# Chapter 8
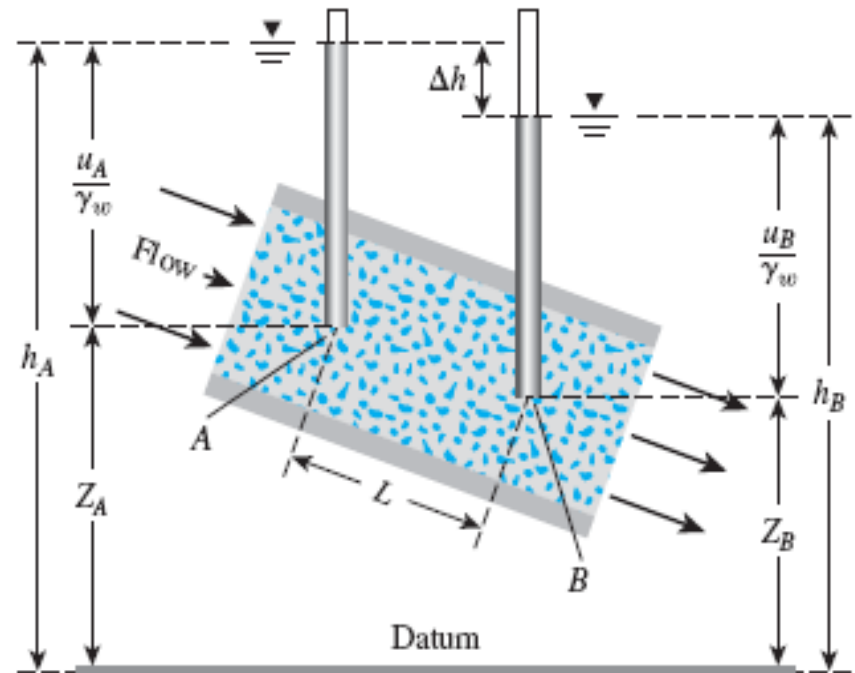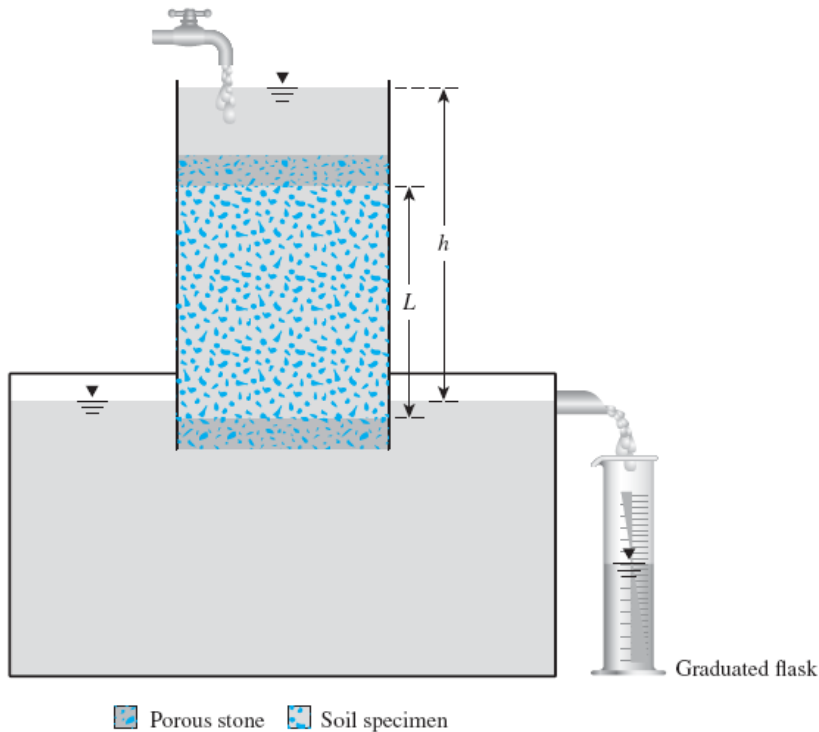# SEEPAGE

**Omitted Section 8.10**

# SEEPAGE

## One-dimensional flow

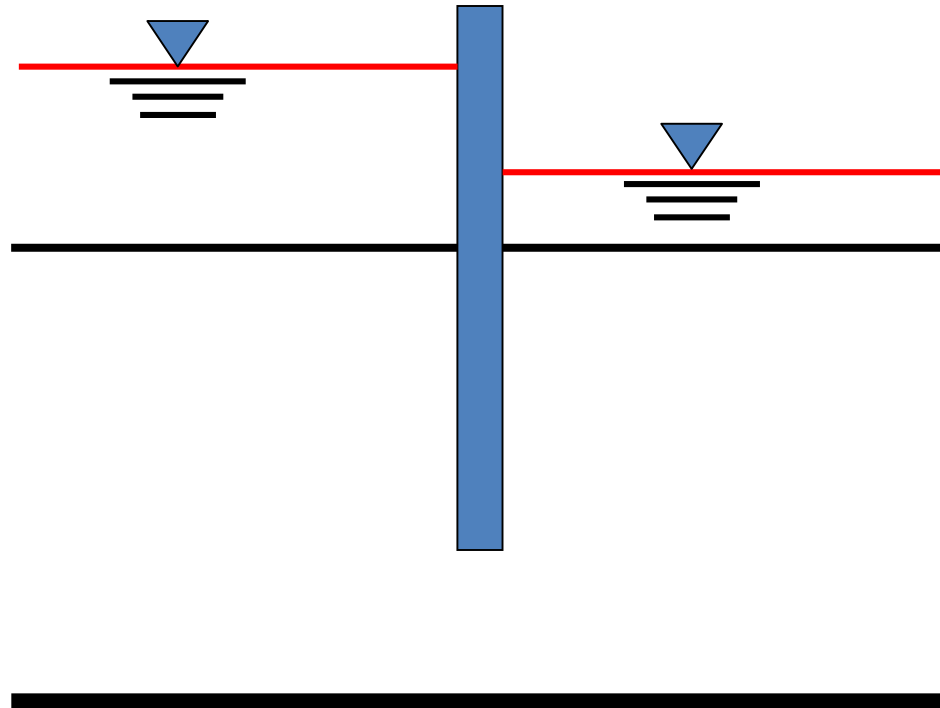**We need only Darcy's law**



$$\text{Flow Rate} = q = v. A = k . i . A = k (h/L). A$$

**Total head at any point can be found through linear interpolation**
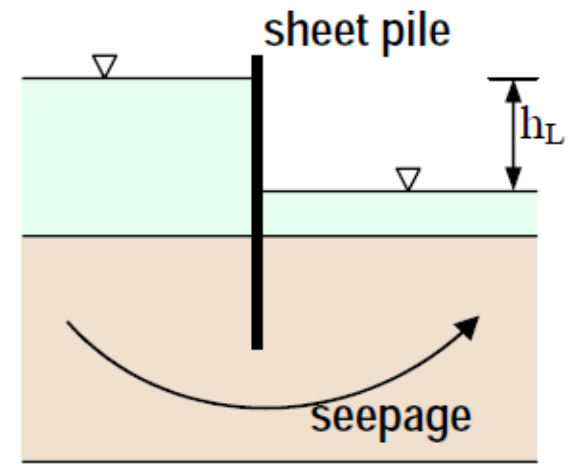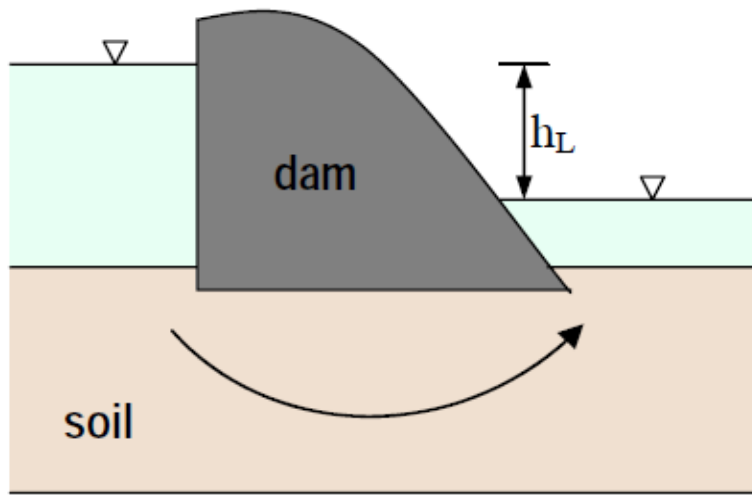
# SEEPAGE

## Two-dimensional flow

In reality, the flow of water through soil is not in **one** direction only, nor is it **uniform** over the entire area perpendicular to the flow.

# SEEPAGE



Seepage beneath (a) a concrete dam (b) a sheet pile

# SEEPAGE

⊙ **In the preceding chapter the flow of water for simple 1-D is calculated through direct application of Darcy's law as already we did in the case of the permeability tests.**

⊙ **As engineers, we will wish to examine and calculate the leakage below the sheet pile or through or beneath the dam and find the distribution of pore water pressure (p.w.p) and effective stress throughout the soil.**

⊙ **If the structure (sheet pile, dam, …etc.) is very long, we may neglect the component of flow in the $3^{rd}$ dimension and consider only the flow through a slice of unit thickness.**

⊙ **This corresponds to plane or two-dimensional flow and we will consider only this case.**

# SEEPAGE

- ⊙ **If p.w.p varies with time the flow will be time-dependent. This is what is so-called non-steady state flow or TRANSIENT SEEPAGE. This may be due to:**

  - ✚ **External cause like change in the head difference between inlet or outlet due to any reason.**

  - ✚ **Internal cause like deformation of soil during the process of seepage.**

- ⊙ **In the second case there is a complex interrelationships between p.w.p, seepage, and deformation. This time-dependent process is known as consolidation and is considered in CE 481 Geotechnical Engineering II.**

# SEEPAGE

- If **p.w.p** does not vary with time, the rate of flow will be constant and the flow is known as **STEADY-STATE-SEEPAGE**.

- During steady state seepage, **p.w.p** remains **constant** and **no** soil **deformation** occur. Hence we will consider <u>**TWO-DIMENSIONAL STEADY-STATE SEEPAGE**</u>.

- The soil may therefore be regarded as <u>**RIGID**</u> and **stationary** with a steady flow of water through the pore spaces.

- Also in this course we will **mostly** consider the simple case of <u>**CONFINED**</u> flow, where the seepage is confined between **two** impervious surfaces, or boundaries.

# Laplace's Equation of Continuity

**The hydrodynamics steady-state fluid flow through porous media follows the same basic laws as the problems of steady-state:**
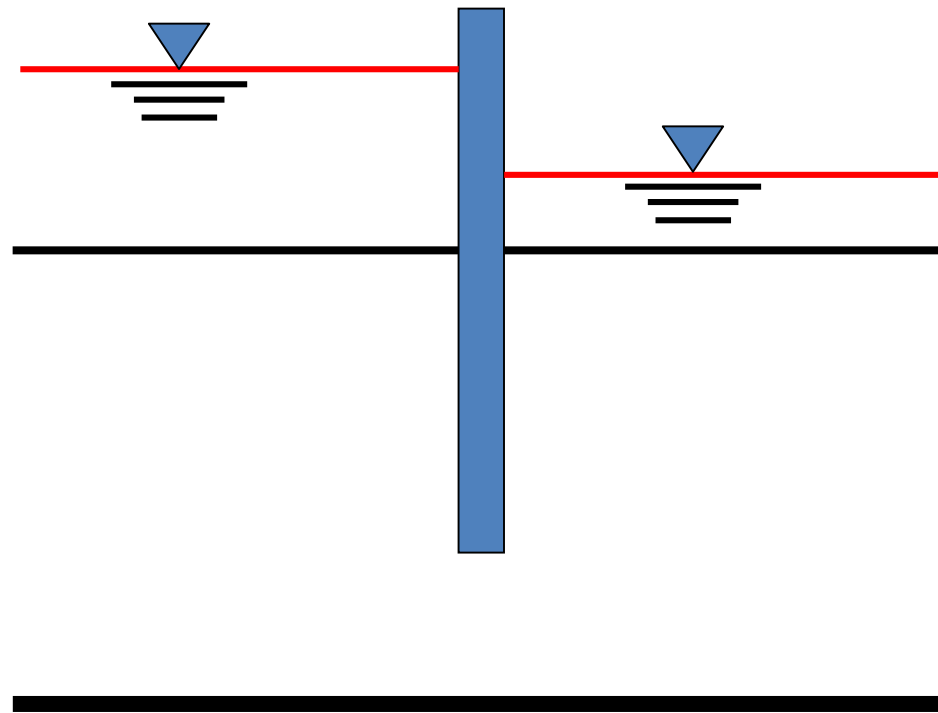
- **Heat flow**

- **Current flow**

**In a continuous contactors**

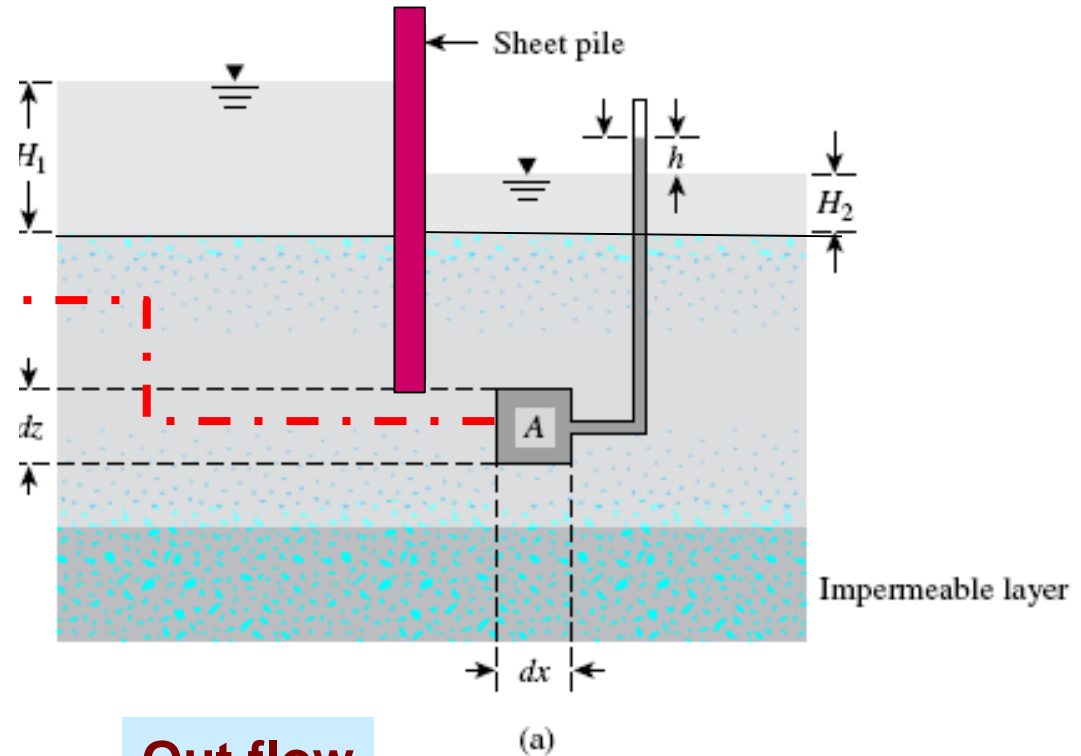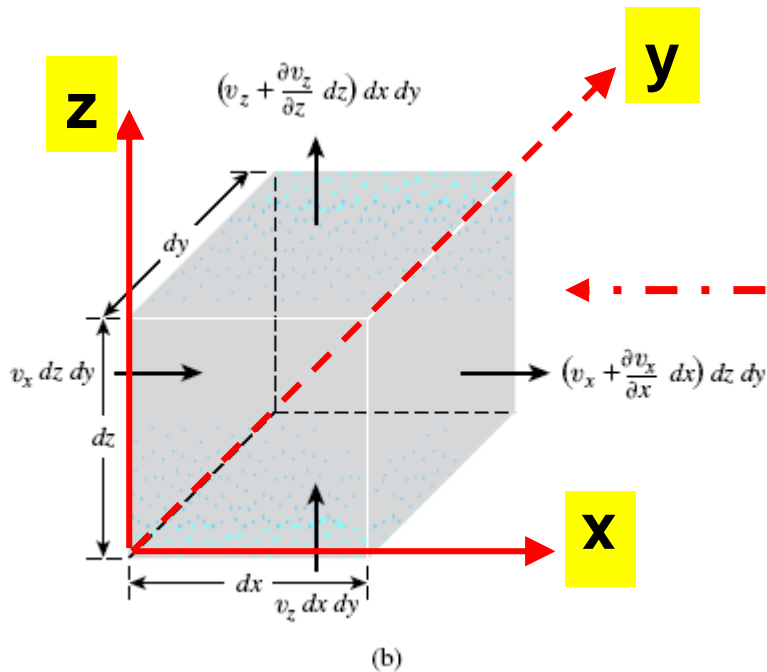**All can be represented by LAPLACE EQUATION.**

**In flow problems Laplace equation is the combination of the equation of continuity and Darcy's law.**

# Laplace's Equation of Continuity



**Steady-State Flow around an impervious Sheet Pile Wall**

# SEEPAGE



(b)

(a)

**In flow**

$q_{in} = v_x . dz dy$    (horizontal direction)
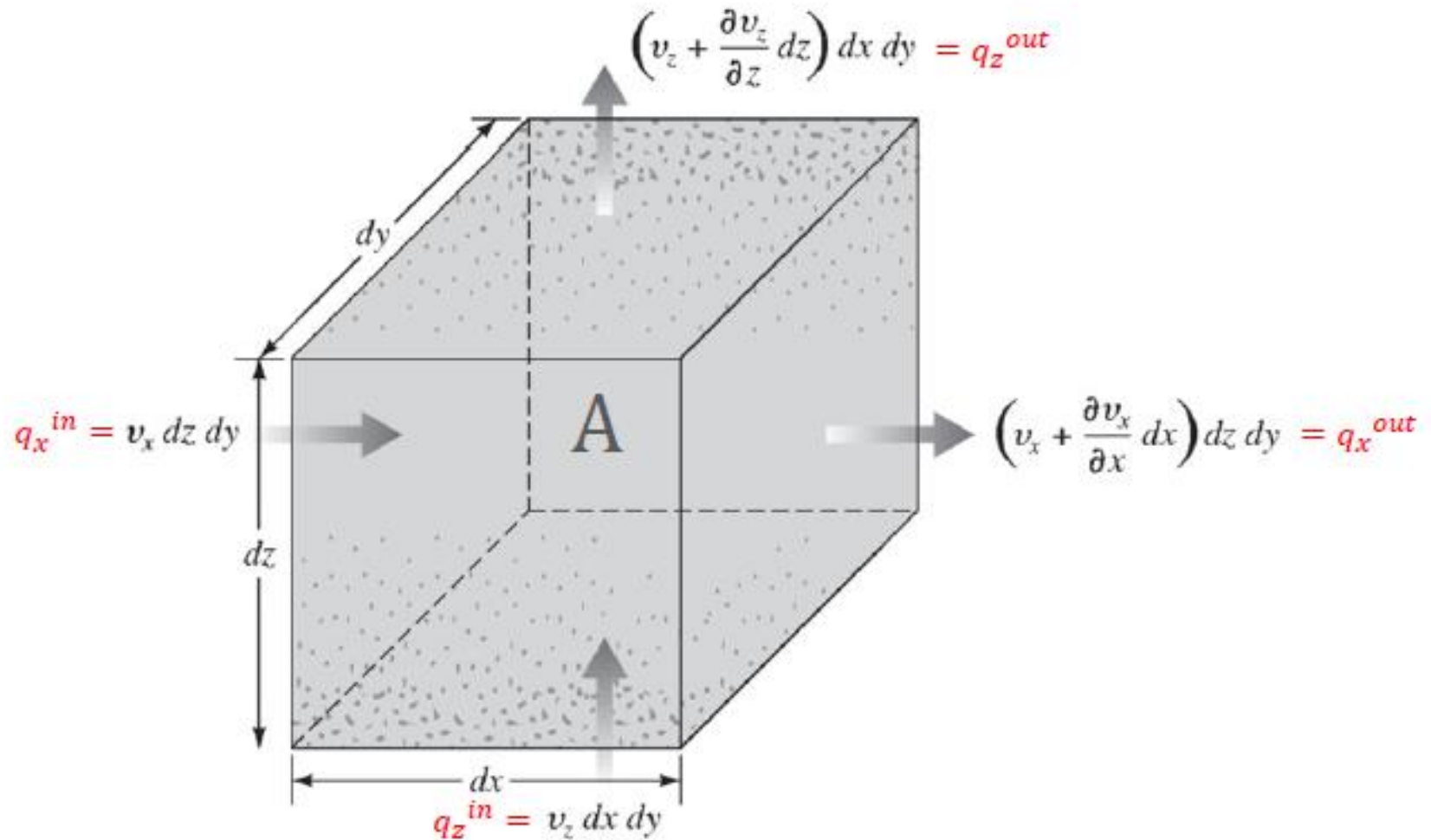
$q_{in} = v_z . dx dy$    (vertical direction)

**Out flow**

$q_{out} = (v_x + \dfrac{\partial v_x}{\partial x} dx) dz dy$ **(horizontal direction)**

$q_{out} = (v_z + \dfrac{\partial v_z}{\partial z} dz) dx dy$    (vertical direction)

# SEEPAGE



$$\left(v_z + \frac{\partial v_z}{\partial z}\, dz\right) dx\, dy = q_z{}^{out}$$

$$q_x{}^{in} = v_x\, dz\, dy$$

$$\left(v_x + \frac{\partial v_x}{\partial x}\, dx\right) dz\, dy = q_x{}^{out}$$

$$q_z{}^{in} = v_z\, dx\, dy$$

$dy$

$dz$

$dx$

A

# Laplace's Equation of Continuity

**Total rate of inflow = Total rate of outflow**

$q_{in} = v_x.dzdy$ (horizontal direction)

$\qquad$ **+** $\qquad q_{in} = v_z.dxdy$ (vertical direction)

$\qquad\qquad\qquad$ **(I)**

$\qquad$ **=** $\qquad q_{out} = (v_x + \dfrac{\partial v_x}{\partial x} dx)dzdy$ **(horizontal direction)**

$\qquad\qquad$ **+** $\quad q_{out} = (v_z + \dfrac{\partial v_z}{\partial z} dz)dxdy$ (vertical direction)

$$\left[ \left( v_x + \frac{\partial v_x}{\partial x} dx \right) dz\, dy + \left( v_z + \frac{\partial v_z}{\partial z} dz \right) dx\, dy \right] - [v_x\, dz\, dy + v_z\, dx\, dy] = 0 \quad \textbf{(II)}$$

# Laplace's Equation of Continuity

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \qquad \text{(III)}$$

$$v_x = k_x i_x = k_x \frac{\partial h}{\partial x} \qquad v_z = k_z i_z = k_z \frac{\partial h}{\partial z} \qquad \text{(IV)}$$

**Substituting Eq. (IV) into Eq. (III) and assuming $k_x = k_z$**

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad \text{(V)}$$

**This equation governs the steady flow condition for a given point in the soil mass**

# Solution of Laplace's Equation

**Laplace's equation can be solved using different methods. The most important are:**

- **Analytical Methods**
  - •Separation of variables
  - •Laplace transform
  - •Fourier transform

- **Numerical Methods**
  - •Finite element
  - •Finite difference

- **Graphical method**    Flow nets    **Trial-and-error sketching**

# Solution of Laplace's Equation

- The **analytical** methods are exact, however they are **complicated** and solutions are available only for simple boundary conditions. Therefore they are less practical.

- Numerical methods vastly developed in the presence of **digital computer** and a number of solutions are now available.

- Graphical method termed **FLOW NET** is preferred by many because it is very versatile and **simple**.

- In this course we will limit discussion to **flow net** through **trial-and-error** sketching.

