

LINEARNA ALGEBRA I GEOMETRIJA

TUTORIJAL Br.7

Zadatak 1.

Da li vektori $\vec{a} = (1,2,1)$, $\vec{b} = (3, -1,0)$ i $\vec{c} = (4,0,2)$ čine bazu vektorskog prostora? Ako čine izraziti vektor $\vec{d} = (1, -1,3)$ u tom prostoru.

Rješenje: DA! $\vec{d} = -\frac{8}{5}\vec{a} - \frac{11}{5}\vec{b} + \frac{23}{10}\vec{c}$.

Zadatak 2.

Zadani su vektori $\vec{a} = (2\lambda, 1, 1 - \lambda)$, $\vec{b} = (-1, 3, 0)$ i $\vec{c} = (5, -1, 8)$.

- Odrediti λ tako da vektor \vec{a} zaklapa jednake uglove s vektorima \vec{b} i \vec{c} .
- Naći zapreminu i jednu od visina paralelopipeda konstruiranog nad tim vektorima.

Rješenje:

- $\lambda = \frac{1}{4}$.
- $V = 9,5$; $H = 2,75$.

Zadatak 3.

Zadani su vrhovi $A(-1, -9, -7)$, $B(13, -3, -11)$ i $C(5, 13, -5)$ paralelograma ABCD. Odrediti koordinate vrha D i izračunati površinu, obim, unutarnje uglove i visinu tog paralelograma.

Rješenje: $D(-9, 7, 1)$; $P_{\square} = 294,43$; $O = 69,24$; $\alpha = 82,26^{\circ}$.

Zadatak 4.

Odrediti zapreminu tetraedra SABC, površinu trougla ABC i odgovoriti na pitanje koje je orijentacije triedar vektora $(\vec{a}, \vec{b}, \vec{c})$. Razložiti vektor $\vec{a} \times \vec{b}$ na komponente u tom triedru. Poznato je $\vec{a} = \vec{SA} = (2, 4, 0)$, $\vec{b} = \vec{SB} = (4, 3, 1)$ i $\vec{c} = \vec{SC} = (3, 0, 4)$.

Rješenje: $V = \frac{14}{3}$; Lijeva orijentacija; $P_{\square} = \sqrt{30}$.

Zadatak 5.

Naći koordinate tačke B kao drugog tjemena paralelograma ABCD, a potom ugao između dijagonala, dužine dijagonala, površinu i zapreminu piramide ABCDS, ako su dane tačke: $A(2, 1, -1)$, $C(1, 2, 0)$, $D(-1, 0, 2)$ i $S(4, -3, -2)$.

Rješenje: $B(4, 3, -3)$; $|\vec{d}_1| = \sqrt{3}$; $|\vec{d}_2| = \sqrt{59}$; $\alpha = 58,254^{\circ}$; $P = 4\sqrt{2}$; $V = \frac{4}{3}$.

Zadatak 6.

Zadana su redom tri uzastopna vrha paralelograma ABCD: $A(-3, 2, \lambda)$, $B(3, -3, 1)$ i $C(5, \lambda, 2)$.

- Odrediti četvrti vrh D.
- Odrediti λ tako da je $|\overline{AD}| = \sqrt{14}$.
- Za veću vrijednost λ , nađenu u tački b), naći linearnu zavisnost između vektora: \overline{AD} , \overline{BC} i \overline{AC} . Razložiti vektor \overline{AC} preko vektora \overline{AD} i \overline{BD} .

Rješenje:

- $D(-1, 5 + \lambda, 1 + \lambda)$.
- $\lambda_1 = 0$; $\lambda_2 = -6$.
- $\overline{AC} = 2\overline{AD} - \overline{BD}$.

