Title

How Proportional are Mixed Compensatory Electoral Systems? Determining the Necessary Share of Compensation Mandates in Mixed Systems

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Introduction: Once a strange German hybrid, today used around the world[*]

The German electoral system is often praised as a genial combination of two opposed worlds of mechanisms, allowing the election of one part of representatives of local districts by first-past-the-post vote in single-seat constituencies, and at the same time, thanks to compensation mandates, allowing a fully proportional outcome.[1] No wonder such mixed compensatory systems (some call them "mixed-member proportional systems", Shugart/Wattenberg 2001a[2]), are praised for combining "the best of both worlds" (Shugart/Wattenberg 2001b: 582-3). And what once was treated as strange German hybrid, today is a bestseller in the charts of worldwide electoral systems: Albania, Bolivia, Lesotho, Mexico, New Zealand, the Philippines, Venezuela adopted it at the national level, the UK for the election of the Scottish and the Welsh regional assemblies.[3] Many more countries introduced very similar mixed electoral systems, in those cases the mandates allocated on party lists employ a pure proportional representation (PR) formula, and no correction formula that would lead to an overall proportional seat allocation.

In the "best of both worlds" view, *mixed compensatory systems* are seen as positive,
- since they allow voters not only to elect a political party, but at the same time to have an influence too on which candidates will be elected (Shugart 2001);
- they represent both local and national interests, due to the election district candidates that rely on the local votes of their constituency, and of additional candidates from party list that are told to represent rather national interests (Shugart 2001; Thames 2005);
- and, the combination of both logics of electoral systems might help to find a compromise between parties, when each of them favours a different electoral system logic (find an example in Schiemann 2001).

And, despite the quick spreading of mixed electoral systems, we know fairly little about their outcomes in practice. Regarding their impact on the party system, previous studies found that they have an impact somewhere in between PR and plurality/majority vote (Kostadinova 2002), although without a much more precise specification. With regards to disproportionality, many scholars are

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[1] In national parliament elections, each German voter has two votes: with the *first vote* she elects the MP of her local district, similar to the logics of a first-past-the-post system; with the *second vote* she elects one of the parties competing at the national level. Half of the 598 seats in the German "Bundestag" (lower chamber of the parliament) are elected in single-seat districts, according the *first vote* (personality vote); the remaining half, the compensatory mandates, is given directly to the parties, according the share of the *second vote* (party vote) they win. However, in the distribution of the compensatory mandates, the seats parties won in the single-seat districts are subtracted, in order that the overall distribution of the 598 seats on the parties will be as proportional as possible. I employ the term *mixed compensatory systems* for such and similar electoral systems, where plurality or majority rules apply in one tier, a second tier corrects disproportionalities, and each voter votes according to both rules. For more extensive classifications, see Massicotte/Blais (1999) and Shugart/Wattenberg (2001a).

[2] Other, less frequent terminologies speak of 'linked systems' (Moser/Scheiner 2004), 'dependent' or 'correction systems' (Massicotte/Blais 346-8), or 'personalised PR' (Moser 1995: 383; Nohlen 2004: 188-90).

3 The application in Italy is not listed, as it was dropped in 2006, and further, it had some different mechanisms. Tunisia is not listed because using only one vote for the election of both tiers (Massicotte and Blais 1999: 354-5).

expecting mixed compensatory systems such as the German one to lead to proportional or almost proportional outcomes and thus comparable to PR.4

Pioneering work by Behnke (2003) and Behnke et al. (2003) has analysed at the case of the German party system the conditions under which there are not enough compensation seats to allow a fully proportional seat allocation. The study showed that 50% of compensation seats are usually roughly enough to proportionalise the seat distribution, however looking only at the German context, thus the German vote distribution. Indeed, empirical experience teaches us that in the Germany 50% of compensation seats are usually enough to allow an (almost) proportional outcome. But does the 50%-rule apply in other party systems too? We might further wonder, if the vote distribution in Germany might rely on a psychological effect of the electoral system: a limited number of compensation seats in an electoral system might have an impact on the number of competing parties.

We still lack a genuine model that explains how many compensation seats are needed for the outcome to be proportional. As proportional I understand that the outcome is comparable to the functioning of proportional representation systems. Without this knowledge we will neither be able to estimate how many parties we expect to compete and to win representation in parliament under mixed compensatory systems; a key question of comparative research on electoral systems and party systems. This paper shall provide a quantitative model that shows under which conditions a mixed compensatory system might lead to proportional vote-seat conversion.

My paper is structured as follows: First, I summarise previous views on the outcome under mixed compensatory systems, then I explain why and how I distinguish semi-compensatory systems from fully compensatory ones, and I propose a quantitative predictive model to make this distinction. Thereafter, I discuss a possible way of estimating the number of parties elected under semi-compensatory systems. In the conclusion, I discuss the chances of the approach and a further research agenda related to the model under study.

Predicting the outcome of mixed compensatory electoral systems

Majority systems, personalised PR, or a hybrid in between?

With the widespread introduction in many democracies, the scholarly view on mixed compensatory systems and their impact on party system has undergone a change in recent years. Regarding simple electoral systems, based on single-seat districts or PR, scholars share widely accepted conclusions: We know that under typical condition of Western Democracies, plurality or majority vote often favours two-party systems,5 whereas PR in multi-member districts allows multi-party systems (Duverger 1951; Rae 1971). In mixed compensatory electoral systems, these incentives are combined, so that in plurality/ majority tier, the outcome will be not proportional, and favour the largest parties in the seat allocation. The PR mandates are there to compensate for the disproportionalities created through these incentives.

