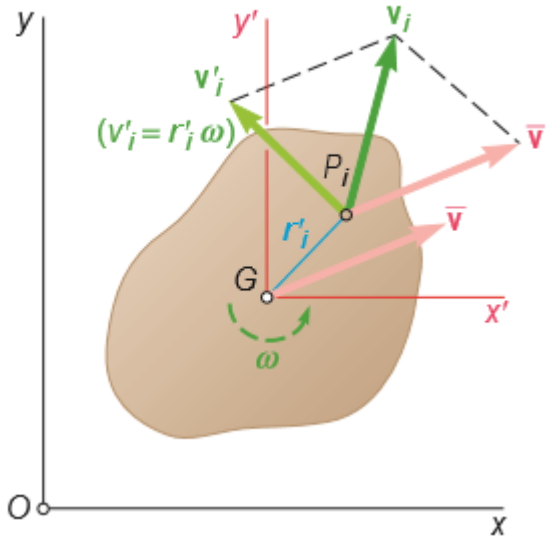
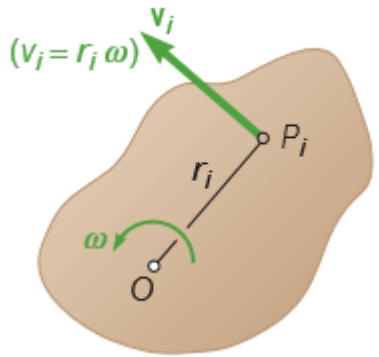
Ενέργεια, ορμή και στροφορμή

Ενέργεια και έργο

$$T_1 + U_{1\mathbf{y}2} = T_2$$

$$T = \frac{1}{2}\sum_{i=1}^{n} \Delta m_i\, v_i^2$$





$$T = \tfrac{1}{2}m\bar{v}^2 + \frac{1}{2}\sum_{i=1}^{n} \Delta m_i v_i'^2$$

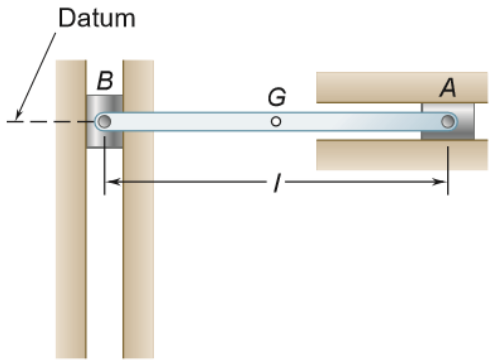
$$T = \tfrac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\sum_{i=1}^{n} r_i'^2\,\Delta m_i\right)w^2$$

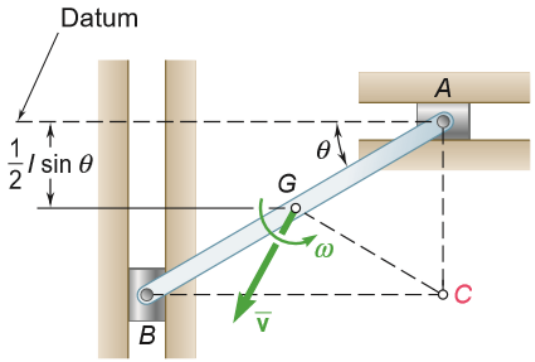
$$T = \tfrac{1}{2}m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2$$

$$T = \tfrac{1}{2}I_O\,\omega^2$$

Διατήρηση Ενέργειας

$$T_1 + V_1 = T_2 + V_2$$





$$T_1 = 0.$$

$$V_1 = 0.$$

$$V_2 = -\tfrac{1}{2}Wl\sin\Theta = -\tfrac{1}{2}mgl\sin\Theta$$

$$\bar{CG} = \tfrac{1}{2}l, \quad \bar{v}_2 = \tfrac{1}{2}l\,\omega$$

$$T_2 = \tfrac{1}{2}m\bar{v}_2^2 + \tfrac{1}{2}\bar{I}\omega_2^2 = \tfrac{1}{2}m(\tfrac{1}{2}l\,\omega)^2 + \tfrac{1}{2}(\tfrac{1}{12}ml^2)\omega^2$$

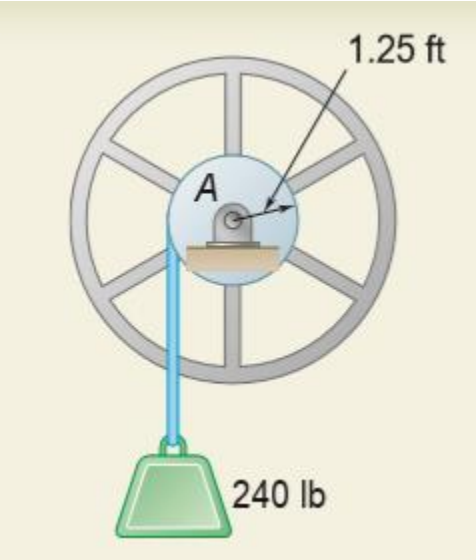
$$= \frac{1}{2}\frac{ml^2}{3}\omega^2$$

Ισχύς

$$\text{Power} = \frac{dU}{dt} = \mathbf{F} \cdot \mathbf{v}$$

$$\text{Power} = \frac{dU}{dt} = \frac{M d\theta}{dt} = M\omega$$

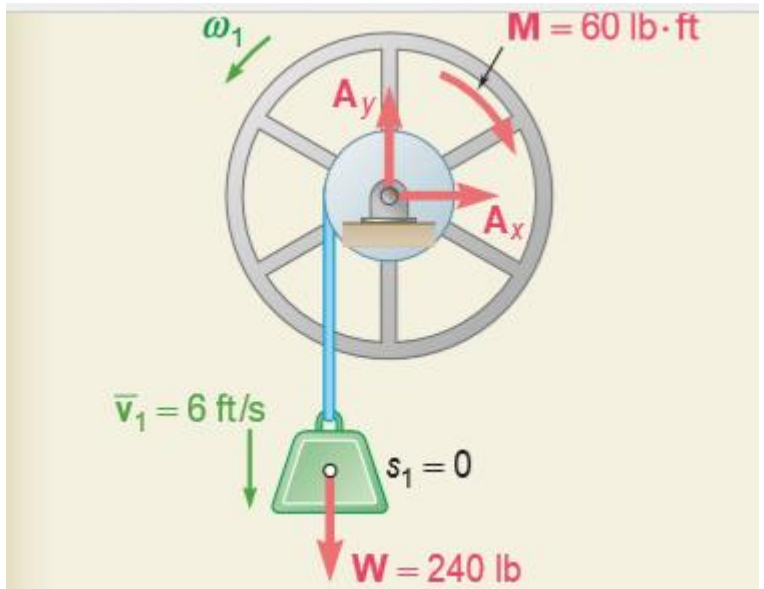
Παράδειγμα



- Το μπλόκ των 240 lb κρέμεται από μη εκτατό καλώδιο γύρο από το τύμπανο με ακτίνα 1.25ft στερεωμένο στο σφόνδυλο. Το τύμπανο και ο σφόνδυλος έχουν ροπή αδρανείας