T7 - Analitička geometrija u ravni

SUBJECT

TEACHER

GRADE

DATE

LAG

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OVERVIEW

$$\begin{aligned} 1) \quad \vec{a} &= (1, 2, 1) \\ \vec{b} &= (3, -1, 0) \\ \vec{c} &= (4, 0, 2) \\ \vec{d} &= (1, -1, 3) \end{aligned}$$

Vektori \vec{a}, \vec{b} i \vec{c} čine bazu vektorskog prostora ako su linearno nezavisni. Vektori su linearno nezavisni ako nisu komplanarni, a komplanarni su ako je $(\vec{a} \times \vec{b}) \cdot \vec{c} = 0$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & 0 \\ 4 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 0 \end{vmatrix} = -2 + 4 - 12 = -10 \neq 0 \Rightarrow \text{Vektori su}$$

nekomplanarni, tj. linearno nezavisni i čine bazu vekt. prostora

$$\vec{d} = \alpha \vec{a} + \beta \vec{b} + \gamma \vec{c}$$

$$\alpha \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

$$\alpha + 3\beta + 4\gamma = 1 \Rightarrow \alpha = \frac{1 - 3\beta - 4\gamma}{1}$$

$$2\alpha - \beta = -1 \Rightarrow \beta = 2\alpha + 1 \Rightarrow \beta = \frac{-11}{5}$$

$$\alpha + 2\gamma = 3 \Rightarrow \gamma = \frac{1}{2}(3 - \alpha) \Rightarrow \gamma = \frac{23}{10}$$

$$\vec{d} = \frac{-8}{5} \vec{a} - \frac{11}{5} \vec{b} + \frac{23}{10} \vec{c}$$

$$2) \vec{a} = (2\lambda, 1, 1-2)$$

$$\vec{b} = (-1, 3, 0)$$

$$\vec{c} = (5, -1, 8)$$

$$a) \lambda = ? \quad \angle \vec{a}, \vec{b} = \angle \vec{a}, \vec{c} = \alpha$$

$$b) V, h = ?$$

a)

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \alpha \Rightarrow \cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\vec{a} \cdot \vec{c} = |\vec{a}| |\vec{c}| \cos \alpha \Rightarrow \cos \alpha = \frac{\vec{a} \cdot \vec{c}}{|\vec{a}| |\vec{c}|}$$

$$\cos \alpha = \cos \alpha$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{\vec{a} \cdot \vec{c}}{|\vec{a}| |\vec{c}|} \Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{\vec{a} \cdot \vec{c}}{|\vec{c}|}$$

$$|\vec{b}| = \sqrt{1+9} = \sqrt{10} \quad ; \quad |\vec{c}| = \sqrt{25+1+64} = \sqrt{90} = 3\sqrt{10}$$

$$\vec{a} \cdot \vec{b} = -2\lambda + 3$$

$$\vec{a} \cdot \vec{c} = 10\lambda - 1 + 8 - 8\lambda \Rightarrow \vec{a} \cdot \vec{c} = 2\lambda + 7$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{\vec{a} \cdot \vec{c}}{|\vec{c}|} \Rightarrow \frac{-2\lambda + 3}{\sqrt{10}} = \frac{2\lambda + 7}{3\sqrt{10}}$$

$$-6\lambda + 9 = 2\lambda + 7$$

$$8\lambda = 2 \Rightarrow \boxed{\lambda = \frac{1}{4}}$$

b)

$$V = (\vec{a} \times \vec{b}) \cdot \vec{c}$$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} 1/2 & 1 & 3/4 \\ -1 & 3 & 0 \\ 5 & -1 & 8 \end{vmatrix} = \frac{1}{4} \begin{vmatrix} 2 & 4 & 3 \\ -1 & 3 & 0 \\ 5 & -1 & 8 \end{vmatrix} = \begin{vmatrix} 2 & 4 \\ -1 & 3 \\ 5 & -1 \end{vmatrix} =$$

$$= \frac{1}{4} \cdot (48 + 3 - 45 + 32) = \frac{1}{4} \cdot 38 = \frac{19}{2} \Rightarrow \boxed{V = 3,45}$$

$$V = B \cdot h \Rightarrow h = \frac{V}{B} \quad ; \quad B = |\vec{a} \times \vec{b}|$$

$$(\vec{a} \times \vec{b}) = \frac{1}{4} \begin{vmatrix} i & j & k \\ 2 & 4 & 3 \\ -1 & 3 & 0 \end{vmatrix} = \frac{1}{4} \left\{ i \begin{vmatrix} 4 & 3 \\ 3 & 0 \end{vmatrix} - j \begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} + k \begin{vmatrix} 2 & 4 \\ -1 & 3 \end{vmatrix} \right\} =$$

$$= \frac{1}{4} (-9\vec{i} - 3\vec{j} + 10\vec{k})$$

$$B = |\vec{a} \times \vec{b}| = \frac{1}{4} \sqrt{81 + 9 + 100} = \frac{1}{4} \sqrt{190} = 3,45$$

$$h = \frac{\frac{19}{2}}{3,45} \Rightarrow \boxed{h = 2,75}$$

3.)

