ECE 255, Diodes and BJT's

25 January 2018

In this lecture, the various applications of diodes will be discussed, and the working of the bipolar junction transistors (BJT's) will be introduced.

0.1 DC Restorer

The clamping circuit together with a capacitor, which has the function of smoothing a signal, can be used as a DC restorer from an alternating signal pulses.

When the voltage v_I is at -6 V, the diode is forward biased and the capacitor is charged up, and $v_C = 6$ V. There will be stored charge Q inside the capacitor to maintain this voltage difference between the two capacitor plates according to the formula $Q = CV$. When the voltage v_I assumes a positive polarity, the diode becomes reverse biased and open circuit. There is no way for the charge Q to leave the capacitor, and hence the constant voltage difference of v_C is maintained between the two plates. As v_I rises to a different value, v_O will increase to maintain this voltage difference. So $v_O = v_I + v_C$. So v_O is a voltage follower, and will rise to the value of 10 V, as shown in Figure 1. The signal repeats itself thereafter.

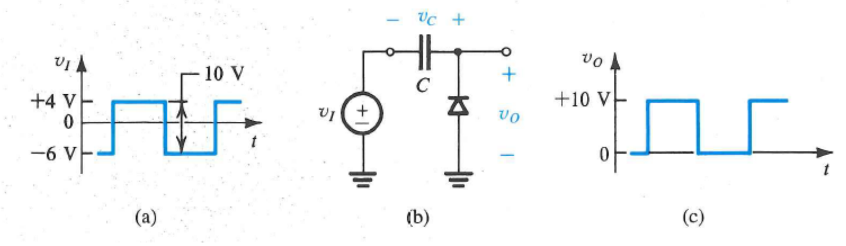

Figure 1: A clamped capacitor can be used as a DC restorer (Courtesy of Sedra and Smith).

Now looking at Figure 2, which is the same as Figure 1 except now, it is loaded with a resistor. Without the resistor, and the diode is assumed to be

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reverse biased or open, v_O is essentially a voltage follower, if the capacitor does not retain any charge: v_O will look like v_I.

But with a resistor, this is not the case. Even if the diode is in reverse bias, the capacitor can now charge and discharge via the resistor. Assume that there is no charge in the capacitor to begin with. Then v_O would have been a faithful voltage follower, but because v_O is positive, it necessitates that a current now flows through the resistor, charging up the capacitor. Because of the charge in the capacitor now, v_O diminishes in value with respect to v_I, in the exponentially decaying manner as shown in Figure 2. One can derive the ordinary differential equation (ODE) that the capacitance current i should satisfy. The capacitance current will also flow through the resistor R. Hence, by KVL (Kirchhoff voltage law), one gets

$$v_I = v_C + iR \tag{0.1}$$

One can take the time derivative of the above. Since v_i is constant between t_0 and t_1, the left-hand side will become zero, yielding the equation

$$0 = \frac{dv_C}{dt} + R\frac{di}{dt} \tag{0.2}$$

Multiplying the above by C, one gets

$$0 = C\frac{dv_C}{dt} + RC\frac{di}{dt} \tag{0.3}$$

Noticing that $i = C\frac{dv_C}{dt}$, one arrives at

$$0 = i + RC\frac{di}{dt} \tag{0.4}$$

The above is the ODE for i. Solving yields

$$i = I_0 e^{-t/(RC)}, \qquad v_O = Ri = RI_0 e^{-t/(RC)} \tag{0.5}$$

The above shows that the voltage v_O decays exponentially with a time constant $\tau = RC$. The above is also known as the homogeneous solution of an ODE. The circuit is a single-time-constant circuit with $\tau = RC$.

If now v_I suddenly drops by a value V_a, then v_O has to follow v_I and drops by the same value since $Q = CV$ and Q cannot change in a short time scale. Then v_O is lower than ground and the diode becomes forward biased, feeding a current that charges the capacitor in the opposite polarity. Notice that the relaxation time of the signal between t_1 and t_2 is shorter than that between t_0 and t_1, because the diode resistance is much smaller than R, and hence, it has a smaller RC time constant.

