DESIGN OF AN AIRCRAFT MAIN WING SPAR

DESIGN PROJECT 2

Final Report

AEE 471 | Davidson

Assigned: October 24th, 2018

Due Date: November 16th, 2018

TYLER VARTABEDIAN

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BACKGROUND AND PROVIDED INFORMATION

The goal of this project is to design and optimize the main wing spar of a concept plane designed for personal use. This plane is currently designed to weigh 15,000 pounds with a 10-foot wingspan per wing. The main wing spar in question will be modeled as cantilever beam estimated to be subjected to a variety of loadings shown in Table 1 below, with the design and loading of the beam modeled in Figure 1 below. The design is rated for 10,000 flights, and the beam will feature a thin-walled channel cross section to make room for fuel tanks, fuel lines, and other integral systems. The beam will be manufactured out of 7075-T6 Aluminum which features material aspects shown in Table 2 below. Note that these values are obtained from the MIL-HDBK-5 and utilize A-basis allowables as specified by the manufacturer. These allowables are taken from the allowables where an area is assumed to be less than 20 square inches, and the extruded beam has a thickness in-between 3.1 and 4.4 inches.

Figure 1 - Geometry and Loading of Main Wing Spar *(AEE 471 Project 2 Handout - Davidson)*

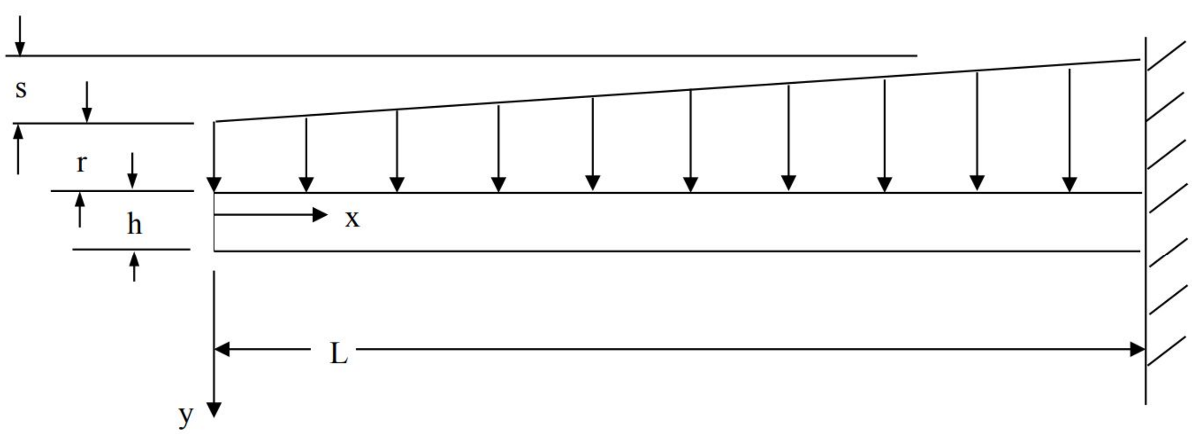


Table 1 - Expected Limit Load Spectrum for 1 Flight *(AEE 471 Project 2 Handout - Davidson)*					
EVENT	**r-min (lb/in)**	**r-max (lb/in)**	**s-min (lb/in)**	**s-max (lb/in)**	**N (cycles)**
Take-Off	26	55	18	48	1
Maneuver 1	24	58	12	48	50
Maneuver 2	20	60	15	50	5
Cruise	34.4	51.6	31.2	46.8	500
Landing Flare	26	55	18	48	1
Landing Touchdown	-40	5	-30	2	2

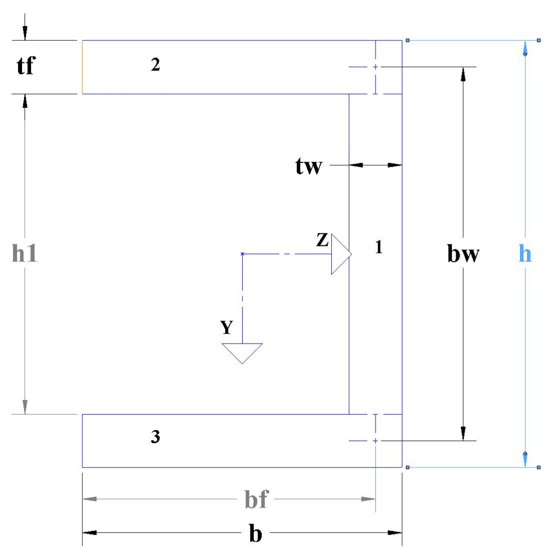
Table 2 - Material Properties			
Density (lb/in^3)	**Young's Modulus (psi)**	**Poisson's Ratio**	**Scatter Factor**
0.101	10400000 (Tensile) 10700000 (Comp.)	0.330	4.00
Compressive Yield Stress	**Compressive Ultimate Stress**	**Tensile Yield Stress**	**Tensile Compressive Stress**
71000 psi	81000 psi	71000 psi	81000 psi

The design of this wing spar is limited to specific design constraints. These constraints are displayed below in Table 3.

Table 3 - Design Constraints *(AEE 471 Project 2 Handout - Davidson)*			
Height (h) in	**Depth (b) in**	**Thickness in**	**tw/tf in**
$4 \leq h \leq 8$	$3 \leq b \leq 6$	Flange or Web Thickness ≥ 0.135	$0.5 \leq tw/tf \leq 2.0$
bf/bw	**Length**	**Yielding Factor of Safety**	**All other Factors of Safety**
bf/bw ≤ 1.00	120 inches	1.250	1.500

Where the dimensions are expressed as shown in a cross section below in Figure 2:

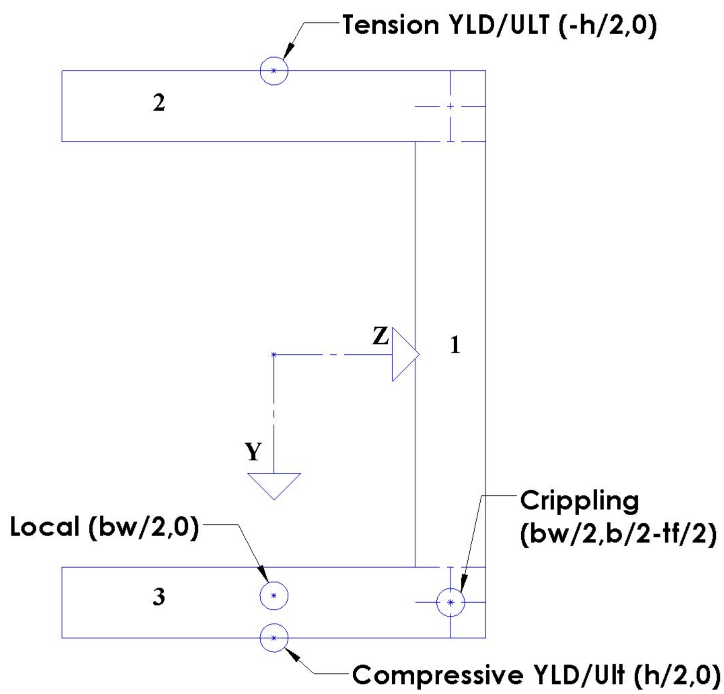
<u>Figure 2 - Main Wing Spar Cross Section Example</u>



Based upon the material properties in Table 2 and the loading in Figure 1, factors of safety will be calculated for every failure mode, including Yielding, Ultimate, Local Buckling, and Crippling. These values will drive the design of the cross section for this beam with the goal of minimizing weight while adhering to the appropriate factors of safety. This will also include tapering the beam to optimize the beam for the lowest weight possible. To do this, factors of safety for every failure mode will be calculated at every cross section. For simplicity, cross sections will be analyzed at every 12" of the beam, starting at the Fuselage and ending at the wingtip. Example calculations are shown in Appendix A (hand calculations pages 1-4.)

The factors of safety for each failure mode are calculated by dividing the critical stress for that failure mode by the calculated stress at that failure location as shown in Equation 1. The bending moment about the Y-axis is equivalent to zero (derived in Appendix A), and there is no axial loading leading to a value of 0 for *Nx*, which simplifies this equation. The failure locations for each failure mode are shown below in Figure 3. The subsequent critical stress equations utilized for local buckling and crippling are shown below in Equations 5 and 6 and 7, respectively. For crippling, the lower value of the two critical stresses is utilized for the Factor of Safety.