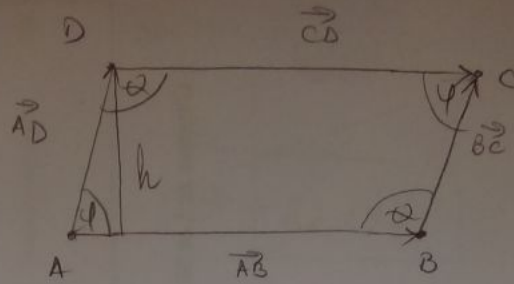
$$A(-1, -9, -4)$$

$$B(13, -3, -11)$$

$$C(5, 13, -5)$$

$$D(x_D, y_D, z_D) = ?$$

$$P, \varphi, \alpha, h = ?$$



$$\vec{AB} = \vec{DC}$$

$$\vec{AB} = (x_B - x_A, y_B - y_A, z_B - z_A) = (14, 6, -4)$$

$$\vec{DC} = (x_C - x_D, y_C - y_D, z_C - z_D) = (5 - x_D, 13 - y_D, -5 - z_D)$$

$$5 - x_D = 14 \Rightarrow x_D = -9$$

$$13 - y_D = 6 \Rightarrow y_D = 7$$

$$-5 - z_D = -4 \Rightarrow z_D = -1$$

$$\Rightarrow \boxed{D(-9, 7, -1)}$$

$$P = |\vec{AB} \times \vec{BC}|$$

$$\vec{BC} = (x_C - x_B, y_C - y_B, z_C - z_B) = (-8, 16, 6)$$

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 14 & 6 & -4 \\ -8 & 16 & 6 \end{vmatrix} = \vec{i} \begin{vmatrix} 6 & -4 \\ 16 & 6 \end{vmatrix} - \vec{j} \begin{vmatrix} 14 & -4 \\ -8 & 6 \end{vmatrix} + \vec{k} \begin{vmatrix} 14 & 6 \\ -8 & 16 \end{vmatrix} =$$

$$= -100\vec{i} - 52\vec{j} - 242\vec{k}$$

$$|\vec{AB} \times \vec{BC}| = \sqrt{86688} = 294,43 \Rightarrow \boxed{P = 294,43}$$

$$O = 2|\vec{AB}| + 2|\vec{BC}| = 31,5 + 37,74 = 69,24$$

$$\boxed{O = 69,24}$$

$$|\vec{AB}| = \sqrt{248} = 15,75 ; \quad |\vec{BC}| = \sqrt{356} = 18,87$$

$$\vec{AB} \times \vec{BC} = |\vec{AB}| |\vec{BC}| \sin \theta \Rightarrow \sin \theta = \frac{|\vec{AB} \times \vec{BC}|}{|\vec{AB}| |\vec{BC}|}$$

$$\sin \theta = \frac{294,43}{297,2} = 0,99 \Rightarrow \boxed{\theta = 82,171}$$

$$\theta + \varphi = 180 \Rightarrow \boxed{\varphi = 97,83}$$

$$P = |\vec{AB}| \cdot h \Rightarrow h = \frac{P}{|\vec{AB}|}$$

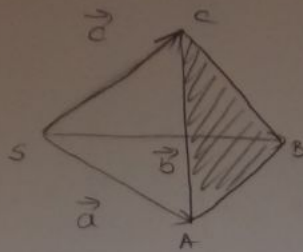
$$h = \frac{294,43}{15,75} \Rightarrow \boxed{h = 18,69}$$