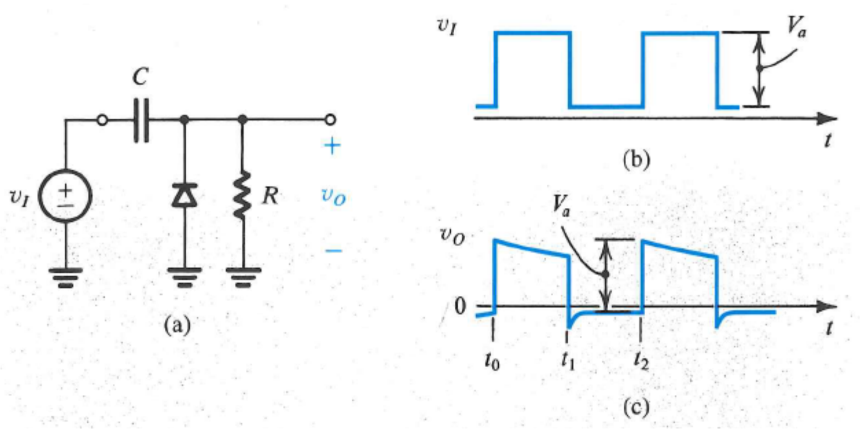
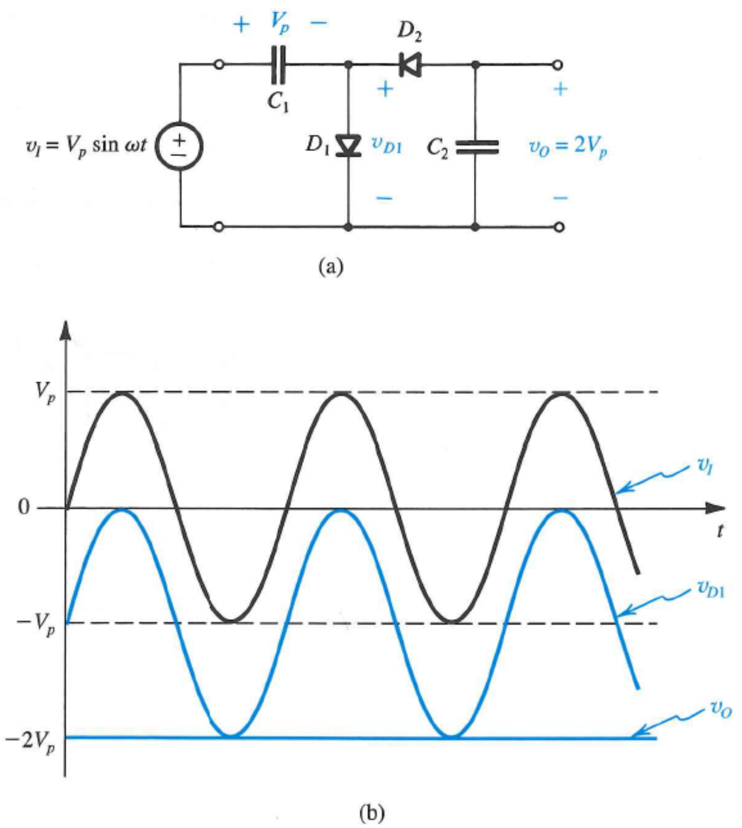


Figure 2: A clamped capacitor can be used as a DC restorer with a load R (Courtesy of Sedra and Smith).

0.2 Voltage Doubler

A clamped capacitor, C_1 and D_1, working together with a peak rectifier, D_2 and C_2, can be used as a voltage doubler as shown in Figure 3. This figure shows the steady-state voltage of the circuit. First, it is easiest to remove D_2 and C_2 from the picture, and focus on the clamping circuit part. Before, it reaches steady state, one can do a Gedanken experiment experiment. The rectifier diode D_1 essentially supplies the circuit with DC current flow, charging up the capacitor C_1, until it is filled to its brim. At this point, the diode will always be reverse biased, and the capacitor is filled with a constant charge Q. Then v_{D1} becomes a faithful voltage follower, safe for the voltage difference caused by Q that will make v_{D1} always negative as shown in Figure 3(b). With the peak rectifier added, it will just convert this rippling voltage of the same polarity to a constant voltage!