- By constructing the **flow net** we can know the values of **head** at any point in the soil and also find the **flow rate**.
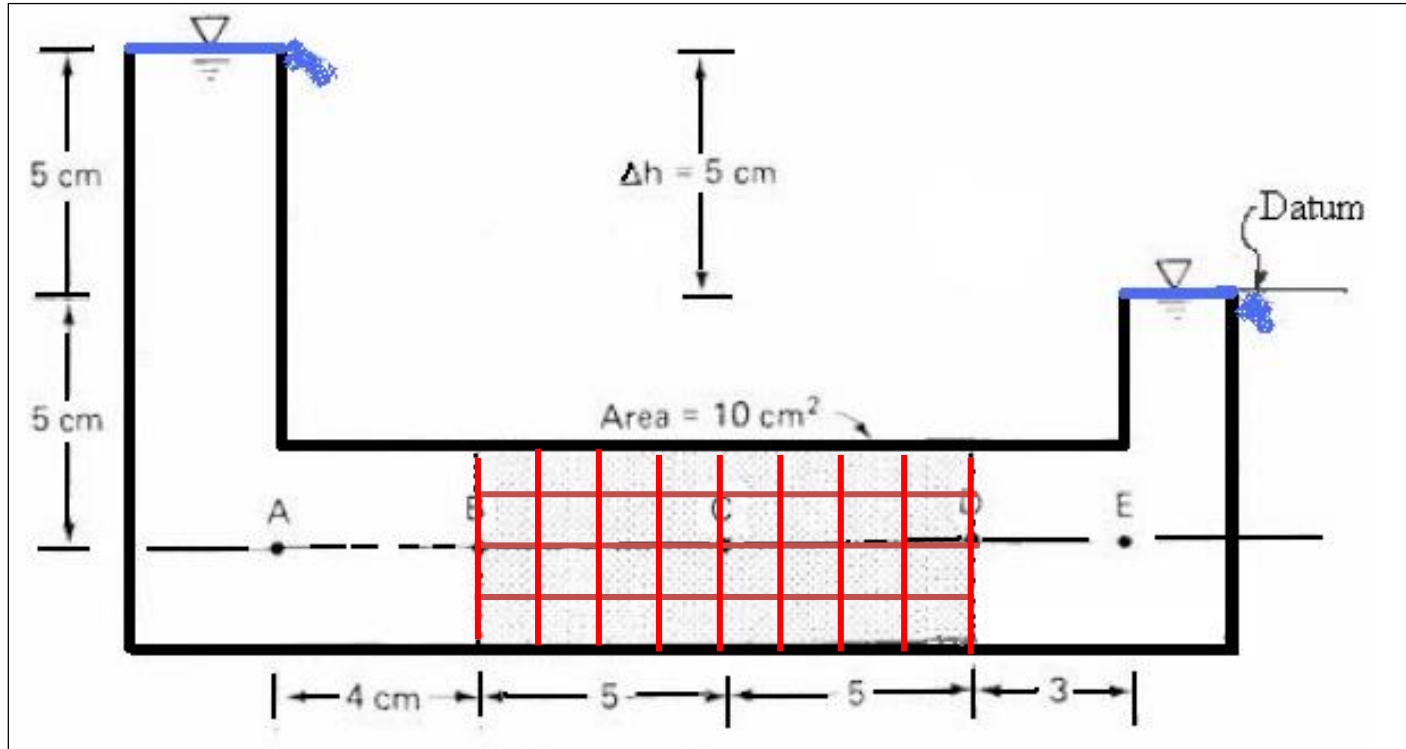
# FLOW NETS

# Basis of Flow Nets

⊙ **Consider flow through the constant head permeameter shown below:**
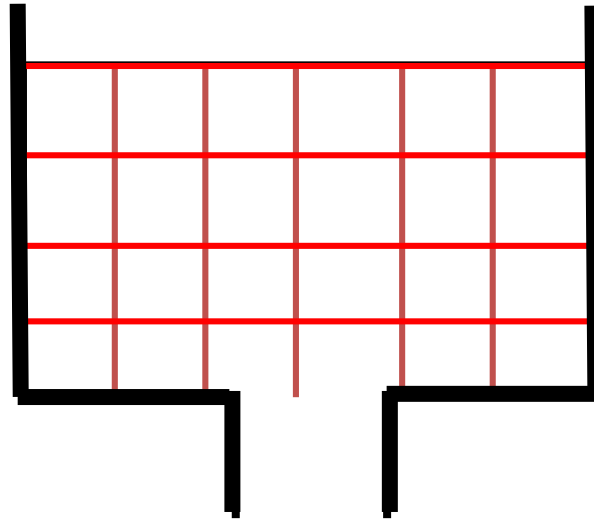


⊙ **We can see that there are two families of lines:**

- **Vertical lines which represent the direction of flow of water particles.**

- **Horizontal lines which represent lines of constant head.**

⊙ **Note: As the water progress through the sample, head is lost at constant rate (but we deal only with steady state, i.e. h is constant)**

# Basis of Flow Nets

⊙ **Consider the other way around (i.e. flow is horizontal)**



**We can see also that there are TWO families of lines:**

- **Horizontal lines which represent the direction of flow of water particles.**
- **Vertical lines which represent lines of constant total head.**

# Theoretical Basis of Flow Nets

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

- ⊙  The governing equation is a **2nd** order **homogeneous**, **partial** differential equation with **constant** coefficients.

- ⊙  The solution of this equation is represented by **two functions** which both satisfies the equation and any relevant boundary conditions.

- ⊙  These functions are the **potential function $\Phi$ (x,z)** and the **flow function $\psi$(x,z).** These functions represent a **family** of equipotential lines and a **family** of flow lines constituting what is referred to as a **FLOW NET**.

- ⊙  These lines are proved to be **orthogonal**.

- ⊙  What we actually do, we by following specific rules find the **FLOW Net** by which we reach the solution of Laplace's equation.

- ⊙  In other word **flow net** is actually a graphical solution of Laplace's Equation in **2-D**.

# FUNDAMENTAL DEFINITIONS

- A line along which a water particle will travel from upstream to the down stream side in the permeable soil medium is called a **FLOW LINE (OR STREAM LINE).**

- A line along which the **TOTAL HEAD** at all points is the same is called **EQUIPOTENTIAL** line.

- If piezometers are placed at different points along an equipotential line, the water level will rise to the same elevation in all of them.



*Different pressure heads but equal total heads*

Sheet pile

$H_1$

Flow line
$k_x = k_z = k$

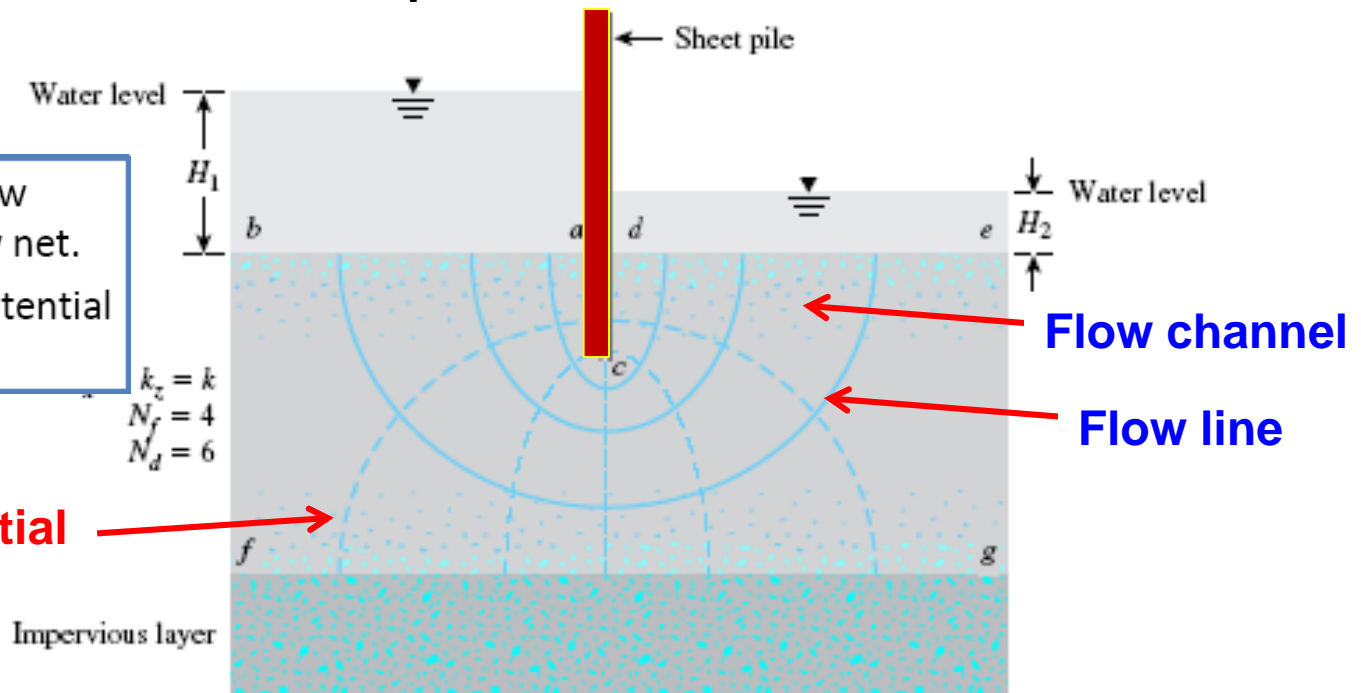Equipotential line

$H_2$

Datum

Impervious layer
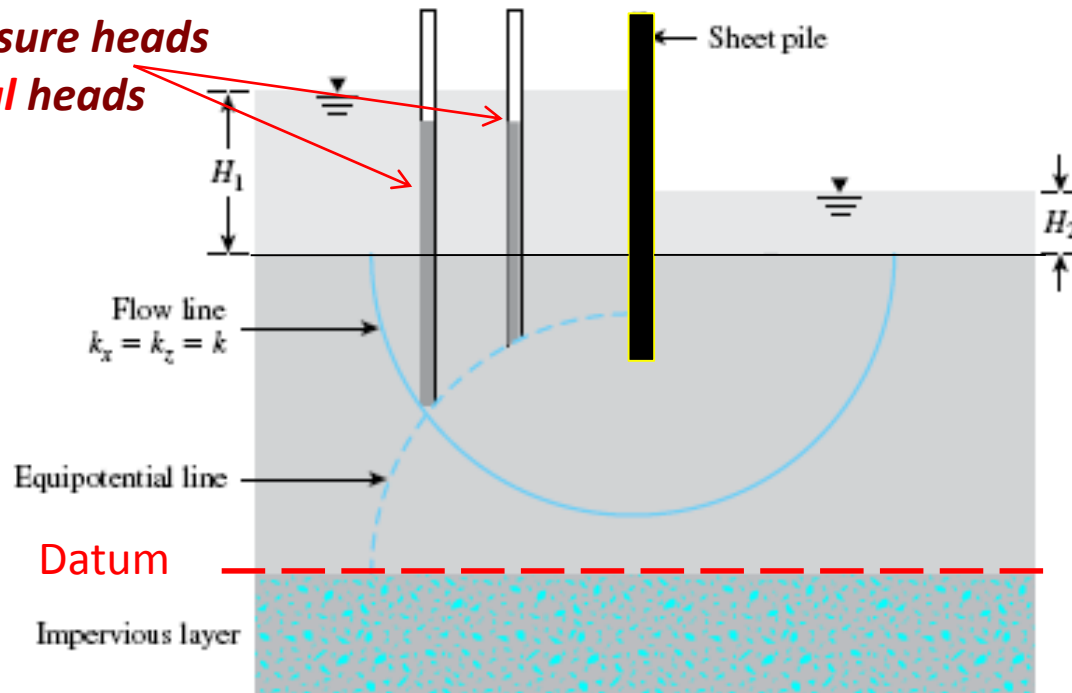
# FUNDAMENTAL DEFINITIONS

⊙ The space between any two adjacent flow lines is called **FLOW PATH, FLOW TUBE, or FLOW CHANNEL**.

⊙ The space between any two equipotential lines is called **EQUIPOTENTIAL SPACE** .

⊙ The mesh made by a number of flow lines and equipotential lines is called a <u>**FLOW NET**</u>.

⊙ The "Phreatic surface" is the top flow line

$N_f$: is the number of flow channels in the flow net.

$N_d$: is the number of potential drops.

$k_z = k$
$N_f = 4$
$N_d = 6$

Water level

$H_1$

Sheet pile

Water level

$H_2$

Flow channel

Flow line

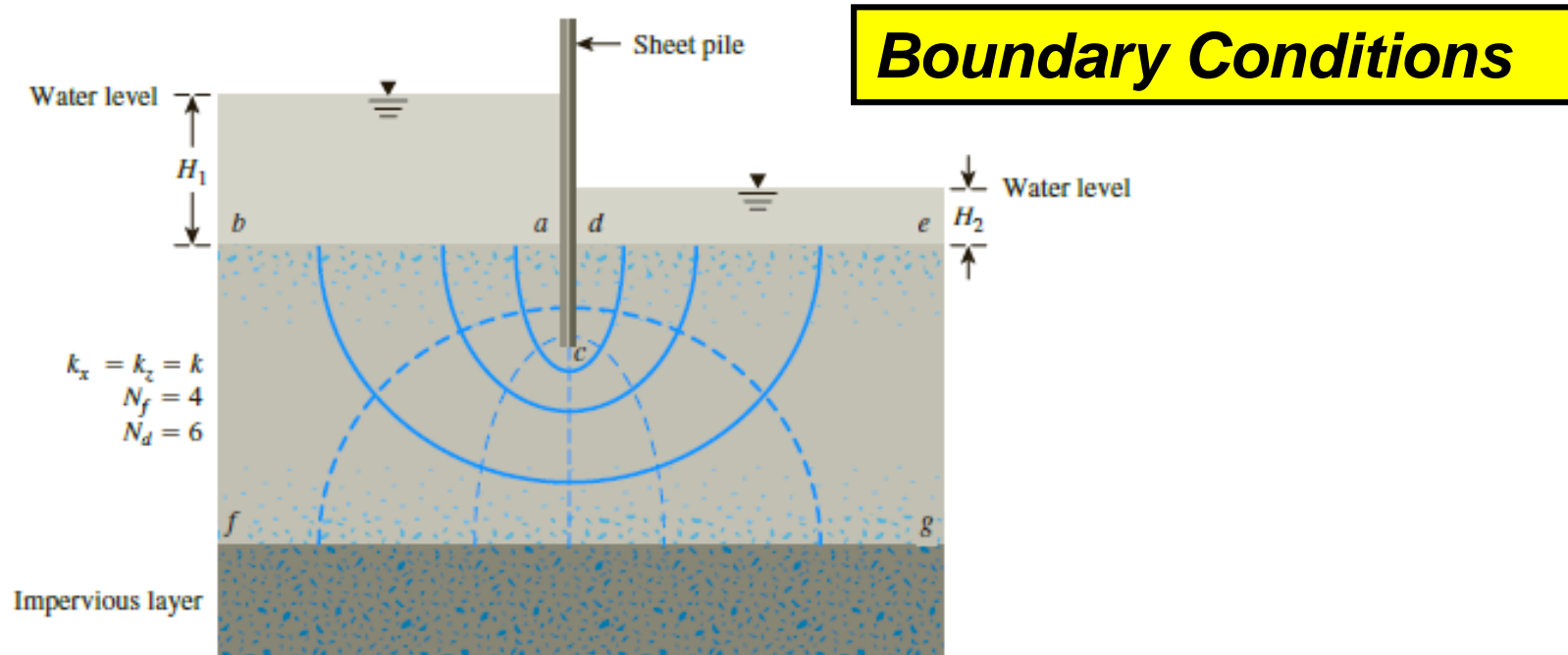Equipotential line

Impervious layer

(b)

# FLOW NETS

**To complete the graphic construction of a flow net, <u>one must draw the flow and equipotential lines in such a way that:</u>**

**1.** **The equipotential lines intersect the flow lines at right angles.**

**2.** **The flow elements formed are approximate squares.**

# FLOW NETS
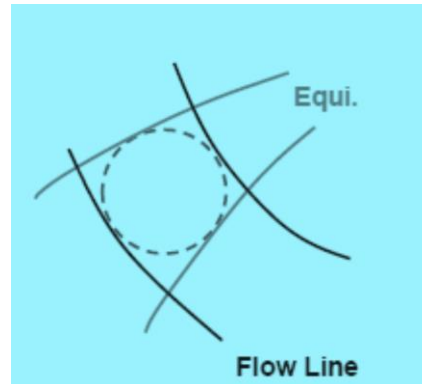


Boundary Conditions

**Condition 1.** The upstream and downstream surfaces of the permeable layer (lines *ab* and *de*) are equipotential lines.

**Condition 2.** Because *ab* and *de* are equipotential lines, all the flow lines intersect them at right angles.

**Condition 3.** The boundary of the impervious layer—that is, line *fg*—is a flow line, and so is the surface of the impervious sheet pile, line *acd*.

**Condition 4.** The equipotential lines intersect *acd* and *fg* at right angles.
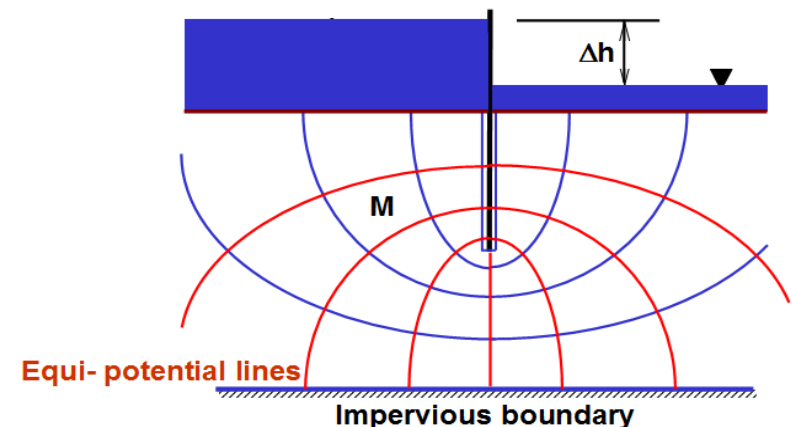
# FLOW NETS



Boundary Conditions
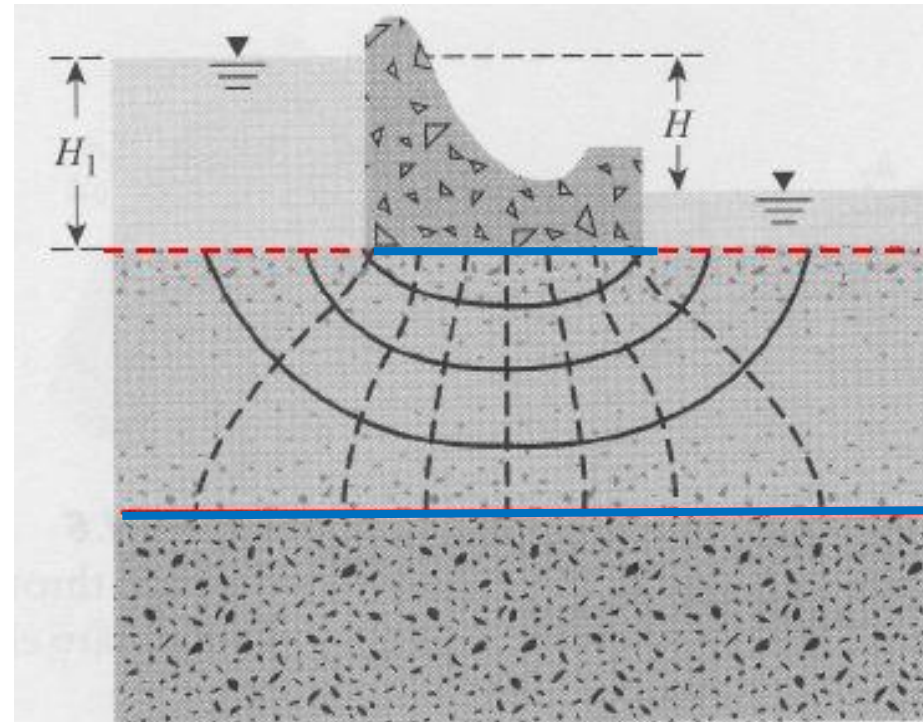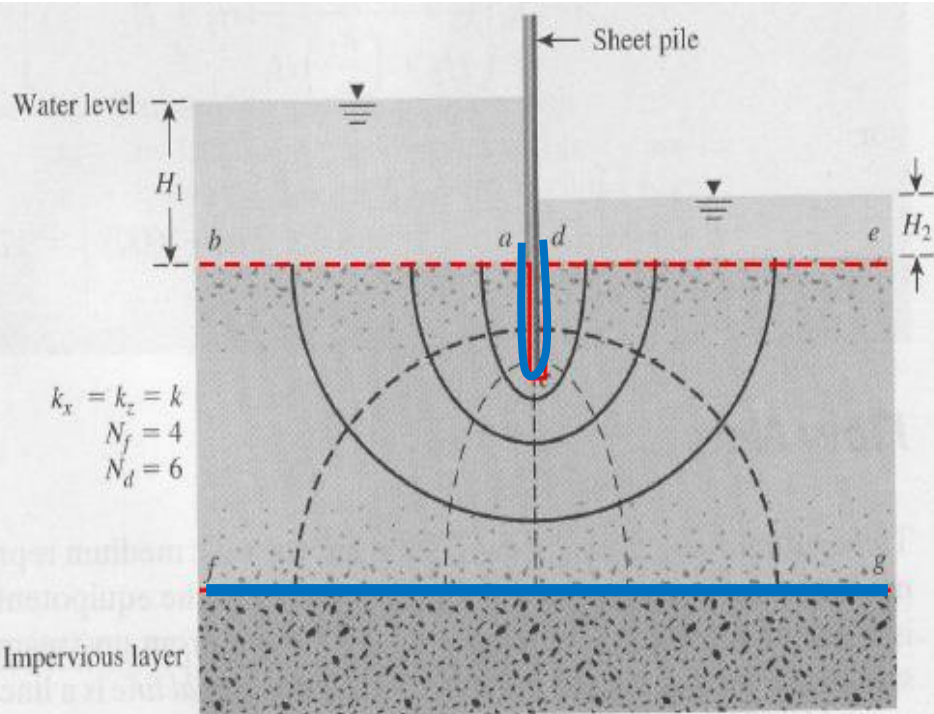
Blue : Flow line
Red :Equipotential line

# Rules Followed in Sketching Flow Nets

1. Flow lines and equipotential lines must intersect at right angles.

2. Constant head boundaries represent initial or final equipotential.

3. Impermeable (no-flow) boundaries are flow lines.

4. Adjacent equipotentials have equal head loss (= $h/N_d$)

5. The same quantity of seepage flows between pairs of flow lines (i.e. equal flow channels).



$\Delta h$

M

Equi- potential lines

Impervious boundary

**6.** **Geometric figures formed by pairs of flow lines and equipotential lines must be essentially <span style="color:red">squares</span>. This is the case when we have a <span style="color:red">true flow net</span>. In more practical case involving <span style="color:red">curvilinear</span> flow, the figures cannot be true squares. However they must have <u>right angles at the corners and the two median dimensions of each figure must be equal</u>.**

**Note:**
**Drawing of square elements is convenient but not always necessary**
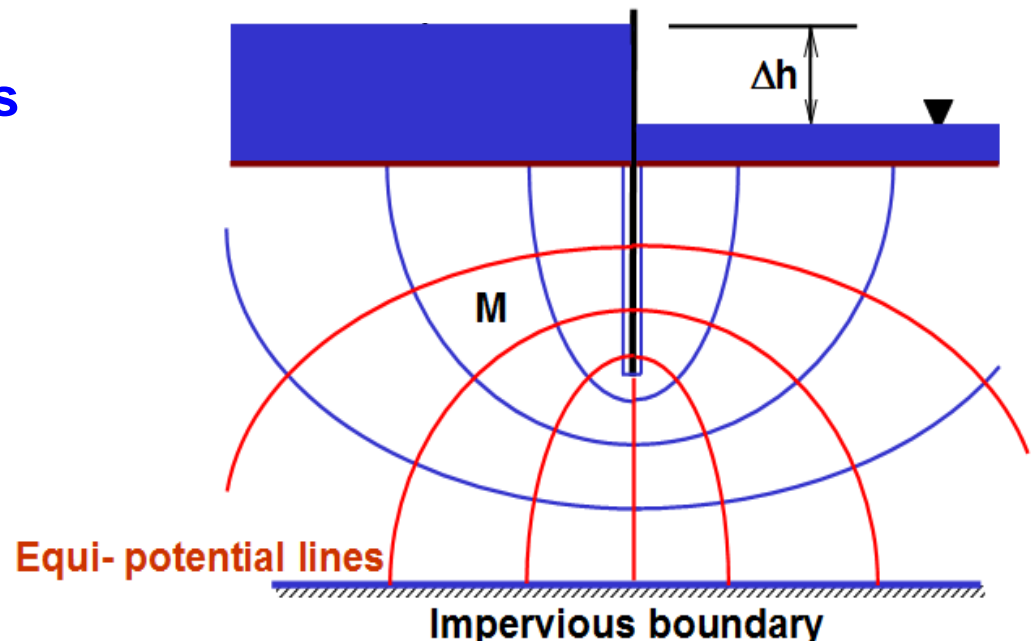
# STEPS IN CONSTRUCTING FLOW NETS

- **Step 1**: Draw to a convenient scale a cross-section of the **medium** and its **boundaries**.

- **Step 2**: Establish the **two** boundary **flow lines** and the **two** boundary **equipotential lines**.

- **Step 3**: By trial and error sketch a network of flow lines and equipotential lines, observing the right-angle intersection and the space figures rules.

  **Where flow direction is a straight line, flow lines are an equal distance apart and parallel.**

- **Step 4**: At first attempt certainly there will be some inconsistencies. Successive trials are made until the net is reasonably consistent throughout.

# GENERAL SUGGESTIONS AND TIPS

- **Be practical in selecting a scale for the drawing. A scale that is too large waste time and eraser.**

- **Before starting the sketch a flow net look for important boundary conditions.**

- **Use as few flow lines (and resulting equipotential lines) as possible. Generally THREE to FIVE lines will be sufficient.**

- **Always watch the appearance of the entire flow net. Do not make fine detail adjustments until the entire flow net is approximately correct.**

- **Try to keep the number of flow channels to a whole number.**

- **Remember, flow lines do not intersect the lower boundary since itself is a flow line.**

- **Obtaining results from a rough flow net is considered adequate. The error committed is relatively small in comparison to the accuracy we obtain for the coefficient of permeability.**
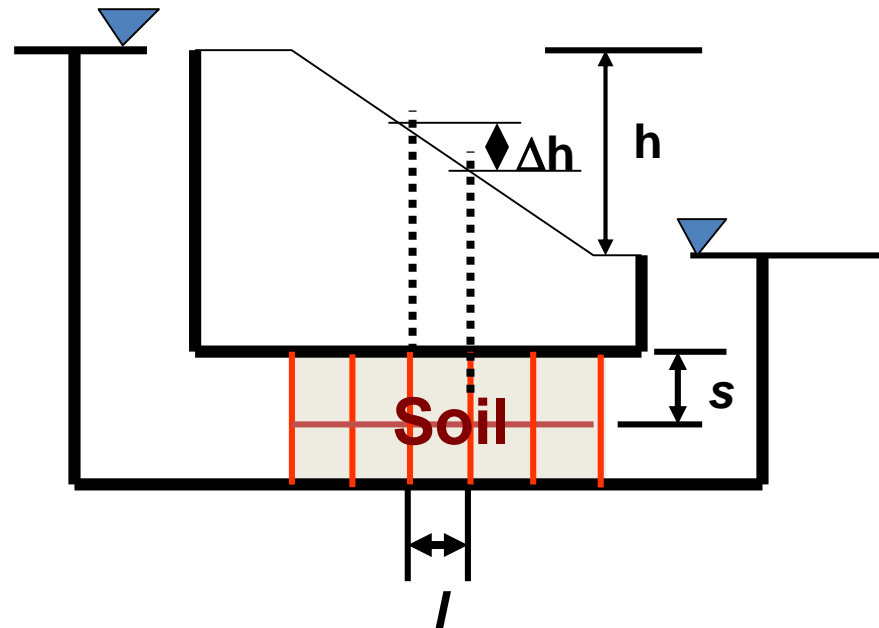
# SEEPAGE CALCULATION FROM FLOW NETS

# SEEPAGE CALCULATION FROM FLOW NETS

Let us first consider the case of **straight** flow and equipotential lines **(i.e. perfect squares)** as shown in the figure below.