Regarding the overall outcome of those systems views are divided, and often imprecise:

- Duverger (1986: 72) sees electoral behaviour in mixed electoral systems widely influenced by the first vote that is cast for the candidates in the single-seat districts, apparently determined by the German case in the pre-1990 period. The electoral system was described leading to two large parties, because only the two main parties the parties may have good chances to win the single-seat districts.
- The view is opposed by a second school that sees the mixed compensatory system as a special form of PR, the type of "personalised PR" (Moser 1995: 383; Kreuzer 2004; Nohlen 2004: 188-

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4 Cf. Shugart/Wattenberg (2001b: 584), Schoen (1999: 475), Nohlen (2004: 188-90), Tiefenbach (2006: 124), Herron/Nishikawa (2001: 65).
5 For a discussion of limits and exceptions to this rule, see Cox (1997).

90; Ferrara et al. 2005: 131). Indeed, in compensatory systems it is the PR component that determines the overall seat distribution in the parliament, and accordingly, this school expects outcomes that are not different from pure PR.

■ Finally, an increasing number of studies treat *mixed electoral systems* as a whole, no matter the rules applied for the seat allocation in the proportional or compensatory tier, and this school would often characterise the mixed electoral systems as being a hybrid between plurality/majority systems and PR (Kostadinova 2002).

There might not only be doubts about the different estimations of the impact of mixed compensatory systems on party systems, the latter approach lacks of a more precise indication of what degree of fractionalisation of the party system we might expect in those electoral system.

The limits of party fractionalisation in mixed compensatory electoral systems

If a compensatory system leads – or, better to say – might lead to PR-like results relies on the vote distribution on the parties and on the amount of compensatory mandates that are at disposal to compensate for possible disproportionalities of the plurality/majority tier. It does not need to be argued thoroughly that in a compensatory electoral system with a very large plurality/majority tier and only a very small number of compensatory mandates, the proportionality might be quite limited. The few compensatory mandates might simply not be enough to compensate many small parties6 for their under-representation in the plurality/majority tier. On the other hand, if a very large part of the mandates is allocated as compensatory seats, they will (under normal conditions) be enough to compensate for all non-proportionalities.

We might introduce a distinction between different kinds of compensatory system, related to their outcome: If the share of compensatory mandates (c) is low, they might not be enough to compensate completely for disproportionalities, and the outcome might still deviate from the proportionality formula applied (cf. figure 1, left part). Even if the compensation tier allows some correction of disproportionalities, the system remains just semi-compensatory. If the share of compensatory mandates (c) is high enough, the system will be fully compensatory, thus there will be enough compensation mandates to make the overall outcome proportional (to the degree that the applied formula of proportionality is proportional) (cf. figure 1, right part). Both types of compensatory systems are separated by the point comp=c, the point where an electoral system is just compensatory: The point is defined through the characteristic that even a single seat is moved from the compensatory to the plurality/majority tier, or if only a single vote is moved from a large party to a small party (that does not win any plurality/majority mandates, but clears the PR threshold), the system will become only semi-compensatory.

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6 In this paper, I use the distinction in small parties and large parties in order to describe different typical outcomes of plurality/majority elections. Typically, in highly nationalised party systems and under the absence of strategic party alliances, small parties will be under-represented and large parties over-represented in the plurality/majority tier (Cox 1997; Bochsler 2005). Some small parties might have regional strongholds and win there district mandates, so that they might be overpaid with seats at the end. The characteristics that in this paper are discussed for large parties would fit for those regional parties too. But to keep things simple, readers might assume high party nationalisation – thus, parties having an approximately homogeneous vote share across all regions of a country. A particular problem is imposed by parties that win direct mandates in single-member districts, but do not clear the threshold imposed in the PR tier. But as long as we do not have very important amounts of systematic vote splitting of the two votes (the case of Venezuela or Lesotho, cf. Bochsler 2007), this problem might remain rather low-scale. The consequences of mixed compensatory systems in countries with low party nationalisation might merit further academic attention.
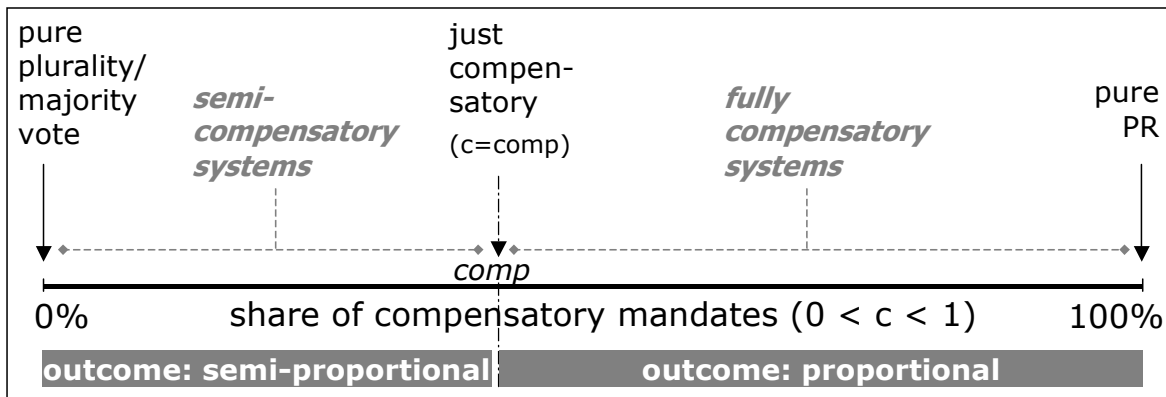
Figure 1: Semi- and fully compensatory systems and break-even point (c=comp), where systems are just compensatory.