(a)

(b)

Figure 3: A pair of clamped capacitor can be used as a voltage doubler (Courtesy of Sedra and Smith).

1 Bipolar Junction Transistors

The transistor is one of the most important inventions of the modern era that changes and enriches our lives. The bipolar-junction transistor (BJT) was invented in 1948 by William Shockley, Walter Brattain, and John Bardeen. Without it, there could be no radio, television, computers, internet, and the modern communication and electronic industries.

The transistor is markedly different from a diode since it has three terminals. One very important feature that it has is its ability to amplify signals. Previously, this amplification ability was achieved by vacuum tubes, but transistors now can be made a billion times smaller than vacuum tubes, ushering in the era of nanotechnologies.

The metal-oxide-semiconductor field-effect transistor (MOSFET), was invented in 1959 by Dawon Kahng and Martin M. (John) Atalla. It now gains popularity in digital circuit design. We will study the BJT first here since it was invented first before MOSFET.

1.1 Device Structure and Physical Operation

The transistor can be a *pnp* type where an n region is sandwiched between two p regions as shown in Figure 4. Both types of transistors comprise three terminals: the emitter (E), the base (B), and the collector (C). Hence, there are two junctions in these transistors, the emitter-base junction (EBJ), and the collector base junction (CBJ).

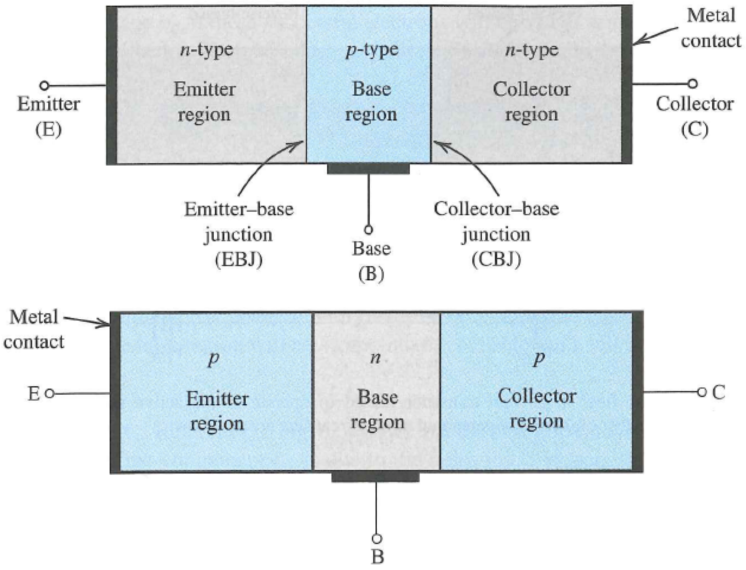


Figure 4: Simplified structures of the *npn* and *pnp* transistors (Courtesy of Sedra and Smith).

A transistor can operate in three modes: (i) **active mode**, (ii)**cutoff mode**, and (iii) **saturation mode** as shown in Figure 5.

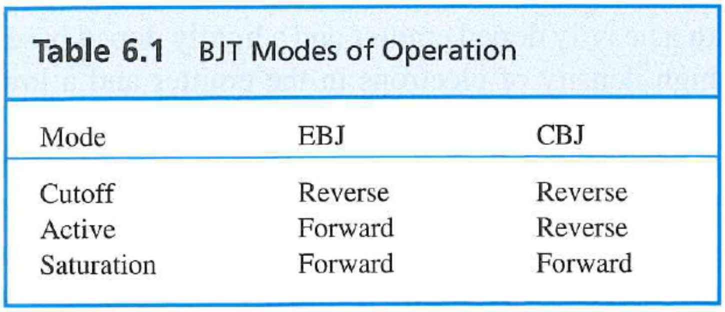


Table 6.1 BJT Modes of Operation

Mode	EBJ	CBJ
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

Figure 5: A table showing different modes of transistor operation. (Courtesy of Sedra and Smith).