- In a flow net, the strip between any two adjacent flow lines is called a <u>flow channel</u>.

- The drop in the total head between any two adjacent equipotential lines is called the **potential drop**.

➡ **Applying Darcy's law, the flow in each flow channel is**

$$\Delta q = kiA \qquad (But \ \ i = \frac{\Delta h}{l}, \quad A = s.1 = s) \longrightarrow \boxed{\Delta q = k\frac{\Delta h}{l}s}$$

➡ **Since the figures are squares (as must be the case in general) _s/l_ =1, and hence** $\Delta q = k\Delta h$

➡ **Since the potential drop between any two adjacent equipotential lines is the same, then** $\Delta h = \dfrac{h}{N_d}$

 **Where N$_d$ is the number of potential drops.**

$$\Delta q = k\frac{h}{N_d}$$

➡ **If the number of flow channels is N$_f$, then the total discharge q per unit depth (perpendicular to the paper) is**

$$q = N_f \Delta q \longrightarrow \boxed{q = kh\frac{N_f}{N_d} \ .......(*)}$$
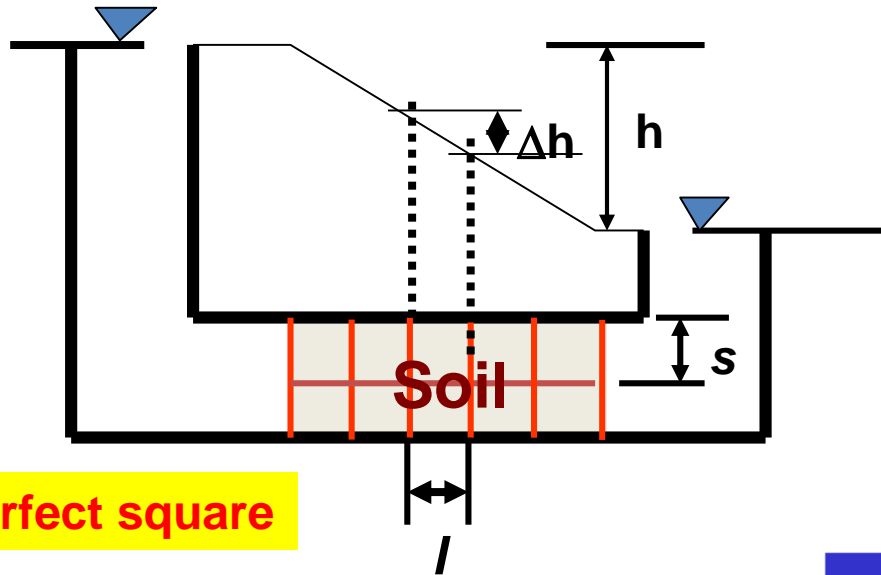
# SEEPAGE CALCULATION FROM FLOW NETS

- **Equation (\*) is the basic equation for computation of seepage quantities from flow net.**

- **The ratio $N_f/N_d$ is called the shape factor.**

- **Equation (\*) for the case when the width of the cross-section of the channel normal to the page is UNITY.**

- **If it is not we have to multiply it by the given value of the width, or**
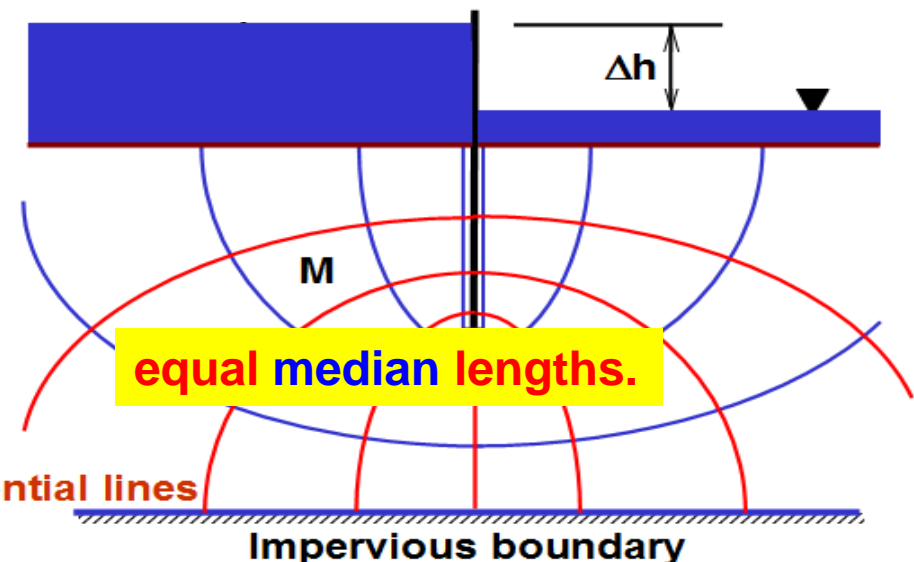
$$q = kh\frac{N_f}{N_d}b$$

**Eq. (\*\*)**

# SEEPAGE CALCULATION FROM FLOW NETS

## CURVILINEAR FLOW AND EQUIPOTENTIAL LINES



$$q = kh\frac{N_f}{N_d} \dots\dots(*)$$

**Perfect square**

**l**

**Soil**

$$q = k\, h\left(\frac{N_f}{N_d}\right)$$

equal median lengths.

Equi- potential lines

Impervious boundary

M

Δh

# SEEPAGE CALCULATION FROM FLOW NETS

## CURVILINEAR FLOW AND EQUIPOTENTIAL LINES

**Case I: Square Elements (medians are equal)**

**Flow channel**

1. **Because there is no flow across the flow line, rate of flow through the flow channel per unit width perpendicular to the flow direction is the same, or**

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \Delta q$$

2. **The potential drop is the same and equal to:**

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \frac{h}{N_d}$$

**Where** $h$: **head difference between the upstream and downstream sides.**
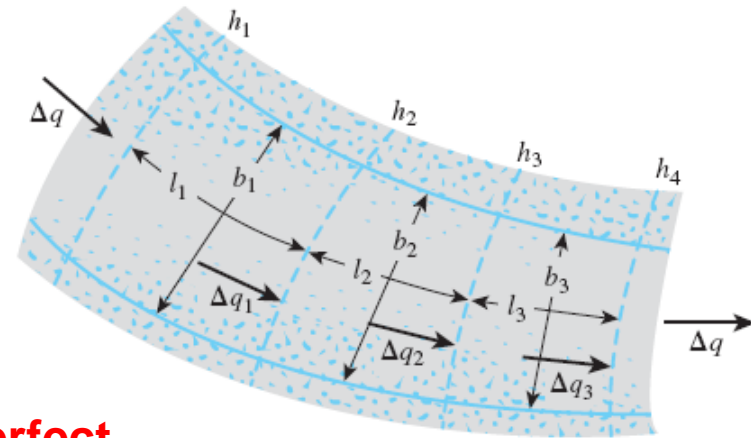$N_d$: **number of potential drops.**
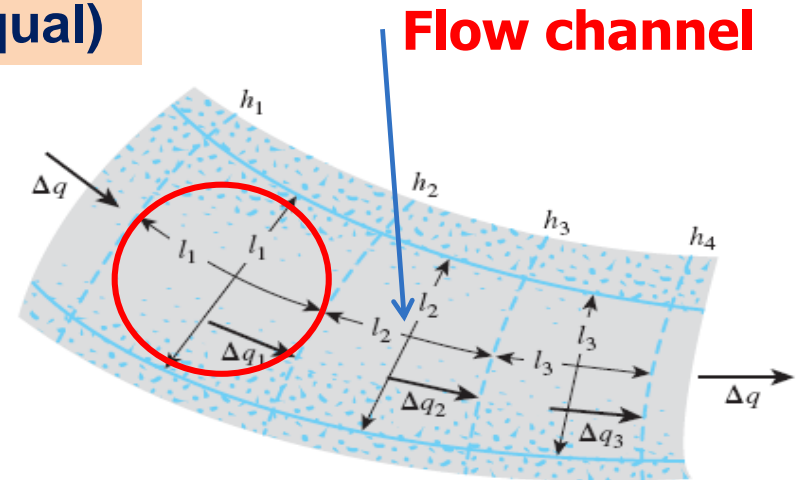
# SEEPAGE CALCULATION FROM FLOW NETS

**From Darcy's Law, the rate of flow is equal to:**

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right)l_1 = k\left(\frac{h_2 - h_3}{l_2}\right)l_2 = k\left(\frac{h_3 - h_4}{l_3}\right)l_3 = \cdots$$

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \cdots = \boxed{\frac{H}{N_d}}$$

$$\Delta q = k\,\frac{H}{N_d}$$

**Note: Here we took the median lengths, for perfect square we have side lengths**

**If the number of flow channels in a flow net is equal to $N_f$, the total rate of flow through all the channels per unit length can be given by:**

$$\boxed{q = k\,h\left(\frac{N_f}{N_d}\right)}$$   **Eq. (\*\*\*) ….same as Eq. (\*)**

**Instead of thinking of perfect square, we always consider the most general curvilinear case and assuming equal median lengths.**

**Case II: Rectangular Elements**

• Drawing of square elements   is convenient but not **always necessary**

• Alternatively, one can draw a rectangular mesh for a flow channel provided that the **width-to-length** ratios for **all** the rectangular elements in the flow net are the same
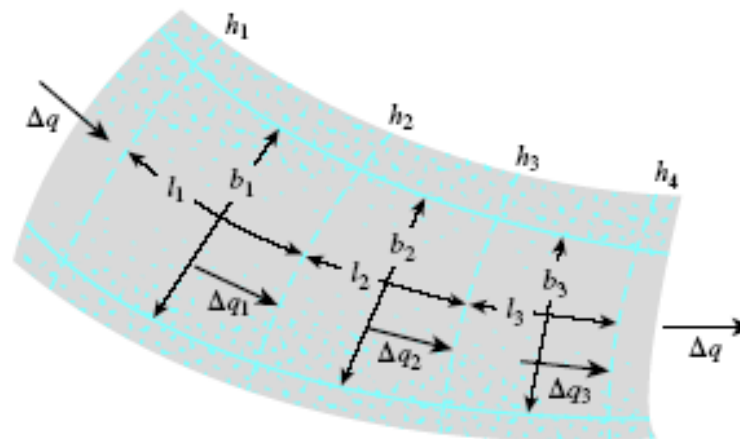
**Case II: Rectangular Elements**

- **In this case the rate of flow through the channel is expressed as**

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right)b_1 = k\left(\frac{h_2 - h_3}{l_2}\right)b_2 = k\left(\frac{h_3 - h_4}{l_3}\right)b_3 = \cdots$$

If $b_1/l_1 = b_2/l_2 = b_3/l_3 = \cdots = n$ (i.e., the elements are not square),

$$\Delta q = kH\left(\frac{n}{N_d}\right)$$

If the number of flow channels in a flow net is equal to $N_f$, the total rate of flow through all the channels per unit length can be given by

$$q = k\,h\left(\frac{N_f}{N_d}\right)n$$

## Three Cases:

1. **Square**

**If not**

2. **Curvilinear square where Medians are equal**

**If not**

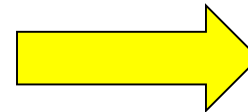3. **Curvilinear rectangular with width-to-length ratio being the same**

# SUMMARY

- **Case of unit depth (perpendicular to the paper)**

$$q = k\,h\left(\frac{N_f}{N_d}\right)$$

- **Case when the width of the cross-section of the channel normal to the page is not UNITY.**
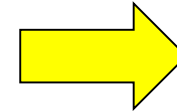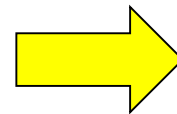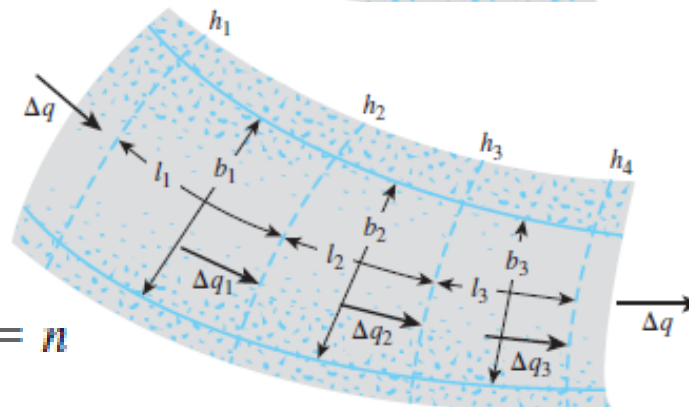
$$q = kh\frac{N_f}{N_d}b$$

- **Case of Square Elements (medians are equal)**
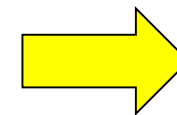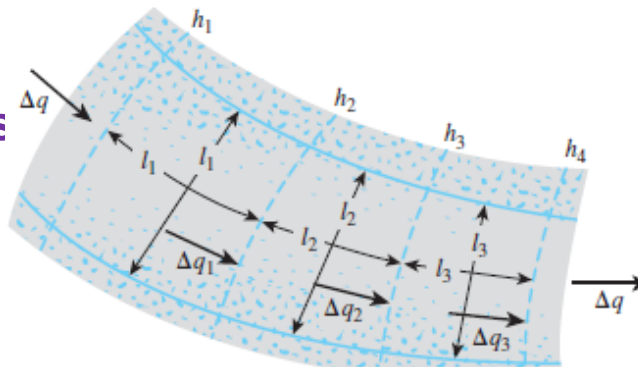
$$q = k\,h\left(\frac{N_f}{N_d}\right)$$

- **Case of Rectangular Elements (medians are not equal)**

$$q = k\,h\left(\frac{N_f}{N_d}\right)n$$
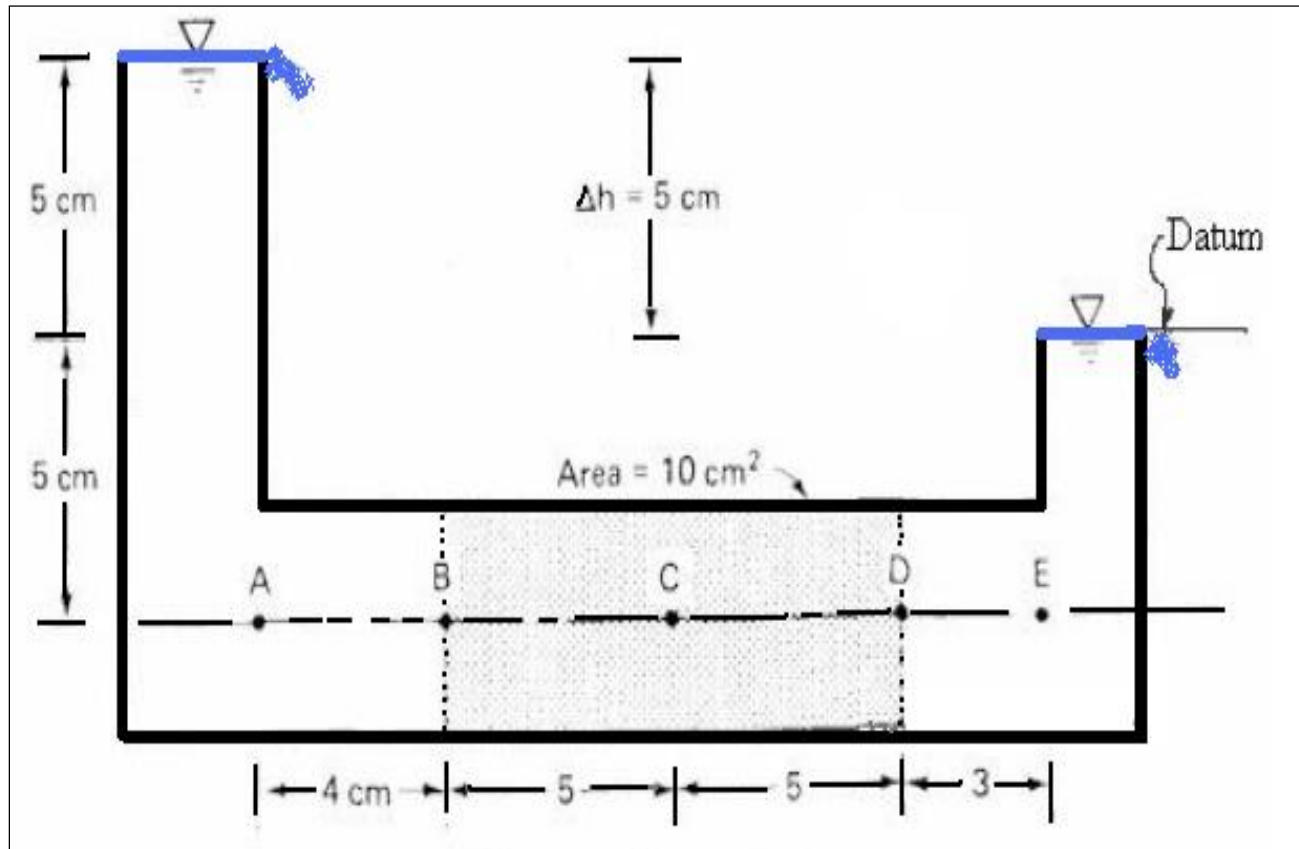
$$b_1/l_1 = b_2/l_2 = b_3/l_3 = \cdots = n$$

- **<u>The general case:</u>**

    o **The cross-section of the channel normal to the page is not UNITY**
    o **Medians are not equal**

$$q = k\, h \left( \frac{N_f}{N_d} \right) \mathrm{n} \; . \; \mathbf{b}$$

# EXAMPLE

If the horizontal cylinder of soil shown below has a coefficient of permeability of 0.01 cm/sec. Calculate the amount of flow through the soil.
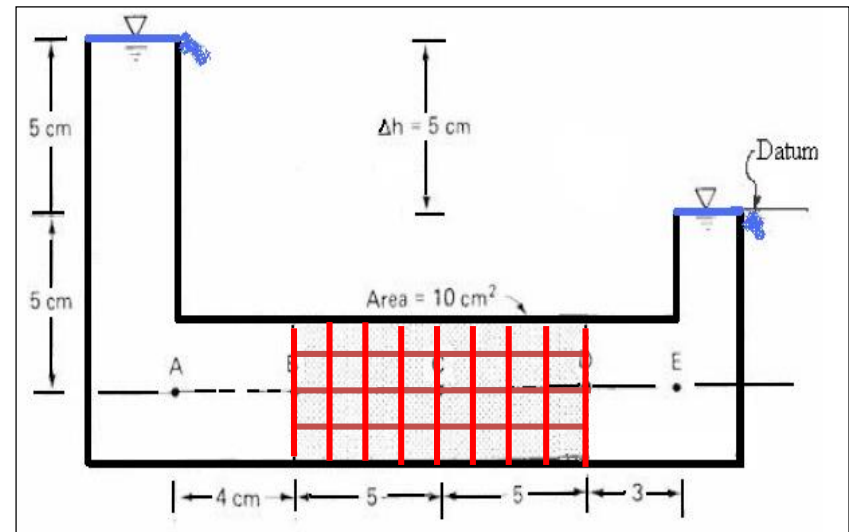
# EXAMPLE

## Considering it as a 1-D problem

**we can apply Darcy's law directly**

$$q = vA = kiA = 0.01 \times (5/10) \times 10 = 0.05 \ cm^3/s$$

## Considering it as a 2-D problem

$$q = kh\frac{N_f}{N_d}b$$

**q = 0.01 X 5 X 4/8 X 2 = 0.05 cm³/s**



According to rule of square mesh the height of the soil must equal 5 cm and hence in the 3rd direction **2 cm.**

# EXAMPLE 8.1

## Example 8.1

A flow net for flow around a single row of sheet piles in a permeable soil layer is shown in Figure 8.6. Given that $k_x = k_z = k = 5 \times 10^{-3}$ cm/sec, determine

a. How high (above the ground surface) the water will rise if piezometers are placed at points $a$ and $b$
b. The total rate of seepage through the permeable layer per unit length
c. The approximate average hydraulic gradient at $c$
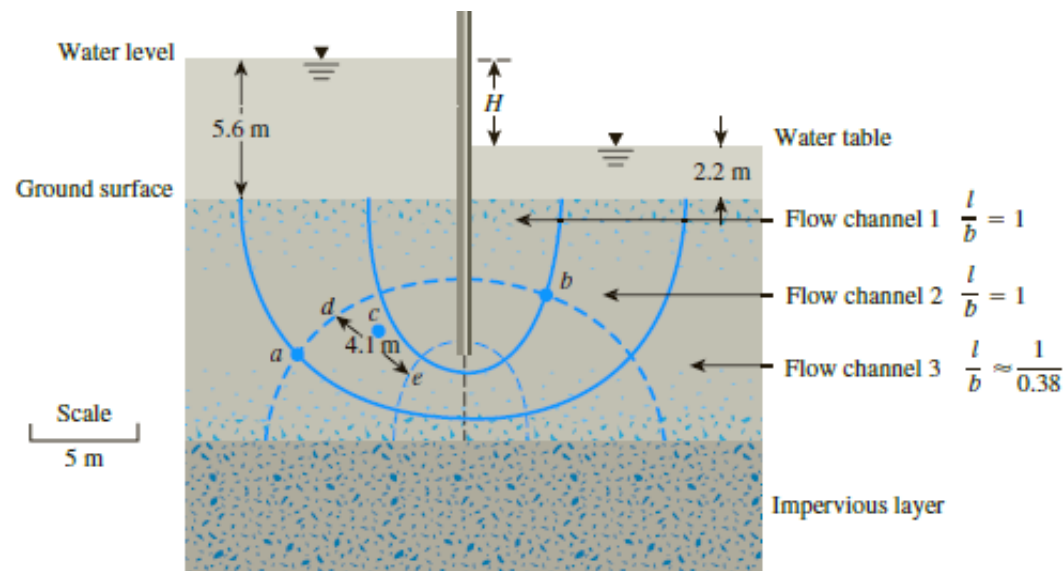
# EXAMPLE 8.1

## Solution

### Part a

From Figure 8.6, we have $N_d = 6$, $H_1 = 5.6$ m, and $H_2 = 2.2$ m. So the head loss of each potential drop is

$$\Delta H = \frac{H_1 - H_2}{N_d} = \frac{5.6 - 2.2}{6} = 0.567 \text{ m}$$

At point $a$, we have gone through one potential drop. So the water in the piezometer will rise to an elevation of

$$(5.6 - 0.567) = \textbf{5.033 m above the ground surface}$$

At point $b$, we have five potential drops. So the water in the piezometer will rise to an elevation of

$$[5.6 - (5)(0.567)] = \textbf{2.765 m above the ground surface}$$

### Part b

From Eq. (8.14),

$$q = 2.38\frac{k(H_1 - H_2)}{N_d} = \frac{(2.38)(5 \times 10^{-5} \text{ m/sec})(5.6 - 2.2)}{6}$$

$$= \textbf{6.74} \times \textbf{10}^{-5}\textbf{m}^3\textbf{/sec/m}$$

### Part c

The average hydraulic gradient at $c$ can be given as

$$i = \frac{\text{head loss}}{\text{average length of flow between } d \text{ and } e} = \frac{\Delta H}{\Delta L} = \frac{0.567 \text{ m}}{4.1 \text{ m}} = \textbf{0.138}$$

(*Note:* The average length of flow has been scaled.)