       For electoral systems where the compensatory tier is smaller than necessary to be just compensatory, the number of mandates won by the largest party in the plurality/majority tier will be larger than the number of mandates that the largest parties usually win in PR systems, and then the electoral system will be only semi-compensatory – a hybrid status between non-compensatory and compensatory systems.

       The position where a system is just compensatory has not been determined. It might be not universal, but depend on the nationalisation of the party system, the size of the parliament, on the PR formula applied in the PR tier, and most of all on the vote distribution on political parties. "Roughly", according to Moser and Scheiner (2004: 580), with 50% of compensatory seats, "the result is a distribution of seats almost fully controlled by the PR vote", whereas with 25% of the mandates it is not. But this estimations along with others (Cox/Schoppa 2002: 1029-30) is mainly founded at the empirical outcomes in Hungary, Italy, and Germany, and they neglect the methodological difficulty to establish the necessary share of compensatory seats at empirical cases, or what would be the equivalent to the psychological effect: To some extent, the vote distribution itself might rely on the compensatory or the semi-compensatory character of the electoral system: Strategically thinking political actors (political entrepreneurs, voters, party factions, party leadership, etc.; see above) anticipate the impact of the electoral system, so that at the end only so many parties compete and gain votes that all the seat-winning parties will be represented proportionally. This effect was demonstrated for simple electoral systems and called the "psychological effect" of electoral systems (Duverger 1951; Sartori 1968).[7]

       Consequently, if measured from the outcome aspect, the same electoral system might once be fully compensatory and another time only semi-compensatory, if the vote distribution changes.

       In order to introduce a conceptual distinction of compensatory systems from semi-compensatory ones, we might thus not rely on real outcomes, because they are all biased by the possibly system-affected vote distribution on parties, but we should rather ask: When do compensatory systems lead to a party system that is similar to a party system in proportional representation systems? Or, how many compensatory mandates do we need, in order to allow proportional results if the vote distribution is similar to that in PR systems? The latest would be the definition of a just compensatory system; at any higher share of compensatory mandates the system

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7 Ferrara et al. (2005: 60) show that the share of SMD districts affects the average number of candidates in mixed electoral systems. However, the results are to be taken with caution, because the authors do not distinguish if this effect works in compensatory systems the same way as in non-compensatory systems (the linear model applied supposes that the average number of candidates will change in both types of systems identically if the share of the SMD tier changes. Although the incentives given by the compensatory and non-compensatory systems are not the same, there might be not sufficient empirical variance to distinguish the effect and to get significant results). It might be interesting investigating this impact not only on the number of candidates in districts, but likewise on the number of effective candidates, thus not only consider the parties' strategic decisions, but the strategic behaviour of voters too.

should be fully compensating, and at any lower share, the system might either produce disproportionalities, the fractionalisation of the party system will decrease in comparison to a pure PR system, so that no more disproportionalities are visible. Speaking about methodology, if we want to operationalise empirically the question of the character of an electoral system, we should not only consider measures of proportionality (such as the Least-Square index by Gallagher, 1991), but also the effective number of parties in parliament.

Compensatory versus semi-compensatory systems: Parliament size and threshold matter

The key characteristics of the functioning of mixed compensatory electoral systems would thus be:
   a) The effective thresholds for party entry set through the PR formula (including legal thresholds, or division of the compensatory tier into districts). Those give incentives for party entry or party concentration, and thus "allow" a certain degree of fractionalisation of the party system.
   b) The seat share accorded in the compensatory tier. The key question related to this measure is: Given the degree of party system fractionalisation as incited through the PR formula and other thresholds in the compensatory tier, and given that the plurality/majority tier will produce disproportionalities in the distribution of the district seats on the competing parties, will the share of compensatory mandates be large enough to correct those disproportionalities and allow an overall proportional seat distribution?

To discuss the compensatory tier's ability to correct disproportionalities, we might either ask if it counts enough mandates to give small parties all the mandates that they deserve proportionally. Or, we might ask the opposite question, if the share of plurality/majority seats is small enough, so that the overrepresentation of large parties in this tier might be corrected through the compensatory tier.

We have more knowledge on the latter aspect, since previous work established accurate estimations of the size of the largest party in a given electoral system. Based on this knowledge, we can estimate the absolute number of mandates that the largest party wins in the plurality/majority tier. The largest party is generally the most over-represented one (the one with the highest seat-vote ratio)[8], and since the function of the compensation seats is to correct for over-representations and under-representations, the seats won by the largest party are thus the relevant number to control for. We can estimate the number of mandates that the largest party of a party system would win, if all the seats of a parliament were accorded by pure PR. As long as the number of mandates won in the plurality/majority tier is not larger than the number of seats the party might win according to the PR rule, the system will be fully compensatory. Based on the considerations about the seat share of largest parties, we can establish a predictive formula that indicates at which level a system will be just compensatory, so that a further increase of the compensatory tier would not increase the proportionality of the system and not increase the number of parties in parliament.

The point where a system is just compensatory is particularly important for our purposes, since it separates fully compensatory systems from only semi-compensatory ones. To find this point, it is important to have more information about the seat share that the largest party in the party system might win – either in the plurality/majority districts, or if the seats are accorded by a proportional formula. At which share of compensatory mandates does the largest party in the party system win exactly the number of seats in the plurality/majority tier that it would be entitled proportionally? This would be the point where an electoral system is just compensatory.