Figure 3 - Main Wing Spar Failure Locations



$$Factor\ of\ Safety\ =\ \sigma_{cr}\ /\ \sigma_x \quad \textbf{(1)}$$

$$\sigma_x = \frac{Nx}{A} - \frac{Mz\bar{y}}{Iz} + \frac{My\bar{z}}{Iy} = -Mz\bar{y}/Iz \quad \textbf{(2)}$$

Where Mz is the Bending Moment in the beam (derived in Appendix A) given by:

$$Mz = r_{max}x^2/2 + s_{max}x^3/(6 * length) \quad \textbf{(3)}$$

And Iz is the moment of inertia about the Z-axis given by:

$$Iz = \sum \tfrac{1}{12}bh^3 + Ad^2 \quad \textbf{(4)}$$

$$\sigma_{cr_{Local\ Buckling}} = kw\pi^2 E/[12(1-\nu^2)] * (tw^2/bw^2) \quad \textbf{(5)}$$

Where kw is taken from Figure C6.4 from the MIL-HDBK-5

$$\sigma_{cr1_{crippling}} = \sigma_{ys_{compressive}}(3.2)[(t_{avg.}{}^2/A)(E/\sigma_{ys_{compressive}})^{1/3}]^{0.75} \quad \textbf{(6)}$$

$$\sigma_{cr2_{crippling}} = 0.8\sigma_{ys_{compressive}} \quad \textbf{(7)}$$

Fatigue due to cyclic loading will also be analyzed. This will be done utilizing the Palmgren-Miner rule. Example calculations are shown in Appendix A (hand calculations pages 1-4.) Here, equivalent stress equations from the MIL-HDBK-5 (Appendix C Fig. C3) will be utilized for simplicity for values of the stress ratio of fatigue loading R between -1 and 1. R is given below in Equation 8. The applicable equivalent stress locations are shown in Equations 9-11.

$$R = \frac{\sigma_{min}}{\sigma_{max}} \quad -1 \le R \le 1 \quad \textbf{(8)}$$

$$\sigma_{max} = Mz\bar{y} \,/\, Iz \quad \textbf{(9)}$$

$$S_{eq} = \sigma_{max}(1-R)^{0.62} \quad \textbf{(10)}$$

$$log(Nf) = 18.21 - 7.73log(Seq - 10) \quad \textbf{(11)}$$

Where *Mz* is the bending moment in the beam, and *Iz* is the Moment of Inertia about the Z axis. Here, an important assumption is made for Equation 4. When *Seq* decreases below 10, the assumption that life is simply 10^10. This is because of the negative value created within the log function which yields an error. These calculations are done for each maneuver specified in Table 1 and then applied to Equations 12 and 13 (with Eq. 12 summing the value of D for every maneuver shown in Table 1 for each individual cross section). This results in the anticipated number of flights before fatigue induced failure.

$$D = \sum n/Nf \quad \textbf{(12)}$$

$$Flights = (1/D) \,/\, Scatter\ Factor \quad \textbf{(13)}$$

It's also important to note that the values utilized to calculate Fatigue failure are based on the design loads for each maneuver, not the expected limit loads as shown in Table 1. Design Load is given by Equation 14, and the new values are presented in Table 4 below.

$$Design\ Load = Limit\ Load * Factor\ of\ Safety \quad \textbf{(14)}$$

Table 4 - Expected Design Load Spectrum for 1 Flight *(AEE 471 Project 2 Handout - Davidson)*					
EVENT	**r-min (lb/in)**	**r-max (lb/in)**	**s-min (lb/in)**	**s-max (lb/in)**	**N (cycles)**
Take-Off	39	82.5	27	72	1
Maneuver 1	36	87	18	72	50
Maneuver 2	30	90	22.5	75	5
Cruise	51.6	77.4	46.8	70.2	500
Landing Flare	39	82.5	27	72	1
Landing Touchdown	-60	7.5	-45	3	2

The company also requests tip deflection be calculated, however, the results will not be a driver of the design. This deflection will be calculated at design loads. Results from this will determine the next iteration of the design. Details on deflection are detailed in Appendix A Page 4 as well as Appendix B that features attached Maple code utilized to solve for tip deflection.

Optimization was a heavy factor in the design of each cross section. The goal was to minimize the area to decrease the weight as much as possible. Here, a focus was applied to minimizing the depth (b) due to having a larger influence on the area compared to the height (h). With a smaller depth, a larger tf could be utilized to balance this out, while a higher height yielded a lower tw. This was the thought process by optimizing the values of each cross section. Cross sections 9, 10, and 11 were all found to be minimized values and still hold the appropriate factors of safety. The method included heavily optimizing the first cross-section at the wall with the maximum height and a minimum depth. Many iterations were tested due to initial errors with bf and bw increasing from cross-section to cross-section. This was an important constraint with optimization, every value, the thicknesses, height, depth, and bw and bf had to decrease from one cross-section to the next. This essentially was the driving factor in the end behind final optimization after locating ballpark values. Making sure bf and bw decreased from section to section was difficult and often guided how values were picked. Cross sections 1 through 6 were guided by Crippling, and 7 through 11 were guided by Local Buckling.

Weight was also calculated, as it was ideally the goal of this project to obtain the lowest weight possible. A general approximation was calculated by multiplying the area of each cross section by its “length” of 12 inches, and then summing these areas and multiplying by the density. This yielded an approximation of a “tapered” beam. A more accurate weight was obtained by integrating the areas over the length. Both Maple and Matlab were utilized, utilizing different methods. The *trapz* function in Matlab integrated to find the volume, while Maple integrated a polynomial line of best fit to a power of 6 that was found in excel. Both yielded similar values that were lower than the approximate weight. This will be discussed more in depth in the results section.

RESULTS

SUMMARY

After optimizing cross-sectional dimensions through analysis of failure mode Factors of Safeties, Fatigue, and Tip Deflection, a final set of dimensions for all 11 cross sections were achieved. A table featuring the values of the optimized cross-section dimensions is shown below in Table 6. Note that all of the subsequent values were calculated using the limit loads described in Table 1, not design loads unless otherwise specified.

Table 6 - Final Cross Section Dimensions (inches)				
Cross Section	**Height h**	**Depth b**	**Web Thick. tw**	**Flange Thick. tf**
1 (wall)	7.994	3.300	0.300	0.500
2	7.575	3.269	0.269	0.450
3	6.145	3.259	0.265	0.431
4	5.139	3.244	0.250	0.400
5	4.864	3.210	0.216	0.337
6	4.778	3.200	0.206	0.251
7	4.119	3.200	0.206	0.195
8	4.045	3.000	0.135	0.180
9	4.000	3.000	0.135	0.135
10	4.000	3.000	0.135	0.135
11 (tip)	4.000	3.000	0.135	0.135

These values follow the dimensional naming guidelines shown in Figure 2 and the dimensional magnitude constraints from Table 3. As stated prior, optimization was based on minimizing the depth b while maximizing the Height h with a focus on making sure the values decreased from one cross-section to the next. These values then went and calculated the secondary dimension values shown in Table 7.

Table 7 - Cross Section Dimensions Continued (inches)						
Cross Section	**bf**	**bw**	**h1**	**tw/tf**	**bf/bw**	**kw**
1 (wall)	3.000	7.494	6.994	0.600	0.420	6.100
2	3.000	7.125	6.675	0.598	0.439	6.100
3	2.994	5.714	5.283	0.615	0.547	5.400
4	2.994	4.739	4.339	0.625	0.658	3.600
5	2.994	4.527	4.190	0.641	0.685	3.300
6	2.994	4.527	4.276	0.821	0.684	2.500
7	2.994	3.924	3.729	1.056	0.789	1.500
8	2.865	3.865	3.685	0.750	0.759	2.300
9	2.865	3.865	3.730	1.000	0.759	1.400
10	2.865	3.865	3.730	1.000	0.759	1.400
11 (tip)	2.865	3.865	3.730	1.000	0.759	1.400

Table 7 values confirm that every dimensional aspect decreases from each cross section. This table also shows the ratios of the thicknesses of the flange and webs and the ratio of the flange length to the web length. These values were then utilized using Figure C6.4 from the AEE Cylindrical Buckling, Local Buckling, and Crippling of Thin-Walled Sections handout in the Local Buckling section for channel cross sections to calculate *kw* for local buckling calculations (Appendix C Fig. C2).

Next, appropriate values were calculated for each cross section. This includes the bending moment, area, moment of inertia, and approximate weight, as seen in Table 8. Equations defining the bending moment and moment of inertia as they vary in length x are shown in Appendix A Hand Calculations (Pages 1-4). The equations for area and weight are also detailed in Appendix A. As stated earlier, the weight calculation here is just a piecewise approximation assuming each cross-section extends straight 12 inches and does not taper.

Table 8 - Length, Bending Moment Mz, Areas , Moment of Inertia, Approx. Weight					
Cross Section	**Length (in)**	**Mz (in lbs)**	**Area (in^2)**	**Iz (in^4)**	**Weight (lbs)**
1 (wall)	120	552000	5.3982	54.95374859	6.5426184
2	108	437400	4.737675	44.05594813	5.7420621
3	96	337920	4.209253	26.23007812	5.101614636
4	84	252840	3.67995	16.30730678	4.4600994
5	72	181440	3.06858	12.4293076	3.71911896
6	60	123000	2.487256	9.580859145	3.014554272
7	48	76800	2.016174	5.698210565	2.443602888
8	36	42120	1.577475	4.599180496	1.9118997
9	24	18240	1.31355	3.610040816	1.5920226
10	12	4440	1.31355	3.610040816	1.5920226
11 (tip)	0	0	1.31355	3.610040816	1.5920226

The next sections feature summaries of all pertinent values of each cross section. The final weight utilizes the best weight calculation (Best Fit, seen in the Weight Section) for each tapered cross section to the next. These tables also summarize the final optimized dimensions. These dimensions are shown in each cross section figure. The weight calculation code is shown in Appendix D Fig. D2. This also states the driving Factor of Safety for each cross section.