# EXAMPLE 8.2

## Example 8.2

Seepage takes place around a retaining wall shown in Figure 8.7. The hydraulic conductivity of the sand is $1.5 \times 10^{-3}$ cm/s. The retaining wall is 50 m long. Determine the quantity of seepage across the entire wall per day.
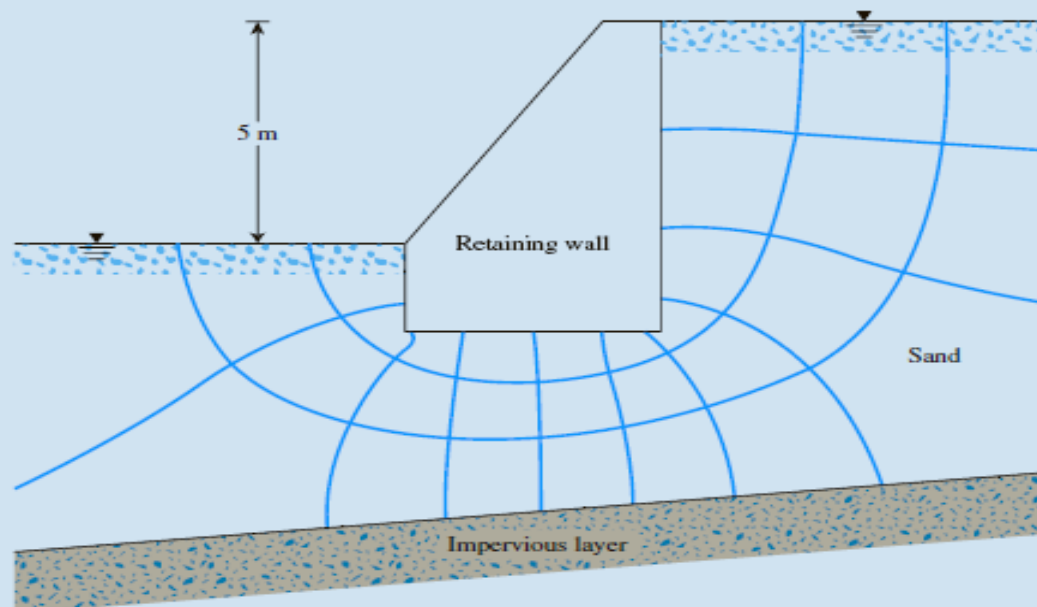
## Solution

For the flow net shown in Figure 8.7, $N_f = 3$ and $N_d = 10$. The total head loss from right to left, $H = 5.0$ m. The flow rate is given by [Eq. (8.10)],

$$q = kH \frac{N_f}{N_d} = (1.5 \times 10^{-5} \text{ m/s})(5.0)\left(\frac{3}{10}\right) = 2.25 \times 10^{-5} \text{ m}^3/\text{s/m}$$

Seepage across the entire wall,

$$Q = 2.25 \times 10^{-5} \times 50.0 \times 24 \times 3600 \text{ m}^3/\text{day} = 97.2 \text{ m}^3/\text{day}$$

# EXAMPLE

If k = 10⁻⁷ m/sec, what would be the flow per day over a 100 m length of wall?
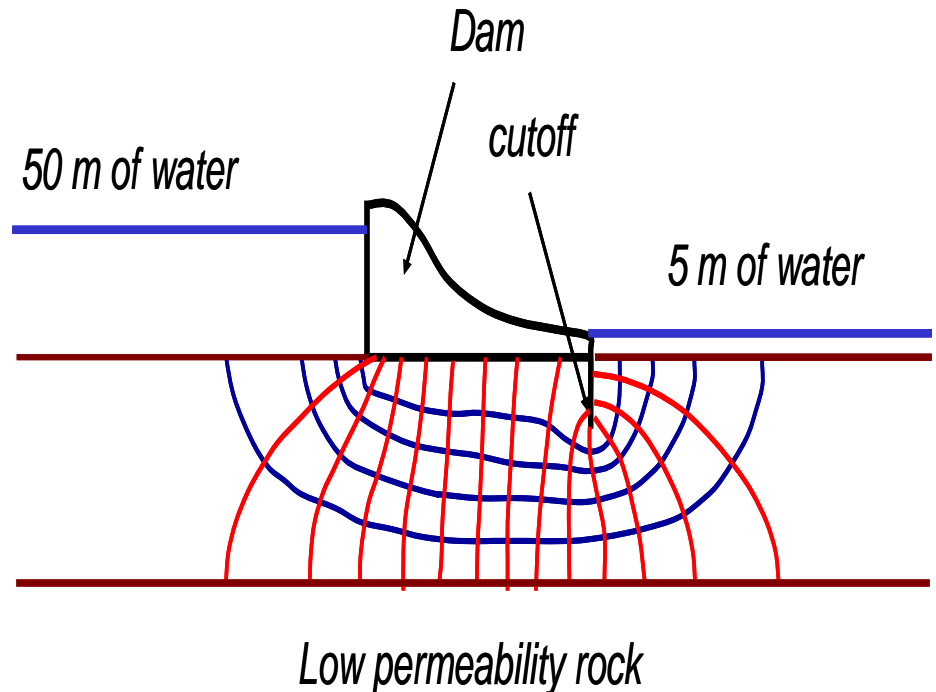
## Solution
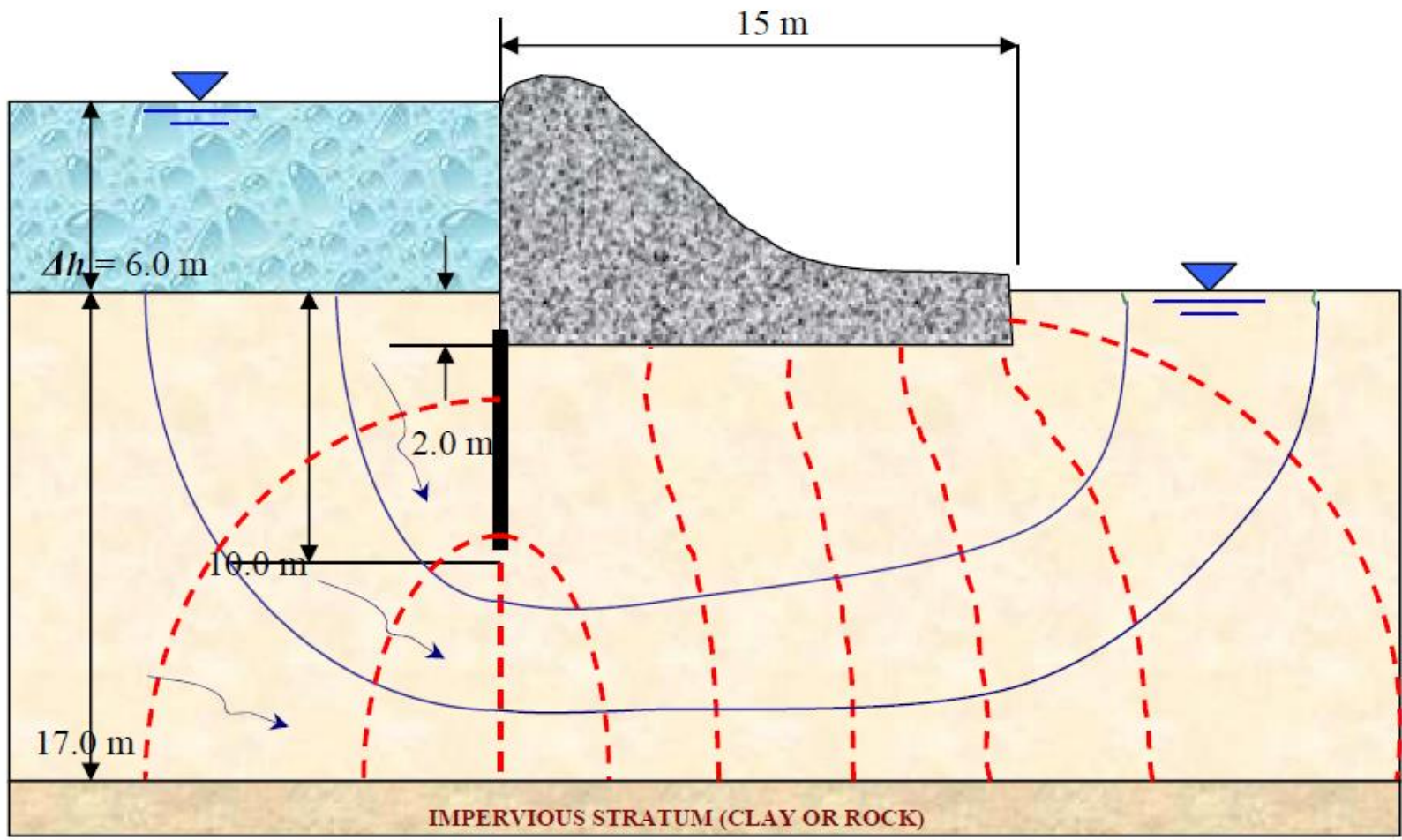
$$q = kh\frac{N_f}{N_d}b$$

$N_f = 5$

$N_d = 14$

$h = 45$ m

$k = 10^{-7}$ m/sec

50 m of water

Dam

cutoff

5 m of water

Low permeability rock

$q = 10^{-7} \times 45 \times (5/14) \times 100$
  $= 0.000161$ m³/sec
  $= 13.9$ m³/day

# FLOW NETS

# Calculation of Heads

**Consider Point X**

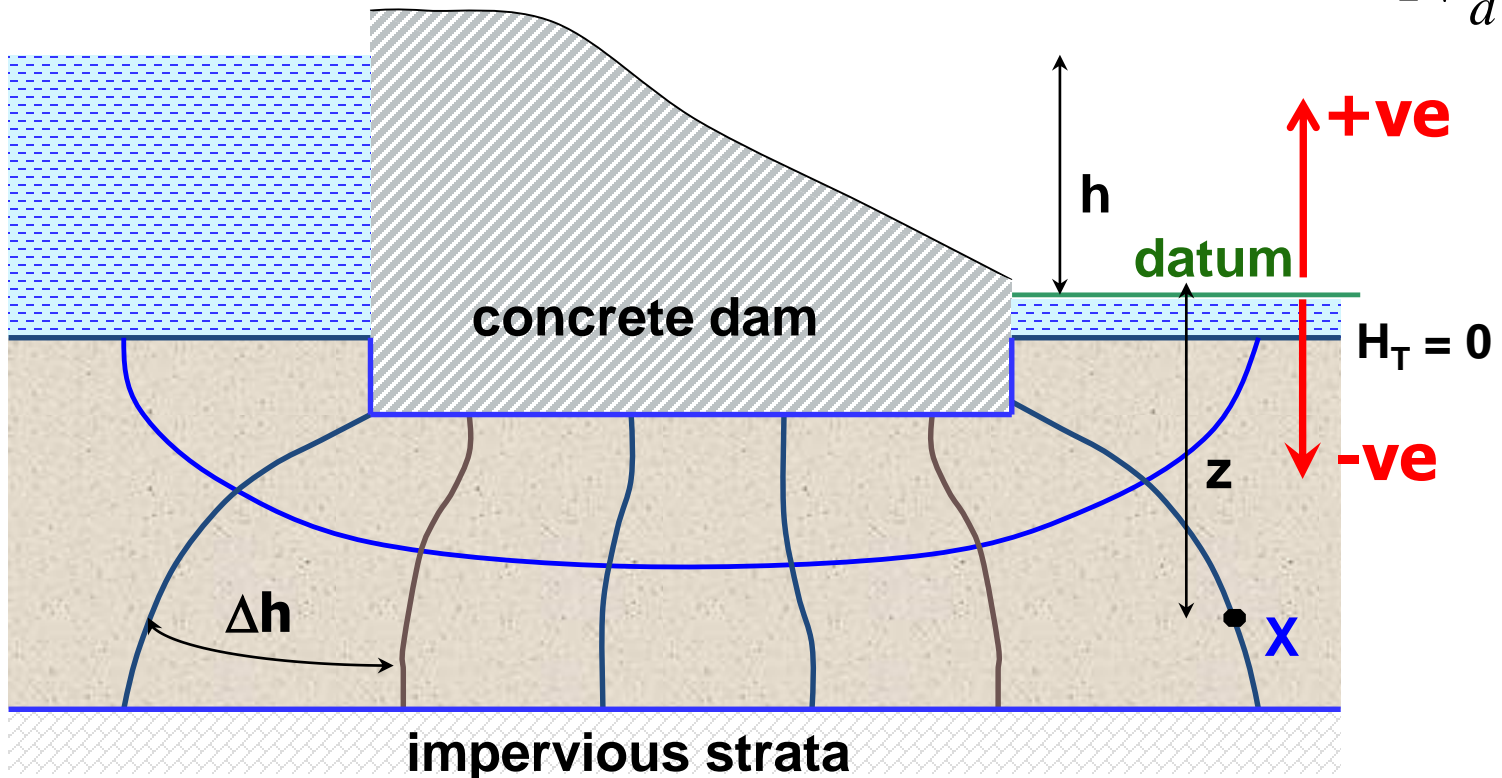**Total head = h - # of drops from upstream x Δh**

**Elevation head = -z**

**Pressure head = Total head − Elevation head**

$$h = \frac{u}{\gamma_w} + z$$

$$= \frac{h}{N_d}$$



concrete dam

datum

+ve

-ve

h

z

Δh

$H_T = 0$

X

impervious strata

# EXAMPLE

**Uplift pressure under hydraulic structures**

H = 7m    N$_d$ = 7
Head Loss at each equipotential line
= 7/7 = 1 m

h$_{(a)}$ = 6 m,  h$_{(b)}$ = 5 m, ……h$_{(f)}$ = 1m
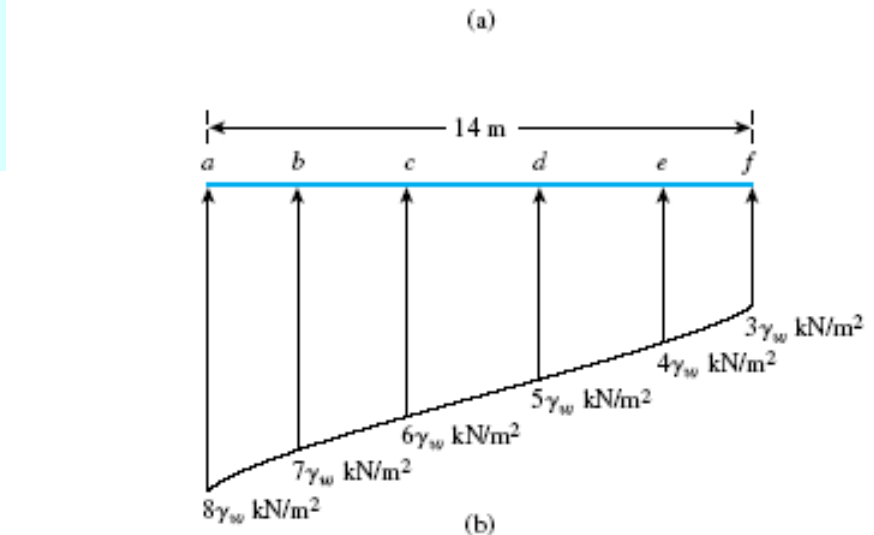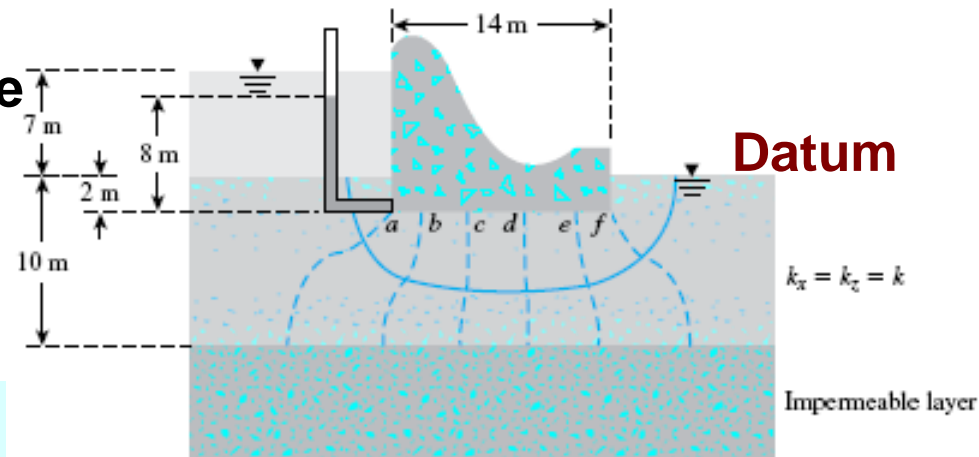
**Recall**

$$h = \frac{u}{\gamma_w} + Z$$

$$u = \gamma_w(h - Z)$$

$$u_{(a)} = \gamma_w(6 - (-2)) = 8\gamma_w$$

$$u_{(b)} = \gamma_w(5 - (-2)) = 7\gamma_w$$



**Figure 8.12** (a) A weir; (b) uplift force under a hydraulic structure

# EXAMPLE

**Consider the datum at the bottom border of the drainage layer**

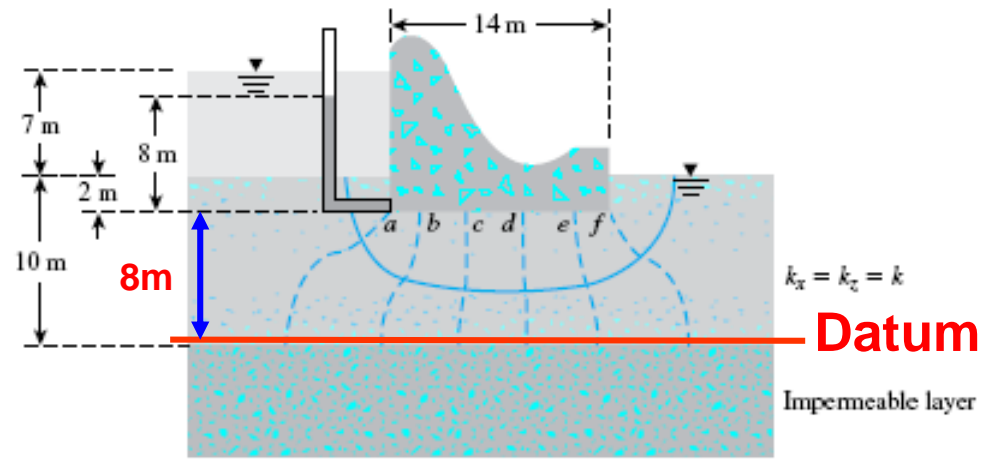H = 7m    $N_d$ = 7
Head Loss at each
equipotential line = 7/7 = 1 m
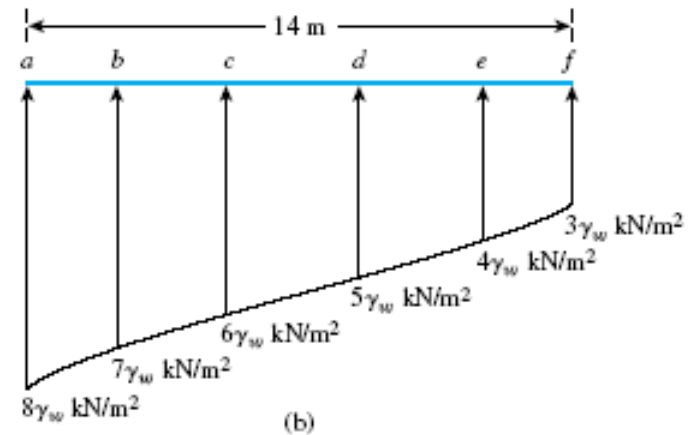
$$h = \frac{u}{\gamma_w} + Z$$

$$\boxed{u = \gamma_w (h - Z)}$$

$$u_{(a)} = \gamma_w (16 - 8) = 8\gamma_w$$

$$u_{(b)} = \gamma_w (15 - 8) = 7\gamma_w$$



(a)



(b)

Figure 8.12 (a) A weir; (b) uplift force under a hydraulic structure

# EXAMPLE

A stiff clay layer underlies a **12 m** thick silty sand deposit. A sheet pile is driven into the sand to a depth of 7 m, and the upstream and downstream water levels are as shown in the figure below.

**k = 8.6x 10⁻⁶ m/sec**
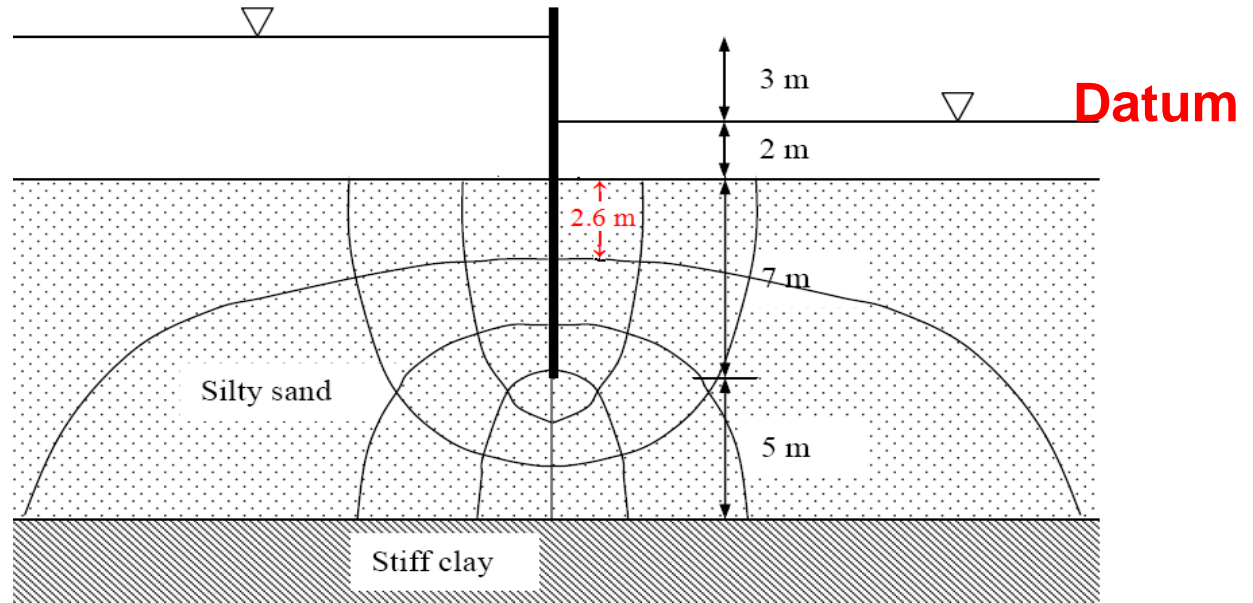


**Estimate**

a)   The seepage beneath the sheet pile in m³/day per **meter**.

b)  What is the pore water pressure at the **tip** of the sheet pile?

# EXAMPLE

## *Solution*

**h**

(a) In the flow net, $N_f = 3$; $N_d = 8$; $\Delta h = 3$ m. The flow (Q) is given by:

$$\mathbf{q} = kh_L \frac{N_f}{N_d} = (8.6 \times 10^{-6})(3)\left(\frac{3}{8}\right)(24 \times 3600) = \underline{0.836\,\mathrm{m}^3} / day \text{ per metre}$$

(b) Taking downstream water level as the datum, at the tip of the sheet pile,

Total head = 1.5 m
Elevation head = -9 m
∴ Pressure head = 1.5 – (-9) = 10.5 m
Pore water pressure = (10.5)(9.81) = <u>103.0 kPa</u>

$$\mathrm{h}_{tip} = \frac{\mathrm{u}_{tip}}{\gamma_w} + z_{tip}$$

$$q = kh \frac{N_f}{N_d} b$$

# EXAMPLE

**Let us try and take the datum at the bottom of the layer**

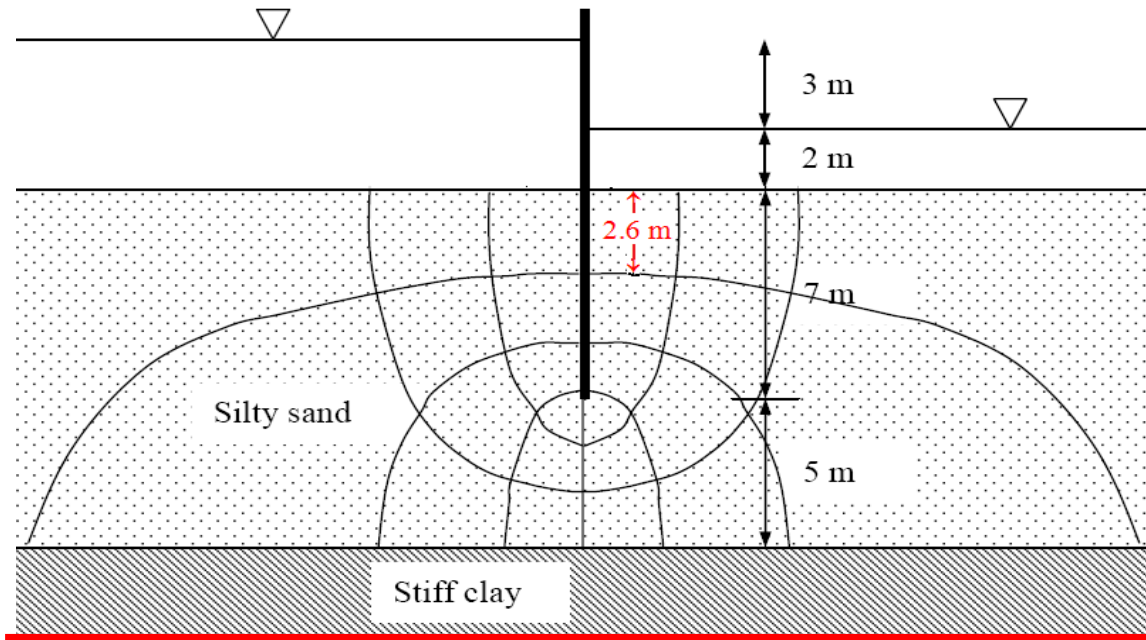**Total head at the tip = 17- 4 x 3/8 =15.5 m**
**Elevation head = 5 m**

→ **Δh**

$$h_{tip} = \frac{u_{tip}}{\gamma_w} + z_{tip}$$

$$15.5 = \frac{u_{tip}}{9.81} + 5$$

$$u_{tip} = (15.5 - 5) \times 9.81 = 103.0 \, kPa$$



3 m

2 m

2.6 m

7 m

Silty sand

5 m

Stiff clay
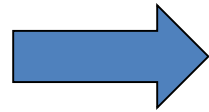
**DATUM**

# EXAMPLE (2nd Midterm Exam Fall 40-41)

Question#3 (35%) [CLO 1= 30%, CLO 3 = 30%, CLO 4 = 40%]

For the concrete dam shown in Figure 3.

    I. Compute the flow under the dam per meter of the dam per day.

    II. To what level water should be allowed to rise above the downstream ground surface so that the existing rate of seepage is reduced by 25%?