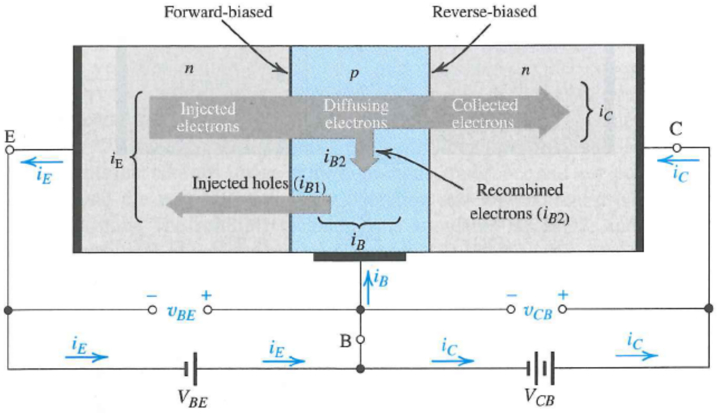


Figure 6: An *npn* transistor in active mode operation (Courtesy of Sedra and Smith).

Figure 6 shows the transistor being biased in the active mode. In this mode, the EBJ is forward biased while the CBJ is reverse biased. To understand the working of this transistor, it is best to recall two features of the *pn* junction:

- When a junction is forward biased, the dominant current that flows across the junction is the diffusion of electron current from the majority carrier region (n region or the emitter E) to the p region where the electrons become minority carrier. In the p region, the electron density n decays exponentially away from the depletion region because of electron-hole pair (EHP) recombination.
- When a junction is in reverse bias, a depletion region is formed where the majority carriers are depleted. The current flow is hence tiny, primarily

due to the drift current. The magnitude to this drift current does not change with respect to the biasing voltage, as the supply of the drift current is due to minority carriers coming from EHP generation in their respective regions.

The above was what we have learnt for a diode *pn* jucntion. Now we can apply this knowledge to understand the transistor. In the BJT, the *p* region is lightly doped in the base region compared to the emitter region, and the base region is made thin. Hence, the diffusion current does not have a chance of being absorbed by EHP recombination before it reaches the CBJ or the collector. The emitter *n* region is heavily doped to give a plentiful supply of electron carriers that are injected into the B region.

Because of the light doping in the base region, the reverse injection of *p* minority carriers from the base to the emitter across the EBJ is small. Nevertheless, both injection of minority diffusion carriers across the EBJ grow exponentially with the applied voltage, namely, proportional to e^{v_{BE}/V_T} due to Boltzmann's law, since the EBJ is forward biased.

Few electron minority carriers will be annihilated by the EHP recombination due to two things:

1. The base region is lightly doped producing less hole carriers that will recombine with the electron carriers. That is, the diffusion length of the electrons in the base region or the hole region is long.

2. With the base region being thin, the recombination of the minority carriers n with the holes, p is further reduced, allowing most of the injected diffusion electron carriers to reach the CBJ or the collector.

At the CBJ, which is in reverse bias, there is a depletion region. An electric field is formed in the depletion region that is of the right polarity that will sweep the minority carriers that enter into it, across the junction from the base region to the collector region.

Hence, the collector current is coming mainly from the injected minority carrier from the EBJ, and it is given as

$$i_C = I_S e^{v_{BE}/V_T} \tag{1.1}$$

where I_S is the saturation current as in the EBJ or the first *pn* junction case.