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[8] Exceptions might apply, but as long as the A-ratio of the most over-represented party is not fundamentally different from the estimations made here or as this regards only very small parties, the overall outcome is not affected.

Estimating necessary shares of compensation seats for systems with legal thresholds

The seat share of the largest party in the plurality/majority tier might be estimated using a formula proposed by Taagepera (2007) for district-based electoral systems: The seat share of the largest party p_1 is given through the number of seats (S) in parliament and the district size (*m*), or $p_1 = 1/\sqrt[8]{mS}$. (A list of variables can be found in the appendix.) In my case of single-seat districts, *m* is equal to 1, and *S* is equal to the number of mandates in the plurality/majority tier of the electoral system S_{SMD}. According to the Taagepera formula, in a 100-seat plurality/majority tier, we would expect the largest party to win 56 seats (56%); in a 24-seat-tier it would win 16 seats (67%), provided that there is no contamination through the mixed electoral system. The number of mandates in the plurality/majority tier is on the other hand a simple transformation of the share of the compensation tier *c* and the overall number of mandates in parliament *S*.

$$p_1 = \frac{1}{\sqrt[8]{m \cdot S}} \qquad \text{[formula according to Taagepera (2007)]}$$

$$p_{1SMD} = \frac{1}{\sqrt[8]{S_{SMD}}} = S_{SMD}^{-\frac{1}{8}} = (S \cdot (1-c))^{-\frac{1}{8}} \qquad [1]$$

To estimate the seat share of the largest party in the PR tier, the type of the PR formula applied matters. In many of the mixed compensatory systems, compensatory PR mandates are accorded in nationwide districts, to parties that pass a national legal threshold. The Taagepera formula of the largest seat share applies for district based electoral systems, there has no corresponding formula been published for electoral systems that are based on national legal thresholds. However, it can be developed in analogy.

Taagepera (2007) is setting an upper and a lower conceptual limit of the number of parties in a party system. The upper limit is given through the limited district size – or, in our case, through the national threshold. In the case of national legal thresholds t, we would have the maximal number of parties, if each party obtains just the amount of votes necessary to pass the threshold. If we divide 100% in shares of t, this results in 1/t shares, and accordingly, we have maximally 1/t parties. But in reality this limit will never be reached. The lower limit however would be reached if only one single party clears the threshold, either because it wins almost all the votes, not leaving enough votes for further parties to clear the threshold, or because the rest of the votes is too fractionalised. In the absence of more precise knowledge, we might expect the number of parties being close to the geometric mean of both values, or the square root of 1/t. And, in analogy to Taagepera, I expect the seat share of the largest party p1 to be exponentially related to the number of effective parties N2. (Further discussion is needed to verify how concepts developed for district-based electoral systems are transferable to electoral systems with national legal thresholds!)

$$p_1 = N_2^{-\frac{3}{4}}$$

$$\text{if } N_2 = \sqrt[2]{\frac{1}{t}} \text{, then accordingly } p_1 = t^{\frac{3}{8}} \qquad [2]$$

A compensatory electoral system is just compensatory, when the share of compensatory mandates c is such that the number of plurality/majority mandates that goes to the largest party is equal to the number of mandates that this party is entitled to proportionally. The number of mandates a party wins can easily be calculated from the party's seat shares (as developed above), and then used to build an equation that will inform us about where the electoral system is just compensatory.

At c=comp, there:

$$S \cdot (1 - comp) \cdot p_{1SMD} = S \cdot p_1$$

$$S \cdot (1 - comp) \cdot (S \cdot (1 - comp))^{-\frac{1}{8}} = S \cdot t^{\frac{3}{8}}$$ including formula [1] and [2]

Transforming this formula, we can resolve it for comp:

$$(1 - comp)^{\frac{7}{8}} = S^{\frac{1}{8}} \cdot t^{\frac{3}{8}}$$, and thus $comp = 1 - S^{\frac{1}{7}} \cdot t^{\frac{3}{7}}$ [3]

For a given number of seats in parliament (S) and a given legal threshold applied in the PR tier (t), the share of compensatory mandates where a system is just compensatory (comp) is given as $comp = 1 - S^{\frac{1}{7}} \cdot t^{\frac{3}{7}}$.

According to this formula, both with an increasing number of seats in the overall parliament, and with an increasing legal threshold in the PR tier, the number of necessary mandates for full compensation decreases. Taagepera's model on which my considerations are based predicts that for a parliaments that are elected in single-seat plurality/majority districts, an increase in seats will result in an increase of the number of elected parties (because the party system might vary from district to district). A side effect of this is that the expected seat share of the largest party in larger parliaments will be smaller. We can apply the same logic on mixed compensatory systems: There, a larger size of the parliament will reduce the share of necessary compensation seats, because the relative overrepresentation of large parties is smaller. Second, a larger legal threshold in the PR tier reduces the number of small parties in the overall system, and in consequence, it increases the share of seats the largest party is entitled to proportionally. Because the largest party is "allowed" to hold more mandates, less compensation seats are needed.

