad \text{and } r < T$$

$$\text{or } w(r) \leq 1 \quad \text{and } r = T$$

Combining Propositions 6 and 7 we have

$$w(s) < 1 \qquad (s < r)$$

In other words, the level of the social insurance contribution is positive, but decreases with age until the socially desired retiring age r, at which the contribution is zero. To continue the equal utility path beyond r would require subsidizing labor. Thus it is sufficient to offer no wage subsidies to have no one planning to work beyond r.

This is a good point at which to compare the properties of the optimum for the present model with the optimum when private saving is not allowed. In our previous paper, we showed that the optimum in the absence of private saving also has the property that the consumer is completely indifferent about retirement age.

We also showed that the lifetime consumption of the consumer increased for each year of work by an amount that is less than the marginal product of labor, but at a rate that increased with the number of years worked. This result for the problem without private saving corresponds to the proposition here that the net wage is an increasing function of years worked.

Also we showed that $u_1'(c(t)) < u_2'(b(t))$ for all $t < r$, and consumption at and after the retirement age is defined by

$$\frac{u_2(b(r)) - u_1(c(r))}{u_2'(b(r))} - b(r) + c(r) = 1 \tag{65}$$

$$u_1'(c(r)) = u_2'(b(r))$$

In our present problem, precisely the same equations hold for $c_r(r)$ and $b_r(r)$ when $r < T$: $(b_r(r), c_r(r))$ take the same values, although the value of r is not generally the same here. Consequently, some of the results from the previous paper carry over here. In particular, the value of $\frac{u_2-u_1}{u_2'} + c - b$

increases monotonically as one moves out the equal marginal utility curve. Thus there is at most one value of $(b_r(r),\ c_r(r))$ satisfying the first order conditions. We have not shown uniqueness of r such that the wage profile optimal for that r induces the appropriate values of b and c. As sufficient conditions for the existence of a (b,c) pair satisfying the first order condition we have the two properties that marginal utilities take on all positive values and that the disutility of labor not be vanishingly small.

Proposition 8. If for $i = 1,2$, $u_i'(0) = \infty$, $u_i'(\infty) = 0$ and if there exists $a > 0$ such that $u_2 - u_1 \geq a$ when $u_1' = u_2'$ Then there exists a pair $(\bar{b},\bar{c})$ satisfying the first order condition (65).

This is proved in our previous paper.

Now let us consider the question of whether the planned retirement date occurs before the (determinate) end of life. (Recall we have assumed $F(t) < 1$ for $t < T$.)

Making the assumptions of Proposition 8, consider the situation if $r = T$. $w(r) \leq 1$, so $c_T(T) \leq \bar{c}$, $b_T(T) \leq \bar{b}$. We have

$$\dot{b}_T(t) = [w(t) + b_T(t) - c_T(t)]\ \frac{1}{T - t} \tag{66}$$

As $t \to T$,

$$\begin{aligned} w(t) + b_T(t) - c_T(t) &\to \frac{u_2\{b_T(T)\} - u_1\{c_T(T)\}}{u_1'\{c_T(T)\}} \\ &\geq \frac{a}{u_1'\{c_T(T)\}} \end{aligned} \tag{67}$$

If $c_T(T) > 0$, integration of the right hand side of (66) up to T yields a divergent integral, which is inconsistent with $b_T(T) \geq \bar{b}$. But if $c_T(T) = 0$ we obviously do not have an optimum: the budget is not exhausted. We conclude:

Proposition 9. Under the assumptions of Proposition 8, the optimal r is less than T.

It is plausible that life expectancy, conditional on disability, remains positive at any age. Thus, in a model where, for simplicity, a known length of life is assumed, it might have been more appropriate to assume that disability must occur by $T' < T$. The only change this would make in our analysis is the introduction of the further possibility that the desired planned retirement age equals T'. With positive life expectancy at T', increases in retirement benefits remain an incentive to work right up to T'. As formulated, the zero retirement period that would occur with planned retirement at T is the source of the result that the desired planned retirement date is less than T.