CROSS SECTION 1 - X=120 (AT WALL)

Figure 4 - Cross Section Dimensions (right)

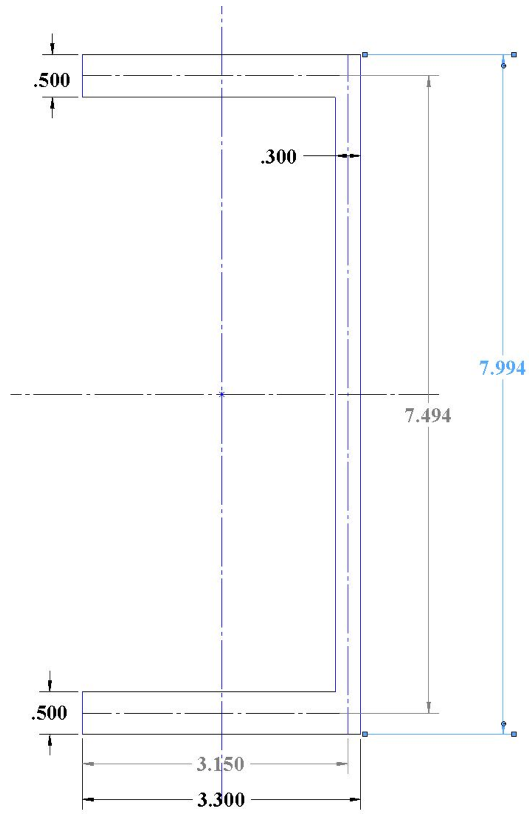


Table 9 - Cross Section 1 Details	
Cross Section 1	
Driving Factor of Safety	Crippling - 1.500
Length (in)	120
Area (in^2)	5.398
Iz (in^4)	54.954
Mz (in-lbs)	552000
Final Weight (lbs)	7.093
Deflection (in)	
Flights	10859
H (in)	7.994
B (in)	3.3
Tw (in)	0.3
Tf (in)	0.5
tw/tf (in)	0.6
bf/bw (in)	0.4

Table 9 - Cross Section 1 Fatigue Life Calculations

Iz = 54.954, X=120, Y=3.997, 1/R for Touchdown = -0.11333

EVENT	Mz Min (in-lb)	Mz Max in-lb)	σ min KSI	σ max KSI	R	Seq (psi)	Log Nf	Nf	n/Nf
Take-Off	345600	766800	25.137	55.772	0.451	38.468	6.968	9.29E+06	1.08E-07
Maneuver 1	302400	799200	21.995	58.129	0.378	43.289	6.443	2.77E+06	1.80E-05
Maneuver 2	270000	828000	19.638	60.224	0.326	47.152	6.074	1.19E+06	4.22E-06
Cruise	360000	540000	26.184	39.276	0.667	19.875	10.522	3.33E+10	1.50E-08
Landing Flare	345600	766800	25.137	55.772	0.451	38.468	6.968	9.29E+06	1.08E-07
Touchdown	-540000	61200	-39.276	4.451	-8.824	41.980	6.577	3.78E+06	5.29E-07

CROSS SECTION 2 - X=108

Figure 5 - Cross Section Dimensions (right)

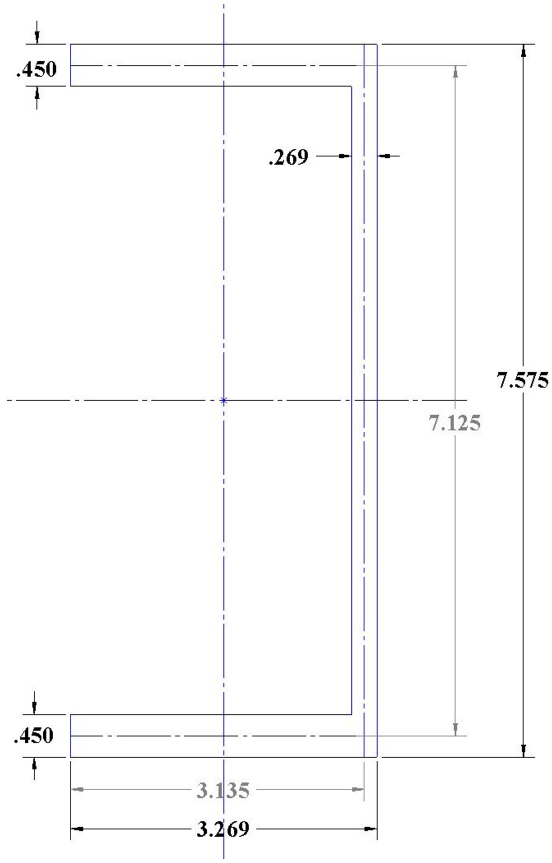


Table 10 - Cross Section 2 Details	
Cross Section 2	
Driving Factor of Safety	Crippling - 1.500
Length (in)	108
Area (in^2)	4.738
Iz (in^4)	44.056
Mz (in-lbs)	437400
Final Weight (lbs)	5.447
Deflection (in)	
Flights	21547
H (in)	7.575
B (in)	3.269
Tw (in)	0.269
Tf (in)	0.450
tw/tf (in)	0.598
bf/bw (in)	0.421

Table 10 - Cross Section 2 Fatigue Life Calculations									
Iz = 44.056, X=108, Y=3.787, 1/R for Touchdown = -0.114									
EVENT	**Mz Min (in-lb)**	**Mz Max in-lb)**	**σ min KSI**	**σ max KSI**	**R**	**Seq (psi)**	**Log Nf**	**Nf**	**n/Nf**
Take-Off	2.75E+05	6.07E+05	23.615	52.193	0.452	35.929	7.282	1.91E+07	5.23E-08
Maneuver 1	2.41E+05	6.33E+05	20.757	54.450	0.381	40.434	6.744	5.54E+06	9.02E-06
Maneuver 2	2.14E+05	6.56E+05	18.426	56.405	0.327	44.139	6.358	2.28E+06	2.19E-06
Cruise	2.83E+05	4.24E+05	24.287	36.430	0.667	18.435	11.051	1.13E+11	4.44E-09
Landing Flare	2.75E+05	6.07E+05	23.615	52.193	0.452	35.929	7.282	1.91E+07	5.23E-08
Touchdown	-4.29E+05	4.90E+04	-36.851	4.212	-8.750	39.409	6.859	7.22E+06	2.77E-07

CROSS SECTION 3 - X=96

Figure 6 - Cross Section Dimensions (right)

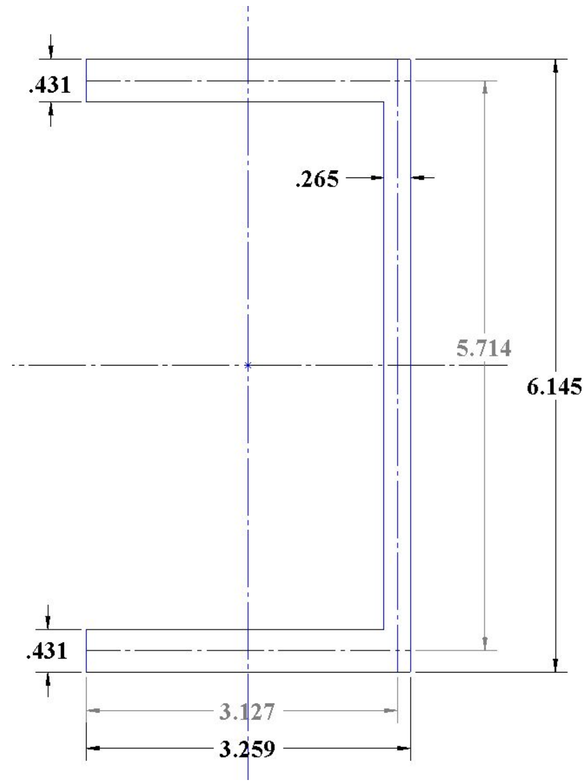


Table 11 - Cross Section 3 Details	
Cross Section 3	
Driving Factor of Safety	Crippling - 1.500
Length (in)	96
Area (in^2)	4.209
Iz (in^4)	26.230
Mz (in-lbs)	337920
Final Weight (lbs)	4.413
Deflection (in)	
Flights	13105
H (in)	6.145
B (in)	3.259
Tw (in)	0.265
Tf (in)	0.431
tw/tf (in)	0.615
bf/bw (in)	0.524

Table 11 - Cross Section 3 Fatigue Life Calculations									
Iz = 26.230, X=96, Y=3.073, 1/R for Touchdown = -0.115									
EVENT	**Mz Min (in-lb)**	**Mz Max in-lb)**	**σ min KSI**	**σ max KSI**	**R**	**Seq (psi)**	**Log Nf**	**Nf**	**n/Nf**
Take-Off	2.13E+05	4.69E+05	24.937	54.894	0.454	37.709	7.059	1.14E+07	8.74E-08
Maneuver 1	1.88E+05	4.89E+05	22.022	57.323	0.384	42.441	6.529	3.38E+06	1.48E-05
Maneuver 2	1.66E+05	5.07E+05	19.432	59.374	0.327	46.436	6.139	1.38E+06	3.63E-06
Cruise	2.16E+05	3.24E+05	25.304	37.956	0.667	19.207	10.757	5.72E+10	8.74E-09
Landing Flare	2.13E+05	4.69E+05	24.937	54.894	0.454	37.709	7.059	1.14E+07	8.74E-08
Touchdown	-3.32E+05	3.82E+04	-38.863	4.480	-8.675	41.583	6.619	4.16E+06	4.81E-07

CROSS SECTION 4 - X=84

Figure 7 - Cross Section Dimensions (right)