Some examples: If a parliament counts 200 seats, and has a 5% threshold, then the largest party under pure PR rules would be expected to win approximately 66 seats. The largest party would win the same number of seats if there were 120 single-seat districts. In consequence, maximally 120 out of 200 seats might be accorded by single-seat plurality, or the minimal share of compensation seats equals 40% in this case. If the threshold would only count 2.5% of the votes, then 56% of the seats needed to be compensatory; if the parliament would count 400 seats instead of 200, and still a 5% threshold would be applied, then only one third of the seats would need to be compensatory ones.9

Estimating necessary shares of compensation seats for systems with districts

Usually national legal thresholds are used for the PR part of mixed compensatory systems. However, we might in analogy develop a formula where the PR part of a mixed compensatory system is organised in multi-member districts, and no threshold applies (or, where the legal threshold is so small that the district size is more relevant for the party system fractionalisation; cf. Lijphart 1994). In this case, the district size of the PR districts determines how many of the district seats need to be compensatory ones, and how many can be allocated in single-seat districts. We can apply the same procedure as in the case of the legal thresholds, however this time with the formula

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9 If the national legal threshold gets very large, and the size of the parliament is hold stable, then the share of necessary compensating mandates would get negative. In this case, my model would predict that the seat share of the largest party would be smaller if all the seats were allocated by PR than if all the seats were allocated by plurality or majority vote. Or, the PR threshold gets the much more important threshold than the district threshold. A negative share of compensatory mandates is not possible. In fact, the value is generated because a previous simplifying assumption gets disturbing: In the case of a very high PR threshold and a large number of seats in parliament, it is possible that small parties fail the PR threshold, but are strong enough in some regions to win a considerable amount of mandates in the districts. I assumed that the number of district mandates won by small parties that fail the threshold is negligible. However, this problem occurs only if extreme values are taken: For an empirically typical national legal threshold of 5%, the share of compensatory seats gets only then negative when the number of seats in parliament outnumbers 8000.

for the largest in district systems, $p_1=1/^8\sqrt{mS}$. The model supposes that every district has the same size; otherwise the formula needs to be applied for each district separately.

At $c=comp$, there:

$$\left| \begin{array}{l} S \cdot (1 - comp) \cdot p_{1SMD} = S \cdot p_1 \\ S \cdot (1 - comp) \cdot (S \cdot (1 - comp))^{-\frac{1}{8}} = S \cdot (mS)^{-\frac{1}{8}} \end{array} \right.$$

Transforming this formula, we can resolve it for comp:

$$\left| (1 - comp)^{\frac{7}{8}} = m^{-\frac{1}{8}} \text{ , and thus } comp = 1 - m^{-\frac{1}{7}} \right. \tag{4}$$

Results need to be interpreted cautiously, since the Taagepera model predicts the seat share of the largest party at the national level; however if the correction applies at the district level.[10] Unknown deviations might lead to an underestimation of the necessary share of compensatory mandates.

The impact of semi-compensatory systems on party system fractionalisation

For electoral systems where the compensatory tier is smaller than the estimated value, the number of mandates won by the largest party in the plurality/majority tier will be larger than the number of mandates that the largest parties usually win in PR systems, and then the electoral system will be only semi-compensatory. With no means, this does mean that the semi-compensatory system will not be able to compensate for the disproportionalities of the basis of a given empirical vote distribution.

For this purpose, we should study the nature of what has been called the "psychological effect of electoral systems" (Duverger 1951). Budge et al. (1997: 238) argue, in line with Duverger, that "the more the voting system discriminates against mini-parties, and the poorer their chances of winning even one seat, the more likely they are to drop out of the contest, leaving mainly the larger parties in the running. This will lower the number of effective parties and increase proportionality in future elections." However, what will happen in mixed semi-compensatory systems?

The functioning of the psychological effect is different: What is special about semi-compensatory systems is that they would allow many small parties to win at least one seat, but they will still be under-represented in parliament, and will have low conversion rates of votes into seats ("A-ratio" lower than 1, cf. Taagepera/Laakso 1980), whereas large parties win more than the seat share that they are entitled to proportionally ("A-ratio" greater than 1). This makes the nature of the psychological effect different. We deal not any more with representation or non-representation, but with over- or under-representation, with A-ratios above or below 1.[11]

In this situation, incentives for small parties are conflicting: Previous work has often focussed on the chances of small parties to win at least one seat, and in this sense, semi-compensatory systems would incite many small parties to compete, because parties – if passing the electoral threshold – will always win mandates in parliament in the compensatory tier, however, not proportionally to their actual voting power.

On the other hand, semi-compensatory systems might bring along negative incentives for small parties' electoral support. For this, we should not look at the chances of the parties to win

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[10] The formula would rely on the strong assumption that at level of the territorial units, where the compensation takes place, the seat shares of the strongest parties at the regional level are comparable to the national level.

11 Simple electoral systems in most of the cases make a sharp distinction between winners and losers – yet parties that win an amount of seats that is often above their vote share and others that do not get represented in parliament. It might be possible that parties narrowly win representation, but not in all the districts, what might lead to severe under-representation, but this can be observed only for parties with a vote share just around the effective threshold, and would rather not be the case if the analysis would have been made at the level of electoral districts. Although A-ratio under 1 might occur for smaller parties due to heterogenous district sizes or heterogeneous vote shares over the country, in each district the A-ratio is very likely to be above 1 or 0 – and intermediate levels are the exception. And it is at this district level, where voters and politicians take the decision for which party they vote or run.

at least one seat, but rather at the chances of winning a proportional share of seats (an A-ratio above one). To understand why the conversion rate of votes into seats matters for the psychological effect, instead of looking at the level of the party leadership, we need to look at the level of individual voters and at the level of politicians seeking offices and choosing for this purpose among different parties. For voters, political activists, and potential candidates, large parties – ceteris paribus – offer a better value than small parties, because they can make more out of their votes or out of their mobilisation force: If small parties get under-represented, and large parties over-represented, a vote for a large party counts more in terms of seats than a vote for a small party, what gives a voter of a large party more weight in parliament and candidates better chances to get elected. Over time, even small differences in the vote-seat conversion rate can provoke an exodus of voters, activists, and politicians towards large parties, and make small parties disappear or merge with large parties. The consequence of semi-compensatory systems might be that the party fractionalisation remains below the level of fractionalisation that we expect in a purely proportional system with the same PR formula applied.