9. Computation of the Optimum

The optimum is defined by a rather unconventional set of equations, which it will be convenient to collect together, in the form of differential equations:

$$\frac{\partial}{\partial t} \{u_1'\{c_s(t)\}[1 - F(t)]\} = - u_2'\{b_s(t)\}f(t) \tag{68}$$

$$\frac{\partial}{\partial t} \{b_s(t)(T - t)\} = w(t) - c_s(t) \tag{69}$$

$$w(t) = k(c_t(t)) \tag{70}$$

where

$$k(c) = \frac{u_2(b) - u_1(c)}{u_2'(b)} - b + c \text{ with } u_2'(b) = u_1'(c) \tag{71}$$

$$u_1'\{c_s(s)\} = u_2'\{b_s(s)\} \tag{72}$$

$$b_s(0) = b_o \tag{73}$$

$$w(r) = 1 \tag{74}$$

$$b_o T - \int_o^r [1 - w(t)][1 - F(t)]dt = A \tag{75}$$

Equations (68) and (72) are derived from (32); equations (69) and (73) from (29); equations (70) and (71) from (63); equation (74) from Propositions 6 and 8; and equation (75) is the budget constraint.

Observe that s does not occur explicitly in equations (68) and (69). This is the key to obtaining a solution. Take some plausible value of b_o. The idea is then to calculate solutions to (68) and (69) for successively larger values of c(0). For each initial c(0), we find what value of s this solution corresponds to by finding when the solution crosses the equal marginal utility curve, $u_1'\{c(s)\} = u_2'\{b(s)\}$. To calculate these solutions we need to know w. Once we have calculated consumption paths for initial $c_t(0)$ up to $c_s(0)$, (70) tells us w(t) for $t \leq s$. Starting from c(0) slightly greater than $c_s(0)$, we can compute c(t) and b(t) for t up to s. Extrapolation then finds the time s', slightly greater than s, at which the path cuts the equal-marginal-utility curve. Using (70), one obtains w(s').

In this way the paths defining the optimum can be built up step by step. The computation ends when w reaches unity.

The equations can be solved for any particular value of b_o. Equation (75) is then used to find the value of A for which the computed solution is optimal. One would guess that resource use increases with b_o, but we have not proved this. Otherwise, there might be more than one solution corresponding to particular values of A. Since

$$V(r) = V(0) = Tu_2(b_o) \tag{76}$$

the solution with the largest value of b_o is the optimal one. In any case (76) shows us that b_o is a good measure of the expected utility provided by the optimum.

In Table 2 we give solutions for the case

$$u_1(c) = \log c - a, \quad u_2(b) = \log b$$

$$f(t) = 1, \qquad T = 1$$

A detailed account of the computational procedure is available from the authors on request. Two values of a, .5 and 1, are used: a = .5 gives the more reasonable figure for the disutility of work.

In Table 1 the expected utility achieved is compared with the first-best and the optimum when private saving is prevented. We also show the utility level in the absence of a social insurance program, with government resources equally distributed, which is possible only when A > 0. Solutions for the first two come from our previous paper. Utility is expressed as the level of consumption of a nonworker which gives the same level of utility. An interesting aspect of these results is the small size of the utility loss from the inability to control savings. In the more interesting case, the value of having a social insurance program is high.

In Table 2 we show the wage profiles and socially desirable retirement age. In the examples, the possibility of private savings lowers the socially desirable retirement ages. As with the previous case, the net wage rises significantly with age. Since the net wage equals take home pay plus the growth in expected pension benefits, many countries already have age related net wages, at least past the minimum eligibility age for retirement benefits. In the U.S., the net wage does not rise monotonically with age. While this may represent a redistribution element missing in a model where everyone is *ex ante* identical, this suggests a need to evaluate critically the parameters of existing social insurance systems.

In Table 3 are displayed the chosen consumption profiles for this model and that analyzed before. It is interesting how close the consumption profiles are to each other until close to the socially desirable retirement age where they diverge significantly. The great variation in the consumption profile is a reflection of the considerable intertemporal substitutability of consumption with an additive lifetime utility function and logarithmic instantaneous utility function.

TABLE 1: UTILITY[1]

Resources	a = .5			a = 1		
	.5	0	-.2	.5	0	-.2
First best	.779	.389	.234	.607	.303	.182
Optimum, no saving	.766	.383	.230	.593	.285	.170
Optimum, private saving	.765	.382	.229	.592	.282	.168
No intervention	.733			.588		

[1]Reported here is the consumption level of a nonworker with the same expected utility.