4)

$$\vec{a} = \vec{SA} = (2, 4, 0)$$

$$\vec{b} = \vec{SB} = (4, 3, 1)$$

$$\vec{c} = \vec{SC} = (3, 0, 4)$$



$$V_{SABC}, P_{ABC} = ?$$

$$V_{SABC} = \frac{1}{6} \cdot V_p = \frac{1}{6} |(\vec{a} \times \vec{b}) \cdot \vec{c}|$$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} 2 & 4 & 0 \\ 4 & 3 & 1 \\ 3 & 0 & 4 \end{vmatrix} = 24 + 12 - 64 = -28 < 0 \Rightarrow$$

lijeva orijentacija!

$$V_{SABC} = \frac{1}{6} \cdot |-28| = 4,67 \Rightarrow \boxed{V_{SABC} = 4,67}$$

$$P_{ABC} = \frac{1}{2} |\vec{AB} \times \vec{BC}|$$

$$\vec{AB} = \vec{b} - \vec{a} \Rightarrow \vec{AB} = (2, -1, 1)$$

$$\vec{BC} = \vec{c} - \vec{b} \Rightarrow \vec{BC} = (-1, -3, 3)$$

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ -1 & -3 & 3 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & 1 \\ -3 & 3 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & -1 \\ -1 & -3 \end{vmatrix} =$$

$$= -7\vec{j} - 7\vec{k}$$

$$P_{ABC} = \frac{1}{2} \cdot \sqrt{7^2 + 7^2} = \frac{1}{2} \cdot 9,90 = 4,95 \Rightarrow \boxed{P_{ABC} = 4,95}$$

$$\vec{a} \times \vec{b} = \lambda \vec{a} + \beta \vec{b} + \gamma \vec{c}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & 0 \\ 4 & 3 & 1 \end{vmatrix} = \vec{i} \begin{vmatrix} 4 & 0 \\ 3 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 0 \\ 4 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 4 \\ 4 & 3 \end{vmatrix} =$$
$$= 4\vec{i} - 2\vec{j} - 10\vec{k}$$

$$\lambda \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ -10 \end{bmatrix}$$

$$2\lambda + 4\beta + 3\gamma = 4$$

$$4\lambda + 3\beta = -2 \Rightarrow \lambda = -\frac{1}{4}(3\beta + 2)$$

$$\beta + 4\gamma = -10 \Rightarrow \gamma = -\frac{1}{4}(\beta + 10)$$

$$\beta = \frac{50}{7} ; \lambda = -\frac{41}{7} ; \gamma = -\frac{30}{7}$$

$$\vec{a} \times \vec{b} = -\frac{41}{7} \vec{a} + \frac{50}{7} \vec{b} - \frac{30}{7} \vec{c}$$

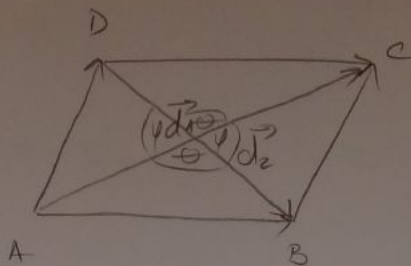
5)

$$A(2, 1, -1)$$

$$C(1, 2, 0)$$

$$D(-1, 0, 2)$$

$$S(4, -3, -2)$$



$$B(x_B, y_B, z_B) = ?$$

$$|\vec{d}_1| = ?$$

$$|\vec{d}_2| = ? \quad \alpha, \varphi = ?$$

$$P_{ABCD}, V_{ABCD} = ?$$

$$\vec{AB} = \vec{DC}$$

$$\vec{DC} = (x_C - x_D, y_C - y_D, z_C - z_D) = (2, 2, -2)$$

$$\vec{AB} = (x_B - x_A, y_B - y_A, z_B - z_A) = (x_B - 2, y_B - 1, z_B + 1)$$

$$x_B - 2 = 2 \Rightarrow x_B = 4$$

$$y_B - 1 = 2 \Rightarrow y_B = 3$$

$$z_B + 1 = -2 \Rightarrow z_B = -3$$

$$\boxed{B(4, 3, -3)} \Rightarrow \vec{AB} = (2, 2, -2)$$

$$\vec{d}_1 = \vec{AB} + \vec{AD} \quad ; \quad \vec{AD} = (x_D - x_A, y_D - y_A, z_D - z_A) = (-3, -1, 3)$$