What would be thus the expected number of parties for a semi-compensatory electoral system? As seen, over time, there is no (or minimal) disproportionality produced, and thus only those parties compete that are likely to win a proportional share of the (overall) seats. This is only the case if the largest party does not win an over-proportional share of the seats; otherwise those surplus mandates would go on the extent of smaller parties. Again, based on the work of Taagepera (2007), we are able to estimate the share of seats that the largest party wins in the plurality/majority tier. Further, we know that the number of seats the largest party wins in the plurality/majority tier is the number that it should win according to its votes in the PR tier. If this is the case, the electoral system manages to accord each party a proportional share of seats (since no party is over-represented). On the other hand, in the case of a larger fractionalisation of the vote, the largest party would win more than a proportional share of seats, and thus disproportionalities would be created. Using the share of compensation mandates (c) and the overall number of seats in the electoral system (S), we can estimate the effective number of parties in parliament (N2) in two ways. The first method is based on the seat share of the largest party, the second on Taagepera's method of the mean between extreme conceptual limits.

The largest seat share approximation is based on the assumption that the largest number of seats a party wins in the plurality/majority tier is equal to the number that would win in a pure PR system according to its seat share.

The largest seat share in the plurality/majority tier was given before, based on the overall number of seats in the parliament and the share of the compensatory tier:

$$p_{1SMD} = (S \cdot (1-c))^{-\frac{1}{8}}, \text{ or as absolute number of seats, } \dot{P}_{1SMD} = (S \cdot (1-c))^{\frac{7}{8}} \qquad [1]$$

Based on Taagepera's work (2007), we know further that the share of the largest party a PR system is related to the effective number of parties in the following way:

$$p_1 = N_2^{-\frac{3}{4}}, \text{ or as absolute number of seats, } \dot{P}_1 = N_2^{-\frac{3}{4}} \cdot S \qquad [2]$$

If thus the number seats the largest party wins in the plurality/majority tier is equal to the number of seats it wins in the overall system, then we can set the outcome of equation [1] and [2] as equal (the same as we did in the first part of the appendix, although this time the share of compensatory seats c is given, and we do not consider the variable t in our model, because we know that the resulting number of parties will be lower than in a pure PR system with a threshold).

If the largest seat share in the plurality/majority tier is known:

$$N_2^{-\frac{3}{4}} \cdot S = \dot{P}_1 \rightarrow N_2 = (p_{1SMD}(1-c))^{-\frac{4}{3}} \qquad \text{[including formula 2]}$$

If the largest seat share in the plurality/majority tier is estimated with Taagepera's formula:

$$\dot{P}_1 = N_2^{-3/4} \cdot S = (S \cdot (1-c))^{7/8} \qquad \text{[including formula 1 and 2]}$$

$$\text{or transformed,} \quad N_2 = \left(S^{-1/8} \cdot (1-c)^{7/8} \right)^{-4/3}$$

$$N_2 = S^{1/6} \cdot (1-c)^{-7/6} \qquad \text{[5]}$$

We can arrive to a very similar result if we consider the conceptual limits of the number of effective parties in mixed semi-compensatory systems, the method used by Taagepera (2007). The upper and lower limits of the number of parties are drawn in figure 2 for a compensatory electoral system with 140 seats and a 2.5% threshold. The upper limit is the result of a simulation of the seat distribution if the political individuals do not take into account the disproportionalites of the electoral system, and cast their vote as if it was perfectly proportional (according to the formula applied in the PR tier). Still, the number of parliamentary parties is heavily dependent on the share of compensatory seats, since if the number of compensatory seats is very small, only the parties that are successful in the plurality/majority tier will get elected.

The lower limit corresponds however to the expected outcome of the plurality/majority tier. If disproportionalities in the seat allocation in a semi-compensatory electoral system would be an important factor for the actors' behaviour, then the outcome would highly correspond to this lower limit.
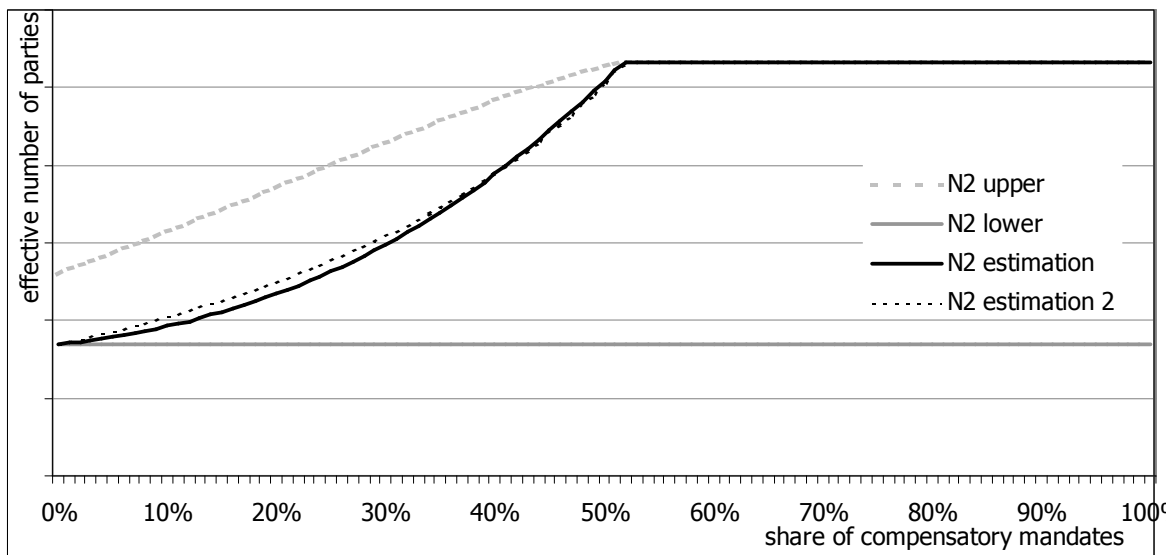