    III. The pore water pressure at points (a) and (b).

    IV. Take the datum along the bottom flow line (line (1)----(2)) and compute the pore water pressure at points (c) and (d).
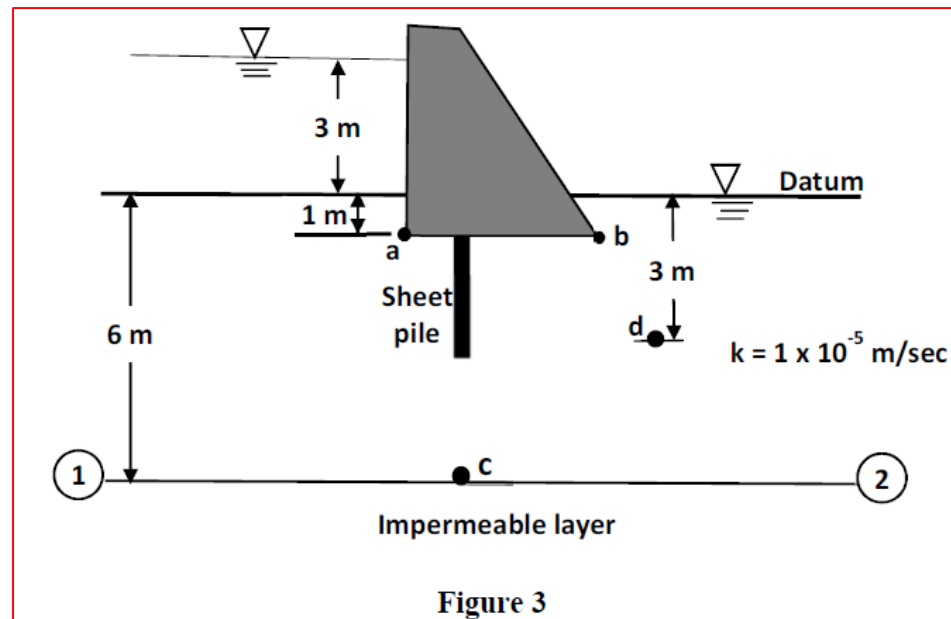


Figure 3

# Common Mistake

$$h_a = \frac{u_a}{\gamma_w} + z_a$$

$$u_a = \gamma_w(h_a - z_a)$$

$$h_a = (h - n \times \Delta h)$$

$$u_a = \gamma_w(h - n \times \Delta h - z_a)$$

$$\Delta h = \frac{h}{n_d}$$

**This is valid only if the DATUM is taken at the downstream water level.**

$$u_a = \gamma_w\left(h - n \times \frac{h}{n_d} - z_a\right)$$

# FLOW NETS

## All times correct Steps

$$\Delta h = \frac{h}{n_d}$$

**h** = the head difference

$$h_a = (h - n \times \Delta h)$$

**h** = the distance from <u>datum</u> to the upstream water level.

$$h_a = \frac{u_a}{\gamma_w} + z_a$$

**h** and **h** are the same if and only if the **datum** is taken at downstream water level.

# EXAMPLE (Final Exam Fall 38-39)

A river bed consists of a layer of **sand 8.25 m** thick overlying impermeable rock; the depth of water is **2.5 m**. A long cofferdam **5.50 m** wide is formed by driving two lines of sheet piling to a depth of **6.0 m** below the level of the river bed and excavation to a depth of **2.0 m** below bed level is carried out within the cofferdam. The section through a dam is shown in Figure 1.
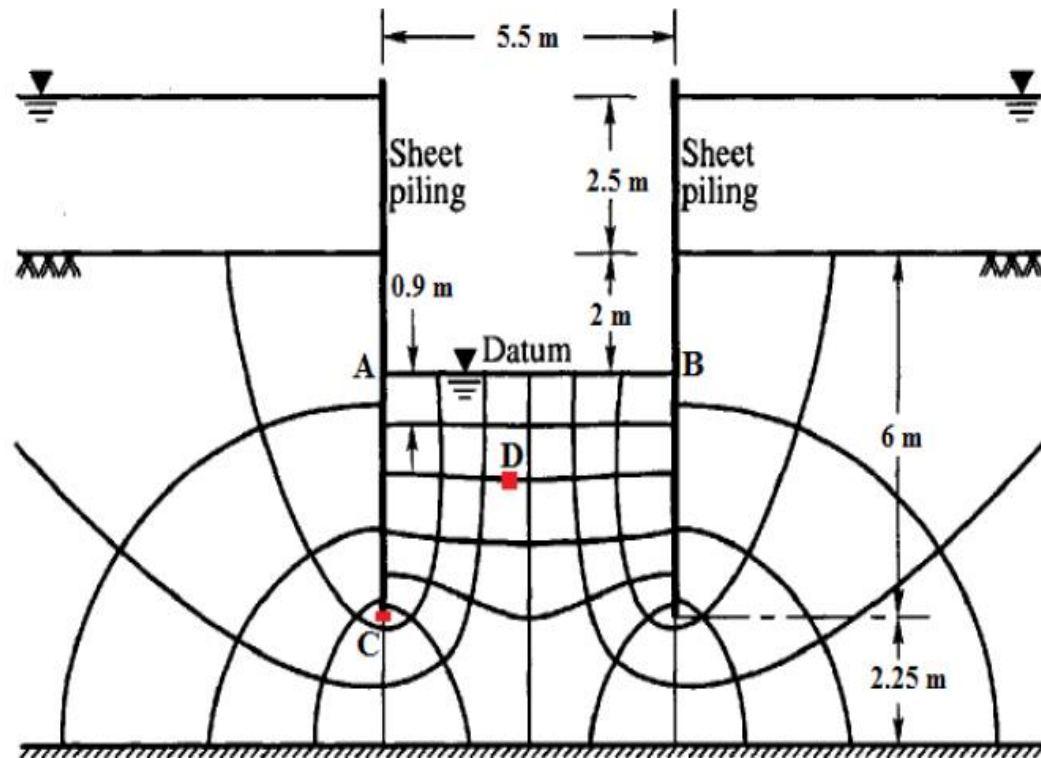
# EXAMPLE 6 (Final Exam Fall 38-39)

## Required:

a) The coefficient of permeability of the sand if the flow of water into the cofferdam is 0.25 m³/h per unit length.

b) The hydraulic gradient immediately below the excavated surface? (i.e. line AB).

c) The pore water pressure at point C.

d) The effective stress at point D (located 1.8 m below line AB) if the unit weight of the sand is 18.0 kN/m³.

e) Repeat part (d) if the water level behind the wall is lowered to the ground surface and the water level rises 2.0 above the river bed.

# Flow Nets in Anisotropic Soil

# Flow Nets in Anisotropic Soil

- The Laplace's equation is based on the assumption that permeability are equal in the **horizontal and vertical** directions.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

- However, most compacted embankments and many natural soil deposits are more or less stratified, often with horizontal bedding that make **horizontal** permeability much **greater** than the vertical.

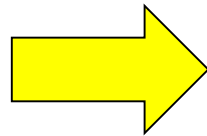- The differential equation in **2-D** for seepage through **anisotropic** soil is

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \qquad \textbf{(*)}$$

# Flow Nets in Anisotropic Soil

- Eq. (*) **is not** a **Laplace** equation and we can no longer obtain solutions to plane seepage problems by drawing "square flow nets". In this case, Eq. (*) represents two families of curves that **do no meet at 90º**.

- Eq. (*) can be written as

$$\frac{\partial^2 h}{(k_z / k_x)\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad \textbf{(**)}$$

$$Let \quad \sqrt{k_z / k_x}.x = \bar{x}$$

$$\frac{\partial^2 h}{\partial \bar{x}^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad \textbf{(***)}$$

- Eq. (***) expresses the anisotropic seepage condition as a **Laplace** equation. Here **x** is replaced by **x-Bar** which is the new transformed coordinate. **This equation can be solved exactly as before by square flow nets.**

- The **x** direction is scaled to transform a given anisotropic flow region into a **fictitious isotropic flow region** in which the Laplace equation is valid.

# Flow Nets in Anisotropic Soil

**To construct the flow net, the following procedures may be followed:**

Step 1. Adopt a vertical scale (that is, $z$ axis) for drawing the cross section.

Step 2. Adopt a horizontal scale (that is, $x$ axis) such that horizontal scale $= \sqrt{k_z/k_x} \times$ vertical scale.

Step 3. With scales adopted as in steps 1 and 2, plot the vertical section through the permeable layer parallel to the direction of flow.

Step 4. Draw the flow net for the permeable layer on the section obtained from step 3, with flow lines intersecting equipotential lines at right angles and the elements as approximate squares.

**The flow through the anisotropic soil is given by**

$$q = \sqrt{k'}\, h \, \frac{N_f}{N_d}$$

**$k'$ = Equivalent coefficient of permeability given by:**

$$k' = \sqrt{k_x k_z}$$

**The final flow net can be obtained by transforming the cross-section including the flow net back to the natural scale.**

# Flow Nets in Anisotropic Soil

## Remarks

- In transformed section the flow lines and equipotential lines are **orthogonal** and the figures are **squares**. However when they redrawn in a true section, they will not intersect at **right angles**, nor will the figures will be squares.

- If the **horizontal** permeability is **greater** than the **vertical**, the transformed section will always be **shrunk** to a narrower horizontal dimension. If the reverse were true, it will be **longer** than horizontally.

# EXAMPLE

A single row of sheet pile structure is shown below. Draw a flow net for the **transformed** section. **Replot** this flow net in the natural scale also. The relationship between the permeabilities is given as $k_x = 6\ k_z$.
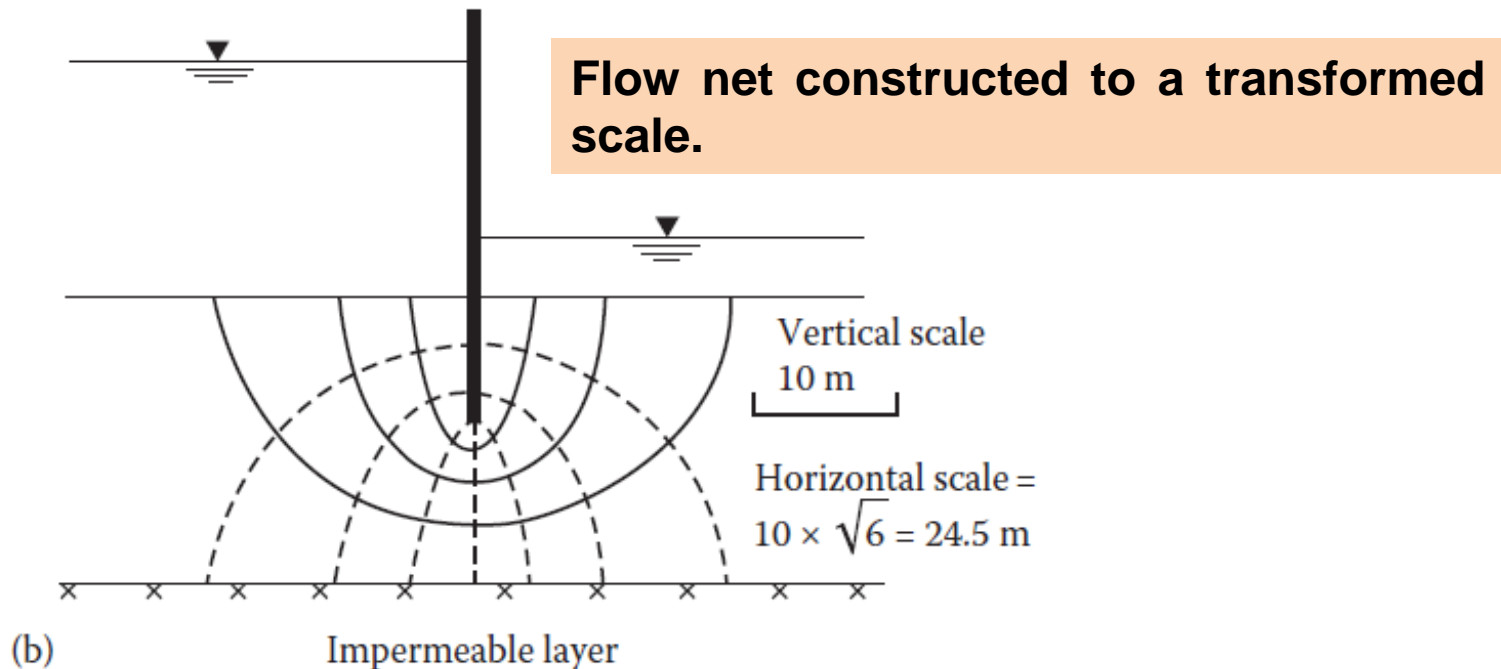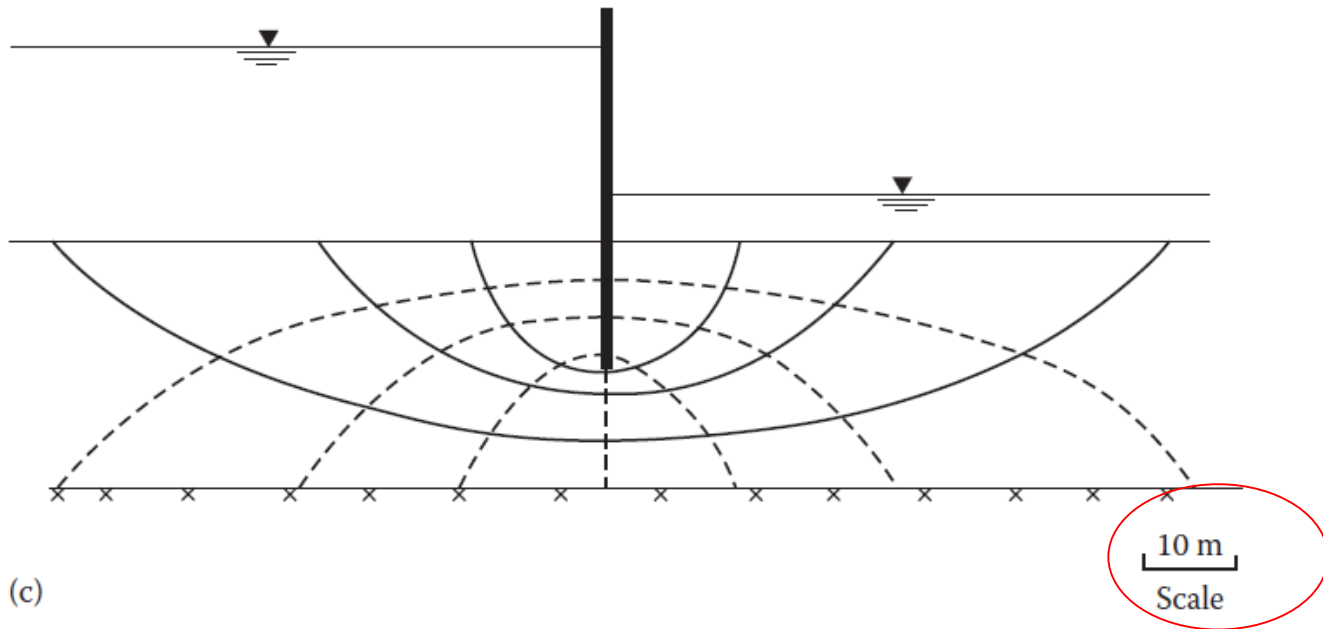


(a)                    Impermeable layer
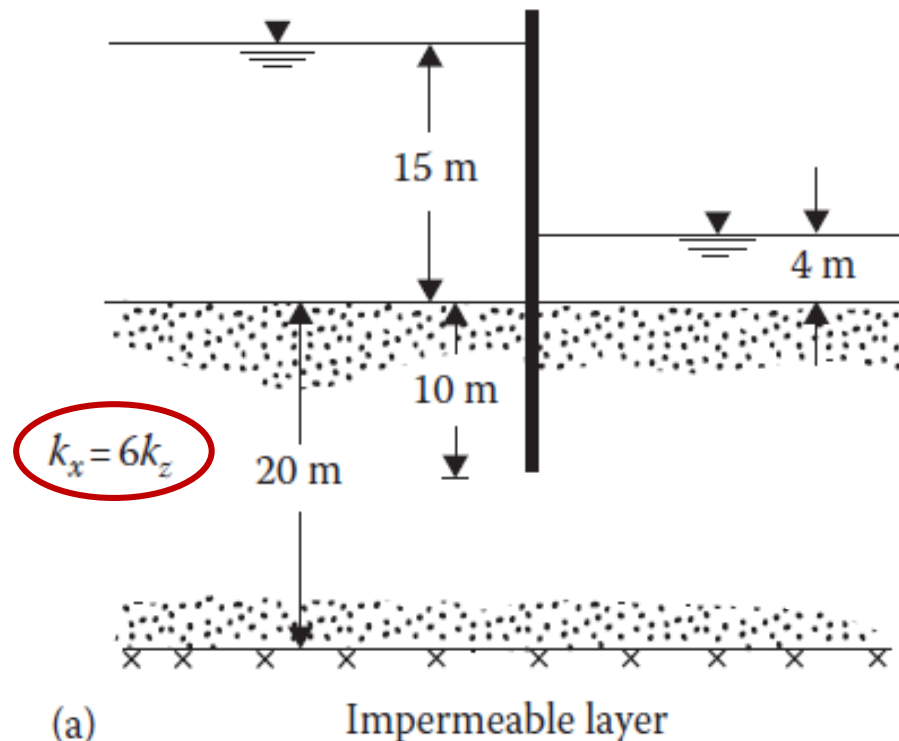
## Solution

For the transformed section

$$\text{Horizontal scale} = \sqrt{\frac{k_z}{k_x}} \text{ (vertical scale)}$$

$$= \frac{1}{\sqrt{6}} \text{ (vertical scale)}$$

Flow net constructed to a transformed scale.

Vertical scale
10 m

Horizontal scale =
$10 \times \sqrt{6} = 24.5$ m

(b)          Impermeable layer

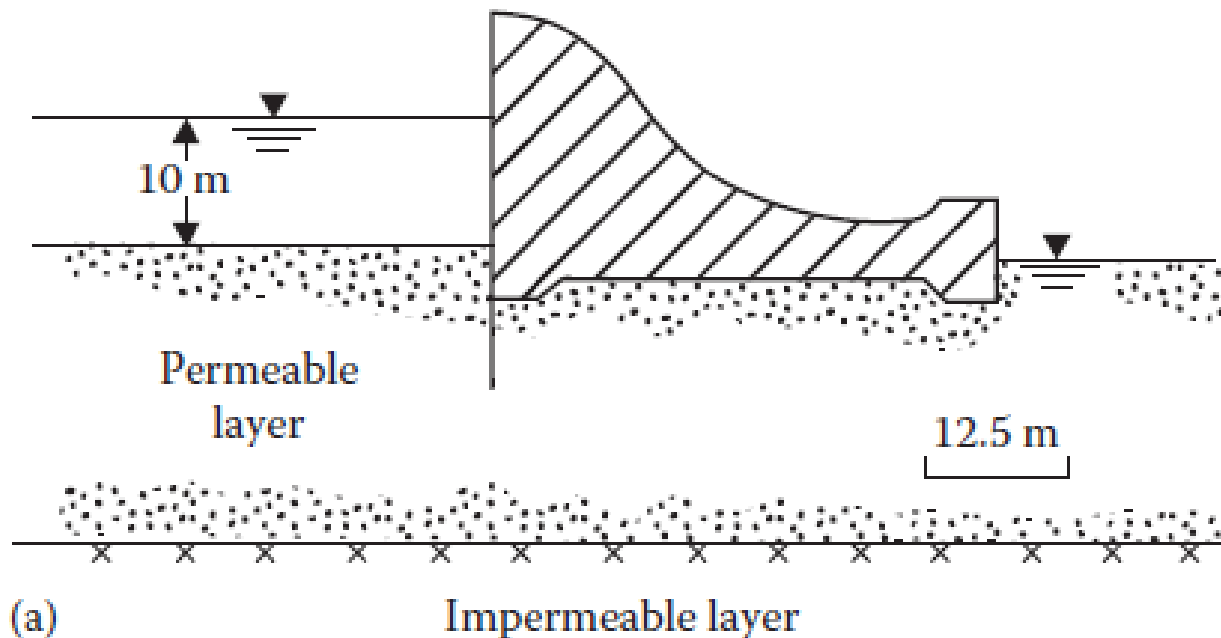# EXAMPLE



(c)

10 m
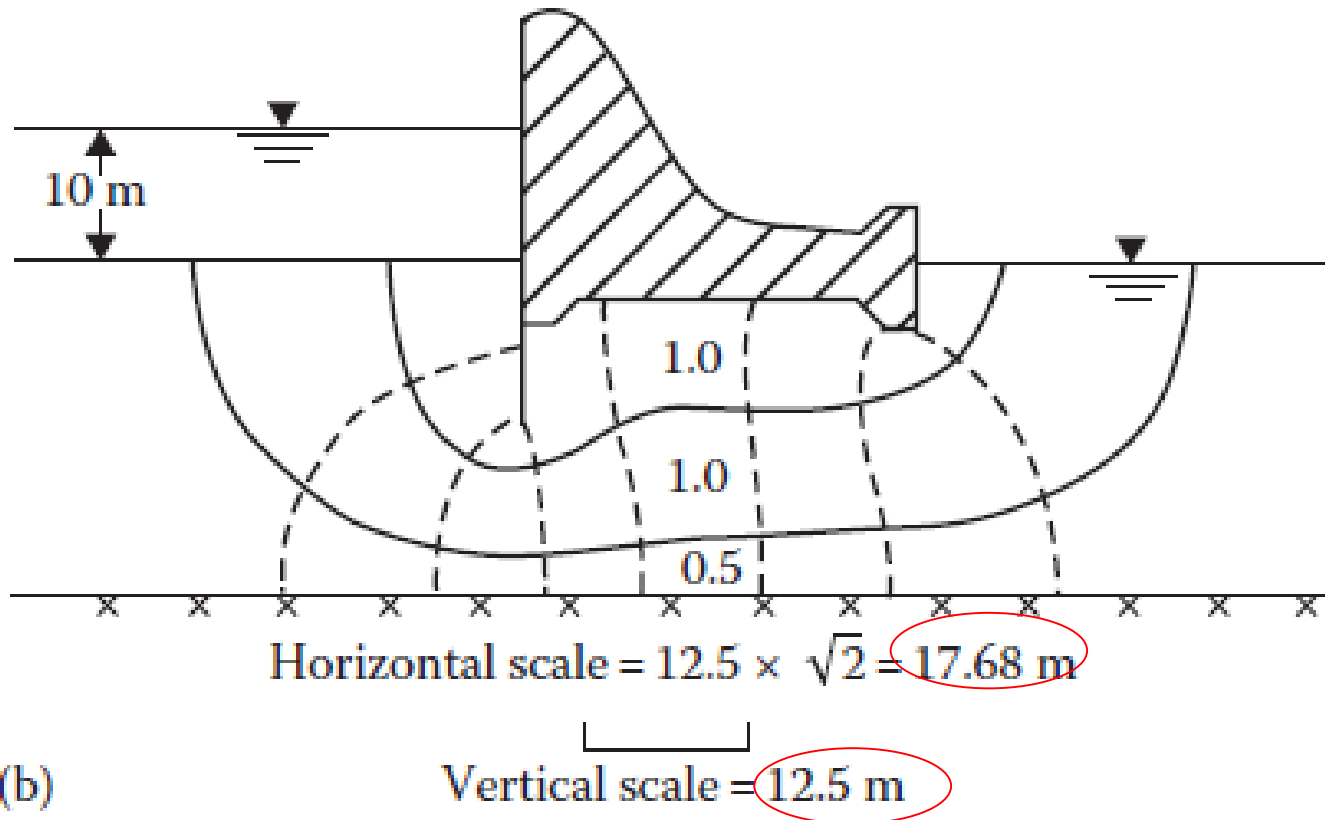Scale

**Flow net constructed to the natural scale.**

# EXAMPLE

A dam section is shown below. The coefficients of permeability of the permeable layer in the **vertical** and **horizontal** directions are **2 × 10⁻²** and **4 × 10⁻²** mm/s, respectively.

Draw a flow net and calculate the seepage loss of the dam in m³/(day. m).