$$\vec{d}_1 = (2, 2, -2) + (-3, -1, 3) = (-1, 1, 1)$$

$$|\vec{d}_1| = \sqrt{1^2 + 1^2 + 1^2} \Rightarrow \boxed{|\vec{d}_1| = \sqrt{3}}$$

$$\vec{d}_2 = \vec{AB} - \vec{AD} = (2, 2, -2) - (-3, -1, 3) = (5, 3, -5)$$

$$|\vec{d}_2| = \sqrt{5^2 + 3^2 + 5^2} \Rightarrow \boxed{|\vec{d}_2| = \sqrt{59}}$$

$$\vec{d}_1 \cdot \vec{d}_2 = |\vec{d}_1| |\vec{d}_2| \cdot \cos \theta \Rightarrow \cos \theta = \frac{\vec{d}_1 \cdot \vec{d}_2}{|\vec{d}_1| |\vec{d}_2|}$$

$$\vec{d}_1 \cdot \vec{d}_2 = (-1, 1, 1) \cdot (5, 3, -5) = -5 + 3 - 5 = -7$$

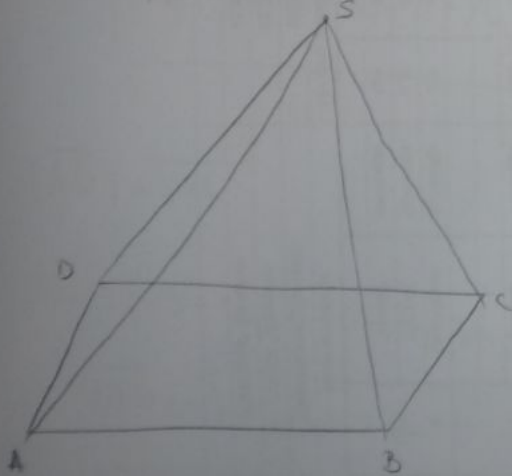
$$\cos \theta = \frac{-7}{\sqrt{3 \cdot 59}} = \frac{-7}{\sqrt{177}} = -0,526$$

$$\boxed{\theta = 121,74^\circ}$$

$$\psi + \theta = 180 \Rightarrow \psi = 180 - \theta \Rightarrow \boxed{\psi = 58,26^\circ}$$

$$P_{ABCD} = |\vec{AB} \times \vec{AD}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & -2 \\ -3 & -1 & 3 \end{vmatrix} = \vec{i} \begin{vmatrix} 2 & -2 \\ -1 & 3 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & -2 \\ -3 & 3 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 2 \\ -3 & -1 \end{vmatrix}$$

$$= |4\vec{i} + 4\vec{k}| = \sqrt{4^2 + 4^2} = \sqrt{39} \Rightarrow \boxed{P_{ABCD} = 4\sqrt{2}}$$



$$V_{SABCD} = \frac{1}{3} V_P$$

$$V_P = |(\vec{AB} \times \vec{AD}) \cdot \vec{AS}|$$

$$\vec{AS} = (x_s - x_A, y_s - y_A, z_s - z_A) = (2, -4, -1)$$

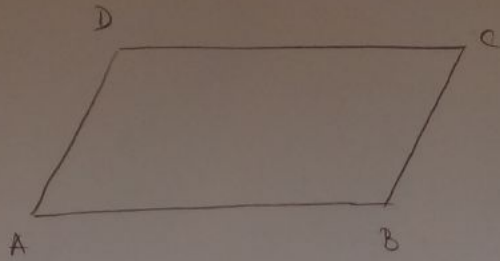
$$(\vec{AB} \times \vec{AD}) \cdot \vec{AS} = \begin{vmatrix} 2 & 2 & -2 \\ -3 & -1 & 3 \\ 2 & -4 & -1 \end{vmatrix} \begin{vmatrix} 2 & 2 \\ -3 & -1 \\ 2 & -4 \end{vmatrix} = 2 + 12 - 24 - 4 + 24 - 6 = 4$$

$$V_{SABCD} = \frac{1}{3} \cdot 4 = \frac{4}{3} \Rightarrow \boxed{V_{SABCD} = 1,333}$$