$$\bar{I} = 10.5\,\text{lb} \cdot \text{ft} \cdot \text{s}^2$$

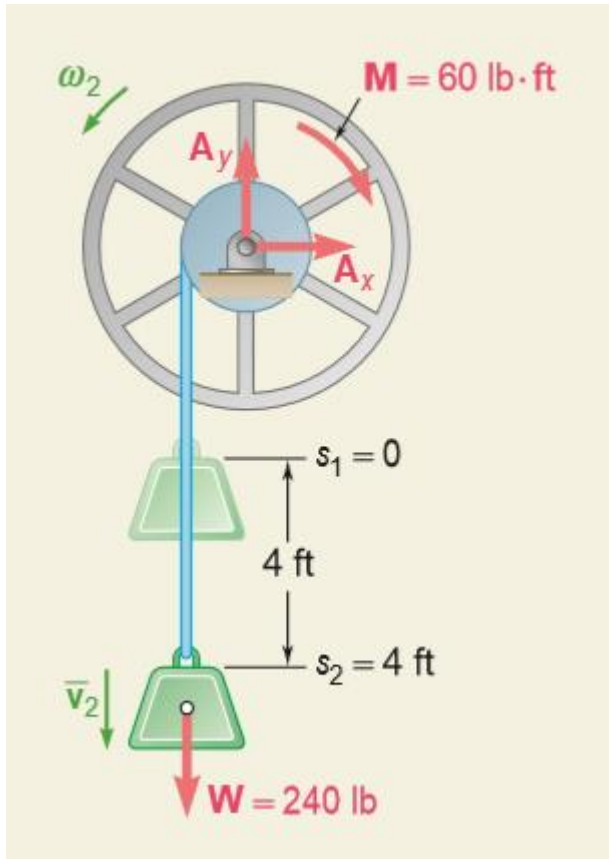
Αν η ταχύτητα του μπλόκ είναι 6 ft/s προς τα κάτω και υπάρχει ροπή αντίδρασης στο ρουλεμάν στην άρθρωση M=60lb*ft, ποια η ταχύτητα του μπλοκ όταν είναι στα 4ft πιο κάτω



$$\bar{v}_1 = 6 \text{ ft/s}$$

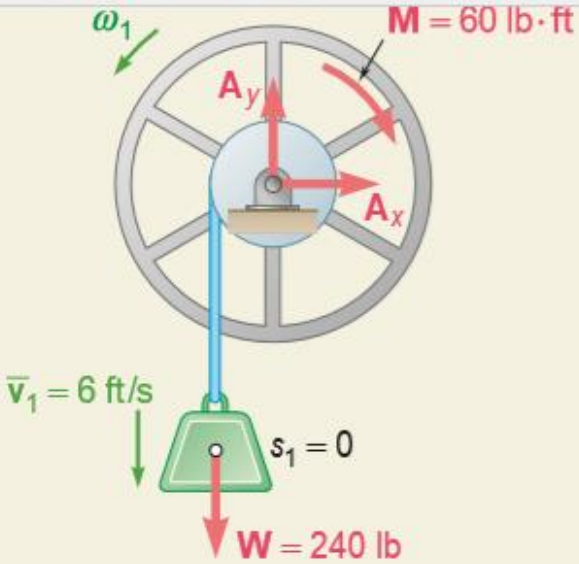
$$w_1 = \frac{\bar{v}_1}{r} = \frac{6 \text{ ft/s}}{1.25 \text{ ft}} = 4.80 \text{ rad/s}$$

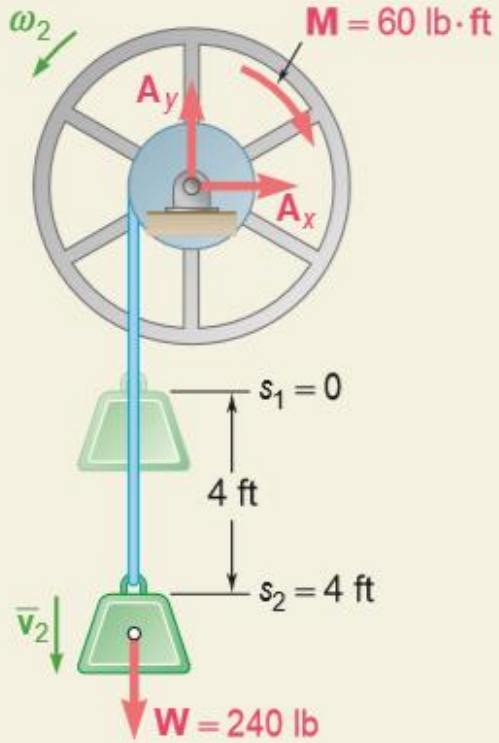
$$\begin{aligned} T_1 &= \tfrac{1}{2} m \bar{v}_1^2 + \tfrac{1}{2} \bar{I} \omega_1^2 \\ &= \frac{1}{2} \frac{240 \text{ lb}}{32.2 \text{ ft/s}^2} (6 \text{ ft/s})^2 + \tfrac{1}{2} (10.5 \text{ lb} \cdot \text{ft} \cdot \text{s}^2)(4.80 \text{ rad/s})^2 \\ &= 255 \text{ ft} \cdot \text{lb} \end{aligned}$$



$$\omega_2 = \bar{v}_2/1.25,$$

$$T_2 = \tfrac{1}{2}m\bar{v}_2^2 + \tfrac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}\frac{240}{32.2}(\bar{v}_2)^2 + (\tfrac{1}{2})(10.5)\left(\frac{\bar{v}_2}{1.25}\right)^2 = 7.09\bar{v}_2^2$$