It is to be noted that the collector current I_C is independent of the biasing voltage v_{CB} of the collector-base junction (CBJ) very similar to a diode in reverse bias. The current through the CBJ is mainly the drift current: in the *pn* junction case, the source of this drift current, which are from minority carriers before they reach the depletion region, are from EHP generation due to thermal effect, and hence is small. However, in this case, the source for the drift current is due to the injection of minority carriers into the base from the emitter region, and the supply of these carriers is large.

Reason for Amplification

The transistor is a good current amplifier if we can keep the base current small. With the EBJ forward biased, a large diffusion current as minority carrier is injected into the base region. Before these carriers can leave the base region, they are consumed by the depletion region in the CBJ, and become drift current they are swept across to the collector, giving rise to a large collector current. Hence, with little current flowing as base current, a small bias in the EBJ can cause a deluge of collector current.

Source for the Base Current

The flow of the base current should be minimized so as to maximize the amplification effect. The base current has two components:

1. The component that has to be injected from base region to the emitter region as minority carriers.
2. As the electrons diffuse into the base region by injection from the emitter, the electrons will combine with the holes in the base region needing a supply of positive current into the base region.

In general, one can write

$$i_C = \beta i_B \tag{1.2}$$

where β, the ratio between the collector current and the base current, is the amplification factor. Or

$$i_B = \frac{I_S}{\beta} e^{v_{BE}/V_T} \tag{1.3}$$

Typically, β ranges from 50 to 200. The β is also called the **common-emitter current gain**.

Emitter Current

It is clear that

$$i_E = i_C + i_B \tag{1.4}$$

Then

$$i_E = \frac{\beta + 1}{\beta} i_C = \frac{\beta + 1}{\beta} I_S e^{v_{BE}/V_T} \tag{1.5}$$

Or that

$$i_C = \alpha i_E \tag{1.6}$$

where $\alpha = \frac{\beta}{\beta+1}$, or that $\beta = \frac{\alpha}{1-\alpha}$. Here, α is the **common-base current gain**. For large β, it is close to 1, but it always less than 1.

Minority Carriers

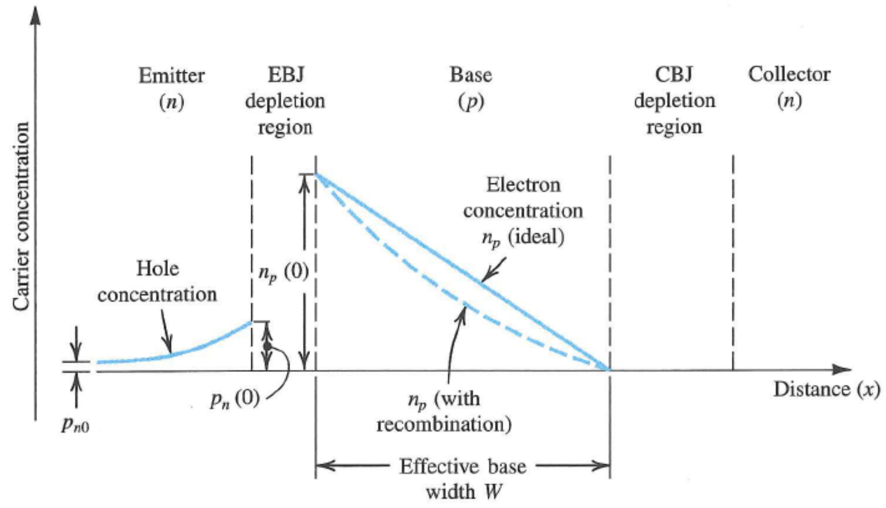


Figure 7: Minority carriers of an *npn* transistor in active mode (Courtesy of Sedra and Smith).