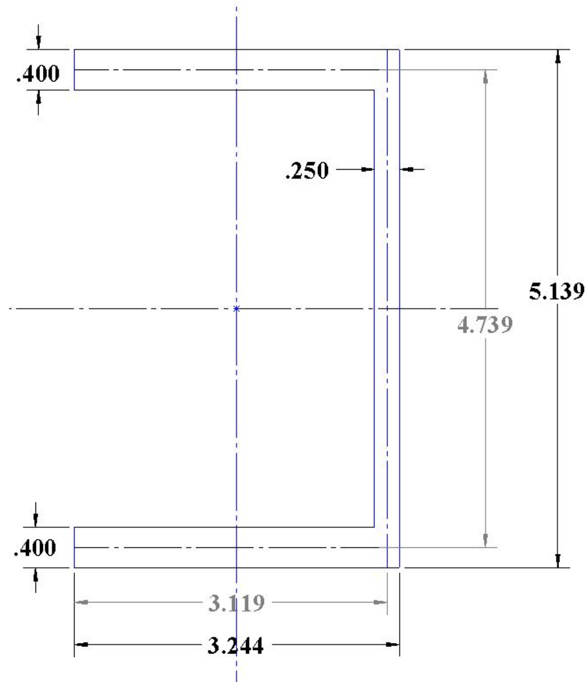


Table 12 - Cross Section 4 Details	
Cross Section 4	
Driving Factor of Safety	Crippling - 1.500
Length (in)	84
Area (in^2)	3.679
Iz (in^4)	16.307
Mz (in-lbs)	252840
Final Weight (lbs)	3.668
Deflection (in)	
Flights	12582
H (in)	5.139
B (in)	3.244
Tw (in)	0.250
Tf (in)	0.400
tw/tf (in)	0.625
bf/bw (in)	0.632

Table 12 - Cross Section 4 Fatigue Life Calculations									
Iz = 16.307, X=84, Y=2.5965, 1/R for Touchdown = -0.116									
EVENT	**Mz Min (in-lb)**	**Mz Max in-lb)**	**σ min KSI**	**σ max KSI**	**R**	**Seq (psi)**	**Log Nf**	**Nf**	**n/Nf**
Take-Off	1.60E+05	3.50E+05	25.182	55.201	0.456	37.837	7.043	1.10E+07	9.06E-08
Maneuver 1	1.42E+05	3.66E+05	22.347	57.702	0.387	42.589	6.514	3.27E+06	1.53E-05
Maneuver 2	1.24E+05	3.79E+05	19.595	59.759	0.328	46.710	6.114	1.30E+06	3.84E-06
Cruise	1.60E+05	2.40E+05	25.193	37.790	0.667	19.123	10.788	6.14E+10	8.15E-09
Landing Flare	1.60E+05	3.50E+05	25.182	55.201	0.456	37.837	7.043	1.10E+07	9.06E-08
Touchdown	-2.49E+05	2.89E+04	-39.191	4.558	-8.598	41.958	6.580	3.80E+06	5.26E-07

CROSS SECTION 5 - X=72

Figure 8 - Cross Section Dimensions (right)

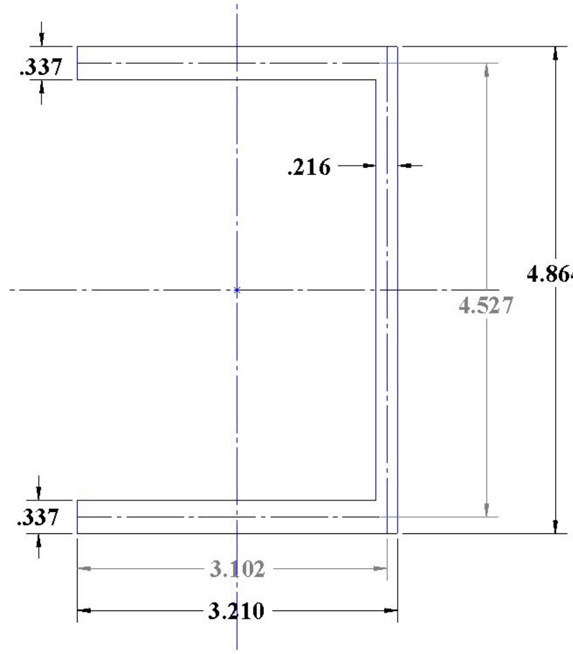


Table 13 - Cross Section 5 Details	
Cross Section 5	
Driving Factor of Safety	Crippling - 1.500
Length (in)	72
Area (in^2)	3.068
Iz (in^4)	12.429
Mz (in-lbs)	181440
Final Weight (lbs)	3.068
Deflection (in)	
Flights	42061
H (in)	4.864
B (in)	3.210
Tw (in)	0.216
Tf (in)	0.337
tw/tf (in)	0.641
bf/bw (in)	0.661

Table 13 - Cross Section 5 Fatigue Life Calculations									
Iz = 12.429, X=72, Y=2.432, 1/R for Touchdown = -0.117									
EVENT	**Mz Min (in-lb)**	**Mz Max in-lb)**	**σ min KSI**	**σ max KSI**	**R**	**Seq (psi)**	**Log Nf**	**Nf**	**n/Nf**
Take-Off	1.15E+05	2.51E+05	22.518	49.145	0.458	33.609	7.596	3.95E+07	2.53E-08
Maneuver 1	1.03E+05	2.63E+05	20.084	51.427	0.391	37.832	7.044	1.11E+07	4.52E-06
Maneuver 2	8.94E+04	2.72E+05	17.497	53.253	0.329	41.599	6.618	4.15E+06	1.21E-06
Cruise	1.13E+05	1.70E+05	22.194	33.290	0.667	16.846	11.752	5.65E+11	8.85E-10
Landing Flare	1.15E+05	2.51E+05	22.518	49.145	0.458	33.609	7.596	3.95E+07	2.53E-08
Touchdown	-1.79E+05	2.10E+04	-34.995	4.108	-8.519	37.488	7.085	1.22E+07	1.64E-07

CROSS SECTION 6 - X=60

Figure 9 - Cross Section Dimensions (right)

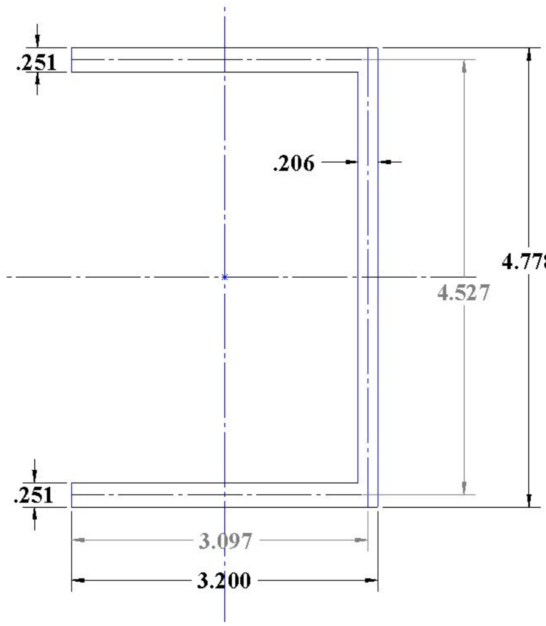


Table 14 - Cross Section 6 Details	
Cross Section 6	
Driving Factor of Safety	Crippling - 1.500
Length (in)	60
Area (in^2)	2.487
Iz (in^4)	9.581
Mz (in-lbs)	123000
Final Weight (lbs)	2.484
Deflection (in)	
Flights	208230
H (in)	4.778
B (in)	3.200
Tw (in)	0.206
Tf (in)	0.251
tw/tf (in)	0.821
bf/bw (in)	0.661

Table 14 - Cross Section 6 Fatigue Life Calculations

Iz = 9.581, X=60, Y=2.389, 1/R for Touchdown = -0.118

EVENT	Mz Min (in-lb)	Mz Max in-lb)	σ min KSI	σ max KSI	R	Seq (psi)	Log Nf	Nf	n/Nf
Take-Off	7.83E+04	1.70E+05	19.524	42.415	0.460	28.936	8.337	2.17E+08	4.61E-09
Maneuver 1	7.02E+04	1.78E+05	17.504	44.434	0.394	32.575	7.747	5.58E+07	8.96E-07
Maneuver 2	6.08E+04	1.85E+05	15.148	46.005	0.329	35.914	7.283	1.92E+07	2.60E-07
Cruise	7.60E+04	1.14E+05	18.941	28.411	0.667	14.377	13.254	1.79E+13	2.79E-11
Landing Flare	7.83E+04	1.70E+05	19.524	42.415	0.460	28.936	8.337	2.17E+08	4.61E-09
Touchdown	-1.22E+05	1.44E+04	-30.296	3.591	-8.438	32.475	7.761	5.77E+07	3.46E-08

CROSS SECTION 7 - X=48

Figure 10-Cross Section Dimensions (right)