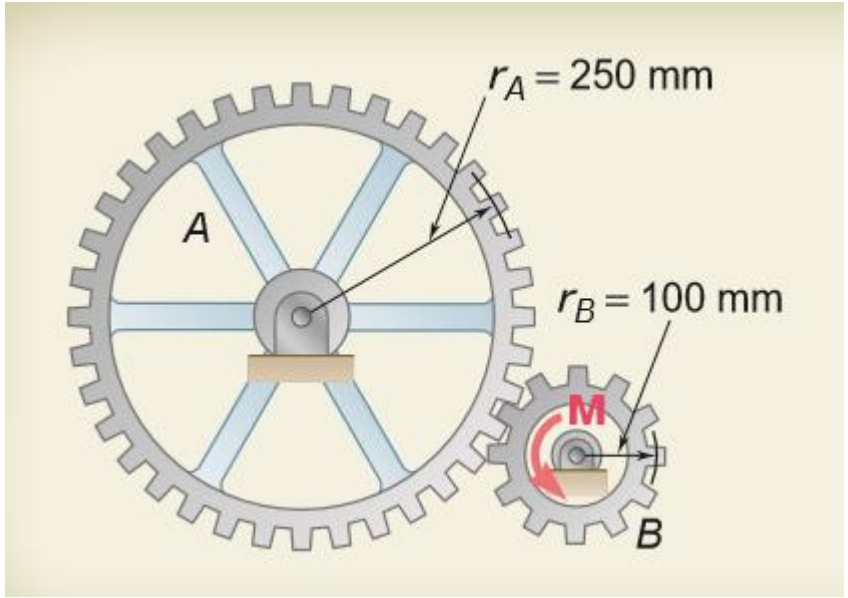


$$s_1 = 0 \qquad s_2 = 4 \text{ ft}$$

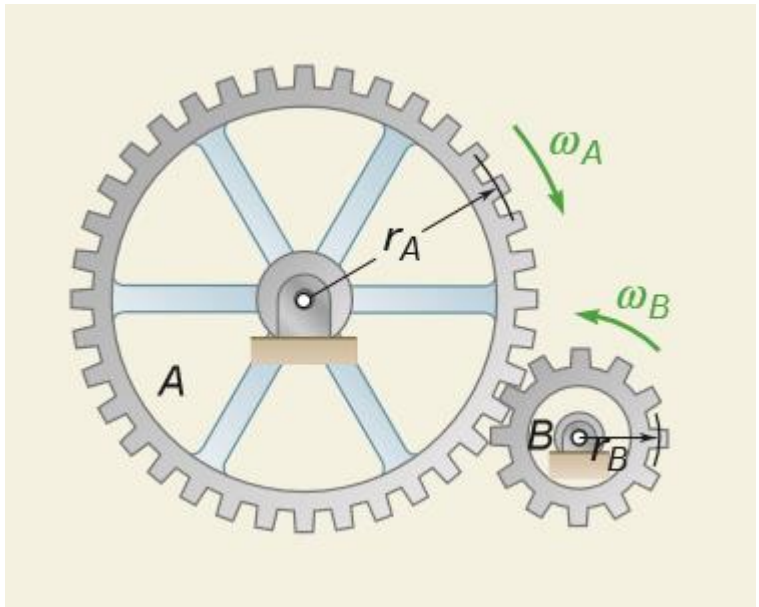
$$u_1 = 0 \qquad u_2 = \frac{s_2}{r} = \frac{4 \text{ ft}}{1.25 \text{ ft}} = 3.20 \text{ rad}$$

$$\begin{aligned} U_{1y2} &= W(s_2 - s_1) - M(u_2 - u_1) \\ &= (240 \text{ lb})(4 \text{ ft}) - (60 \text{ lb} \cdot \text{ft})(3.20 \text{ rad}) \\ &= 768 \text{ ft} \cdot \text{lb} \end{aligned}$$

$$T_1 + U_{1\mathbf{y}2} = T_2$$
$$255\ \text{ft} \cdot \text{lb} + 768\ \text{ft} \cdot \text{lb} = 7.09\bar{v}_2^2$$
$$\bar{v}_2 = 12.01\ \text{ft/s} \qquad \bar{\mathbf{v}}_2 = 12.01\ \text{ft/s}$$



- Το γρανάζι Α έχει μάζα 10 κιλά και ακτίνα περιστροφής 200 χιλιοστά. Το Β μάζα 3 κιλά και ακτίνα περιστροφής 80 χιλιοστά. Το σύστημα ηρεμεί όταν ασκείται ροπή M=6N*m στο Β. Αμελώντας τη τριβή, να βρεθεί ο αριθμός περιστροφών που το Β θέλει για να φτάσει τα 600 rpm και η εφαπτομενική δύναμη από το Β στο Α



$$r_A\omega_A = r_B\omega_B \qquad \omega_A = \omega_B\frac{r_B}{r_A} = \omega_B\frac{100\text{ mm}}{250\text{ mm}} = 0.40\omega_B$$

$\omega_B = 600$ rpm,

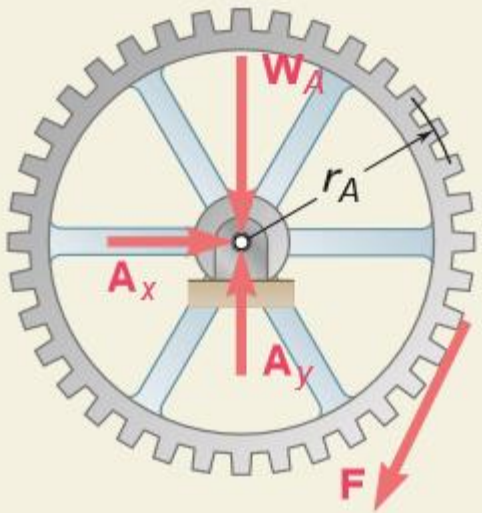
$$\omega_B = 62.8\text{ rad/s} \qquad \omega_A = 0.40\,\omega_B = 25.1\text{ rad/s}$$
$$\bar{I}_A = m_A\bar{k}_A^2 = (10\text{ kg})(0.200\text{ m})^2 = 0.400\text{ kg}\cdot\text{m}^2$$
$$\bar{I}_B = m_B\bar{k}_B^2 = (3\text{ kg})(0.080\text{ m})^2 = 0.0192\text{ kg}\cdot\text{m}^2$$

$T_1 = 0.$

$$T_2 = \tfrac{1}{2}\bar{I}_A\omega_A^2 + \tfrac{1}{2}\bar{I}_B\omega_B^2$$
$$= \tfrac{1}{2}(0.400\text{ kg}\cdot\text{m}^2)(25.1\text{ rad/s})^2 + \tfrac{1}{2}(0.0192\text{ kg}\cdot\text{m}^2)(62.8\text{ rad/s})^2$$
$$= 163.9\text{ J}$$

$$U_{1\gamma 2} = M\theta_B = (6\text{N}\cdot\text{m})(\text{u}_B\text{ rad}) = (6\theta_B)\text{ J}$$

$$T_1 + U_{1y2} = T_2$$
$$0 + (6\theta_B)\ \text{J} = 163.9\ \text{J}$$
$$\theta_B = 27.32\ \text{rad}$$

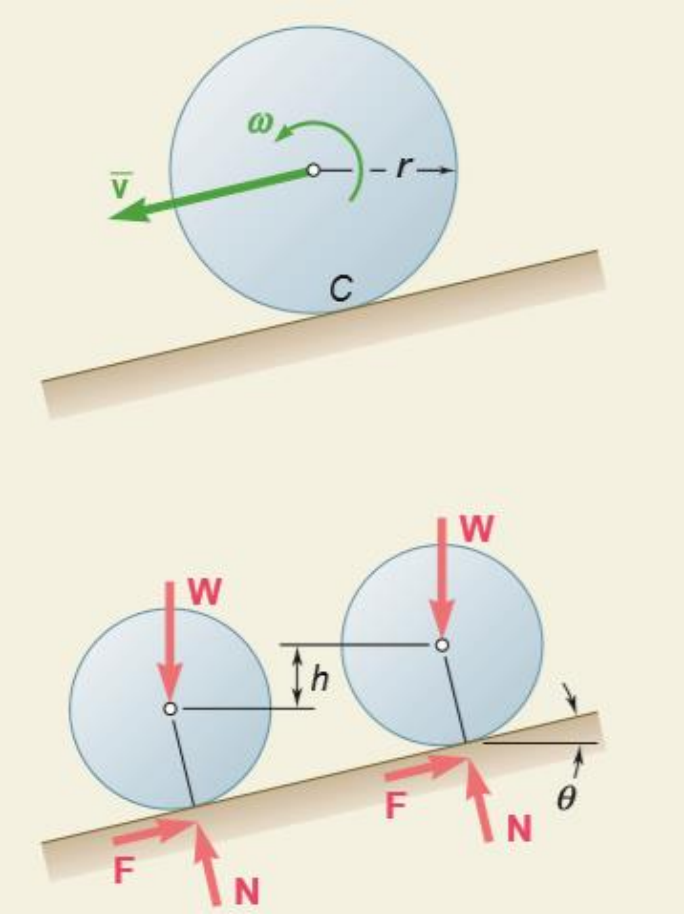


$$T_2 = \tfrac{1}{2}\bar{I}_A\omega_A^2 = \tfrac{1}{2}(0.400\ \text{kg}\cdot\text{m}^2)(25.1\ \text{rad/s})^2 = 126.0\ \text{J}$$