The minority carrier distribution is as shown in Figure 7, where in this figure, the doping concentration in the emitter region is assumed to be much larger than that in the base region. That is, the hole injection into the emitter region is small, while the electron injection into the base is large. The electron concentration at the beginning of the base region can be written as

$$n_p(0) = n_{p0}e^{v_{BE}/V_T} \tag{1.7}$$

according to Boltzmann's law. It is assumed that there is little or no recombination process in the base region, so that diffusion current is a constant across the base region. In this case, gradient of the electron concentration is a constant, and is represented by a straight line. From this, one gathers that the electron current, which is diffusion current in nature, is the electron current density times the cross section area. It becomes

$$I_n = A_E q D_n \frac{dn_p(x)}{dx} = A_E q D_n \left(-\frac{n_p(0)}{W}\right) \tag{1.8}$$

where A_E is the cross-section area of the junction. One can assume that this diffusion current is not reduced by EHP recombination, and all of it is swept to become the collector current.

Substituting for $n_p(0)$ from (1.7), one gets the expression for the collector current to be

$$i_C = I_S e^{v_{BE}/V_T}, \text{ where } I_S = A_E q D_n n_{p0}/W = \frac{A_E q D_n n_i^2}{N_A W} \tag{1.9}$$

where I_S ranges from 10^{-12} A to 10^{-18} A, and is temperature dependence because n_i is. It doubles for every 5°C rise in temperature.

2 Equivalent Circuit Model for Transistors

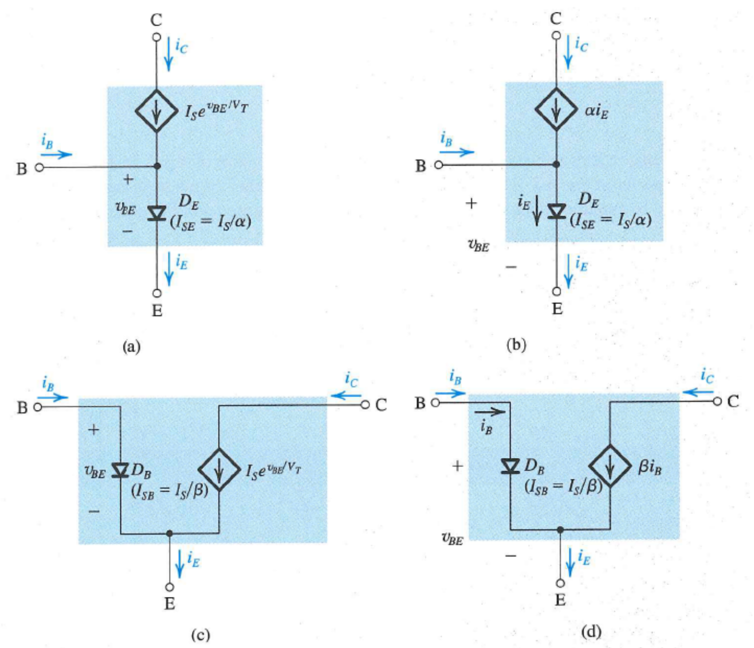


Figure 8: Different equivalent circuit models of an *npn* transistor in active mode (Courtesy of Sedra and Smith).

Equivalent circuit models are good because they represent a simplified picture of the transistor operation. Moreover, many software use this simplified picture, and it is easier to integrate the circuit model into commercial circuit analysis software such as SPICE.

Figure 8(a) shows a voltage-controlled current source for the collector current. On the other hand, Figure 8(b) shows a current-controlled current source. Other equivalent circuit models are shown in 8(c) and 8(d).

3 Real World Transistor

A real transistor is made by very sophisticated and complicated epitaxy, lithography, and diffusion doping processes. Instead of making a transistor looking like your Subway sandwich, it looks more like one shown in Figure 9. Notice that the EBJ contact area is a lot smaller than that of the CBJ contact area.

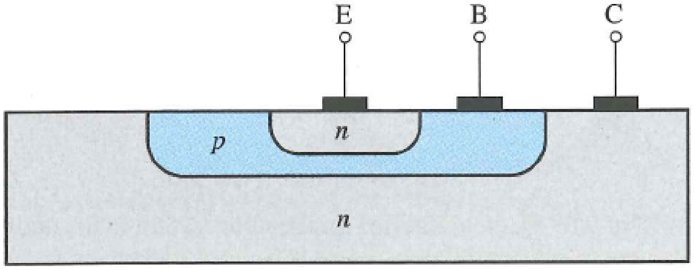


Figure 9: A real world transistor grown by epitaxy, lithography, and doping processes (Courtesy of Sedra and Smith).