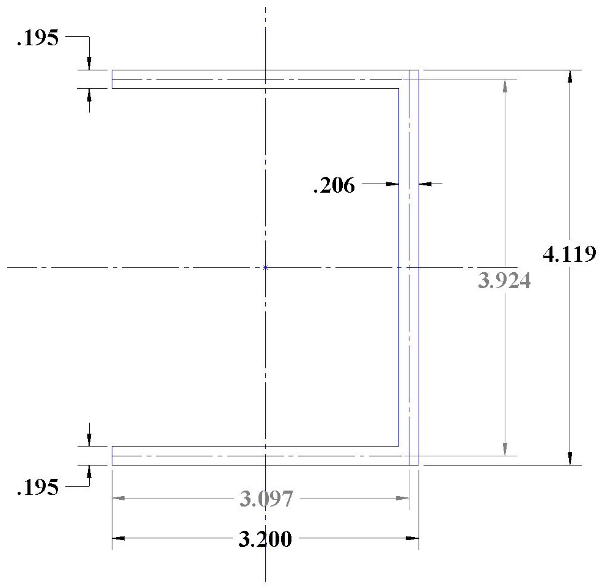


Table 15 - Cross Section 7 Details	
Cross Section 7	
Driving Factor of Safety	Local - 1.501
Length (in)	48
Area (in^2)	2.016
Iz (in^4)	5.698
Mz (in-lbs)	76800
Final Weight (lbs)	2.018
Deflection (in)	
Flights	811429
H (in)	4.119
B (in)	3.200
Tw (in)	0.206
Tf (in)	0.195
tw/tf (in)	1.056
bf/bw (in)	0.763

Table 15 - Cross Section 7 Fatigue Life Calculations

Iz = 5.698, X=48, Y=2.059, 1/R for Touchdown = -0.098

EVENT	Mz Min (in-lb)	Mz Max in-lb)	σ min KSI	σ max KSI	R	Seq (psi)	Log Nf	Nf	n/Nf
Take-Off	5.89E+04	1.06E+05	21.297	38.347	0.555	23.200	9.548	3.53E+09	2.83E-10
Maneuver 1	5.08E+04	1.11E+05	18.362	40.221	0.457	27.559	8.590	3.89E+08	1.29E-07
Maneuver 2	4.62E+04	1.15E+05	16.707	41.637	0.401	30.295	8.104	1.27E+08	3.94E-08
Cruise	6.39E+04	7.02E+04	23.092	25.382	0.910	5.712	#NUM!	1.00E+10	5.00E-08
Landing Flare	5.89E+04	1.06E+05	21.297	38.347	0.555	23.200	9.548	3.53E+09	2.83E-10

Touchdown	-9.24E+04	9.10E+03	-33.413	3.289	-10.158	35.416	7.348	2.23E+07	8.96E-08

CROSS SECTION 8 - X=36

Figure 11-Cross Section Dimensions (right)

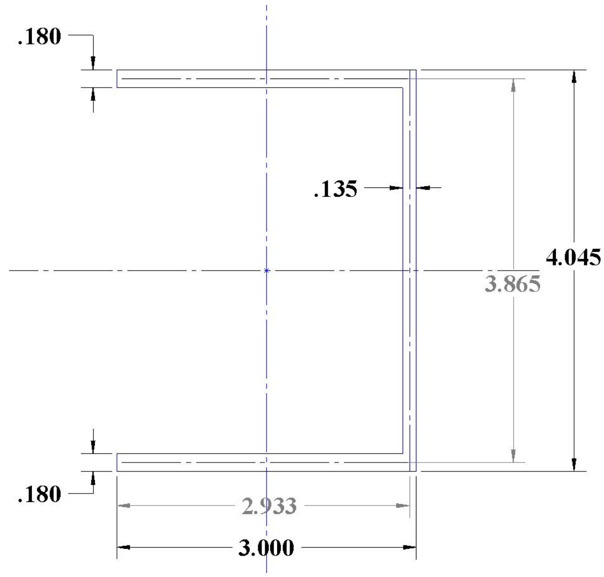


Table 16 - Cross Section 8 Details	
Cross Section 8	
Driving Factor of Safety	Local - 1.522
Length (in)	36
Area (in^2)	1.577
Iz (in^4)	4.599
Mz (in-lbs)	42120
Final Weight (lbs)	1.702
Deflection (in)	
Flights	4980079
H (in)	4.045
B (in)	3.000
Tw (in)	0.135
Tf (in)	0.180
tw/tf (in)	0.75
bf/bw (in)	0.741

Table 16 - Cross Section 8 Fatigue Life Calculations

Iz = 4.599, X=36, Y=2.022, 1/R for Touchdown = -0.121

EVENT	Mz Min (in-lb)	Mz Max in-lb)	σ min KSI	σ max KSI	R	Seq (psi)	Log Nf	Nf	n/Nf
Take-Off	2.70E+04	5.81E+04	11.883	25.561	0.465	17.347	11.515	3.27E+11	3.05E-12
Maneuver 1	2.45E+04	6.10E+04	10.771	26.843	0.401	19.531	10.641	4.38E+10	1.14E-09
Maneuver 2	2.09E+04	6.32E+04	9.190	27.784	0.331	21.659	9.965	9.22E+09	5.42E-10
Cruise	2.53E+04	3.80E+04	11.136	16.704	0.667	8.453	#NUM!	1.00E+10	5.00E-08
Landing Flare	2.70E+04	5.81E+04	11.883	25.561	0.465	17.347	11.515	3.27E+11	3.05E-12
Touchdown	-4.18E+04	5.05E+03	-18.380	2.223	-8.269	19.728	10.573	3.74E+10	5.35E-11

CROSS SECTION 9 - X=24

Figure 12-Cross Section Dimensions (right)

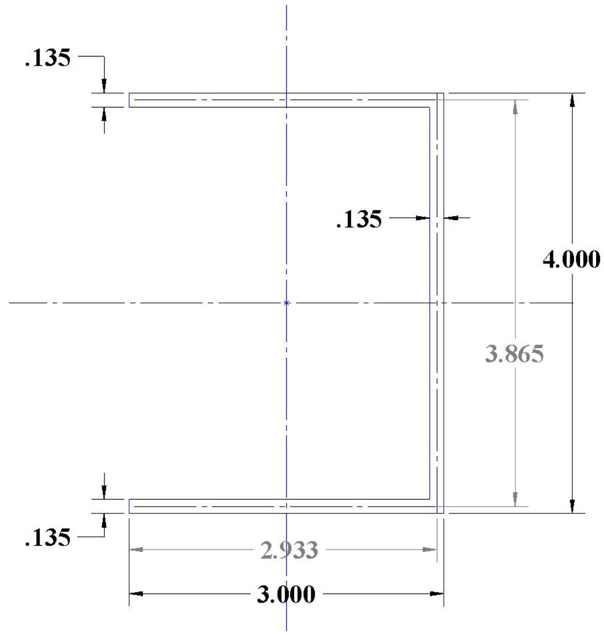


Table 17 - Cross Section 9 Details	
Cross Section 9	
Driving Factor of Safety	Local - 1.679
Length (in)	24
Area (in^2)	1.314
Iz (in^4)	3.610
Mz (in-lbs)	18240
Final Weight (lbs)	1.571
Deflection (in)	
Flights	4472271
H (in)	4.000
B (in)	3.000
Tw (in)	0.135
Tf (in)	0.135
tw/tf (in)	1
bf/bw (in)	0.741

Table 17 - Cross Section 9 Fatigue Life Calculations									
Iz = 3.610, X=24, Y=2, 1/R for Touchdown = -0.122									
EVENT	**Mz Min (in-lb)**	**Mz Max in-lb)**	**σ min KSI**	**σ max KSI**	**R**	**Seq (psi)**	**Log Nf**	**Nf**	**n/Nf**
Take-Off	1.18E+04	2.51E+04	6.510	13.929	0.467	9.426	#NUM!	1.00E+10	1.00E-10
Maneuver 1	1.07E+04	2.64E+04	5.935	14.647	0.405	10.613	19.851	7.10E+19	7.04E-19
Maneuver 2	9.07E+03	2.74E+04	5.026	15.158	0.332	11.808	16.222	1.67E+16	3.00E-16
Cruise	1.08E+04	1.62E+04	5.987	8.980	0.667	4.544	#NUM!	1.00E+10	5.00E-08
Landing Flare	1.18E+04	2.51E+04	6.510	13.929	0.467	9.426	#NUM!	1.00E+10	1.00E-10

Touchdown	-1.81E+04	2.22E+03	-10.052	1.229	-8.182	10.797	18.972	9.38E+18	2.13E-19

CROSS SECTION 10 - X=12

Figure 13-Cross Section Dimensions (right)

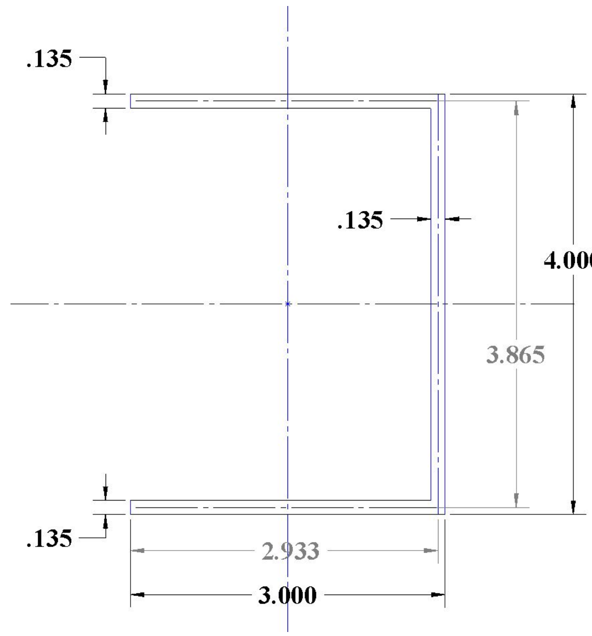


Table 18 - Cross Section 10 Details	
Cross Section 10	
Driving Factor of Safety	Local - 6.898
Length (in)	12
Area (in^2)	1.314
Iz (in^4)	3.610
Mz (in-lbs)	4440
Final Weight (lbs)	1.581
Deflection (in)	
Flights	4990019
H (in)	4.000
B (in)	3.000
Tw (in)	0.135
Tf (in)	0.135
tw/tf (in)	1
bf/bw (in)	0.741