$$U_{1y2} = F(\theta_B r_B) = F(27.3\ \text{rad})(0.100\ \text{m}) = F(2.73\ \text{m})$$

$$T_1 + U_{1y2} = T_2$$
$$0 + F(2.73\ \text{m}) = 126.0\ \text{J}$$
$$F = +46.2\ \text{N} \qquad \mathbf{F = 46.2\ N}$$

Μια σφαίρα, ένας κύλινδρος και ένας δίσκος με ίδια ακτίνα αφήνονται από την ηρεμία σε κεκλιμένο επίπεδο. Ποια η ταχύτητα τους όταν διανύσουν μια απόσταση που αντιστοιχεί σε ύψος h



$$\omega = \frac{\bar{v}}{r}$$

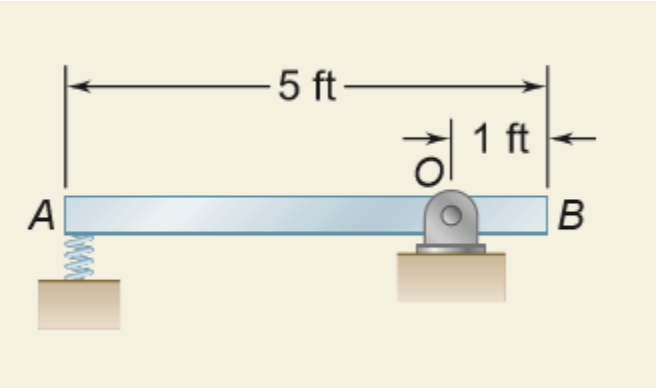
$$T_1 = 0$$
$$T_2 = \tfrac{1}{2} m\bar{v}^2 + \tfrac{1}{2}\bar{I}\omega^2$$
$$= \tfrac{1}{2} m\bar{v}^2 + \tfrac{1}{2}\bar{I}\left(\frac{\bar{v}}{r}\right)^2 = \tfrac{1}{2}\left(m + \frac{\bar{I}}{r^2}\right)\bar{v}^2$$

$$U_{1y2} = Wh$$

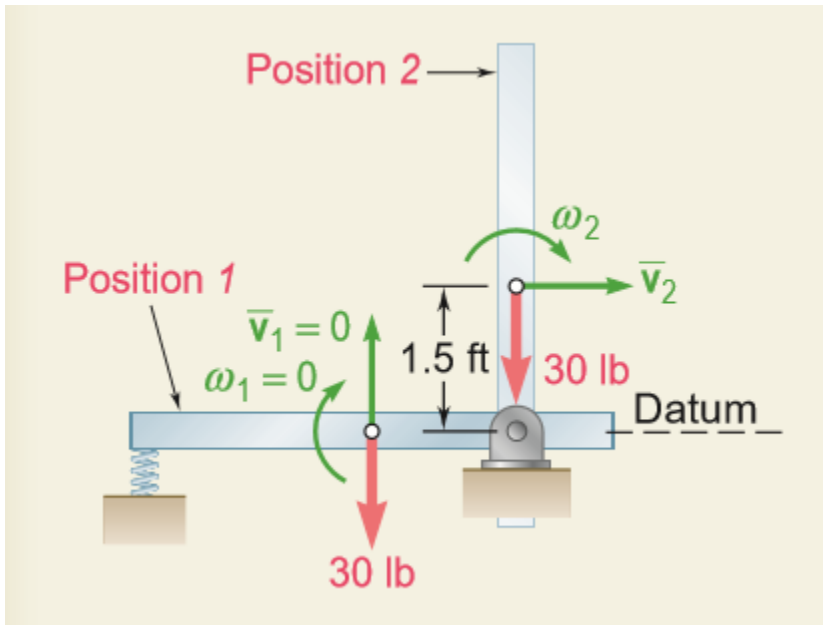
$$T_1 + U_{1y2} = T_2$$
$$0 + Wh = \tfrac{1}{2}\left(m + \frac{\bar{I}}{r^2}\right)\bar{v}^2 \qquad \bar{v}^2 = \frac{2Wh}{m + \bar{I}/r^2}$$

$$\bar{v}^2 = \frac{2gh}{1 + \bar{I}/mr^2}$$

Sphere:	$\bar{I} = \frac{2}{5} mr^2$	$\bar{v} = 0.845\sqrt{2gh}$
Cylinder:	$\bar{I} = \frac{1}{2} mr^2$	$\bar{v} = 0.816\sqrt{2gh}$
Hoop:	$\bar{I} = mr^2$	$\bar{v} = 0.707\sqrt{2gh}$



- Η ράβδος ΑΒ των 30 lb με μήκος 5 ft περιστρέφεται γύρω από το Ο, που βρίσκεται 1 ft από το Β. Το ελατήριο στο Α έχει σταθερά 1800 lb/in. Η περιστροφή γίνεται μέχρι το ελατήριο να έχει συμπιεστεί για 1 in, οπότε η ράβδος είναι σε οριζόντια θέση. Αν αφεθεί, ποια η γωνιακή της ταχύτητα και η αντίδραση στο Ο όταν περνά από την κάθετη θέση;



Θέση 1

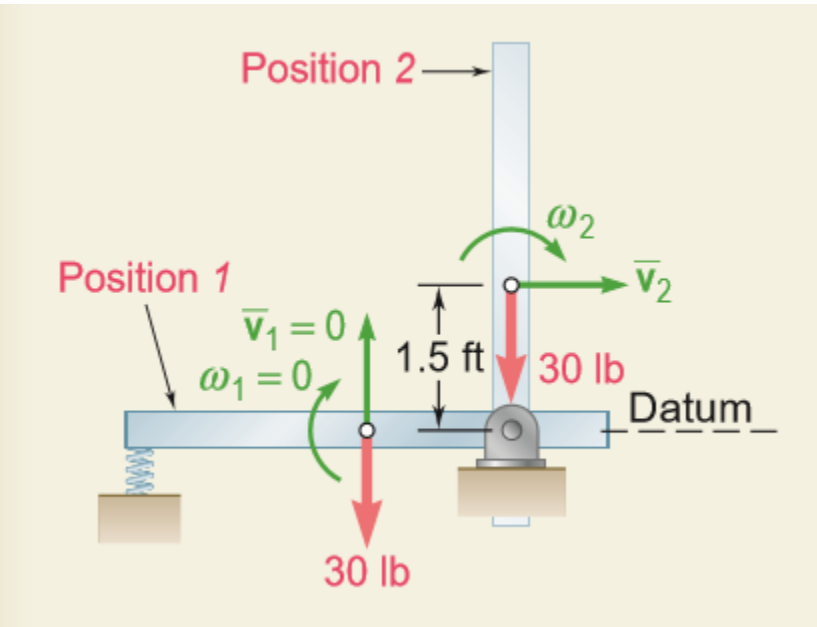
$$V_e = \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}(1800\ \text{lb/in.})(1\ \text{in.})^2 = 900\ \text{in} \cdot \text{lb}$$