TABLE 2: NET WAGES

Resources	a = .5				a = 1			
	.5	.2	0	-.2	.5	.2	0	-.2
Age								
0	.38	.27	.19	.11	.59	.40	.28	.17
.1	.40	.28	.20	.12	.65	.44	.31	.19
.2	.42	.30	.21	.13	.72	.49	.34	.21
.3	.45	.31	.22	.13	.80	.54	.38	.23
.4	.47	.33	.23	.14	.90	.61	.43	.25
.5	.50	.35	.25	.15		.69	.48	.29
.6	.54	.37	.27	.16		.79	.56	.33
.7	.58	.41	.29	.17		.95	.67	.40
.8	.66	.46	.33	.20			.86	.51
.9	.80	.56	.40	.24				.78
.95	.97	.68	.48	.29				
.99			.76	.45				
Compulsory Retirement	.956 (.969)	.988	.996 (.998)	.999 (1.000)	.486 (.494)	.724	.845 (.882)	.934 (.963)

Bracketed figures refer to the case with no private saving.

a = .5

Resources	.5				0				-.2			
	c		b		c		b		c		b	
Age												
0	.97	(1.00)	.76	(.77)	.48	(.50)	.38	(.38)	.29	(.30)	.23	(.23)
.1	.99	(1.03)	.79	(.78)	.50	(.51)	.39	(.39)	.30	(.31)	.23	(.24)
.2	1.03	(1.05)	.81	(.81)	.51	(.53)	.40	(.40)	.31	(.32)	.24	(.24)
.3	1.06	(1.09)	.84	(.83)	.53	(.54)	.42	(.42)	.32	(.33)	.25	(.25)
.4	1.11	(1.13)	.87	(.86)	.56	(.56)	.44	(.43)	.33	(.34)	.26	(.26)
.5	1.17	(1.17)	.92	(.90)	.58	(.59)	.46	(.45)	.35	(.35)	.27	(.27)
.6	1.24	(1.24)	.97	(.95)	.62	(.62)	.49	(.47)	.37	(.37)	.29	(.28)
.7	1.35	(1.32)	1.06	(1.02)	.67	(.66)	.53	(.51)	.40	(.40)	.32	(.31)
.8	1.50	(1.45)	1.18	(1.12)	.75	(.73)	.59	(.56)	.45	(.44)	.35	(.33)
.9	1.79	(1.69)	1.46	(1.34)	.91	(.86)	.71	(.66)	.55	(.51)	.43	(.39)
.95	1.99	(1.92)	1.89	(1.64)	1.11	(1.01)	.87	(.77)	.67	(.60)	.52	(.46)
.99					1.71	(1.46)	1.37	(1.13)	1.04	(.88)	.81	(.67)

a = 1

Resources	.5				0				-.2			
	c		b		c		b		c		b	
Age												
0	.84	(.87)	.59	(.59)	.44	(.49)	.28	(.28)	.26	(.30)	.17	(.17)
.1	.88	(.90)	.63	(.63)	.46	(.52)	.30	(.30)	.28	(.31)	.18	(.18)
.2	.92	(.93)	.69	(.69)	.49	(.54)	.32	(.32)	.30	(.33)	.19	(.19)
.3	.96	(.96)	.76	(.75)	.53	(.57)	.34	(.34)	.32	(.35)	.20	(.20)
.4	.99	(.99)	.86	(.85)	.58	(.61)	.37	(.36)	.35	(.37)	.22	(.21)
.5					.65	(.66)	.41	(.39)	.39	(.40)	.24	(.23)
.6					.73	(.72)	.48	(.44)	.44	(.44)	.28	(.25)
.7					.85	(.80)	.59	(.51)	.53	(.50)	.34	(.29)
.8					.97	(.91)	.81	(.65)	.67	(.59)	.43	(.35)
.9									.93	(.78)	.69	(.49)
.95										(.96)		(.76)

Bracketed figures refer to the case with no private saving.

References

P.A. Diamond and J.A. Hausman, "Retirement Savings and Behavior," Report to Department of Labor, unpublished, 1981.

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