Figure 2: Effective number of parties in a mixed electoral system with a varying number of compensatory mandates, upper and lower limit.

Our estimation must be somewhere in between the both extremes. However we lack of more precise information to locate the outcome more precisely. Only when the number of compensatory mandates is very small or when at the contrary the number of compensatory reaches almost the necessary number for a fully compensatory system, the number of parties might be easier to predict:

- If there would just be a marginally small number of compensatory mandates, let's say one mandate in a large assembly, that would be accorded to the most under-represented party, reasonable voters and parties would cast their plurality/majority vote not very different than in a pure plurality/majority system: If there is only a marginal compensation, why count on it? Yet, the outcome of a system with a very small number of compensation mandates can be approximated with the outcome of a pure plurality/majority system.

- If the system would be almost compensatory (let's say one compensation mandate less than in a perfectly compensatory system), why should voters and parties not behave as in a fully compensatory system? It might be that one party would be underrepresented by one mandate, but with such a marginal difference (and difficulties to predict which party would be hit by the disproportionality), there is no systematic reason to defeat from smaller parties, and the electoral outcome can very well be approximated with the upper limit.

Based on this knowledge, I draw an expected function for the number of parliamentary parties in a compensatory electoral system (with some given specifications), based on the share of compensatory mandates (figure 2, bold line), starting from a simple plurality/majority system at c=0%, and then gradually with an increase of the compensatory share approaching the upper limit (interpolation of both limits).

The graph corresponds widely with the formula that estimates the effective number of parties as

$$N_2 = S^{1/6} \cdot (1-c)^{-7/6}.$$

Conclusion

Mixed compensatory electoral systems have often been discussed as a special case of PR, but the conditions for PR-like outcomes remained unclear: How many mandates are necessary in the compensatory tier in order to guarantee that the personality vote in the plurality/majority tier does not disadvantage small parties from competing?

The present paper looks at the distinction of fully compensatory systems (those that have possibly PR-like outcomes) and semi-compensatory systems. For the first time, it offers a way to predict the necessary share of compensatory mandates to make a mixed system fully compensatory or PR-like. The value depends on the overall number of seats in the parliament and on the level of the threshold applied in the PR tier. If the national legal thresholds applied in PR are low, quite a large share of compensation seats, sometimes even more than half of the seats in parliament, are necessary for full compensation. In single cases, it depends on the seat share that large parties win in the single-seat districts and on their nationwide vote share.

My exploratory estimation of the necessary share of compensation mandates, and of the number of parties that get elected under semi-compensatory systems, relies on models that were developed for simple electoral systems. I needed to make a few assumptions in order to use the formulas that were originally thought for district-based electoral systems, such as single-seat plurality or multimember-district PR, in this context. We need to ask: Can formula and methods that were developed for district-based electoral systems be borrowed for the estimation of party system characteristics under mixed compensatory electoral systems, or for electoral systems with national legal thresholds? And, to what extent can results that were obtained for party systems at the national level be employed at the level of electoral districts? There are good reasons to do so: national party systems are nothing else than the sum or regional party systems. And mixed compensatory electoral systems are nothing else than a complicated combination of different simple electoral systems. Further research might however examine to what extent knowledge about vote shares and seat shares in district-based electoral systems can be used for mixed compensatory systems.

Appendix: List of symbols used for variable abbreviations

c   Share of compensatory mandates in mixed electoral systems [$c = 1 - S_{SMD}/S$]

comp   Necessary share of compensatory mandates for a just compensatory system

m   number of seats per electoral district

N_0 Absolute number of parties

N_2 Effective number of parties (Laakso/Taagepera 1979)

N_V Effective number of parties, based on their vote share

p_1 Largest seat share of single party

p_{1SMD} Largest seat share of single party in the plurality/majority tier of mixed electoral systems

$\dot{P}_1$ Absolute number of seats hold by largest party in parliament [$\dot{P}_1 = p_1 * S$]

$\dot{P}_{1SMD}$ Absolute number of seats hold by largest party in plurality/majority tier [$\dot{P}_{1SMD} = p_{1SMD} * S_{SMD}$]

S   Overall number of mandates in parliament (or elected body)

S_{PR} Number of mandates in proportional or compensatory tier of mixed electoral systems

S_{SMD} Number of mandates in plurality/majority tier of mixed electoral systems [$S_{SMD} = S \cdot (1 - c)$]

t   Legal threshold

References

Behnke, Joachim (2003): Ein integrales Modell der Ursachen von Überhangsmandaten; in: Politische Vierteljahresschrift 44(1). 41-65.