$$V_1 = V_e + V_g = 900\ \text{in} \cdot \text{lb} = 75\ \text{ft} \cdot \text{lb}$$

$$V_g = 0$$

$$T_1 = 0$$

Θέση 2



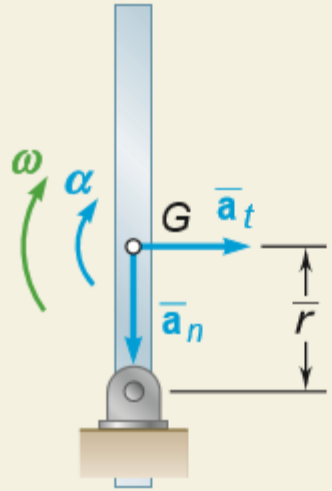
$$V_g = (30\text{ lb})(+1.5\text{ ft}) = 45\text{ ft}\cdot\text{lb}$$
$$V_2 = V_e + V_g = 45\text{ ft}\cdot\text{lb}$$

$$\bar{v}_2 = \bar{r}\omega_2 = 1.5\omega_2.$$

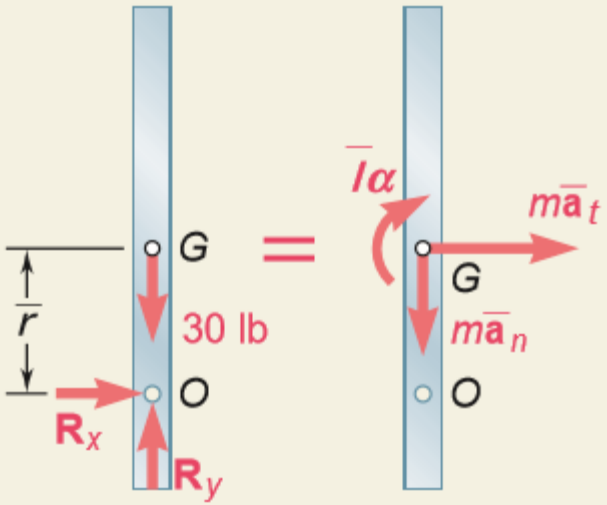
$$\bar{I} = \tfrac{1}{12}ml^2 = \frac{1}{12}\frac{30\text{ lb}}{32.2\text{ ft/s}^2}(5\text{ ft})^2 = 1.941\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$T_2 = \tfrac{1}{2}m\bar{v}_2^2 + \tfrac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\frac{30}{32.2}(1.5\omega_2)^2 + \tfrac{1}{2}(1.941)\omega_2^2 = 2.019\,\omega_2^2$$

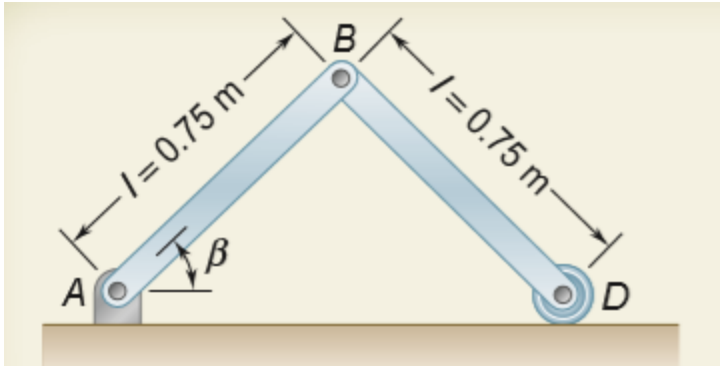
$$T_1 + V_1 = T_2 + V_2$$
$$0 + 75\text{ ft}\cdot\text{lb} = 2.019\omega_2^2 + 45\text{ ft}\cdot\text{lb}$$
$$\omega_2 = 3.86\text{ rad/s}$$



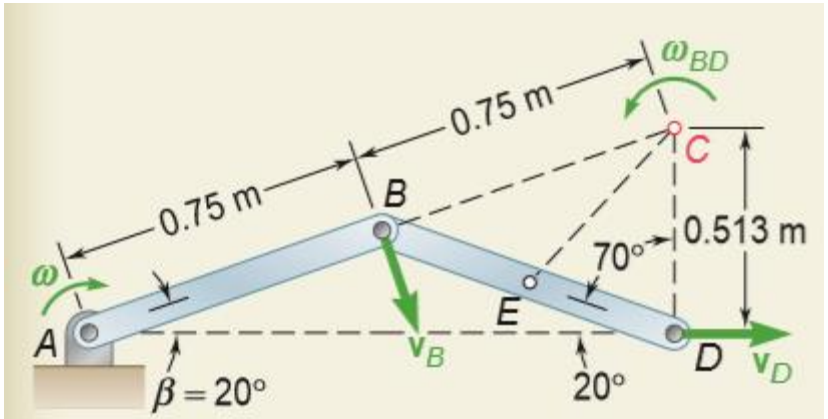
$$\bar{a}_n = \bar{r}\omega_2^2 = (1.5\text{ ft})(3.86\text{ rad/s})^2 = 22.3\text{ ft/s}^2 \qquad \bar{\mathbf{a}}_n = 22.3\text{ ft/s}^2$$
$$\bar{a}_t = \bar{r}\mathsf{a} \qquad \bar{\mathbf{a}}_t = \bar{r}\mathsf{a}$$



$$\Sigma M_O = \Sigma(M_O)_{\text{eff}}: \qquad 0 = \bar{I}\mathsf{a} + m(\bar{r}\mathsf{a})\bar{r} \qquad \mathsf{a} = 0$$
$$\Sigma F_x = \Sigma(F_x)_{\text{eff}}: \qquad R_x = m(\bar{r}\mathsf{a}) \qquad R_x = 0$$
$$\Sigma F_y = \Sigma(F_y)_{\text{eff}}: \qquad R_y - 30\text{ lb} = -m\bar{a}_n$$
$$R_y - 30\text{ lb} = -\frac{30\text{ lb}}{32.2\text{ ft/s}^2}(22.3\text{ ft/s}^2)$$
$$R_y = +9.22\text{ lb} \qquad \mathbf{R} = 9.22\text{ lb}$$