4 Saturation Mode of a Transistor

In order to be in the active mode, the CBJ has to be in reverse bias. However, not knowingly, one may design a circuit where this reverse bias is absent, and the CBJ may even be in forward bias. In this case, the transistor operates in the saturation mode, and its efficiency as a signal amplifier is greatly hampered.

Figure 10 shows the collector current i_C versus the the CBJ biasing voltage v_{CB}. When v_{CB} is positive, the junction is in reverse bias, and the transistor is in active mode. However, as v_{CB} diminishes, and even when it is negative, but above -0.4 V, the junction is still in reverse bias. However, if v_{CB} drops below -0.4 V, then junction becomes forward biased, and the transistor leaves the active mode and goes into the saturation mode.

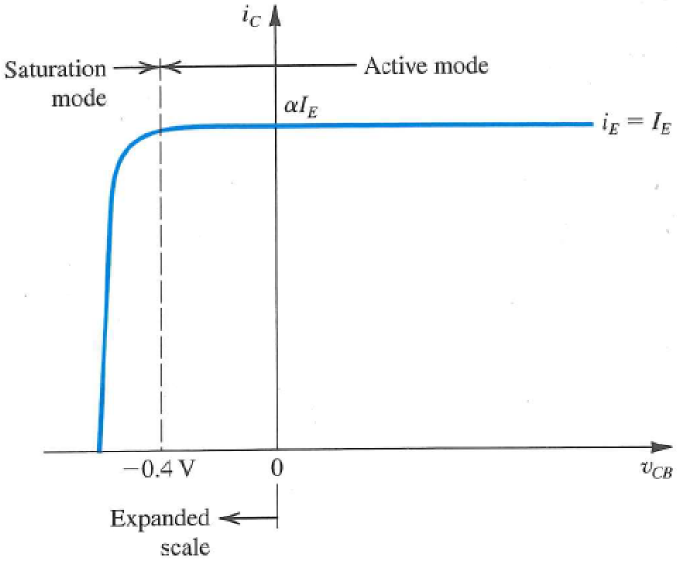


Figure 10: The i-v characteristic of the CBJ showing when the transistor enters into a saturation mode (Courtesy of Sedra and Smith).

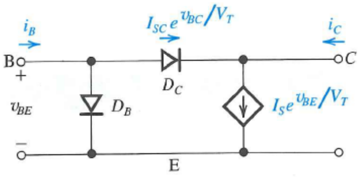


Figure 11: The equivalent circuit model of a transistor operating in the saturation mode (Courtesy of Sedra and Smith).

In the saturation mode, there is a base to collector current leakage, and this can be represented by the circuit model of Figure 11. The total collector current then is

$$i_C = I_S e^{v_{BE}/V_T} - I_{SC} e^{v_{BC}/V_T} \tag{4.1}$$

The leakage current can null the collector current, greatly reducing it. The base current also increases as

$$i_B = (I_S/\beta) e^{v_{BE}/V_T} + I_{SC} e^{v_{BC}/V_T} \tag{4.2}$$

Overall, the β factor decreases, and the new β is known as **forced** β. Therefore,

$$\beta_{\text{forced}} = \left.\frac{i_C}{i_B}\right|_{\text{saturation}} \leq \beta \tag{4.3}$$

In general,

$$V_{CE\text{sat}} = V_{BE} - V_{BC} \tag{4.4}$$

Because of the much larger cross-sectional area of the CBJ compared to the EBJ, V_{BC} is smaller than V_{BE} by 0.1 to 0.3 V. Hence, $V_{CE} \approx 0.1$ to 0.3 V.