(a)

10 m

Permeable layer

12.5 m

Impermeable layer

# EXAMPLE



10 m

1.0

1.0

0.5

Horizontal scale = 12.5 × √2 = 17.68 m

Vertical scale = 12.5 m

(b)

# EXAMPLE 8.4

## Example 8.4

A dam section is shown in Figure 8.11a. The hydraulic conductivity of the permeable layer in the vertical and horizontal directions are $2 \times 10^{-2}$ mm/s and $4 \times 10^{-2}$ mm/s, respectively. Draw a flow net and calculate the seepage loss of the dam in ft³/day/ft

### Solution

From the given data,

$$k_z = 2 \times 10^{-2} \text{ mm/s} = 5.67 \text{ ft/day}$$
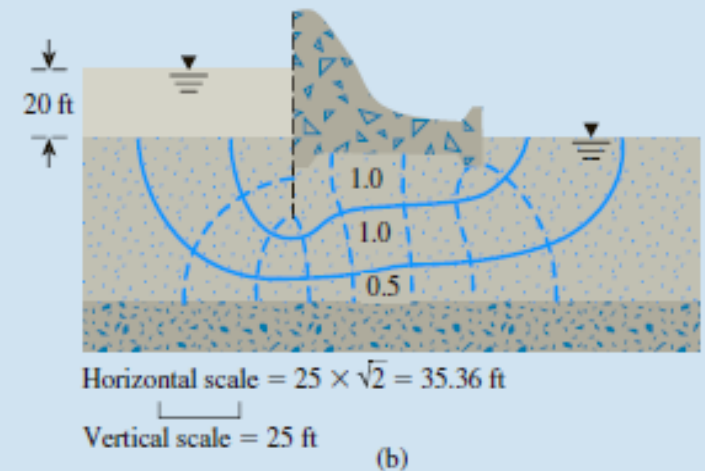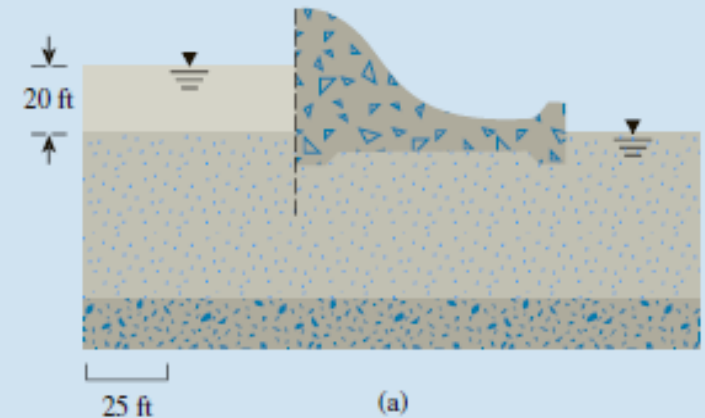
$$k_x = 4 \times 10^{-2} \text{ mm/s} = 11.34 \text{ ft/day}$$

and $H = 20$ ft. For drawing the flow net,

$$\text{Horizontal scale} = \sqrt{\frac{2 \times 10^{-2}}{4 \times 10^{-2}}} \text{ (vertical scale)}$$

$$= \frac{1}{\sqrt{2}} \text{ (vertical scale)}$$

On the basis of this, the dam section is replotted, and the flow net drawn as in Figure 8.11b. The rate of seepage is given by $q = \sqrt{k_x k_z} \, H(N_f / N_d)$. From Figure 8.11b, $N_d = 8$ and $N_f = 2.5$ (the lowermost flow channel has a width-to-length ratio of 0.5). So,

$$q = \sqrt{(5.67)(11.34)}(20)(2.5/8) = \mathbf{50.12} \text{ ft}^3\text{/day/ft}$$



20 ft
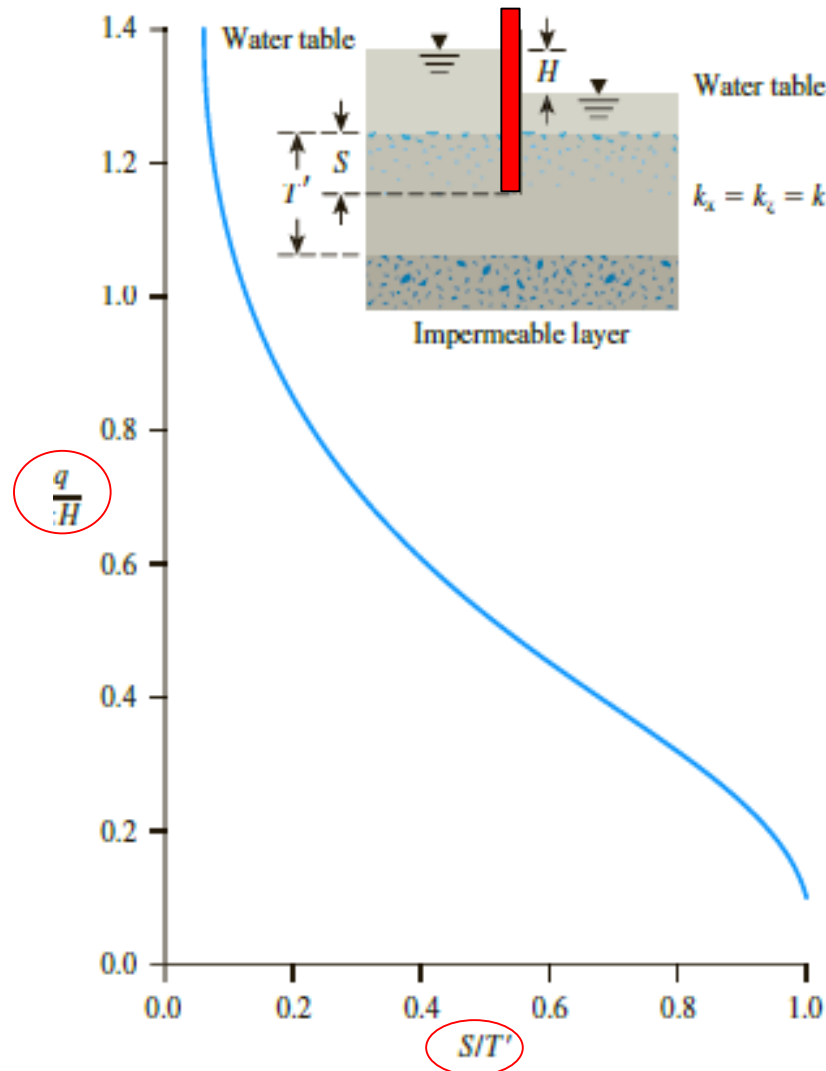
25 ft      (a)

20 ft

1.0

1.0

0.5

Horizontal scale $= 25 \times \sqrt{2} = 35.36$ ft

Vertical scale $= 25$ ft

(b)

☐ Permeable layer    ▨ Impermeable layer

# Mathematical Solution for Seepage

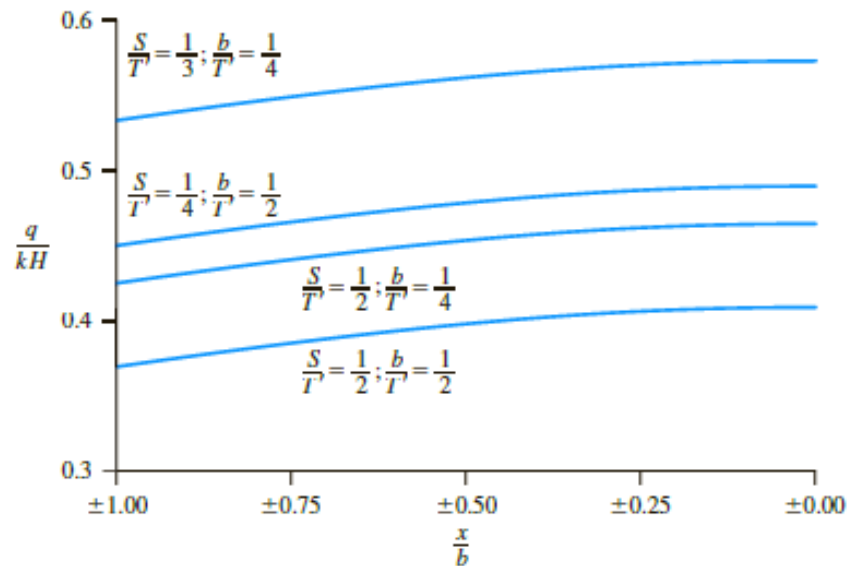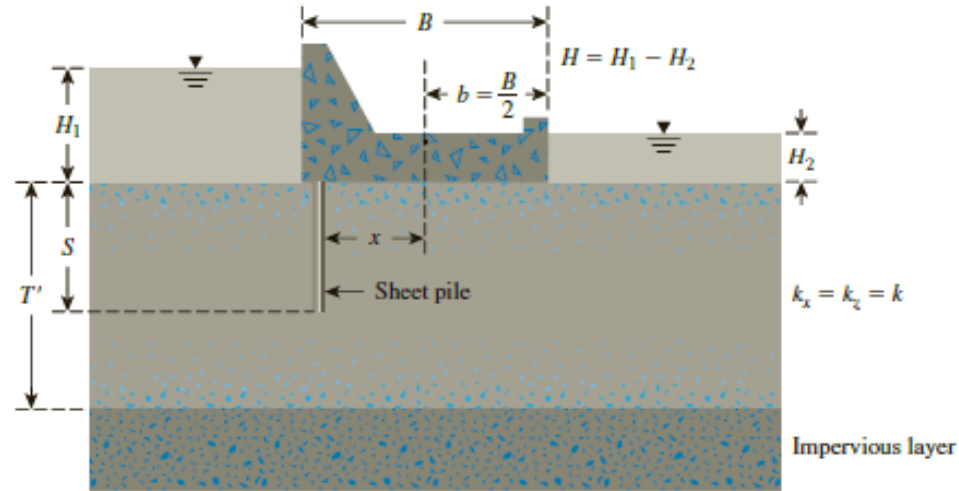## Seepage Around a Single Row of Sheet Piles

**Figure 8.13** Seepage under a dam (*After Harr, 1962. By permission of Dover Publications, Inc.*)

# EXAMPLE 8.5

## Example 8.5

Refer to Figure 8.13. Given; the width of the dam, $B = 6$ m; length of the dam, $L = 120$ m; $S = 3$ m; $T' = 6$ m; $x = 2.4$ m; and $H_1 - H_2 = 5$ m. If the hydraulic conductivity of the permeable layer is 0.008 cm/sec, estimate the seepage under the dam $(Q)$ in m³/day.

## Solution

Given that $B = 6$ m, $T' = 6$ m, and $S = 3$ m, so $b = B/2 = 3$ m.

$$\frac{b}{T'} = \frac{3}{6} = 0.5$$
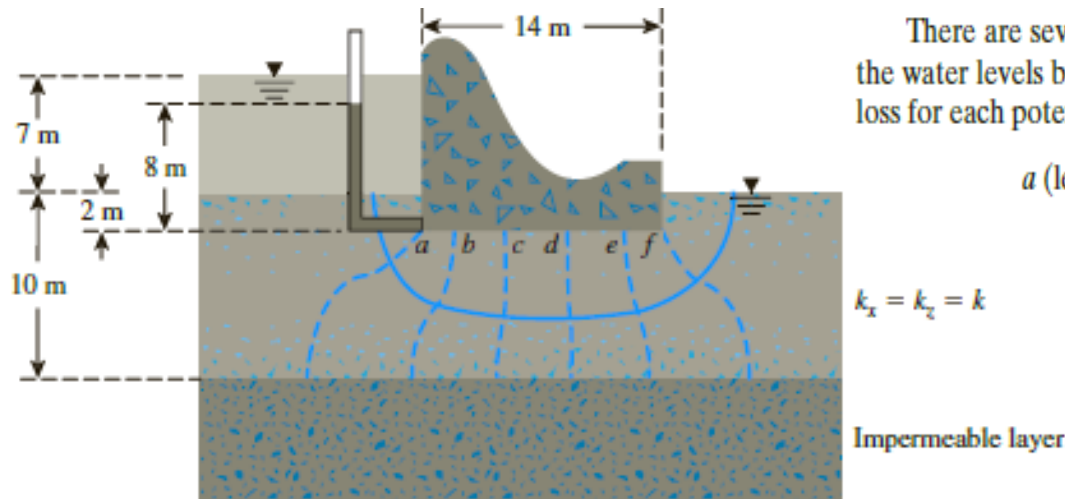
$$\frac{S}{T'} = \frac{3}{6} = 0.5$$

$$\frac{x}{b} = \frac{2.4}{3} = 0.8$$

From Figure 8.13, for $b/T' = 0.5$, $S/T' = 0.5$, and $x/b = 0.8$, the value of $q/kH \approx 0.378$.

Thus,

$$Q = q\,L = 0.378\,k\,HL = (0.378)(0.008 \times 10^{-2} \times 60 \times 60 \times 24 \text{ m/day})(5)(120)$$
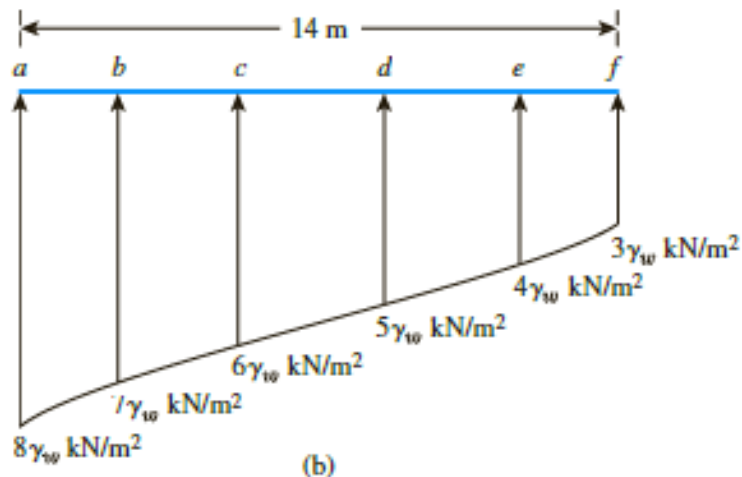$$= 1567.64 \text{ m}^3/\text{day}$$

(a)

There are seven equipotential drops ($N_d$) in the flow net, and the difference in the water levels between the upstream and downstream sides is $H = 7$ m. The head loss for each potential drop is $H/7 = 7/7 = 1$ m. The uplift pressure at

$$a \text{ (left corner of the base)} = \text{(Pressure head at } a) \times (\gamma_w)$$
$$= [(7+2) - 1]\gamma_w = 8\gamma_w$$

$k_x = k_z = k$

uplift pressure at

$$b = [9 - (2)(1)]\gamma_w = 7\gamma_w$$

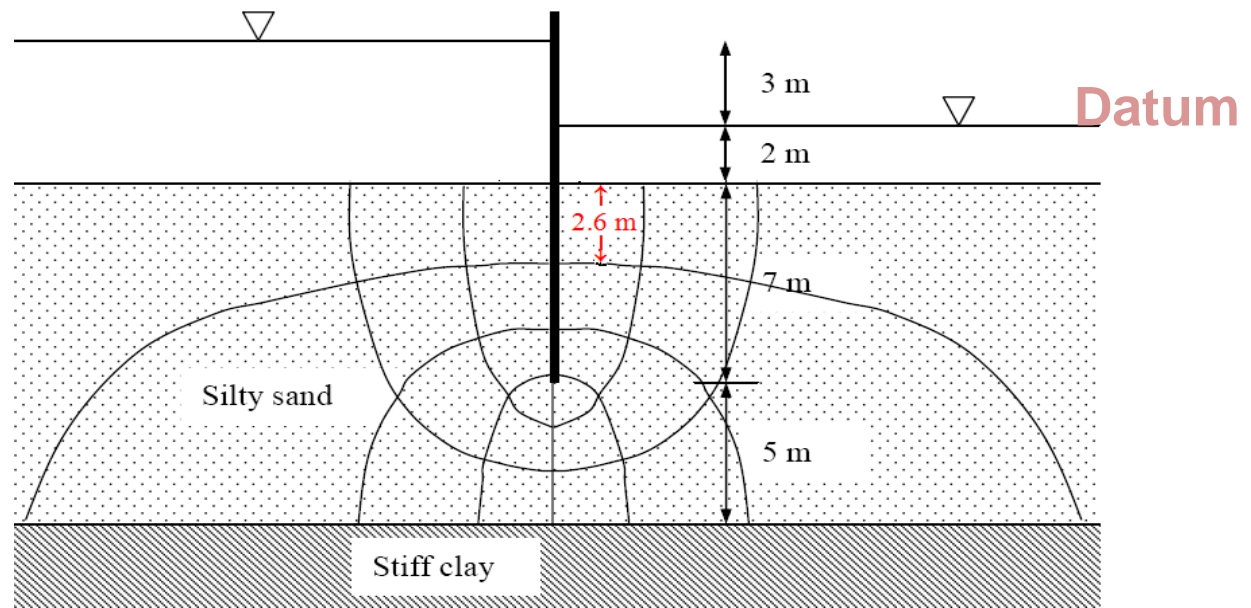$$f = [9 - (6)(1)]\gamma_w = 3\gamma_w$$

Impermeable layer



(b)

# Example

A stiff clay layer underlies a 12 m thick silty sand deposit. A sheet pile is driven into the sand to a depth of 7 m, and the upstream and downstream water levels are as shown in the figure below.

**k = 8.6x 10⁻⁶ m/sec**



## Estimate

**a) The seepage beneath the sheet pile in m³/day per meter.**

# Example

Recall from flow net q = 0.836 m³/ day

H = 3 m
S = 7 m
T' = 12 m
S/T' = 0.58

k = 8.6x 10⁻⁶ m/sec
    = 0.743 m/day

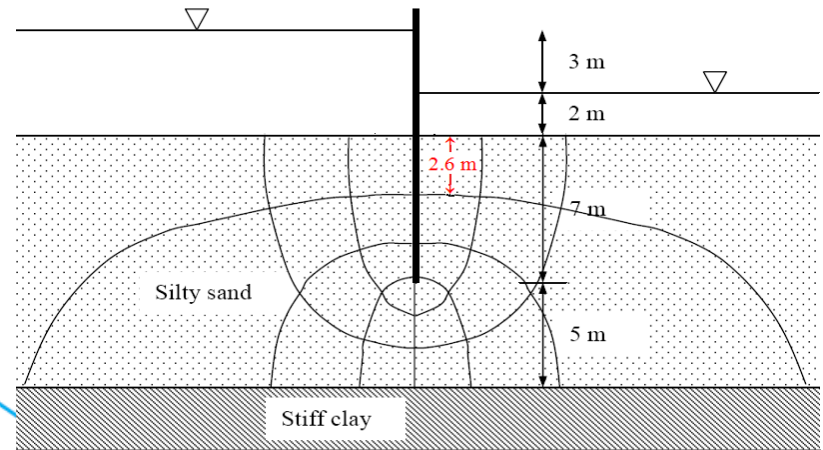q = 0.743 x 3 x 0.47    = 0.47

q = 1.05 m³/day

Difference = ?
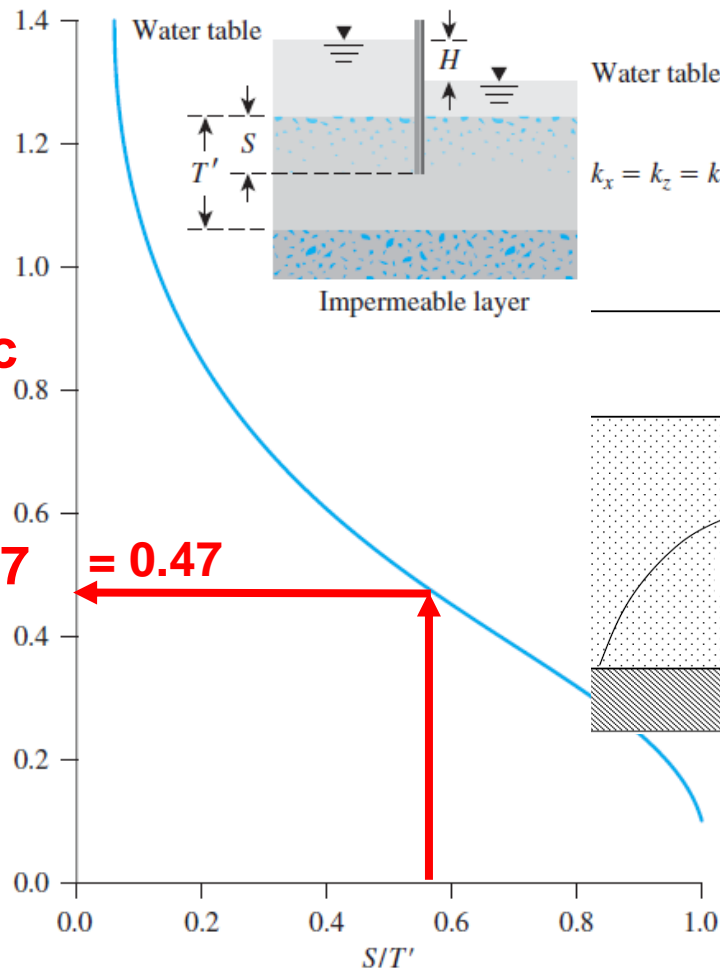


Figure 8.10 Plot of q/kH against S/T' for flow around a single row of sheet piles (After Harr, 1962. By permission of Dover Publications, Inc.)

# FLOW THROUGH EARTH DAMS RESTING ON AN IMPERVIOUS BASE

# Confined Flow
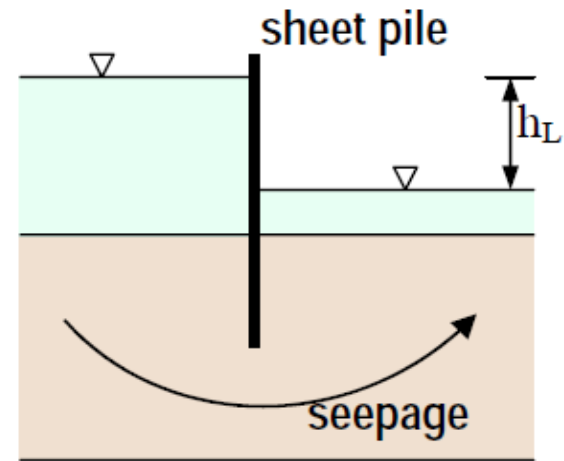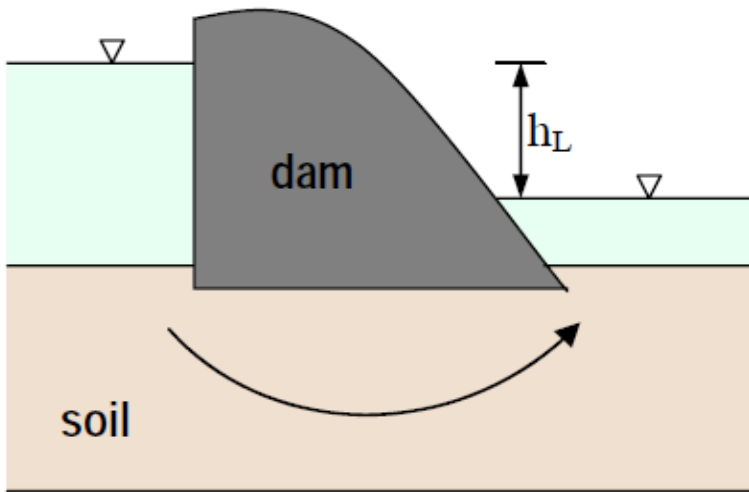


Seepage beneath (a) a concrete dam (b) a sheet pile

# Unconfined Flow

- **Seepage through <span style="color:red">earth dams</span> is an example of unconfined seepage.**



**Pervious foundation**

**Impervious boundary**
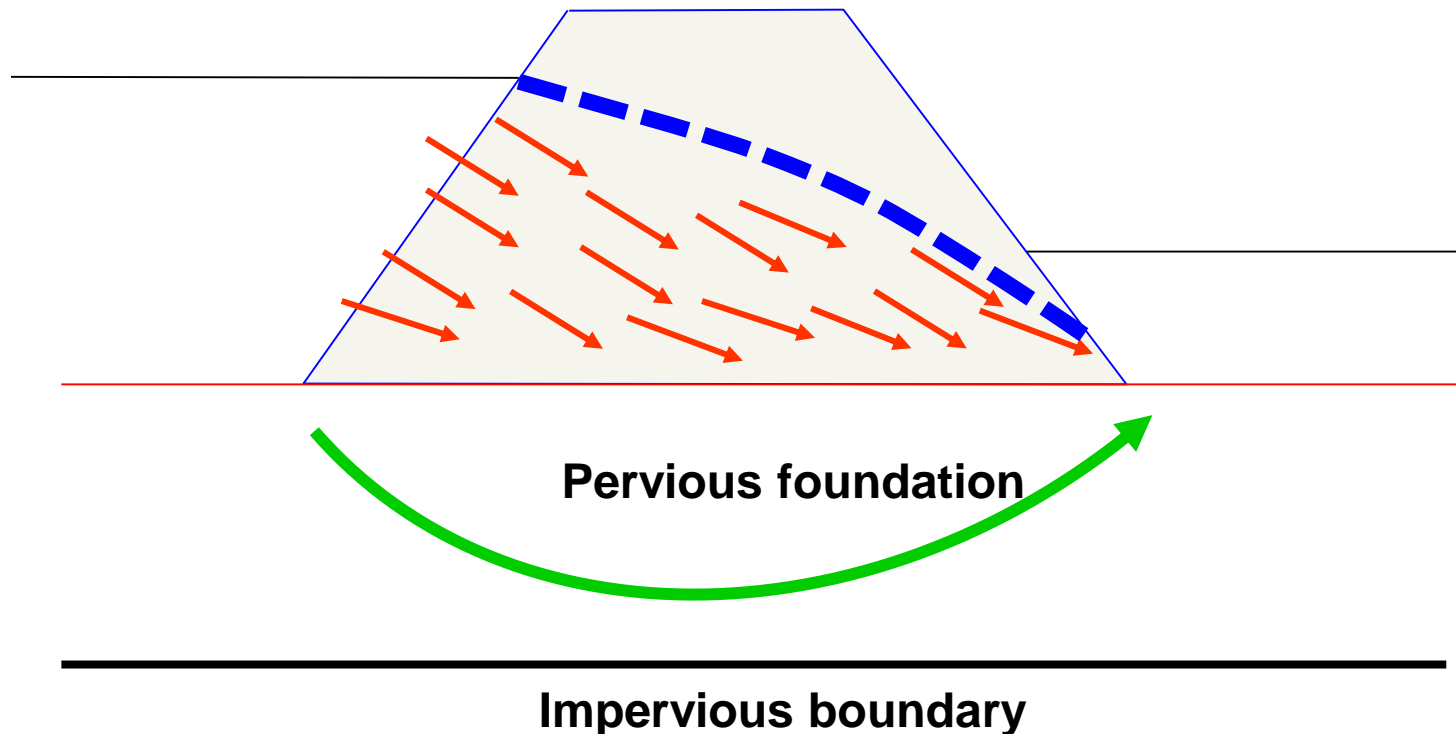
**Earth fill dam with flow through both the dam and the foundation**

# What we will consider?

- We will consider a section of a **trapezoidal** dam of **homogeneous** soils.

- For simplicity it is assumed that the dam rests on **impervious** foundation and that all seepage water therefore flows through the **dam**. (**opposite of concrete dam**).

- Then the line of contact with the foundation is one boundary flow line.