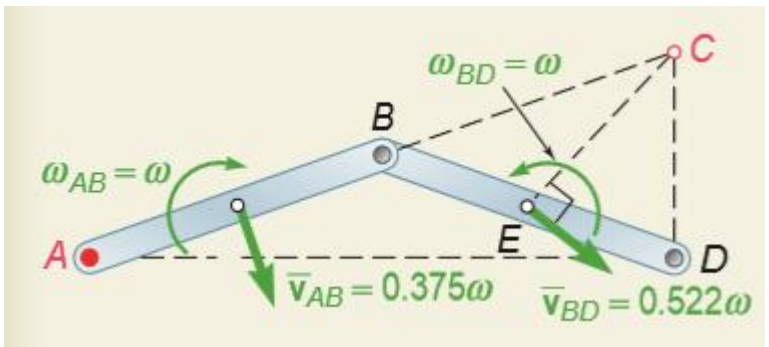


- Η μάζα κάθε ράβδου είναι 6 κιλά. Αν το σύστημα ελευθερωθεί με β=60 ποια η γωνιακή ταχύτητα της ΑΒ αν β=20, και ποια η ταχύτητα του D στο ίδιο σημείο



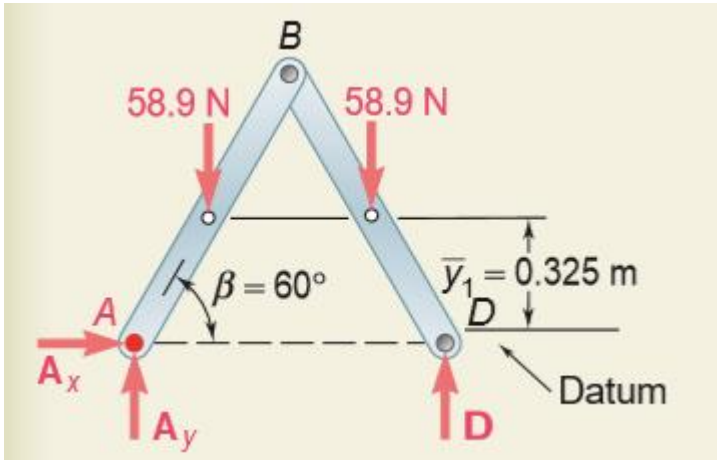
$$BC = 0.75 \text{ m} \qquad CD = 2(0.75 \text{ m}) \sin 20° = 0.513 \text{ m}$$

$$EC = 0.522 \text{ m}.$$



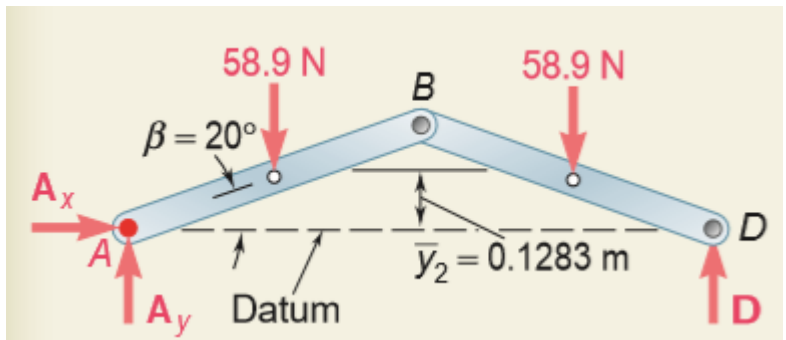
$$v_B = (BC)\omega_{BD} \qquad (0.75 \text{ m})\omega = (0.75 \text{ m})\omega_{BD} \qquad V_{BD} = \omega 1$$

$$\bar{v}_{BD} = (EC)\omega_{BD} = (0.522 \text{ m})\omega \qquad \bar{\omega}_{BD} = 0.522\,\omega$$

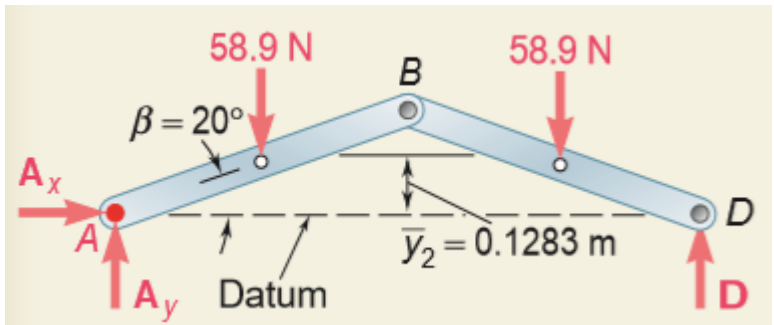


$$W = (6\ \text{kg})(9.81\ \text{m/s}^2) = 58.86\ \text{N},$$

$$V_1 = 2W\bar{y}_1 = 2(58.86\ \text{N})(0.325\ \text{m}) = 38.26\ \text{J}$$



$$V_2 = 2W\bar{y}_2 = 2(58.86\ \text{N})(0.1283\ \text{m}) = 15.10\ \text{J}$$

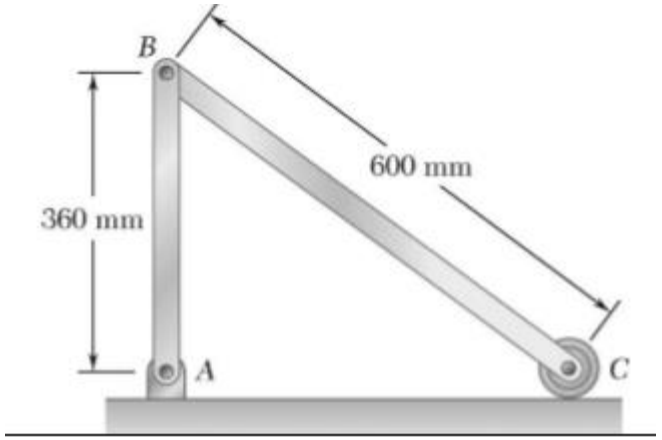


$$I_{AB} = \bar{I}_{BD} = \tfrac{1}{12} ml^2 = \tfrac{1}{12}(6\text{ kg})(0.75\text{ m})^2 = 0.281\text{ kg}\cdot\text{m}^2$$

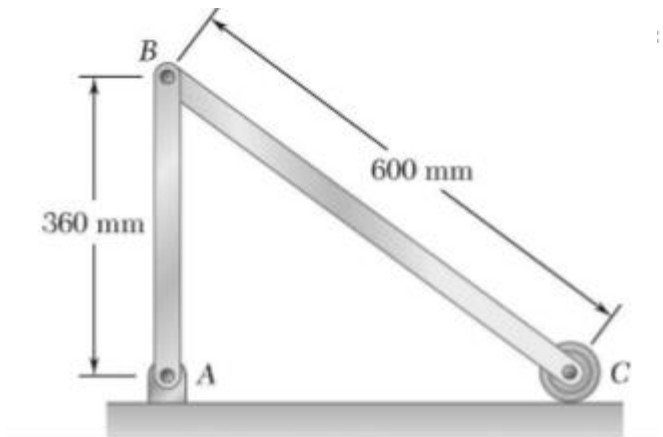
$$\begin{aligned} T_2 &= \tfrac{1}{2} m\bar{v}_{AB}^2 + \tfrac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \tfrac{1}{2} m\bar{v}_{BD}^2 + \tfrac{1}{2}\bar{I}_{BD}\omega_{BD}^2 \\ &= \tfrac{1}{2}(6)(0.375\omega)^2 + \tfrac{1}{2}(0.281)\omega^2 + \tfrac{1}{2}(6)(0.522\omega)^2 + \tfrac{1}{2}(0.281)\omega^2 \\ &= 1.520\omega^2 \end{aligned}$$