Table 18 - Cross Section 10 Fatigue Life Calculations									
Iz = 3.610, X=12, Y=2, 1/R for Touchdown = -0.124									
EVENT	**Mz Min (in-lb)**	**Mz Max in-lb)**	**σ min KSI**	**σ max KSI**	**R**	**Seq (psi)**	**Log Nf**	**Nf**	**n/Nf**
Take-Off	2.87E+03	6.11E+03	1.592	3.387	0.470	2.285	#NUM!	1.00E+10	1.00E-10
Maneuver 1	2.64E+03	6.44E+03	1.460	3.566	0.409	2.573	#NUM!	1.00E+10	5.00E-09
Maneuver 2	2.21E+03	6.66E+03	1.227	3.690	0.332	2.872	#NUM!	1.00E+10	5.00E-10
Cruise	2.59E+03	3.88E+03	1.434	2.152	0.667	1.089	#NUM!	1.00E+10	5.00E-08
Landing Flare	2.87E+03	6.11E+03	1.592	3.387	0.470	2.285	#NUM!	1.00E+10	1.00E-10
Touchdown	-4.43E+03	5.47E+02	-2.453	0.303	-8.092	2.637	#NUM!	1.00E+10	2.00E-10

CROSS SECTION 11 - X=0 (AT WING TIP)

Figure 14-Cross Section Dimensions (right)

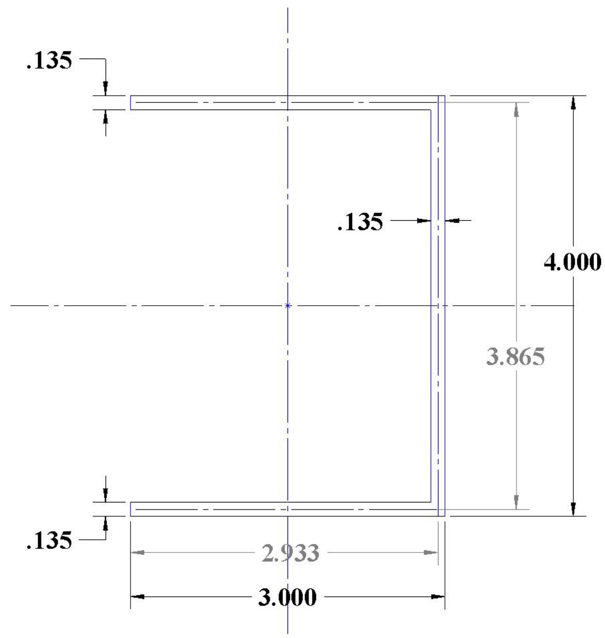


Table 19 - Cross Section 11 Details	
Cross Section 11	
Driving Factor of Safety	N/A
Length (in)	0
Area (in^2)	1.314
Iz (in^4)	3.610
Mz (in-lbs)	0
Final Weight (lbs)	N/A
Deflection (in)	
Flights	N/A
H (in)	4.000
B (in)	3.000
Tw (in)	0.135
Tf (in)	0.135
tw/tf (in)	1
bf/bw (in)	0.741

Fatigue: N/A

STRESS CALCULATIONS and FACTORS OF SAFETY

Table 20 - Failure Locations					
Cross Section	**Yield/Ult Tension Y (in)**	**Yield/Ult Comp. Y (in)**	**Local Y (in)**	**Crippling Y (in)**	**Crippling Z (in)**
1	-3.997	3.997	3.747	3.747	3.497
2	-3.788	3.788	3.563	3.563	3.338
3	-3.073	3.073	2.857	2.857	2.642
4	-2.570	2.570	2.370	2.370	2.170
5	-2.432	2.432	2.264	2.264	2.095
6	-2.389	2.389	2.264	2.264	2.138
7	-2.060	2.060	1.962	1.962	1.865
8	-2.023	2.023	1.933	1.933	1.843
9	-2.000	2.000	1.933	1.933	1.865
10	-2.000	2.000	1.933	1.933	1.865
11	-2.000	2.000	1.933	1.933	1.865

Table 21-Stress Calculations Factor of Safety Applications in Yielding, Ultimate, Local, and Crippling (psi)							
	Yielding/Ultimate		**Local**		**Crippling**		
Cross Section	**σx - tension**	**σx - compression**	**σx**	**σ - critical**	**σx**	**σx - critical 1**	**σx -criticall 2**
1	40149.108	-40149.108	-37637.906	93836.189	-37637.906	56461.880	56800.000
2	37603.379	-37603.379	-35369.515	83462.243	-35369.515	53055.018	56800.000
3	39582.772	-39582.772	-36806.503	111488.592	-36806.503	55216.283	56800.000
4	39839.342	-39839.342	-36738.401	96168.866	-36738.401	55118.283	56800.000
5	35501.743	-35501.743	-33042.021	72115.040	-33042.021	49566.892	56800.000
6	30670.214	-30670.214	-29059.033	49691.128	-29059.033	43590.416	56800.000
7	27757.767	-27757.767	-26443.670	39681.958	-26443.670	41939.923	56800.000
8	18522.365	-18522.365	-17698.131	26935.297	-17698.131	35099.510	56800.000
9	10105.149	-10105.149	-9764.100	16395.398	-9764.100	31953.451	56800.000
10	2459.806	-2459.806	-2376.788	16395.398	-2376.788	31953.451	56800.000
11	0.000	0.000	0.000	16395.398	0.000	31953.451	56800.000

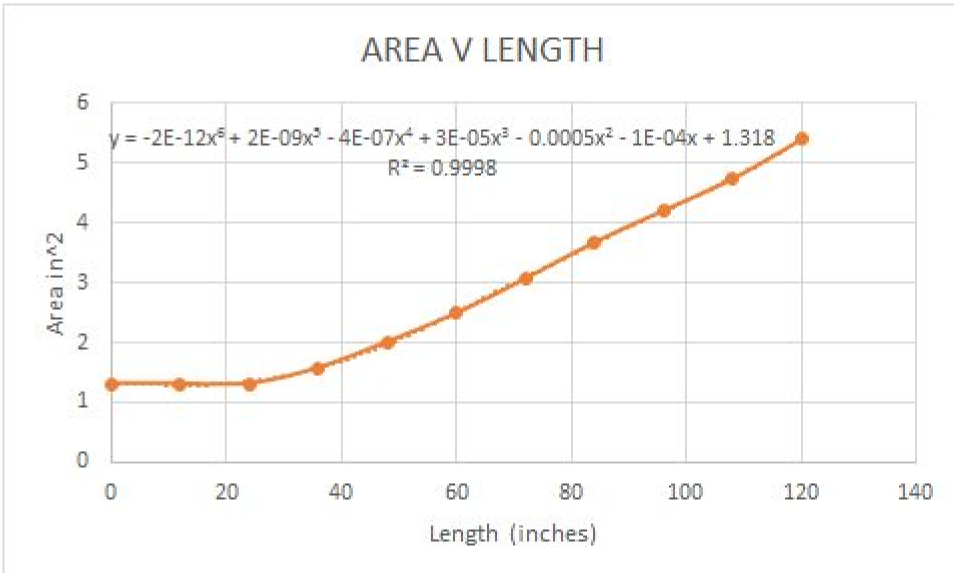
The stresses calculated above were calculated using limit load factors. Sample Calculations are shown in Appendix A (Hand calculations Pages 1-4). The associate Factors of Safety for each failure mode are shown below in Table 22. Note that Crippling is the driving F.O.S. for the first 1 through 6 cross sections, and local buckling is the driving F.O.S. for the 7th to 10th cross section. The 11th cross section reveals infinite factors of safety due to zero magnitude stress calculations as shown above, so it is not shown in the following table.

Table 22 - Factors of Safety

	Tension		Compression			
Cross Section	**Yield**	**Ultimate**	**Yield**	**Ultimate**	**Local**	**Crippling**
1	1.768	2.017	1.768	2.017	2.493	**1.5**
2	1.888	2.154	1.888	2.154	2.36	**1.5**
3	1.794	2.046	1.794	2.046	3.029	**1.5**
4	1.782	2.033	1.782	2.033	2.618	**1.5**
5	2	2.282	2	2.282	2.183	**1.5**
6	2.315	2.641	2.315	2.641	1.71	**1.5**
7	2.558	2.918	2.558	2.918	**1.501**	1.586
8	3.833	4.373	3.833	4.373	**1.522**	1.983
9	7.026	8.016	7.026	8.016	**1.679**	3.273
10	28.864	32.929	28.864	32.929	**6.898**	13.444

WEIGHT CALCULATION

Figure 15 - Area and Length Plot for Main Wing Spar Best Fit
(Where the solid line is the original values and the dotted line is the Best Fit - Legend Error in Excel)



Three weight values were calculated for comparison. Linear, piecewise, and best fit. The best fit utilized a line of best fit utilizing the polynomial trendline tool in excel over the plot of areas versus length (shown above in Figure 15.) Here, the line of best fit calculated and is shown as Y and is shown below in Equation 15. This is then plotted versus x and yields an efficient relation between each cross section. This equation was then integrated over the length of zero to 120 in Maple (See Appendix D Fig. D1). The Linear solution utilized the *trapz* function in Matlab and integrated beneath the linear plot of area versus length done in Matlab (Code in Appendix D.) This isn't as accurate as the line of best fit, but it is a quick and easy integration process that yields close results. This was also plotted utilizing the Matlab Polyfit function and is shown below in Figure 16. The piecewise weight was just the simple approximation done in Excel where the areas of each cross section were multiplied by the length of 12 inches and summed together. This is not a reliable weight calculation due to the poor assumption of a constant cross-sectional area between points. The results are shown below in Table 23.

