QUESTIONS FOR ORAL EXAM

† **1** Examples of compact and complex Lie groups (classical Lie groups).

† **2** Symplectic group: relation between compact, complex and real version.

† **3** Lie algebra of a Lie group (definition of the comutator [,]).

† **4** Properties of the exponential map.

† **5** A closed subgroup of a Lie group is a Lie groups (proof).

† **6** Lie theorem (some elements of a proof).

† **7** Adjoint representations Ad and ad, $ad_X Y = [X, Y]$

† **K** Reductive groups, Cartan involution, polar decomposition.

⊛ **1** Invariant measure for a compact group.

⊛ **2** Properties of characters of compact groups.

⊛ **3** How to construct an ad-invariant scalar product in $\mathfrak{g}$, Killing form.

⊛ **4** Lie algebras of compact/reductive group: decomposition into the center and semisimple part.

⊛ **5** Representations of compact groups decompose into irreducible representations.

⊛ **6** Representations of tori. Representation ring for a torus.

⊛ **7** Maximal tori of compact groups (all are conjugate).

⊛ **K** Decomposition of a Lie algebra into root subspaces (examples for classical groups).

‖ **1** Representations of $\mathfrak{sl}_2(\mathbb{C})$.

‖ **2** Examples of $\mathfrak{sl}_3(\mathbb{C})$ representations.

‖ **3** Rank one groups.

‖ **4** Properties of root systems of compact/reductive Lie groups, abstract root systems.

‖ **5** Dynkin diagrams and classification of compact/reductive Lie groups.

‖ **6** Weyl group and its action on Weyl chambers

‖ **7** The center and the fundamental group of a Lie group.

‖ **K** Highest weight of a representation and classification of irreducible representations of a compact/reductive Lie algebras and groups.

⋃ **1** Low rank Lie groups SL_2, Sp_2, $SO(4)$, $SO(5)$: describe their irreducible representation.

⋃ **2** Irreducible representations of SL_n. How to construct them? Pieri formula: examples of aplications.

⋃ **3** How to compute the dimensions of a weight subspace of an irreducible representation.

⋃ **4** Weil Character formula: the case of SL_n – Schur functions, the case of Sp_n (examples)

⋃ **5** Clifford algebra and Spin group.

⋃ **6** Representations of $Spin(n)$ – spinors.

⋃ **7** Special properies of $\mathfrak{so}(8)$.

⋃ **K** The exceptional group G_2.