$$\begin{aligned} T_1 + V_1 &= T_2 + V_2 \\ 0 + 38.26\text{ J} &= 1.520\omega^2 + 15.10\text{ J} \\ \omega &= 3.90\text{ rad/s} \end{aligned}$$

$$v_D = (CD)\omega = (0.513\text{ m})(3.90\text{ rad/s}) = 2.00\text{ m/s}$$



- Οι ράβδοι έχουν μάζες 2.4 κιλά και 4 κιλά. Ο τροχός έχει αμελητέα μάζα. Αν στην θέση αυτή η ταχύτητα του τροχού είναι 2m/s δεξιά, ποια η ταχύτητα του B αν η AB έχει περιστραφεί κατά 90 μοίρες



$$I_A = \frac{1}{3} m_{AB} L_{AB}^2 = \frac{1}{3}(2.4)(0.36)^2 = 0.10368 \text{ kg} \cdot \text{m}^2$$

$$\frac{1}{12} m_{BC} L_{BC^2} = \frac{1}{12}(4)(0.600)^2 = 0.1200 \text{ kg} \cdot \text{m}^2$$

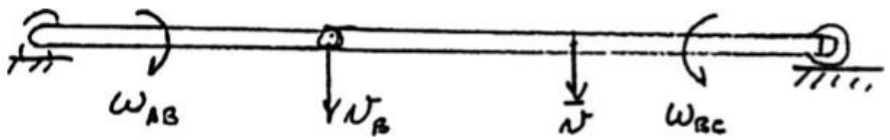
$$\begin{aligned} V_1 &= W_{AB} h_{AB} + W_{BC} h_{BC} \\ &= (2.4)(9.81)(0.180) + (4)(9.81)(0.180) \\ &= 11.301 \text{ J} \end{aligned}$$

$$\omega_{BC} = 0$$

$$\bar{v} = v_G = v_B = v_C = 2 \text{ m/s}$$

$$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{2}{0.36} = 5.5556 \text{ rad/s}$$

$$\begin{aligned} T_1 &= \frac{1}{2} I_A \omega_{AB}^2 + \frac{1}{2} m_{BC} \bar{v}^2 + \frac{1}{2} I \omega_{BC}^2 \\ &= \frac{1}{2}(0.10368)(5.5556)^2 + \frac{1}{2}(4)(2)^2 + 0 \\ &= 9.6 \text{ J} \end{aligned}$$



$V_2 = 0$

$$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{v_B}{0.36}$$

$$\omega_{BC} = \frac{v_B}{L_{BC}} = \frac{v_B}{0.60}$$

$$\bar{v} = \frac{1}{2} v_B$$

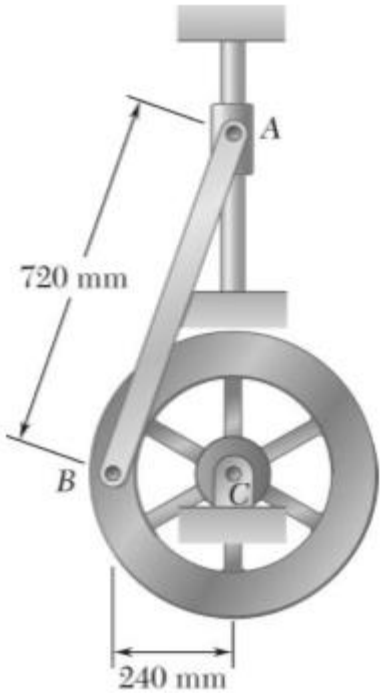
$$T_2 = \frac{1}{2} I_A \omega_{AB}^2 + \frac{1}{2} m_{BC} \bar{v}^2 + \frac{1}{2} \bar{I} \omega_{BC}^2$$

$$= \frac{1}{2}(0.10368)\left(\frac{v_B}{0.36}\right)^2 + \frac{1}{2}(4)\left(\frac{1}{2} v_B\right)^2 + \frac{1}{2}(0.12)\left(\frac{v_B}{0.60}\right)^2$$

$$= 1.0667 v_B^2$$

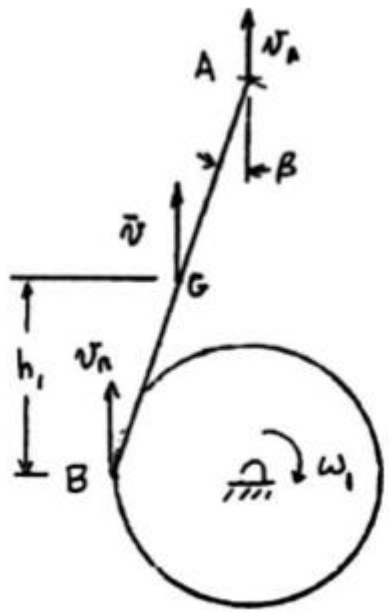
$$T_1 + V_1 = T_2 + V_2: \quad 9.6 + 11.301 = 1.0667 v_B^2 + 0$$

$$v_B = 4.4266 \text{ m/s}$$



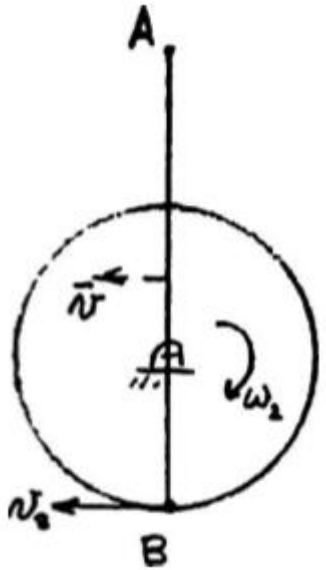
- Η ράβδος ΑΒ των 4 κιλών συνδέεται με το κολάρο στο Α και ένα σφόνδυλο στο Β, μάζας 16 κιλών και ακτίνας περιστροφής 180 χιλιοστών. Αν στην θέση αυτή η ταχύτητα του σφονδύλου είναι 60 rpm ωρολογιακά, να βρεθεί΄ η ταχύτητα του όταν το Β είναι κάτω από το C

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2$$
$$= \frac{1}{12}(4\text{ kg})(0.72\text{ m})^2$$
$$= 0.1728\text{ kg}\cdot\text{m}^2$$
$$I_C = m\bar{k}^2$$
$$= (16\text{ kg})(0.18\text{ m})^2$$
$$= 0.5184\ \text{kg}\cdot\text{m}^2$$



$$\omega = \omega_1 \curvearrowright$$
$$\sin\beta = \frac{0.24}{0.72} \quad \beta = 19.471^\circ$$
$$h_1 = \frac{1}{2}(0.72)\cos\beta = 0.33941\text{ m}$$
$$V_1 = W_{AB} h_1$$
$$= (4)(9.81)(0.33941)$$
$$= 13.3185\text{ J}$$
$$v_B = r\omega_1 = 0.24\omega_1$$

$$\omega_{AB} = 0, \quad \bar{v} = v_B$$
$$T_1 = \frac{1}{2} m_{AB} \bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2} I_C \omega_1^2$$
$$= \frac{1}{2}(4)(0.24\omega_1)^2 + 0 + \frac{1}{2}(0.5184)\omega_1^2$$
$$= 0.3744\omega_1^2$$



$$h_2 = \frac{1}{2}L_{AB} - r$$

$$= \frac{1}{2}(0.72) - 0.24$$

$$= 0.12 \text{ m}$$

$$V_2 = W_{AB}h_2$$

$$= (4)($$

$$= 4.7($$

$$T_1 + V_1 = T_2 + V_2: \quad 0.3744\omega_1^2 + 13.3185 = 0.2976\omega_2^2 + 4.7088$$

$$\omega_1 = 60 \text{ rpm} = 2\pi \text{ rad/s}$$

$$v_B = r\omega_2$$

$$\omega_2 = 8.8655 \text{ rad/s}$$

$$\omega_{AB} = \frac{v_B}{0.72} = 0.33333\omega_2$$

$$\bar{v} = \frac{1}{2}v_B = 0.12\omega_2$$

$$T_2 = \frac{1}{2}m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}I_C\omega_2^2$$