<u>Figure 16 - Area and Length Plot for Main Wing Spar Linear Approximation</u>

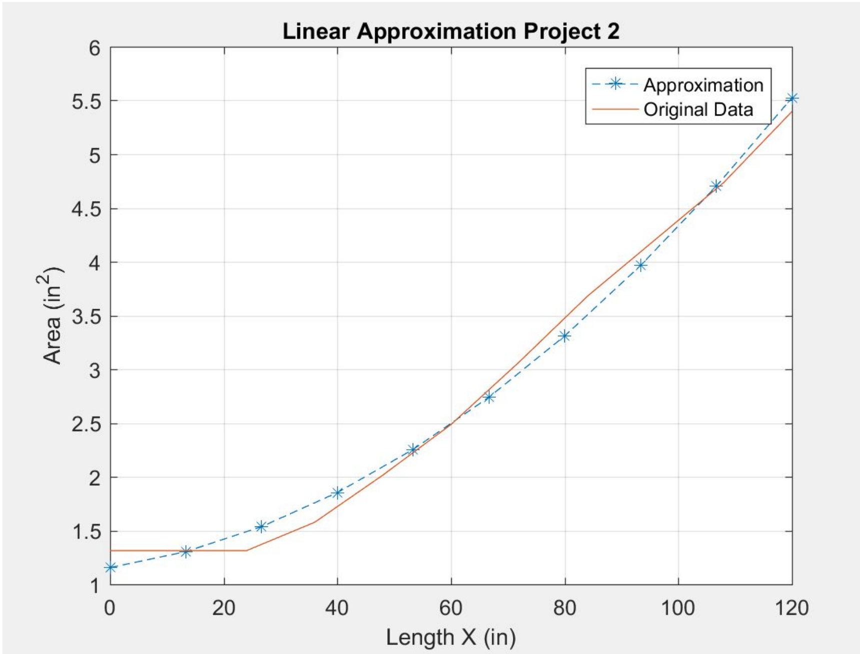


Table 23 - Weight Calculations			
	Piecewise	**Linear**	**Best Fit**
Weight (lbs)	36.120	33.644	33.020

Despite 3 methods utilized <u>strictly for comparison</u>, the chosen and true weight is the Best Fit weight. Utilizing a high order polynomial trendline over the area vs length plot, this yielded a far better and accurate function when compared to the linear *trapz* model. When integrating this equation of best fit from 0 to 120 and multiplying by the density, an believable and accurate weight of just over 33 pounds is obtained.

<u>Weight = 33.020 lbs</u>

FATIGUE

Fatigue was calculated and determined to not be a driving design factor. Example equations and derivations can be seen in Appendix A (hand calculations Page 4.) Fatigue utilized the Palmgren-Miner Rule to predict fatigue life. The aircraft is rated for 10,000 flights, and then Palmgren-Miner Rule should predict a life equal to or greater than 40,000 flights. The following calculations did not, but they did predict a life that would survive the 10,000 flight design requirement. Note that Fatigue is examined at the Design Loads, given by Equation 14 and utilizes the values in Table 4 for each maneuver. Fatigue calculations were required at the most tensile and compressive points (top and bottom of cross section - same location as Tensile and Compressive Yield and Ultimate failure locations as shown in Figure 3 and given in Table 20. It was proved that the Fatigue life on the top and bottom are equivalent, as shown in the following table calculations. Individual cross section fatigue data for each maneuver is shown in the above individual cross section sections. Table 24 shows the final results for life, each exceeding the rated 10,000 flights after a scatter factor of 4 is applied (as specified by the manufacturer). Tables 25 and 26 show the calculations for the top and bottom locations for fatigue life in the first cross section at the provided design loads. Note the rules explained prior in the background information section regarding when an R value sits outside the typical zone of negative 1 to 1 (take the inverse and replace sigma max with sigma minimum in Equation 10). The appropriate equations utilized for analyzing fatigue are given by Equations 8 through 13.

Table 24 - Fatigue Life Summary of Results for Cross Sections

TOP				
Length (in)	**Cross Section**	**D**	**1/D**	**Flights**
120	**1**	2.30E-05	43437.75103	10859.438
108	**2**	1.16E-05	86191.90565	21547.976
96	**3**	1.91E-05	52423.83268	13105.958
84	**4**	1.99E-05	50329.08298	12582.271
72	**5**	5.94E-06	168246.0044	42061.501
60	**6**	1.20E-06	832921.6689	208230.42
48	**7**	3.08E-07	3245717.473	811429.37
36	**8**	5.02E-08	19920318.61	4980079.7
24	**9**	5.59E-08	17889087.66	4472271.9
12	**10**	5.01E-08	19960079.72	4990019.9
0	**11**	#DIV/0!	#DIV/0!	#DIV/0!
BOTTOM				
Length (in)	**Cross Section**	**D**	**1/D**	**Flights**
120	**1**	2.30E-05	43437.75103	10859.438

Tables 25 and 26 below show how fatigue at the top and bottom of the channel is equivalent, therefore, only the singular list of fatigue values needed to be reported in Table 24 above. Sample calculations at the first cross section are shown below that prove this.

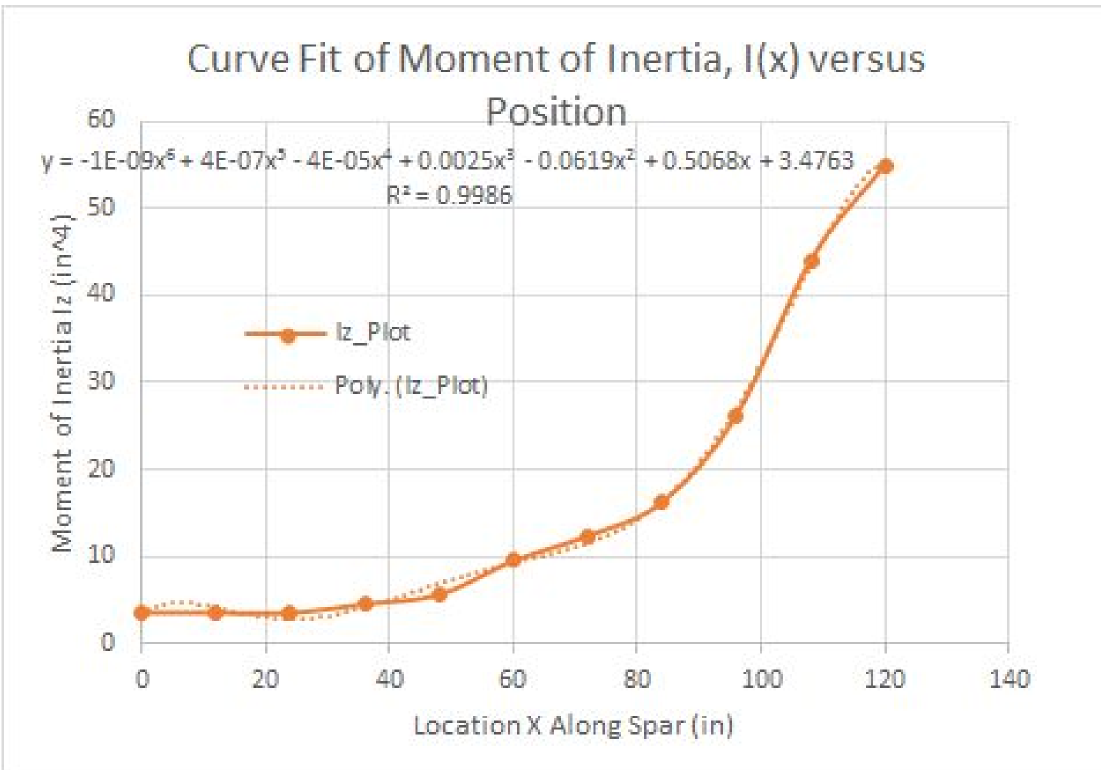
Table 25 - Cross Section 1 Fatigue Calculations based off Design Loads for the BOTTOM Portion of the Cross Section									
	Iz (in^4) =	54.9537486		Length (in) =	120		Y (in) =	3.997	
EVENT	Mz Min	Mz Max	σ min KSI	σ max KSI	R	Seq	Log Nf	Nf	n/Nf
Take-Off	3.46E+05	7.67E+05	25.137	55.772	0.451	38.468	6.968	9.29E+06	1.08E-07
Maneuver 1	3.02E+05	7.99E+05	21.995	58.129	0.378	43.289	6.443	2.77E+06	1.80E-05
Maneuver 2	2.70E+05	8.28E+05	19.638	60.224	0.326	47.152	6.074	1.19E+06	4.22E-06
Cruise	3.60E+05	5.40E+05	26.184	39.276	0.667	19.875	10.522	3.33E+10	1.50E-08
Landing Flare	3.46E+05	7.67E+05	25.137	55.772	0.451	38.468	6.968	9.29E+06	1.08E-07
Touchdown	-5.40E+05	6.12E+04	-39.276	4.451	-8.824	41.980	6.577	3.78E+06	5.29E-07
				1/R	-0.113				