**Impervious foundation**
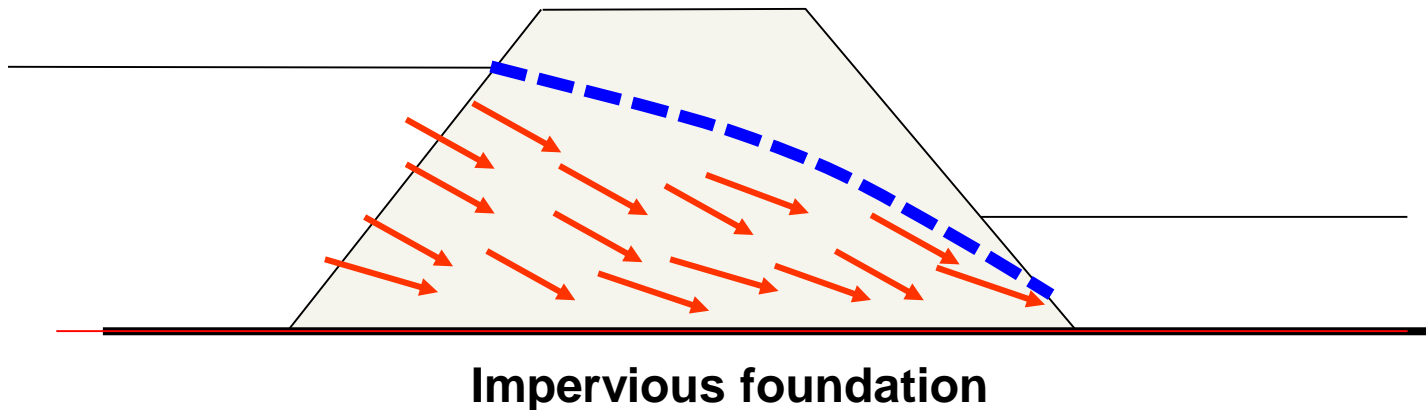
**Earth fill dam with flow through the dam only**

# FLOW THROUGH EARTH DAMS

- The <u>major problem</u> is to establish the shape of the **top line** of seepage.

- The location of the free surface depends on the **flow regime** and the **flow regime** depends on its turn on the position of the free surface.

- Several procedures for obtaining the top flow line are available.

- Unconfined flow problems are often considered more **<u>difficult</u>** to analyze because the determination of the location of the phreatic surface.

# FLOW THROUGH EARTH DAMS

## Schaffernak's (1917), Casagrande (1937)

Considering the triangle *cde*, we can give the rate of seepage per unit length of the dam (at right angles to the cross section shown in Figure) as:

$$i \simeq \frac{dz}{dx}$$

$$q = kiA$$

**Consider Plane ce**

$$i = \frac{dz}{dx} = \tan \alpha$$

$$A = (\overline{ce})(1) = L \sin \alpha$$

$$q = k(\tan \alpha)(L \sin \alpha) = kL \tan \alpha \sin \alpha \quad (*)$$

Rate of seepage (per unit length of the dam) through the section *bf* is

$$q = kiA = k\left(\frac{dz}{dx}\right)(z \times 1) = kz\frac{dz}{dx} \quad (**)$$
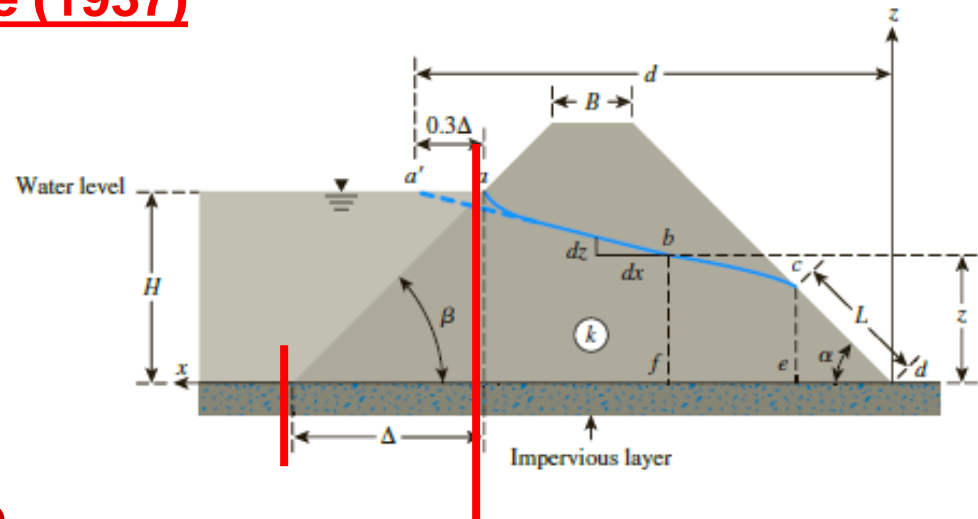
For continuous flow,

Eq (*) = Eq (**)

$$kz\frac{dz}{dx} = kL \tan \alpha \sin \alpha$$

$$\int_{z=L\sin\alpha}^{z=H} kz \, dz = \int_{x=L\cos\alpha}^{x=d} (kL \tan \alpha \sin \alpha) \, dx$$

$$L = \frac{d}{\cos \alpha} - \sqrt{\frac{d^2}{\cos^2 \alpha} - \frac{H^2}{\sin^2 \alpha}} \quad (***)$$

# FLOW THROUGH EARTH DAMS

**Following is a step-by-step procedure to obtain the seepage rate *q* (per unit length of the dam):**

**Step 1. Obtain** $\alpha$**.**

**Step 2. Calculate** $\Delta$ **(from Figure) and then 0.3**$\Delta$**.**

**Step 3. Calculate *d*.**

**Step 4. With known values of** $\alpha$ **, *H* and *d*, calculate *L* from Eq. (\*\*\*)**

$$L = \frac{d}{\cos \alpha} - \sqrt{\frac{d^2}{\cos^2 \alpha} - \frac{H^2}{\sin^2 \alpha}} \qquad (\text{\*\*\*})$$

**Step 5. With known value of *L*, calculate *q* from Eq. (\*).**

$$q = k(\tan \alpha)(L \sin \alpha) = kL \tan \alpha \sin \alpha \qquad (\text{\*})$$

# FLOW THROUGH EARTH DAMS

## L. Casagrande's Solution for Seepage through an Earth Dam
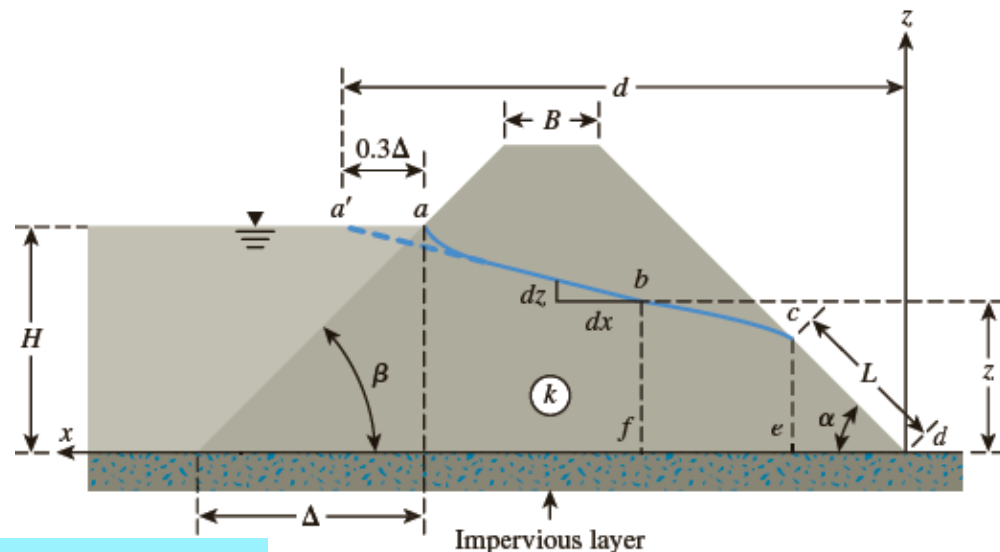
- Equation **(***)** is derived on the basis of Dupuit's assumption (i.e., i < dz/dx).

- when the downstream slope angle becomes greater than **30°**, deviations from Dupuit's assumption become more noticeable

$$i = \frac{dz}{ds} = \sin \alpha$$

where $ds = \sqrt{dx^2 + dz^2}$.

**Eq. (*) becomes**

$$q = kiA = k\sin\alpha(L\sin\alpha) = kL\sin^2\alpha \qquad \text{(****)}$$

# FLOW THROUGH EARTH DAMS

## L. Casagrande's Solution for Seepage through an Earth Dam

$$q = kiA = k\left(\frac{dz}{ds}\right)(1 \times z)$$

$$q_{(Eq **)} = q_{(Eq ****)}$$

$$\int_{L\sin\alpha}^{H} z\, dz = \int_{L}^{S} L\sin^2\alpha\, ds \quad \Longrightarrow \quad L = s - \sqrt{s^2 - \frac{H^2}{\sin^2\alpha}}$$

where **s** is the length of the **curve** *a'bc*

With about a 4–5% error, the dimension **s** differs only slightly from the straight line *a'c.* Therefore, the distance **s** is *approximated as*
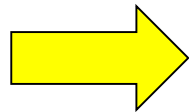
$$s = \sqrt{d^2 + H^2} \quad \Longrightarrow \quad \boxed{L = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2\cot^2\alpha}}$$

$$\boxed{q = kL\sin^2\alpha}$$

# Alternative <u>graphical</u> procedure for obtaining L

## Solution by L. Casagrande's method based on Gilboy's solution
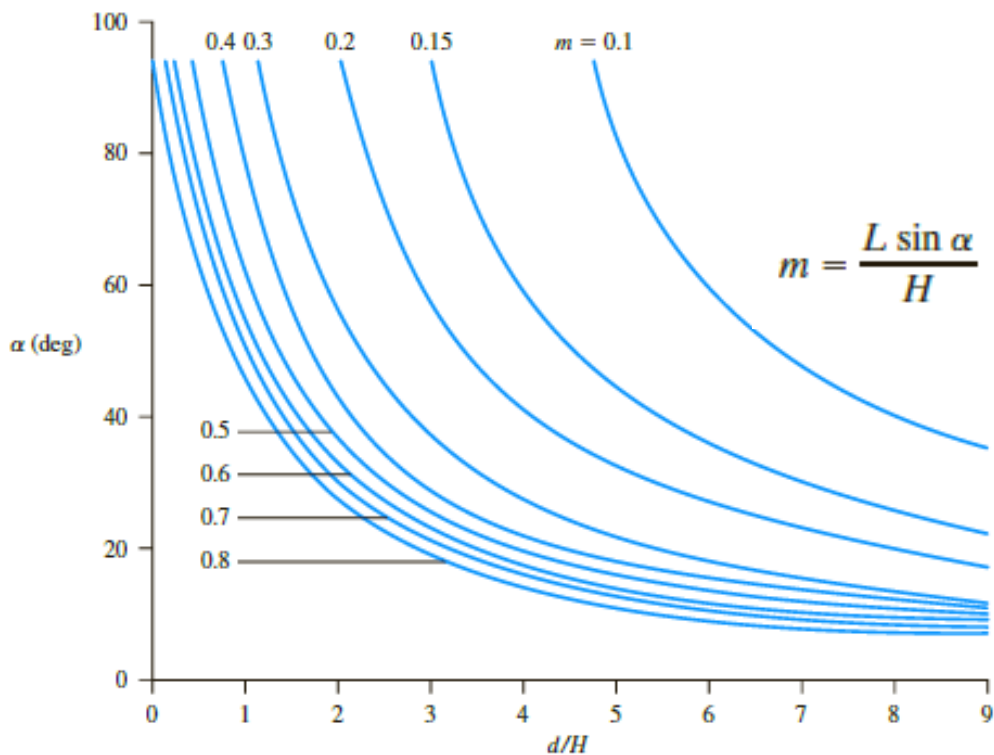


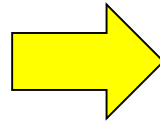$$m = \frac{L \sin \alpha}{H}$$

In order to use the graph,

**Step 1.** Determine $d/H$.

**Step 2.** For a given $d/H$ and $\alpha$, determine $m$.

**Step 3.** Calculate $L = \dfrac{mH}{\sin \alpha}$.

**Step 4.** Calculate $q = kL \sin^2 \alpha$.

# L. Casagrande's (1932) Solution

$$q = kL \sin^2 \alpha$$

o **Approximate solution for L**  $\Longrightarrow$  $L = \sqrt{d^2 + H^2} - \sqrt{d^2 - H^2 \cot^2 \alpha}$

- - - - - - - - - - - - - - - - - - - - - - -

o **Graphical solution for L**
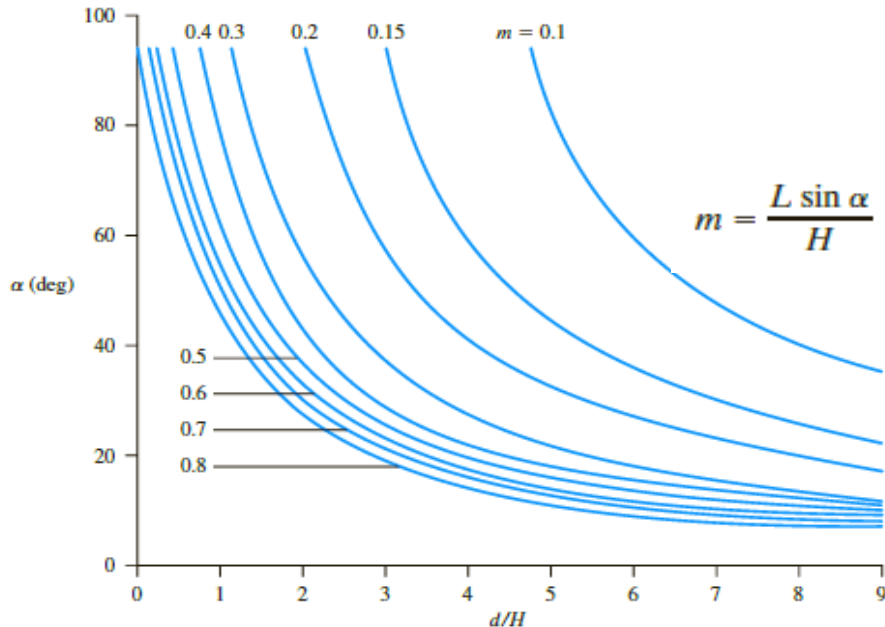


$$m = \frac{L \sin \alpha}{H}$$

In order to use the graph,

**Step 1.** Determine $d/H$.

**Step 2.** For a given $d/H$ and $\alpha$, determine $m$.

**Step 3.** Calculate $L = \dfrac{mH}{\sin \alpha}$.

**Step 4.** Calculate $q = kL \sin^2 \alpha$.

# Piping Failures

# Piping Failures

# Exit Hydraulic Gradient



At the downstream, near the dam,

the exit hydraulic gradient $i_{exit} = \dfrac{\Delta h}{\Delta l}$

$h_L$

datum

concrete
dam

$\Delta l$

$\Delta h$ = total head drop

soil

impervious strata
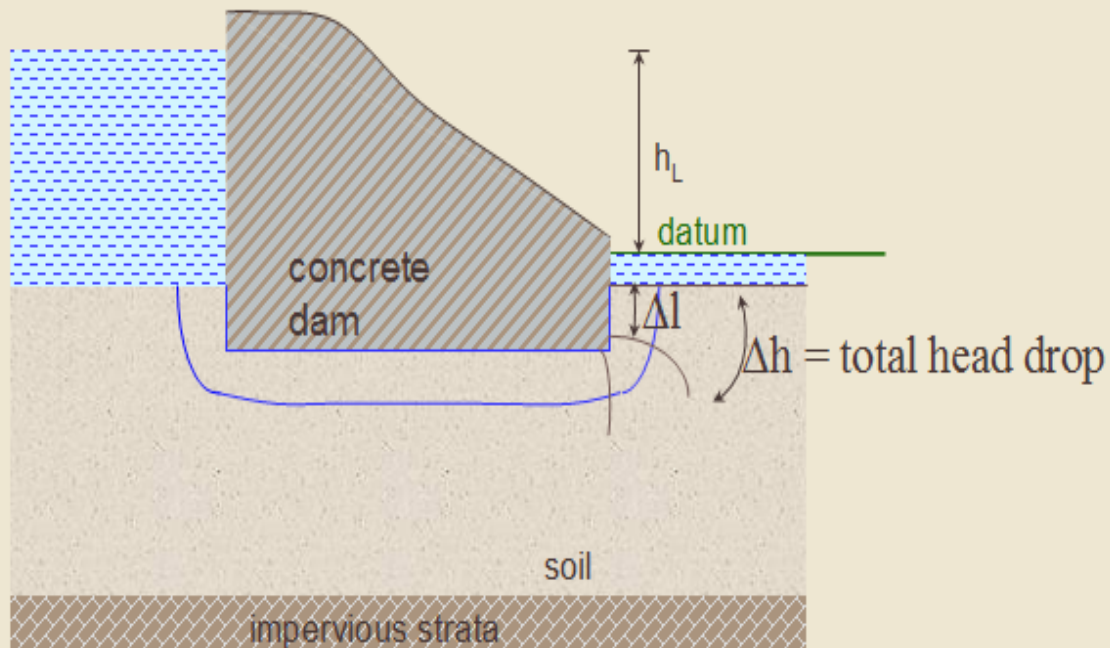
# Critical Hydraulic Gradient

o **The critical hydraulic gradient ($i_c$), is given by**

$$i_c = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1 + e}$$

o **If exit gradient is greater than critical hydraulic gradient:**

Consequences:
no stress to hold granular soils together
∴ soil may flow ⇒
"boiling" or "piping" = EROSION

# Critical Hydraulic Gradient

If $i_{exit}$ exceeds the critical hydraulic gradient ($i_c$), firstly the soil grains at exit get washed away.

This phenomenon progresses towards the upstream, forming a free passage of water ("pipe").

datum

$h_L$

concrete dam

no soil; all water

soil

impervious strata
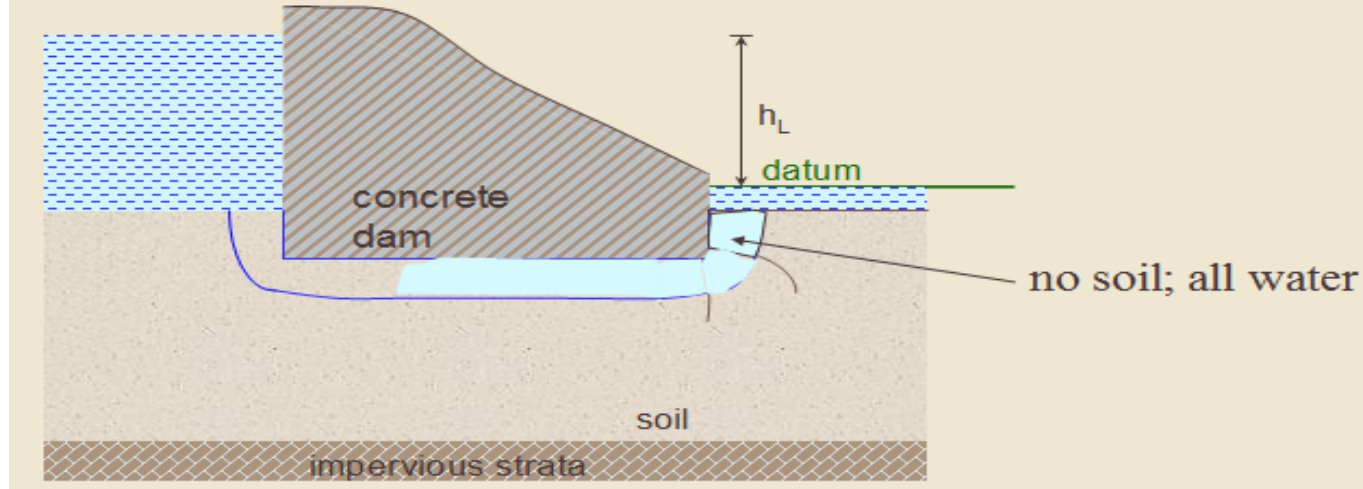
**Factor against piping**

$$F_{piping} = \frac{i_c}{i_{exit}}$$

**Typically 5 to 6**

**Note: we use high value for the factor of safety because of the disastrous consequences of failure.**

# Example

Exit gradient in the last "square"
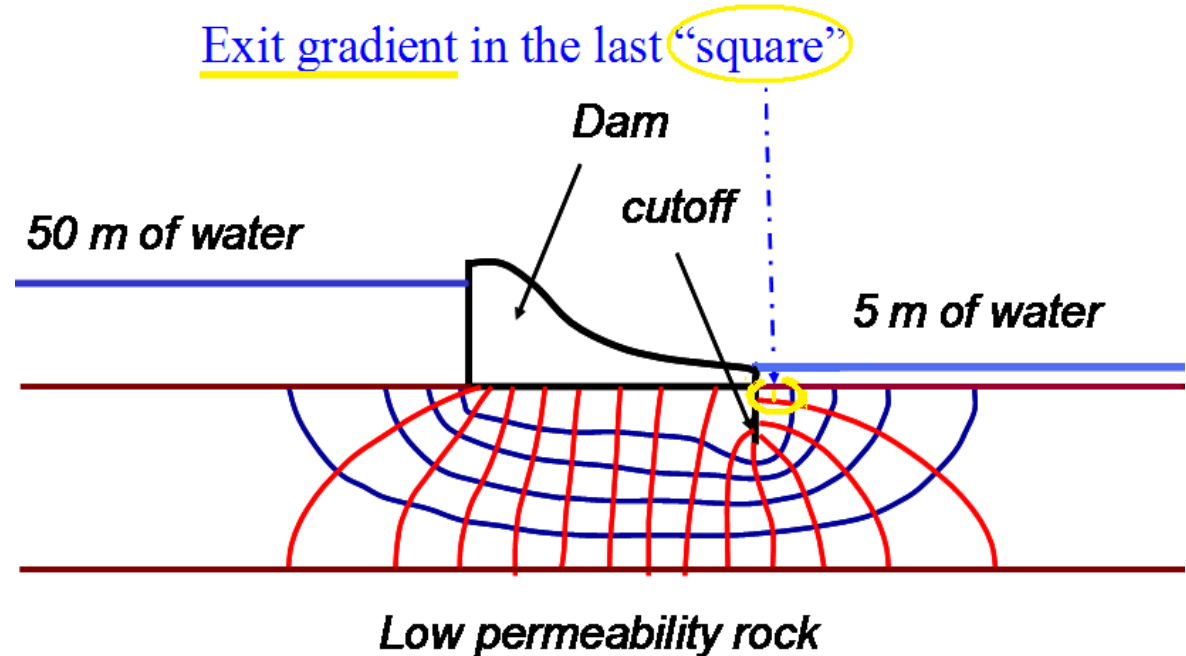
Dam

cutoff

50 m of water

5 m of water

**Is the arrangement safe against piping?**

Low permeability rock

- $[h / N_d] = 45/14 =$ **3.2 m** head per drop
- Average length of last element is about **3 m**
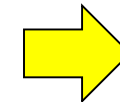
$I_{exit} = 3.2/3 \approx 1.1$

$$i_c = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1 + e}$$

- For most soils $0.9 < i_c < 1.1$ with an average of 1.0.