$$a) A(-3, 2, \lambda)$$

$$B(3, -3, 1)$$

$$C(5, \lambda, 2)$$



$$a) D(x_0, y_0, z_0) = ?$$

$$b) \lambda = ? \quad |\vec{AD}| = \sqrt{14}$$

$$c) \vec{AD}, \vec{BC}, \vec{AC}$$

\vec{AC} preko \vec{AD} i \vec{BD}

$$a) \vec{AB} = \vec{DC}$$

$$\vec{AB} = (x_B - x_A, y_B - y_A, z_B - z_A) = (6, -5, 1 - \lambda)$$

$$\vec{DC} = (x_C - x_D, y_C - y_D, z_C - z_D) = (5 - x_0, \lambda - y_0, 2 - z_0)$$

$$5 - x_0 = 6 \Rightarrow x_0 = -1$$

$$\lambda - y_0 = -5 \Rightarrow y_0 = 5 + \lambda$$

$$2 - z_0 = 1 - \lambda \Rightarrow z_0 = 1 + \lambda$$

$$D(-1, 5 + \lambda, 1 + \lambda)$$

$$b) |\vec{AD}| = \sqrt{14}$$

$$\vec{AD} = (-3 - (-1), 2 - (5 + \lambda), \lambda - (1 + \lambda)) = (-4, -3 - \lambda, -1)$$

$$|\vec{AD}| = \sqrt{4^2 + (-3 - \lambda)^2 + 1^2}$$

$$\sqrt{14} = \sqrt{5 + (-3 - \lambda)^2} \quad |^2$$

$$14 - 5 = (-3 - \lambda)^2$$

$$14 - 5 = 9 + 2 \cdot (-3) \cdot (-\lambda) + \lambda^2$$

$$9 = 9 + 6\lambda + \lambda^2$$

$$6\lambda + \lambda^2 = 0$$

$$\lambda(6 + \lambda) = 0$$

$$\lambda_1 = 0$$

$$\lambda_2 = -6$$

c) Jedna vrijednost $\Rightarrow \lambda = 0$

$$A(-3, 2, 0)$$

$$B(3, -3, 1)$$

$$C(5, 0, 2)$$

$$D(-1, 5, 1)$$

$$\vec{AB} = (-4, -3, -1)$$

$$\vec{BC} = (2, 3, 1)$$

$$\vec{AC} = (8, -2, 2)$$

$$\begin{bmatrix} -4 & -3 & -1 \\ 2 & 3 & 1 \\ 8 & -2 & 2 \end{bmatrix} \begin{bmatrix} \lambda \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-4\lambda - 3\beta - \gamma = 0$$

$$2\lambda + 3\beta + \gamma = 0$$

$$8\lambda - 2\beta + 2\gamma = 0 \Rightarrow 2\gamma = 2\beta - 8\lambda \Rightarrow \gamma = \frac{\beta - 8\lambda}{2} \Rightarrow \gamma = 0$$

$$2\lambda + 3\beta + \frac{\beta - 8\lambda}{2} = 0 \quad | \cdot 2$$

$$4\lambda + 6\beta + \beta - 8\lambda = 0$$

$$-4\lambda + 7\beta = 0 \Rightarrow \beta = \frac{4\lambda}{7} \Rightarrow \beta = 0$$

$$(1) \Rightarrow -4\lambda - 3 \cdot \frac{4\lambda}{7} - \frac{\beta - 8\lambda}{2} = 0 \quad | \cdot 14$$

$$-56\lambda - 24\lambda - 7\beta + 56\lambda = 0$$

$$-24\lambda - 7 \cdot \frac{4\lambda}{7} = 0 \Rightarrow -24\lambda - 4\lambda = 0 \Rightarrow \lambda = 0$$

Vektori \vec{AB} , \vec{BC} , \vec{AC} su linearno nezavisni!

\vec{AC} preko \vec{AD} i \vec{BD}

$$\vec{AC} = \vec{AB} + \vec{AD}$$

$$\vec{BD} = -(\vec{AB} - \vec{AD}) \Rightarrow \vec{AB} = -\vec{BD} + \vec{AD}$$

$$\vec{AC} = -\vec{BD} + \vec{AD} + \vec{AD}$$

$$\boxed{\vec{AC} = 2\vec{AD} - \vec{BD}}$$