Table 26 - Cross Section 1 Fatigue Calculations based off Design Loads for the TOP Portion of the Cross Section									
	Iz (in^4) =	54.9537486		Length (in) =	120		Y (in) =	-3.997	
EVENT	Mz Min	Mz Max	σ min KSI	σ max KSI	R	Seq	Log Nf	Nf	n/Nf
Take-Off	3.46E+05	7.67E+05	-25.137	-55.772	0.451	38.468	6.968	9.29E+06	1.08E-07
Maneuver 1	3.02E+05	7.99E+05	-21.995	-58.129	0.378	43.289	6.443	2.77E+06	1.80E-05
Maneuver 2	2.70E+05	8.28E+05	-19.638	-60.224	0.326	47.152	6.074	1.19E+06	4.22E-06
Cruise	3.60E+05	5.40E+05	-26.184	-39.276	0.667	19.875	10.522	3.33E+10	1.50E-08
Landing Flare	3.46E+05	7.67E+05	-25.137	-55.772	0.451	38.468	6.968	9.29E+06	1.08E-07
Touchdown	-5.40E+05	6.12E+04	39.276	-4.451	-8.824	41.980	6.577	3.78E+06	5.29E-07
				1/R	-0.113				

The Fatigue Factor of Safety was considered by applying the calculations to the design loads rather than the limit loads, as well as anticipating and attempting to design for 40,000 flights rather than the stated 10,000 flight expectancy (FOS of four) along with the application of a scatter factor of 4. Nf was again replaced by 10^10 whenever an error occurred due to a necessary assumption due to errors in taking the log of a negative value. This did scue the results largely in cross sections 6 to 10. It was an unfortunate assumption to make but was stated and allowed for results to still be calculated. Even if an error wasn't stated in the equation, the log function approaches an error as it nears zero or negative values. This deals with calculated stresses not lining up with the R value in Figure C1 in Appendix C.

TIP DEFLECTION

Figure 17 - Curve Fit of Moment of Inertia versus Position of Main Spar



Tip deflection was calculated utilizing Maple software to solve multiple indefinite integrals. First, a "check" behind the theory was calculated and included in Appendix B. Essentially, After generating the 6th order polynomial of the best fit curve for the Moment of Inertia plot seen in Figure 17. This Equation is listed below as Equation 15.

$$-10\hat{}(-9)*x\hat{}6+4*10\hat{}(-7)*x\hat{}5-4*10\hat{}(-5)*x\hat{}4+0.25e-2*x\hat{}3-0.619e-1*x\hat{}2-.5068*x+3.4763$$

Equation (15)

This equation was integrated along with the bending moment and moment of inertia as shown in Appendix B and Appendix A Page 4. This indefinite integral was given constants after integration manually due to the lack of software ability to do so. This constant was solved by applying knowns at the edge of the system, where the value of x was known. This equation was then integrated again and the same process was applied to solve for another constant of integration. These values were then applied to yield the tip deflection, solved as:

$$.3026378790 - .4452269990 * I \qquad \textbf{(16)}$$

This contains an imaginary number and an exact solution is not yielded. The answer should typically be given in inches as the tip deflects.

APPENDIX A: HAND CALCULATIONS

APPENDIX B: TIP DEFLECTION CALCULATIONS

Fig B1: Closed Form Check Utilizing Maple (screenshot of work)

Fig B2: Integration of Classical Beam Theory Equations Using Maple to solve for Deflection

APPENDIX C: SUPPORTING FIGURES

Figure C1 *(MIL5-1 for 7075)*

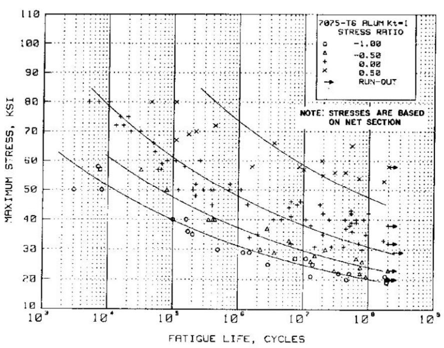


FIGURE 3.7.4.1.8(a). *Best-fit S/N curves for unnotched 7075-T6 aluminum alloy, various product forms, longitudinal direction.*

Figure C2 (AEE 471 Local Buckling Handout)

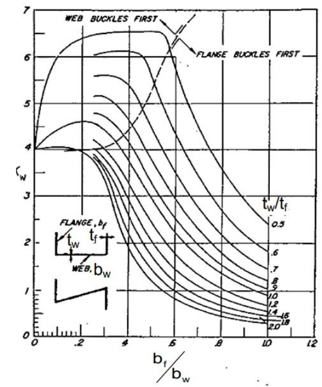


Fig. C6.4 (Ref. 2) Channel- and Z-section stiffeners.

Figure C3 *(MIL5-2 for 7075)*

TABLE 3.7.4.0(g_1). *Design Mechanical and Physical Properties of 7075 Aluminum Alloy Extrusion*

Specification	QQ-A-200/11														
Form	Extrusion (rod, bar, and shapes)														
Temper	T6, T6510, T6511, and T62[c]														
Cross-sectional area, in.^{2}	≤20												>20, ≤32	≤32	
Thickness, in.[a]	Up to 0.249		0.250-0.499		0.500-0.749		0.750-1.499		1.500-2.999		3.000-4.499			4.500-5.000	
Basis	A	B	A	B	A	B	A	B	A	B	A	B	S	A	B
Mechanical Properties:															
F_{tu}, ksi:															
L	78	82	81	85	81	85	81	85	81	85	81	84	78	78	81
LT	76	80	78	81	76	80	74	78	70	74	67	70	65	65	67
F_{ty}, ksi:															
L	70	74	73	77	72	76	72	76	72	76	71	74	70	68	71
LT	66	70	68	72	66	70	65	68	61	65	56	58	55	50	53
F_{cy}, ksi:															
L	70	74	73	77	72	76	72	76	72	76	71	74	70	68	71
LT	72	76	74	78	72	76	71	74	67	71	61	64	60	56	58
F_{su}, ksi	42	44	43	45	43	45	42	44	41	43	40	41	38	37	39
F_{bru}[b], ksi:															
(e/D = 1.5)	112	118	117	122	117	122	116	122	115	120	109	113	105	100	104
(e/D = 2.0)	141	148	146	153	146	153	145	152	144	151	142	147	136	135	140
F_{bry}[b], ksi:															
(e/D = 1.5)	94	99	97	103	96	101	95	100	93	98	89	92	87	83	87
(e/D = 2.0)	110	117	115	121	113	119	112	118	110	116	105	110	104	99	103
e, percent (S-basis):															
L	7	...	7	...	7	...	7	...	7	...	7	...	6	6	...
E, 10^3 ksi	10.4														
E_c, 10^3 ksi	10.7														
G, 10^3 ksi	4.0														
μ	0.33														
Physical Properties:															
ω, lb/in.^{3}	0.101														
C, K, and α	See Figure 3.7.4.0														

[a] For extrusions with outstanding legs, the load-carrying ability of such legs shall be determined on the basis of the properties in the appropriate column to the leg thickness.
[b] Bearing values are "dry pin" values per Section 1.4.7.1.
[c] The allowables shown for these tempers are based on and have been determined from the results obtained on testing of T6, T6510, and T6511 temper material and on the testing of T62 temper samples for specification conformance. These allowables also apply when samples of material supplied in the O or F temper are heat treated to demonstrate response to heat treatment. Properties obtained by the user, however, may be lower than those listed if the material has been formed or otherwise cold or hot worked, particularly in the annealed temper, prior to solution heat treatment.

APPENDIX D: SUPPORTING CODE

Figure D1 - Maple Integration Code for Line of Best Fit Weight Calculation

> *#TYLER VARTABEDIAN*

> *#STRUCTURES PROJECT 2*

> $-2\text{E-}12\, x6 + 2\text{E-}09\, x5 - 4\text{E-}07\, x4 + 3\text{E-}05\, x3 - 0.0005\, x2 - 1\text{E-}04\, x + 1.318$

> $$\int_0^{120} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx$$

326.9354057

> $326.9354057\cdot .101$

33.02047598

>

Figure D2 - Maple Integration Code for Line of Best Fit Individual Weight Calculation

> $$\left(\int_0^{12} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

1.581397252 **(3)**

> $$\left(\int_{12}^{24} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

1.571105714 **(4)**

> $$\left(\int_{24}^{36} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

1.702590292 **(5)**

> $$\left(\int_{36}^{48} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

2.018434699 **(6)**

> $$\left(\int_{48}^{60} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

2.483534513 **(7)**

> $$\left(\int_{60}^{72} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

3.041843292 **(8)**

> $$\left(\int_{72}^{84} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

3.667907316 **(9)**

> $$\left(\int_{84}^{96} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

4.413188941 **(10)**

> $$\left(\int_{96}^{108} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

5.447178593 **(11)**

> $$\left(\int_{108}^{120} \left(-2\cdot 10^{-12}\cdot x^6 + 2\cdot 10^{-9}\cdot x^5 - 4\cdot 10^{-7}\cdot x^4 + 0.00003\cdot x^3 - 0.0005\cdot x^2 - 0.0001\cdot x + 1.318\right) dx\right)\cdot .101$$

7.093295366 **(12)**