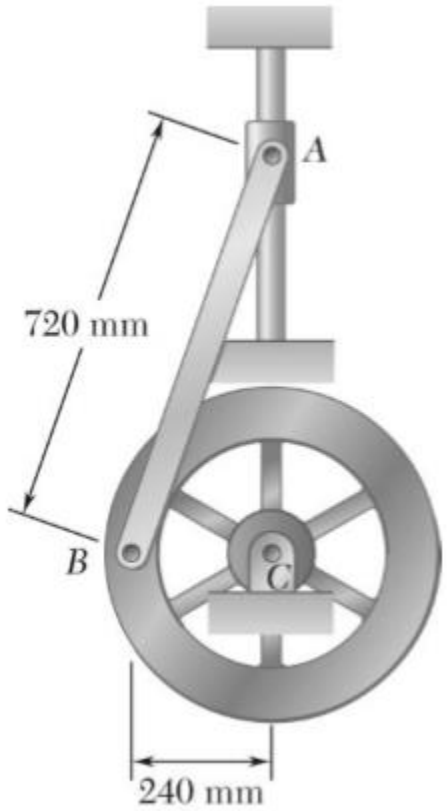
$$= \frac{1}{2}(4)(0.12\omega_2)^2 + \frac{1}{2}(0.1728)(0.33333\omega_2)^2 + \frac{1}{2}(0.5184)\omega_2^2$$

$$= 0.2976\omega_2^2$$

$$T_1 + V_1 = T_2 + V_2: \quad 0.3744\omega_1^2 + 13.3185 = 0.2976\omega_2^2 + 4.7088$$

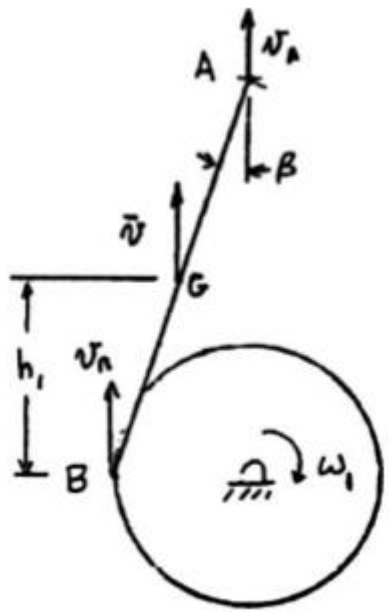
$$\omega_1 = 60 \text{ rpm} = 2\pi \text{ rad/s}$$

$$\omega_2 = 8.8655 \text{ rad/s}$$



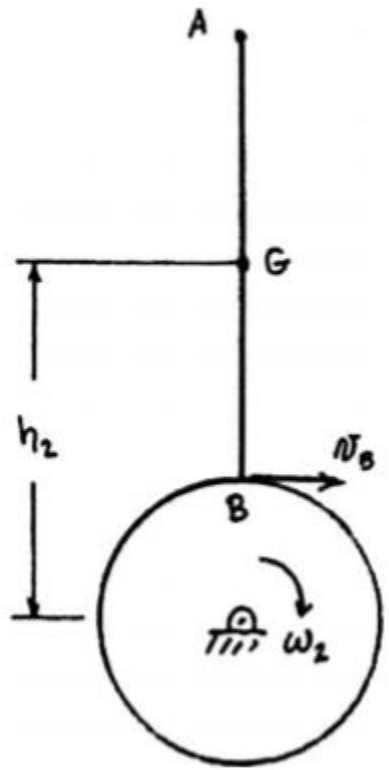
- Τα δεδομένα ίδια με την προηγούμενη άσκηση. Αν ο σφόνδυλος πρέπει να έχει την ίδια γων. Ταχύτητα στην θέση του σχήματος και όταν το Β είναι ακριβώς πάνω από το C ποια πρέπει να είναι αυτή;

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2$$
$$= \frac{1}{12}(4\text{ kg})(0.72\text{ m})^2$$
$$= 0.1728\text{ kg}\cdot\text{m}^2$$
$$I_C = m\bar{k}^2$$
$$= (16\text{ kg})(0.18\text{ m})^2$$
$$= 0.5184\ \text{kg}\cdot\text{m}^2$$



$$\omega = \omega_1 \curvearrowright$$
$$\sin\beta = \frac{0.24}{0.72} \quad \beta = 19.471^\circ$$
$$h_1 = \frac{1}{2}(0.72)\cos\beta = 0.33941\text{ m}$$
$$V_1 = W_{AB} h_1$$
$$= (4)(9.81)(0.33941)$$
$$= 13.3185\text{ J}$$
$$v_B = r\omega_1 = 0.24\omega_1$$

$$\omega_{AB} = 0, \quad \bar{v} = v_B$$
$$T_1 = \frac{1}{2} m_{AB} \bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2} I_C \omega_1^2$$
$$= \frac{1}{2}(4)(0.24\omega_1)^2 + 0 + \frac{1}{2}(0.5184)\omega_1^2$$
$$= 0.3744\omega_1^2$$



$$h_2 = \frac{1}{2}L_{AB} + r$$
$$= \frac{1}{2}(0.72) + 0.24$$
$$= 0.6 \text{ m}$$
$$V_2 = W_{AB}h_2$$
$$= (4)(9.81)(0.6)$$
$$= 23.544 \text{ J}$$

$$v_B = r\omega_2 = 0.24\omega_2$$

$$\omega_{AB} = \frac{v_B}{0.72} = 0.33333\omega_2$$

$$\bar{v} = \frac{1}{2}v_B = 0.12\omega_2$$

$$T_2 = \frac{1}{2}m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}I_C\omega_2^2$$
$$= \frac{1}{2}(4)(0.12\omega_2)^2 + \frac{1}{2}(0.1728)(0.33333\omega_2)^2 + \frac{1}{2}(0.5184)\omega_2^2$$
$$= 0.2976\omega_2^2$$

$$T_1 + V_1 = T_2 + V_2: \quad 0.3744\omega_1^2 + 13.3135 = 0.2976\omega_2^2 + 23.544$$

$$\omega_2 = \omega_1$$

$$0.0760\omega_1^2 = +0.4105$$

$$\omega_1 = 11.602 \text{ rad/s}$$