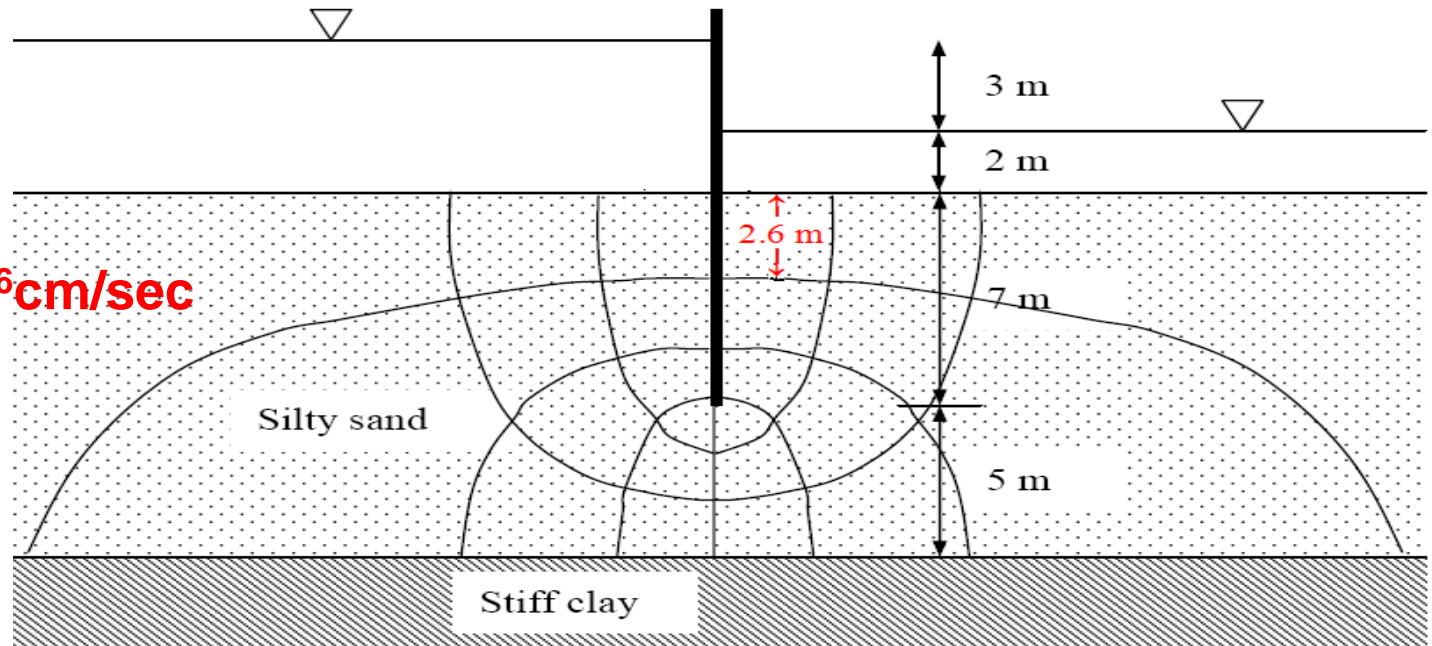- $F_{piping} = 1/1.1 \approx 0.9$ ⟶ **Very dangerous**

# Example

A stiff clay layer underlies a 12 m thick silty sand deposit. A sheet pile is driven into the sand to a depth of 7 m, and the upstream and downstream water levels are as shown in the figure below.



k = 8.6x 10⁻⁶cm/sec  
e =0.72  
G$_s$= 2.65

## Required

## Is the arrangement safe against piping?

# Example



e =0.72
G$_s$= 2.65
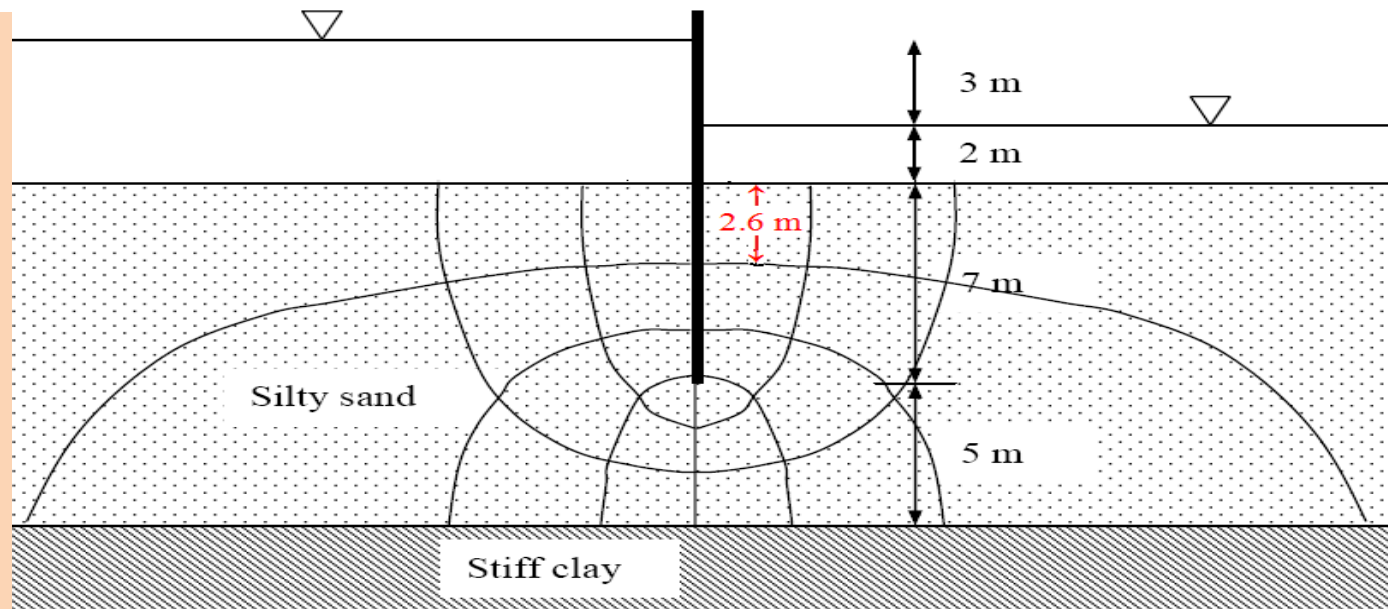
(c)   Head loss per equipotential drop, $\Delta h = 3/8 = \underline{0.375 \text{ m}}$

The maximum exit hydraulic gradient (near the sheet pile) = $0.375/2.6 = \underline{0.144}$

The critical hydraulic gradient ($i_c$) is given by:

$$i_c = \frac{G_s - 1}{1 + e} = \frac{2.65 - 1}{1 + 0.72} = \underline{0.96}$$

$\therefore$ Safety factor with respect to piping $F = 0.96/0.144 = 6.7 > 5$

The arrangement is quite safe with respect to piping.

# Preventing Piping

- As mentioned previously, **piping** and **erosion** are a possibility if, somewhere in the porous medium, the **<u>gradient</u>** exceeded the **<u>critical</u>** gradient.

- Piping can occur any place in the system, but usually it occurs where **the flow is <u>concentrated</u>**.

- We may have:

  o **Washing of the fine material or**

  o **Clogging of voids and buildup of p.w.p.**

# Filters

- There are several methods to control seepage and to prevent erosion and piping, one of which is to use a **protective filter**.

- A properly designed coarser material is called a **FILTER**.

- A filter consists of one or more layers of **free-draining granular** materials placed in less pervious foundation or base materials to:

  - **Prevent the movement** of soil particles that are susceptible to piping.

  - While at the same time **allowing the seepage water** to escape with relatively little head loss.

# Filters
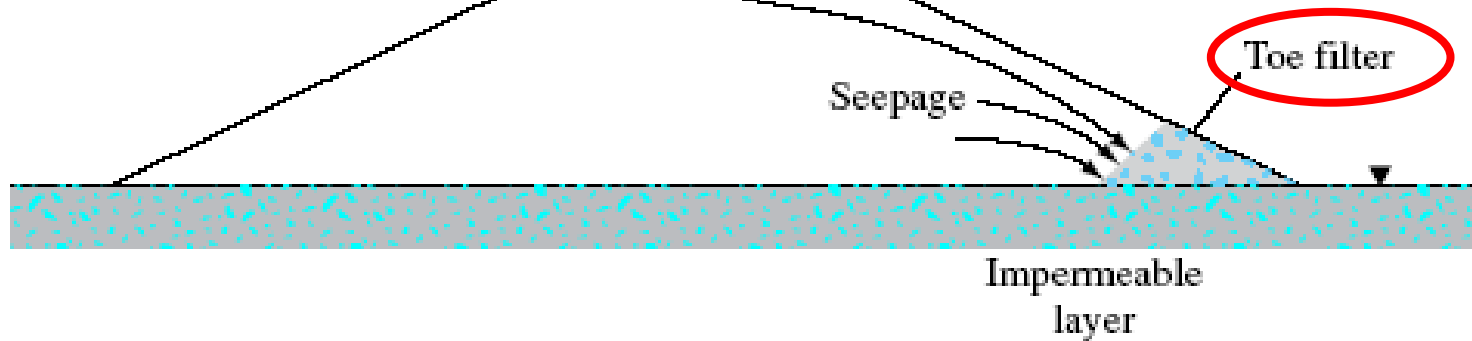


Steady-state seepage in an earth dam with a toe filter

**Filters used for:**
- **Facilitating drainage**
- **preventing fines from being washed away**

**Used in:**

- **Earth dams**

- **Retaining walls**

**Filter Materials:**

- **Granular soils**

- **Geotextiles**

# Filters


FIGURE 9.11 A horizontal drainage blanket at the toe of an earth dam.
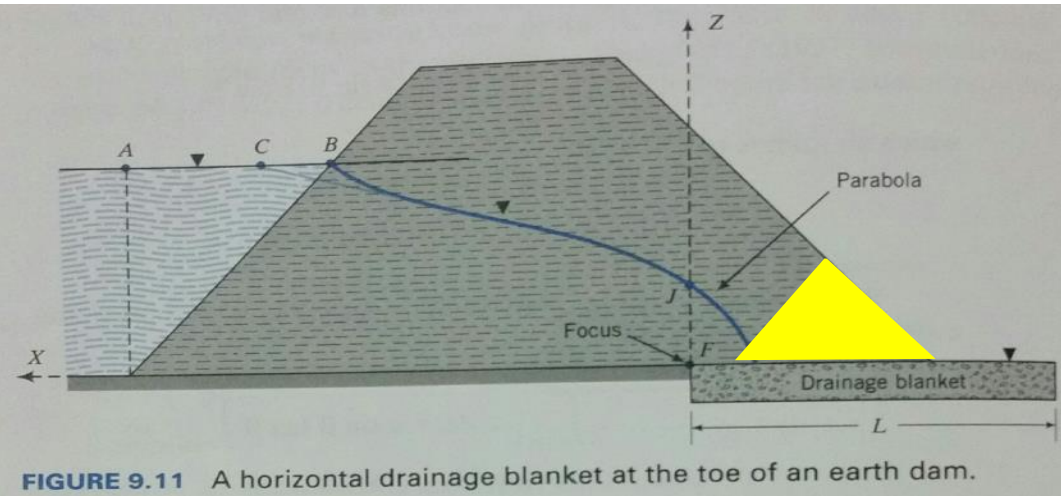
**Dams with Triangular Toe Drain**

**Dams with Drainage Blanket**


FIGURE 9.11 A horizontal drainage blanket at the toe of an earth dam.
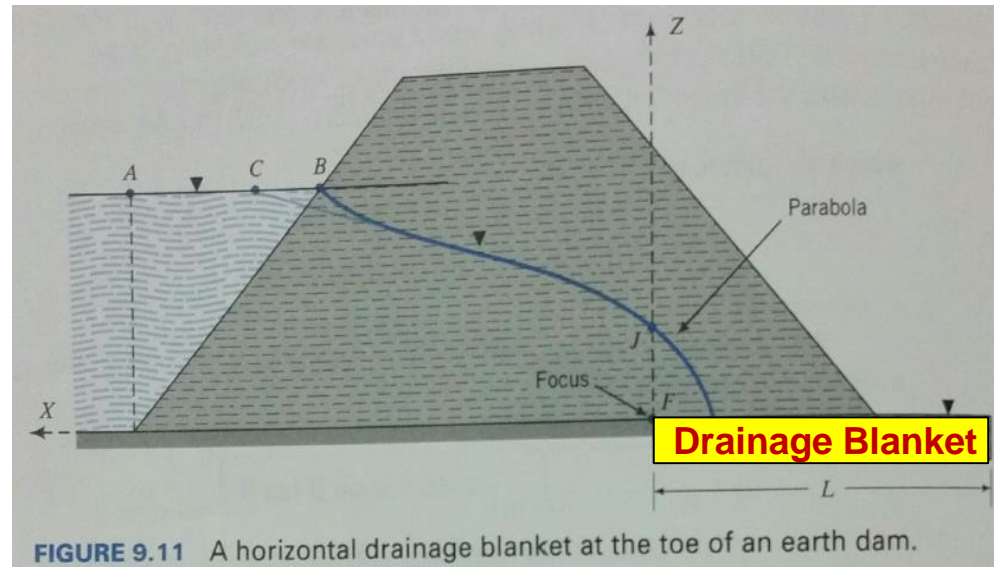
# Filter Design

**Two major criteria:**

(a)   **Retention** Criteria

- to prevent washing out of fines

∴ Filter grains must not be too coarse

(b)   **Permeability** Criteria

- to facilitate drainage and thus avoid build-up of pore pressures

∴ Filter grains must not be too fine

granular filter

# Filters

**For proper selection of the filter material, two conditions should be kept in mind:**

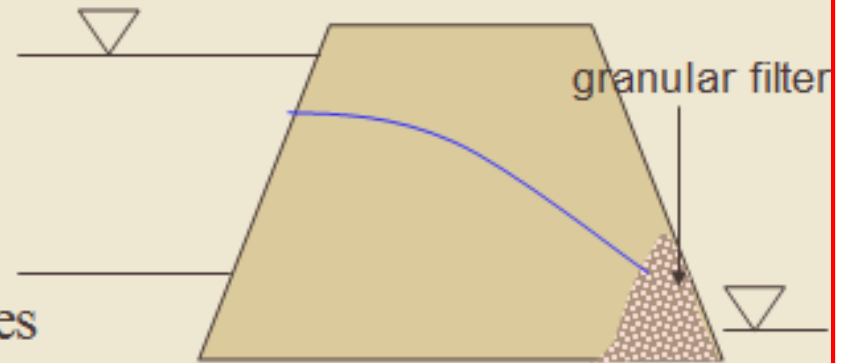**Condition 1. The size of the voids in the filter material should be small enough to hold the larger particles of the protected material in place.**

**Condition 2. The filter material should have a high hydraulic conductivity to prevent buildup of large seepage forces and hydrostatic pressures in the filters.**

$$If \quad D_{LS} > 6.5 D_{SS}$$

**The small sphere can move through the void spaces of the larger one.**



Large sphere

Small sphere

(a)
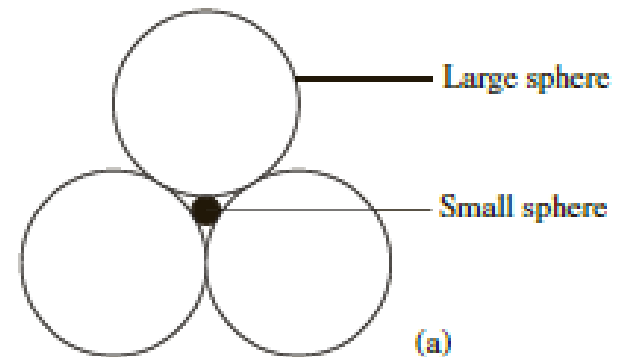
**Large spheres with diameters of 6.5 times the diameter of the small sphere;**

# Filters

$$\frac{D_{15(F)}}{D_{85(S)}} \leq 4\,to\,5 \qquad (\text{to satisfy Condition 1})$$
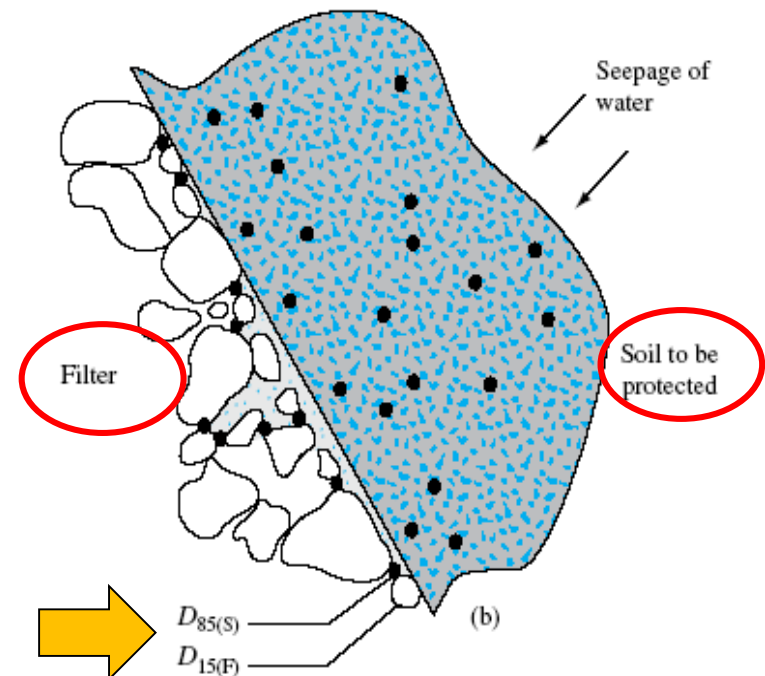
$$\frac{D_{15(F)}}{D_{15(S)}} \geq 4\,to\,5 \qquad (\text{to satisfy Condition 2})$$

where $D_{15(F)}$ = diameter through which 15% of filter material will pass
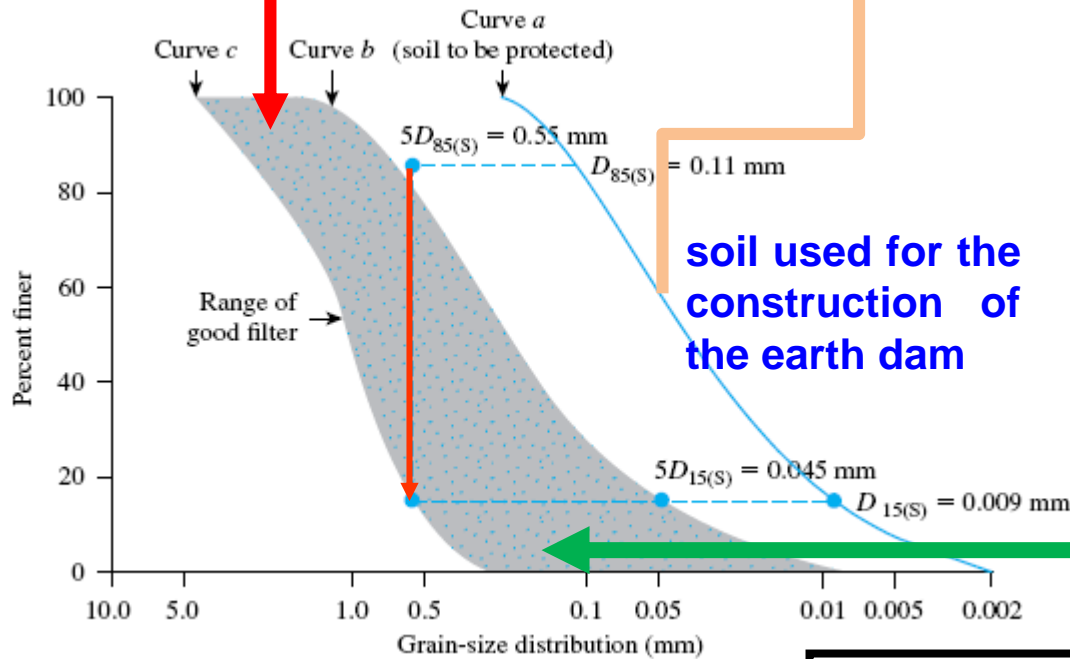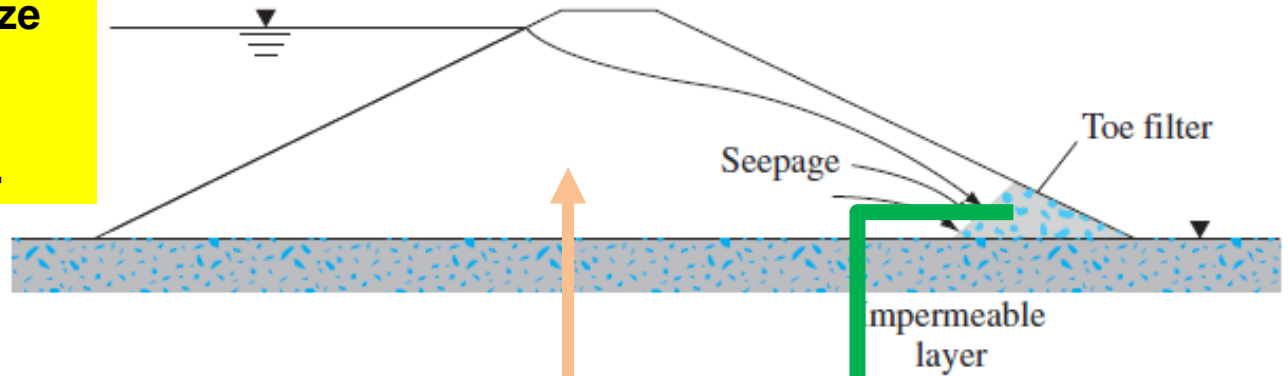$D_{15(S)}$ = diameter through which 15% of soil to be protected will pass
$D_{85(S)}$ = diameter through which 85% of soil to be protected will pass

**Terzaghi and Peck (1948)**

**The acceptable grain-size distribution of the filter material will have to lie within the shaded zone.**

Toe filter

Seepage

Impermeable layer

Curve a (soil to be protected)

Curve c    Curve b

Percent finer

$5D_{85(S)} = 0.55$ mm

$D_{85(S)} = 0.11$ mm

**soil used for the construction of the earth dam**

Range of good filter

$5D_{15(S)} = 0.045$ mm

$D_{15(S)} = 0.009$ mm

100

80

60

40

20

0

10.0   5.0          1.0     0.5          0.1    0.05          0.01  0.005    0.002

Grain-size distribution (mm)

$$\frac{D_{15(F)}}{D_{85(S)}} \leq 4 \text{ to } 5 \quad \text{(to satisfy Condition 1)}$$

$$\frac{D_{15(F)}}{D_{15(S)}} \geq 4 \text{ to } 5 \quad \text{(to satisfy Condition 2)}$$

# The US Navy (1948)

**The following conditions are required for the design of filters:**

*Condition 1:* For avoiding the movement of the particles of the protected soil:

$$\frac{D_{15(F)}}{D_{85(S)}} < 5$$

$$\frac{D_{50\,(F)}}{D_{50\,(S)}} < 25$$

$$\frac{D_{15(F)}}{D_{15(S)}} < 20$$

**Extra**

$$\frac{D_{15(F)}}{D_{85(S)}} \leq 4\,\text{to}\,5 \qquad (\text{to satisfy Condition 1})$$

If the uniformity coefficient $C_u$ of the protected soil is less than 1.5, $D_{15(F)}/D_{85(S)}$ may be increased to 6. Also, if $C_u$ of the protected soil is greater than 4, $D_{15(F)}/D_{15(S)}$ may be increased to 40.

# The US Navy (1948)

**The following conditions are required for the design of filters:**

*Condition 2:* For avoiding buildup of large seepage force in the filter:

$$\frac{D_{15(F)}}{D_{15(S)}} > 4 \qquad \frac{D_{15(F)}}{D_{15(S)}} \geq 4\,\text{to}\,5 \qquad \text{(to satisfy Condition 2)}$$

*Condition 3:* The filter material should not have grain sizes greater than 76.2 mm (3 in.). (This is to avoid segregation of particles in the filter.)

*Condition 4:* To avoid internal movement of fines in the filter, it should have no more than 5% passing a No. 200 sieve.

# The US Navy (1948)

*Condition 5:* When perforated pipes are used for collecting seepage water, filters also are used around the pipes to protect the fine-grained soil from being washed into the pipes. To avoid the movement of the filter material into the drain-pipe perforations, the following additional conditions should be met:

**Slots** ➡ $$\frac{D_{85(F)}}{\text{slot width}} > 1.2 \text{ to } 1.4$$

**Holes** ➡ $$\frac{D_{85(F)}}{\text{hole diameter}} > 1.0 \text{ to } 1.2$$

# Example 8.8

Example 8.8

## Example 8.8

The grain-size distribution of a soil to be protected is shown as curve $a$ in Figure 8.22. Given for the soil: $D_{15(S)} = 0.009$ mm, $D_{50(S)} = 0.05$ mm, and $D_{85(S)} = 0.11$ mm. Using Eqs. (8.38) through (8.41), determine the zone of the grain-size distribution of the filter material.
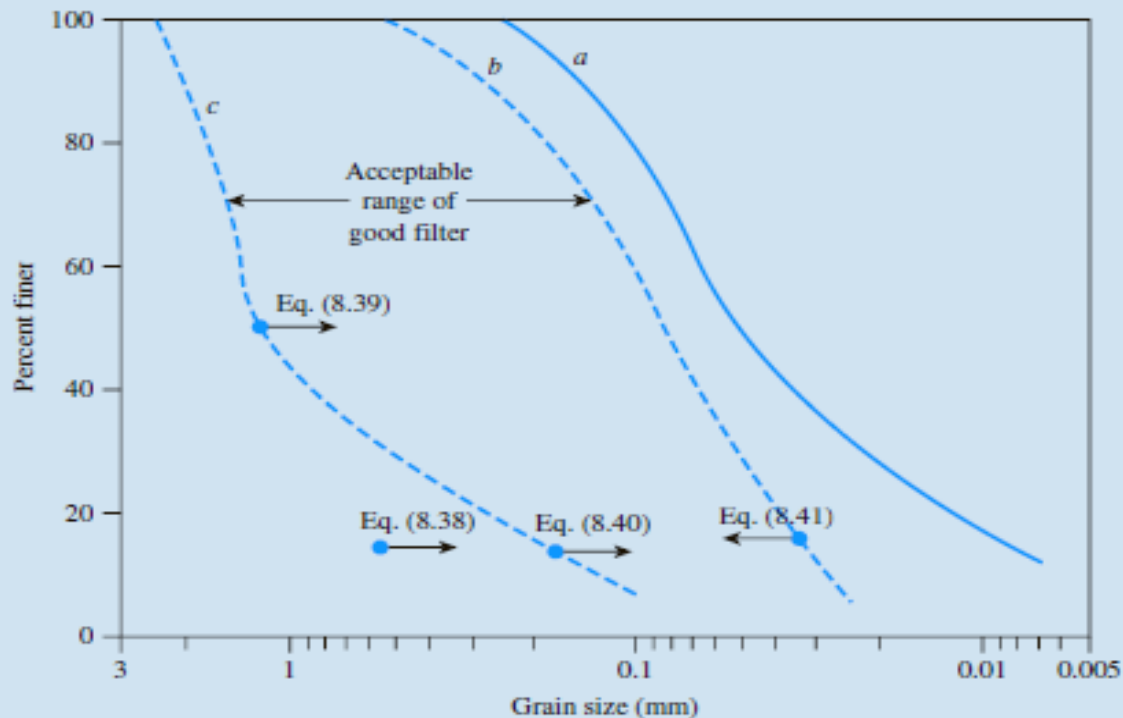


Figure 8.22

# Example 8.8

# Example 8.8

**Solution**

From Eq. (8.38),

$$\frac{D_{15(F)}}{D_{85(S)}} < 5$$

or

$$D_{15(F)} < 5D_{85(S)} = (5)(0.11) = 0.55 \text{ mm}$$

From Eq. (8.39),

$$\frac{D_{50(F)}}{D_{50(S)}} < 25$$

or

$$D_{50(F)} < 25D_{50(S)} = (25)(0.05) = 1.25 \text{ mm}$$

From Eq. (8.40),

$$\frac{D_{15(F)}}{D_{15(S)}} < 20$$

or

$$D_{15(F)} < 20D_{15(S)} = (20)(0.009) = 0.18 \text{ mm}$$

From Eq. (8.41),

$$\frac{D_{15(F)}}{D_{85(S)}} > 4$$

or

$$D_{15(F)} > 4D_{15(S)} = (4)(0.009) = 0.036 \text{ mm}$$

The above calculations have been plotted in Figure 8.22. Curves *b* and *c* are approximately the same shape as curve *a*. **The acceptable range of good filter falls between curves *b* and *c*.**

# THE END