Last but not least, we show the *pnp* transistor in active mode operation in Figure 12. Other than interchanging the p and n regions, and reversing the polarity of the biasing voltages, the physics of its operation very similar to that of a *npn* transistor. The *pnp* transistor can some time be used in tandem with an *npn* transistor, because of the different polarity they are connected. Also, *npn* transistors tend to be more agile than *pnp* transistors, since in *npn* transistors, the carriers are mainly electrons which have a higher mobility than holes.

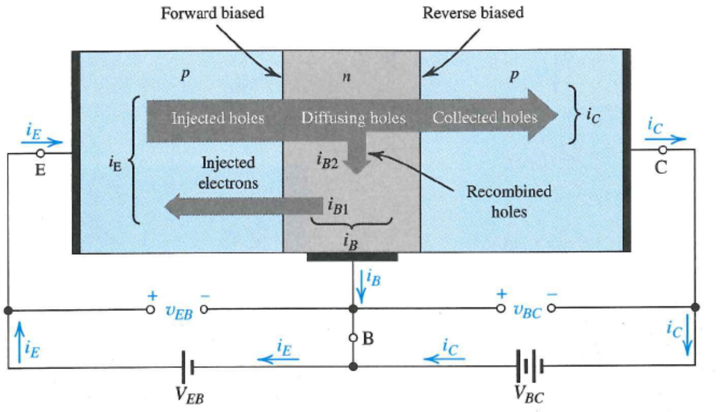


Figure 12: A *pnp* transistor in active mode operation (Courtesy of Sedra and Smith).

5 Vacuum Tubes

Before we leave this lecture, it will be interesting to revisit vacuum tubes as shown in Figure 13. They are the precursors to transistors. Vacuum tubes are still used by avid HIFI (high fidelity) sound system lovers since they have low noise.

Figure 14 shows the dissected view of a vacuum tube, as well as its simplified symbol. A vacuum tube works by a heater element that heats up the cathode in a vacuum. The cathode emits electrons and are attracted to the anode which is positively charged. A grid element, which is mainly "porous" to electrons, is placed in between the cathode and the anode working very much like the base in a transistor. By varying the potential between the grid and the cathode, electrons are attracted to the grid, but most of them are not captured by it, but pass through it. The deluge of electrons that misses the grid element are

Figure 13: An array of modern vacuun tubes (Courtesy of Wikipedia).

captured by the anode. In this way, a small current in the grid can give rise to a large current in the anode, or through the vacuum tube.

The reason for the low noise level in the vacuum tube is that the electrons hardly collide with anything when they travel from the cathode to the anode. Whereas in a transistor, the carriers or electrons collide with impurities in the semiconductor material, causing their path to zig-zag through the material. As a result, the emerged current in a transistor is not as smooth as that in a vacuum tube.

Vacuum tube technology is important in a number of high power microwave sources. They are also coming back as alternative sources for THz electromagnetic field.

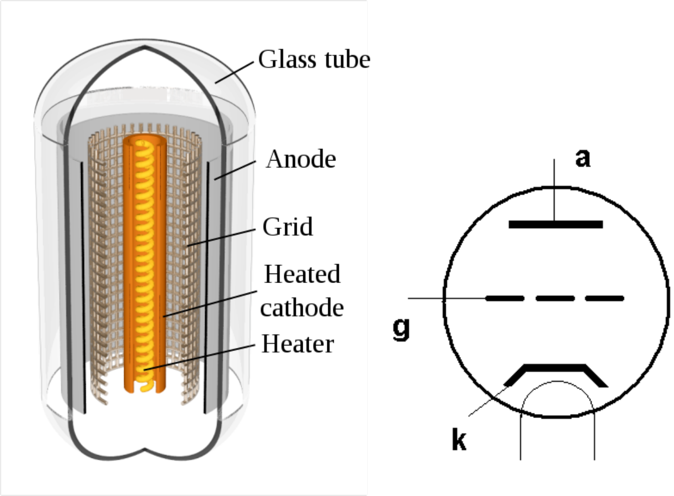


Figure 14: A dissected vacuum tube picture, and its simplified symbol (Courtesy of Wikipedia).