Behnke, Joachim; Kamm, Ruth; Sommerer, Thomas (2003): Der Effekt der Neueinteilung der Wahlkreise auf die Entstehung von Überhangmandaten; in: Zeitschrift für Parlamentsfragen 1/2003. 122-145.

Bochsler, Daniel (2005): Electoral systems and party systems in the new European democracies. Is party nationalisation the key variable? University of Geneva, political science department. Mémoire préliminaire.

Bochsler, Daniel (2007): If Albanians, Venezuelans and Lesothans can figure out how to do it, why not Germans and everybody else? – The hidden potential for manipulating mixed compensatory electoral systems. Manuscript.

Budge, Ian et al. (1997): The Politics of the New Europe. Atlantic to Urals. Longman, London/New York.

Cox, Gary W. (1997): Making Votes Count. Strategic Coordination in the World's Electoral Systems. Cambridge University Press, Cambridge.

Cox, Karen E.; Schoppa, Leonard J.(2002): Interaction Effects in Mixed-Member Electoral Systems: Theory and Evidence from Germany, Japan and Italy; in: Comparative Political Studies 35. 1027-1053.

Duverger, Maurice (1951): Les partis politiques. Colin, Paris.

Duverger, Maurice (1986): Duverger's Law: Forty Years Later, in: Bernard Grofman und Arend Lijphart (Eds.): Electoral Laws and Their Political Consequences. Agathon Press, New York. 69-84.

Ferrara, Federico; Herron, Erik S.; Nishikawa, Misa (2005): Mixed Electoral Systems. Contamination and Its Consequences. Palgrave, New York.

Gallagher, Michael (1991): Proportionality, Disproportionality and Electoral Systems; in: Electoral Studies 10(1). 33-51.

Herron, Erik S.; Nishikawa, Misa (2001): Contamination effects and the number of parties in mixed-superposition electoral systems; in: Electoral Studies 20 (1). 63-86.

Kostadinova, Tatiana (2002): Do mixed electoral systems matter?: a cross-national analysis of their effects in Eastern Europe; in: Electoral Studies 21 (1). 23-34.

Kreuzer, Marcus (2004): Germany: Partisan Engineering of Personalized Proportional Representation; in: Colomer, Josep M. (ed.): Handbook of Electoral System Choice. Palgrave, Houndsmills. 222-236.

Lijphart, Arend (1994): Electoral Systems and Party Systems. A Study of Twenty-Seven Democracies 1945-1990. Oxford University Press, Oxford.

Massicotte, Louis; Blais, André (1999): Mixed electoral systems: a conceptual and empirical survey: Electoral Studies 18 (3). 341-366.

Moser, Robert G. (1995): The Impact of the Electoral System on Post-Communist Party Development: the Case of the 1993 Russian Parliamentary Elections; in: Electoral Studies 14 (4). 377-398.

Moser, Robert G.; Scheiner, Ethan (2004): Mixed electoral systems and electoral system effects: controlled comparison and cross-national analysis; in: Electoral Studies 23 (4). 575-599.

Nohlen, Dieter (2004): Wahlrecht und Parteiensystem. Leske + Budrich, Opladen.

Rae, Douglas W. (1971): The political consequences of electoral laws. Yale University Press, New Haven, London.

Sartori, Giovanni (1968): Political Development and Political Engineering; in: Public Policy 17: 261–98.

Schiemann, John W. (2001): Hedging Against Uncertainty: Regime Change and the Origins of Hungary's Mixed-Member System; in: Shugart, Matthew Soberg; Wattenberg, Martin P. (eds.): Mixed-Member Electoral Systems. The Best of Both Worlds? Oxford University Press, Oxford. 231-255.

Schoen, Harald (1999): Split voting in German federal elections 1953-90: an example of sophisticated balloting; in: Electoral Studies 18 (4). 473-796.

Shugart, Matthew Soberg (2001): "Extreme" Electoral Systems and the Appeal of the Mixed-Member Alternative; in: Shugart, Matthew Soberg; Wattenberg, Martin P. (eds.): Mixed-Member Electoral Systems. The Best of Both Worlds? Oxford University Press, Oxford. 25-51.

Shugart, Matthew Soberg; Wattenberg, Martin P. (2001a): Mixed-Member Electoral Systems: A Definition and Typology; in: Shugart, Matthew Soberg; Wattenberg, Martin P. (eds.): Mixed-Member Electoral Systems. The Best of Both Worlds? Oxford University Press, Oxford. 9-24.

Shugart, Matthew Soberg; Wattenberg, Martin P. (2001b): Conclusion: Are Mixed-Member Systems the Best of Both Worlds?; in: Shugart, Matthew Soberg; Wattenberg, Martin P. (eds.): Mixed-Member Electoral Systems. The Best of Both Worlds? Oxford University Press, Oxford. 571-596.

Taagepera, Rein (2007, forthcoming): The Logic of Simple Electoral Systems. Forthcoming in: Oxford University Press, Oxford.

Taagepera, Rein; Laakso, Markku (1980): Proportionality Profiles of West European Electoral Systems; in: European Journal of Political Research 8. 423-446.

Thames, Frank C. (2005): A House Divided. Party Strength and the Mandate Divide in Hungary, Russia, and Ukraine; in: Comparative Political Studies 38 (3). 282-303.

Tiefenbach, Paul (2006): Kumulieren, Panaschieren, Mehrmandatswahlkreise - mehr Demokratie beim Wahlrecht; in: Zeitschrift für Parlamentsfragen 1/2006: 115-125.